Week 11: Assignment 11 Solution

1. Interpolation is a process for
   a) extracting feasible data set from a given set of data
   b) finding a value between two points on a line or curve.
   c) removing unnecessary points from a curve
   d) all of the mentioned

Solution: (b) Interpolation is the process of finding a value between two points on a line or curve.

2. Given two data points $(a, f(a))$ and $(b, f(b))$, the linear Lagrange polynomial $f(x)$ that passes through these two points are given as
   a) $f(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{a-b} f(b)$
   b) $f(x) = \frac{x}{a-b} f(a) + \frac{x}{b-a} f(b)$
   c) $f(x) = f(a) + \frac{f(b)-f(a)}{b-a} f(b)$
   d) $f(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

Solution: (d)
Given a set of n points, Lagrange interpolation formula is

$$f(x) = \sum_{i=0}^{n-1} L_i(x) f(x_i)$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n-1} \frac{x - x_j}{x_i - x_j}$$

Thus, $f(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

3. To solve a differential equation using Runge-Kutta method, necessary inputs from user to the algorithm is/are
   a) the differential equation dy/dx in the form x and y
   b) the step size based on which the iterations are executed.
   c) the initial value of y.
   d) all the above

Solution: (d) The differential equation, step size and the initial value of y are required to solve differential equation using Runge-Kutta method.

4. A Lagrange polynomial passes through three data points as given below

x	10	15	20
$f(x)$	3	5.2	6.8

The polynomial is determined as $f(x) = L_0(x).3 + L_1(x).(5.2) + L_2(x).(6.8)$
The value of $L_1(x)$ at $x = 18$ is
a) 0.64
b) 3.33
c) 2.67
d) 0.56

Solution: (a)

$$L_1(x) = \prod_{\substack{j=0 \\ j \neq 1}}^{2} \frac{x - x_j}{x_1 - x_j} = \frac{(18-10)(18-20)}{(15-10)(15-20)} = \frac{16}{25} = 0.64$$

5. The value of $\int_0^{3.2} xe^x dx$ by using one segment trapezoidal rule is
   a) 172.7
   b) 125.6
   c) 136.2
   d) 142.8

Solution: (b)

$$\int_a^b f(x)dx = (b-a)\frac{f(b)-f(a)}{2}$$

Here, $a = 0, b = 3.2, f(a) = 0$ and $f(b) = 78.5$. Hence, $\int_0^{3.2} xe^x dx = 125.6$

6. Solve the ordinary differential equation below using Runge-Kutta4th order method. Step size h=0.2.

$$5\frac{dy}{dx} + xy^3 = \cos(x), y(0) = 3$$

   The value of y(0.2) is (upto two decimal points)
   a) 2.86
   b) 2.93
   c) 3.13
   d) 3.08

Solution: (b)

7. Using Bisection method, negative root of x3 - 4x + 9 = 0 correct to three decimal places is
   a) -2.506
   b) -2.706
   c) -2.406
   d) None

Solution: (b) -2.706

8. Match the following

A. Newton Method	1. Integration
B. Lagrange Polynomial	2. Root finding
C. Trapezoidal Method	3. Differential Equation
D. RungeKutta Method	4. Interpolation

   a) A-2, B-4, C-1, D-3
   b) A-3, B-1, C-2, D-4
   c) A-1, B-4, C-3, D-2
   d) A-2, B-3, C-4, D-1

Solution: (a)

9. The value of $\int_{2.5}^{4} \ln x$ dx calculated using the Trapezoidal rule with five subintervals is (* range is given in output rather than single value to avoid approximation error)
   a) 1.45 to 1.47
   b) 1.74 to 1.76
   c) 1.54 to 1.56
   d) 1.63 to 1.65

Solution: (b) 1.74 to 1.76

10. Consider the same recursive C function that takes two arguments

```
unsignedintfunc(unsigned int n, unsigned int r)
{
  if (n  > 0) return (n%r +  func (n/r, r ));
  else return 0;
}
```

What is the return value of the function foo when it is called as func(513, 2)?

a) 9
b) 8
c) 5
d) 2

Solution: (d) 2

func(513, 2) will return 1 + func(256, 2). All subsequent recursive calls (including func(256, 2)) will return 0 + func(n/2, 2) except the last call func(1, 2) . The last call func(1, 2) returns 1. So, the value returned by func(513, 2) is 1 + 0 + 0…. + 0 + 1=2.