Lab. 1

Forward Modelling of Self Potential Data: Introduction

Forward Modelling of Self-Potential Method

Forward modelling deals with the computation of theoretical response for assumed model parameters. It depends on two sets of variables. One set of variables deals with the physical property and geometry of the subsurface structure and another set is locations of the observation points. In forward modelling both sets of variables are known. However, first set of variables are unknown during interpretation of field data and they are determined either by manual matching using repeated forward modelling or by automatic inversion. Further, second set of variables is known and its location of each observation point.

Forward Modelling of 2-D Inclined Sheet Type Body

An inclined sheet-type structure (Figure 1) in two dimensions can be described by a set of five model parameters, namely, electric dipole density $k = I\rho/2\pi$ (I is the current density of the medium and ρ is the resistivity of the sheet), x coordinate of the centre of the sheet x_0, depth of the centre of the sheet h, half-width of the sheet a and inclination angle α.

The general equation of SP anomaly $V(x)$ at any point P on a profile perpendicular to the strike of a 2-D inclined sheet (Murthy and Haricharan, 1985; Sundararajan et al., 1998) is written as:

$$V(x) = k \ln\left[\frac{\{(x - x_0) - a\cos\alpha\}^2 + (h - a\sin\alpha)^2}{\{(x - x_0) + a\cos\alpha\}^2 + (h + a\sin\alpha)^2}\right]$$

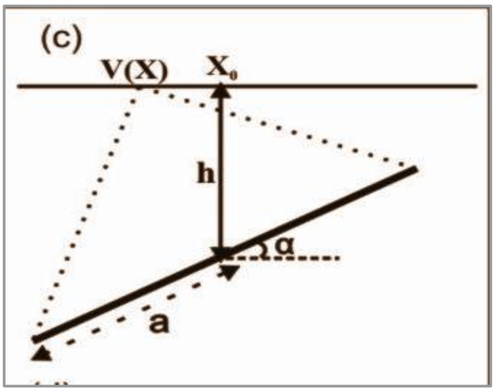


Figure 1.1: Geometrical shaped body 2D inclined sheet geometry.

Exercises

1. Write down a computer program to compute the SP response using the equation 1

```
clc
clear all;
close all;
x=0:10:1000;          %horizontal distance along the profile from 0 to 1km at 10m interval
k=50;                 %electric dipole density
x0=500;               %coordinate of centre of the sheet
h=20;                 %depth of centre of the sheet
a=50;                 %half-width of the sheet
alpha=60;             %inclination angle with horizontal

numr=((x-x0)-a*cosd(alpha)).^2 + (h-a*sind(alpha)).^2
denom=((x-x0)+a*cosd(alpha)).^2 + (h+a*sind(alpha)).^2
v=k*log(numr./denom)              %evaluation of value in Eqn 1

figure
plot (x,v)                        %plotting the graph V(x) vs x
title('Variation of Potential response with x')
xlabel('Distance')
ylabel('Potential')
```

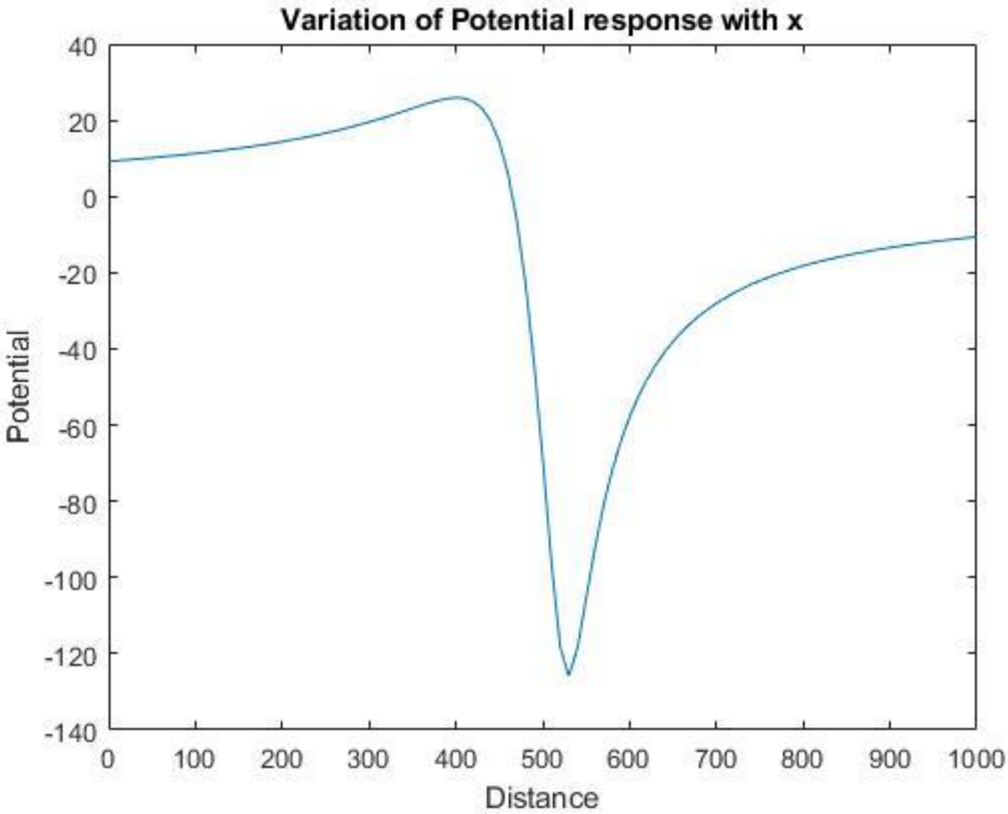


Fig 1.2: Graph showing the variation of Self-Potential response **V(x)** with position along the profile **x** with following parameters fixed: electric dipole density $\boldsymbol{k}$; depth of center of sheet $\boldsymbol{h}$; position of center of the sheet $\boldsymbol{x_0}$; half width of the sheet $\boldsymbol{a}$; angle of inclination from horizontal $\boldsymbol{\alpha}$; with values, $\boldsymbol{k}$= 50, $\boldsymbol{h}$= 20 m, $\boldsymbol{a}$=50 m, $\boldsymbol{x_0}$=500 m, $\boldsymbol{\alpha}$=60^0

2. Write down the variation of the response curves with variation in the parameters: x_0, depth, half width, inclination angle and electric dipole density

i) Variation of Potential response curve with changing x_0

The distance x_0 in Equation (1), acts as the offset distance with respect to the scale of profiling. In other words, the distance x_0 puts no effect on the intrinsic characteristics in measurements, but simply shows a lateral shift in response in the direction of profiling. In the following set of curves, with all other parameters fixed, the responses with exactly same shape is found to be simply shifted towards the +*x*-direction with an increasing positive value of the co-ordinate of the center of the sheet (x_0).

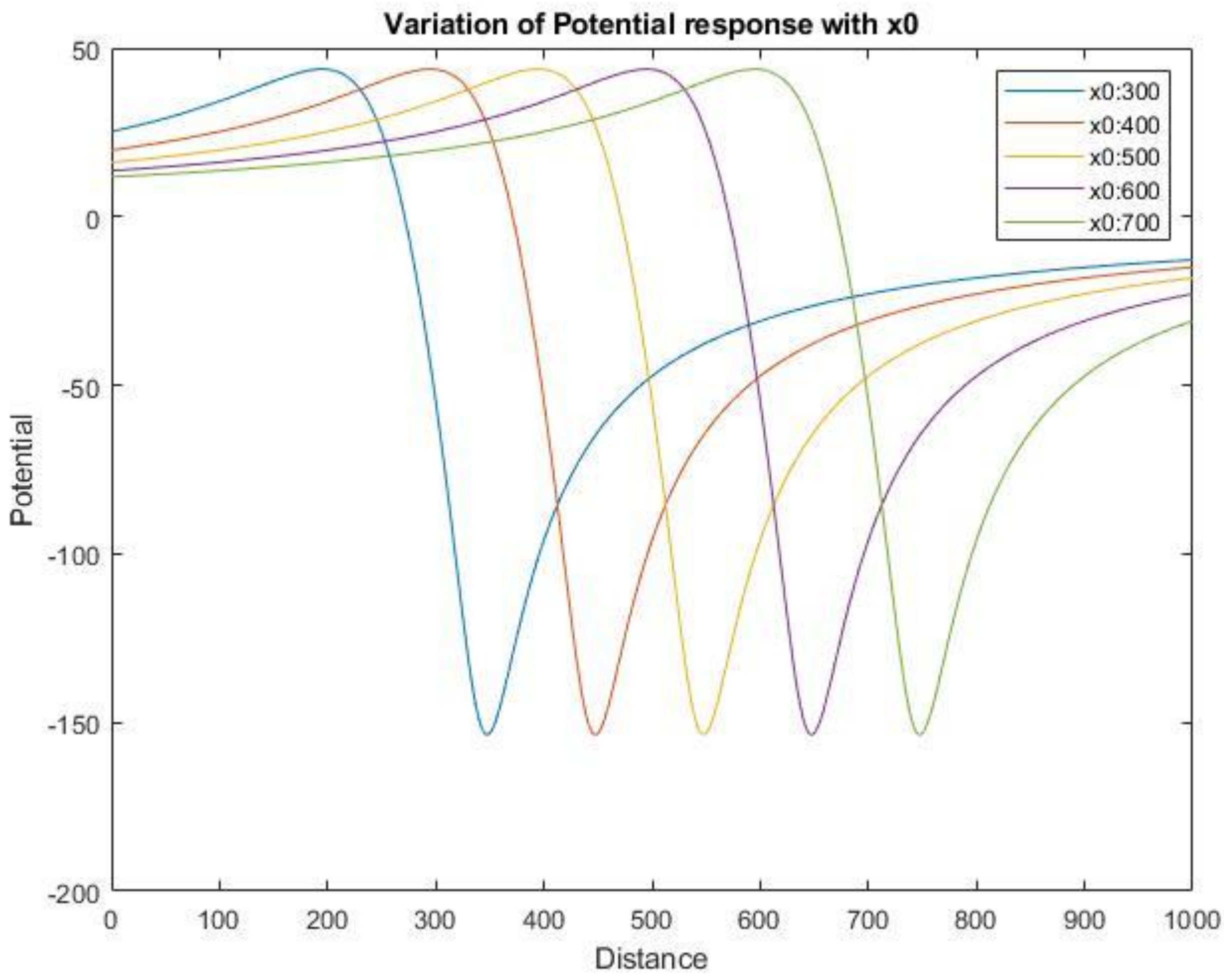


Fig 1.3: Potential response against x with changing $\mathbf{x_0}$ at $\boldsymbol{k}$= 50, $\boldsymbol{h}$= 50 m, $\boldsymbol{a}$=50 m, $\boldsymbol{\alpha}$=30^0

ii) Variation of Potential response curve with changing depth

The depth ‘h’ in Equation (1) can be identified to be related to the magnitude of generated Self-Potential from the derived set of curves. As can be implied from the curve the peak values of Self-Potential keeps on reducing with increasing values of ‘h’. We see that not only the peak values of curves are reduced with an increasing ‘h’, but also the response becomes more flat in this trend. In other words, the responses are steeper and of higher magnitude at lower values of ‘h’ (when the body is near the surface) and they become comparatively smooth, having a low magnitude at higher ‘h’ values (when the body is deep-seeded).

