Methodology ZJU
2018–19

This practice sheet is similar to the final examination in length, structure and style of questions. Use it for practice.

Practice Examination

There are 50 questions. Each question is worth 2% of the grade.

A. Decision theory and game theory

Consider the following decision problem. You are going to a party and you do not know whether it is formal and whether there will be dancing. You have to decide which shoes to wear. Your outcomes are as follows:

	informal		formal	
	dancing	no dancing	dancing	no dancing
high heels	1	3	6	7
sneakers	9	9	6	6
flip flops	7	10	0	2
boots	7	8	9	5

1. What does the Maximax rule recommend? Briefly justify your answer.
2. What does the Maximin rule recommend? Briefly justify your answer.
3. What does the Minimax Regret rule recommend? Briefly justify your answer.

Suppose your deciding between buying a woolen coat or a jacket for the winter. The winter will be either cold or mild. The utility of the woolen coat is 3 if the winter is cold, 0 if mild. The utility of the jacket is 2 if the winter cold, 1 if mild. The probability of a cold winter is $1/4$.

4. What is your expected utility that for buying a woolen coat? What is your expected utility of wearing a jacket?
5. What does the principle of maximizing expected utility recommend?
6. What probability of a cold winter would make the expected utility of wool coat and jacket equal?

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7. State the dominance principle.

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8. Li is going out for a walk in winter. She wonders whether to wear a scarf. She reasons as follows:

   Either I will catch a cold or I won't. If I catch a cold, wearing a scarf is useless. If I don't, wearing a scarf is superfluous. Either way, I shouldn't wear a scarf.

   What's wrong with Li's reasoning?

Xia and Qiang are both selling ice cream and food on the beach. They know that their respective outcomes are as follows, depending on which position they adopt. (Xia's outcomes are the left ones):

		Qiang		
		position 1	position 2	position 3
Xia	position 1	20, 20	10, 25	10, 15
	position 2	25, 15	15, 15	20, 10
	position 3	30, 10	10, 20	30, 15

9. Is there an action that Xia should select? Briefly justify your answer.
10. Is there an action that Qiang should select? Briefly justify your answer.

Consider the following game. Rowie's outcomes are the left ones.

		Collie	
		sell apples	sell peaches
Rowie	sell oranges	£5, £5	£11, £4
	sell pears	£4, £14	£10, £12

11. Does the game have a unique solution? If yes, why? If no, why not?
12. What is special about that game?

B. Probability theory

I toss three fair coins once. What is the probability that I get:

13. Three tails.
14. Exactly two tails.
15. At least one tail.

Let the probability that it rains be $7/10$, the probability that it's cold be $6/10$ and the probability that it's cold and it rains be $5/10$. What is the probability that:

16. It doesn't rain.
17. It's cold and it does not rain.
18. It's rain or it's cold.

19. It's cold or it rains but not both.

Suppose I have the following colony of pets. *Dogs*: 12 out of which 3 long-haired, the rest short-haired. *Sheep:* 8 out of which 4 long-haired, the rest short-haired. I pick up a pet at random. Give the following probabilities.

20. It's short-haired *conditional on* its being a sheep.
21. It's long-haired *conditional on* its being a dog.
22. It's a dog *conditional on* its being short-haired.

Suppose that $1/10$ of the population plays the guitar. Suppose the probability of being in a band conditional on playing the guitar is $1/2$ and the probability of being in a band conditional on not playing the guitar is $1/9$.

23. What is the probability that a randomly selected person is in a band?
24. Use Bayes's theorem to calculate the probability that somebody plays the guitar conditional on their being in a band.

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25. Suppose I have a bucket containing two types of coins: two-headed coins and two-tailed coins. I pick a coin from the bucket at random. I flip it. Is the probability that the coin falls on heads $1/2$, or it is 0 or 1 but we cannot tell which? Briefly justify your answer.
    5 sentences maximum. If you are not confident writing in English, you can write your answer in Chinese.

C. Set theory and mereology

Write down the union ($\cup$) of these sets:

26. {Alberta, Ontario, Quebec} and {Manitoba, Saksatchewan}.
27. $\{x : x$ weighs between 1 and 5 lbs$\}$ and $\{x : x$ weighs between 3 and 7 lbs$\}$.

Write down the intersection ($\cap$) of these sets:

28. {Havana, Kingston, Port-au-Prince} and {Kingston, Havana}.
29. $\{x : x$ weights between 1 and 5 lbs$\}$ and $\{x : x$ weighs between 3 and 4 lbs$\}$.

