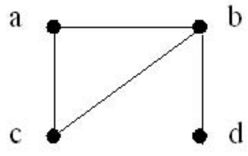
DISCRETE MATHEMATICS
HOMEWORK 11 SOL

Undergraduate Course
College of Computer Science
ZHEJIANG UNIVERSITY
Fall-Winter, 2013

HOMEWORK 11

P596

3-9. Determine whether the graph shown is a simple graph, a multigraph (and not a simple graph), a pseudograph (not a multigraph), a directed graph, or a directed multigraph (and not a directed graph).



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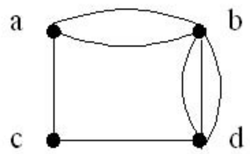

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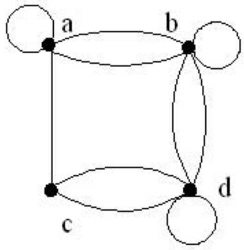

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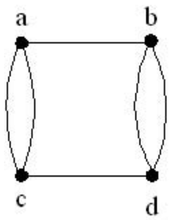

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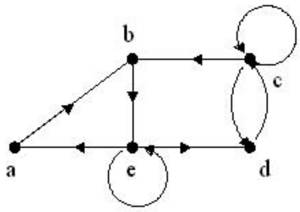

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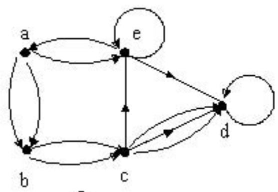

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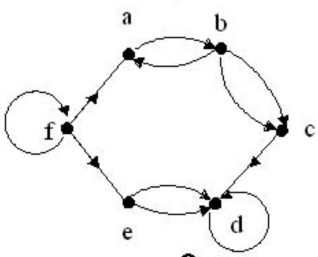

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Solution:

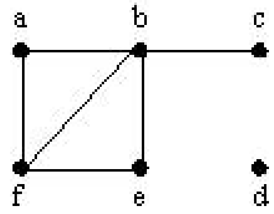
3) A simple graph
4) A multigraph
5) A pseudograph
6) A multigraph

7) A directed graph
8) A directed multigraph
9) A directed multigraph

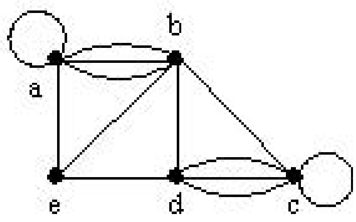
P608

1-3. In exercises 1-3 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

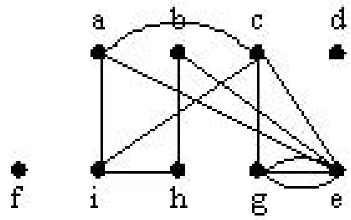
1.



2



3.



Solution :
1. $v = 6$, $e = 6$; $deg(a) = 2$, $deg(b) = 4$, $deg(c) = 1$, $deg(d) = 0$, $deg(e) = 2$, $deg(f) = 3$; c is the pendant vertex, d is the isolated vertex.
2. $v = 5$, $e = 13$; $deg(a) = 6$, $deg(b) = 6$, $deg(c) = 6$, $deg(d)$=5, $deg(e) = 3$; no pendant and isolated vertices.
3. $v = 9$, $e = 12$; $deg(a) = 3$, $deg(b) = 2$, $deg(c) = 4$, $deg(d) = 0$, $deg(e) = 6$, $deg(f) = 0$, $deg(g) = 4$, $deg(h) = 2$, $deg(i) = 3$; d and f are the isolated vertex.

5. Can a simple graph exist with 15 vertices each of degree 5 ?

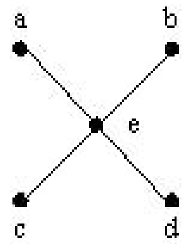
Solution: No, it can't. The sum of degrees of all vertices is even, but we can see

$$\sum_{v \in V} deg(v) = 15 \times 5 = 75$$

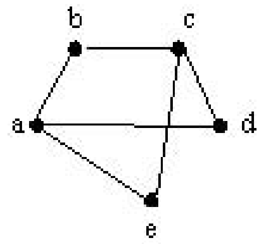
is odd.

21-25. In exercises 13-17 determine whether the graph is bipartite.

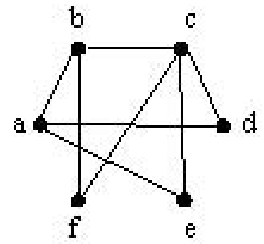
13.



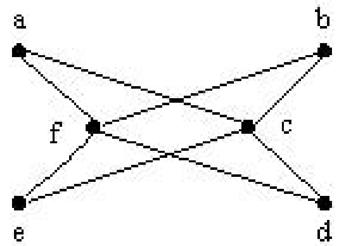
14.



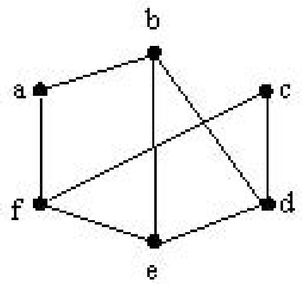
15.



16.



17.



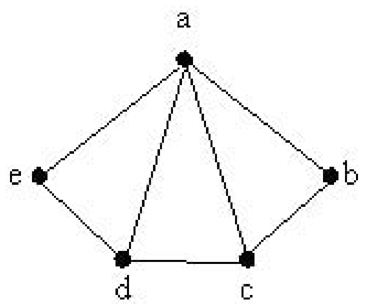
Solution:

The graphs in Exercises 13, 14 and 16 are bipartite.

The graphs in Exercises 15 and 17 are not bipartite.

34. How many edges does a graph have if it has vertices of degree 4, 3, 3, 2, 2 ? Draw such a graph.

Solution: The graph has $(4 + 3 + 3 + 2 + 2)/2 = 7$ edges.



53. The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. Find these graphs.

a) $\overline{K_n}$ b) $\overline{K_{m,n}}$ c) $\overline{C_n}$ d) $\overline{Q_n}$

Solution:

a) The graph with n vertices and no edges.

b) The disjoint union of K_m and K_n.
c) The graph with vertices $\{V_1, \cdots, V_n\}$ with an edge between V_i and V_j unless $i \equiv j + 1(\text{mod} n)$.
d) The graph whose vertices are represented by bit strings of length n with an edge between two vertices if the associated bit strings differ in more than on bit.

54. If G is a simple graph with 15 edges and $\overline{G}$ has 13 edges, how many vertices does G have ?

Solution: Assume G has n vertices, then $C(n, 2) = 2 \times (15 + 13) = n(n1)/2$, $n = 8$.

55. If the simple graph G has v vertices and e edges, how many edges does $\overline{G}$ have ?

Solution: $C(v, 2) - e$.