Lesson 2: Elementary Signals

About this presentation

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You can view the notes for this presentation in HTML and PDF.

The source code of this presentation is available in Markdown format from GitHub: Introduction.md.

The GitHub repository EG-247 Resources also contains the source code for all the Matlab/Simulink examples.

Review of Homework Problem from Lesson 1

Consider a signal

$$x = f(t) = \begin{cases} 0 & : \; t < -1 \\ t+1 & : \; -1 \le t \le 1 \\ 0 & : \; t > 1 \end{cases}$$

Sketch this signal

Sketch the effect on this signal of applying the following basic signal operations

Amplitude scaling

- $2f(t)$
- $0.5f(t)$

Time scaling

- $f(2t)$
- $f(0.5t)$

Mirroring

- $-f(t)$
- $f(-t)$
- $-f(-t)$

Time shifting - delay and advance

- $f(t-1)$
- $f(t+1)$

Try this

A combination of transformations

- $-2f(-t+2)$

Note that this involves *amplitude scaling*, *amplitude mirroring*, *time mirroring*, and a *time shift*. Each operation can be performed in sequence in any order.

Sketch of signal

Amplitude scaling

Time scaling

Mirroring

Time shifting - delay and advance

Quiz: consider this circuit:

Elementary signals

Unit Step Function

Definition

$$u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

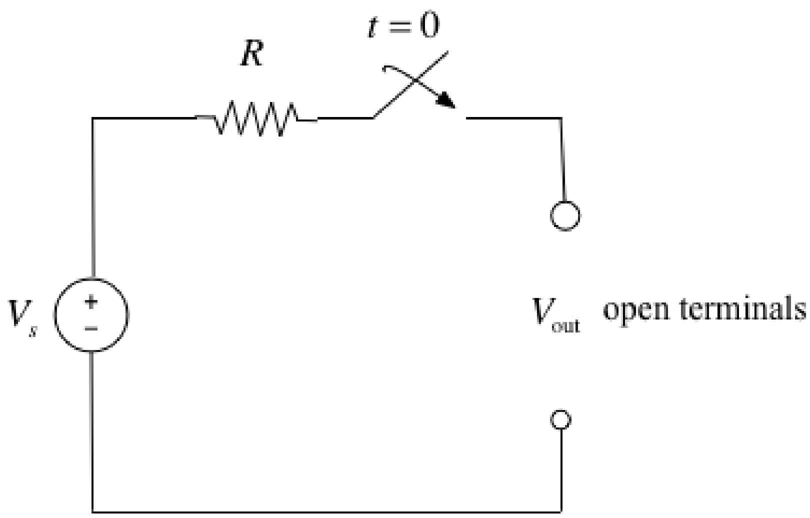

Figure 1: Circuit for quiz

Sketch

Computing/Plotting in Matlab

In Matlab, we use the `heaviside` function (Named after Oliver Heaviside).

```
syms t
ezplot(heaviside(t),[-1,1])
```

See: heaviside_function.m

Result

Note that, so it can be plotted, Matlab defines the *heaviside function* slightly differently from the mathematically ideal unit step:

$$\mathrm{heaviside}(t) = \begin{cases} 0 & t < 0 \\ 1/2 & t = 0 \\ 1 & t > 0 \end{cases}$$