Say whether the following are true. (Recall that $\in$ stands for membership, $\subseteq$ for inclusion, $\varnothing$ for the empty set and $\mathcal{P}(X)$ for the power set of a set X.)

30. $\{b, c\} \subseteq \{a, b, c\}$.
31. $\{3\} \in \{x : x$ is a natural number greater than 1 and smaller than $9\}$.
32. $\{2\} \in \mathcal{P}(\{x : x$ is a natural number greater than 1 and smaller than $9\})$.
33. $\varnothing \in \{a, b, c\}$.

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34. How do we define an infinite set?
35. Are the natural numbers a subset of the real numbers?
36. Can we pair the integers with the natural numbers? If so, how? If not, why not?
37. Is the power set of the set of natural numbers finite, countably finite, or uncountably finite? Briefly justify your answer.

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38. Answer **ONE** question below.
    Be brief: 10 sentences maximum. If you are not confident writing in English, you can write your answer in Chinese.
    (a) Present the paradox of the Ship of Theseus.

    OR

    (a) Present one solution to the paradox of the Statue and the Lump. (Only present one solution. Do not argue for it, simply explain what the solution says.)

D. Truth and reference

39. What is a analytic truth?
40. What is a necessary truth?
41. What is an empirical (*a posteriori*) truth?

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42. Are logical truths a priori or a posteriori? Are they analytic or synthetic? Are they necessary or contingent?
43. Give a standard example of a synthetic *a priori* truth. (Whether there are such truths is debated; give an example that some philosophers take to be a synthetic a priori truth.)
44. Define contingency in terms of truths at possible worlds.

Say whether the following implications are correct:

45. If p is false then p is not necessary.
46. If p is not possible then not-p is true.
47. If not-p is possible then p is possible.

Consider the following sentence:

Someone brought food to everyone.

48. State at least two different readings of this sentence using parentheses or logical notation.
49. Describe a scenario in which all of your proposed readings are true.
50. Describe a scenario in which one of the sentence's readings is true and another false. Say which reading is true and which reading is false in that scenario. (To answer this question it is useful to give a label or number to each reading.)

Model answers

Decision theory and game theory

1. Flip flops. It has the highest best possible outcome (10).
2. Sneakers. It has the highest worst possible outcome (6).
3. Boots. It has the lowest possible regret (2).
4. The expected utility of buying a coat is:

$$\begin{aligned} eu(\text{coat}) &= Pr(\text{cold}) \times u(\text{coat \& cold}) + Pr(\text{mild}) \times u(\text{coat \& mild}) \\ &= \frac{1}{4} \times 3 + \frac{3}{4} \times 0 \\ &= \frac{3}{4} \end{aligned}$$

   The expected utility of wearing a jacket is:

$$\begin{aligned} eu(\text{jacket}) &= Pr(\text{cold}) \times u(\text{jacket \& cold}) + Pr(\text{mild}) \times u(\text{jacket \& mild}) \\ &= \frac{1}{4} \times 2 + \frac{3}{4} \times 1 \\ &= \frac{2}{4} + \frac{3}{4} = \frac{5}{4} \end{aligned}$$

5. Wearing a jacket.
6. Let x be the probability that the winter is cold. We have:

$$\begin{aligned} eu(\text{coat}) &= Pr(\text{cold}) \times u(\text{coat \& cold}) + Pr(\text{mild}) \times u(\text{coat \& mild}) \\ &= x \times 3 + (1 - x) \times 0 \\ &= 3x \end{aligned}$$

$$\begin{aligned} eu(\text{jacket}) &= Pr(\text{cold}) \times u(\text{jacket \& cold}) + Pr(\text{mild}) \times u(\text{jacket \& mild}) \\ &= x \times 2 + (1 - x) \times 1 \\ &= 2x + 1 - x \\ &= x + 1 \end{aligned}$$

   We are looking for the value of $x$that makes them equal:

$$\begin{aligned} 3x &= x + 1 \\ 3x - x &= 1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

   So a probability $^1/_2$ in the winter being cold would give both the same expected utility.

