CS_547 HW4
Group 37
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Colab link: https://colab.research.google.com/github/052D/CS547_SP2021/blob/main/HW4/CS547_HW4.ipynb?authuser=1

Problem 1

$$f(m) = e^{\pi \cos(2mx)}$$
$$\frac{\partial f(m)}{\partial m} = -2\pi x \sin(2mx) e^{\pi \cos(2mx)}$$

substitute x=5 and m=10, we have

$$f'(10) = -10\pi \sin(100) e^{\pi \cos(100)} = 238.8589841$$

Problem 2

$$\phi_1 = \cos(2x) \quad \phi_2 = \cos(4x) \quad \phi_3 = \cos(8x) \quad L = e^{\pi x}$$
$$\phi_1' = -2\sin(2x) \quad \phi_2' = -4\sin(4x) \quad \phi_3' = -8\sin(8x) \quad L' = \pi e^{\pi x}$$

Now compute a_n

$$a_1 = m_1 x = 10 \times 5 = 50 \quad a_2 = m_2 \phi_1(a_1) = 9\cos(100) = 7.76087 \quad a_3 = m_3 \phi_2(a_2) = 8\cos(4a_2) = 7.451517$$

Compute δ_n

$$\delta_3 = L'(\phi_3(a_3))\phi_3'(a_3) = \pi e^{\pi \cos(8a_3)} \times -8\sin(8a_3) = -8\pi \sin(8a_3) e^{\pi \cos(8a_3)} = -0.08557955$$
$$\delta_2 = \delta_3 m_3 \phi_2'(a_2) = -32\delta_3 \sin(4a_2) = -0.9965439$$
$$\delta_1 = \delta_2 m_2 \phi_1'(a_1) = -18\delta_2 \sin(2a_1) = -9.083080635$$

(a)

$$\frac{\partial f_5}{\partial m_3}(10, 9, 8) = \delta_3 \phi_2(a_2) = \delta_3 \cos(4a_2) = -0.0797122$$

(b)

$$\frac{\partial f_5}{\partial m_2}(10, 9, 8) = \delta_2 \phi_1(a_1) = \delta_2 \cos(2a_1) = -0.8593386$$

(c)

$$\frac{\partial f_5}{\partial m_1}(10, 9, 8) = \delta_1 x = 5\delta_1 = -45.4154$$

Problem 3

ϕ_n is defined the same as question 2. Now compute a_n

$$a_1 = m_1 x + b_1 = 10 \times 5 + 11 = 61 \quad a_2 = m_2 \phi_1(a_1) + b_2 = 9\cos(122) + 19 = 11.199096 \quad a_3 = m_3 \phi_2(a_2) + b_3 = 8\cos(4a_2) + 18 = 23.49226$$

Compute δ_n

$$\delta_3 = L'(\phi_3(a_3))\phi_3'(a_3) = \pi e^{\pi \cos(8a_3)} \times -8\sin(8a_3) = -8\pi \sin(8a_3) e^{\pi \cos(8a_3)} = 191.22014$$
$$\delta_2 = \delta_3 m_3 \phi_2'(a_2) = -32\delta_3 \sin(4a_2) = -4449.151142$$
$$\delta_1 = \delta_2 m_2 \phi_1'(a_1) = -18\delta_2 \sin(2a_1) = 39939.30357$$

(a)

$$\frac{\partial f_5}{\partial m_3}(10, 11, 9, 19, 8, 18) = \delta_3 \phi_2(a_2) = \delta_3 \cos(4a_2) = 131.2788657$$

(b)

$$\frac{\partial f5}{\partial m2}(10,11,9,19,8,18) = \delta_2\phi_1(a_1) = \delta_2\cos(2a_1) = 3856.377793$$

(c)

$$\frac{\partial f5}{\partial m1}(10,11,9,19,8,18) = \delta_1 x = 5\delta_1 = 199696.5179$$

Problem 4

In [56…

```
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
```

Define the functions for this problem.