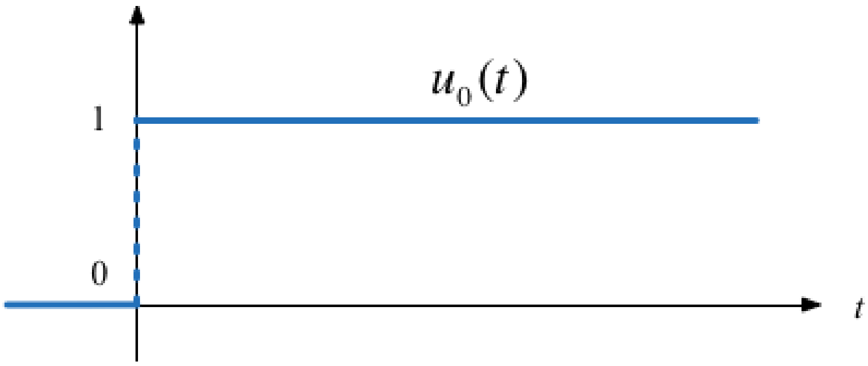

Figure 2: Unit step function

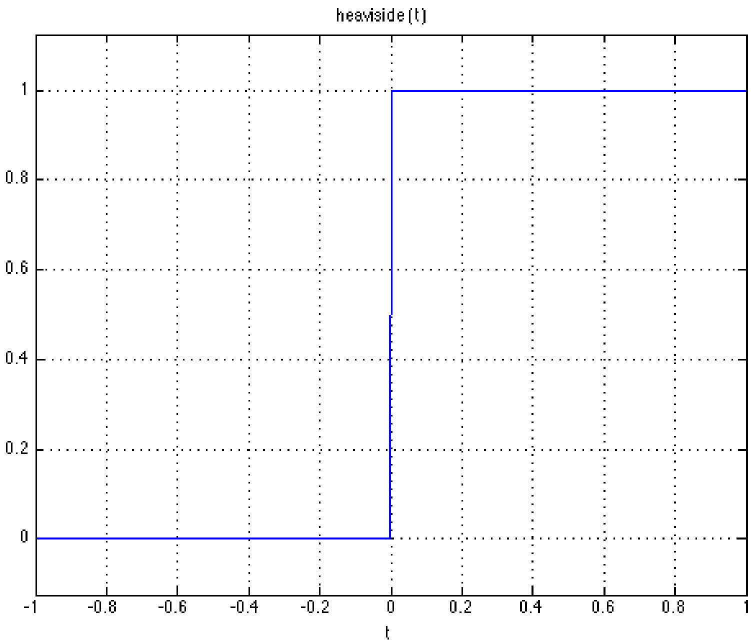

Figure 3: The Heaviside function (unit step)

Circuit Revisited

Consider the network shown below, where the switch is closed at time $t = T$.

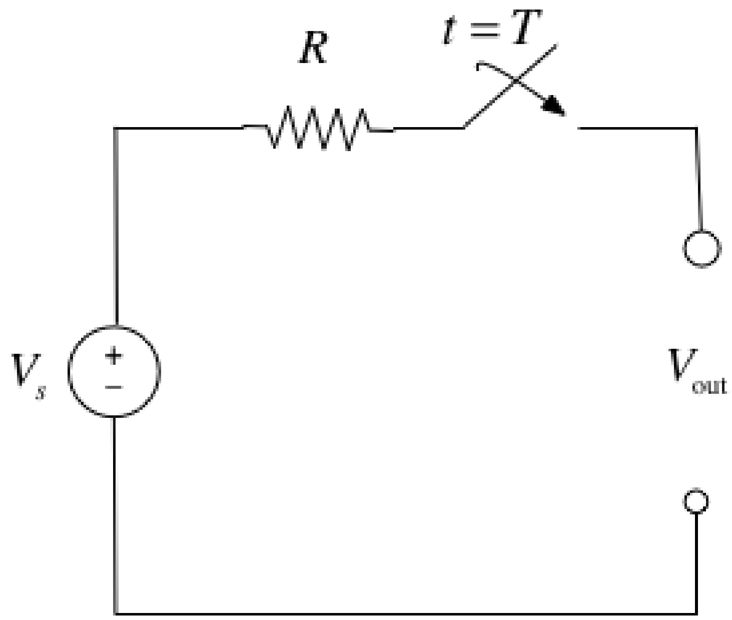

Figure 4: The circuit revisited

Express the output voltage v_{out} as a function of the unit step function, and sketch the appropriate waveform.

