	Undergraduate Course
DISCRETE MATHEMATICS	College of Computer Science
HOMEWORK 1-3 SOL	**Zhejiang University**
	Fall-Winter 2014

HOMEWORK 1

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p16

4. Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

Express each of the following propositions as an English sentence.

f) $\neg p \to \neg q$

g) $\neg p \wedge \neg q$

h) $\neg p \vee (p \wedge q)$

Solution : f) If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

g) I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.

h) Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

10. Let p q and r be the propositions

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Writing the following propositions using p, q and r and logical connectives.

a) You get an A in this class, but You do not do every exercise in this book.

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

c) To get an A in this class, it is necessary for you to get an A on the final.

d) You get an A on the final, but you do not every exercise in this book; nevertheless, you get an A in this class.

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Solution : a) $r \wedge \neg q$ b) $p \wedge q \wedge r$ c) $r \rightarrow p$ d) $p \wedge \neg q \wedge r$ e) $(p \wedge q) \rightarrow r$ f) $r \leftrightarrow (q \vee p)$

20. Write each of the following statements in the form "if p then q" in English.
a) I will remember to send you the address only if you send me a E-Mail message.
b) To be a citizen of this country, it is sufficient that you were born in the United States.
c) If you keep your textbook, it will be a useful reference in your future course.
d) The Red Wings will win the Stanley Cup if their goalie plays well.
e) That you get the job implies that you had the best credentials.
f) The beach erodes whenever there is a storm.
g) It is necessary to have a valid password to log on the sever.

Solution: a) If I am to remember to send you the address then you will have to send me a E-Mail message.
b) If you were born in the United States, then you are a citizen of this country.
c) If you keep your textbook, then it will be a useful reference in your future course.
d) If their goaltender plays well, then the Red Wings will win the Stanley Cup.
e) If you get the job, then you had the best credentials.
f) If there is a storm, then the beach erodes.
g) If you log on the sever, then you have a valid password.

30. Construct a truth table for each of the following compound propositions
c) $p \oplus \neg q$

Solution :

p	q	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

38. Evaluate the each of the following expressions.
d) $11011 \vee 01010) \wedge (10001 \vee 11011)$

Solution : $11011 \vee 01010) \wedge (10001 \vee 11011) = 11011 \wedge 11011 = 11011$

HOMEWORK 2

P28

10. Show that each of the following implications is tautology by using truth table.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$

d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Solution : a)

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

d)

p	q	r	$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$	$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

12. Show the each implication in Exercise 10 is a tautology without using truth tables.

Solution :

a)
$$
\begin{aligned}
[\neg p \wedge (p \vee q)] \rightarrow q &\iff \neg[\neg p \wedge (p \vee q)] \vee q \\
&\iff [p \vee (\neg p \wedge \neg q)] \vee q \\
&\iff [(p \vee \neg p) \wedge (p \vee \neg q)] \vee q \\
&\iff (p \vee \neg q) \vee q \\
&\iff p \vee (\neg q \vee q) \\
&\iff p \vee \mathsf{T} \\
&\iff \mathsf{T}
\end{aligned}
$$

d)
$$
\begin{aligned}
[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r &\iff [(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \rightarrow r \\
&\iff [(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \rightarrow r \\
&\iff [(p \vee q) \wedge ((\neg p \wedge \neg q) \vee r)] \rightarrow r
\end{aligned}
$$

$$
\begin{aligned}
&\iff [(p \vee q) \wedge ((\neg(p \vee \ q) \vee r)] \to r \\
&\iff [((p \vee q) \wedge \neg(p \vee \ q)) \vee ((p \vee q) \wedge r)] \to r \\
&\iff [\mathsf{F} \vee ((p \vee q) \wedge r)] \to r \\
&\iff [(p \vee q) \wedge r] \to r \\
&\iff \neg[(p \vee q) \wedge r] \vee r \\
&\iff [\neg(p \vee q) \vee \neg r] \vee r \\
&\iff \neg(p \vee q) \vee (\neg r \vee r) \\
&\iff \neg(p \vee q) \vee \mathsf{T} \\
&\iff \mathsf{T}
\end{aligned}
$$

Convert the following formula into a full disjunctive normal form.

