Follow the Pack: Information Acquisition in the Presence of Institutional Activism *

Paula Cocoma† & Jinyuan Zhang‡

This version: October, 2020

Abstract

We provide a theoretical framework to understand the implications of institutional activism for the composition of the asset management industry. We enable investors to engage institutional activism in an, otherwise standard, information model. We find that such opportunity gives rise to a novel conflict of interest among investors, generating new strategic complementarities in investors' decisions to acquire information. Such strategic complementarities lead to an increase in the proportion of passive investors in equilibrium, which can rationalize the rising share of passive investment. Our results are robust to the various approaches that passive investors can take to engage in institutional activism. Moreover, we generate testable predictions on price informativeness, return volatility, and product competition to test the implications of institutional activism.

Keywords: mutual funds, passive investment, institutional activism, information acquisition, strategic complementarities, corporate governance

*This paper benefits tremendously from discussions with Adrian Buss, Bernard Dumas, Thierry Foucault, Sergei Glebkin, Naveen Gondhi, John Kuong, Joel Peress, Timothy Van Zandt and Bart Yueshen Zhou.

†INSEAD; paula.cocoma@insead.edu; Boulevard de Constance, Fontainebleau 77300, France.

‡INSEAD; jinyuan.zhang@insead.edu; Boulevard de Constance, Fontainebleau 77300, France.

1 Introduction

During the last decade, there has been a dramatic increase in the share of institutional ownership of publicly listed companies.[1] The sizable ownership stakes of institutional investors give them the potential to play a pivotal role when carrying out institutional activism. Activist investors search to impound changes within a company to maximize their portfolio return. Nevertheless, the portfolios of institutional investors differ depending on their investment approach and information; for example, portfolios might be actively or passively managed. As a result, the same corporate strategy may not maximize the portfolio return of all institutional investors simultaneously. As a consequence, there exists a natural conflict of interest among investors when engaging in institutional activism: each institutional investor may champion a conflicting corporate strategy to maximize their portfolio return.

In this context, a number of questions arise naturally: How does investors' capability to engage in institutional activism affect their portfolio allocation decisions? What does this imply for the composition of the asset management industry? And, how do investors' decisions impact financial and product markets?

We seek to address these and related questions by modelling the endogenous choice of investors to become active or passive investors when they take into account their impact on institutional activism.[2] The analysis generates a novel insight: there are strategic complementarities in investors' decisions to become active or passive. In other words, an investor's incentive to become active or passive increases when other investors take the same choice. Consequently, the model generates an amplification effect in the equilibrium share of passive investors, which rationalizes the increasing

[1] As of 2017, institutions own about 80% of the large-cap S&P 500 index, and 58% of the companies in the S&P Euro index.

[2] There is a broad spectrum of what it means to be an active or passive investor. For clarity of the argument, we focus on the two extremes where passive investors hold the whole market and active investors pick firms based on information.

share of passive investment in the asset management industry. Furthermore, we provide empirical predictions on price informativeness, return volatility and product market competition to test whether institutional activism affects the decision to become active or passive.

Our results provide a new perspective into the consequences of passive investment in financial markets. Under the traditional role of substitutability of information, theoretical models predict that there should be a limit to the growth of passive investing. Our main mechanism demonstrates that this may not be true and that the growth might be unbounded to the point that every investor prefers to hold a passive portfolio. Such an equilibrium outcome has detrimental welfare implications for price informativeness. Our results highlight that the inclusion of strategic complementarities discourages active investment and represents a shift in the equilibrium composition of the asset management industry. Notably, our identified strategic complementarities prevail even as we sidestep the heated debate on the anti-competitive consequences of common ownership, which has focalized much of the criticism to passive investment.[3]

We present a model that has the novelty of allowing investors to engage in *institutional activism* on top of acquiring information and forming portfolios. The economy is composed of two firms that produce a homogeneous good and compete with each other.[4] We focus on competition because this creates a strong type of strategic interaction that makes information acquisition meaningful.[5] We introduce information asymmetry in the model by assuming that firms are of an efficient or inefficient type when producing the good but look ex-ante identical for an investor without information. Investors can pay a cost to acquire information about the type of firms to allocate their

[3] The U.S. Department of Justice Antitrust Division and the Federal Trade Commission defines common ownership as "the simultaneous ownership of stock in competing companies by a single investor, where none of the stock holdings is large enough to give the owner control of any of those companies."

[4] We focus on Cournot competition. However, our results hold using very loose constraints on the type of competition as long as they are of the type of the Prisoner's Dilemma, which subsumes Bertrand competition or Stackelberg competition. Further details in Appendix C.

[5] Portfolios formed on information are tilted to over-weight and under-weight companies or industries. Therefore, information is maximized for a pair of competing firms.

capital efficiently. We define as *active* investors those who choose to acquire information about the firms and as *passive* investors those who do not. We call uninformed investors passive investors because they optimally hold the market portfolio. We introduce institutional activism by allowing investors to affect the strategy that a firm uses to compete in the market. For simplicity, we limit the decision to a high or a low aggressive competition strategy.[6] Investors nudge firms to act in their best interest by voting their shares, where majority voting decides the competition strategies that the firms follow.[7]

After solving for the equilibrium, we find that two sources affect the incentives of investors to acquire information and become active: 1) the *information asymmetry* gains, that arise from trading against an efficient market maker, and 2) the *institutional activism* gains, that arise from the capability of shareholders to influence corporate decisions. The latter being the main contribution of our paper, where strategic complementarities arise.

In our setting, the maximization of a firm's value may not maximize the portfolio return of both passive and active investors simultaneously. Our model is an example where the Fisher Separation Theorem, which states that in a complete market shareholders search for the firm to maximize its value, does not hold (Milne, 1974). The assumption of a complete market breaks down because of the information asymmetry between active and passive investors. As a result, the two hold different portfolios and have a conflict of interest regarding the desired corporate strategies that maximize their own portfolio return (e.g., Kelsey and Milne (1996)). For example, an investor who holds the market portfolio, a value-weighted portfolio of both firms, maximizes his portfolio return by advocating for cooperation between the firms and low aggressive competition. In contrast, an

[6] The strategy taken by the firm can be very general, such as capacity expansion, product innovation, a strategic merge, predatory pricing, or even an advertisement campaign. Importantly, any such actions can affect how aggressive competition is in the market.

[7] We focus on institutional activism by voice in the sense that we allow shareholders to vote their shares to influence corporate decisions. Other types of activism by voice, like taking legal actions, selecting individual members for the board or directly raising concerns by calling the management are also supported in our mechanism.

investor with a portfolio that is long the efficient firm and short the inefficient firm maximizes her return when the firm that she has over-weighted competes aggressively and steals market share.[8] Therefore, the information asymmetry, and as a consequence, the difference in portfolio holdings, creates a conflict of interest among shareholders of a firm.[9]

The conflict of interest between shareholders of a firm, active and passive, is the main mechanism that generates strategic complementarities. Recall that such conflict of interest arises naturally from the different portfolio allocation spawn by information asymmetry. Related to the two sources of profit identified, investors have a higher incentive to acquire information when they have more power to influence firms' strategies through *institutional activism*. Nevertheless, the strategies that active and passive investors prefer are likely to be different due to their different portfolio holdings. A rise in the share of passive investors reduces the power that active investors have to influence a firm's strategy to maximize their portfolio payoff. As a result, the expected gains from information, through institutional activism, are reduced, and more investors endogenously choose to be passive. Note that information in our setting has the role of coordinating investors with the same interests.[10] Therefore, including institutional activism gains generates strategic complementarities in investors' decision to acquire information. By coupling such strategic complementarities with the traditional substitution role of information, we provide an amplification mechanism that exacerbates the amount of passive investment in equilibrium. The intuition is that our model highlights a source by which passive investors are strictly detrimental to active investors returns. This finding can rationalize the

[8] This occurs because, compared to a highly aggressive strategy, a low aggressive strategy can only reduce the payoff of the firm that the investor has over-weighted and increase the payoff of the firm that the investor has under-weighted, which leads to lower portfolio payoffs in aggregate.

[9] This channel is supported by empirical evidence. Christoffersen et al. (2007) document that asymmetric information motivates the vote trade. Cvijanović, Dasgupta, and Zachariadis (2016) show that mutual funds' proxy voting is influenced by business ties with firms in the portfolio. He, Huang, and Zhao (2019) and Huang (2018) find that cross-ownership affects institutional investors' institutional activism.

[10] In the extreme case that the information is perfectly precise, all active investors hold exactly the same portfolio. Therefore, all active investors engage in institutional activism in the same way, which increases the probability of their preferred strategy to be implemented.

rising share of passive investment in the asset management industry.

Notably, the above results hold true no matter how passive investors engage in institutional activism—follow management blindly, do not engage in institutional activism, or engage in institutional activism optimally by maximizing their portfolio payoff. Regardless of how passive investors vote their shares, more passive investors mean that the balance of power among shareholders is shifted such that active investors have less influence in firms strategies. Therefore, more passive investors make it difficult for active investors to coordinate and generate less expected profit from institutional activism. As a result, there are less incentives to acquire information and more passive investors in equilibrium.

Since our main result identifies new strategic complementarities that go against the traditional strategic substitution of information, we can generate sharp empirical predictions. Based on comparative statics, we exploit the contrasting outcomes for price informativeness and returns volatility of a model with and without institutional activism. As a result, we develop a set of testable predictions that can identify the strength of institutional activism gains influencing information acquisition decisions.

Furthermore, our model sheds light on the reason why empirical tests on passive investors' effect in the financial markets can result in conflicting conclusions. The share of passive investors is an equilibrium outcome. This means that depending on which exogenous variable shifts the equilibrium share of passive investors, the effects in the financial market may even be the opposite. Moreover, the researches which adopt exogenous variations in passive holdings, such as Heath et al. (2019), may lack the capability to be generalized since they make inferences based on a small proportion of firms that experience shocks. Our approach remedies both these limitations while exploiting the fact that it is more accurate to measure market outcomes, such as price informativeness, return volatility, or product quantity, than institutional activism. In this manner, our results can complement and extend the existing empirical evidence.

The rest of this paper is organized as follows. In section 2, we present a simple and very general model that is enough to highlight the main mechanism that this paper offers. In section 3, we describe our solution method and characterize the equilibrium of the economy. In section 4, we analyze the equilibria found and identify the two sources that can affect the incentives to acquire information. In section 5, we develop a comparative statics exercise and offer our empirical predictions. Finally, section 6 contains the concluding remarks.

Literature Review Our paper contributes to understanding the effects of institutional activism in information acquisition. The existing theoretical work on institutional activism mostly assumes an exogenous shareholder composition. For example, Maug and Rydqvist (2009) study strategic voting decisions of shareholders with heterogeneous information and examine the effectiveness of information aggregation through voting. Cvijanovic, Groen-Xu, and Zachariadis (2017) and Meirowitz and Pi (2019) investigate how investors' voting decisions rely on the likelihood that their voting is pivotal. The notable exception is Levit, Malenko, and Maug (2019), where the authors study the link between trading and voting and find that trading and voting are complementary. In contrast to these papers, our paper provides a model where investors can engage in institutional activism, by affecting firms competition in equilibrium, while keeping endogenously both their decisions to acquire information and their trading in the financial market. One strand of literature, which connects information, trading and institutional activism, focuses on blockholders. Edmans (2009) builds a link between blockhodlers' governance and managerial myopia. Back et al. (2018) study how market liquidity affects blockholders' efforts to affect the firm value. See more details in survey paper by Edmans (2014) and Edmans and Holderness (2017). Most of these papers model a single blockholder, which prevents them from capturing the collective and strategical behaviour of investors that arise in our model.

Secondly, our theoretical model provides a new mechanism that generates strategic complementarities in the asset management industry. The result stands in contrast to the traditional substitution effect of information acquisition. We consider the feedback literature as the closest type of strategic complementarities to our paper. In contrast to the standard mechanism in feedback models where managers learn information from prices, our model assumes that shareholders can directly influence firms' payoffs by voting the shares they own. Gondhi and Davis (2017) adopt a feedback model to study agency problems. Edmans, Goldstein, and Jiang (2015) draw attention to possible limit to arbitrage generated by the feedback from stock markets to investments. Further detailed discussion on the real effects of the financial market can be found in Bond, Edmans, and Goldstein (2012). Dow, Goldstein, and Alexander (2015) study information acquisition for investors in an economy where the firm's investment decision depends on the information revealed from stock prices. Their model forms market breakdowns arising from the strategic complementarities between information production and efficiency in investment. Our paper differs from Dow, Goldstein, and Alexander (2015) in two main ways. First, the feedback in our model is through the voting on corporate decisions, instead of managers' learning from price. Second, we extend the model to include interactions of two firms, which allows us to generate rich empirical implications on product quantities, return volatility.

Finally, our paper contributes to providing some guidance on empirical studies to test the existence of institutional activism channel. Researchers document that passive institutional investors, instead of being passive owners, affect firms' choices (e.g., Appel, Gormley, and Keim, 2016a), managerial incentives (e.g, Appel, Gormley, and Keim, 2016b), and market competition (e.g., Huang, 2018; Azar, Schmalz, and Tecu, 2018). However, these views are challenged by the opposite evidence. For example, Heath et al. (2019) do not find evidence that index funds vote against firms' management and use exit mechanism to enforce institutional activism. Gilje, Gormley, and Levit (2017) document a week associating between the rise of passive investing and managerial incentives using newly developed common ownership measure. Our model lays out rich predictions on price informativeness, return volatility and product competition to test whether institutional activism impacts information acquisition decisions. Furthermore, we also develop predictions on the approach that passive investors take to institutional activism.

2 Model

This section presents a simplified model with only two firms that compete with each other.[11] The key feature of the model is that investors can engage in *institutional activism* on top of deciding to acquire information and trade in the financial market.

This static model has four stages. First, investors decide to acquire information about the type of the firms. Second, based on the information acquired, investors choose portfolios and trade in the financial market with a competitive market maker and liquidity traders. Third, after becoming shareholders of a firm, investors engage in institutional activism and influence the actions that the firms take in order to maximize their portfolio's payoff. Finally, the type of firms is revealed, and investors' payoff is determined. We now proceed first to describe each component of the model and then devote the section 2.2 to discuss in detail how investors engage in institutional activism with its implications.

2.1 Model Setup

2.1.1 Firms

The economy is composed of two firms, X and Y, that compete with each other for market share. Each firm is equally likely to be of an efficient (denoted as "E") or an inefficient type (denoted as "I"), and the realization is independent for each firm. We denote as c_X and c_Y the type of firm X and firm Y, respectively. There are four possible realizations: $(c_X, c_Y) \in \mathbb{S}^c = \{(E, E), (E, I), (I, E), (I, I)\}$,

[11] We focus on competition because it highlights a strong type of strategic interaction between the firms, such that information acquisition is meaningful.

each equally likely ex-ante.

Both firms take action A to compete for market share. This action A can be very general, such as capacity expansion, product innovation, a strategic merge, predatory pricing, or even an advertisement campaign. For simplicity, we interpret A as a corporate strategy, which can be either a highly aggressive strategy (A^H) or a low aggressive one (A^L). The decision of which strategy to follow by each firm is made by its shareholders via institutional activism. There are four possible combinations of corporate strategies for firms X and Y: $(A_X, A_Y) \in \mathbb{S}^A = \{(A^H, A^H), (A^H, A^L), (A^L, A^H), (A^L, A^L)\}$.

The final payoff of each firm depends on the realization of the types and the chosen corporate strategy for each firm. We denote $V_j(A_X(c_X), A_Y(c_Y))$ as the payoff of firm $j \in \{X, Y\}$. Importantly, since the firms compete with each other, their payoffs are correlated even though their type distribution is independent of each other.

2.1.2 Competition

To establish a functional form for the corporate strategies A^L and A^H, we assume that the two firms produce a homogeneous good and engage in quantity competition á la Cournot. Our results do not rely on the assumption of Cournot competition but hold for a very general type of competition.[12]

Using Cournot competition, we assume the efficiency of a firm is captured by its marginal cost, which can be either c^E for an efficient firm or c^I an inefficient one, where $c^E < c^I$. For simplicity, we assume a linear demand function for the homogeneous good where the price of the good G is determined by the quantities that each firm produces Q_X and Q_Y as:

$$G(Q_X, Q_Y) = a - b(Q_X + Q_Y)$$

The most aggressive strategy one firm can adopt is to produce the Cournot competition quantity.

[12] In the Appendix C, we show that our results prevail for any Prisoner Dilemma-like game, which includes Bertrand and Stackelberg competition.

