浙江大学 2006-2007 学年 秋冬 季学期

研究生《计算理论》课程期末考试试卷

考试时间: 2007 年 1 月22日，所需时间:120 分钟，任课教师: ________

考生姓名：________________ 学号：________________ 专业：________________

题序	1	2	3	4	5	6	7	8	总分
得分									
评卷人									

Zhejiang University
Theory of Computation, Fall-Winter 2006
Final Exam

1. **(30%) Determine whether the following statements are true or false. If it is true write a ✓ otherwise a × in the bracket before the statement.**

   (a) ( ) A countable union of regular languages is necessarily regular.

   (b) ( ) If a DFA M contains a self-loop on some state q, then M must accept an infinite language.

   (c) ( ) Language $\{ucv \mid u, v \in \{a,b\}^* \text{ and } |v| < |u| < 2|v|\}$ is context-free.

   (d) ( ) For a given context-free language L and a string x, the decision problem for whether $x \in \overline{L}$ is decidable.

   (e) ( ) The complement of every recursive enumerable language is necessarily nonrecursive enumerable.

   (f) ( ) If one can list the elements of a language in order, then the language must be recursive.

   (g) ( ) Languages $\{$"M" : Turing machine M accepts more than 2007 distinct inputs$\}$ is recursive enumerable.

   (h) ( ) Let L be a language and there is a Turing machine M halts on x for every $x \in L$, then L is decidable.

   (i) ( ) If L is polynomial time reducible to a finite language, then L is in $\mathcal{P}$.

   (j) ( ) If $A \leq_p B$, $B \leq_p C$ and both A and C are $\mathcal{NP}$-complete, then B is $\mathcal{NP}$-complete.

2. **On Regular Languages**
   (12%) Decide whether the following languages are regular or not and provide a formal proof for your answer.

   (a) $L = \{w \in \{a,b\}^* : |n_a(w) - n_b(w)| \mod 2 \neq 0\}$

(b) $L = \{w \in \{a,b\}^* : |n_a(w) - n_b(w)| \neq 0\}$
where $n_a(w)$ and $n_b(w)$ give the number of a and b in w respectively.

3. **On Context-free Languages**
(15%) Consider the pushdown automaton $M = \{K, \Sigma, \Gamma, \Delta, s, F\}$ where $K = \{s, f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{b\}$, $F = \{f\}$ and Δ is given by the following table

$(p, a, \beta), (q, \gamma)$
$((s, a, e), (f, e))$
$((s, b, e), (s, b))$
$((s, a, b), (s, b))$
$((s, e, e), (f, e))$
$((f, a, e), (f, e))$
$((f, b, e), (s, b))$

(a) Can PDA M accept string $aaaaababa$?

(b) Describe the language accepted by M;

(c) Give a Turing machine that decides the same language.

4. **On Primitive Recursive Functions**
   (11%) Show function

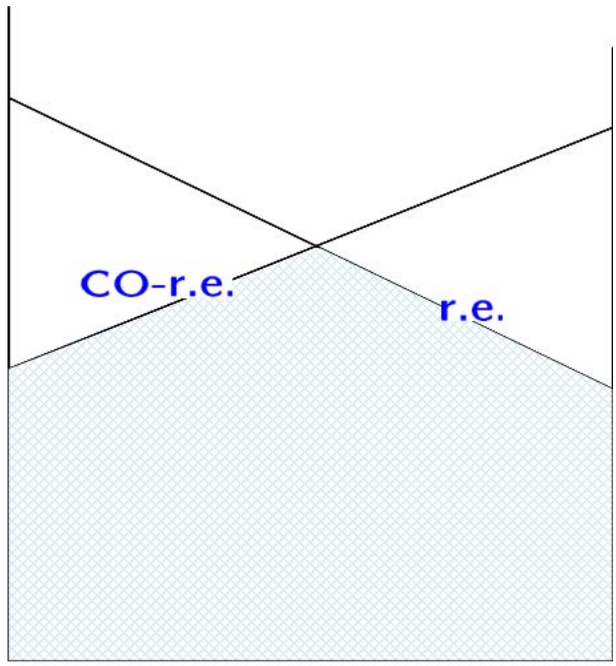
$$f(x,y) = \begin{cases} x+y, & \text{if } y \text{ is odd} \\ x \sim \frac{y}{2}, & \text{if } y \text{ is even} \end{cases}$$

   is primitive recursive.

5. **On Turing Machines**
   (10%) Show that the computable functions are closed under composition, using the definition of computation of the Turing machine. That is, if f and g are computable functions, show that the function φ given by $\varphi(x) = f(g(x))$ is a computable function.

6. **On Undecidability**
(12%) Let $K_0 = \{$"M""w" : M halts on input string w $\}$, $K_1 = \{$"M" : M halts on input string "M" $\}$. On the assumption that $\mathcal{P} \neq \mathcal{NP}$, try to sign languages K_0 , $\overline{K_1}$ and sets of languages recursive, $\mathcal{P}$, $\mathcal{NP}$ and $\mathcal{NP}$-Complete to the corresponding zone of the following figure:

   Note: r.e. is the set of recursive enumerable languages and CO-r.e. $=\{L$: complement of L is r.e. $\}$.



7. **On $\mathcal{P}$ and $\mathcal{NP}$ Problems**
(10%) The SET-PACKING problem is defined as follows: given a set S with n sets and a number $k \leq n$, does S contains k disjoint sets?

   (a) Prove that SET-PACKING problem is $\mathcal{NP}$ Problem.

   (b) Prove that SET-PACKING problem is $\mathcal{NP}$-complete.

   For showing hardness, you can assume that the VERTEX-COVER problem is $\mathcal{NP}$-complete.