1) $((p \to q) \leftrightarrow (\neg q \to \neg p)) \wedge r$

2) $(\neg r \wedge (q \to p) \to (p \to (q \vee r))$

Solution :

1)
$$
\begin{aligned}
&((p \to q) \leftrightarrow (\neg q \to \neg p)) \wedge r \\
&\iff ((\neg p \vee q) \leftrightarrow (\neg\neg q \vee \neg p)) \wedge r \\
&\iff ((\neg p \vee q) \leftrightarrow (\neg\neg q \vee \neg p)) \wedge r \\
&\iff ((\neg p \vee q) \leftrightarrow (\neg p \vee q)) \wedge r \\
&\iff \mathsf{T} \wedge r \\
&\iff (\neg p \vee p) \wedge (\neg q \vee q) \wedge r \\
&\iff (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r) \\
&\iff m_1 \vee m_3 \vee m_5 \vee m_7 \\
&\iff \sum(1, 3, 5, 7)
\end{aligned}
$$

2)
$$
\begin{aligned}
&(\neg r \wedge (q \to p) \to (p \to (q \vee r)) \\
&\iff \neg r \wedge (\neg q \vee p) \to (\neg p \vee q \vee r) \\
&\iff \neg(\neg r \wedge (\neg q \vee p)) \vee (\neg p \vee q \vee r) \\
&\iff (r \vee (q \wedge \neg p)) \vee (\neg p \vee q \vee r) \\
&\iff (\neg p \wedge q) \vee \neg p \vee q \vee r \\
&\iff \neg p \vee q \vee r \\
&\iff m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_5 \vee m_6 \vee m_7 \\
&\iff \sum(0, 1, 2, 3, 5, 6, 7)
\end{aligned}
$$

P72

5. Construct an argument using rules of inference to show that the hypothesis " Randy works hard," "If Randy works hard, then he is a dull boy," and " If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job".

Solution : Let h be the proposition "Randy works hard," let d be the proposition "he is a dull boy," and let j be the proposition "he will got the job."

We are given premises $h, h \to d$ and $d \to \neg j$. We want to conclude $\neg j$.

Step	**Reason**
1. h	Hypothesis
2. $h \to d$	Hypothesis
3. d	Modus tollens using Steps 1 and 2
4. $d \to \neg j$	Hypothesis
5. $\neg j$	Modus tollens using Steps 3 and 4

6. Construct an argument using rules of inference to show that the hypothesis " If it does not rain or if it is not foggy, then the sailing race will be held and the life saving demonstration will go on," "If sailing race is held, the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained".

Solution : Let r be the proposition "It rains," let s be the proposition "the sailing race will be held," let f be the proposition "it is foggy," let l be the proposition "the life saving demonstration will go on," and let t be the proposition "the trophy will be awarded."

We are given premises $(\neg r \vee \neg f) \to (s \wedge l), s \to t$ and $\neg t$. We want to conclude r.

Step	**Reason**
1. $\neg t$	Hypothesis
2. $s \to t$	Hypothesis
3. $\neg s$	Modus tollens using Steps 1 and 2
4. $(\neg r \vee \neg f) \to (s \wedge l)$	Hypothesis
5. $\neg s \vee \neg l) \to (r \wedge f)$	Contrapositive of step 4 and De Morgan laws
6. $\neg s \vee \neg l$	Addition Using step 3
7. $r \wedge f$	Modus ponens using Steps 5 and 6
8. r	Simplification using 7

Prove the following results:

(1) $\neg p \vee q$, $\neg q \vee r$, $r \to s \Rightarrow p \to s$

(2) $r \to \neg q$, $r \vee s$, $s \to \neg q$, $p \to q \Rightarrow \neg p$

Solution : (1)

Step	**Reason**
1. p	Hypothesis
2. $\neg p \vee q$	Hypothesis
3. q	Disjunctive Syllogism using Steps 1 and 2
4. $\neg q \vee r$	Hypothesis

5. r	Disjunctive Syllogism using Steps 3 and 4
6. $r \to s$	Hypothesis
7. s	Modus ponens using Steps 5 and 6

(2)

Step	**Reason**
1. $\neg(\neg p) = p$	Hypothesis
2. $p \to q$	Hypothesis
3. q	Modus ponens using Steps 1 and 2
4. $s \to \neg q$	Hypothesis
5. $\neg s$	Modus tollens using Steps 3 and 4
6. $r \vee s$	Hypothesis
7. r	Disjunctive Syllogism using Steps 5 and 6
8. $r \to \neg q$	Hypothesis
9. $\neg q$	Modus ponens using Steps 7 and 8
10. $q \wedge \neg q \Leftrightarrow F$	Conjunction of steps 3 and 9

—— **Contradiction!**

HOMEWORK 3

P47-50

9. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C^{++}". Express each of the following sentence in terms of $P(x), Q(x)$, quantifiers, and logical connectives. For the universe of discourse for quantifiers use the set of all students at your school.

a) There is a student at your school who can speak Russian and who knows C^{++}.

b)a) There is a student at your school who can speak Russian but who does not know C^{++}.

c) Every student at your school either can speak Russian or knows C^{++}.

d) No student at your school speak Russian or knows C^{++}.

