浙江大学 2005-2006 学年 秋冬 学期期末考试

《计算理论》课程试卷

考试时间：120 分钟　开课学院：计算机学院　专业：＿＿＿＿＿＿

任课教师：＿＿＿＿＿＿＿＿　姓名：＿＿＿＿＿＿＿＿　学号：＿＿＿＿＿＿＿＿

题序	1	2	3	4	5	6	7	总分
得分								
评阅人								

Zhejiang University
Theory of Computation, Fall-Winter 2005
Final Exam

1. **(30%) Determine whether the following statements are true or false. If it is true write a ✓ otherwise a × in the bracket before the statement.**

   (a) (　) Language $\{a^m(bc)^n : m, n \in \mathbb{N}\}$ is not regular.

   (b) (　) Language $\{a^i b^j c^k \mid i, j, k \geq 0, i \geq j + k\}$ is context-free.

   (c) (　) Let $F = \{f : f$ be a primitive recursive function from $\mathbb{N}$ to $\mathbb{N}\}$, then 2^F (Power set of F) is uncountable.

   (d) (　) Let $L_1, L_2, \cdots, L_i, \cdots$ be all regular languages, so is $\cup_{i=1}^{\infty} L_i$.

   (e) (　) Suppose language L is context-free and L' is a regular, then $L^* L'^*$ is context-free.

   (f) (　) Every computable function is primitive recursive.

   (g) (　) The complement of every recursive enumerable language is recursive enumerable.

   (h) (　) $a^* b^* \cap c^* d^* = \emptyset^*$.

   (i) (　) Every regular language is recursively enumerable.

   (j) (　) Let L be a language and there is a Turing machine M halts on x for every $x \in L$, then L is decidable.

2. **(14%)** Decide whether the following languages are regular or not and provide a formal proof for your answer.

   (a) $L_1 = \{a^n b^m : m \equiv n \mod 2\}$

(b) $L_2 = \{w \in \{a,b\}^* : w \neq w^R\}$

3. **(18%)**

(a) Give a Context-Free Grammar that generates the language

$$L_3 = \{xy \mid x, y \in \{a,b\}^*, |x| = |y| \text{ and } x \text{ and } y^R \text{ differ in one position }\}.$$

For example, $abbbbaba, abbbbbbb \in L_3$, but $aababb \notin L_3$.

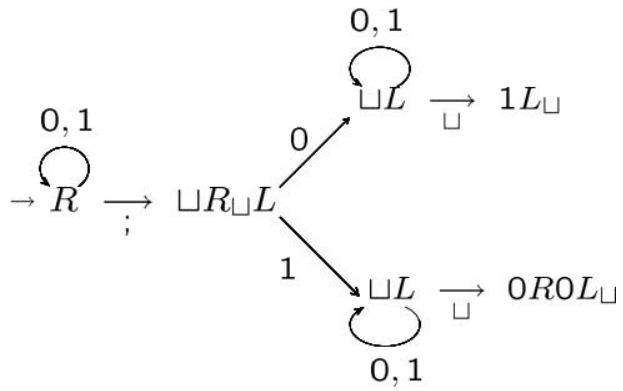
(b) Design a PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ accepting the language L_3.

Solution: (a)

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

	(q, σ, β)	(p, γ)
$K =$_____		
$\Sigma = \{a, b\}$		
$\Gamma =$_____		
$s =$_____		
$F =$_____		

4. **(16%)** Let the following Turing machine M computes $f(x, y)$, the alphabet is $\{0, 1, ;\}$. The head of M begins from the most left blank; $\sqcup$ is the symbol of blank; x and y are presented by binary strings respectively and separated with ;.



(a) Describe the key configurations when M started from the configuration $\triangleright\underline{\sqcup}10111;111$.

(b) Try to give the function $f(x, y)$ that M can compute.

5. **(12%)** Let $P(x, y)$ be primitive recursive predicate. Prove the following predicate

$$\exists y_{\leq u} P(x, y),\ \forall u \in \mathbb{N}$$

is also primitive recursive.

6. **(10%)** Show that the following language

$$H = \{\text{“}M\text{”} \mid M \text{ is a Turing Machine and halts on empty string}\}$$

is recursively enumerable. An informal description suffices.