浙江大学 2015-2016 学年 秋冬 学期

研究生《计算理论》课程期终考试试卷

考试形式：闭卷, 考试时间: 2016 年 1 月15日，所需时间:120 分钟

学号：__________ 姓名：__________ 专业：__________ 任课教师: 金小刚

题序	1	2	3	4	5	6	7	总分
得分								
评卷人								

Zhejiang University
Theory of Computation, Fall-Winter 2015
Final Exam

1. **(20%) Determine whether the following statements are true or false. If it is true write a ◯ otherwise a $\times$ in the bracket before the statement.**

   (a) ( ) Let L be any regular language. Then the number of equivalence classes respect to language L (i.e. $x \approx_L y$, if for all $z \in \Sigma^*$, $xz \in L$ iff $yz \in L$) is finite.

   (b) ( ) The language $\{a^n b^n w | w \in \{a,b\}^*, n \in \mathbb{N},$ and $|w| = 2n\}$ is context-free.

   (c) ( ) The language $\{$"M""w"$|M$ accepts w in less than 2016 steps $\}$ is not recursive.

   (d) ( ) The language $\{$"M" $|$ M is a TM and $L(M)$ is Context-free but $L(M)$ is not regular$\}$ is not recursive.

   (e) ( ) The language $\{$"M_1""M_2" $|$ M_1 and M_2 are TMs, and M_1 halts on e but M_2 doesn't halt on $e\}$ is recursively enumerable, but not recursive.

   (f) ( ) The set of all primitive recursive functions is a proper subset of the set of all μ-recursive functions.

   (g) ( ) Let A and B be two disjoint recursively enumerable languages. If $\overline{A \cup B}$ is also be recursively enumerable, then it is possible that neither A nor B is decidable.

   (h) ( ) Let $H_{10} = \{$"M"$|M$ is a TM and $10 \in L(M)\}$ and τ_1 and τ_2 are recursive functions. If $H_{10} \leq_{\tau_1} L$ and $\overline{H_{10}} \leq_{\tau_2} L$, then L is recursive enumerable but not recursive.

   (i) ( ) Suppose **SAT** $\leq_P L$ and $L \in \mathbb{P}$. Then $\mathbb{P} = \mathbb{NP}$.

   (j) ( ) Let $H = \{$"M""w"$|$ TM M halts on input string $w\}$, then H is $\mathbb{NP}$-complete.

2. **(18%) On FA and Regular Languages**
   Say whether each of the following languages is regular or not regular? Prove your answers.
   (a) $L_1 = \{w | w \in \{0,1\}^*$ and w has an equal number of 0s and 1s$\}$.
   (b) $L_2 = \{w | w \in \{0,1\}^*$ and w has an equal number of 01s and 10s$\}$.

3. **(20%) On PDA and Context-Free Languages**
   Let $L_3 = \{wca^m b^n | w \in \{a,b\}^*,$ where $w = w^R,$ and $m, n \in \mathbb{N}, n \leq m \leq 2n\}$.
   (a) Give a context-free grammar for the language L_3.
   (b) Design a PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ accepting the language L_3.

   Solution: (a)

   (b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

$K = \{$ ________________ $\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{$ $\}$

$s =$ ________________

$F = \{$ ________________ $\}$

(q, σ, β)	(p, γ)

4. **(18%) On Undecidability**

   Classify whether each of the following languages are *recursive*, *recursively enumerable but- not-recursive*, or *non-recursively-enumerable*. Prove your answers, but you may not simply appeal to Rice's theorem.

   (a) $L_4 = \{\text{“}M\text{”} | M \text{ is a Turing Machine, and } |L(M)| > 2\}$.

   (b) $L_5 = \{\text{“}M\text{”} | M \text{ is a Turing Machine, and } |L(M)| \leq 2\}$.

5. **(15%) On $\mathbb{P}$ and $\mathbb{NP}$ Problems**
In an undirected graph $G = (V, E)$, a **Sovereign set** is a vertex subset $S \subseteq V$ such every edge in E has at most one endpoint in S. Let **Sovereign Set** $= \{(G, k) : G \text{ is an undirected graph that contains a sovereign set of at least } k \text{ vertices}\}$.
   (a) Give the definition of the class $\mathbb{P}$ and $\mathbb{NP}$.
   (b) Show that **Sovereign Set** is a $\mathbb{NP}$ problem.
   (c) Prove that **Sovereign Set** is $\mathbb{NP}$-complete. You may reduce from **Clique** problem.

6. **(9%) On Computability and Computational Complexity**

   (a) Draw a diagram illustrating the relations between $\mathbb{P}$, $\mathbb{NP}$, *recursive*, *recursively enumerable*(**r.e.**), *co-r.e.* on the hypothesis that $\mathbb{P} \neq \mathbb{NP}$, where *co-r.e.* is the set of languages that its complement is recursively enumerable;

   (b) Try to mark the languages H, $\overline{H}$, H_1 and $\overline{H_1}$ in the given diagram in part(a), where $H = \{\text{“}M\text{”}\,\text{“}w\text{”} \mid \text{Turing Machine } M \text{ halts on } w\}$ and $H_1 = \{\text{“}M\text{”} \mid \text{Turing Machine } M \text{ halts on “}M\text{”}\}$, respectively.

Enjoy your Spring Festival !