In contrast, the least aggressive strategy for both firms is a collusive strategy where the production is set by a monopolist who considers the two firms as a single entity. Therefore, under the demand function $G(\cdot)$, we interpret the high aggressive strategy A^H as the Cournot competition quantity function and the low aggressive strategy A^L as the monopolist quantity function:

$$A^H(c) = \frac{a-c}{3b}, \qquad A^L(c) = \frac{a-c}{4b}, \quad \forall c < a.$$

For the specific case of Cournot competition, we furthermore assume that the different in types has a stronger effect than the difference in corporate strategy chosen. Such assumption translates to the very weak restriction $12(c^I - c^E > a - c^I)$.[13] Clearly, the quantity produced for each firm is jointly determined by its own type (the marginal cost) and the corporate strategy chosen, that is, $Q_x = A_X(c_X)$ and $Q_Y = A_Y(c_Y)$. The payoff for firm $j \in \{X, Y\}$ can be written as

$$V_j\Big(A_X(c_X), A_Y(c_Y)\Big) = A_j(c_j)\Big(a - b(A_X(c_X) + A_Y(c_Y)) - c_j\Big), \quad \forall A_j \in \{A^H, A^L\} \tag{1}$$

2.1.3 Traders

There are three types of agents trading in the financial market for firm X and Y: investors, liquidity traders, and a competitive market maker.

Investors The investors are risk-neutral with a unit mass, indexed by $i \in [0, 1]$. Each investor can acquire a noisy signal, $S_i = (S^i_X, S^i_Y)$, about the type of the firms, $\mathbb{S}^c$, for a cost ψ. [14] Denote $\gamma > \frac{1}{2}$ as the probability of a correct signal:[15]

$$\mathbb{P}(S^i_j = S^E | c_j = c^E) = \mathbb{P}(S^i_j = S^I | c_j = c^I) = \gamma, \quad j \in \{X, Y\}.$$

[13] Without this assumption, the expected payoff of two inefficient firms, can be inflated by the chance that they choose low aggressive strategies and form a cartel.

[14] Because firms' payoff depends on the strategic interactions between firms, we assume investors can learn signals about both firms. Relaxing this assumption does not affect our main mechanism because the conflict of interest between investors prevails and is weaker among active investors.

[15] The assumption of an imprecise signal is simply meant to portray realism, while our results are stronger for a perfectly precise signal.

Since the type of firms is independent, we assume S_X^i and S_Y^i are also independent of each other.

Each investor decides to acquire information or not by comparing the expected gains from acquiring information with the cost ψ. We denote λ as the mass of investors who, endogenously, do *not* acquire information.

After observing the signal, investors choose portfolios by holding θ_i shares of each firm. We assume that informed investors have access to a leveraging technology denoted by κ.[16] Therefore, informed investors trade an amount $\theta_i \in \{-\kappa, \kappa\}$ for each firm based on their information.[17] The uninformed investors do not have access to such leveraging technology and optimally choose to purchase *one* share of each firm, since both firms look identical from their perspective. In other words, uninformed investors hold the market; this is the reason why we call uninformed investors "passive investors" for the rest of the paper. In contrast, informed investors choose their portfolio allocation conditional on information, and we call them "active investors".

Liquidity traders Liquidity traders arrive at the market randomly and do not engage in institutional activism. The existence of liquidity traders prevents the prices from being fully revealing as in Grossman and Stiglitz (1976). We assume that the total orders submitted by liquidity traders for firm X and Y are N_X^F and N_Y^F, which follow two independent Gaussian distributions with mean 0 and variance σ_N^2.

A market maker As in Kyle (1985), we assume that there exists a competitive market maker who observes order flows for both firms, F_X and F_Y. It sets an efficient price for each firm.

[16] The introduction of a leveraging technology is not necessary for our main mechanism. It only introduces an exogenous parameter that affects informed investors but not uninformed investors, which is useful to generate sharp empirical predictions.

[17] One can interpret this assumption as a constraint on the trade size.

2.1.4 Private owners

In reality, not all shares of a firm are available to trade in the financial market, and not every shareholder of a firm is willing to engage in institutional activism. To capture this feature, we assume that a known mass of shares, $\bar{\varphi}$, is held by private owners for each firm. These private owners may be founders, members of the management team or even employees. We assume they are endowed with such shares and do not trade in the financial market. For simplicity, we do not allow these private owners to crossly hold both firms' shares.

Private owners can actively engage in institutional activism. However, actively participating in institutional activism can be costly for small shareholders. Therefore, we assume that only a random fraction of private owners of each firm, denoted as φ_X or φ_Y, engage in institutional activism. We assume that φ_X, φ_Y are independent random variables uniformly distributed between $[0, \bar{\varphi}]$. Such randomness makes the corporate strategy chosen by each firm a random variable.

The notation and timeline of the economy are summarized in Figure 1.

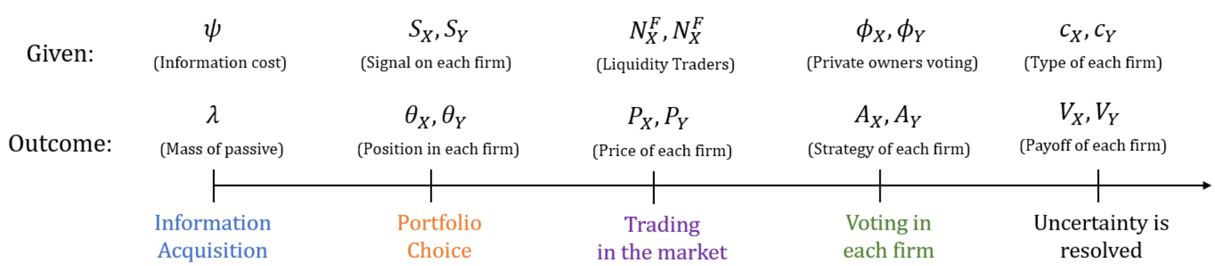

Figure 1: Time-line of the model and notation used

2.2 Institutional Activism

This section specifies the mechanism by which investors engage in institutional activism. institutional activism aims to mitigate conflicts of interests among shareholders of a firm. In our model, we have two sources of principal-principal conflict of interests, first, between investors and private owners, and, second, among investors with different portfolios.

To mitigate the conflicts of interest, we choose *majority voting* as the method for shareholders to engage in institutional activism.[18] We make two assumptions regarding how voting is implemented. First, both firms adopt a one-share-one-vote policy. Second, liquidity traders do not vote. This means that only private owners and investors with a long position in the firm can participate in institutional activism.

2.2.1 Active investors' voting decision

We begin by studying the optimal voting decision conditional on an active investor's portfolio. Each active investor votes for the strategy that maximizes her portfolio payoff. There are three possible portfolios she may hold: i) go long both firms, ii) go long one and short the other, or iii) go short both firms. Since only investors with a long position can vote, the last portfolio choice gives no voting rights. In what follows, we focus on the first two cases.

When an investor holds both firms, she can view the two firms as a single entity. Therefore, the strategy that gives the highest payoff is a collusive strategy, which corresponds to the A^L strategy in our setting. Hence, voting (A^L, A^L) maximizes the payoff of an investor that holds both shares. When an investor holds solely firm, she wants that firm to win as much market share as possible to maximize her payoff. Under Cournot competition, analogous to a Prisoner's Dilemma game, the dominant strategy is to cast her vote as A^H.[19] We summarize the optimal voting decision in Table 1 where "–" represent no voting.

[18] In reality, institutional activism can be achieved through various channels, for example, by taking legal actions, by championing specific members for the board of directors, by providing monetary incentives for a specific management actions. We believe voting is the most straightforward mechanism that preserves realism while allowing all shareholders to express their interest and simultaneously giving more power to investors with a higher ownership share.

[19] Suppose an investor holds firm X and shorts firm Y. Note that only if she believes $c_X \leq c_Y$, she will take a long position solely in firm X. Following the Equation (1), the payoff of voting A^H for firm X dominates voting A^L

Portfolio Choice		Optimal Voting	
firm X	Firm Y	firm X	Firm Y
1	-1/0	A^H	-
-1/0	1	-	A^H
1	1	A^L	A^L
-1/0	-1/0	-	-

Table 1: Active investors' voting decision conditional on their portfolio holdings.

2.2.2 Passive investors' voting decision

Given the emphasis of our paper, we devote this section to discuss three possible ways in which passive investors may engage in institutional activism. The three possible scenarios are indicated by the parameter ζ.

Case 1: $\zeta = 1$ corresponds to the case that passive investors engage in institutional activism to maximize their portfolio payoff. Given that passive investors hold both firms, they optimally vote for the strategy (A^L, A^L). In this case, passive investors, even though they do not actively pick firms, are not passive owners. This assumption is in line with the anti-competitive effects of passive investors' institutional activism highlighted in the common ownership literature.

Case 2: $\zeta = 0$ corresponds to the case that passive investors do not engage in institutional activism and simply hold shares. In this case, all passive investors do not vote; though, as we show below, this does not mean that their presence does not affect the voting outcome.

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regardless of firm Y's strategy:

$$V_X\Big(A^H(c_X), A^H(c_Y)\Big) - V_X\Big(A^L(c_X), A^H(c_Y)\Big) = \frac{(a-c_X)(a-c_X+4(c_Y-c_X))}{144b} > 0,$$

$$V_X\Big(A^H(c_X), A^L(c_Y)\Big) - V_X\Big(A^L(c_X), A^L(c_Y)\Big) = \frac{(a-c_X)(2(a-c_X)+3(c_Y-c_X))}{144b} > 0.$$

Case 3: $\zeta = -1$ corresponds to the case that passive investors vote their shares in line with management. This is another case by which passive investors are not actively engaged in institutional activism but using their shares to support management decisions. In our setting, we assume the management always vote to maximize the firm's payoff even when this decision does not maximize the payoff of the investor's portfolio.

2.2.3 Conjectured portfolio allocation

To obtain the voting outcome, it is necessary to establish the portfolio allocation of investors. Our conjectured portfolio allocation for active investors is as follow:

Conjecture 1. *An passive investor takes a long position in both firms. An active investor takes*

- *a long position in both firms for signal* $(S_X, S_Y) = (S^E, S^E)$*;*
- *a long position in firm X and short one in firm Y for signal* $(S_X, S_Y) = (S^E, S^I)$*;*
- *short position in firm X and long one in firm Y for signal* $(S_X, S_Y) = (S^I, S^E)$*;*
- *a short position in both firms for signal* $(S_X, S_Y) = (S^I, S^I)$.

After solving for the equilibrium in section 3, we show that this conjecture is valid.

2.2.4 Voting outcome

With conjecture portfolio allocation, we now calculate the mass of votes for each strategy. The votes come from active investors, passive investors and private owners. Among all three shareholders, only active investors' votes depend on information. Table 2 summarizes the proportion of active investors that receive a particular signal and their corresponding optimal allocation and voting decisions. For illustration, we take as an example the case that the firm's type realization is efficient, $(c_X, c_Y) = (E, E)$. Since an active investor has a probability γ^2 of receiving the correct signal (S^E, S^E), there are, by the law of large numbers, γ^2 fraction of such active investors. For each of them, the optimal portfolio allocation is (κ, κ), and the optimal voting strategy is (A^L, A^L). The same reasoning is used for each type realization to complete the columns in Table 2.

Signal	Allocation	Vote	Firm's type realization			
		per share	(E, E)	(E, I)	(I, E)	(I, I)
(S^E, S^E)	(κ,κ)	(A^L, A^L)	γ^2	$\gamma(1-\gamma)$	$\gamma(1-\gamma)$	$(1-\gamma)^2$
(S^E, S^I)	$(\kappa,-\kappa)$	$(A^H, -)$	$\gamma(1-\gamma)$	γ^2	$(1-\gamma)^2$	$\gamma(1-\gamma)$
(S^I, S^E)	$(-\kappa,\kappa)$	$(-, A^H)$	$\gamma(1-\gamma)$	$(1-\gamma)^2$	γ^2	$\gamma(1-\gamma)$
(S^I, S^I)	$(-\kappa,-\kappa)$	(-, -)	$(1-\gamma)^2$	$\gamma(1-\gamma)$	$\gamma(1-\gamma)$	γ^2

Table 2: The left panel lists the optimal allocation and voting choice conditional on an active investor's signal. The right panel summarizes the proportion of active investors that receive a certain signal given the realization of the firm's type.

We aggregate the total amount of investors (passive, active and private owners) that vote for each strategy, conditional on each approach that passive investors can take to vote their shares. As an example for the type realization (E, E) and $\zeta = 1$, the mass of investors that vote for A^H in firm X is: The realization ψ_X of private owners that participate in institutional activism, and $\gamma(1-\gamma)$ fraction of $(1-\lambda)$ active investors who receive the incorrect signal (S^E, S^I). We list all cases, conditional of passive investors voting, for the realization (E, E) below:

$$\text{If } \zeta = 1: \quad \text{For } A^H: \underbrace{\varphi_X}_{\substack{\text{Private}\\\text{owners}}} + \kappa \underbrace{(1-\lambda)}_{\substack{\text{Active}\\\text{investors}}} \underbrace{\gamma(1-\gamma)}_{\substack{\text{Proportion}\\\text{that votes for } A^H}} \qquad \text{For } A^L: \underbrace{\lambda}_{\substack{\text{Passive}\\\text{investors}}} + \kappa \underbrace{(1-\lambda)}_{\substack{\text{Active}\\\text{investors}}} \underbrace{\gamma^2}_{\substack{\text{Proportion}\\\text{that votes for } A^L}};$$

$$\text{If } \zeta = 0: \quad \text{For } A^H: \varphi_X + \kappa(1-\lambda)\gamma(1-\gamma) \qquad \text{For } A^L: \kappa(1-\lambda)\gamma^2;$$

$$\text{If } \zeta = -1: \quad \text{For } A^H: \varphi_X + \kappa(1-\lambda)\gamma(1-\gamma) + \lambda \qquad \text{For } A^L: \kappa(1-\lambda)\gamma^2.$$

The total mass of investors that vote for strategy A^L minus the mass for A^H in firm X is summarized in Table 3 for each realization of the firms type.

Realization of firm type	Mass difference in votes $A^L - A^H$ for firm X
(E,E)	$\zeta\lambda + \kappa(1-\lambda)\gamma^2 - (\varphi_X + \kappa(1-\lambda)\gamma(1-\gamma))$
(E,I)	$\zeta\lambda + \kappa(1-\lambda)\gamma(1-\gamma) - \big(\varphi_X + \kappa(1-\lambda)\gamma^2\big)$
(I,E)	$\zeta\lambda + \kappa(1-\lambda)\gamma(1-\gamma) - \big(\varphi_X + \kappa(1-\lambda)(1-\gamma)^2\big)$
(I,I)	$\zeta\lambda + \kappa(1-\lambda)(1-\gamma)^2 - (\varphi_X + \kappa(1-\lambda)\gamma(1-\gamma))$

Table 3: The mass difference in votes $A^L - A^H$ for firm X conditional on the realization of firms type.

We can then determine the probability that the strategy A^H is chosen. Given the majority voting rule, if the total mass of votes for A^H is higher than A^L, then the strategy A^H is chosen. We define q_{EE}, q_{EI}, q_{IE} and q_{II} as the probability that A^H is the strategy chosen for firm X for each possible realization of firm types $\{(E,E),(E,I),(I,E),(I,I)\}$, respectively.[20] For the type realization (E,E) and $\zeta = 1$, $q_E E$ is calculated as:

$$
\begin{aligned}
q_{EE} &= \mathbb{P}(\underbrace{\varphi_X + \kappa(1-\lambda)\gamma(1-\gamma)}_{A^H} > \underbrace{\lambda + \kappa(1-\lambda)\gamma^2}_{A^L}) \\
&= \mathbb{P}\Big(\varphi_X > \zeta\lambda + \kappa\gamma(1-\lambda)(2\gamma-1)\Big) = 1 - \frac{1}{\bar{\varphi}}\Big(\zeta\lambda + \kappa\gamma(1-\lambda)(2\gamma-1)\Big).
\end{aligned}
$$

Table 4 summarizes the probability that A^L is the strategy chosen conditional on ζ. For all cases, $1 \geq q_{EI} \geq q_{II} > q_{IE} > q_{EE} \geq 0$.

We now highlight how our model nests the anti-competitive features of passive investors' institutional activism present in the common ownership literature. The following proposition directly follows from Table 4:

[20] By means of symmetry, q_{EE}, q_{IE}, q_{EI} and q_{II} determine the probability that A^H strategy is chosen for firm Y for each possible realization of firm type $\{(E,E),(E,I),(I,E),(I,I)\}$, respectively.

