Assignment 3, Compute vision CSC8830 Saitejaswi chakravaram 002762775

2)

a) Derive the motion tracking equation from fundamental principle. Select 2 consecutive frames from the set from problem1 and compute the motion function estimates

we have taken 2 equations which will make the problem constrained so that it will be solved : we use 2 equation $I(x+\delta x, y+\delta y, t+\delta t) = I(x,y,t)$, $\delta x, \delta y$ and δt are small

(I) $I(x+\delta x, y+\delta y, t+\delta t) = I(x,y,t)$

For the point (x,y) in the image at frame at frame point of t, the intensity, brightness or pixel values does not change when it is moved by $(\delta x, \delta y)$ later at frame $t+\delta t$.

Taylor series Explains:- when $f(x+\delta x) = f(x) + \frac{\delta f}{\delta x}\delta x + \frac{\delta^2 f}{\delta^2 x}\frac{\delta x^2}{2!} + \cdots \frac{\delta^n f}{\delta x^n}\frac{\delta x^n}{n!}$ where δx is small

$$f(x+\delta x) = f(x) + \frac{\delta f}{\delta n}\delta x$$

Here is the next one

(II) $\delta x, \delta y$ and δt are small

$$\Rightarrow I(x+\delta x, y+\delta y, t+\delta t)$$

$$= I(x,y,t) + \underbrace{\frac{\delta I}{\delta x}\delta x}_{I_x} + \underbrace{\frac{\delta I}{\delta y}\delta y}_{I_y} + \underbrace{\frac{\delta I}{\delta t}\delta t}_{I_t}$$

Substituting (I) from (II)

$$I_x\delta x + I_y\delta y + I_t\delta t = 0$$

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dividing by δt and taking the limits $\delta t \to 0$:

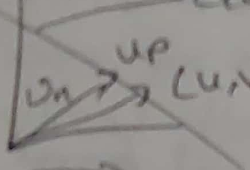
$$I_x \frac{\delta x}{\delta t} + I_y \frac{\delta y}{\delta t} + I_t = 0 \Rightarrow I_x u + I_y v + I_t = 0.$$

↳ $\frac{\delta x}{\delta t}, \frac{\delta y}{\delta t}$ - velocity of the point, i.e. - option flow of (u,v)

The equation of points (u,v) is a straight line

We already know that $\vec{u_n}, \vec{u_p}$ can be anything $\vec{u_n}$ is our estimation

equation $I_x u + I_y v + I_t = 0$



$\vec{u_n}$, the normal optical flow vector can be computed as magnitude: $\frac{|I_t|}{\sqrt{I_x^2+I_y^2}}$ direction: $\frac{(I_x, I_y)}{\sqrt{I_x^2+I_y^2}}$

2b) Derive the procedure for performing Lucas-Kanade algorithm for motion tracking when the motion is known to the affine $u(x,y) = a1*x + b1*y + c1$; $v(x,y) = a2*x + b2*y + c2$ (the number are subscripts, not power)

When the motion is affine, i.e. representable by translation rotation, etc. Lucas-Kanade algorithm works effectively for motion tracking. As the optical flow is underconstrained problem, the Lucas Kanade algorithm assumes that the optical flow. motion field within a small window is the same for all the pixels and further uses this to solve the optical flow estimation problem for all the points $(i,j) \in w$;

$$I_x(i,j)u + I_y(i,j)v + I_t(i,j) = 0$$

for a window size of m by m :-

$$\begin{bmatrix} I_x^{(1,1)} & I_y^{(1,1)} \\ \vdots & \\ I_x^{(m,m)} & I_y^{(m,m)} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} I_t^{(1,1)} \\ I_t^{(m,m)} \end{bmatrix}$$

general Lucas kanade equations

known $(m^2 \times 2)$ unknown (2×1) known $(m^2 + 1)$

As the motion is assumed to be affine we can write for each pixel (i,j) in w as: the generalize from above,

$$\begin{bmatrix} I_x^{(1,1)} & I_y^{(1,1)} & 1 \\ I_x(i,j) & I_y & 1 \\ I_x^{(m,m)} & I_y^{(m,m)} & \vdots \\ & & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} I_t^{(1,1)} \\ \vdots \\ I_t^{(m,m)} \end{bmatrix}$$

Similar equation for $a_2\ b_2\ c_2$

$$\begin{bmatrix} I_x^{(1,1)} & I_y^{(1,1)} & 1 \\ I_x(i,j) & I_y(i,j) & 1 \\ I_x^{(m,m)} & I_y^{(m,m)} & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} I_t^{(1,1)} \\ \vdots \\ I_t^{(m,m)} \end{bmatrix}$$

These ~~system~~ of equations ~~are~~ solvable via atleast with least squares. once a_1, b_1, c_1 a_2, b_2, c_2 are obtained, $u(x,y)$ $v(x,y)$ are computable for eq.

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