7. An act is dominated iff there is another act that is at least at good as every state, and better at some state. The dominance principle says that we should avoid dominated acts.
8. Li is ignoring the fact that the state of whether she will get a cold is dependent on the acts. That is, whether she will get a cold depends in part on whether she wears a scarf.
9. Position 2. For Qiang, position 3 is strongly dominated (by 2), so that is ruled out. Once Qiang position 3 is ruled out, for Xia position 3 is dominated, so that is ruled out too. Once that is ruled out, for Qiang position 1 is (weakly dominated). So Qiang should choose position 2.
10. Position 2. By the above, Qiang chosing positions 1 and 3 is ruled out (and Xia can know it). If only Qiang position 2 is left, for Xia positions 1 and 3 are dominated. So Xia should choose position 2.
11. Yes. Pears is dominated for Rowie and peaches is dominated for Collie so Rowie will sell oranges and Collie will sell peaches.
12. It is a prisonner's dilemma: the rational solution is one such that there is outcome that would be better for all players.

Probability theory

13. Three tails. $1/8$.
14. Exactly two tails. $3/8$.
15. At least one tail. $7/8$.

For questions 16-19 you should help yourself with a table:

		$7/10$*	$3/10$
		rain	no rain
$6/10$*	cold	$5/10$*	$1/10$
$4/10$	not cold	$2/10$	$2/10$

16. It doesn't rain. $3/10$
17. It's cold and it does not rain. $1/10$
18. It's rain or it's cold. $8/10$
19. It's cold or it rains but not both. $3/10$
20. It's short-haired *conditional on* its being a sheep. $4/8 = 1/2$
21. It's long-haired *conditional on* its being a dog. $3/12 = 1/4$
22. It's a dog *conditional on* its being short-haired. $9/13$
23. Answer in the weekly exercises.
24. Answer in the weekly exercises.
25. On the one hand, the coin in my hand is either two-tailed or two-headed. If the former, the probability that it falls on heads is 0, if the latter, it is 1. So given the facts about the coin, the probability is either 0 or 1. On the other hand, I cannot know what kind of coin it is. Given my information, it is equally likely to be either two-tailed or two-headed. So given my information, the probability is $1/2$. We should thus distinguish the objective probability - probability given the facts - from the subjective probability - probability given our information.

Set theory and mereology

26. {Alberta, Ontario, Quebec, Manitoba, Saksatchewan}.
27. $\{x : x$ weighs between 1 and 7 lbs$\}$.
28. {Havana, Kingston}.
29. $\{x : x$ weighs between 3 and 4 lbs$\}$.
30. False.
31. False.
32. True.
33. False.
34. An infinite set is a set that can be put in one-to-one correspondence with one of its proper subsets.
35. Yes.
36. Yes, we can pair them in the following way:

natural numbers	0	1	2	3	4	5	6	7	...
integers	0	1	-1	2	-2	3	-3	4	...

37. Uncountably infinite. By Cantor's power set theorem, the power set of a set cannot be put in one-to-one correspondence with it. So the power set of natural numbers cannot be put in one-to-one correspondence with the set of natural numbers, or a subset of it. So it is uncountable.
38. Example for (a). A the starting time we have a ship, S1. Through journeys its parts are replaced bit by bit. In the end we have a repaired ship, S2, that has none of the initial parts left. At the same time, we have kept the original parts and built a ship from them, the rebuilt ship S3. The problem is that the three following claims seem plausible:
    (a) S2 is identical with S1, because it is the result of continuous small repairs to a ship, and small repairs do not make a ship cease to exist.
    (b) S3 is identical with S1, because they are made of the same parts as the ship S1 was, put in the same order.
    (c) S2 and S3 are distinct, they are two distinct ships, located at different places.

    But the claims cannot all be true, because (by Leibniz's law) if S2 is identical to S1 and S1 is identical to S3, then S2 is identical to S1.

Truth and reference

39. A statement whose truth is guaranteed by the meaning of the words used to make it.
40. A statement whose truth could not have been otherwise.
41. A statement whose truth cannot be known prior to experience.
42. Logical truths are a priori, analytic and necessary.
43. The angles of a triangle add up to 180 degrees (two right angles).
    Another possible example. Nothing is red and green all over.
44. p is contingent iff p is true at some world and p is not true at some (other) world.
45. Correct.
46. Correct.

47. Incorrect.
48. Two readings:
    (a) (Some person x) (for every person y) (x brought food to y).
    In logical notation: $\exists x \forall y(Bxy)$.
    (b) (For every person x) (for some person y) (y brought food to x).
    In logical notation: $\forall x \exists y(Byx)$.
49. Wei brings food to everyone. In that scenario it is true that for some person x, that person brings food to every one (reading (a)). It is also true that for every person, someone brings food to them (reading (b)).
50. Wei brings food to half of the people, Ye brings food to the rest. In that scenario it is true that for every person, someone brings food to them (reading (b)). But it is not true that there is some person who brings food to everyone (reading (a)).