Simple Signal Operations

Amplitude Scaling

Sketch $Au_0(t)$ and $-Au_0(t)$

Time Reversal

Sketch $u_0(-t)$

Time Delay and Advance

Sketch $u_0(t-T)$ and $u_0(t+T)$

Example 1

Which of these signals represents $-Au_0(t+T)$?

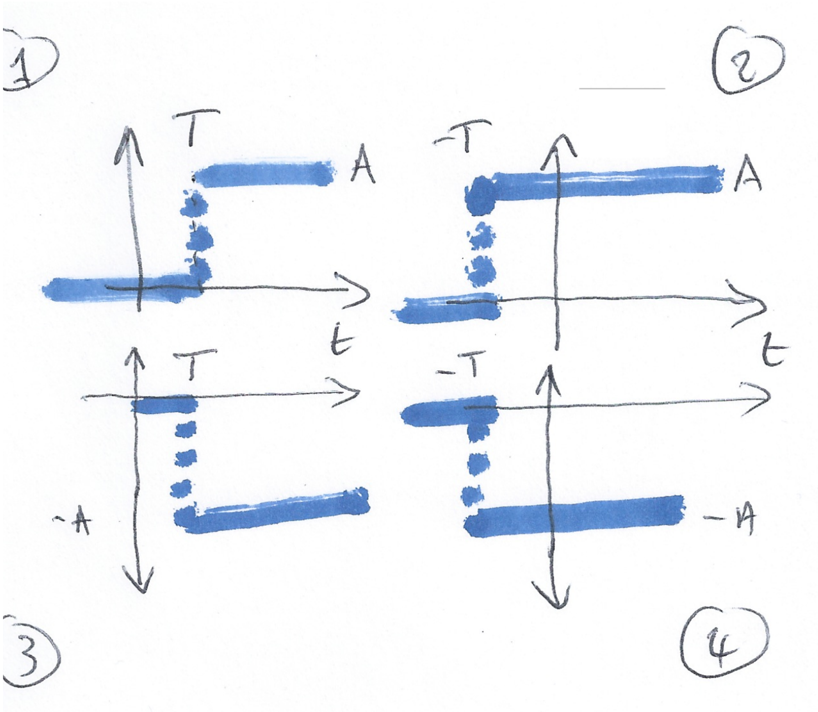

Figure 5: Example 1

Example 2

What is represented by

1. $-Au_0(t-T)$
2. $-Au_0(-t+T)$
3. $-Au_0(-t-T)$
4. $-Au_0(t-T)$

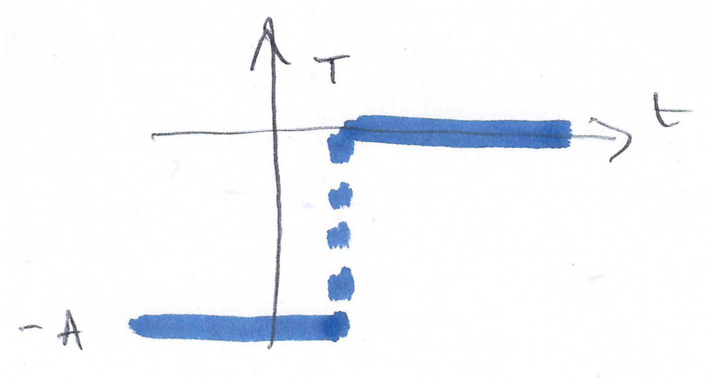

Figure 6: Example 2

Synthesis of Signals from Unit Step

Unit step functions can be used to represent other time-varying functions such as rectangular pulses, square waves and triangular pulses.

