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HW9-1

prove that $\sum_{i=1}^{n} w_i \max(a_i - x_i, 0) \geq \max\left(\sum_{i=1}^{n}(w_i a_i - w_i x_i), 0\right)$

整體避險較各別避險便宜

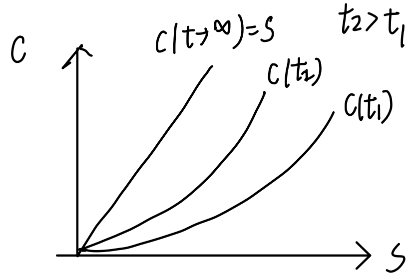
By Jensen's Inequality $E(\ell(x)) \geq \ell(E(x))$ if $\ell(x)$ is convex

$\ell(x) = \max(\cdot)$ 取大小

$E(\cdot) =$ 期望值

$E(\ell(x)) = \sum_{i=1}^{n} w_i \ell(x)$, 其中 $\ell(x) = \max(a_i - x_i, 0)$

$\ell(E(x)) = \max(E(x), 0)$, 其中 $E(x) = \sum_{i=1}^{n}(w_i a_i - w_i x_i)$



其中 $\ell(x)$ 為取 call option 正的部份

曲線為 convex

因此可使用 Jensen's Inequality

得證 $\sum_{i=1}^{n} w_i \max(a_i - x_i, 0) \geq \max\left(\sum_{i=1}^{n}(w_i a_i - w_i x_i), 0\right)$

the call on the portfolio with a strikeprice $X = \sum_i w_i x_i$

has a value at most $\sum_i w_i C_i$

HW9-2

Denote the prices for call and put options with strike price X and maturity T as V_c and V_p

Denote the price for the future matured at T as V_f

1. Long a call and short a put
2. Short a future

strategies	Initial cost	maturity return
1	$V_p - V_c$	$\cancel{S_T} - X$
2	0	$V_f - \cancel{S_T}$

$$V_p - V_c + (V_f - X)e^{-r_f T} \overset{?}{=} 0$$

← 假設 continuous Compounding，以 r_f 折現

$$\Rightarrow \quad V_c + Xe^{-r_f T} = V_p + V_f e^{-r_f T}$$

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