浙江大学 2008-2009 学年 秋冬 学期

研究生《计算理论》课程期终考试试卷

开课学院：计算机学院　考试形式：闭卷，允许带 ________ 入场

考试时间: 2009 年 1 月 11 日，所需时间: 120 分钟，任课教师: ________

考生姓名：________________ 学号：________________ 专业：________________

题序	1	2	3	4	5	6	7	总分
得分								
评卷人								

Zhejiang University
Theory of Computation, Fall-Winter 2008
Final Exam

1. **(20%) Determine whether the following statements are true or false. If it is true write a T otherwise a F in the bracket before the statement.**
   (a) ( ) For all languages L_1, L_2 and L_3, if $L_1 \subseteq L_2 \subseteq L_3$ and both L_1 and L_3 are regular, then L_2 is also regular.
   (b) ( ) For a given context-free language L and a string x, the decision problem for whether $x \in \overline{L}$ is decidable.
   (c) ( ) The complement of every recursive enumerable language is necessarily nonrecursive enumerable.
   (d) ( ) Languages $\{$"M" : Turing machine M accepts at least 2009 distinct inputs$\}$ is recursive enumerable.
   (e) ( ) Let L be a language and there is a Turing machine M halts on x for every $x \in L$, then L is decidable.
   (f) ( ) For all languages L_1 and L_2, if L_1 is in $\mathcal{P}$ and $L_1 <_p L_2$, then L_2 is in $\mathcal{NP}$.
   (g) ( ) The class $\mathcal{NP}$ is closed under complementation.
   (h) ( ) Checking equivalence of two propositional formulas is in $\mathcal{NP}$.
   (i) ( ) If L is polynomial time reducible to a finite language, then L is in $\mathcal{P}$.
   (j) ( ) If SAT reduces to a language L, then L is $\mathcal{NP}$-complete.

2. **On Regular Languages**
   (12%) Consider two deterministic finite automata $M_1 = (K_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (K_2, \Sigma, \delta_2, p_1, F_2)$, where

$$\Sigma = \{a, b\}$$

$$K_1 = \{q_1, q_2\} \text{ and } K_2 = \{p_1, p_2, p_3\}$$

$$F_1 = \{q_2\} \text{ and } F_2 = \{p_3\}$$

and δ_1 and δ_2 are the functions tabulated below.

δ_1	a	b
q_1	q_2	q_1
q_2	q_1	q_2

δ_2	a	b
p_1	p_1	p_3
p_2	p_3	p_2
p_3	p_2	p_1

Use the Cartesian product(笛卡尔积) construction to produce deterministic automata accepting the union of the two languages accepted by these automata.

3. **On Context-free Languages (18%)**

   (a) Give a context-free grammar for language:

$$L_1 = \{a^m b^n c w w^R | m, n \in \mathbb{N}, n \leq m \leq 2n, w \in \{a, b\}^*\}$$

   (b) Design a PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ accepting the language L_1.
   Solution: (b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

	(q, σ, β)	(p, γ)
$K =$_______		
$\Sigma = \{a, b, c\}$		
$\Gamma =$_______		
$s =$_______		
$F =$_______		

4. **On Turing Machines**
   (10%) Design a Turing Machine to decide the following language:

$$L_2 = \{ww^R | w \in \{a, b, c\}^*\}$$

   where the initial configuration in form of $\triangleright\underline{\sqcup}ww^R$.

5. **On Undecidability**
   (16%) Consider the language $L_3 = \{$"M"$|M$ when started on a blank tape eventually writes a 1 somewhere on the tape$\}$.
   (a) Show that L_3 is recursively enumerable.
   (b) Show that L_3 is not recursive by a reduction from the halting problem.

6. **On Primitive Recursive Functions**
   (10%) Let $g(x,y)$ be a primitive recursive function. Show that the function

$$e(x,y) = \begin{cases} 1, & \text{if } \exists t_{(0 \le t < y)}(g(x,t) = 0) \\ 0, & \text{otherwise} \end{cases}$$

   is primitive recursive.

7. **On $\mathcal{P}$ and $\mathcal{NP}$ Problems**
   (14%) The **clique** problem is defined as follows: given a graph $G = (V, E)$ with n vertices, is there a set of k vertices such that there is edge between any two vertices in the set?

   (a) Prove that **clique** problem is $\mathcal{NP}$ Problem.

   (b) Prove that **clique** problem is $\mathcal{NP}$-complete.

   For showing hardness, you can assume that the VERTEX-COVER problem is $\mathcal{NP}$-complete.