CS547 HW3 Group37

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Single Quadrant Case (1st quadrant)

Alternative Representation 1:

$$\mathbf{1}_{\mathbb{R}_+ \times \mathbb{R}_+}(x, y) \approx s_\varepsilon(s_\varepsilon(-s_\varepsilon(-x) + s_\varepsilon(y) - \frac{1}{2}) - \frac{1}{2})$$

$$M^{(1)} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \Big/ \varepsilon \quad B^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M^{(2)} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \Big/ \varepsilon \quad B^{(2)} = \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} \Big/ \varepsilon$$

$$M^{(3)} = [1 \quad 0] / \varepsilon \quad B^{(3)} = -\frac{1}{2\varepsilon}$$

Alternative Representation 2:

$$\mathbf{1}_{\mathbb{R}_+ \times \mathbb{R}_+}(x, y) \approx s_\varepsilon(s_\varepsilon(s_\varepsilon(x) - s_\varepsilon(-y) - \frac{1}{2}) - \frac{1}{2})$$

$$M^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Big/ \varepsilon \quad B^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M^{(2)} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \Big/ \varepsilon \quad B^{(2)} = \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} \Big/ \varepsilon$$

$$M^{(3)} = [1 \quad 0] / \varepsilon \quad B^{(3)} = -\frac{1}{2\varepsilon}$$

Alternative Representation 3:

$$\mathbf{1}_{\mathbb{R}_+ \times \mathbb{R}_+}(x, y) \approx s_\varepsilon(s_\varepsilon(-s_\varepsilon(-x) - s_\varepsilon(-y) + \frac{1}{2}) - \frac{1}{2})$$

$$M^{(1)} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Big/ \varepsilon \quad B^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M^{(2)} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} \Big/ \varepsilon \quad B^{(2)} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} \Big/ \varepsilon$$

$$M^{(3)} = [1 \quad 0] / \varepsilon \quad B^{(3)} = -\frac{1}{2\varepsilon}$$

Complements of Single Quadrants Case (1st quadrants 0)

Alternative Representation 1:

$$\mathbf{1}_{\mathbb{R}^2 \setminus (\mathbb{R}_+ \times \mathbb{R}_+)}(x,y) \approx s_\varepsilon(s_\varepsilon(-s_\varepsilon(x) - s_\varepsilon(y) + \frac{3}{2}) - \frac{1}{2})$$

$$M^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Big/ \varepsilon \quad B^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M^{(2)} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} \Big/ \varepsilon \quad B^{(2)} = \begin{bmatrix} \frac{3}{2} \\ 0 \end{bmatrix} \Big/ \varepsilon$$

$$M^{(3)} = \begin{bmatrix} 1 & 0 \end{bmatrix} / \varepsilon \quad B^{(3)} = -\frac{1}{2\varepsilon}$$

Alternative Representation 2:

$$\mathbf{1}_{\mathbb{R}^2 \setminus (\mathbb{R}_+ \times \mathbb{R}_+)}(x,y) \approx s_\varepsilon(s_\varepsilon(-s_\varepsilon(x) + s_\varepsilon(-y) + \frac{1}{2}) - \frac{1}{2})$$

$$M^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Big/ \varepsilon \quad B^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M^{(2)} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \Big/ \varepsilon \quad B^{(2)} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} \Big/ \varepsilon$$

$$M^{(3)} = \begin{bmatrix} 1 & 0 \end{bmatrix} / \varepsilon \quad B^{(3)} = -\frac{1}{2\varepsilon}$$