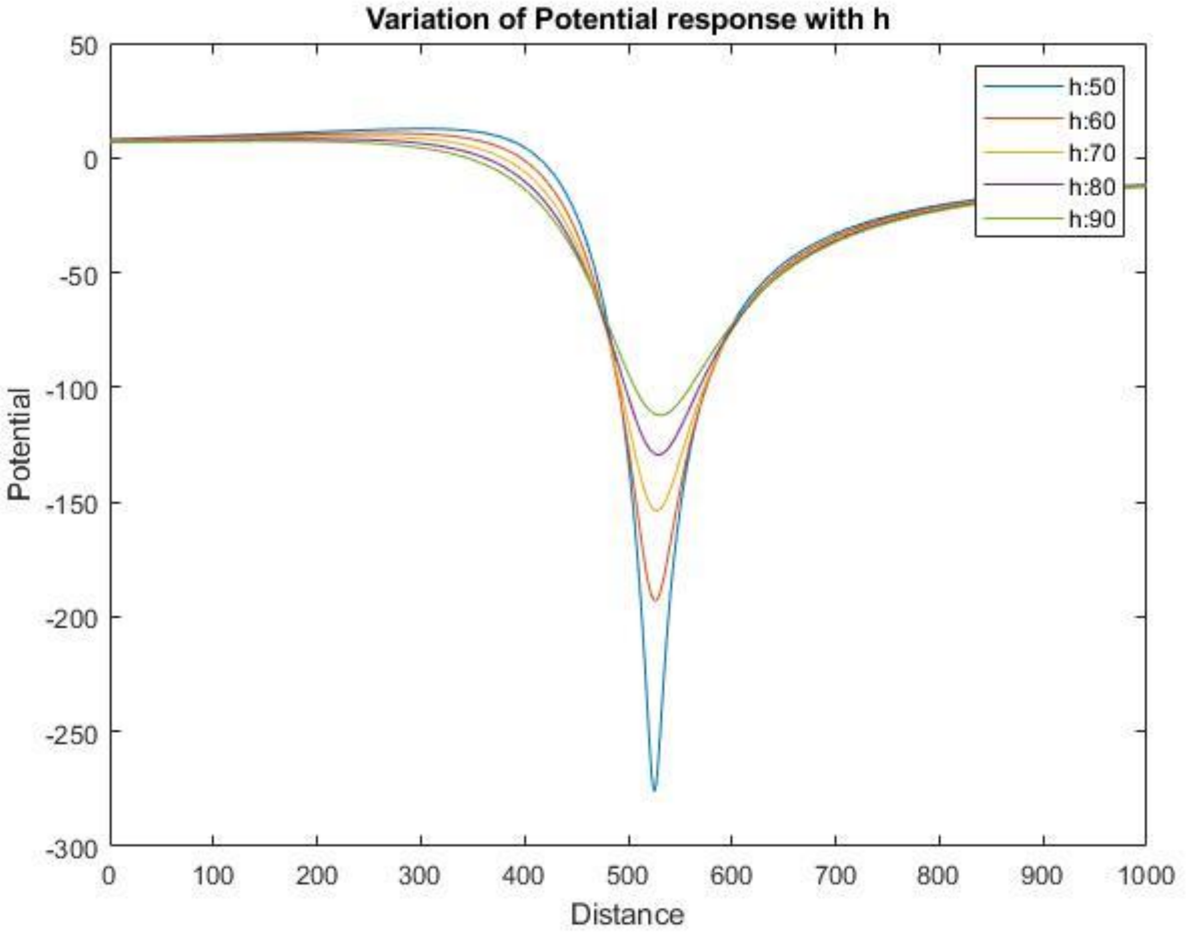


Fig 1.4: Potential response against x with changing **h** at ***k***= 50, ***a***=50 m, x_0=500 m, α=60^0

iii) Variation of Potential Response curve with changing half-width

The half-width 'a' in Equation (1) effects the value of the Self-Potential because of the fact that it is related to the size of the body. So when the body has large value of 'a' (hence a larger size), higher magnitude of Self-Potential contribution is expected, and that with smaller size, contribution be only up to a small extent. This feature is clearly visible in the following set of curves with varying half-width: the lower values of 'a' has a comparatively flat curve with a small magnitude than those with a higher value of 'a', having a steep response with larger magnitude.

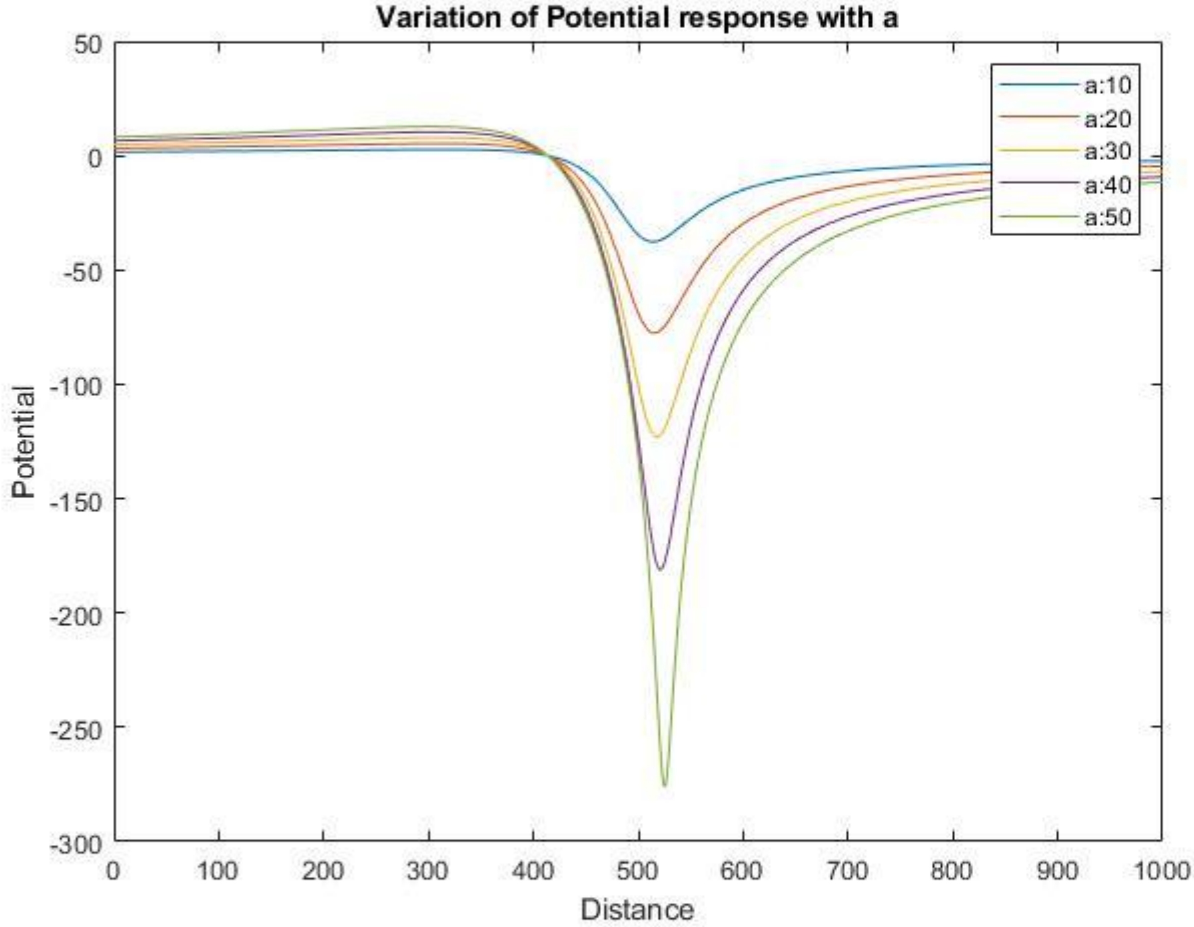


Fig 1.5: Potential response against x with changing **a** at ***k***= 50***,*** ***h***= 50 m***,*** x_0=500 m***,*** α=60^0

iv) *Variation of Potential response with changing inclination angle*

The inclination angle ‘α’ of the inclined planar body in Equation (1) can be seen to be effecting the shape of the response from the generated set of curves. The S-P responses are less intense and asymmetric at lower inclination angles, whereas they become more intense and symmetric towards higher values of ‘α’ (tending to the right angle). As the value of ‘α’ exceeds 90^0, the S-P responses are found to be the mirror image of that of their supplementary ‘α’ (supplementary angles sum up to total of 180^0).

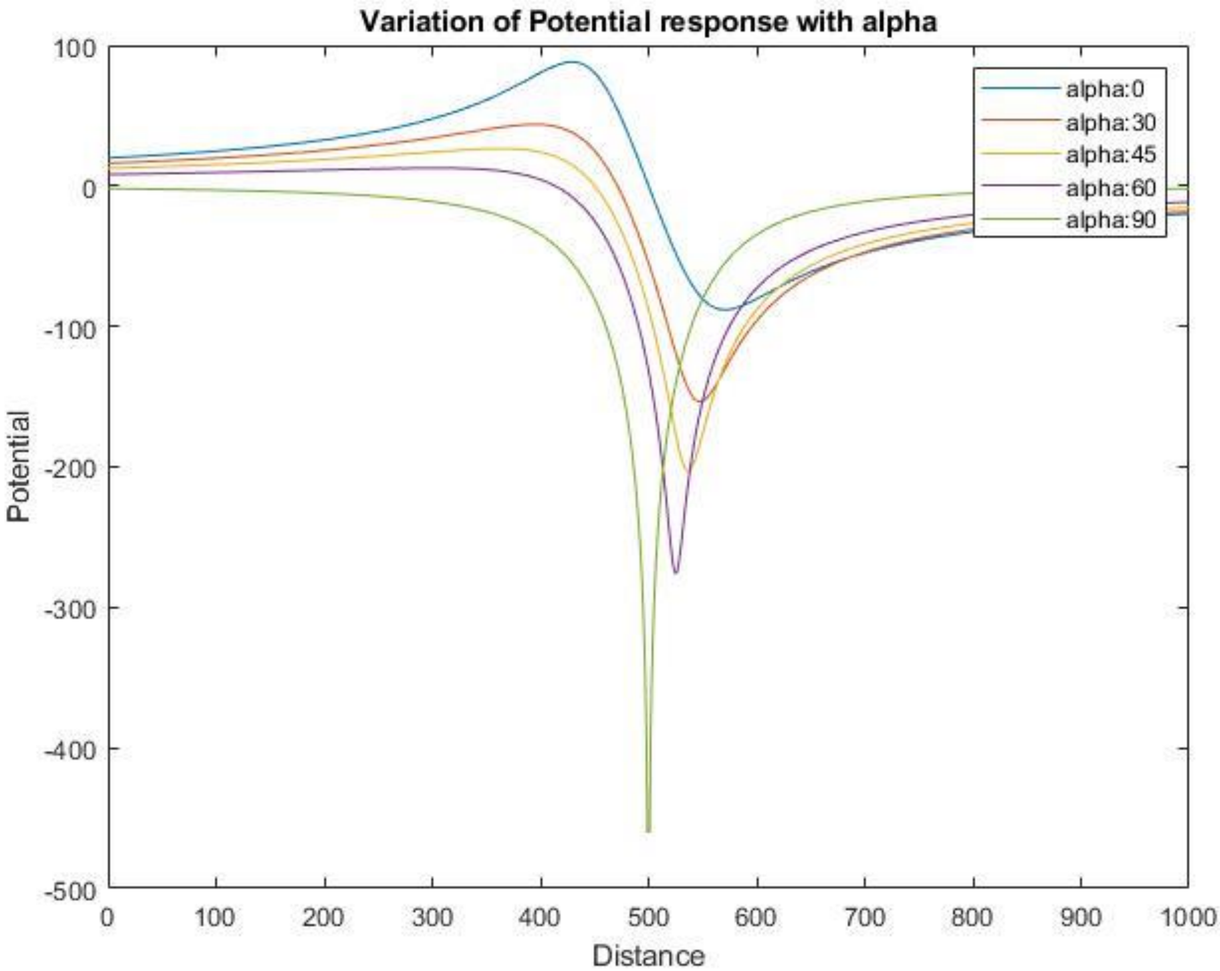


Fig 1.6: Potential response against x with changing **α** at ***k***= 50***,*** ***h***= 50 m***,*** ***a***=50 m***,*** x_0=500 m

v) Variation of Potential Response with changing electric dipole density

The electric dipole density ‘k’ appears as the product term on the RHS of the Equation (1) to determine the Self-Potential. Therefore, the Self-Potential generated is directly proportional to ‘k’. As seen in the following set of curves, for fixed values of all other parameters, an increasing value of ‘k’ simply increases the magnitude of the generated Self-Potential, and vice-versa.

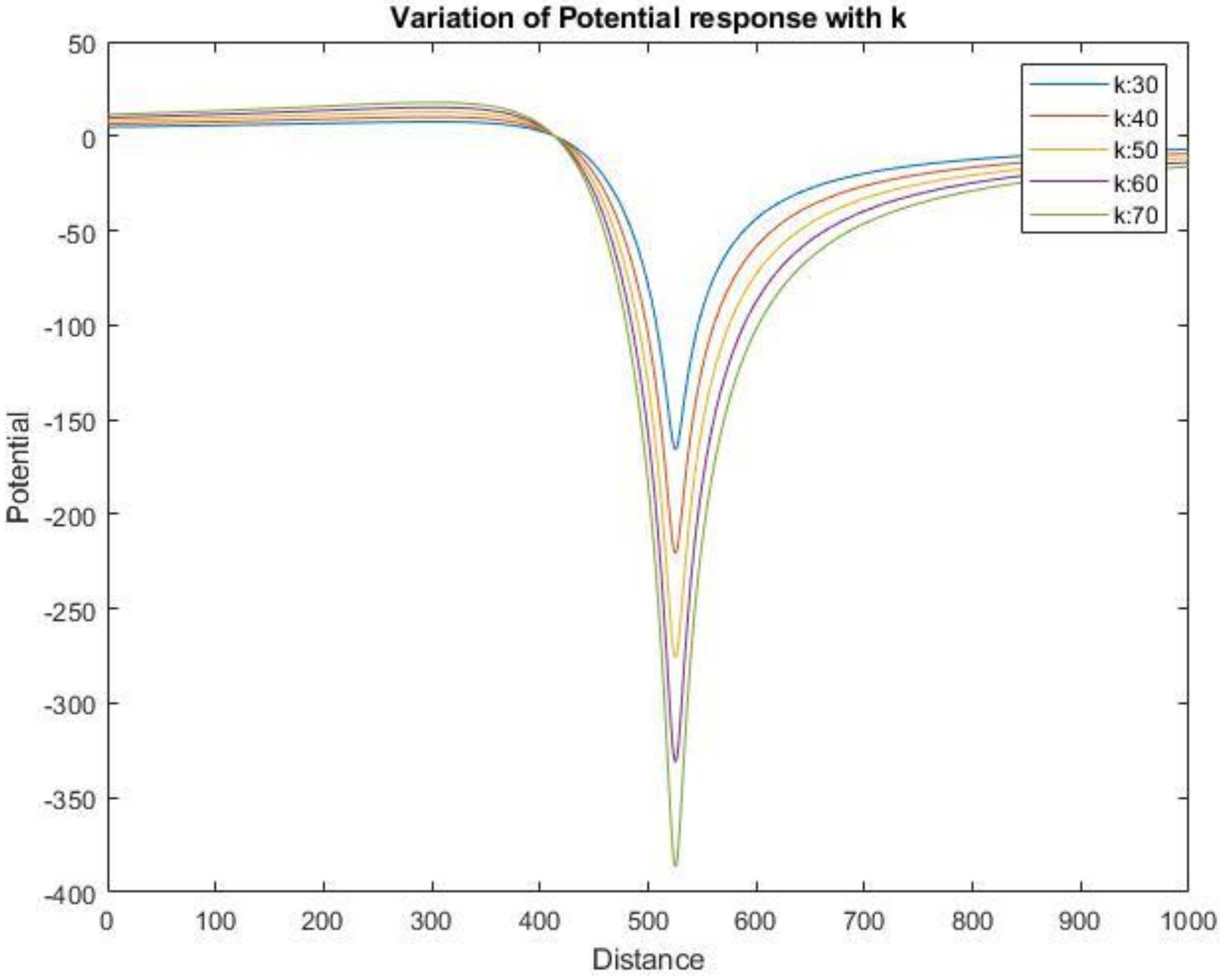


Fig 1.7: Potential response against x with changing **k** at $\boldsymbol{h}$= 50 m, $\boldsymbol{a}$=50 m, $\boldsymbol{x_0}$=500 m, $\boldsymbol{\alpha}$=60^0