Synthesize Rectangular Pulse

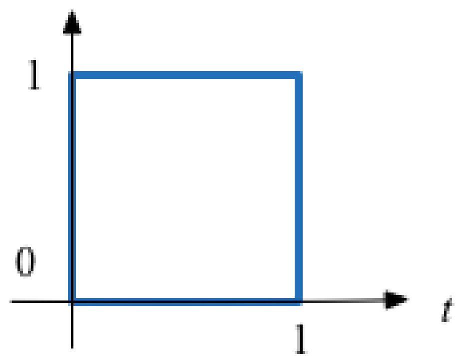

Figure 7: Rectangle function

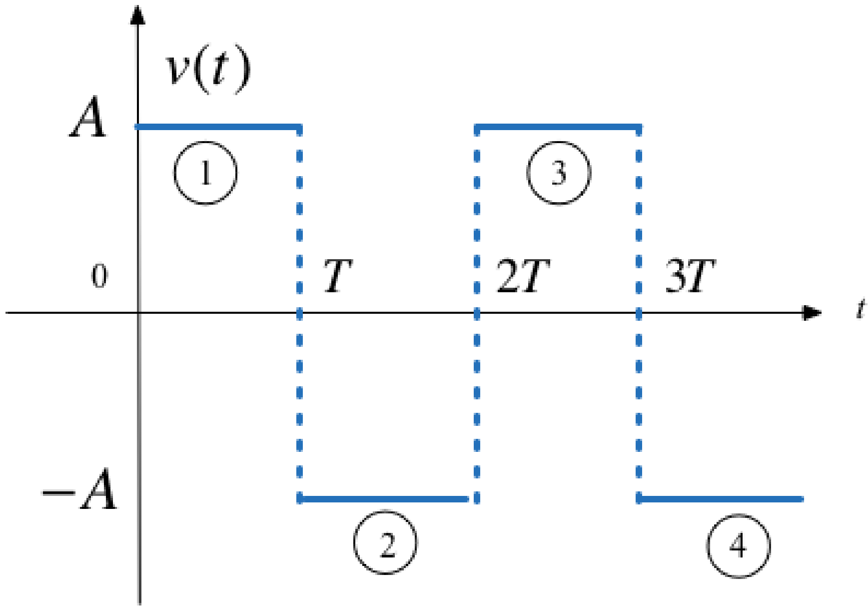

Figure 8: Square wave

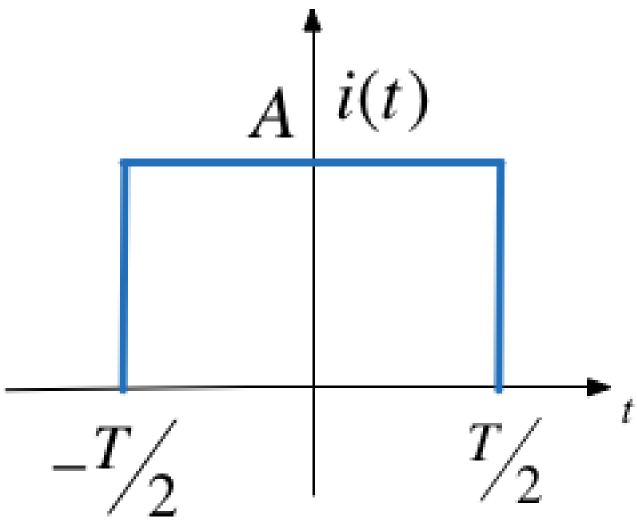

Figure 9: Symmetric rectangular pulse

Synthesize Square Wave

Synthesize Symmetric Rectangular Pulse

Synthesize Symmetric Triangular Pulse

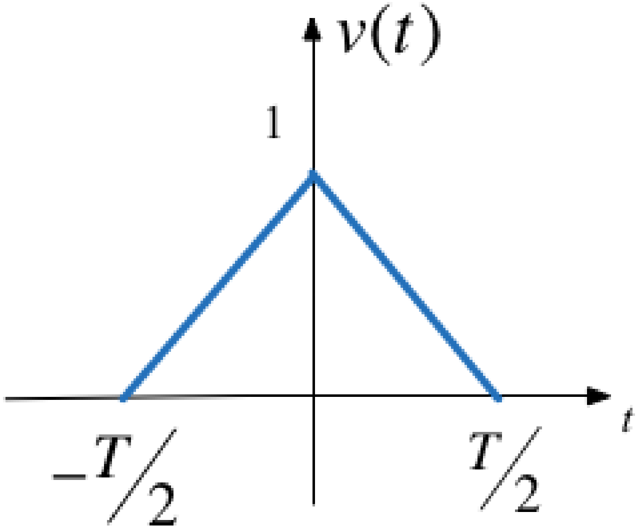

Figure 10: Symmetric triangular pulse

Homework

Show that the waveform shown below

