	Undergraduate Course
DISCRETE MATHEMATICS	Chukechen Honors College
HOMEWORK 9 SOL	Zhejiang University
	Fall-Winter 2014

Homework 9:

P527

3. For each of the following relations on the set $\{1,2,3,4\}$, decide whether it is reflexive, whether it is symmetric, whether is antisymmetric, and whether it is transitive.

(a) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$
(b) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$
(c) $\{(2,4),(4,2)\}$
(d) $\{(1,2),(2,3),(3,4)\}$
(e) $\{(1,1),(2,2),(3,3),(4,4)\}$
(f) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

Solution :

a) Neither reflexive nor irreflexive; Neither symmetric nor antisymmetric; transitive
b) Reflexive; symmetric; transitive
c) Irreflexive; symmetric; not transitive
d) Irreflexive; Anti-symmetric, not transitive
e) Reflexive; either symmetric and anti-symmetric; transitive
f) Irreflexive; neither symmetric nor antisymmetric; not transitive

7. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x,y) \in R$ if and only if

a) $x \neq y$
c) $x = y+1$ or $x = y-1$
h) $x \geq y^2$

Solution:

a) Irreflexive; symmetric; not transitive
c) Irreflexive; symmetric; not transitive($|x-y|=1$)
h) Neither reflexive nor irreflexive; antisymmetric; transitive

P543

13. Let R be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the matrix representing

a) R^{-1} **b)** $\overline{R}$ **c)** R^2

Solution:

a) $$M_{R^{-1}} = {M_R}^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

b) $$M_{\overline{R}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

c) $$M_{R^2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{(2)} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

P553

2. Let R be the relation $\{(a, b) \mid a \neq b\}$ on the set of integers. What is the reflexive closure of R?

Solution:

When we add all the pairs (x, x) to the given relation we have all of $\mathbb{Z} \times \mathbb{Z}$; in other words, we have $r(R) = \mathbb{Z} \times \mathbb{Z}$.

3. Let R be the relation $\{(a, b) \mid a$ adivides $b\}$ on the set of integers. What is the symmetric closure of R?

Solution:

$$s(R) = \{(a, b) \mid a \text{ adivides } b \text{ or } b \text{ adivides } a\}$$

19. Let R be the relation on the set $\{1, 2, 3, 4, 5\}$ containing the ordered pairs $(1, 3), (2, 4)$, $(3, 1), (3, 5)$, $(4, 3), (5, 1), (5, 2)$ and $(5, 4)$. Find

b) R^3 **f)** R^*

Solution:

b) $R^3 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 5), (3, 1), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2),$
$(4, 3), (4, 4), (5, 1), (5, 3), (5, 5)\}$

f) $R^* = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 1), (3, 2), (3, 3),$
$(3, 4), (3, 5), (4, 1), (4, 2)(4, 3), (4, 4), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)\}$

24. Suppose that the relation R is irreflexive. Is the relation R^2 necessarily irreflexive?

Solution : It is certainly possibly for R^2 to contain some pairs (a, a). For example, let $R = \{(1, 2), (2, 1)\}$.

27. Using Warshall's algorithm to find the transitive closures of the relation R in Exercise 25(b).

Solution : Warshall's algorithm compute M_{R^*} by efficiently computing $W_0 \to W_1 \to \cdots \to W_4 = M_{R^*}$.

$$W_0 = W_1 = M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \quad W_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad W_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$