浙江大学 2014-2015 学年 秋冬 学期

研究生《计算理论》课程期终考试试卷

考试形式：闭卷, 考试时间: 2015 年 1 月20日，所需时间:120 分钟

学号：__________ 姓名：__________ 专业：__________ 任课教师: 金小刚

题序	1	2	3	4	5	6	7	总分
得分								
评卷人								

Zhejiang University
Theory of Computation, Fall-Winter 2014
Final Exam

1. **(24%) Determine whether the following statements are true or false. If it is true write a $\bigcirc$ otherwise a $\times$ in the bracket before the statement.**

   (a) ( ) The complement of any finite language is recursive.

   (b) ( ) Let L be any language. Then the equivalence class $[e]$ respect to language L (i. e. $x \approx_L y$, if for all $z \in \Sigma^*$, $xz \in L$ iff $yz \in L$) that either contains no other strings, or contains infinitely many strings.

   (c) ( ) Let A be a context-free language and $B \subseteq A$, then B is context-free.

   (d) ( ) The language $\{$"M_1""M_2" $\mid M_1$ is a PDA and M_2 is a DFA and $L(M_1) \subseteq L(M_2)$ $\}$ is recursive, where "M_1" and "M_2" are encodings of PDA M_1 or DFA M_2, just as Turing machine's encoding.

   (e) ( )There's a function φ such that φ can be computed by some Turing ma chines, yet φ is not a primitive recursive function.

   (f) ( ) Let A and B be two disjoint, recursively enumerable languages. If $\overline{A \cup B}$ is also be recursively enumerable, then both A and B are decidable.

   (g) ( ) Let A be a recursive language and B be a recursively enumerable language, then $A \oplus B$ is recursively enumerable, where $A \oplus B = (A - B) \cup (B - A)$.

   (h) ( ) Let $H_e = \{$"M"$\mid$ Turing machine$M$ halts on empty string$\}$ and τ_1 and τ_2 are two recursive function. If $H_e \leq_{\tau_1} L$ and $H_e \leq_{\tau_2} \overline{L}$, then L is recursive enumerable but not recursive.

   (i) ( ) There are some languages that cannot be semi-decided by any Turing machine.

   (j) ( ) A language L is recursive if and only if it is Turing-enumerable.

   (k) ( ) For any languages A, B and C. If $A \leq_p C$, $B \leq_p C$ and $C \in \mathbb{P}$, then $A \oplus B \in \mathbb{P}$, where $A \oplus B = (A - B) \cup (B - A)$.

   (l) ( ) Let L be a language and $L \in \mathbb{NP}$. If there is a polynomial time reduction from language L to SAT, then L is $\mathbb{NP}$-complete.

2. **(18%) On FA and Regular Languages**
   Say whether each of the following languages is regular or not regular? Prove your answers.
   (a) $L_1 = \{wxw^R | w \in \{a,b\}^+, \text{ and } x \in \{a,b\}\}$.
   (b) $L_2 = \{wxw^R | w \in \{a,b\}^+, \text{ and } x \in \{a,b\}^+\}$.

3. **(18%) On PDA and Context-Free Languages**
   Let $L_3 = \{a^i b^j c^k | i,j,k \in \mathbb{N} \text{ and } j \leq i+k\}$.
   (a) Give a context-free grammar for the language L_3.
   (b) Design a PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ accepting the language L_3.
   Solution: (a)

   (b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

$K = \{$____________________ $\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{$ $\}$

$s =$____________________

$F = \{$ ____________________ $\}$

(q, σ, β)	(p, γ)

4. **(12%) On Turing Machines**

   Construct a Turing machine that decides the following language:

   $$L_4 = \{uvcww^R | u, v, w \in \{a, b\}^*, \text{ and } |u| = |v|\}$$

   When describing the Turing machines above, you can use the elementary Turing machines described in textbook. Always assume that the Turing machines start computation from the configuration $\triangleright\underline{\sqcup}x$ where $x \in \{a, b, c\}^*$ is the input string.

5. **(12%) On Undecidability**

   Let

   $\mathbf{ODD}_{TM} = \{\text{“}M\text{”} | M \text{ is a TM, and } L(M) \text{ doesn't contain any string of odd length}\}.$

   Classify whether the languages $\mathbf{ODD}_{TM}$ and $\overline{\mathbf{ODD}_{TM}}$ are *recursive*, *recursively enumerable-but-not-recursive*, or *non-recursively enumerable*, respectively. Prove your answers, but you may not simply appeal to Rice's theorem.

6. **(16%) On $\mathbb{P}$ and $\mathbb{NP}$ Problems**
   Let 4-**SAT** be the satisfiability formulae in conjunctive normal form(CNF) with exactly four literals per clause, i.e.,

   $$4\text{-}\mathbf{SAT} = \{F | F \text{ is a Boolean formula in 4-CNF that is satisfiable}\}.$$

   (a) Give the definition of the class $\mathbb{P}$ and $\mathbb{NP}$.
   (b) Show that 4-**SAT** is a $\mathbb{NP}$ problem.
   (c) Show that 4-**SAT** is $\mathbb{NP}$ -Complete by giving a reduction from 3-**SAT**.

Enjoy your Spring Festival !