can be represented by the function

$$v(t) = (2t + 1)u_0(t) - 2(t - 1)u_0(t - 1) - tu_0(t - 2) + (t - 3)u_o(t - 3)$$

The Ramp Function

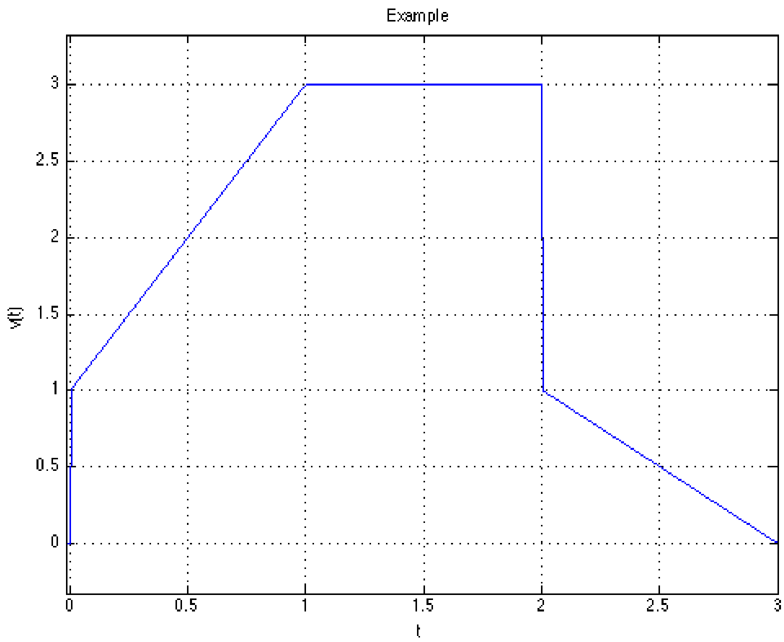

Figure 11: Homework

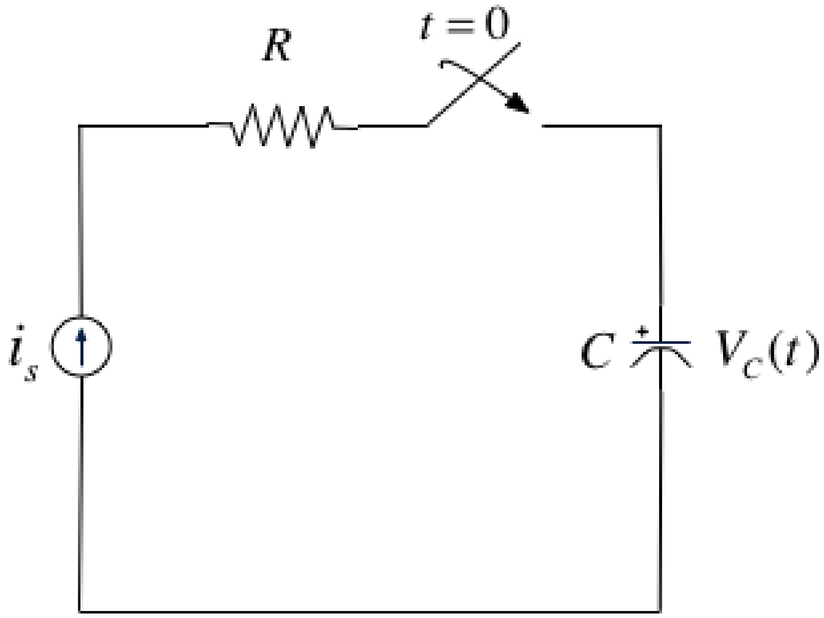

Figure 12: RC circuit

In the circuit shown in the previous slide i_s is a constant current source and the switch is closed at time $t = 0$. Show that the voltage across the capacitor can be represented as

$$v_C(t) = \frac{i_s}{C} t u_0(t)$$

and sketch the wave form.

