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Numerical Methods

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Taylor Series

Taylor series

- A Taylor series is a polynomial of infinite degree that can be used to represent many different functions, particularly functions that aren't polynomials.
- Taylor series has applications ranging from classical and modern physics to the computations that your hand-held calculator makes when evaluating trigonometric expressions.

Taylor series (2)

- Let $f(x)$ be a real-valued function that is infinitely differentiable at $x = x_0$.
- The Taylor series expansion for the function $f(x)$ centered around the point $x = x_0$ is given by

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(x_0)\frac{(x - x_0)^n}{n!}$$

Note that $f^{(n)}(x_0)$ represents the n^{th} derivative of $f(x)$ at $x = x_0$

Taylor series (3)

$$f(x) = f(x_0) + \frac{(x - x_0)}{1!} f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) \ldots + \frac{(x - x_0)^m}{m!} f^m(x_0) + \cdots$$

Ex:

- We will now use the definition above to construct a graceful polynomial equivalency to cos (x).
- Because the formula for the Taylor series given in the definition above contains $f^{(n)}(x_0)$, we should build a list containing the values of $f(x)$ and its first four derivatives at $x = 0$:

Taylor series (4)

$$f(x) = f(x_0) + \frac{(x - x_0)}{1!} f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) \ldots + \frac{(x - x_0)^m}{m!} f^m(x_0)$$

Ex:

- f(0) = cos (0) = 1
- f'(0) = - sin (0) = 0
- f''(0) = - cos (0) = -1
- f'''(0) = sin (0) = 0
- f^4(0) = cos (0) = 1

Taylor series (5)

We begin assembling the Taylor series by writing f(x) = [the first number in our list] $\frac{(x-x_0)^0}{0!}$ like so: $$f(x) = 1 \cdot \frac{(x-0)^0}{0!} = 1.$$

So far, our constructed function f(x) = 1 looks nothing like f(x) = cos *x*. They merely have f(0) = 1 in common, but we shall add more terms. We add the next term from our list above, this time multiplied by $\frac{(x-x_0)^1}{1!}$

$$f(x) = 1 \cdot \frac{(x-0)^0}{0!} + \boxed{0 \cdot \frac{(x-0)^1}{1!}} = 1.$$

Taylor series (6)

$$f(x) = 1 \cdot \frac{(x-0)^0}{0!} + 0 \cdot \frac{(x-0)^1}{1!} + \boxed{(-1) \cdot \frac{(x-0)^2}{2!}} = 1 - \frac{x^2}{2!}.$$

$$f(x) = 1 \cdot \frac{(x-0)^0}{0!} + 0 \cdot \frac{(x-0)^1}{1!} + (-1) \cdot \frac{(x-0)^2}{2!} + 0 \cdot \frac{(x-0)^3}{3!} + 1 \cdot \frac{(x-0)^4}{4!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}.$$

n=0 n=1 n=2 n=3 n=4

Taylor series (7)

- Lets prove it:
- Using the calculator for x=1 or cos (1) ->0.540302305
- Using the Taylor series: $$1-\frac{x^2}{2!}+\frac{x^4}{4!}.$$
- $\cos(1)=1-\frac{1^2}{2!}+\frac{1^4}{4!}$

$$\cos(1)=1-\frac{1}{2}+\frac{1}{24}=\frac{1}{2}+\frac{1}{24}=\frac{13}{24}=0.541666666$$

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Assignment

In the same way, find the sin(x) from the expansion of Taylor series with the 5 derivative!

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Thank you