	$\zeta = 1$	$\zeta = 0$	$\zeta = -1$
$1 - q_{EE}$	$\frac{1}{\bar{\varphi}}\Big(\lambda + \kappa\gamma(1-\lambda)(2\gamma-1)\Big)$	$\frac{1}{\bar{\varphi}}\Big(\kappa\gamma(1-\lambda)(2\gamma-1)\Big)$	$\frac{1}{\bar{\varphi}}\Big(-\lambda + \kappa\gamma(1-\lambda)(2\gamma-1)\Big)$
$1 - q_{EI}$	$\frac{1}{\bar{\varphi}}\Big(\lambda - \kappa\gamma(1-\lambda)(2\gamma-1)\Big)$	0	0
$1 - q_{IE}$	$\frac{1}{\bar{\varphi}}\Big(\lambda + \kappa(1-\gamma)(1-\lambda)(2\gamma-1)\Big)$	$\frac{1}{\bar{\varphi}}\Big(\kappa(1-\gamma)(1-\lambda)(2\gamma-1)\Big)$	$\frac{1}{\bar{\varphi}}\Big(-\lambda + \kappa(1-\gamma)(1-\lambda)(2\gamma-1)\Big)$
$1 - q_{II}$	$\frac{1}{\bar{\varphi}}\Big(\lambda - \kappa(1-\gamma)(1-\lambda)(2\gamma-1)\Big)$	0	0

Table 4: Probability that the A^L strategy is chosen conditional on how passive investors engage in institutional activism.

> **Proposition 1.** *When passive investors engage in institutional activism, i.e., $\zeta = 1$, the likelihood that a low aggressive strategy is chosen is higher than if they do not engage in institutional activism.*

The intuition of the proposition is that passive investors prefer less aggressive competition because of their portfolio allocation. If they vote optimally to maximize their portfolio payoff, they tilt the firms' strategies towards collusion.

As an illustration, we assign numerical values to all variables of the model and plot the probability of the strategy A^L being chosen for each possible realization of firm type under the three different approaches that passive investors can take to institutional activism.[21]

[21] The parameters used are: $\varphi = 1, \lambda = \frac{1}{2}, \gamma = 0.7, \kappa = 4$.

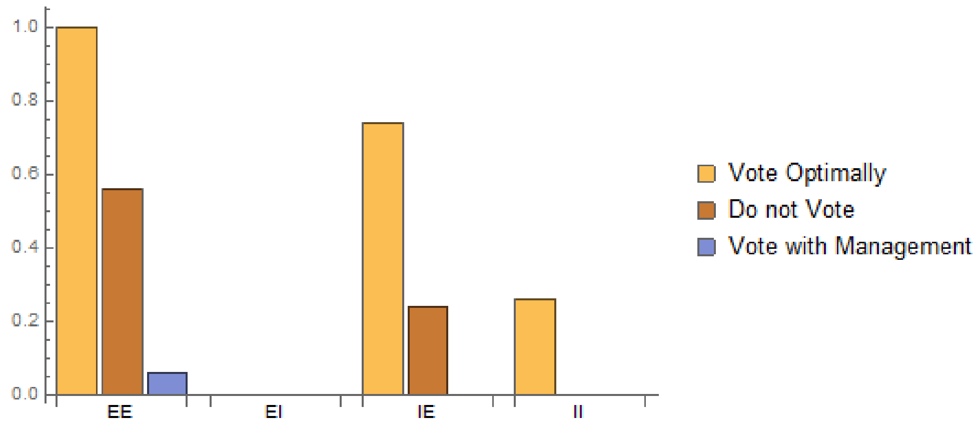

Figure 2: Illustration of the probability that the low aggressive strategy A^L is chosen for the different approaches of passive investors' institutional activism.

3 Characterization of Equilibrium

In this section, we characterize the equilibrium of the model by using backward induction. The analysis is conducted in four steps. First, we derive the expected payoff of each firm, taking into account the randomness of voting outcomes. Second, we determine the efficient prices quoted by the competitive market maker, who observes order flows. Third, we verify our conjectured portfolio of an investor (passive or active), specified in section 2.2.3. Lastly, we solve for the endogenous information acquisition decision, which determines the proportion of passive vs active investors. The following analysis focuses on firm X, and the same steps are applied for firm Y.

3.1 Firms' Expected Payoff

The payoff of each firm is a random variable depending on the voting outcome, i.e., the chosen corporate strategy. Using the definitions in Table 4, we denote $\pi_X(c_X, c_Y)$ as the expected payoff of firm X for a realization of firm type $(c_X, c_Y) \in \mathbb{S}^c$, specified as:

$$
\begin{aligned}
\pi_X(c_X, c_Y) = {} & q_{c_X c_Y} q_{c_Y c_X} V_X\Big(A^H(c_X), A^H(c_Y)\Big) + (1 - q_{c_X c_Y})(1 - q_{c_Y c_X}) V_X\Big(A^L(c_X), A^L(c_Y)\Big) \\
& + q_{c_X c_Y}(1 - q_{c_Y c_X}) V_X\Big(A^H(c_X), A^L(c_Y)\Big) + (1 - q_{c_X c_Y}) q_{c_Y c_X} V_X\Big(A^L(c_X), A^H(c_Y)\Big)
\end{aligned} \tag{2}
$$

Note that Equation (2) nests all three cases of passive investors' institutional activism, where the vales of $q_{c_X c_Y}$ change depending on the parameter ζ. Therefore, the value of expectation changes depending on whether passive investors vote optimally, do not vote, or vote with management. At the same time, Equation (2) holds for all cases.

3.2 Stock Prices

The competitive market maker observes the order flows for both firms, F_X and F_Y, updates his beliefs about the realization of firm's types, and sets the efficient prices as:

$$
P_j = \mathbb{E}[\pi_j | F_X = x, F_Y = y], \qquad j \in \{X, Y\} \tag{3}
$$

Where x and y are the observed order flows for each firm. We show steps to determine P_X, and the same follows for firm Y. If firm X is an efficient firm, a measure $(1-\lambda)\gamma$ of active investors receives a signal S^E and purchases κ shares; and a measure $(1-\lambda)(1-\gamma)$ receives a S^I signal and sell κ shares. Passive investors always buys one share from each firm. So, the aggregate order for firm X is $F_X = \lambda + N_X^F + \kappa(1-\lambda)\gamma - \kappa(1-\lambda)(1-\gamma)$. On the contrary, when firm X is an inefficient firm, the aggregate order flow is $F_X = \lambda + N_X^F + \kappa(1-\lambda)(1-\gamma) - \kappa(1-\lambda)\gamma$. Therefore, the order flow F_X that the market maker observes is:

$$
F_x = \begin{cases} \lambda + N_X^F + \kappa(1-\lambda)(2\gamma - 1), & \text{if } c_X = E \\ \lambda + N_X^F - \kappa(1-\lambda)(2\gamma - 1), & \text{if } c_X = I. \end{cases}
$$

Note that the actions that an investor follows for one firm does not depend on the information of the other firm. According to our conjecture, an investor chooses to go long firm X with the signal

$S_X = S^E$ regardless of what the signal for firm Y is. Therefore, the order flow of firm Y is not informative for firm X's type and can be ignored when determining the price of firm X.

After observing order flow $F_X = x$, the market maker updates his belief, based on Bayes' rule, on firm X's type to a posterior probability denoted as $\rho(x)$:

$$\begin{aligned}\rho(x) = \mathbb{P}(c_X = E|F_X = x) &= \frac{\mathbb{P}(F_X = x|c_X = E)\mathbb{P}(c_X = E)}{\mathbb{P}(F_X = x)} \\ &= \frac{\phi\left(\frac{x-\lambda-\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}{\phi\left(\frac{x-\lambda-\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right) + \phi\left(\frac{x-\lambda-\kappa(1-\lambda)(1-2\gamma)}{\sigma_N}\right)},\end{aligned}$$

where $\phi(\cdot)$ represents the probability density function of the normal distribution with mean 0 and variance 1. The efficient price that the market maker sets is the expectation over all possible firm's types realizations, resulting in:

$$\begin{aligned}P_X(F_X = x, F_Y = y) = \rho(x)\rho(y)\pi_X(E,E) + \rho(x)(1-\rho(y))\pi_X(E,I) \\ + (1-\rho(x))\rho(y)\pi_X\Big(A_X(I)), A_Y(E)\Big) + (1-\rho(x))(1-\rho(y))\pi_X(I,I),\end{aligned}$$

where $\pi_X\Big((A_X(c_X), A_Y(c_Y))\Big)$ corresponds to the expected payoff of firm X given its type realization as per Equation (2). It is worth noting that even though the liquidity traders, the type realization and the signals received by active investors are independent for both firms, the stock prices are *not* independent. This is because firm X's payoff is affected by the strategy adopted by firm Y. Hence, the market maker needs to infer the joint realization of the two firms' types to determine efficient stock prices.

3.3 Verifying the Optimal Trading Strategy

We now verify our conjectured trading strategy in Section 2.2.3. The optimal trading strategy depends on the expected trading profit, i.e., the payoff - the price. As a consequence, investors need to form an expectation of the price because they make trading decisions before the market maker sets the price. For this purpose, investors form an expectation about how likely is the market maker to extract the correct information from the order flow. We denote such expectation given the *true* type of the firm as ξ:[22]

$$\xi(\lambda, \gamma, \kappa, \sigma_N) \equiv \mathbb{E}[\rho(F_x)|c_X = E] = 1 - \mathbb{E}[\rho(F_x)|c_X = I] = \mathbb{E}\left[\frac{\phi\left(\frac{N_X^F}{\sigma_N}\right)}{\phi\left(\frac{N_X^F}{\sigma_N}\right) + \phi\left(\frac{N_X^F + 2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}\right].$$

Based on the total law of expectation, we can derive the expected belief about market makers information given investors' signal.[23]

Then, we can prove the conjectured portfolio allocation and calculate the expected profit for active investors (denoted as Ω), summarized in Proposition 2.[24]

Proposition 2. *An active investor's optimal trading strategy follows the conjecture 1, and the expected profit is given by*

$$\Omega(\lambda, \gamma, \kappa, \sigma_N, \bar{\varphi}) = (2\gamma - 1)(1 - \xi(\lambda, \gamma, \kappa, \sigma_N))\Pi(\lambda, \gamma, \kappa, \sigma_N, \bar{\varphi}), \tag{4}$$

$$\textit{where} \quad \Pi(\lambda, \gamma, \kappa, \sigma_N, \bar{\varphi}) = \pi_X(E, E) - \pi_X(I, E) + \pi_X(E, I) - \pi_X(I, I) > 0.$$

Note further that passive investors can be seen as receiving a signal of informativeness $\gamma = \frac{1}{2}$. It is then straightforward to show that passive investors have zero expected profit, making the conjecture of holding the whole market valid.

3.4 Information Acquisition

Each investor decides whether to acquire information by comparing the gain from information acquisition, $\Omega(\cdot)$ and the cost, ψ. The proportion of passive investors is determined by the point $\hat{\lambda}$

[22] The proof of the equivalence $\mathbb{E}[\rho(F_x)|c_X = E] = 1 - \mathbb{E}[\rho(F_x)|c_X = I]$ is found in the Appendix B.1.

[23] For example, $\mathbb{E}[\rho(F_x)|S^E = E] = \mathbb{E}\Big[\mathbb{E}[\rho(F_x)|c_X = E]|S^E = E\Big]$.

[24] Proof is in the Appendix A.1.

such that a marginal investor is indifferent between acquiring information, and paying the cost ψ, and not acquiring it:

$$\Omega(\hat{\lambda}, \gamma, \kappa, \sigma_N, \bar{\varphi}) - \psi = 0. \tag{5}$$

There may be a corner solution $\hat{\lambda}$ depending on the cost of information acquisition. When the cost, ψ, is greater than the highest expected profit of active investors, no investor wants to become active and $\hat{\lambda} = 1$. On the contrary, the opposite corner solution occurs for a very small ψ, where every investor acquires information and $\hat{\lambda} = 0$. In the following, we focus on the range of ψ where an interior solutions exist, i.e., $\hat{\lambda} \in (0, 1)$.

4 Equilibrium Results

This section analyzes the equilibrium outcomes of the model. We first describe the interplay between the two possible sources of expected profit for active investors—information asymmetry vs institutional activism—which determine the equilibrium share of passive investors. Here we highlight our main result: the existence of strategic complementarities, which provide an amplification mechanism that exacerbates the amount of passive investment in equilibrium. Importantly, such strategic complementarities exist *regardless* of the way passive investors engage in institutional activism. We follow with a section that characterizes the potential multiple equilibria of the model and their stability. Finally, we close with a discussion on our model assumptions and their implications for our main mechanism.

4.1 Incentives to Acquire Information

We decompose the expected profit from information acquisition in Equation (5) into two components as follows:

$$\Omega = \underbrace{(2\gamma - 1)(1 - \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N))}_{\substack{\text{Information}\\\text{asymmetry}}} \underbrace{\Pi(\hat{\lambda}, \gamma, \kappa, \sigma_N, \bar{\varphi})}_{\substack{\text{Institutional}\\\text{activism}}}.$$

Given this decomposition, we analyze how each component reacts to changes in the equilibrium share of passive investors, summarized in Proposition 3.[25]

Proposition 3. *It holds that*

$$\frac{\partial(1 - \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N))(2\gamma - 1)}{\partial(1 - \lambda)} < 0, \qquad \frac{\partial \Pi(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial(1 - \lambda)} > 0, \quad \forall \zeta \in \{-1, 0, 1\}.$$

The first component, which we denote *information asymmetry*, is standard in models of informed trading (e.g., Grossman and Stiglitz, 1976).[26] As the fraction of active investors, $1 - \lambda$, increases, the market maker can extract more information from the order flows and sets the price closer to the payoff of the firms. Therefore, more active investors make the expected profit Ω of the active investors decrease. This generates the traditional substitution effect in investors' decisions to acquire information.

The second component, $\Pi(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})$, is the main contribution of the paper, which we denote *institutional activism*. In a model that ignores the gains from institutional activism, $\Pi(\cdot)$ is a constant. In our model, in contrast, it includes the expectation over the possible corporate strategies of each firm conditional on the firm's type, as defined in Equation (2). Therefore, it depends on the parameters $\{\hat{\lambda}, \gamma, \kappa, \bar{\varphi}\}$ that can affect the probability that a high or low aggressive strategy is chosen for each firm. From Proposition 3, as the proportion of active investors increases, the

[25] The proof can be found in the Appendix A.2

[26] The term on information asymmetry has two parts: First, $(2\gamma - 1)$ captures the informativeness of the signal. Second, $1 - \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N)$ captures the adverse selection between the active investors and the market maker.

gain from institutional activism increases as well. The reason is that there is a conflict of interest between shareholders of the firm that arises from the different portfolio choice of active investors, passive investors and private owners of a firm. The more active investors appear in the market, the higher is the likelihood that they can influence the corporate strategies to maximize their portfolio payoff. This mechanism generates novel strategic complementaries in investors' decisions to acquire information.

Importantly, this result holds regardless of how passive investors vote—vote optimally by maximizing their portfolio payoff, do not vote, or vote with management.[27] The reason is that, regardless of how passive investors vote their shares, the portfolio allocation is different across the shareholders of a firm, and hence the conflict of interests prevails. If passive investors vote optimally, they influence corporations to pursue strategies that maximize their portfolio returns.[28] Since the portfolios of active and passive investors' differ, their interests are not aligned, and the optimal voting for passive investors reduces expected profits of active investors.[29] If passive investors do not vote, they still hold shares that active investors cannot use to participate in institutional activism. As before, the rise in passive investors takes away voting power from active investors and reduces their expected profit from information. The same logic applies to the case when passive investors vote in line with management. The interests of active investors and private owners are not aligned because of their different portfolio holdings. Therefore, only when the size of active investors increases, can active investors increase their power in institutional activism and obtain higher expected profits.[30] As a result, the strategic complementarities that arise from the institutional activism gains are robust to the approach taken by passive investors to institutional activism.

[27] Proof is in Appendix A.3

[28] This coincides with a higher likelihood of low aggressive strategies being chosen as per Proposition 1.

[29] It can be the case that coincidentally passive and active investors hold the same portfolio if the latter receives the signal pair (S^E, S^E).

[30] There may exist a conflict in interests among active investors unless the signal is perfectly accurate, i.e., $\gamma = 1$. However, because the signal is informative, the interests among active investors, are, on average, more aligned, compared to those between active and passive investors.