The unit ramp function

The unit ramp function is defined as

$$u_1(t) = \int_{-\infty}^{t} u_0(\tau) d\tau$$

so

$$u_1(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

and

$$u_0(t) = \frac{d}{dt} u_1(t)$$

Note

Higher order functions of t can be generated by the repeated integration of the unit step function.

For future reference, you should determine $u_2(t)$, $u_3(t)$ and $u_n(t)$ for yourself and make a note of the general rule:

$$u_{n-1} = \frac{1}{n} \frac{d}{dt} u_n(t)$$

Details are given in equations 1.26—1.29 in the textbook.

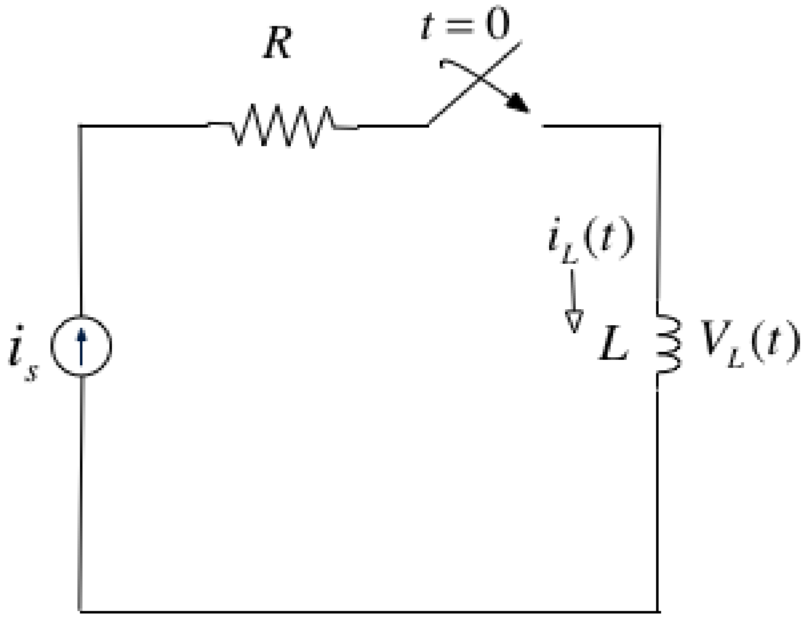

Figure 13: RL circuit

The Dirac Delta Function

In the circuit shown on the previous slide, the switch is closed at time $t = 0$ and $i_L(t) = 0$ for $t < 0$. Express the inductor current $i_L(t)$ in terms of the unit step function and hence derive an expression for $v_L(t)$.

Note

To solve this problem we need to invent a function that represents the derivative of the unit step function. This function is called $\delta(t)$ or the *dirac delta* function (named after Paul Dirac).

The delta function

The *unit impulse* or the *delta function*, denoted as $\delta(t)$, is the derivative of the unit step.

This function is tricky because $u_0(t)$ is discontinuous at $t = 0$ but it must have the properties

$$\int_{-\infty}^{t} \delta(\tau) d\tau = u_0(t)$$

and

$\delta(t) = 0$ for all $t \neq 0$.