Lab. 2

Forward Modelling of Self Potential Data: Modelling

Exercise

Interpret the given three self-potential sounding datasets (see the attached excel sheet). Write the model parameters of each self-potential datasets and show the fittings between the measured data and modelled data sets. Compute the root mean square error between measured data and modelled datasets:

Distance	SP1(mv)	SP2(mv)	SP3(mv)
0	-10.176	8.5057	-26.63
5	-10.298	8.5899	-27.046
10	-10.422	8.6757	-27.474
15	-10.55	8.7632	-27.916
20	-10.681	8.8525	-28.372
25	-10.815	8.9435	-28.842
30	-10.953	9.0363	-29.329
35	-11.094	9.131	-29.831
40	-11.238	9.2276	-30.351
45	-11.386	9.3262	-30.889
50	-11.539	9.4269	-31.446
55	-11.695	9.5297	-32.023
60	-11.856	9.6346	-32.62
65	-12.021	9.7418	-33.241
…	…	…	…
…	…	…	…
…	…	…	…
800	10.561	-13.315	17.011

Program for the given Problem

```
x=datasp{:,1};                 %importing values of x
sp1=datasp{:,2};               %importing the response for SP1
sp2=datasp{:,3};               %importing the response for SP2
sp3=datasp{:,4};               %importing the response for SP3
k=100;                         %electric dipole density
x0=350;                        %coordinate of centre of the sheet
h=40;                          %depth of centre of the sheet
a=30;                          %half-width of the sheet
alpha=135;                     %inclination angle with horizontal

numr=((x-x0)-a*cosd(alpha)).^2 + (h-a*sind(alpha)).^2;
denom=((x-x0)+a*cosd(alpha)).^2 + (h+a*sind(alpha)).^2;
v=k*log(numr./denom);          %evaluation of value in Eqn 1

err=sp3-v;                     %%
sq_err=err.^2;                  %%   evaluation of rms of error between calculated and
avg=mean(sq_err);               %%   measured data
rms=avg.^0.5;                  %%

plot(x,sp3,'r*');
hold on;
plot(x,v);
title('Variation of Self-Potential response with x');
legend('measured model','calculated model');
xlabel('Distance (m)');
ylabel('Self Potential (mV)');
```

In the above program we change the values for k, x_0, a and alpha so as to find the appropriate model parameters to fit the given curve. The program here is written to model the SP3 potential response, and can be modified easily to model the responses SP1 and SP2 also.

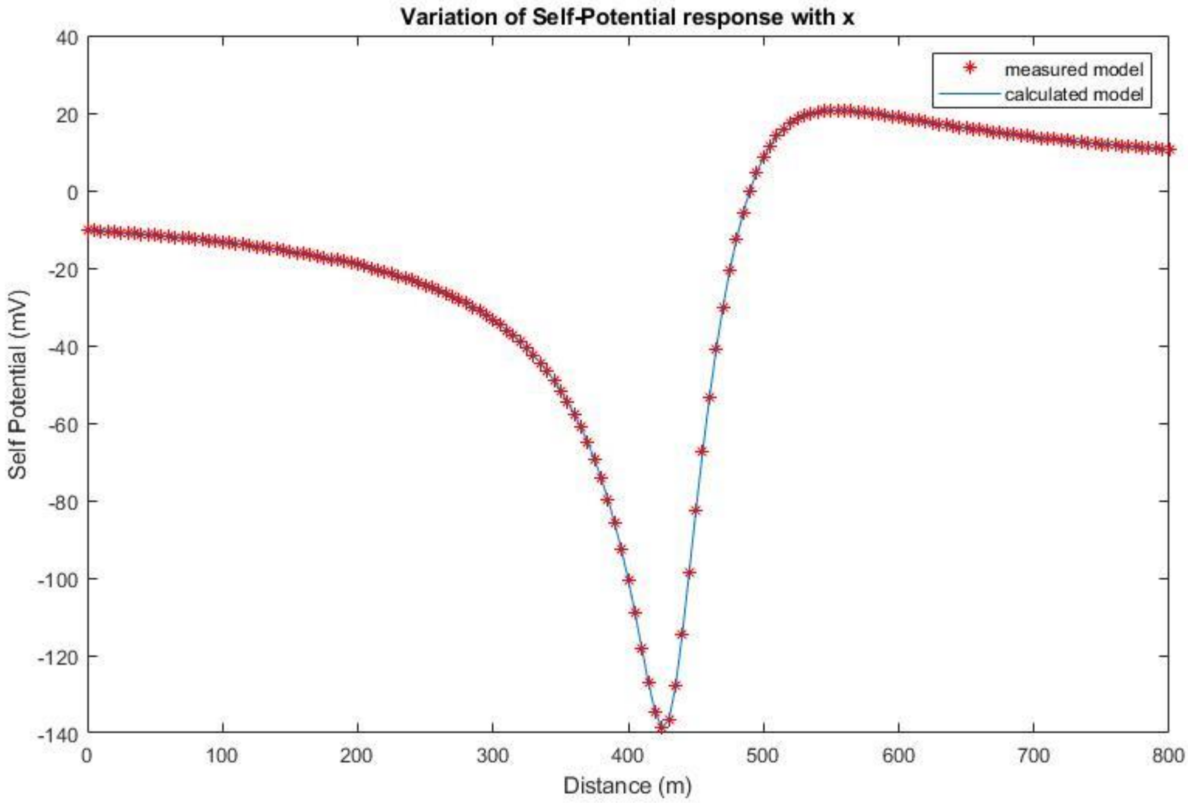
Here we also determine the rms value of the error. We aim to find model parameter in such a way that the rms value of the error be minimized for the given response.

Curve Matching and determination of Model-parameters for the given Data-sets:

For the purpose of curve-fitting, the following were taken in account (with a given fixed value of h= 40 m):

i) For matching the peak value of S-P response, the value of co-ordinate of center of the sheet (x_0) is adjusted.
ii) For matching the shape of the response, the value of inclination angle (α) is checked.
iii) For matching the magnitude of the response curve, the values of electric dipole density (k) and the half-width (a) were adjusted.

1. **Model parameter for Self-Potential sounding data-set SP1**
   Electric dipole density (k) = 50
   Co-ordinate of center of the sheet (x_0) = 450 m
   Inclination angle (α) = 135^0
   Half-width of the sheet (a) = 30m
   Depth of center of the sheet (h) = 40 m



The Root Mean Square (rms) error is calculated to be: **$error_{rms}$= 7.0629 × 10^{-4}**

2. **Model parameter for Self-Potential sounding data-set SP2**

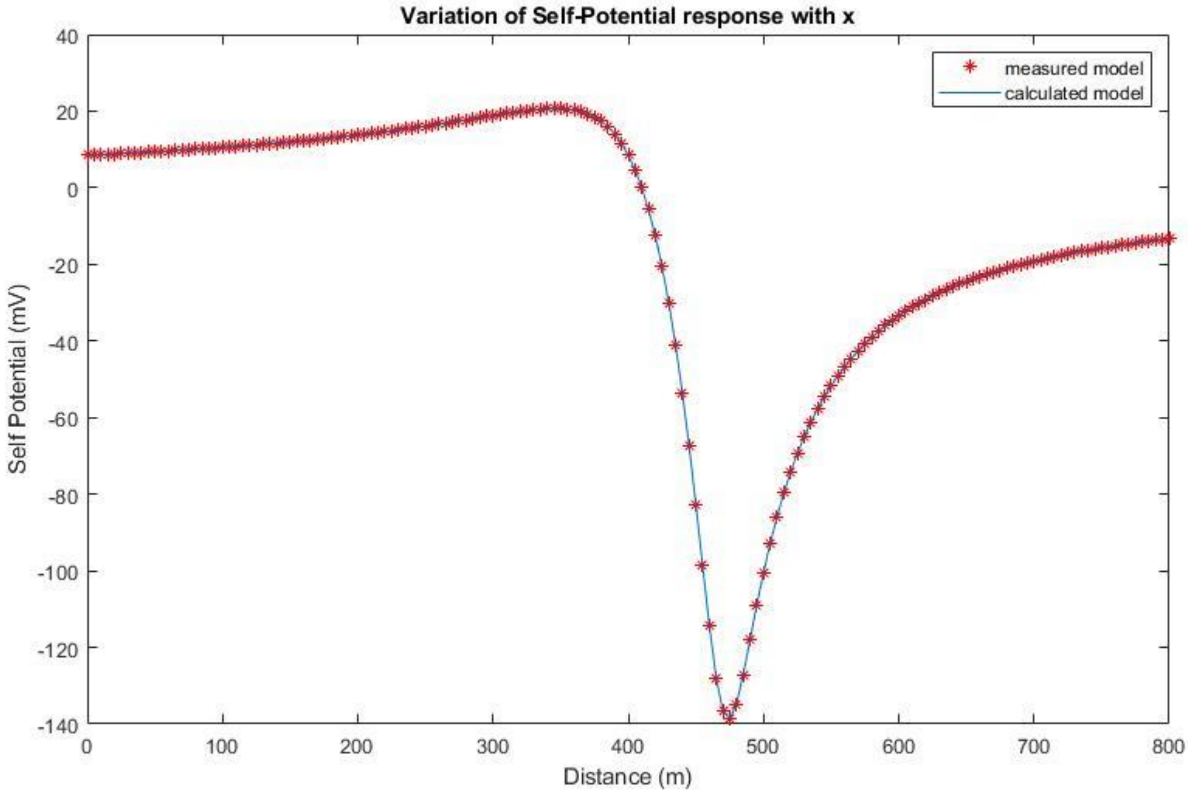
Electric dipole density (k) = 50

Co-ordinate of center of the sheet (x_0) = 450 m

Inclination angle (α) = 45^0

Half-width of the sheet (a) = 30m

Depth of center of the sheet (h) = 40 m



The Root Mean Square (rms) error is calculated to be: **$error_{rms}$= 7.0014 × 10^{-4}**

3. **Model parameter for Self-Potential sounding data-set SP3**

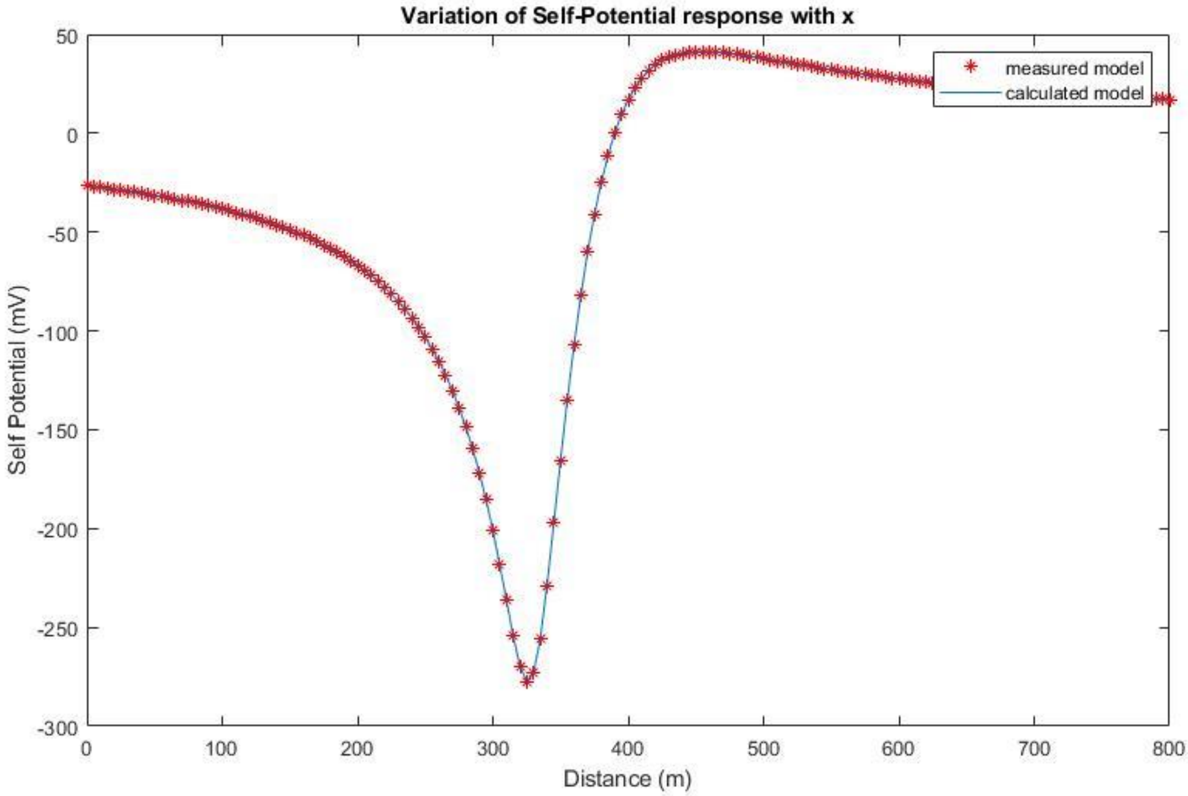
Electric dipole density (k) = 100

Co-ordinate of center of the sheet (x_0) = 350 m

Inclination angle (α) = 135^0

Half-width of the sheet (a) = 30m

Depth of center of the sheet (h) = 40 m



The Root Mean Square (rms) error is calculated to be: **$error_{rms}$= 0.0012**

Lab. 3

Resistivity modelling of the sub-surface: Wenner configuration

Exercise

Given a Wenner configuration data set consisting of electrodes separation (a), potential measured (V) and current measured (I). Find out following by plotting the values:

1. The resistivity of the top layer
2. Thickness of the top layer

Describe upon the pattern of the resistivity.