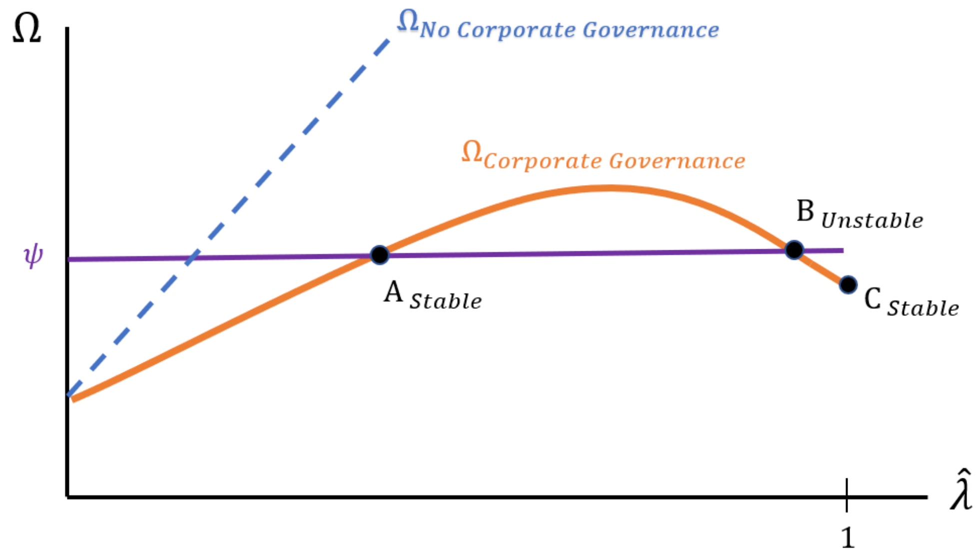

Figure 3: Expected profit for informed investors Ω for different values of λ.

4.2 Equilibrium Outcomes

To illustrate the effects in expected profits of both the information asymmetry and institutional activism gains, we plot, as an example, the expected profit of active investors, Ω, against the share of passive investors $\hat{\lambda}$ in Figure 3. The equilibrium $\hat{\lambda}$ is determined by the point at which the expected profit equals the cost of information ψ as per Equation (5). Note that a model that ignores the variability of the institutional activism gains with respect to λ, dotted blue line, has a steeper slope because $\Pi(\cdot)$ is a constant. In contrast, including how the institutional activism gains change with λ results in a less steep slope because $\frac{\partial \Pi(\cdot)}{\partial \lambda} < 0$.

At this point, it is worthwhile mentioning that the strategic complementaries of the institutional activism channel generate an amplification mechanism for the share of passive investors, compared to the model without such channel. This may rationalize the increasing share of passive investment in the asset management industry. Moreover, the presence of multiple equilibria speaks to the empirical research that documents market fragility as a result of the rise of passive investors, see Anadu et al. (2018).

It is the non-monotonicity in Ω what potentially gives rise to multiple equilibria.[31] As shown in Figure 3, the horizontal line representing the cost of information can interact twice with a hump-shape curve of Ω, resulting in two interior solutions, A and B. We proceed to select the stable equilibrium in the presence of multiplicity. A stable equilibrium requires that the share of passive investment λ reverts back to the equilibrium point for small derivations in investors' beliefs on λ. [32] The solution A satisfies such condition while B does not. At point B, when investors believe the fraction of passive is slightly higher, the expected profit of being active decreases. As a consequence, more investors would choose not to acquire information and eventually converge to the corner equilibrium of 100% passive investors, denoted as point C.

4.3 Discussion

In this section, we devote to discussing our model assumptions around four main topics. Specifically, we focus on the risk neutrality of investors in our model, the assumption that there is no learning from the price, our focus in competition between the firms and on voting as the approach taken to institutional activism.

Risk Neutrality: The assumption of risk neutrality and a fixed trade size is a widely used technique to obtain closed-form solutions in a model where firms' future payoffs can be endogenously affected, see Dow, Goldstein, and Alexander (2015); Edmans, Goldstein, and Jiang (2015) among others. Suppose we assume investors are risk-averse instead of risk-neutral. In that case, two opposing effects can arise in the model at the voting stage. On the one hand, an active investor

[31] When considering the information asymmetry channel solely, we can find at most one unique equilibrium because of monotonicity of function Ω in λ, as indicated in Proposition 3.

[32] Put differently, a stable equilibrium is the solution Equation (5) where the profit function Ω is decreasing in $\hat{\lambda}$ at the point.

may not want to vote for highly aggressive strategy in the firms, because of the fear of potentially high volatile payoffs that result from fierce competition. In this case, risk aversion would weaken our identified strategic complementarities. On the other hand, it is known that information is more valuable for firms with higher volatility, as in Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016). If investors are risk-averse and care about volatility, they have more incentives to acquire information in the presence of volatile payoffs. Therefore, risk aversion would strengthens our strategic complementarities since a rise in passive investors, reduces firms volatility (as in Peress (2010)) and leads to even more investors that choose to become passive. In aggregate, as long as active investors are not so risk averse that they vote in exactly the same way that passive investors do, the introduction of risk aversion only reinforces our story.

learning from the price: The tractability of the model is compromised if we allow investors to learn from the price. Nevertheless, it is worthwhile discussing what this knowledge implies; especially for passive investors, given that they are uninformed. Suppose passive investors can infer the type of the firm from the prices. In that case, it may occur, under a restricted parameter set, that the voting strategy that maximizes their payoffs is not a low aggressive one for both firms. In contrast, they may prefer the efficient firm to compete aggressively at the expense of the inefficient firm if the difference in type is large enough. Such a feature can attenuate the information asymmetry, and hence the conflict of interest, between active and passive investors but does not eliminate it. The reason is that the information asymmetry prevails because active investors obtain information from a highly precise source, the signal. In contrast, passive investors would obtain information from a highly imprecise source, the price which includes noise traders.

Competition between the firms: The focus of this paper is on a very strong type of strategic interaction: competition. We are aware that in reality, the interactions of the firms' population might be milder than those implied by competition. Still, we argue that our model can capture a strong sense of realism when assuming competition because active investors do not tend to hold such a diversified portfolio as passive investors do. Therefore, active investors quite frequently must face the decision to pick up winners in an industry, or across industries, to achieve higher returns. In this setting, it is dramatically more valuable to analyze competitor firms, rather than firms with reinforcing or weaker interactions. The reason is that the rent that can be extracted from information increases when dealing with competitor firms, making the whole information extraction more meaningful.

Voting as institutional activism: In our model, we select voting as the mechanism to allow shareholders to influence firms' strategies. However, voting of shares needs not the be the sole mechanism possible to influence a firm. For example, shareholders could also influence the firm by writing a management's contract on stock prices or forcing the management to maximize shareholders' values assuming, unrealistically, that the management has full knowledge of the portfolio holdings of each shareholder of the firm. We select voting because it is the most straightforward and more realistic mechanism that provides the following two features. First, all shareholders can express their views, and, second, a higher ownership stake in the company results in more power to influence the firms' decisions.

5 Comparative Statics

We now proceed to perform a comparative statics analysis in the stable equilibrium, aiming to highlight and contrast the effects of the information asymmetry gains and the institutional activism gains. We begin by analyzing the equilibrium amount of passive investors, measured by $\hat{\lambda}$, and then the price informativeness, measured by the amount of informed shares $\kappa(1-\hat{\lambda})$. Then we study the comparative statics on firms' returns volatility and product market competition.

The general equilibrium model we introduce is parsimonious in the sense that it has a limited amount of exogenous variables. We focus on exogenous variables that generate contrasting effects in the market outcomes to guide empirical predictions. The equilibrium equation with endogenous $\hat{\lambda}$, referred throughout this section, follows from Equation (5) and is:

$$H(\hat{\lambda}, \gamma, \kappa, \sigma_N, \bar{\varphi}) = (2\gamma - 1)(1 - \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N))\Pi(\hat{\lambda}, \gamma, \kappa, \sigma_N \bar{\varphi}) - \psi. \tag{6}$$

To access the effect of two channels, we take the partial derivatives of the equilibrium equation (6) with respect to each exogenous variable $x \in \{\gamma, \kappa, \sigma_N, \bar{\varphi}\}$. The following term emerges:

$$\underbrace{\Pi(\cdot)\frac{\partial \xi(\cdot)}{\partial x}}_{\substack{\text{Information}\\ \text{asymmetry}}} - \underbrace{(1 - \xi(\cdot))\frac{\partial \Pi(\cdot)}{\partial x}}_{\substack{\text{Institutional}\\ \text{activism}}},$$

This term, measures the relative strength of the information asymmetry vs the institutional activism sensitivity to the exogenous variable x. For the rest of this paper, we assume that if the institutional activism effect is stronger, its sensitivity is higher than that of the information asymmetry effect, making the term negative. This assumption helps make a sharp contrast for empirical predictions.

5.1 Informativeness

Proposition 4 establishes the effect of each exogenous variable on stable equilibrium $\hat{\lambda}$, the amount of passive investors, and $\kappa(1 - \hat{\lambda})$, informativeness measured by informed shares.[33] Proposition 4 summarizes that the size of passive investors decreases with an increase in the cost of information ψ or the amount of private owners φ. In contrast, it increases with higher noise traders σ_N regardless of the relative strength of the information asymmetry effect or institutional activism. Mechanically, the effects are precisely the reverse for informativeness. The intuition is straightforward. The high cost of information, more share of private owners, lower active investors leverage and low signal precision all reduce investors' incentives to acquire information, resulting in a large proportion of

[33] The proof can be found in Appendix A.4

passive investors and low informativeness. #jy: More intuition.

Proposition 4. *When the effect of information asymmetry dominates: #jy: change to only information asymmetry? If yes, then change effect on informativeness to be 0.*

- *The amount of passive investors:* $\frac{\partial \hat{\lambda}}{\partial \psi} > 0, \quad \frac{\partial \hat{\lambda}}{\partial \gamma} = \pm, \quad \frac{\partial \hat{\lambda}}{\partial \sigma_N} < 0, \quad \frac{\partial \hat{\lambda}}{\partial \bar{\varphi}} > 0, \quad \frac{\partial \hat{\lambda}}{\partial \kappa} > 0;$
- *Informativeness:* $\frac{\partial \kappa(1-\hat{\lambda})}{\partial \psi} < 0, \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \gamma} = \pm, \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \sigma_N} > 0, \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \bar{\varphi}} < 0, \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \kappa} = \pm.$

When the effect of institutional activism dominates:

- *The amount of passive investors:* $\frac{\partial \hat{\lambda}}{\partial \psi} > 0, \quad \frac{\partial \hat{\lambda}}{\partial \gamma} < 0, \quad \frac{\partial \hat{\lambda}}{\partial \sigma_N} < 0, \quad \frac{\partial \hat{\lambda}}{\partial \bar{\varphi}} > 0; \quad \frac{\partial \hat{\lambda}}{\partial \kappa} < 0;$
- *Informativeness:* $\frac{\partial \kappa(1-\hat{\lambda})}{\partial \psi} < 0, \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \gamma} > 0, \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \sigma_N} > 0, \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \bar{\varphi}} < 0; \quad \frac{\partial \kappa(1-\hat{\lambda})}{\partial \kappa} > 0.$

The effect on the leverage of active investors κ is interesting. A change in leverage κ makes the size of passive investors increase if the institutional activism effect dominates and decrease if the information asymmetry effect dominates; while the opposite holds for informativeness. To understand the intuition of this result, it is useful first to assume that there are no institutional activism gains, that is, to assume that $\Pi(\cdot)$ a constant. In a model with *only* information asymmetry, the equilibrium amount of informed shares, $\kappa(1-\hat{\lambda})$, is a constant. This is because an increase in κ gives active investors the possibility to buy more shares, which reveals more information to the market maker and reduces the profit from information asymmetry. In equilibrium, investors internalize this change by reducing the size of active investors to the point that no one can profit from information any further. Therefore, κ and $(1-\hat{\lambda})$ move in the exact opposite direction. In contrast, including gains from institutional activism, higher leverage κ results in more shares, and more power, to active investors to influence corporations. As a result, when the gains from institutional activism dominate, higher leverage gives incentives to more investors to become active, resulting in a larger proportion of active investors $(1-\hat{\lambda})$ and a more informativeness.

#jy: Emphasize the effects are the same under three cases.

5.2 Return Variance

We then proceed to understand the effects of each exogenous variable on the return variance. Based on the law of total variance, we derive an expression for return variance of firm X (the same for firm Y) as:

$$Var[R_X] = \frac{1}{2}(1-\xi)^2\Big((\pi_{EE}-\pi_{II})^2 + (\pi_{EI}-\pi_{IE})^2 + 2\xi^2(\pi_{EE}-\pi_{EI}-\pi_{IE}+\pi_{II})^2\Big). \tag{7}$$

We decompose the overall effect to two components: the direct effect which captures the effect of a change in variance x on return variance, and the indirect effect, which captures the effect through the change in equilibrium share $\hat{\lambda}$.

$$\frac{dVar(R_X)}{dx} = \underbrace{\frac{\partial Var(R_X)}{\partial x}}_{\text{Direct Effect}} + \underbrace{\frac{\partial Var(R_X)}{\partial \hat{\lambda}}\frac{\partial \hat{\lambda}}{\partial x}}_{\text{Indirect Effect}}.$$

Proposition 5 summaries results for exogenous variables $x \in \{\phi, \kappa, \gamma\}$ that generate sharp comparisons between information asymmetry and institutional activism.[34]

Proposition 5. *When information asymmetry dominates:*

$$\frac{\partial Var(R_X)}{\partial \psi} > 0, \qquad \frac{\partial Var(R_X)}{\partial \kappa} < 0, \qquad \frac{\partial Var(R_X)}{\partial \gamma} < 0;$$

When institutional activism dominates:

$$\frac{\partial Var(R_X)}{\partial \psi} < 0, \qquad \frac{\partial Var(R_X)}{\partial \kappa} > 0, \qquad \frac{\partial Var(R_X)}{\partial \gamma} > 0.$$

We start the analysis with the most simple case, the cost of information ψ. Since ψ does not directly affect variance, as per Equation (7), it only affects it indirectly through changing the equilibrium share of passive investors $\hat{\lambda}$. Its indirect effect through information asymmetry is positive. This is because a rise in ψ decreases the share of active investment and informativeness,

[34] The proof can be found in Appendix A.5

see Proposition ??. It follows from an increase in information asymmetry that the accuracy of the market maker to set price decreases, which increases in return variance. In contrast, the indirect effect of institutional activism is negative. The reason is that the resulting increase in the passive shares due to the higher cost of information reduces the ability of active investors to influence the actions of the firms conditional on their information. As a consequence, the return variance decreases because the strategies chosen by the firms do not change based on the firms' type, making firms' payoff less sensitive to its type. Importantly, such effect persists irrespective of how passive investors engage in institutional activism. No matter how they vote, once the passive investment rises, firms strategic interactions always become less dependent on their type and hence return variance decreases. The contrasting effects are illustrated in the left panel of Figure 4, where the solid line illustrates the positive effect through information asymmetry and the dotted line illustrates the negative effect of institutional activism.

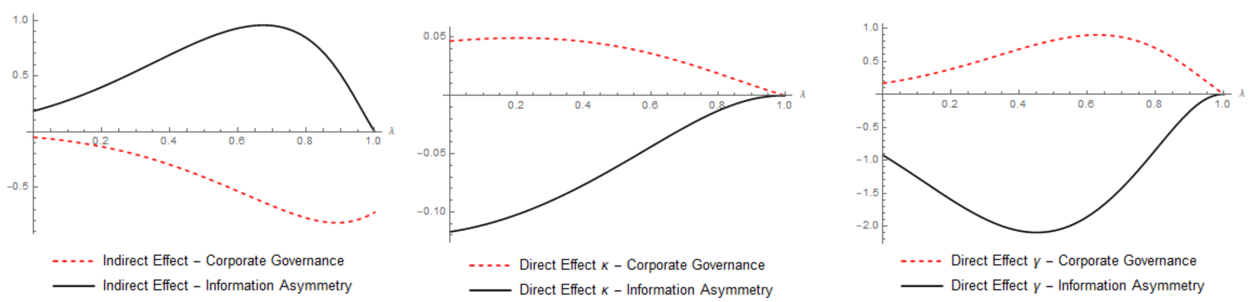

Figure 4: Direct and Indirect effect through institutional activism and Information Asymmetry for different equilibrium share of passive investors λ

Changes in κ and γ include both a direct and an indirect effect. We first discuss the case that institutional activism dominates. It is the case that both higher leverage or a more precise signal directly entitles active investors more power to influence firms' strategies conditional on their information, enlarging return variance through the direct effect. Through the indirect effect, we know from Proposition A.4 that the amount of passive investors decreases in equilibrium when

κ and γ increase. This also leads to an increase in return variance because more active investors can influence firms' strategies based on their information. In contrast, when we only have available the information asymmetry the results reverse. The direct impact of an increase in the leverage of active investors and increased information precision is the reduction in information asymmetry, which lowers return variance. Moreover, through the indirect effect disappears in equilibrium because the share $\kappa(1-\hat{\lambda})$, is a constant. The contrasting effects are illustrated in the centre and right panel of Figure 4 for κ and γ respectively, where the solid line illustrates the negative effect through information asymmetry and the dotted line illustrates the positive effect institutional activism.

