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Metode Numerik

"In the same way, find then $\sin(x)$ from the expansion of Taylor Series with the 5 derivatives!"

Jawaban.

$$\text{Taylor Series : } f(x) \approx f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots + \frac{f^n(a)}{n!} (x-a)^n$$

STEP 1. "Sin"

$$f(x) = \sin x \rightarrow f(0) = \sin 0 = 0$$

$$f'(x) = \cos x \rightarrow f'(0) = \cos 0 = 1$$

$$f''(x) = -\sin x \rightarrow f''(0) = -\sin 0 = 0$$

$$f'''(x) = -\cos x \rightarrow f'''(0) = -\cos 0 = -1$$

$$f^{(4)}(x) = \sin x \rightarrow f^{(4)}(0) = \sin 0 = 0$$

$$f^{(5)}(x) = \cos x \rightarrow f^{(5)}(0) = \cos 0 = 1$$

STEP 2. "5 derivatives"

$$f(x) = \frac{0 \cdot (x-0)^0}{0!} + \frac{1 \cdot (x-0)^1}{1!} + \frac{0 \cdot (x-0)^2}{2!} + \frac{(-1) \cdot (x-0)^3}{3!} + \frac{0 \cdot (x-0)^4}{4!} + \frac{1 \cdot (x-0)^5}{5!}$$

$$f(x) = 0 + x + 0 + \left(\frac{-x^3}{6} \right) + 0 + \frac{x^5}{120} = x - \frac{x^3}{6} + \frac{x^5}{120}$$

Pembuktian $x=1$

$$\sin(x) = \sin(1) = x - \frac{x^3}{6} + \frac{x^5}{120}$$

$$= 1 - \frac{1^3}{6} + \frac{1^5}{120} = 1 - \frac{1}{6} + \frac{1}{120}$$

$$= \frac{101}{120} \approx 0,8416666$$

STEP 1 "e^x"

$$x=0$$

$$f(x) = e^x$$

$$f(0) = e^0 = 1$$

$$f'(x) = e^x$$

$$f'(0) = e^0 = 1$$

$$f''(x) = e^x$$

$$f''(0) = e^0 = 1$$

$$f'''(x) = e^x$$

$$f'''(0) = e^0 = 1$$

$$f^{(4)}(x) = e^x$$

$$f^{(4)}(0) = e^0 = 1$$

$$f^{(5)}(x) = e^x$$

$$f^{(5)}(0) = e^0 = 1$$

STEP 2 "5 derivatives"

$$f(x) = 1 + \frac{1 \cdot (x-0)}{1!} + \frac{1 \cdot (x-0)^2}{2!} + \frac{1 \cdot (x-0)^3}{3!} + \frac{1 \cdot (x-0)^4}{4!} + \frac{1 \cdot (x-0)^5}{5!}$$

$$= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$$

Pembuktian x=1

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$$

$$= 1 + 1 + \frac{1^2}{2} + \frac{1^3}{6} + \frac{1^4}{24} + \frac{1^5}{120} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120}$$

$$= \frac{325}{120} = 2,716666$$