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| **DISCRETE MATHEMATICS** | Undergraduate Course |
| HOMEWORK 4 SOL | College of Computer Science |
| | **Zhejiang University** |
| | Fall-Winter 2014 |

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## HOMEWORK 4

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**P120**

**9 Determine whether each of these statements is true or false.**

a) $x \in \{x\}$ b) $\{x\} \subseteq \{x\}$ c) $\{x\} \in \{x\}$ d) $\{x\} \in \{\{x\}\}$ e) $\emptyset \subseteq \{x\}$ f) $\emptyset \in \{x\}$

*Solution* :

a) **T** b) **T** c) **F** d) **T** e) **T** f) **F**

**16. Find two sets $A$ and $B$ such that $A \in B$ and $A \subseteq B$.**

*Solution*:

Let $A = \emptyset$ and $B = \{\emptyset\}$, then $A \subseteq B$ and $A \in B$.

**20. Can you conclude that $A = B$ if $A$ and $B$ are two sets with the same power set?**

*Solution*:

We can conclude that $A = B$ if $A$ and $B$ are two sets with the same power set.

Because

$$2^A = 2^B \Rightarrow A \in 2^A = 2^B \Rightarrow A \subseteq B$$

and

$$2^A = 2^B \Rightarrow B \in 2^B = 2^A \Rightarrow B \subseteq A$$

Hence, $A = B$.

**P131**

**24. Let $A, B$ and $C$ be sets. Show that** $(A - B) - C = (A - B) - (B - c)$ *Solution*:

$$\begin{aligned}(A - C) - (B - C) &= A \cap \overline{C} \cap \overline{B \cap \overline{C}} = (A \cap \overline{C}) \cap (\overline{B} \cup C) \\ &= (A \cap \overline{C} \cap \overline{B}) \cup (A \cap \overline{C} \cap C) \\ &= (A \cap \overline{C} \cap \overline{B}) \cup \emptyset \\ &= (A \cap \overline{B} \cap \overline{C}) \\ &= (A - B) \cap \overline{C}) \\ &= (A - B) - C\end{aligned}$$

**39. What can you say about the sets $A$ and $B$ if $A \oplus B = A$**

*Solution*:(Omited)

**P162-163**

**40. Show that the union of two countable sets is countable.**

*Solution*: Let $A$ and $B$ be the given countable sets, and let us list their elements, $a_1, a_2, \cdots, a_n, \cdots$ and $b_1, b_2, \cdots, b_n, \cdots$. Then we can list the elements of their union as $a_1, b_1, a_2, b_2, \cdots, a_n, b_n, \cdots$ except that we do not list any element that has already appeared in this list (in case $A \cap B \neq \emptyset$, and if one or both of the original lists (in case $A$ or $B$ is finite, then of course we do not list nonexistent terms. Since we have displayed $A \cap B$ as a list, we conclude that it is countable.

**46. Show that the set of functions from the positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is uncountable.**

*Solution*: From the Example 17, we know that the set of real numbers between 0 and 1( denoted by $(0, 1)$ is uncountable. Let us associate to each real number $[0, 1)$ a function from the set of positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ as follows:

If $x$ is a real number whose decimal representation is $0.d_1d_2d_3\cdots$ (with ambiguity resolved by forbidding the decimal to end with a infinite string of 9's). The we associate to the $x$ the function whose rule is given by $f(n) = d_n$.

Clearly, this is a one to one function from [0,1) and a subset of all functions from positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Two different real numbers must have different decimal representation, so the corresponding functions are different.(A few functions are left out, because of forbidding representations such as $0.2399999\cdots$).

Since (0,1) is uncountable, the subset of functions we have associated with them must be uncountable. But the set of all such functions has at least this cardinality, so it, too, must be uncountable.