S. No.	V (m Volt)	I (m Amp)	a (cm)
1.	20.3	70	1.5
2.	9.6	45	3
3.	9.8	65	5
4.	7.6	46	8
5.	9.9	56	10
6.	7.8	38	12
7.	6.5	25	15
8.	6.5	18	18
9.	5.9	17	20
10.	6.2	12	24

Hint:

1. Use formula $k = 2 \times \pi \times a$ (geometric factor)

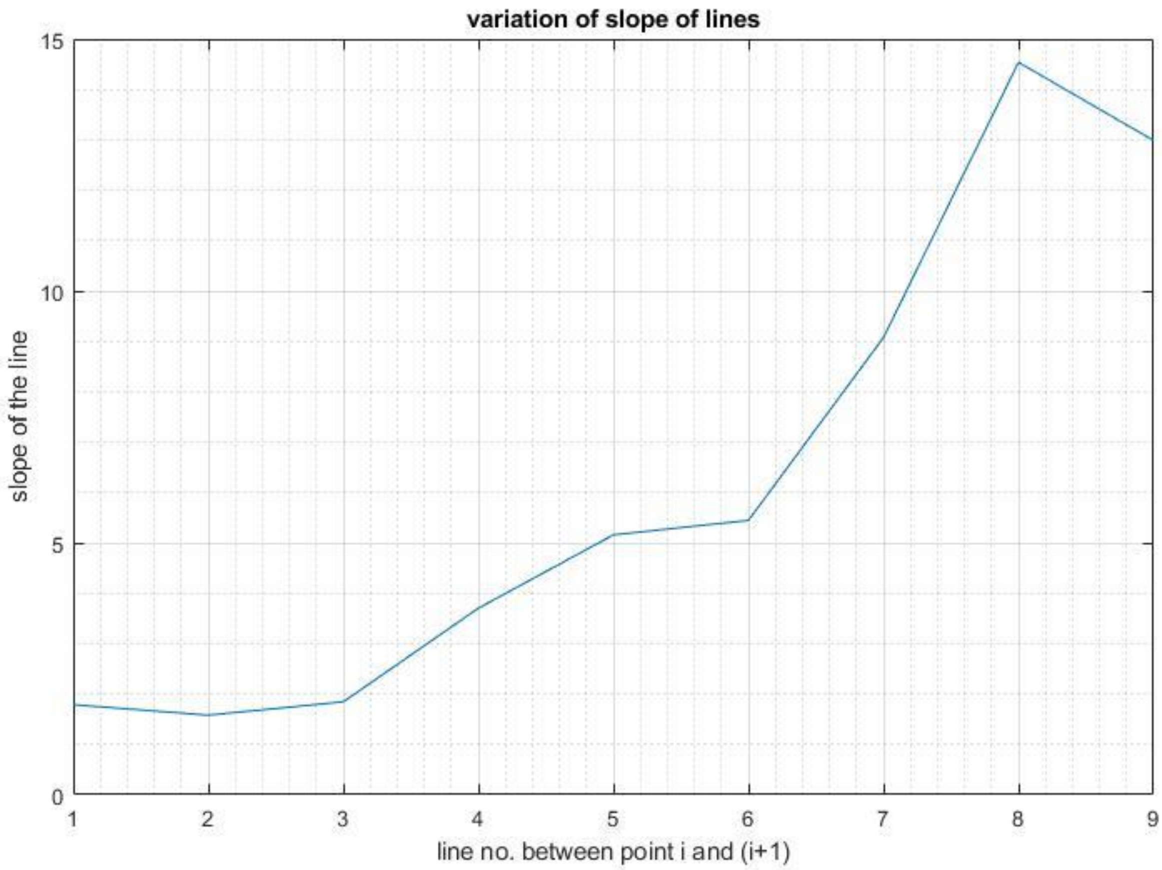
2. Plot cumulative resistivity vs 3*a/2. And calculate depth of first layer by the intersection of the two tangents drawn at the points of maximum slope change.

3. Resistivity of the first layer should be the apparent resistivity measured at small spacing (Think of some averaging).

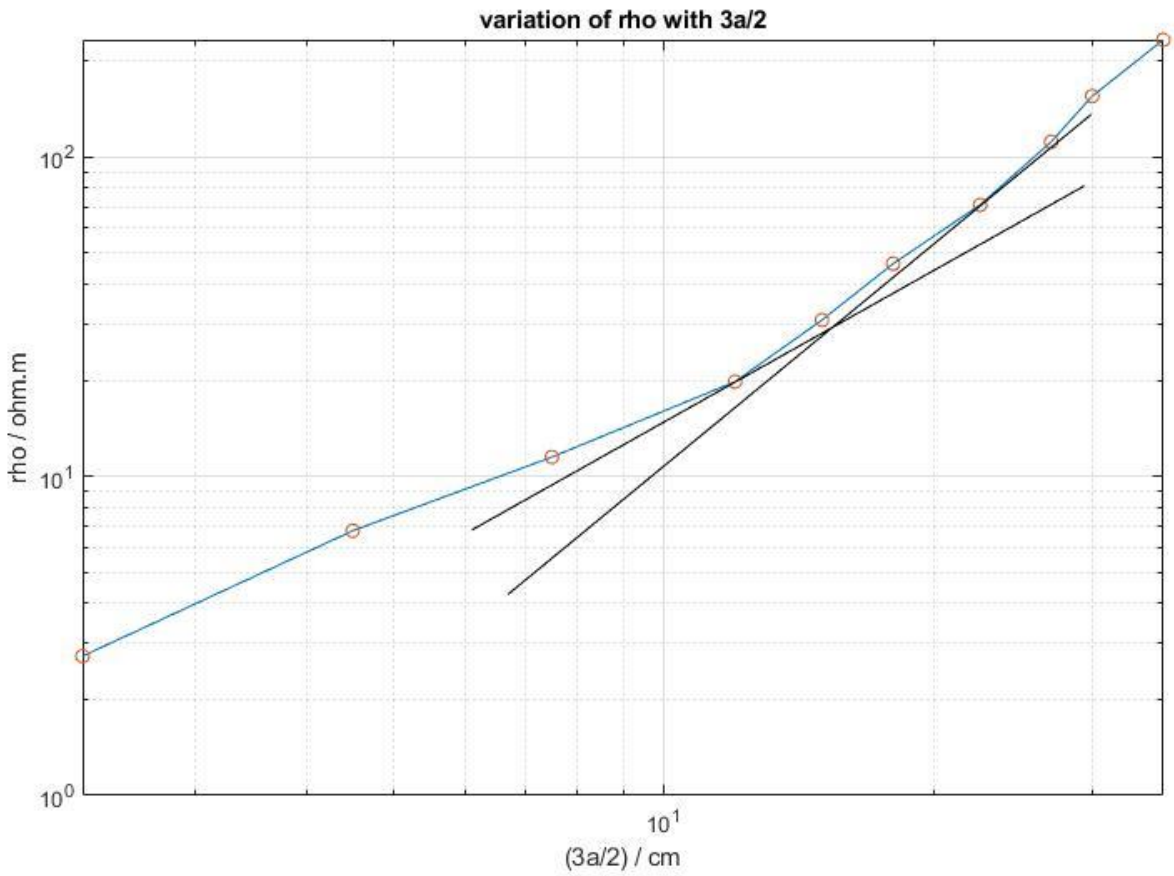
Program for the given Problem

```
v=[20.3 9.6 9.8 7.6 9.9 7.8 6.5 6.5 5.9 6.2];
v=v';                                                %inputting V column
i=[70 45 65 46 56 38 25 18 17 12];
i=i';                                                %inputting I column
a=[1.5 3 5 8 10 12 15 18 20 24];
a=a';                                                %inputting a column
rho=2*pi*a.*v./i;                        %calculation of rho for different layers
newx=3*a/2;                              %column vector with values of (3a/2)
newrho=cumsum(rho);                      %calculating cumulative resistivity
figure
loglog(newx,newrho);                     %plotting cum. resistivity vs (3a/2)
hold on;
scatter(newx,newrho);                    %highlighting the data points
xlabel('(3a/2) / cm');
ylabel('rho / ohm.m');
title('variation of rho with 3a/2');
slope=diff(newrho)./diff(newx);          %calculation of slope between consecutive points
hold off;
plot(slope);                             %plotting the values of the slope
xlabel('line no. between point i and (i+1)');
ylabel('slope of the line');
title('variation of slope of lines')
```

Plotting the values of slopes for identifying points of tangents to determine depth of first layer:



Plotting the values of cumulative values of ρ vs (3a/2) and determination of depth of first layer:



1. **Thickness of the top layer**

The thickness of the top layer is determined by drawing tangent in the plot of cumulative ρ vs (3a/2), at the points with maximum slope variations and determine the point of intersection from the plot. Therefore from the graph we determined:

Point of intersection (from the log-log plot): 17 cm (x-coordinate)

Therefore the thickness of the first layer can be implied to be **17 cm**.

2. **The resistivity of the top layer**

The resistivity of the top layer can be determined by the following expression:

$$\rho = 2\pi a \times \frac{V}{I}$$

where, V and I are the potential and current values corresponding to the depth of penetration of the current in the Wenner method. Value of a is equal to 17 cm (Explained later).

Therefore, the value is determined to be ρ= 0.3857 Ω.m

Justification: Since, we know that the depth of penetration of currents in Wenner configuration for DC resistivity methods is one-third of the separation of the current electrodes. Therefore,

$$\text{Depth of penetration} = 1/3 \times (3a) = a$$

Hence, the value of a= 17 cm.

<u>Why are we representing the resistivity data on logarithmic scale?</u>

The values of resistivity variations has a large range of coverage. Also the rate of change of the values of resistivity are very slow. So it is difficult to notice any significant changes in ρ along with changing parameter (here 3a/2). Hence we represent our resistivity data in the logarithmic scale in order to obtain information about change in behavior of the resistivity along with changing electrode separation.

Lab. 4

Computation of apparent resistivity for N-layer resistivity model for Schlumberger configuration

Write a program to compute apparent resistivity for schlumberger array for N-layer resistivity model.

Theory- First, for a particular value of electrode separation 's_i, $i=1,ns$ (s is half of current electrode separation), determine Resistivity transform $T_1(\lambda_j)$, $j=1, M$ using the relation (M number of filter coefficients) $\lambda_j = 10^{(a_j - log_{10} s_i)}$ where a_j are the base10 abscissa values of filter coefficients in given table below.

The resistivity transform for a N layer case for a particular value of λ_j is given by the recurrence relation

$$T_{k-1}(\lambda_j) = \frac{T_k + \rho_{k-1}\tanh(\lambda_j h_{k-1})}{1 + \frac{T_k \tanh(\lambda_j h_{k-1})}{\rho_{k-1}}}$$

$k=N,N-1,\ldots\ldots2$. ρ_k and h_k are resistivity and thickness of k^{th} layers. Resistivity transform $T_N=\rho_N$

The Schlumberger apparent resistivity is then given by

$$\rho_a(s_i) = \sum_{j=1}^{M} f_j\, T_1(\lambda_j)$$

f_j, $j=1$, M are filter coefficients.