5.3 Product Competition

Finally, we move to understand the impact on product competition. It is worth emphasizing that our result on strategic complementarities in information acquisition holds regardless of the degree of product competition between two firms. Nevertheless, our model nests the results from the common ownership literature where passive investors reduce competition. To cast light on empirical tests of how passive investors engage in institutional activism, we generate comparative statics on the changes in competition with shocks to the exogenous variables given different approaches of passive investors' activism.

It is important to highlight that in our Cournot competition setting, we can proxy competition by either the total quantity produced by both firms or the product price. Since the product price is a linear transformation of the total quantity produced, the predictions are the same. We choose total quantity as the proxy for competition where more quantity produced is analogous to more competition in the market.

The expected quantity produced in the market can be written as:

$$\bar{Q} = E[Q_X + Q_Y] = \frac{1}{24b}\big(6(2a - c^E - c^I) + (a - c^E)(q_{EE} + q_{EI}) + (a - c^I)(q_{IE} + q_{II})\big)$$

We now proceed to take the partial derivatives of $\bar{Q}$ with respect to each exogenous variable $x \in \{\kappa, \gamma, \sigma_N, \bar{\varphi}\}$. As in the analysis of return variance, the change in the exogenous variables has two effects, one direct effect and one indirect on through the change in equilibrium share $\hat{\lambda}$ as:

$$\frac{d\bar{Q}}{dx} = \underbrace{\frac{\partial \bar{Q}}{\partial x}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \bar{Q}}{\partial \hat{\lambda}} \frac{\partial \hat{\lambda}}{\partial x}}_{\text{Indirect Effect}} .$$

We repeat this exercise under the three different approaches that passive investors can take to participate in institutional activism, and summarize the results in the the following proposition.[35]

Proposition 6. *When passive investors vote optimally, to maximize portfolio return:*

$$\frac{d\bar{Q}}{d\psi} < 0, \qquad \frac{d\bar{Q}}{d\kappa} > 0, \qquad \frac{d\bar{Q}}{d\gamma} > 0, \qquad \frac{d\bar{Q}}{d\sigma_N} > 0, \qquad \frac{d\bar{Q}}{d\varphi} = \pm;$$

When passive investors do not vote, don't do institutional activism:

$$\frac{d\bar{Q}}{d\psi} > 0, \qquad \frac{d\bar{Q}}{d\kappa} < 0, \qquad \frac{d\bar{Q}}{d\gamma} < 0, \qquad \frac{d\bar{Q}}{d\sigma_N} < 0, \qquad \frac{d\bar{Q}}{d\varphi} = \pm;$$

When passive investors vote with the management of the firm:

$$\frac{d\bar{Q}}{d\psi} > 0, \qquad \frac{d\bar{Q}}{d\kappa} < 0, \qquad \frac{d\bar{Q}}{d\gamma} < 0, \qquad \frac{d\bar{Q}}{d\sigma_N} < 0, \qquad \frac{d\bar{Q}}{d\varphi} = \pm.$$

The resulting market competition only depends on the likelihood that an aggressive strategy is chosen. Intuitively, when leverage and information precision increase, more active investors hold the same portfolio and the likelihood of an aggressive strategy chosen increases. Thus, product competition increases. The approach that passive investors take to activism plays a role in the indirect effect through $\hat{\lambda}$. When passive investors vote for less aggressive strategies (the optimal strategy), then low competition follows from the rise of passive investment in equilibrium (see Proportion A.4). Otherwise, when passive investors do not vote or vote with management, the opposite results emerge. Changes in the amount of noise traders σ_N only affect competition

[35] The proof can be found in Appendix A.6.

through the indirect effect. An increase in noise traders increases the share of active investors because information asymmetry increases and they can better hide their information from the market maker. A higher share of active investors increases competition if passive investors vote optimally, reducing competition. However, the same increase in the share of active investors reduces competition if passive investors do not vote or vote with the management. The intuition of this last result is that when passive investors do not vote or vote with the management, they implicitly give more power to private owners, who, in contrast to active investors, always vote for more competition.

5.4 Empirical predictions

In this subsection, we collect sharp empirical predictions that arise from our comparative statics exercise. We can generate two different types of tests. First, we generate predictions on the strength of institutional activism vs information asymmetry gains. Second, we generate predictions that disentangle whether passive investors engage in institutional activism optimally or do not engage in activism or directly follow the firm's management.

5.4.1 institutional activism vs. Information Asymmetry

The following list summarizes the predictions in terms of price informativeness and returns variance depending on the shocks to each exogenous variables.

Prediction 1: When the cost of information ψ increases, the variance of returns decreases if institutional activism is stronger; otherwise it increases.

Prediction 2: When the leverage of active investors κ increases, the price informativeness increases if institutional activism is stronger; otherwise it decreases.

Prediction 3: When the leverage of active investors κ increases, the variance of returns increases if institutional activism is stronger; otherwise it decreases.

Prediction 4: When the precision of information γ increases, the variance of returns increases if institutional activism is stronger; otherwise it decreases.

5.4.2 Optimal institutional activism vs no activism or follow management

The following list summarizes the predictions in terms of competition depending on the shocks to each exogenous variables.

Prediction 5: When the cost of information ψ increases, competition decreases if passive investors engage in institutional activism as to maximize portfolio payoff; otherwise it increases.

Prediction 6: When the leverage of active investors κ increases, competition increases if passive investors engage in institutional activism as to maximize portfolio payoff; otherwise it decreases.

Prediction 7: When the precision of information γ increases, competition increases if passive investors engage in institutional activism as to maximize portfolio payoff; otherwise it decreases.

Prediction 8: When the amount of noise traders σ_N increases, competition increases if passive investors engage in institutional activism as to maximize portfolio payoff; otherwise it decreases.

6 Conclusion

This paper contributes to understanding the effects of institutional activism for information acquisition. We introduce a fully rational model in which institutional investors maximize their portfolio payoff conditional on their information. We allow all investors to influence the corporations they own and study in detail the contrasting implications that the gains from *institutional activism* have for the financial market. Our main contribution is that we identify a novel strategic complementarities mechanism where more passive investors increase the incentives of other investors to become passive as well.

Our main result is novel because the profits of active investors decreasing with the rise of passive goes against the traditional sustainability role of information, which a canonical information model implies. By including the interaction of both information asymmetry and institutional activism, our model sustains the existence of multiple equilibria. We relate such multiplicity to the view that passive investors increase market fragility. Each of the sources of profit in isolation is monotone and does not sustain multiple equilibria.

Furthermore, our main result does not depend on the approach passive investors take to institutional activism nor on the level of competition in the product market. In this sense, our model nest the common-ownership literature results without relying on them.

We provide empirical predictions to identify the prevalence of institutional activism gains in influencing information acquisition decisions. Instead of using noisy proxies to measure institutional activism, we provide predictions on financial market outcomes such as price informativeness and return volatility. We leave further tests open for future empirical research.

We highlight the need for a model, such as the one in this paper, to address the fact that the shares of active and passive investment are equilibrium outcomes. As such, they cannot be used directly in empirical tests since they provide contradicting results depending on which exogenous variable gave rise to the initial shift in equilibrium. In this sense, we speak to the seemingly contradicting empirical evidence that surrounds the rise of passive investment, and that uses this rise directly as a left-hand variable.

Lastly, this paper also contributes to an explanation for the rise in the share of passive investment over the last two decades. The rise of passive is a puzzling phenomenon because the empirical evidence points towards a steady decrease in the costs, such as expense ratios, of active investment. At the same time, there is no evidence of significant changes in the information structure of the market. The institutional activism gains introduced in our paper provides a mechanism by which a rise in passive investing endogenously creates a decrease in the expected profits of information acquisition and, therefore, gives rise to more investors choosing to become passive in equilibrium.

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A Appendix: Proofs

A.1 Proof for Proposition 2

Proof. To simplify notation we write the expected profit conditional on each realization of firms type as:

$$\pi_{EE} \equiv \pi_X(E,E) \qquad \pi_{EI} \equiv \pi_X(E,I)$$

$$\pi_{IE} \equiv \pi_X(I,E) \qquad \pi_{II} \equiv \pi_X(I,I)$$

$$\text{Where: } (A_X, A_Y) \in \mathbb{S}^A = \{(A^H, A^H), (A^H, A^L), (A^L, A^H), (A^L, A^L)\}$$

Given each realization of firms' type, we define the return as the payoff minus the expected prices, which is calculated by using the expectation about the market maker's belief $\xi(\lambda, \gamma)$. The table below summarizes the return for firm X and the results for firm Y is symmetric.

Realization of firms' type	Payoff_X - $\mathbb{E}[\text{Price}_X]$
(E,E)	$\text{Ret}^X_{EE} = \pi_{EE} - \xi^2\pi_{EE} - \xi(1-\xi)\pi_{EI} - \xi(1-\xi)\pi_{IE} - (1-\xi)^2\pi_{II}$
(E,I)	$\text{Ret}^X_{EI} = \pi_{EI} - \xi(1-\xi)\pi_{EE} - \xi^2\pi_{EI} - (1-\xi)^2\pi_{IE} - \xi(1-\xi)\pi_{II}$
(I,E)	$\text{Ret}^X_{IE} = \pi_{IE} - \xi(1-\xi)\pi_{EE} - (1-\xi)^2\pi_{EI} - \xi^2\pi_{IE} - \xi(1-\xi)\pi_{II}$
(I,I)	$\text{Ret}^X_{II} = \pi_{II} - (1-\xi)^2\pi_{EE} - \xi(1-\xi)\pi_{EI} - \xi(1-\xi)\pi_{IE} - \xi^2\pi_{II}$

Table 5: Payoff of the firm X minus the expected price for each realization of firm type

To prove our conjecture 1, we need to show that after receiving one signal pair, our conjectured strategy dominates all other choices, that is, active investors get the maximized return following the conjectured manual. In the following, we take the signal pair $\{S^E, S^E\}$ as an example. An investor with signal $\{S^E, S^E\}$ should take a long position on both firms, following the conjecture.

Since the prices quoted for firms are different in each realization of firms' type, active investors must form an expectation for each realization of firms' type conditional on the signal. With the signal $\{S^E, S^E\}$, his posterior on firms' type is $\{\gamma^2, \gamma(1-\gamma), \gamma(1-\gamma), (1-\gamma)^2\}$ for $\{(E, E), (E, I), (I, E), (I, I)\}$, respectively (see Table 2). Therefore, his expected return of taking a long position on both firms is

$$\begin{aligned}
\mathbb{E}[\mathrm{Ret}|(S^E, S^E)] &= \gamma^2(\mathrm{Ret}^X_{EE} + \mathrm{Ret}^Y_{EE}) + \gamma(1-\gamma)(\mathrm{Ret}^X_{EI} + \mathrm{Ret}^Y_{Ei}) \\
&\quad + (1-\gamma)\gamma(\mathrm{Ret}^X_{IE} + \mathrm{Ret}^Y_{IE}) + (1-\gamma)^2(\mathrm{Ret}^X_{II} + \mathrm{Ret}^Y_{II}) \\
&= 2(2\gamma-1)(1-\xi)\Big((2\gamma-1)\xi(\pi_{EE} - \pi_{EI} - \pi_{IE} + \pi_{II}) + \pi_{EE} - \pi_{II}\Big).
\end{aligned}$$

To prove such strategy is optimal, we need to show that the above expected return is higher than the strategy of short-selling both firms or holding one and short-selling the other. The expected return of holding one and short-selling the other is 0 because of the symmetry property. Alternatively, we can show that

$$\gamma^2(\mathrm{Ret}^X_{EE} - \mathrm{Ret}^Y_{EE}) + \gamma(1-\gamma)(\mathrm{Ret}^X_{EI} - \mathrm{Ret}^Y_{Ei}) + (1-\gamma)\gamma(\mathrm{Ret}^X_{IE} - \mathrm{Ret}^Y_{IE}) + (1-\gamma)^2(\mathrm{Ret}^X_{II} - \mathrm{Ret}^Y_{II}) = 0.$$

The expected return of short-selling both firms equals to $-\mathbb{E}[\mathrm{Ret}|(S^E, S^E)]$. Therefore, once $\mathbb{E}[\mathrm{Ret}|(S^E, S^E)]$ is positive, we can prove that taking a long position on both firms is the optimal strategy. Under Cournot competition, $\mathbb{E}[\mathrm{Ret}|(S^E, S^E)] > 0$ holds true, see proof B.2.

We repeat the same analysis for the remaining three possible signal pairs, and get expected returns following the conjectured strategy as follows:

$$\begin{aligned}
\mathbb{E}[\mathrm{Ret}|(S^E, S^I)] &= 2(2\gamma-1)(1-\xi)(\pi_{EI} - \pi_{IE}); \\
\mathbb{E}[\mathrm{Ret}|(S^I, S^E)] &= 2(2\gamma-1)(1-\xi)(\pi_{EI} - \pi_{IE}); \\
\mathbb{E}[\mathrm{Ret}|(S^I, S^I)] &= 2(2\gamma-1)(1-\xi)\Big(\pi_{EE} - \pi_{II} - (2\gamma-1)\xi(\pi_{EE} - \pi_{EI} - \pi_{IE} + \pi_{II})\Big).
\end{aligned}$$

After walking though detailed steps, we can prove that our conjectured strategy holds with the condition $12(c_I - c_E) > (a - cI)$. This means that two firms need to be slightly different from each other such that it worth for active investors to acquire information, identify the efficient firm and promote product competition.

Next, we calculate the ex-ante expected return for active investors:

$$\begin{aligned}\Omega &= \frac{1}{4}\Big(\mathbb{E}[\mathrm{Ret}|(S^E, S^E)] + \mathbb{E}[\mathrm{Ret}|(S^E, S^I)] + \mathbb{E}[\mathrm{Ret}|(S^I, S^E)] + \mathbb{E}[\mathrm{Ret}|(S^I, S^I)]\Big)\\ &= (1-\xi)(2\gamma - 1)(\pi_{EE} + \pi_{EI} - \pi_{IE} - \pi_{II}).\end{aligned}$$

We denote $\Pi(\lambda, \gamma, \kappa, \sigma_N, \bar{\varphi}) = \pi_{EE} + \pi_{EI} - \pi_{IE} - \pi_{II}$. To prove that $\Pi(\cdot) > 0$, we calculate the expected profits for each realization of firms' type:

$$\begin{aligned}\pi_{EE} &= (q_{EE})^2\frac{(a-c^E)^2}{9b} + q_{EE}(1-q_{EE})\left(\frac{5(a-c^E)^2}{36b} + \frac{5(a-c^E)^2}{48b}\right) + (1-q_{EE})^2\frac{(a-c^E)^2}{8b},\\ \pi_{EI} &= q_{EI}q_{IE}\frac{(a-c^E)(a-2c^E+c^I)}{9b} + q_{EI}(1-q_{IE})\frac{(a-c^E)(5a-8c^E+3c^I)}{36b},\\ &\quad + q_{IE}(1-q_{EI})\frac{(a-c^E)(5a-9c^E+4c^I)}{48b} + (1-q_{EI})(1-q_{IE})\frac{(a-c^E)(2a-3c^E+c^I)}{16b},\\ \pi_{IE} &= q_{EI}q_{IE}\frac{(a-c^I)(a-2c^I+c^E)}{9b} + q_{IE}(1-q_{EI})\frac{(a-c^I)(5a-8c^I+3c^E)}{36b},\\ &\quad + q_{EI}(1-q_{IE})\frac{(a-c^I)(5a-9c^I+4c^E)}{48b} + (1-q_{EI})(1-q_{IE})\frac{(a-c^I)(2a-3c^I+c^E)}{16b},\\ \pi_{II} &= (q_{II})^2\frac{(a-c^I)^2}{9b} + q_{II}(1-q_{II})\left(\frac{5(a-c^I)^2}{36b} + \frac{5(a-c^I)^2}{48b}\right) + (1-q_{II})^2\frac{(a-c^I)^2}{8b}.\end{aligned}$$

Then $\Pi(\cdot)$ can be written as:

$$\begin{aligned}\Pi(\cdot) = \frac{1}{144b}\big[&(2a - c^E - c^I)(c^I - c^E)(40 + (5 - q_{EE})(1 + q_{EI}) + q_{EE}(q_{EI} - q_{EE}))\\ &+ (a - c^I)^2(5(q_{EI} - q_{IE}) + (q_{II} - q_{EE})(1 + q_{II} + q_{EE}))\big].\end{aligned}$$

Since $q_{EI} > q_{II} > q_{IE} > q_{EE}$ and $c^I > c^E$ it follows that all the terms are positive and $\Pi(\cdot) > 0$.

