浙江大学 2014-2015 学年 秋冬 学期

《计算理论》课程期末考试试卷

课程号：21120520　开课学院：计算机学院

考试试卷：☑ A卷　☐ B卷

考试形式：☑ 闭卷　☐ 开卷，允许带 ＿＿＿＿＿入场

考试日期: 2015 年 1 月 27 日，考试时间：120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名＿＿＿＿＿＿＿＿ 学号＿＿＿＿＿＿＿＿ 所属院系＿＿＿＿＿＿＿＿

题序	1	2	3	4	5	6	总分
得分							
评卷人							

Zhejiang University
Theory of Computation, Fall-Winter 2014
Final Exam

1. **(24%) Determine whether the following statements are true or false. If it is true fill a ◯ otherwise a $\times$ in the bracket before the statement.**

   (a) (　) If L is any language, then the language LL^R must be equal to $\{ww^R \mid w \in L\}$.

   (b) (　) Language $\{a^ib^jc^k|i,j,k \in \mathbb{N} \text{ and } i+j \not\equiv k(\bmod 3)\}$ is not regular.

   (c) (　) If A is non-regular and both of B and $A \cap B$ are regular, then $A \cup B$ is non-regular.

   (d) (　) For all languages L_1, L_2 and L_3, if $L_1 \subseteq L_2 \subseteq L_3$ and both L_1 and L_3 are regular, then L_2 is also regular.

   (e) (　) Language $\{xcy|x,y \in \{a,b\}^* \text{ and } |x| \le |y| \le 2|x|\}$ is context-free.

   (f) (　) Let L_1 be a regular language and L_2 be a context-free language, then $\{uv|u \in L_1, v \in L_2 \text{ and } |u| = |v|\}$ is also context-free.

   (g) (　) Let $\mathbf{D}_{\text{DFA}} = \{$"$M$"$|$ DFA M rejects "M", where "M" is the encoding of DFA M, just as Turing Machine$\}$, then $\mathbf{D}_{\text{DFA}}$ is recursively enumerable but not regular.

   (h) (　) Let L be a language and there is a Turing machine M halts on x for every $x \in L$, then L is decidable.

   (i) (　) Every countably infinite language is recursively enumerable.

   (j) (　) A language is recursively enumerable if and only if it is Turing enumerable.

   (k) (　) Let A be a recursively enumerable language and $A \le_\tau \overline{A}$, then A is recursive.

   (l) (　) There are countably many Turing machines, and uncountably many languages, so most languages are not recursively enumerable.

2. **(20%)** Decide whether the following languages are regular or not and provide a formal proof for your answer. Let $\#_a(u)$ and $\#_a(v)$ be the number of a in string u and v, respectively.

   (a) $L_1 = \{ucv \mid u, v \in \{a,b\}^*, \#_a(u) = 2 \cdot \#_a(v)\}$

   (b) $L_2 = \{uv \mid u, v \in \{a,b\}^*, \#_a(u) = 2 \cdot \#_a(v)\}$

3. **(24%) On PDA and Context-Free Languages**
   Let $L_3 = \{xcy | x, y \in \{a,b\}^*, |x| = |y|, \text{ and } x \neq y^R \}$.

   (a) Construct a context-free grammar that generates the language L_3.

   (b) Construct a pushdown automata that accepts L_3.

   Solution:

   (a)

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

	(q, σ, β)	(p, γ)
$K =$ _____		
$\Sigma = \{a, b, c\}$		
$\Gamma =$ _____		
$s =$ _____		
$F =$ _____		

4. **(20%)** The function $\varphi : \mathbb{N} \to \mathbb{N}$ given by

$$\varphi(x) = \begin{cases} 4x, & \text{if } x < 8 \\ x + 2, & \text{if } x \geq 8 \end{cases}$$

(a) Try to construct a Turing Machine to compute the function $\varphi(x)$. When describing the Turing machines, you can use the elementary Turing machines described in textbook. Always assume that the Turing machines start computation from the configuration $\triangleright\underline{\sqcup}x$ where x is represented by binary string, i.e. $x \in \{0, 1\}^*$.

(b) Show that the function $\varphi(x)$ is primitive recursive.

5. **(12%)** Consider the language

$$\textbf{Non-Empty} = \{\text{“}M\text{”} \mid \text{Turing machine } M \text{ halts some strings}\}$$

Show that **Non-Empty** is recursively enumerable. Justify your answer, and an informal description suffices.

Enjoy Your Spring Festival!