Table: Nineteen point filter

	Abscissa of filter coefficients (a_j)	Filter coefficients (f_j)
1	-0.980685	0.00097112
2	-0.771995	-0.00102152
3	-0.563305	0.00906965
4	-0.354615	0.01404316
5	-0.145925	0.09012
6	0.062765	0.30171582
7	0.271455	0.99627084
8	0.480145	1.3690832
9	0.688835	-2.99681171
10	0.897525	1.65463068
11	1.106215	-0.59399277
12	1.314905	0.22329813
13	1.523595	-0.10119309
14	1.732285	0.05186135
15	1.940975	-0.02748647
16	2.149665	0.01384932
17	2.358355	-0.00599074
18	2.567045	0.00190463
19	2.775735	-0.0003216

Exercise

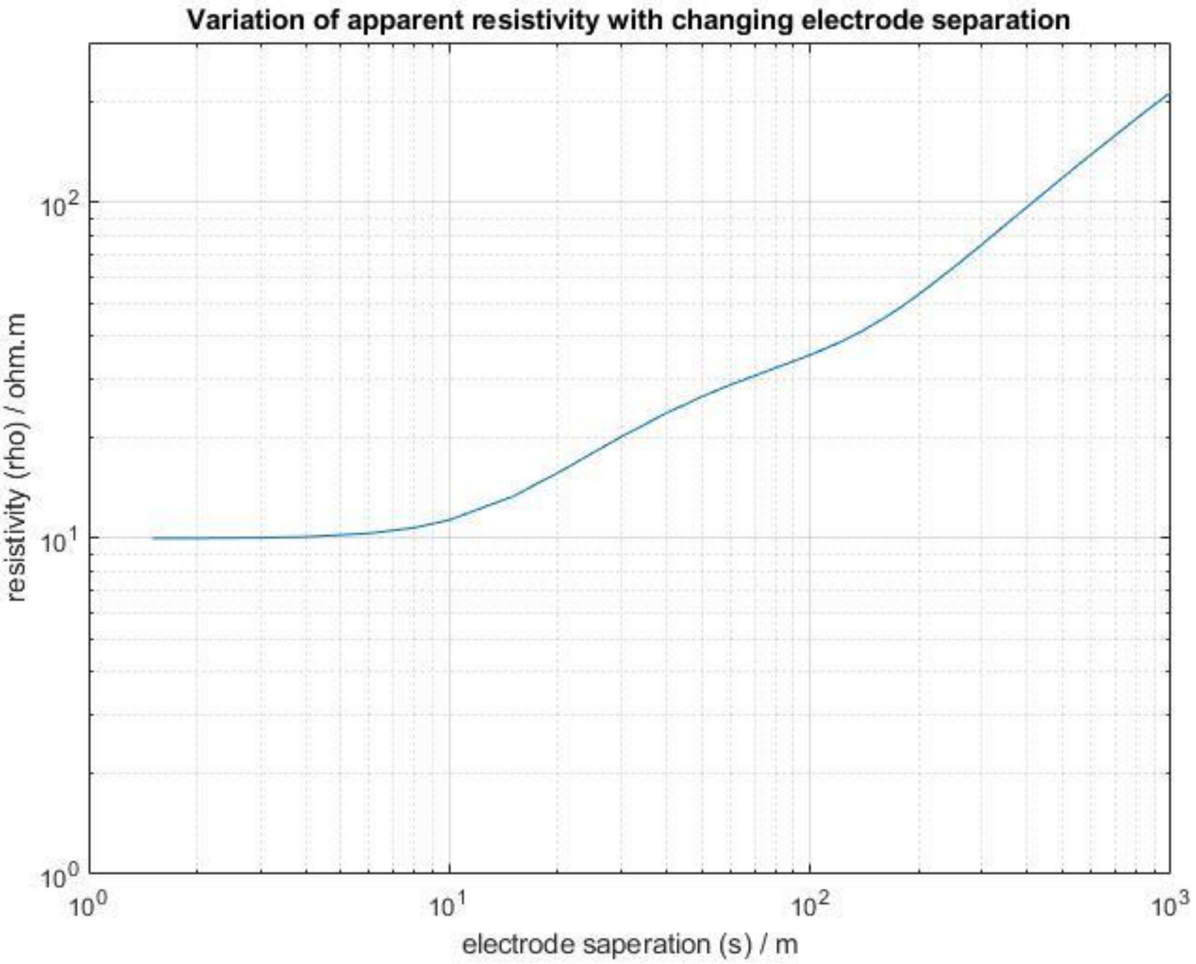
Use **s**=1.5, 2, 3, 4, 6, 8, 10, 15, 20, 25, 30, 40, 50, 60, 80, 100, 120, 140, 160, 180, 200, 250, 300, 350, 400, 500, 600, 800, 1000. m

Plot s verses ρ_a on log-log scale.

MATLAB program:

```
a=[-0.980685 -.771995 -.563305 -.354615 ...
   -.145925  .062765  .271455  .480145  .688835 ...
   0.897525 1.106215 1.314905 1.523595 1.732285 ...        %inputting abscissa of filter coefficient
   1.940975 2.149665 2.358355 2.567045 2.775735];
f=[ 0.00097112 -0.00102152  0.00906965  0.01404316 ...
    0.09012     0.30171582  0.99627084  1.3690832  -2.99681171 ...
    1.65463068 -0.59399277  0.22329813 -0.10119309  0.05186135 ...    %inputting filter coefficients
    -0.02748647  0.01384932 -0.00599074  0.00190463 -0.0003216 ];
s=[1.5 2 3 4 6 8 10 15 20 25 30 40 50 60 80 100 ...             %inputting the electrode separations
   120 140 160 180 200 250 300 350 400 500 600 800 1000];       % (half of current electrode seprn)
ns=29;
rhoa=zeros([1 29]);                          %vector to store apparent resistivity for different ‘s’
rhot=[10 50 20 700 350 1000];                %true resistivity of all layers (considering 6 layers)
h=[10 30 40 50 70];                    %thickness of top 5 resistivity layers (considering 6 layers)
lambda=zeros([1 19]);
t=zeros([1 6]);                         %vector to store Resistivity transform for each value of ‘s’
sumn=0;
for i=1:length(s)                             %loop for calculation of resistivity for different ‘s’
  sumn=0;
  t=zeros([1 6]);
  for j=1:length(a)                                %loop for summing values from filter coefficients
    lambda(j)=10.^(a(j)-log10(s(i)));
    t(6)=rhot(6);
    for k=1:5                                                 %loop for determining T1 for each ‘s’
      num=t(7-k)+(rhot(6-k)*tanh(lambda(j)*h(6-k)));
      den=1+(t(7-k)*tanh(lambda(j)*h(6-k))/rhot(6-k));
      t(6-k)=num/den;
    end
    sumn=sumn+(f(j)*t(1));
  end
  rhoa(i)=sumn;
end
loglog(s,rhoa);                                      %plotting the rho vs ‘s’ curve in log-log scale
title('Variation of apparent resistivity with changing electrode separation');
xlabel('electrode saperation (s) / m');
ylabel('resistivity (rho) / ohm.m');
grid on
ylim([1, 300]);                                      %limiting the extent of y-scale
```

<u>Output result:</u>



Lab. 5

Forward Modelling of DC resistivity data

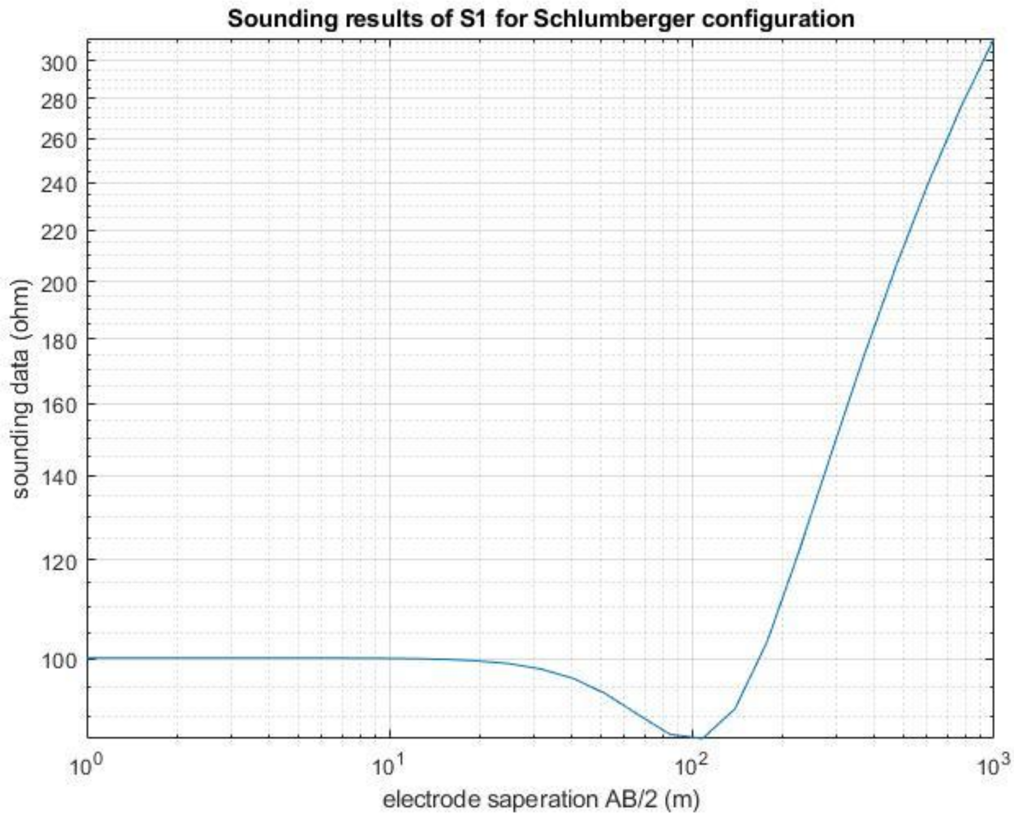
Exercise

Q1. Interpret the given Schlumberger sounding data in terms of curve types (for eg. H, K etc.)

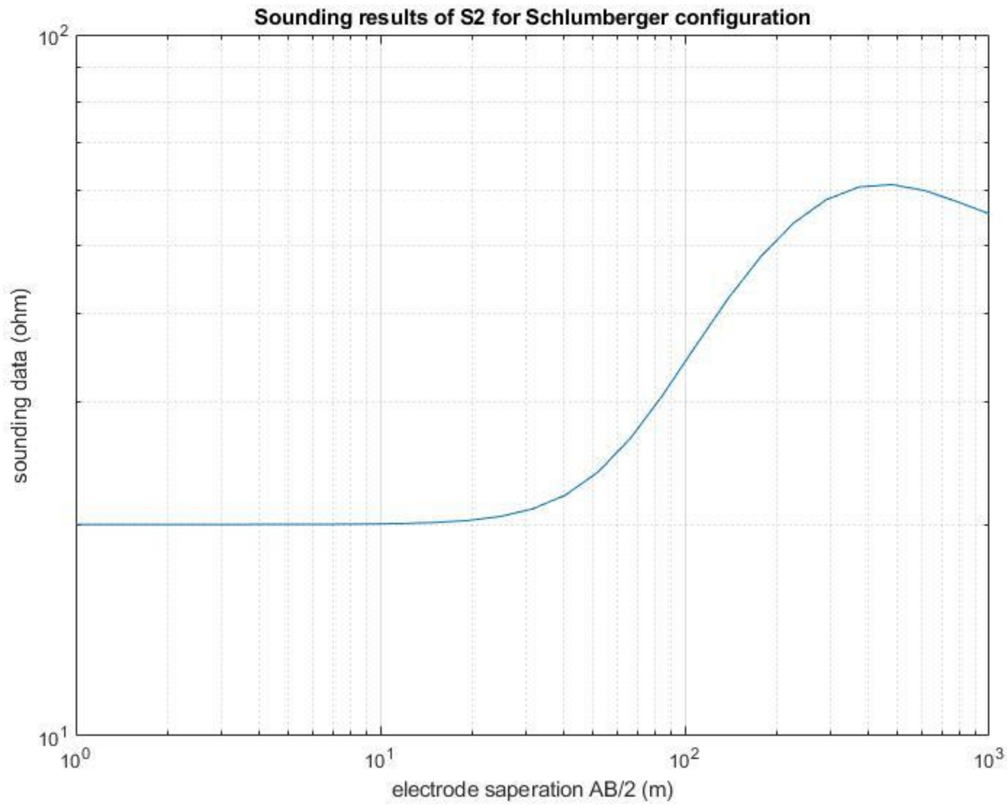
MATLAB Program:

```
x=datadc{:,1};                    % inputting distance along profile from datadc.csv
s1=datadc{:,2};                   % inputting values of profile 1
s2=datadc{:,3};                   % inputting values of profile 2
s3=datadc{:,4};                   % inputting values of profile 3
s4=datadc{:,5};                   % inputting values of profile 4

loglog(x,s1);                     % plotting the data in log-log scale
grid on;
xlabel('electrode saperation AB/2 (m)');
ylabel('sounding data (ohm)');
title('Sounding results of S2 for Schlumberger configuration')
```