Solution : a) $\exists x(P(x) \wedge Q(x))$

b) $\exists x(P(x) \wedge \neg Q(x))$

c) $\forall x(P(x) \vee Q(x))$

d) $\forall x \neg(P(x) \vee Q(x))$

62. Let $P(x)$, $Q(x)$, $R(x)$ and $S(x)$ be the statement "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express the following statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$ and $S(x)$.

a) No ducks are willing to waltz.

b) No officers ever decline to waltz.

c) All my poultry are ducks

d) My poultry are not officers

e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

$Solution:$ a) $\forall x(P(x) \to \neg S(x)$

b) $\neg\exists x(R(x) \wedge \neg S(x))$ or $\forall x(R(x) \to S(x))$

c) $\forall x(Q(x) \to P(x)$

d) $\forall x(Q(x) \to \neg R(x)$

e) Yes. If x is one of my poultry, then he is a duck (by part (c)), hence not willing to waltz(Part(a)). Since officers are always willing to waltz(Part(a)), x is not an officer.

P59-61

9. Let $L(x, y)$ be the statement "x loves y," where the universe of discourse for x and y is the set of all people in the world. Use quantifiers to express each of the following statements:

a) Every body loves Jerry.

c) There is somebody whom everybody loves.

e) There is somebody whom Linda does not love.

g) There is exactly one person whom everyone loves.

i) Everybody loves himself or herself.

$Solution:$ a) $\forall x L(x, Jerry)$

c) $\exists x \forall y L(y, x)$

e) $\exists x \neg L(Linda, x)$

g) $\exists x(\forall y L(y, x) \wedge \forall z(\forall w(L(w, z) \to (z = x)))$

i) $\forall x L(x, x)$

28. Determine the truth value of each of the following statements if the universe of discourse for each variables is the set of real numbers.

a) $\exists x(x^2 = 2)$

b) $\exists x(x^2 = -1)$

c) $\forall x \exists y(x^2 = y)$

d) $\forall x \exists y(x = y^2)$

e) $\exists x \forall y(xy = 0)$

f) $\exists x \exists y(x + y \neq y + x)$

g) $\forall x \neq 0 \exists y(xy = 1)$

h) $\exists x \forall y \neq 0(xy = 1)$

i) $\forall x \exists y(x + y = 1)$

j) $\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$

k) $\forall x \exists y(x + 2y = 2 \wedge 2x - y = 1)$

l) $\forall x \forall y \exists z(z = (x + y)/2)$

$Solution:$ a) T b) F c) T d) F e) T f) F g) T h) F i) T j) F k) F l) T

Prove the following formulas:

1) $\exists x A(x) \to \forall B(x) \Rightarrow \forall x(A(x) \to B(x))$

Solution: (i) $\exists x A(x) \to \forall B(x) \Leftrightarrow \neg(\exists x A(x)) \vee (\forall x B(x))$

$$\Leftrightarrow \forall x \neg A(x) \vee \forall x B(x)$$
$$\Rightarrow \forall x(\neg A(x) \vee B(x))$$
$$\Leftrightarrow \forall x(A(x) \to B(x))$$

(ii)

Step	Reason
1. $\neg(\forall x(A(x) \to B(x)))$	Hypothesis
2. $\exists x(A(x) \wedge \neg B(x))$	Equivalence from (1)
3. $A(e) \wedge \neg B(e)$	EI(2)
4. $A(e)$	Simplification from (3)
5. $\exists x A(x)$	EG(4)
6. $\exists x A(x) \to \forall B(x)$	Hypothesis
7. $\forall B(x)$	Modus ponens using (5) and (6)
8. $B(e)$	UI (7)
9. $\neg B(e)$	Simplification from (3)
10. $\neg B(e) \wedge B(e)$**(Contradiction!)**	Conjunction (8) and (9)

2) $\forall x(A(x) \vee B(x))$, $\forall x(B(x) \to \neg C(x))$, $\forall x C(x) \Rightarrow \forall x A(x)$

Solution:

Step	Reason
1. $\forall x(A(x) \vee B(x))$	Hypothesis
2. $A(e) \vee B(e)$	UI(1)
3. $\forall x(B(x) \to \neg C(x))$	Hypothesis
4. $B(e) \to \neg C(e)$	UI(3)
5. $\forall x C(x)$	Hypothesis
6. $C(e)$	UI(5)
7. $\neg B(e)$	Modus tollens using (4) and (6)
8. $A(e)$	Disjunctive syllogism (2) and (7)
9. $\forall x A(x)$	UG(8)