MODULO-2 ARITHMETIC

$0 \pm 0 = 0$

$1 \pm 0 = 1$ $\qquad a \pm b \equiv a \text{ XOR } b.$

$0 \pm 1 = 1$

$1 \pm 1 = 0$

$$D = 1x^5 + 0x^4 + 1x^3 + 1x^2 + 1x + 0$$

$$G = 1x^3 + 0x^2 + 0x + 1$$

$r = 3$ $\qquad r$ = degree of G

$$\frac{D \cdot 2^r}{G} = Q + \frac{R}{G}$$

$$D \cdot 2^r = QG + R$$

Divisible by G. ✓✓

$$\underbrace{D \cdot 2^r - R} = QG$$

$$[D, \underline{000}] - [0, R] = QG$$

* $\qquad$ 6 zeros

$$\underline{[D, R]} = QG$$

$$\text{rem}([D, R], G) = \text{rem}(QG, G) = 0$$

"remainder after division"

* Here, we've used the fact that "-1" is "$+1$" in modulo-2 arithmetic

Please compare with example on Slide 6-15.

$$\cancel{\big)\,(x^5 + x^3 + x^2 + x)\,x^3}$$

$$
\begin{array}{r|l}
 & x^5 + x^3 + x + 1 \\
\hline
x^3 + 1 & 1x^8 + x^6 + x^5 + x^4 \\
 & -(1x^8 + x^5) \\
\hline
 & 0 \quad x^6 + x^4 \\
 & -(x^6 + x^3) \\
\hline
 & x^4 + x^3 \\
 & -(x^4 + x) \\
\hline
 & x^3 + x \\
 & -(x^3 + 1) \\
\hline
 & x + 1
\end{array}
$$

Modulo-2 arithmetic

$$
\begin{array}{c}
0x^3 \\
-1x^3 \\
\hline
(0-1)x^3 \\
\hline
1\,x^3
\end{array}
$$

coefficient on x^2

$$R = 1x + 1 \overset{**}{=} [\,0 \;\; 1 \;\; 1\,]$$

Result on Slide 6-15.

$$[D, R]$$

$$D2^3 = QG + R$$

$$[D, R] = QG.$$

** Horrible abuse of notation.