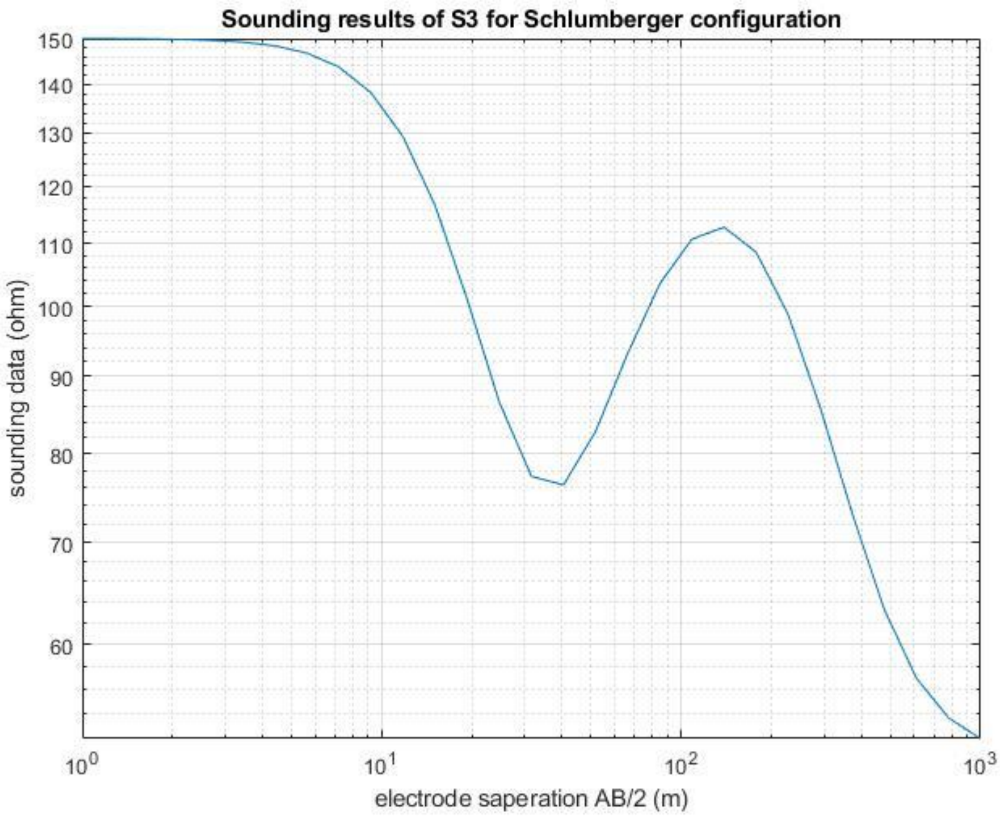
This program can be modified with different values (s2, s3 & s4) in 6th line to obtain sounding curve for different profiles for the given data for Schlumberger configuration



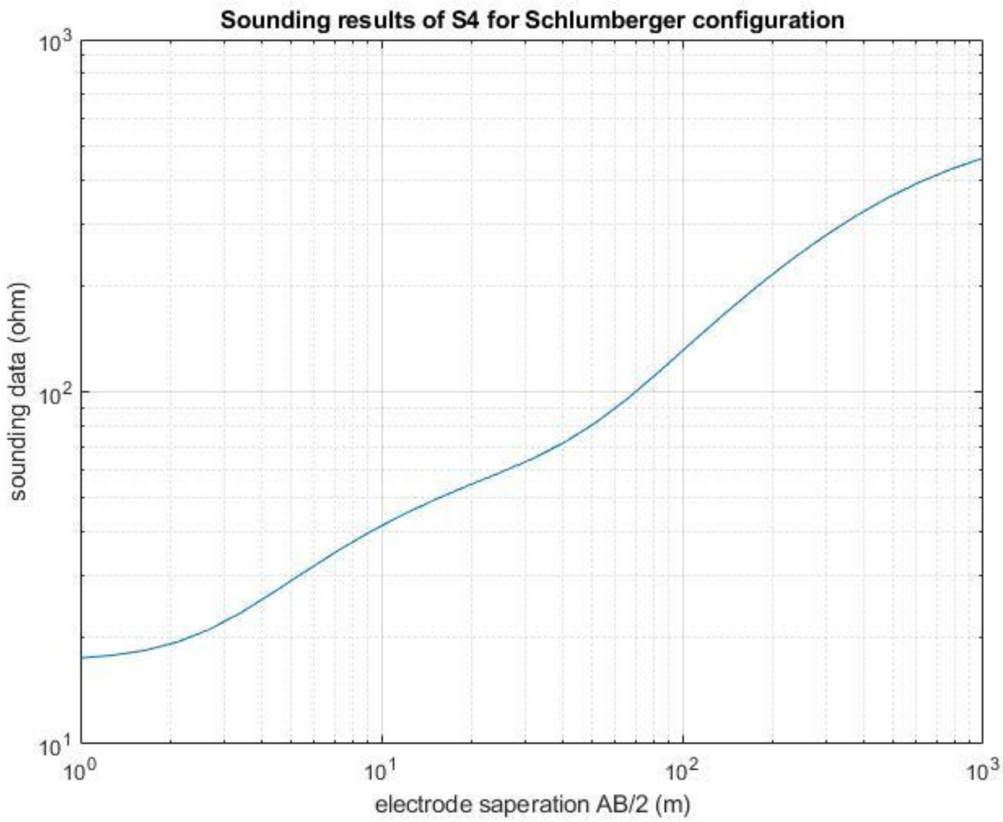
Interpretation for S1: The following curve can be observed to be belonging to **H-type** curve. The given data can be implied to be resulted from 3-layer subsurface model.



Interpretation for S2: The following curve can be observed to be belonging to **K-type** curve. The given data can be implied to be resulted from 3-layer subsurface model.



Interpretation for S3: The following curve can be observed to be belonging to **H-K-type** curve. The given data can be implied to be resulted from 4-layer subsurface model.



Interpretation for S4: The following curve can be observed to be belonging to **A-A-type** curve. The given data can be implied to be resulted from 4-layer subsurface model.

Q2. Interpret the given Schlumberger sounding data. Write the model parameters (layers and thickness) and show the fittings between the measured data and model data by calculating rms error.

Following is the program used to model different profile data given. The program is written for the profile data S4, in which we are modelling the subsurface with 4-layers, by trying different values for resistivities and layer thicknesses. The RMS value of the error between the collected data and the modelled data is then determined for each attempt of modelling.

The interpreted subsurface model is determined by minimizing the values of RMS error between both of the data. The modelled data for which the RMS error is found to be minimum is known to be "the best fit" for the given profile data.

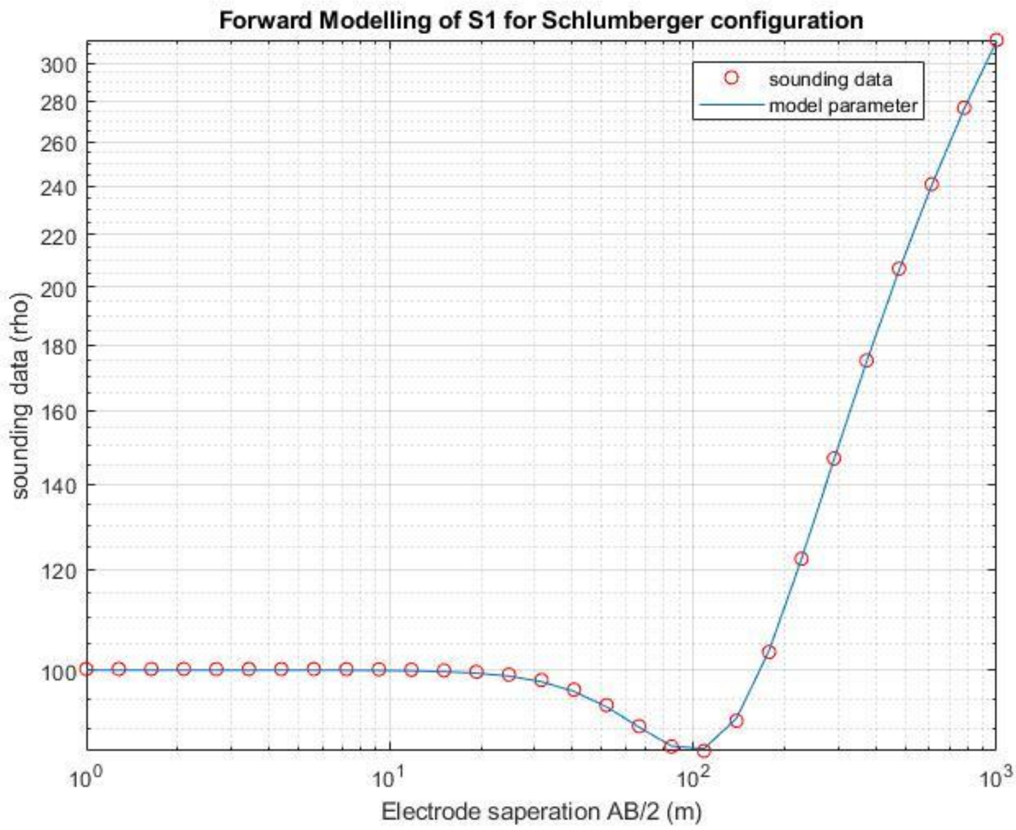
Similarly, the other three profile data (S1, S2 and S3) can be modelled by replacing the values of `'rhot'` and `'h'` in the given program, with a few modifications (size of loop, size of arrays: for n-layered scenario, these values are needed to be set to 'n').

MATLAB program:

```
x=datadc{:,1};                          % inputting distance along profile from datadc.csv
s1=datadc{:,2};                         % inputting values of profile 1
s2=datadc{:,3};                         % inputting values of profile 2
s3=datadc{:,4};                         % inputting values of profile 3
s4=datadc{:,5};                         % inputting values of profile 4
a=[-0.980685 -.771995 -.563305 -.354615 ...
   -.145925  .062765  .271455  .480145  .688835 ...
   0.897525 1.106215 1.314905 1.523595 1.732285 ...        %inputting abscissa of filter coefficient
   1.940975 2.149665 2.358355 2.567045 2.775735];
f=[ 0.00097112 -0.00102152  0.00906965  0.01404316 ...
    0.09012     0.30171582  0.99627084  1.3690832  -2.99681171 ...
    1.65463068 -0.59399277  0.22329813 -0.10119309  0.05186135 ...    %inputting filter coefficients
   -0.02748647  0.01384932 -0.00599074  0.00190463 -0.0003216 ];
ns=29;
rhoa=zeros([1 29]);                     %vector to store apparent resistivity for different x
rhot=[17 64 71 560];                    %true resistivities of all layer (considering 4 layers)
h=[2 11 24];                            %thickness of top 3 resistivity layers (for 4 layers)
lambda=zeros([1 19]);                   %
t=zeros([1 4]);                         %vector to store Resistivity transform for each value of x
sumn=0;
for i=1:length(x)                       % loop for calculation of resistivity for different x
  sumn=0;
  t=zeros([1 4]);
  for j=1:length(a)                     %loop for summing values from filter coefficints
    lambda(j)=10.^(a(j)-log10(x(i)));
    t(4)=rhot(4);
    for k=1:3                           %loop for determining T1 for each x
      num=t(5-k)+(rhot(4-k)*tanh(lambda(j)*h(4-k)));
      den=1+(t(5-k)*tanh(lambda(j)*h(4-k))/rhot(4-k));
      t(4-k)=num/den;
    end
    sumn=sumn+(f(j)*t(1));
  end
  rhoa(i)=sumn;
end
rhoa=rhoa';                             %
err=s4-rhoa;                            % Determining RMS error between observed data
sq_err=err.^2;                          %  and the model
avg=mean(sq_err);                       %
rms=avg.^0.5;
loglog(x,s4,'r*',x,rhoa);               % plotting the data in log-log scale
grid on;
legend('sounding data','model parameter');
title('Forward Modelling of S4 for Schlumberger configuration')
xlabel('Electrode saperation AB/2 (m)');
ylabel('sounding data (rho)');
```

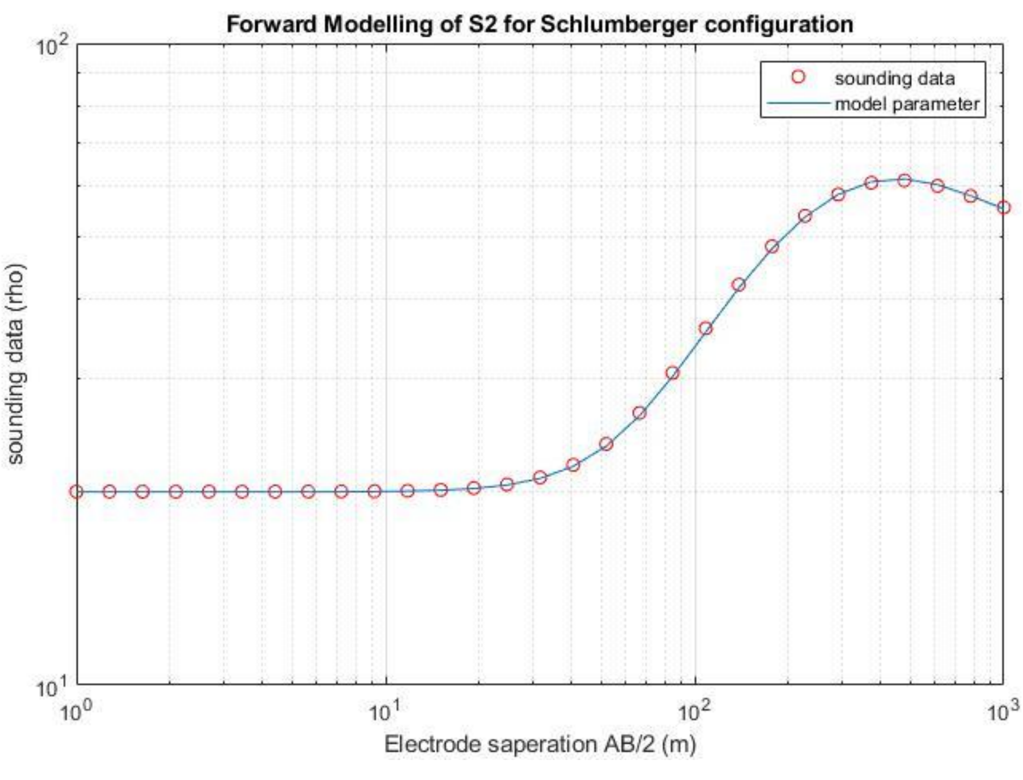
- **Interpretation for sounding profile S1:**
  The given sounding profile can be implied to have a 3-layer subsurface model with H-type curve. The model parameters of the subsurface determined from the model can be given as:

  ρ= [100   42   500]
  h= [45   45]
  RMS error= 0.3121



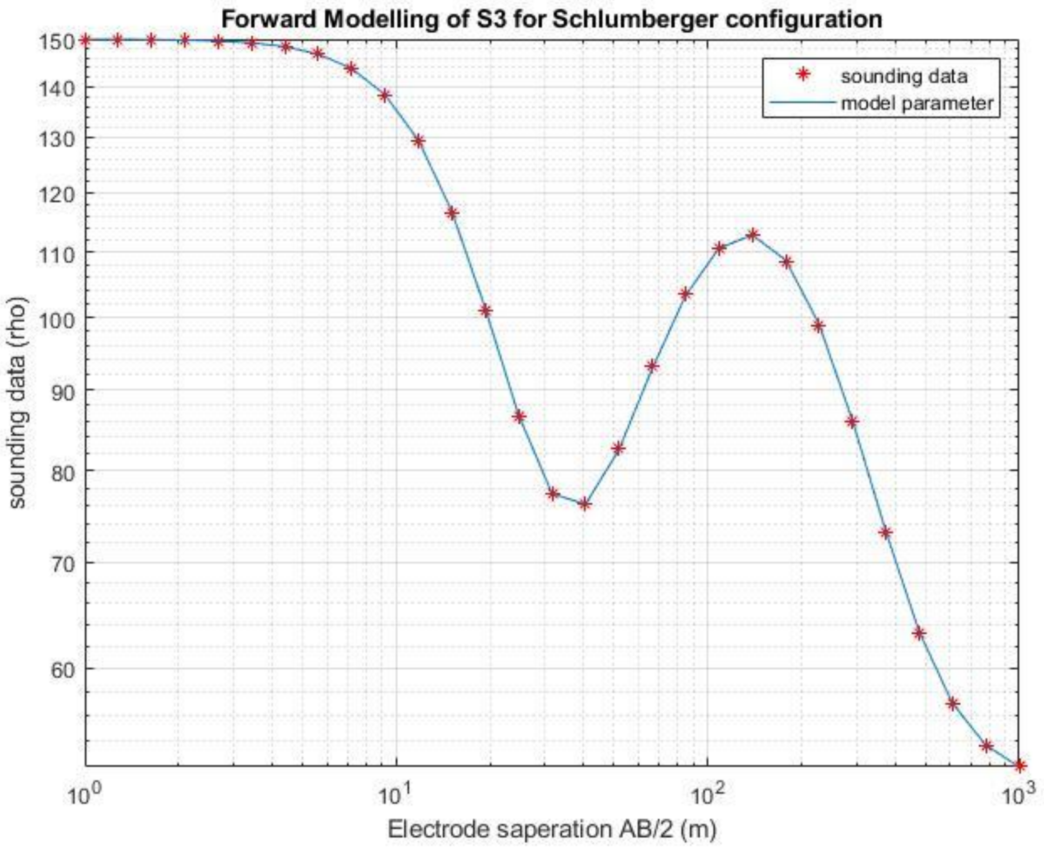
- **Interpretation for sounding profile S2:**
  The given sounding profile can be implied to have a 3-layer subsurface model with K-type curve. The model parameters of the subsurface determined from the model can be given as:

  ρ= [20 225 49]
  h= [50 50]
  RMS error= 0.2101



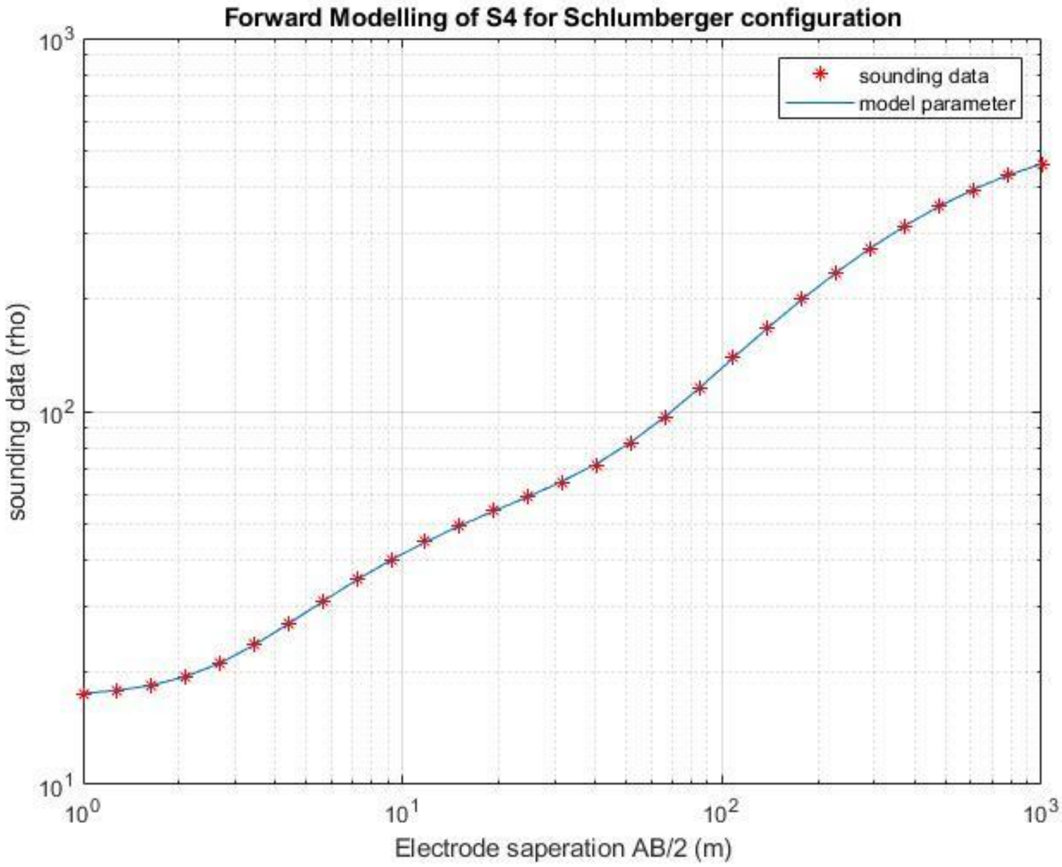
- **Interpretation for sounding profile S3:**
  The given sounding profile can be implied to have a 4-layer subsurface model with HK-type curve. The model parameters of the subsurface determined from the model can be given as:

  ρ= [150   36   700   50]
  h= [10   16   16]
  RMS error= 0.1608



- **Interpretation for sounding profile S4:**
  The given sounding profile can be implied to have a 3-layer subsurface model with AA-type curve. The model parameters of the subsurface determined from the model can be given as:

  ρ= [17   64   71   560]
  h= [2   11   24]
  RMS error= 0.2656



Q3. What kind of ambiguity could be there in your interpretation, mention that?

The ambiguity in resistivity sounding for determination of layer thickness and layer resistivity is governed by :

i) Principle of Suppression: The principle of suppression applies when an inter-related layer has a resistivity intermediate between the resistivities of the layers above and below. In this case, the layer has an insignificant effect on the sounding curve unless it is very thick.

ii) Principle of Equivalance: It says that it is impossible to arrive at a unique solution for the layer parameters (thickness and resistivities). For conductive layers, only thickness/resistivity ratio can be determined, whereas, for highly resistive layers only the product of thickness and resistivity can be determined. This means that when resistivity and the thickness vary within a certain limits, their product remaining constant, no differences can be seen in the sounding curve. Thickness and resistivity are coupled in both cases of equivalence and cannot be uniquely independently determined.

Lab. 6

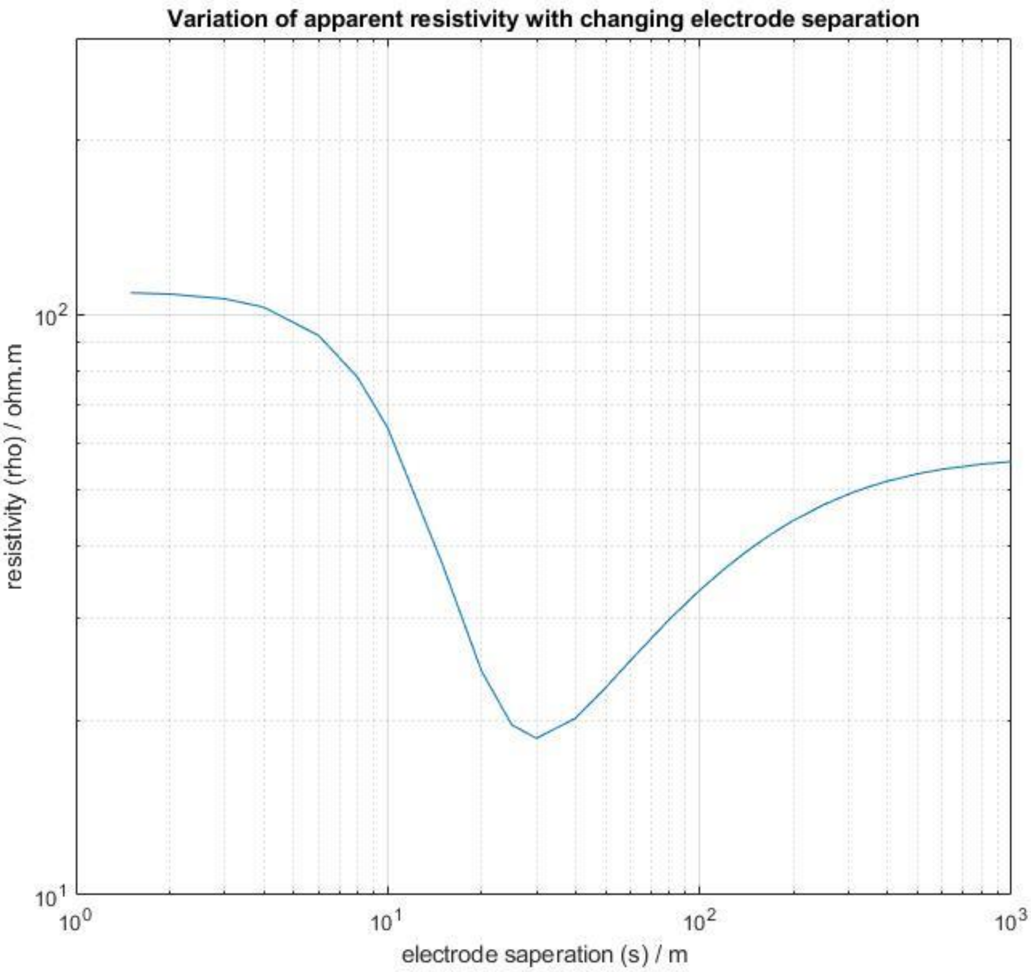
Effect of 2-D Anisotropy in resistivity determination

Q1. Write a program accounting the effect of 2-D anisotropy in resistivity for three layer earth model case, with assumed anisotropy in the direction of 'X', with the following model:

ρ_z= [120 14 65]; ρ_x= [100 10 50]; h=[5 15]