$\square$

A.2 Proof of Proposition 3

Proof.

$$\frac{\partial(1-\xi(\lambda,\gamma))(2\gamma-1)}{\partial\lambda} > 0 \qquad \text{and} \qquad \frac{\partial\Pi(\lambda,\gamma,\kappa)}{\partial\lambda} < 0.$$

We begin by showing first (i) $\frac{\partial(1-\xi(\lambda,\gamma))(2\gamma-1)}{\partial\lambda} > 0$ and we then proceed to (ii) $\frac{\partial\Pi(\lambda,\gamma,\kappa)}{\partial\lambda} < 0$.

(i)

By the chain rule we can write:

$$\frac{\partial(1-\xi(\lambda,\gamma))(2\gamma-1)}{\partial\lambda} = -(2\gamma-1)\frac{\partial\xi(\lambda,\gamma)}{\partial\lambda}$$

Given the definition of $\xi(\lambda,\gamma)$ and the p.d.f. of the standard normal distribution. We have that:

$$\frac{\partial\xi(\lambda,\gamma)}{\partial\lambda} = (1-2\gamma)\kappa(2(2\gamma-1)\kappa(1-\lambda)+N_X^F)\frac{\left(2\exp\left[\frac{(N_X^F-2(2\gamma-1)\kappa(\lambda-1))^2+(N_X^F)^2}{2\sigma_N^2}\right]\right)}{\sigma_N^2\left(e^{\frac{(N_X^F-2(2\gamma-1)\kappa(\lambda-1))^2}{2\sigma_N^2}}+e^{\frac{(N_X^F)^2}{2\sigma_N^2}}\right)^2}$$

Since $\gamma > \frac{1}{2}$, it is the case that $\frac{\partial\xi(\lambda,\gamma)}{\partial\lambda} < 0$. Therefore $\frac{\partial(1-\xi(\lambda,\gamma))(2\gamma-1)}{\partial\lambda} > 0$.

(ii)

We begin by spelling out $\Pi(\lambda, \gamma, \kappa)$ as:

$$\Pi(\lambda, \gamma, \kappa) = \frac{1}{144b}\big[(2a - c^E - c^I)(c^I - c^E)(40 + (5 - q_{EE})(1 + q_{EI}) + q_{EE}(q_{EI} - q_{EE})) + (a - c^I)^2(5(q_{EI} - q_{IE}) + (q_{II} - q_{EE})(1 + q_{II} + q_{EE}))\big]$$

Note that the different probabilities of voting outcomes q_{EE}, q_{EI}, q_{IE} and q_{II} defined in Equation (**??**) are actually functions of λ and γ. The derivatives of q_{EE}, q_{EI}, q_{IE} and q_{II} can be written as:

$$\begin{aligned}
\frac{\partial q_{EE}}{\partial \lambda} &= \frac{\gamma(2\gamma - 1)\kappa - 1}{\bar{\varphi}} \\
\frac{\partial q_{EI}}{\partial \lambda} &= \frac{-\gamma(2\gamma - 1)\kappa - 1}{\bar{\varphi}} \\
\frac{\partial q_{IE}}{\partial \lambda} &= \frac{(1-\gamma)(2\gamma - 1)\kappa - 1}{\bar{\varphi}} \\
\frac{\partial q_{II}}{\partial \lambda} &= \frac{-(1-\gamma)(2\gamma - 1)\kappa - 1}{\bar{\varphi}}
\end{aligned}$$

Taking the derivative of the spelled out $\Pi(\lambda, \gamma, \kappa)$ with respect of λ gives:

$$\begin{aligned}
\frac{\partial \Pi(\lambda, \gamma, \kappa)}{\partial \lambda} &= \frac{1}{144b}\left((a - c^E)^2\left(5\frac{\partial q_{EI}}{\partial \lambda} - (1 + 2q_{EE})\frac{\partial q_{EE}}{\partial \lambda}\right) - (a - c^I)^2\left(5\frac{\partial q_{IE}}{\partial \lambda} - (1 + 2q_{II})\frac{\partial q_{II}}{\partial \lambda}\right)\right) \\
&= \frac{1}{144b}\left((c^I - c^E)(2a - c^I - c^E)\left(2(2 - q_{EE})\frac{\partial q_{EI}}{\partial \lambda} + (1 + 2q_{EE})\left(\frac{\partial q_{EI}}{\partial \lambda} - \frac{\partial q_{EE}}{\partial \lambda}\right)\right)\right. \\
&\quad \left. + (a - c^I)^2\left(5\left(\frac{\partial q_{EI}}{\partial \lambda} - \frac{\partial q_{IE}}{\partial \lambda}\right) + (1 + 2q_{EE})\left(\frac{\partial q_{II}}{\partial \lambda} - \frac{\partial q_{EE}}{\partial \lambda}\right) + 2(q_{II} - q_{EE})\frac{\partial q_{II}}{\partial \lambda}\right)\right)
\end{aligned}$$

Where: $\frac{\partial q_{EE}}{\partial \lambda} + \frac{\partial q_{EI}}{\partial \lambda} < 0, \frac{\partial q_{EI}}{\partial \lambda} < 0, \frac{\partial q_{II}}{\partial \lambda} < 0 \qquad \frac{\partial q_{EI}}{\partial \lambda} < \frac{\partial q_{II}}{\partial \lambda} < \frac{\partial q_{IE}}{\partial \lambda} < \frac{\partial q_{EE}}{\partial \lambda}$

All the terms are negative, which implies that $\frac{\partial \Pi(\lambda, \gamma, \kappa)}{\partial \lambda} < 0$. □

A.3 Proof of Proposition 3 holds for different ζ

Proof. The case for $\zeta = 1$ is available in section A.2. In this proof we focus on the cases where $\zeta = 0$ or $\zeta = -1$. The derivatives of The derivatives of q_{EE}, q_{EI}, q_{IE} and q_{II} can be written as:

For $\zeta = 0$

$$\frac{\partial q_{EE}}{\partial \lambda} = \frac{\gamma(2\gamma-1)\kappa}{\bar{\varphi}} \qquad \frac{\partial q_{EI}}{\partial \lambda} = 0 \qquad \frac{\partial q_{IE}}{\partial \lambda} = \frac{(1-\gamma)(2\gamma-1)\kappa}{\bar{\varphi}} \qquad \frac{\partial q_{II}}{\partial \lambda} = 0$$

For $\zeta = -1$

$$\frac{\partial q_{EE}}{\partial \lambda} = \frac{\gamma(2\gamma-1)\kappa+1}{\bar{\varphi}} \qquad \frac{\partial q_{EI}}{\partial \lambda} = 0 \qquad \frac{\partial q_{IE}}{\partial \lambda} = \frac{(1-\gamma)(2\gamma-1)\kappa+1}{\bar{\varphi}} \qquad \frac{\partial q_{II}}{\partial \lambda} = 0$$

Taking the derivative of the spelled out $\Pi(\lambda, \gamma, \kappa)$ with respect of λ gives:

$$\frac{\partial \Pi(\lambda,\gamma,\kappa)}{\partial \lambda} = -\frac{1}{144b}\left((a-c^E)^2\left((1+2q_{EE})\frac{\partial q_{EE}}{\partial \lambda}\right) + (a-c^I)^2\left(5\frac{\partial q_{IE}}{\partial \lambda}\right)\right)$$

All the terms after the minus sign are positive, which implies that $\frac{\partial \Pi(\lambda,\gamma,\kappa)}{\partial \lambda} < 0$ for $\zeta = 0$ or $\zeta = -1$. □

A.4 Proof of Proposition 4

Proof. By the implicit function theorem, we can derive the partial derivative of endogenous $\hat{\lambda}$ on each exogenous variable $x \in \{\phi, \kappa, \gamma, \sigma_N, \bar{\varphi}\}$ as:

$$\frac{\partial \hat{\lambda}}{\partial x} = -\left(\frac{\partial H(\cdot)}{\partial x}\right) / \left(\frac{\partial H(\cdot)}{\partial \hat{\lambda}}\right)$$

For the equilibrium to be stable, the share $\hat{\lambda}$ has to be on the upwards slopping segment of the equilibrium function $H(\cdot)$ therefore $\frac{\partial H(\cdot)}{\partial \hat{\lambda}} > 0$. As a consequence, the derivatives of the equilibrium parameter $\hat{\lambda}$ with respect to each exogenous variable $x \in \{\phi, \kappa, \gamma, \sigma_N, \bar{\varphi}\}$ can be obtained by multiplying by -1 for the partial derivatives of the equilibrium equation listed below:

$$\frac{\partial H(\cdot)}{\partial \psi} = -1 < 0;$$
$$\frac{\partial H(\cdot)}{\partial \kappa} = -2(2\gamma - 1)\left(\Pi(\cdot)\frac{\partial \xi(\cdot)}{\partial \kappa} - (1 - \xi(\cdot))\frac{\partial \Pi(\cdot)}{\partial \kappa}\right);$$
$$\frac{\partial H(\cdot)}{\partial \gamma} = 4\Pi(\cdot)(1 - \xi(\cdot)) - 2(2\gamma - 1)\left(\Pi(\cdot)\frac{\partial \xi(\cdot)}{\partial \gamma} - (1 - \xi(\cdot))\frac{\partial \Pi(\cdot)}{\partial \gamma}\right);$$
$$\frac{\partial H(\cdot)}{\partial \sigma_N} = -2(2\gamma - 1)\Pi(\cdot)\frac{\partial \xi(\cdot)}{\partial \sigma_N} > 0;$$
$$\frac{\partial H(\cdot)}{\partial \bar{\varphi}} = 2(2\gamma - 1)(1 - \xi(\cdot))\frac{\partial \Pi(\cdot)}{\partial \bar{\varphi}} < 0.$$

□

A.5 Proof of Proposition 5

We study the total variance of returns by using the law of total variance. By denoting as (S_X, S_Y) the signal set, the variance of return for firm X, which is the same as for firm Y, can be written as:

$$\begin{aligned} Var[R_X] &= E[Var[R_X|S_X, S_Y]] + Var[E[R_X|S_X, S_Y]] \\ &= \frac{1}{2}(1 - \xi)^2\Big((\pi_{EE} - \pi_{II})^2 + (\pi_{EI} - \pi_{IE})^2 + 2\xi^2(\pi_{EE} - \pi_{EI} - \pi_{IE} + \pi_{II})^2\Big) \end{aligned}$$

We proceed to re-write the variance in terms of the function $\Pi(\cdot)$ and two new auxiliary function defined as:

$$\Pi_1(\hat{\lambda}, \gamma, \kappa, \bar{\varphi}) = \pi_{EI}(\cdot) - \pi_{II}(\cdot) \qquad \Pi_2(\hat{\lambda}, \gamma, \kappa, \bar{\varphi}) = \pi_{EI}(\cdot) - \pi_{IE}(\cdot)$$

Making the variance of return equivalent to:

$$Var[R_X] = \frac{1}{2}(1 - \xi)^2\Big((\Pi - \Pi_2)^2 + (\Pi_2)^2 + 2\xi^2(\Pi - 2\Pi_1)^2\Big)$$

By applying the chain rule to the variance function we obtain:

$$\frac{\partial Var(R_X)}{\partial x} = \frac{\partial Var(R_X)}{\partial \xi}\left(\frac{\partial \xi}{\partial x} + \frac{\partial \xi}{\partial \hat{\lambda}}\frac{\partial \hat{\lambda}}{\partial x}\right) + \frac{\partial Var(R_X)}{\partial \Pi}\left(\frac{\partial \Pi}{\partial x} + \frac{\partial \Pi}{\partial \hat{\lambda}}\frac{\partial \hat{\lambda}}{\partial x}\right)$$
$$+ \frac{\partial Var(R_X)}{\partial \Pi_1}\left(\frac{\partial \Pi_1}{\partial x} + \frac{\partial \Pi_1}{\partial \hat{\lambda}}\frac{\partial \hat{\lambda}}{\partial x}\right) + \frac{\partial Var(R_X)}{\partial \Pi_2}\left(\frac{\partial \Pi_2}{\partial x} + \frac{\partial \Pi_2}{\partial \hat{\lambda}}\frac{\partial \hat{\lambda}}{\partial x}\right)$$

We denote all the terms related to Π, Π_1 and Π_2 as institutional activism since they are all zero in a model that does not include institutional activism. By reorganizing the terms, we decompose the effect on variance from information asymmetry and from institutional activism as:

$$\frac{\partial Var(R_X)}{\partial x} = \frac{\partial Var(R_X)}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial Var(R_X)}{\partial \xi}\frac{\partial \xi}{\partial \hat{\lambda}}\frac{\partial \hat{\lambda}}{\partial x}$$
$$+ \frac{\partial Var(R_X)}{\partial \Pi}\frac{\partial \Pi}{\partial x} + \frac{\partial Var(R_X)}{\partial \Pi_1}\frac{\partial \Pi_1}{\partial x} + \frac{\partial Var(R_X)}{\partial \Pi_2}\frac{\partial \Pi_2}{\partial x}$$
$$+ \left(\frac{\partial Var(R_X)}{\partial \Pi}\frac{\partial \Pi}{\partial \hat{\lambda}} + \frac{\partial Var(R_X)}{\partial \Pi_1}\frac{\partial \Pi_1}{\partial \hat{\lambda}} + \frac{\partial Var(R_X)}{\partial \Pi_2}\frac{\partial \Pi_2}{\partial \hat{\lambda}}\right)\frac{\partial \hat{\lambda}}{\partial x}$$

The first line represents the information asymmetry, the direct plus the indirect effect. The second and third line are the direct and the indirect effect through institutional activism respectively.[36]

A.6 Proof of Proposition 6

Proof. We first drive the following lemma[37]:

[36] All the terms related to information asymmetry and various components of institutional activism can be signed analytically and are available in the Appendix B.3.

[37] Proof in the Appendix

Lemma 1.

Vote optimally	*Do not vote*	*Vote pro-management*
$\frac{\partial \bar{Q}}{\partial \lambda} < 0$	$\frac{\partial \bar{Q}}{\partial \lambda} > 0$	$\frac{\partial \bar{Q}}{\partial \lambda} > 0$
$\frac{\partial \bar{Q}}{\partial \kappa} = 0$	$\frac{\partial \bar{Q}}{\partial \kappa} < 0$	$\frac{\partial \bar{Q}}{\partial \kappa} < 0$
$\frac{\partial \bar{Q}}{\partial \gamma} = 0$	$\frac{\partial \bar{Q}}{\partial \gamma} < 0$	$\frac{\partial \bar{Q}}{\partial \gamma} < 0$
$\frac{\partial \bar{Q}}{\partial \varphi} > 0$	$\frac{\partial \bar{Q}}{\partial \varphi} > 0$	$\frac{\partial \bar{Q}}{\partial \varphi} = \pm$

Using the results from lemma 4 and lemma 1 we can aggregate the full effect in competition depending on how passive investors engage in institutional activism. □

B Appendix: Auxiliary proofs

B.1 Proof of $1 - \mathbb{E}[\rho(F_x)|c_X = E] = \mathbb{E}[\rho(F_x)|c_X = I]$

Proof. Recall that $F_x = \lambda + N_X^F + \kappa(1-\lambda)(2\gamma-1)$ when $c_X = E$ and $F_x = \lambda + N_X^F - \kappa(1-\lambda)(2\gamma-1)$ when $c_X = I$.

$$1 - \mathbb{E}[\rho(F_x)|c_X = E] = \mathbb{E}\left[\frac{\phi\left(\frac{N_X^F+2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}{\phi\left(\frac{N_X^F}{\sigma_N}\right)+\phi\left(\frac{N_X^F+2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}\right] = \int_{-\infty}^{\infty}\frac{\phi\left(\frac{n+2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}{\phi\left(\frac{n}{\sigma_N}\right)+\phi\left(\frac{n+2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}\phi\left(\frac{n}{\sigma_N}\right)dn$$

$$\overset{x=n+\kappa(1-\lambda)(2\gamma-1)}{=\!=\!=} \int_{-\infty}^{\infty}\frac{\phi\left(\frac{x+\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}{\phi\left(\frac{x-\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)+\phi\left(\frac{x+\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}\phi\left(\frac{x-\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)dx$$

$$\mathbb{E}[\rho(F_x)|c_X = I] = \mathbb{E}\left[\frac{\phi\left(\frac{N_X^F-2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}{\phi\left(\frac{N_X^F}{\sigma_N}\right)+\phi\left(\frac{N_X^F-2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}\right] = \int_{-\infty}^{\infty}\frac{\phi\left(\frac{n-2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}{\phi\left(\frac{n}{\sigma_N}\right)+\phi\left(\frac{n-2\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}\phi\left(\frac{n}{\sigma_N}\right)dn$$

$$\overset{x=n-\kappa(1-\lambda)(2\gamma-1)}{=\!=\!=} \int_{-\infty}^{\infty}\frac{\phi\left(\frac{x-\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}{\phi\left(\frac{x-\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)+\phi\left(\frac{x+\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)}\phi\left(\frac{x+\kappa(1-\lambda)(2\gamma-1)}{\sigma_N}\right)dx$$