Sketch of the delta function

Important properties of the delta function

Sampling Property

The *sampling property* of the delta function states that

$$f(t)\delta(t-a) = f(a)\delta(t-a)$$

or, when $a = 0$,

$$f(t)\delta(t) = f(0)\delta(t)$$

Figure 14: The delta function

Multiplication of any function $f(t)$ by the delta function $\delta(t)$ results in sampling the function at the time instants for which the delta function is not zero.

The study of descrete-time (sampled) systems is based on this property.

You should work through the proof for youself.

Sifting Property

The *sifting property* of the delta function states that

$$\int_{-\infty}^{\infty} f(t)\delta(t-\alpha)dt = f(\alpha)$$

That is, if multiply any function $f(t)$ by $\delta(t-\alpha)$, and integrate from $-\infty$ to $+\infty$, we will get the value of $f(t)$ evaluated at $t = \alpha$.

You should also work through the proof for yourself.

Higher Order Delta Fuctions

the n*th*-order *delta function* is defined as the n*th* derivative of $u_0(t)$, that is

$$\delta^n(t) = \frac{d^n}{dt^n}[u_0(t)]$$

The function $\delta'(t)$ is called the *doublet*, $\delta''(t)$ is called the *triplet* and so on.

By a procedure similar to the derivation of the sampling property we can show that

$$f(t)\delta'(t-a) = f(a)\delta'(t-a) - f'(t)\delta(t-a)$$

Also, derivation of the sifting property can be extended to show that

$$\int_{-\infty}^{infty} f(t)\delta^n(t-\alpha)dt = (-1)^n \frac{d^n}{dt^n}[f(t)]\bigg|_{t=\alpha}$$

Examples

Example 3

Evaluate the following expressions

1. $$3t^4\delta(t-1)$$
2. $$\int_{-\infty}^{\infty} t\delta(t-2)dt$$
3. $$t^2\delta'(t-3)$$

Example 4

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1. Express the voltage waveform $v(t)$ shown above as sum of unit step functions for the time interval $-1 < t < 7$ s
2. Using the result of part (1), compute the derivative of $v(t)$ and sketch it's waveform.

Self-study

Do the end of the chapter exercises (Section 1.7) from the textbook. Don't look at the answers until you have attempted the problems.

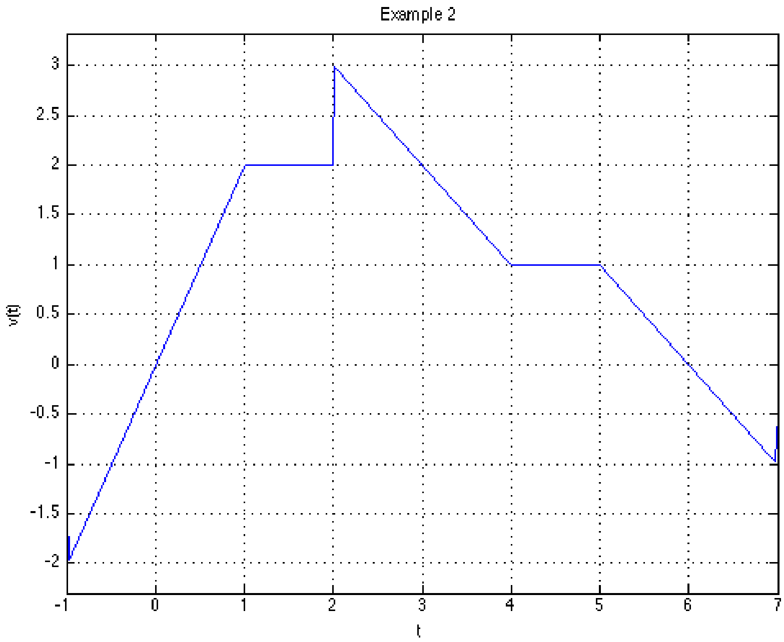

Figure 15: Example 4

Lab Work

In the lab, a week on Friday, we will solve Example 2 using Matlab/Simulink following the procedure given between pages 1-17 and 1-22 of the textbook. We will also explore the `heaviside` and `dirac` functions.