In [51…

```
def phi_n(n, x):
    return np.cos(2**n*x)


def f5(m1=10, m2=9, m3=8,
       b1 = 0, b2 = 0, b3 = 0,
       x=5):
    return np.exp(np.pi * phi_n(3, m3 * phi_n(2, m2 * phi_n(1, m1 * x + b1) + b2) + b3))
```

Perturb m_3

In [48…

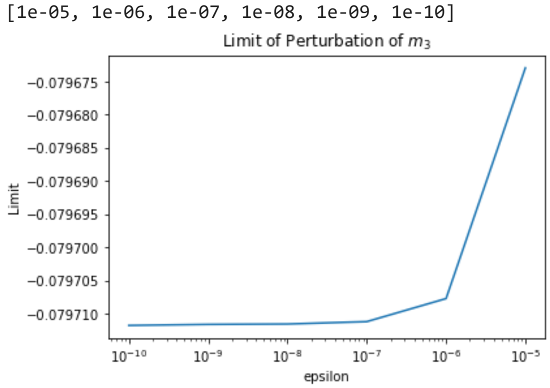
```
epsilon = [10.**item for item in np.arange(-5, -11, -1)]
display(epsilon)
res_m3 = np.ndarray((len(epsilon)))

for i, item in enumerate(epsilon):
    res_m3[i] = (f5(m3 = 8. + item) - f5())/item

plt.figure()
plt.plot(epsilon, res_m3)
plt.xlabel("epsilon")
plt.ylabel("Limit")
plt.xscale("log")
plt.title(f'Limit of Perturbation of $m_3$')
plt.show()
plt.close()

display(res_m3[3])
```

```
[1e-05, 1e-06, 1e-07, 1e-08, 1e-09, 1e-10]
```




```
-0.0797115824735517
```

It can be observed that the limit of the gradient $\frac{\partial f5}{\partial m3}$ at $x=5$ tends to be stable when $\epsilon \leq 10^{-8}$. Hence, the result is taken at $\epsilon = 10^{-8}$, being **-0.0797115824735517**.

Perturb m_2

In [49…

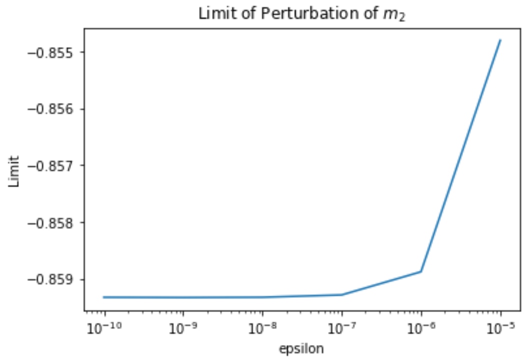
```
epsilon = [10.**item for item in np.arange(-5, -11, -1)]
display(epsilon)
res_m2 = np.ndarray((len(epsilon)))

for i, item in enumerate(epsilon):
    res_m2[i] = (f5(m2 = 9. + item) - f5())/item
```

```
plt.figure()
plt.plot(epsilon, res_m2)
plt.xlabel("epsilon")
plt.ylabel("Limit")
plt.xscale("log")
plt.title(f'Limit of Perturbation of $m_2$')
plt.show()
plt.close()

display(res_m2[3])
```

```
[1e-05, 1e-06, 1e-07, 1e-08, 1e-09, 1e-10]
```




```
-0.8593292973035904
```

It can be observed that the limit of the gradient $\frac{\partial f5}{\partial m2}$ at $x = 5$ tends to be stable when $\epsilon \leq 10^{-8}$. Hence, the result is taken at $\epsilon = 10^{-8}$, being **-0.8593292973035904**.

Perturb m_1

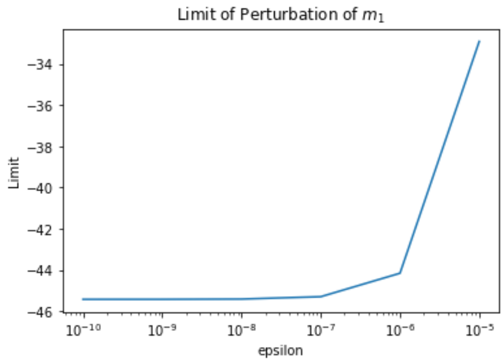
In [50…
```
epsilon = [10.**item for item in np.arange(-5, -11, -1)]
display(epsilon)
res_m1 = np.ndarray((len(epsilon)))

for i, item in enumerate(epsilon):
    res_m1[i] = (f5(m1 = 10. + item) - f5())/item


plt.figure()
plt.plot(epsilon, res_m1)
plt.xlabel("epsilon")
plt.ylabel("Limit")
plt.xscale("log")
plt.title(f'Limit of Perturbation of $m_1$')
plt.show()
plt.close()

display(res_m1[3])
```

```
[1e-05, 1e-06, 1e-07, 1e-08, 1e-09, 1e-10]
```




```
-45.40249018136766
```

It can be observed that the limit of the gradient $\frac{\partial f5}{\partial m1}$ at $x = 5$ tends to be stable when $\epsilon \leq 10^{-8}$. Hence, the result is taken at $\epsilon = 10^{-8}$, being **-45.40249018136766**.