Therefore, $1 - \mathbb{E}[\rho(F_x)|c_X = E] = \mathbb{E}[\rho(F_x)|c_X = I]$. □

B.2 Additional proofs for Proposition A.2

Proof.

$$\mathbb{E}[\text{Ret}|(S^E, S^E)] = (2\gamma-1)\xi(2\pi_{EE} - \pi_{EI} - \pi_{IE}) + (1-(2\gamma-1)\xi)(\pi_{EE} - \pi_{II})$$

$\mathbb{E}[\text{Ret}|(S^E, S^E)]$ is positive because both $\pi_{EE} - \pi_{II}$ and $2\pi_{EE} - \pi_{EI} - \pi_{IE}$ are positive, see below.

$$\pi_{EE} - \pi_{II} = \frac{\left(-q_{EE}^2 - q_{EE} + 18\right)\left((a-c_E)^2 - (a-c_I)^2\right) + (a-c_I)^2\left(-q_{EE}^2 - q_{EE} + q_{II}^2 + q_{II}\right)}{144b} > 0.$$

$$\begin{aligned}2\pi_{EE} - \pi_{EI} - \pi_{IE} = \frac{1}{144b}\Big((a-c_I)(c_I-c_E)\Big(2\Big(1-q_{EE}^2\Big) + (q_{IE}-q_{EE}) + 6(1-q_{EI}) + 4q_{IE} + 28\Big) \\ + 2(a-c_E)(a-c_I)(q_{IE}(q_{EI}-q_{EE}) + q_{EE}(q_{IE}-q_{EE})) \\ + (a-c_E)(a-c_I)(q_{IE}-q_{EE}) + (a-c_E)^2(q_{EI}-q_{EE}) \\ + (c_I-c_E)^2(2(1-q_{EE}^2) + (1-q_{EE}) + 6(1-q_{EI}))\Big) > 0.\end{aligned}$$

□

B.3 Proof of Variance Components

We are able to sign analytically the following components of the variance of returns:

$$\frac{\partial \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N)}{\partial \hat{\lambda}} < 0 \qquad \frac{\partial \Pi(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial \hat{\lambda}} < 0 \qquad \frac{\partial \Pi_2(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial \hat{\lambda}} < 0$$

$$\frac{\partial \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N)}{\partial \kappa} > 0 \qquad \frac{\partial \Pi(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial \kappa} > 0 \qquad \frac{\partial \Pi_1(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial \kappa} > 0 \qquad \frac{\partial \Pi_2(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial \kappa} > 0$$

$$\frac{\partial \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N)}{\partial \gamma} > 0 \qquad \frac{\partial \Pi(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial \gamma} > 0$$

The proofs for $\frac{\partial \xi(\hat{\lambda},\gamma,\kappa,\sigma_N)}{\partial \hat{\lambda}} < 0$ and $\frac{\partial \Pi(\hat{\lambda},\gamma,\kappa,\bar{\varphi})}{\partial \hat{\lambda}} < 0$ are available in A.2 for the remaining terms we have:

$$\frac{\partial \Pi_2(\hat{\lambda}, \gamma, \kappa\bar{\varphi})}{\partial \lambda} = \frac{5}{144b}\left(\left((a-c^E)^2 - (a-c^I)^2\right)\left(\frac{\partial q_{EI}}{\partial \lambda}\right) + (a-c^I)^2\left(\frac{\partial q_{EI}}{\partial \lambda} - \frac{\partial q_{IE}}{\partial \lambda}\right)\right) < 0$$

To sign the derivatives with respect to the other exogenous variables we first determine the partial derivatives of q_{EE}, q_{EI}, q_{IE} and q_{II} with respect to κ and γ as:

For $\zeta = 1$

$$\frac{\partial q_{EI}}{\partial \kappa} \geq \frac{\partial q_{II}}{\partial \kappa} \geq 0 \geq \frac{\partial q_{IE}}{\partial \kappa} \geq \frac{\partial q_{EE}}{\partial \kappa}$$

$$\frac{\partial q_{EI}}{\partial \gamma} \geq \frac{\partial q_{II}}{\partial \gamma}\frac{\partial q_{IE}}{\partial \gamma} \geq \frac{\partial q_{EE}}{\partial \gamma} \qquad \frac{\partial q_{EI}}{\partial \gamma} \geq 0 \geq \frac{\partial q_{EE}}{\partial \gamma}$$

For $\zeta = 0$or $\zeta = -1$

$$\frac{\partial q_{EI}}{\partial \kappa} = \frac{\partial q_{II}}{\partial \kappa} = 0 \geq \frac{\partial q_{IE}}{\partial \kappa} \geq \frac{\partial q_{EE}}{\partial \kappa}$$

$$\frac{\partial q_{EI}}{\partial \gamma} = \frac{\partial q_{II}}{\partial \gamma} = 0 \geq \frac{\partial q_{EE}}{\partial \gamma} \qquad \frac{\partial q_{IE}}{\partial \gamma} \geq \frac{\partial q_{EE}}{\partial \gamma}$$

The derivatives with respect to κ are:

$$\frac{\partial \Pi(\hat{\lambda}, \gamma, \kappa\bar{\varphi})}{\partial \kappa} = \frac{1}{144b}\left((a-c^E)^2\left(5\frac{\partial q_{EI}}{\partial \kappa} - (1+2q_{EE})\frac{\partial q_{EE}}{\partial \kappa}\right) - (a-c^I)^2\left(5\frac{\partial q_{IE}}{\partial \kappa} - (1+2q_{II})\frac{\partial q_{II}}{\partial \kappa}\right)\right) > 0$$

$$\begin{aligned}\frac{\partial \Pi_1(\hat{\lambda}, \gamma, \kappa\bar{\varphi})}{\partial \kappa} =& \frac{1}{144b}\Big((a-c^E)\left(5(c^I-c^E)+a-c^I\right)\frac{\partial q_{EI}}{\partial \kappa} + (a-c^E)(a-c^I)(1-q_{IE})\frac{\partial q_{EI}}{\partial \kappa}. \\ & - (a-c^I)(a-c^E)(3+q_{EI})\frac{\partial q_{IE}}{\partial \kappa} + (a-c^I)^2(1+2q_{II})\frac{\partial q_{II}}{\partial \kappa}\Big) > 0\end{aligned}$$

$$\frac{\partial \Pi_2(\hat{\lambda}, \gamma, \kappa\bar{\varphi})}{\partial \kappa} = \frac{5}{144b}\left((a-c^E)^2\left(\frac{\partial q_{EI}}{\partial \kappa}\right) - (a-c^I)^2\left(\frac{\partial q_{IE}}{\partial \kappa}\right)\right) > 0$$

$$\frac{\partial \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N)}{\partial \kappa} = 2(2\gamma - 1)(1 - \lambda)(2(2\gamma - 1)\kappa(1 - \lambda) + N_X^F) \frac{\exp\left(\frac{(N_X^F - 2(2\gamma-1)\kappa(\lambda-1))^2 + (N_X^F)^2}{2\sigma_N^2}\right)}{\sigma_N^2 \left(e^{\frac{(N_X^F - 2(2\gamma-1)\kappa(\lambda-1))^2}{2\sigma_N^2}} + e^{\frac{(N_X^F)^2}{2\sigma_N^2}}\right)^2} > 0$$

$$\begin{aligned} \frac{\partial \Pi(\hat{\lambda}, \gamma, \kappa, \bar{\varphi})}{\partial \gamma} = \frac{1}{144b} \Bigg(& \Big((a - c^E)^2 - (a - c^I)^2 \Big)(2q_{EE} + 1)\left(-\frac{\partial q_{EE}}{\partial \gamma}\right) + 5\Big((a - c^E)^2 - (a - c^I)^2\Big)\frac{\partial q_{EI}}{\partial \gamma} \\ & + (a - c^I)^2(2q_{II} - 4)\left(\frac{\partial q_{II}}{\partial \gamma} - \frac{\partial q_{EE}}{\partial \gamma}\right) + (a - c^I)^2\left(-\frac{\partial q_{EE}}{\partial \gamma}\right)(2q_{EE} - 2q_{II} + 5) \\ & + 5(a - c^I)^2\left(\frac{\partial q_{EI}}{\partial \gamma} - \frac{\partial q_{IE}}{\partial \gamma} + \frac{\partial q_{II}}{\partial \gamma}\right)\Bigg) > 0 \end{aligned}$$

$$\frac{\partial \xi(\hat{\lambda}, \gamma, \kappa, \sigma_N)}{\partial \gamma} = 4\kappa(1 - \lambda)(2(2\gamma - 1)\kappa(1 - \lambda) + N_X^F) \frac{\exp\left(\frac{(N_X^F - 2(2\gamma-1)\kappa(\lambda-1))^2 + (N_X^F)^2}{2\sigma_N^2}\right)}{\sigma_N^2 \left(e^{\frac{(N_X^F - 2(2\gamma-1)\kappa(\lambda-1))^2}{2\sigma_N^2}} + e^{\frac{(N_X^F)^2}{2\sigma_N^2}}\right)^2} > 0$$

B.4 Proof of Lemma 1

We begin by writing the derivative with respect to λ

$$\frac{\partial \bar{Q}}{\partial \lambda} = \frac{1}{24b}\left((a - c^E)\left(\frac{\partial q_{EE}}{\partial \lambda} + \frac{\partial q_{EI}}{\partial \lambda}\right) + (a - c^I)\left(\frac{\partial q_{IE}}{\partial \lambda} + \frac{\partial q_{II}}{\partial \lambda}\right)\right)$$

For the case that passive investors vote optimally:

$$\left(\frac{\partial q_{EE}}{\partial \lambda} + \frac{\partial q_{EI}}{\partial \lambda}\right) = \left(\frac{\partial q_{IE}}{\partial \lambda} + \frac{\partial q_{IE}}{\partial \lambda}\right) = \frac{-2}{\varphi} < 0$$

For the case that passive investors do not vote or vote with management:

$$\left(\frac{\partial q_{EE}}{\partial \lambda}\right) > 0 \qquad \left(\frac{\partial q_{IE}}{\partial \lambda}\right) > 0$$

For the derivative with respect to κ

$$\frac{\partial \bar{Q}}{\partial \kappa} = \frac{1}{24b}\left((a - c^E)\left(\frac{\partial q_{EE}}{\partial \kappa} + \frac{\partial q_{EI}}{\partial \kappa}\right) + (a - c^I)\left(\frac{\partial q_{IE}}{\partial \kappa} + \frac{\partial q_{II}}{\partial \kappa}\right)\right)$$

For the case that passive investors vote optimally:

$$\left(\frac{\partial q_{EE}}{\partial \kappa} + \frac{\partial q_{EI}}{\partial \kappa}\right) = \left(\frac{\partial q_{IE}}{\partial \kappa} + \frac{\partial q_{IE}}{\partial \kappa}\right) = \frac{-2}{\varphi} = 0$$

For the case that passive investors do not vote or vote with management:

$$\left(\frac{\partial q_{EE}}{\partial \kappa}\right) < 0 \qquad \left(\frac{\partial q_{IE}}{\partial \kappa}\right) < 0$$

For the derivative with respect to γ

$$\frac{\partial \bar{Q}}{\partial \gamma} = \frac{1}{24b}\left((c^I - c^E)\left(\frac{\partial q_{EE}}{\partial \gamma} + \frac{\partial q_{EI}}{\partial \gamma}\right) + (a - c^I)\left(\frac{\partial q_{EE}}{\partial \gamma} + \frac{\partial q_{EI}}{\partial \gamma} + \frac{\partial q_{IE}}{\partial \gamma} + \frac{\partial q_{II}}{\partial \gamma}\right)\right)$$

For the case that passive investors vote optimally:

$$\left(\frac{\partial q_{EE}}{\partial \gamma} + \frac{\partial q_{EI}}{\partial \gamma}\right) = \left(\frac{\partial q_{IE}}{\partial \gamma} + \frac{\partial q_{IE}}{\partial \gamma}\right) = \frac{-2}{\varphi} = 0$$

For the case that passive investors do not vote or vote with management:

$$\left(\frac{\partial q_{EE}}{\partial \gamma}\right) < 0 \qquad \left(\frac{\partial q_{EE}}{\partial \gamma} + \frac{\partial q_{IE}}{\partial \gamma}\right) < 0$$

For the derivative with respect to φ

$$\frac{\partial \bar{Q}}{\partial \varphi} = \frac{1}{24b}\left((a - c^E)\left(\frac{\partial q_{EE}}{\partial \varphi} + \frac{\partial q_{EI}}{\partial \varphi}\right) + (a - c^I)\left(\frac{\partial q_{IE}}{\partial \varphi} + \frac{\partial q_{II}}{\partial \varphi}\right)\right)$$

For the case that passive investors vote optimally:

$$\left(\frac{\partial q_{EE}}{\partial \varphi} + \frac{\partial q_{EI}}{\partial \varphi}\right) = \left(\frac{\partial q_{IE}}{\partial \varphi} + \frac{\partial q_{IE}}{\partial \varphi}\right) = \frac{-2}{\varphi} = \frac{2\lambda}{\varphi^2} > 0$$

For the case that passive investors do not vote:

$$\left(\frac{\partial q_{EE}}{\partial \varphi}\right) = \frac{\gamma(2\gamma - 1)\kappa(1 - \lambda)}{\varphi^2} > 0 \qquad \left(\frac{\partial q_{IE}}{\partial \varphi}\right) = \frac{(1 - \gamma)(2\gamma - 1)\kappa(1 - \lambda)}{\varphi^2} > 0$$

For the case that passive investors vote with management:

$$\left(\frac{\partial q_{EE}}{\partial \varphi}\right) = \frac{\gamma(2\gamma - 1)\kappa(1 - \lambda) - l}{\varphi^2} \qquad \left(\frac{\partial q_{IE}}{\partial \varphi}\right) = \frac{(1 - \gamma)(2\gamma - 1)\kappa(1 - \lambda) - l}{\varphi^2}$$

C Appendix: On types of imperfect competition

In this section we devote to test our assumption of Cournot competition and linear pricing. We begin by identifying the most general conditions on the type of imperfect competition that we need to obtain our results. We show that only three conditions are needed for the main results of this paper, i.e. so that Proposition 2 holds. We then proceed to show that a Prisoner Dilemma game, quadratic demand function for Cournot competition, Bertrand competition and Stackelberg competition all satisfy such conditions.

The conditions are:

(C1) $V_X\Big(A^L(c_X), A^L(c_Y)\Big) + V_Y\Big(A^L(c_X), A^L(c_Y)\Big)$ is the highest aggregate payoff when $c_X = c_Y$

(C2)

$$V_X\Big(A^H(c_X), A^L(c_Y)\Big) > V_X\Big(A^L(c_X), A^L(c_Y)\Big) \text{ and } V_X\Big(A^H(c_X), A^H(c_Y)\Big) > V_X\Big(A^L(c_X), A^H(c_Y)\Big)$$

Moreover, efficient firms should win more given the same strategy adopted by two firms:

(C3) $$V_X\Big(A^L(E), A^L(c_Y)\Big) > V_X\Big(A^L(I), A^L(c_Y)\Big) \text{ where } c_Y \in \{E, I\}$$

To show that 2 holds we only need to show that $\Pi(\lambda, \gamma, \kappa) = \pi_{EE} + \pi_{EI} - \pi_{IE} - \pi_{II} > 0$ as long as conditions C1, C2 and C3. The proof is as follows:

Proof. To show that $\pi_{EE} > \pi_{IE}$ and $\pi_{EI} > \pi_{II}$ we show that, (i) first, when private owners cannot vote and the voting outcome is deterministic $V_{EE} > V_{IE}$ and $V_{EI} > V_{II}$ and (ii) second, when private owners can tilt the voting outcome towards A^H strategies making V a random variable the ranking is still preserved.

