浙江大学 2006-2007 学年 秋冬 季学期

《计算理论》课程期末考试试卷

开课学院：计算机学院 考试形式：闭卷，允许带 _____入场

考试时间: 2007 年 1 月24日，所需时间:120 分钟，任课教师: _____

考生姓名：_____ 学号：_____ 专业：_____

题序	1	2	3	4	5	6	7	8	总分
得分									
评卷人									

Zhejiang University
Theory of Computation, Fall-Winter 2006
Final Exam

1. **(30%) Determine whether the following statements are true or false. If it is true write a ✓ otherwise a × in the bracket before the statement.**

   (a) ( ) If a DFA M contains a self-loop on some state q, then M must accept an infinite language.

   (b) ( ) Let A and B be two regular languages, so is $A \oplus B$, where $A \oplus B = (A - B) \cup (B - A)$.

   (c) ( ) Let $L_1, L_2, \cdots, L_i, \cdots$ be all regular languages, so is $\cup_{i=1}^{\infty} L_i$.

   (d) ( ) Language $\{ucv \mid u, v \in \{a, b\}^* \text{ and } |u| = 2|v|\}$ is context free.

   (e) ( ) Suppose that L is a context-free language and R is regular, $L - R$ is context-free language.

   (f) ( ) Every computable function is primitive recursive.

   (g) ( ) The complement of every recursive enumerable language is recursive enumerable.

   (h) ( ) Let L be a language and there is a Turing machine M halts on x for every $x \in L$, then L is decidable.

   (i) ( ) If one can list the elements of a language in order, then the language must be recursive.

   (j) ( ) Languages $\{\text{“}M\text{”} : \text{Turing machine } M \text{ accepts at least 2007 distinct inputs}\}$ is recursive enumerable.

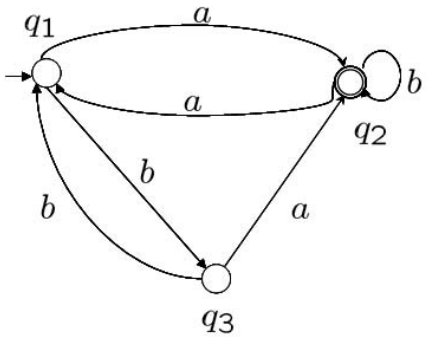
2. **(12%)** Decide whether the following languages are regular or not and provide a formal proof for your answer.

(a) $L_1 = \{a^k u \mid u \in \{a, b\}^* \text{ and } u \text{ contains at least } k \text{ } a\text{'s, for } k \geq 1\}$.

(b) $L_2 = \{a^k u \mid u \in \{a, b\}^* \text{ and } u \text{ contains at most } k \text{ } a\text{'s, for } k \geq 1\}$.

3. **(12%)**

(a) Describe the language accepted by the following finite automaton;



(b) Give a Context-free Grammar for the language.

4. **(14%)** Consider the pushdown automaton $M = \{K, \Sigma, \Gamma, \Delta, s, F\}$ where $K = \{s, f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{b\}$, $F = \{f\}$ and Δ is given by the following table

$(p, a, \beta), (q, \gamma)$
$((s, a, e), (f, e))$
$((s, b, e), (s, b))$
$((s, a, b), (s, b))$
$((s, e, e), (f, e))$
$((f, a, e), (f, e))$
$((f, b, e), (s, b))$

   (a) Can PDA M accept string *aababa*?

   (b) Describe the language accepted by M;

   (c) Give a Turing machine that decides the same language.

5. **(12%)** Show the following function

$$f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ 4x, & \text{if } x \text{ is even} \end{cases}$$

   is primitive recursive.

6. **(10%)** Explain that why the following language

$$H = \{\text{“}M\text{”} : \text{Turing machine M halts on empty string } e\}$$

is recursively enumerable. An informal description suffices.

7. **(10%)** Let $K_0 = \{\text{“}M\text{”}\,\text{“}w\text{”} : M \text{ halts on input string } w\}$, $K_1 = \{\text{“}M\text{”} : M \text{ halts on input string “}M\text{”}\}$. Try to sign languages K_0 , $\overline{K_1}$ and sets of languages recursive, context-free language and regular to the corresponding zone of the following figure:

Note: r.e. is the set of recursive enumerable languages and CO-r.e. $=\{L : \text{complement of } L \text{ is r.e.}\}$.