(i)

Define restriction $R1$ and $R2$ as follows:

$$R_1 : \quad \frac{\kappa(2\gamma^2 - \gamma)}{1 - \kappa\gamma + 2\kappa\gamma^2} > \lambda \qquad R_2 : \quad \frac{\kappa(1 - 3\gamma + 2\gamma^2)}{\kappa - 1 + \kappa\gamma(2\gamma - 3)} > \lambda$$

If the the voting outcome is deterministic. It fully depends on the parameter combination of γ and λ. Ignoring the masses ϕ_X and ϕ_Y from table 3 gives the following voting outcomes in each realization of the firms type:

Realization of the firms typed	Region I $\sim R_1$ & $\sim R_2$		Region II R_1 & $\sim R_2$		Region III R_1 & R_2	
	firm X	Firm Y	firm X	Firm Y	firm X	Firm Y
(E, E)	A^L	A^L	A^L	A^L	A^L	A^L
(E, I)	A^L	A^L	A^H	A^L	A^H	A^L
(I, E)	A^L	A^L	A^L	A^H	A^L	A^H
(I, I)	A^L	A^L	A^L	A^L	A^H	A^H

Table 6: Voting results conditional on the parameter region for each realization of the firms type.

In Region I, where the voting outcome is A^L for both firms, i.e., $A_X = A^L, A_Y = A^L$, the result directly follows from condition C3.

In Region II, where the voting outcome is (A^H,A^L) in the state of the world (E, I) and (A^L,A^H) in in the state of the world (I, E) the relationship is even stronger than in Region I. The reason is that the payoff in the state of the world (I, E) is even lower than the same state of the world of Region I, because condition C2 implies that $V_X(A^L(I), A^H(E)) < V_X(A^L(I), A^L(E))$. This occurs because firm X' market share is reduced by Firm Y's more aggressive strategy. Hence, for Region II, the inequality $V_{EE} > V_{IE}$ is even stronger than in Region I. Similarly, if firm X produces aggressively, while Firm Y does not, firm X's can steal market share and achieve the highest payoff possible.

Therefore, in for Region II, the inequality $V_{EI} > V_{II}$ is even stronger than in Region I as well. Therefore, $\Pi(\lambda, \gamma, \kappa)$ is larger in Region II and than in Region I.

The same occurs in Region III, since the only voting outcome that changes is (A^H, A^H) in the state of the world (I, I). We only need to show that $V_X(A^L(I), A^L(I)) > V_X(A^H(I), A^H(I))$ holds to have a stronger inequality than for Region II. For this case, condition C1 reveals this ranking directly since both firms can obtain a higher profit from colluding and reducing competition.

(ii)

When private owners do vote, the payoff $V_X\Big(A_X(c_X), A_Y(c_Y)\Big)$ becomes the random variable $\pi_X(c_X, c_Y)$ by using the probabilities from Equation (**??**). Given that $V_{EE} > V_{IE}$ and $q_{EE} < q_{IE}$ the ranking of $\pi_{EE} > \pi_{IE}$ is preserved since a low q_{EE} implies that the collusion outcome is more likely, which implies higher profits from condition C1. Since in the state of the world IE, firm X is of the inefficient type, any chance of deviating to a A^H strategy, as implied by high q_{IE} can only reduce the profits of the firm because of condition C3.

Using $V_{EI} > V_{II}$ and $q_{II} < q_{EI}$ the ranking of $\pi_{EI} > \pi_{II}$ is preserved as well. Note that a high q_{EI} implies higher likelihood of strategy A^H to be chosen in the state of the world EI, this leads to a higher profit for firm X because in this state of the world an A^H would be highly beneficial from condition C2. The increase from taking advantage of a A^H strategy in the EI case is larger than in the II case, as per $q_{II} < q_{EI}$, and is at the same time more beneficial for firm X because of condition C3.

In conclusion, the increased chances that strategies A^H get chosen because of the presence of private owners only accentuates the results $V_{EE} > V_{IE}$ and $V_{EI} > V_{II}$. Therefore, $\pi_{EE} > \pi_{IE}$ and $\pi_{EI} > \pi_{II}$ hold and $\Pi(\cdot) > 0$ □

C.1 Prisoner Dilemma

A Prisoner Dilemma game satisfies the conditions C1, C2 and C3 as a perfect example of the type of imperfect competition needed. It is mutually beneficial for both firms to cooperate and choose low aggressive strategies as in C1. However, this is never a Nash equilibrium because each firm can benefit more from selecting the high aggressive strategy to undercut the competing firm, steal market share, and achieve a higher payoff as in C2. Therefore, the Nash equilibrium is the one in which both firms engage in a highly aggressive strategy where the more efficient firm has a higher payoff as in C3.

C.2 Cournot competition quadratic

In this section we analyze a quadratic demand function to apply the conditions C1, C2 and C3 in a Cournot competition setting. Assume that the demand for the final good is quadratic in the form:

$$P(Q_X, Q_Y) = a - b(Q_X + Q_Y)^2$$

The payoff for each firm under this setting is of the form:

$$V_X(A_X(c_X), A_Y(c_Y)) = Q_X(P(A_X(c_X), A_Y(c_Y)) - c_X) \qquad V_Y(A_X(c_X), A_Y(c_Y)) = Q_Y(P(A_X(c_X), A_Y(c_Y)) - c_Y)$$

We define the strategies that firms can choose $(A_H(c), A_L(c))$ as the prices under a competitive setting and the monopolistic quantities respectively. A highly aggressive strategy can be interpreted as the quantity that arise from pure Cournot competition, whereas a low aggressive strategy results from setting monopolistic quantities to produce. Specifically:

$$A^H(c) = \frac{1}{2}\sqrt{\frac{a-c}{2b}} \qquad A^L(c) = \frac{1}{2}\sqrt{\frac{a-c}{3b}}$$

(C1)

We show that in this imperfect competition specification the payoff from a low aggressive strategy for an investor that holds both firms is higher than any other choice of strategy:

$$
\begin{aligned}
&V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^H(c), A^H(c)) + V_Y(A^H(c), A^H(c))\Big) \\
&= \frac{1}{36}\Big(8\sqrt{3} - 9\sqrt{2}\Big) b \Big(\frac{a-c}{b}\Big)^{3/2} > 0 \\
&V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^H(c), A^L(c)) + V_Y(A^H(c), A^L(c))\Big) \\
&= V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^L(c), A^H(c)) + V_Y(A^L(c), A^H(c))\Big) \\
&= \frac{1}{288}\Big(38\sqrt{3} - 45\sqrt{2}\Big) b \Big(\frac{a-c}{b}\Big)^{3/2} > 0
\end{aligned}
$$

(C2)

We show that in this imperfect competition specification the payoff from a high aggressive strategy is higher than a low aggressive strategy, conditional on the same choice by the competitor firm. This is based on the rationale that an informed investor holds a portfolio tilted towards one firm only if she considers that firm to be of an efficient and the competition to be of the inefficient type where $c^I > c^E$. The incremental profit from choosing a highly aggressive strategy is:

$$
\begin{aligned}
&V_X(A^H(c^E), A^H(c^I)) - \Big(V_X(A^L(c^E), A^H(c^I)))\Big) \\
&= \frac{1}{288}\sqrt{\frac{a-c^E}{b}}\Big(6\sqrt{2}\sqrt{a-c^E}\Big(\sqrt{a-c^E} - \sqrt{a-c^I}\Big) + \Big(48\sqrt{2} - 38\sqrt{3}\Big)(a-c^E) + \Big(9\sqrt{2} - 6\sqrt{3}\Big)(c^I - c^E)\Big) > 0 \\
&V_X(A^H(c^E), A^L(c^I)) - \Big(V_X(A^L(c^E), A^L(c^I)))\Big) \\
&= \frac{1}{288}\sqrt{\frac{a-c^E}{b}}\Big(4\sqrt{3}\sqrt{a-c^E}\Big(\sqrt{a-c^E} - \sqrt{a-c^I}\Big) + \Big(57\sqrt{2} - 44\sqrt{3}\Big)(a-c^E) + \Big(6\sqrt{2} - 4\sqrt{3}\Big)(c^I - c^E)\Big) > 0
\end{aligned}
$$

(C3)

We show that in this imperfect competition specification the payoff form being efficient firm is higher than that of being inefficient, given the same type for the competitor firm and when both firms choose the strategy of low aggressiveness as:

$$V_X(A^L(c^E), A^L(c^E)) - \left(V_X(A^L(c^I), A^L(c^E)))\right)$$
$$= \frac{10(a-c^I)\left(\sqrt{\frac{a-c^E}{b}} - \sqrt{\frac{a-c^I}{b}}\right) + (c^I - c^E)\left(8\sqrt{\frac{a-c^E}{b}} + \sqrt{\frac{a-c^I}{b}}\right)}{24\sqrt{3}} > 0$$
$$V_X(A^L(c^E), A^L(c^I)) - \left(V_X(A^L(c^I), A^L(c^I)))\right)$$
$$= \frac{10(a-c^E)\left(\sqrt{\frac{a-c^E}{b}} - \sqrt{\frac{a-c^I}{b}}\right) + (c^I - c^E)\left(\sqrt{\frac{a-c^E}{b}} + 8\sqrt{\frac{a-c^I}{b}}\right)}{24\sqrt{3}} > 0$$

Bertrand competition

In a Bertrand competition setting where firms compete on an homogeneous goods and have capacity constraints, the outcome is the same as in Cournot competition as in Kreps and Scheinkman (1983). We will therefore focus on a setting with differentiated goods that are substitutes. Assume the demand function for the products produced by firm X and firm Y are:

$$Q_X(P_X, P_Y) = a - bP_X + dP_Y \qquad Q_Y(P_X, P_Y) = a - bP_Y + dP_X$$

Where $d > 0$ representing that the goods produced are substitutes while $b > d$. The payoff for each firm under this setting is of the form:

$$V_X(A_X(c), A_Y(c)) = Q_X(A_X(c), A_Y(c))(P_X - c) \qquad V_X(A_X(c), A_Y(c)) = Q_X(A_X(c), A_Y(c))(P_Y - c)$$

We define the strategies that firms can choose $(A_H(c), A_L(c))$ as the prices under a competitive setting and the monopolistic prices respectively. A highly aggressive strategy can be interpreted as prices that arise from Bertrand competition, whereas a low aggressive strategy results from monopolistic prices. Specifically:

$$A^H(c) = \frac{a + bc}{2b - d} \qquad A^L(c) = \frac{a + (b - d)c}{2(b - d)}$$

From the restriction that prices have to be higher than the marginal cost, since otherwise one firm would exit the market in the short run, following conditions in the parameters set arise:

$$\frac{a + c^E(b - d)}{2(b - d)} - c^I > 0 \quad \rightarrow \quad a - (b - d)(2c^I - c^E) > 0$$
$$\frac{a + c(b - d)}{2(b - d)} - c > 0 \quad \rightarrow \quad a - (b - d)c > 0$$
$$\frac{a + c^E b}{2b - d} - c^I > 0 \quad \rightarrow \quad a + bc^E - c^I(2b - d) > 0$$

(C1)

We show that in this imperfect competition specification the payoff from a low aggressive strategy for an investor that holds both firms is higher than any other choice of strategy:

$$\begin{aligned}
&V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^H(c), A^H(c)) + V_Y(A^H(c), A^H(c))\Big) \\
&= \frac{d^2(a - c(b - d))^2}{2(b - d)(2b - d)^2} > 0 \\
&V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^H(c), A^L(c)) + V_Y(A^H(c), A^L(c))\Big) \\
&= V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^L(c), A^H(c)) + V_Y(A^L(c), A^H(c))\Big) \\
&= \frac{bd^2(a - c^E(b - d))^2}{4\big(2b^2 - 3bd + d^2\big)^2} > 0
\end{aligned}$$

(C2)

We show that in this imperfect competition specification the payoff from a high aggressive strategy is higher than a low aggressive strategy, conditional on the same choice by the competitor firm. This is based on the rationale that an informed investor holds a portfolio tilted towards one firm only if she considers that firm to be of an efficient and the competition to be of the inefficient type where $c^I > c^E$. The incremental profit from choosing a highly aggressive strategy is:

$$V_X(A^H(c^E), A^H(c^I)) - \Big(V_X(A^L(c^E), A^H(c^I))\Big) = \frac{bd^2(a - c^E(b-d))(a-(b-d)(2c^I - c^E))}{4\big(2b^2 - 3bd + d^2\big)^2} > 0$$

$$V_X(A^H(c^E), A^L(c^I)) - \Big(V_X(A^L(c^E), A^L(c^I))\Big) = \frac{d^2(a - c^E(b-d))(a + bc^E - c^I(2b-d))}{4(b-d)(2b-d)^2} > 0$$

(C3)

We show that in this imperfect competition specification the payoff form being efficient firm is higher than that of being inefficient, given the same type for the competitor firm and when both firms choose the strategy of low aggressiveness as:

$$\begin{aligned}
&V_X(A^L(c^E), A^L(c^E)) - \Big(V_X(A^L(c^I), A^L(c^E))\Big) \\
&= \frac{(c^I - c^E)((b-d)(2a-(b-d)(c^E + c^I)) + d(a - c^I(b-d)))}{4(b-d)} > 0 \\
&V_X(A^L(c^E), A^L(c^I)) - \Big(V_X(A^L(c^I), A^L(c^I))\Big) \\
&= \frac{(c^I - c^E)((b-d)(2a-(b-d)(c^E + c^I)) + d(a - c^E(b-d)))}{4(b-d)} > 0
\end{aligned}$$

Stackelberg competition

In this section we analyze the case were the firm Y is the leader firm that moves first and then the follower firm, firm X, move sequentially. The argument can be reversed to take firm Y's perspective. Assume that the demand for the final good is linear of the form:

$$P(Q_X, Q_Y) = a - b(Q_X + Q_Y)$$

The payoff for each firm under this setting is of the form:

$$V_X(A_X(c_X), A_Y(c_Y)) = Q_X(P(A_X(c_X), A_Y(c_Y)) - c_X) \qquad V_Y(A_X(c_X), A_Y(c_Y)) = Q_Y(P(A_X(c_X), A_Y(c_Y)) - c_Y)$$

We define the strategies that firms can choose $(A_H(c), A_L(c))$ as the prices under a stackelberg setting and the monopolistic quantities respectively. A highly aggressive strategy can be interpreted as the quantity that arise from Stackelberg competition, whereas a low aggressive strategy results from setting monopolistic quantities to produce. Given the best response of firm X, firm Y chooses to produce $\frac{a-c}{2b}$. Therefore, the strategies chosen are:

$$A^H(c) = \begin{cases} \frac{a-c}{2b}, & \text{for firm Y} \\ \frac{a-c}{4b}, & \text{for firm } X \end{cases} \qquad A^L(c) = \frac{a-c}{4b}$$

(C1)

We show that in this imperfect competition specification the payoff from a low aggressive strategy for an investor that holds both firms is higher than any other choice of strategy:

$$\begin{aligned}
&V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^H(c), A^H(c)) + V_Y(A^H(c), A^H(c))\Big) \\
&= \frac{(a-c)^2}{8b} > 0 \\
&V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^H(c), A^L(c)) + V_Y(A^H(c), A^L(c))\Big) \\
&= V_X(A^L(c), A^L(c)) + V_Y(A^L(c), A^L(c)) - \Big(V_X(A^L(c), A^H(c)) + V_Y(A^L(c), A^H(c))\Big) \\
&= \frac{(a-c)^2}{16b} > 0
\end{aligned}$$

(C2)

We show that in this imperfect competition specification the payoff from a high aggressive strategy is at least the same higher than a low aggressive strategy, conditional on the same choice by the competitor firm. This is based on the rationale that an informed investor holds a portfolio tilted towards one firm only if she considers that firm to be of an efficient and the competition to be of the inefficient type where $c^I > c^E$. The incremental profit from choosing a highly aggressive strategy is:

$$V_X(A^H(c^E), A^H(c^I)) - \Big(V_X(A^L(c^E), A^H(c^I)))\Big) = 0$$

$$V_X(A^H(c^E), A^L(c^I)) - \Big(V_X(A^L(c^E), A^L(c^I)))\Big) = 0$$

(C3)

We show that in this imperfect competition specification the payoff form being efficient firm is higher than that of being inefficient, given the same type for the competitor firm and when both firms choose the strategy of low aggressiveness as:

$$\begin{aligned}
&V_X(A^L(c^E), A^L(c^E)) - \Big(V_X(A^L(c^I), A^L(c^E)))\Big) \\
&= \frac{(c^I - c^E)(2a - c^E - c^I)}{8b} > 0 \\
&V_X(A^L(c^E), A^L(c^I)) - \Big(V_X(A^L(c^I), A^L(c^I)))\Big) \\
&= \frac{(c^I - c^E)(2a - c^E - c^I)}{8b} > 0
\end{aligned}$$