MATLAB program:

```
a=[-0.980685 -.771995 -.563305 -.354615 ...
   -.145925  .062765  .271455  .480145  .688835 ...
   0.897525 1.106215 1.314905 1.523595 1.732285 ...       %inputting abscissa of filter coefficient
   1.940975 2.149665 2.358355 2.567045 2.775735];
f=[ 0.00097112 -0.00102152  0.00906965  0.01404316 ...
    0.09012     0.30171582  0.99627084  1.3690832  -2.99681171 ...
    1.65463068 -0.59399277  0.22329813 -0.10119309  0.05186135 ... %inputting filter coefficients
    -0.02748647  0.01384932 -0.00599074  0.00190463 -0.0003216 ];
s=[1.5 2 3 4 6 8 10 15 20 25 30 40 50 60 80 100 ...        %inputting the electrode separations
   120 140 160 180 200 250 300 350 400 500 600 800 1000];  %  (half of current electrode separation)
ns=29;
rhoa=zeros([1 29]);                          %vector to store apparent resistivity for different s
rhoth=[100 10 50];                           %true resistivities of all layer (considering 6 layers)
rhotv=[120 14 65];
rhot=sqrt(rhoth.*rhotv);                     % effective resistivity
coeff=sqrt(rhotv./rhoth);                    % coefficient of anisotropy (f)
hpre=[5 15];
for m=1:length(hpre)
h(m)=hpre(m).*coeff(m);                  %thickness of top 5 resistivity layers (considering 6 layers)
end
lambda=zeros([1 19]);                        %
t=zeros([1 3]);                              %vector to store Resistivity transform for each value of s
sumn=0;
for i=1:length(s)                            % loop for calculation of resistivity for different s
  sumn=0;
  t=zeros([1 3]);
  for j=1:length(a)                          %loop for summing values from filter coefficients
    lambda(j)=10.^(a(j)-log10(s(i)));
    t(3)=rhot(3);
    for k=1:2                                %loop for determining T1 for each s
      num=t(4-k)+(rhot(3-k)*tanh(lambda(j)*h(3-k)));
      den=1+(t(4-k)*tanh(lambda(j)*h(3-k))/rhot(3-k));
      t(3-k)=num/den;
    end
    sumn=sumn+(f(j)*t(1));
  end
  rhoa(i)=sumn;
end
loglog(s,rhoa);                              %plotting the rho vs s curve in log-log scale
title('Variation of apparent resistivity with changing electrode separation');
xlabel('electrode saperation (s) / m');
ylabel('resistivity (rho) / ohm.m');
grid on
ylim([10, 300]);                              %limiting the extent of y-scale
```



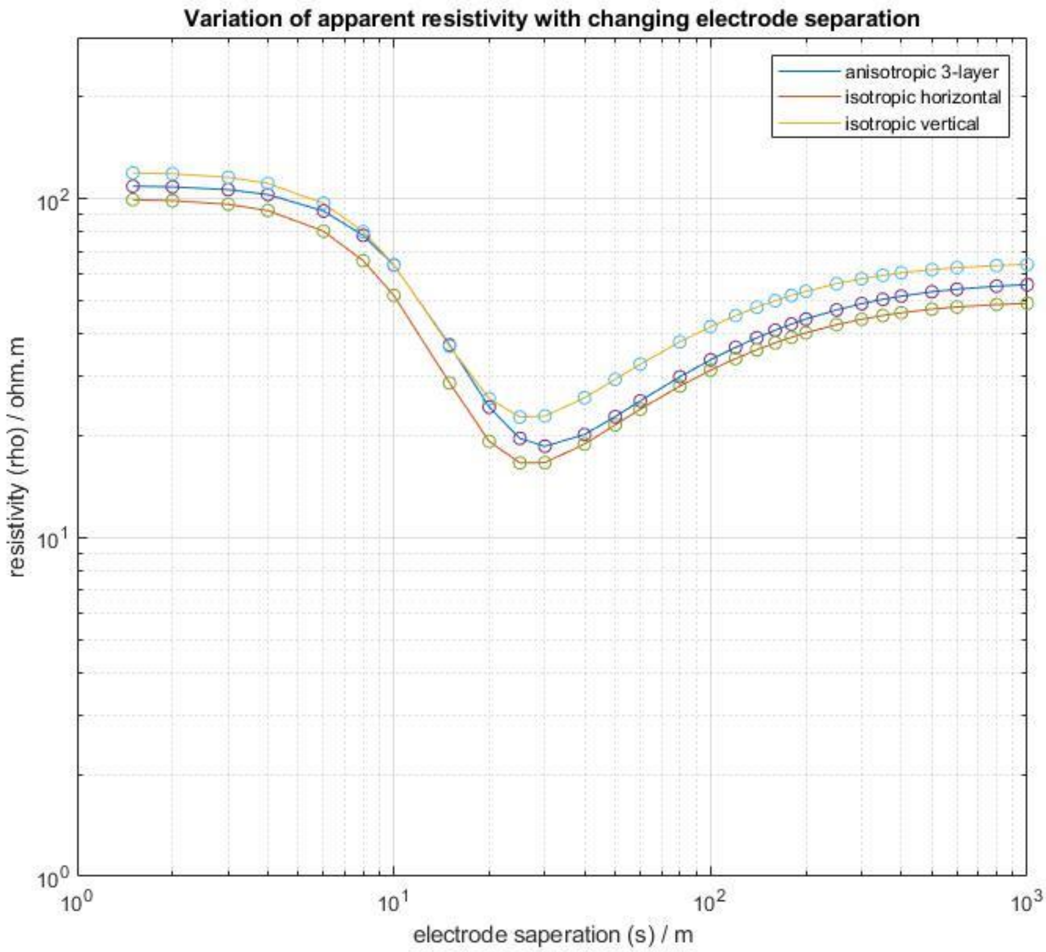
Q2. Write a program accounting the effect of 2-D anisotropy in resistivity for three layer earth model case, with assumed anisotropy in the direction of 'X', with the previous model and show the variation of resistivity response for (i) anisotropic in horizontal direction only; (ii) anisotropic in vertical direction only; (iii) 2-D anisotropy (along x- and z- direction)

MATLAB program:

```
a=[-0.980685 -.771995 -.563305 -.354615 ...
   -.145925  .062765  .271455  .480145  .688835 ...
   0.897525 1.106215 1.314905 1.523595 1.732285 ...       %inputting abscissa of filter coefficient
   1.940975 2.149665 2.358355 2.567045 2.775735];
f=[ 0.00097112 -0.00102152  0.00906965  0.01404316 ...
    0.09012     0.30171582  0.99627084  1.3690832  -2.99681171 ...
    1.65463068 -0.59399277  0.22329813 -0.10119309  0.05186135 ...    %inputting filter coefficients
    -0.02748647  0.01384932 -0.00599074  0.00190463 -0.0003216 ];
s=[1.5 2 3 4 6 8 10 15 20 25 30 40 50 60 80 100 ...             %inputting the electrode separations
   120 140 160 180 200 250 300 350 400 500 600 800 1000];       % (half of current electrode seprn)

ns=29;
rhoa=zeros([1 29]);                      %vector to store apparent resistivity for different s
rhoh=zeros([1 29]);
rhov=zeros([1 29]);
rhoth=[100 10 50];                       %true resistivities of all layer (considering 6 layers)
rhotv=[120 14 65];
```

```
rhot=sqrt(rhoth.*rhotv)

coeff=sqrt(rhotv./rhoth);
hpre=[5 15];

for m=1:length(hpre)
h(m)=hpre(m).*coeff(m);                %thickness of top 5 resistivity layers (considering 6 layers)
end

lambda=zeros([1 19]);                  %
for i=1:length(s)                      % loop for calculation of resistivity for different s
  sumn=0;
  t=zeros([1 3]);
  for j=1:length(a)                    %loop for summing values from filter coefficients
    lambda(j)=10.^(a(j)-log10(s(i)));
    t(3)=rhot(3);
    for k=1:2                          %loop for determining T1 for each s
      num=t(4-k)+(rhot(3-k)*tanh(lambda(j)*h(3-k)));
      den=1+(t(4-k)*tanh(lambda(j)*h(3-k))/rhot(3-k));
      t(3-k)=num/den;
    end
    sumn=sumn+(f(j)*t(1));
  end
  rhoa(i)=sumn;                        % determination of resistivity considering Anisotropy

end

lambda=zeros([1 19]);                  %
for i=1:length(s)                      % loop for calculation of resistivity for different s
  sumn=0;
  t=zeros([1 3]);
  for j=1:length(a)                    %loop for summing values from filter coefficients
    lambda(j)=10.^(a(j)-log10(s(i)));
    t(3)=rhoth(3);
    for k=1:2                          %loop for determining T1 for each s
      num=t(4-k)+(rhoth(3-k)*tanh(lambda(j)*hpre(3-k)));
      den=1+(t(4-k)*tanh(lambda(j)*hpre(3-k))/rhoth(3-k));
      t(3-k)=num/den;
    end
    sumn=sumn+(f(j)*t(1));
  end
  rhoh(i)=sumn;                        % determination of resistivity for horizontal direction
end

lambda=zeros([1 19]);                  %
for i=1:length(s)                      % loop for calculation of resistivity for different s
  sumn=0;
  t=zeros([1 3]);
  for j=1:length(a)                    %loop for summing values from filter coefficients
    lambda(j)=10.^(a(j)-log10(s(i)));
    t(3)=rhotv(3);
    for k=1:2                          %loop for determining T1 for each s
      num=t(4-k)+(rhotv(3-k)*tanh(lambda(j)*hpre(3-k)));
      den=1+(t(4-k)*tanh(lambda(j)*hpre(3-k))/rhotv(3-k));
      t(3-k)=num/den;
    end
    sumn=sumn+(f(j)*t(1));
  end
  rhov(i)=sumn;                        % determination of resistivity for vertical direction
```

```
loglog(s,rhoa,s,rhoh,s,rhov); hold on;     %plotting the rho vs s curve in log-log scale
scatter(s,rhoa); hold on;
scatter(s,rhoh); hold on;
scatter(s,rhov); hold on;
title('Variation of apparent resistivity with changing electrode separation');
legend('anisotropic 3-layer','isotropic horizontal','isotropic vertical');
xlabel('electrode saperation (s) / m');
ylabel('resistivity (rho) / ohm.m');
grid on
ylim([1, 300]);                            %limiting the extent of y-scale
```



Lab. 7

Forward modelling for 1-D Magneto-Telluric (MT) data

The MT impedance Z_0 (w) at the surface of an n-layered earth for an incident plane wave with angular frequency w can be written as:

$$\mathbf{Z}_{j-1}(\omega) = \frac{\mathrm{i}\omega\mu}{\mathrm{k_j}} \coth\left[\mathrm{i k_j h_j} + \coth^{-1}\left(\frac{\mathrm{k_j}}{\mathrm{i}\omega\mu}\mathbf{Z}_j(\omega)\right)\right] \tag{4}$$

with

$$Z_{n-1}(\omega) = \frac{i\omega\mu}{k_n}$$

where $\mathrm{k_j} = \sqrt{-\mathrm{i}\omega\mu\sigma_\mathrm{j}}$ represents the wave number of the jth layer of conductivity and thickness, σ_j and $\mathrm{h_j}$, respectively. The symbol μ represents the permeability of free space and $\mathrm{i} = \sqrt{-1}$.

The apparent resistivity $\rho_\mathrm{a}(\omega)$ and apparent phase $\phi_\mathrm{a}(\omega)$ are related to the MT impedance as

$$\rho_\mathrm{a}(\omega) = \frac{1}{\omega\mu}[\mathrm{Z_0}(\omega)\mathrm{Z_0^*}(\omega)], \tag{5}$$

and

$$\varphi_\mathrm{a}(\omega) = \arg[\mathrm{Z_0}(\omega)], \tag{6}$$

where the superscript * represents the complex conjugate.

Assignment -1 Interpret the given MT sounding data (data_mt.csv). Write the model parameters and show the fittings between the measured data and model data by calculating RMS error. What kind of ambiguity could be there in your interpretation, mention that?

Lab. 8

Forward modelling for VLF data

Q. Write a program for modelling 2-D body in the sub-surface for the given VLF data and determine the RMS error.

The expression for vertical component of the magnetic field is given by:

$$\Delta Hz'(0,0) = \frac{j_y}{4\pi} \left\{ z_2 \, ln\left(\frac{z_2^2+x_2^2}{z_2^2+x_1^2}\right) - z_1 \, ln\left(\frac{z_1^2+x_2^2}{z_1^2+x_1^2}\right) + 2x_2 \tan^{-1}\left(\frac{x_2(z_2-z_1)}{x_2^2+z_1 z_2}\right) - 2x_1 \tan^{-1}\left(\frac{x_1(z_2-z_1)}{x_1^2+z_1 z_2}\right) \right\}$$

MATLAB program:

```
x0= 10:10:1000; x0=x0';                   % initialising profile of 10 unit upto 1000
vlf1=datavlf{:,2};                        %importing vlf dataset 1
vlf2=datavlf{:,3};                        %importing vlf dataset 2
x1=500;                                   %%% x-coordinate constraints for the body
x2=540;                                   %%%
z1=40;                                    % z-coordinate constraints for the body
z2=60;                                    %
j=.0025                                   %% current density
for i=1:100
    x1n=x1-x0(i);
    x2n=x2-x0(i);
    t1=z2*log((x2n^2+z2^2)/(x1n^2+z2^2));
    t2=z1*log((x2n^2+z1^2)/(x1n^2+z1^2));
    t3=2*x2n*atan(x2n*(z2-z1)/(x2n^2+z1*z2));
    t4=2*x1n*atan(x1n*(z2-z1)/(x1n^2+z1*z2));
    hz(i)=-1*j*100/4/pi*(t1-t2+t3-t4);  % determination of ΔHz due to the constrained body
end
hz=hz';
plot(x0,vlf2,x0,hz,'*');
grid on;
title('VLF data Modelling for a 2-dimensional body')
legend('calculated data','experimental data');
xlabel('Profile length (m)');
ylabel('Δ Hz');
del=hz-vlf2;
del=del.^2;
tot=sum(del)/100;
rms=sqrt(tot)                             %rms error calculation
```

The above program is written for the modelling the dataset vlf2; the program can be modified easily to model the dataset vlf1 and determine the model parameters

Data modelling for the dataset vlf1:

x1= 400m;   x2=450m;
z1=20m;   z2=40m;
The determined error from the vlf1 model was found to be RMS= $1.0233163 \times 10^{-16}$

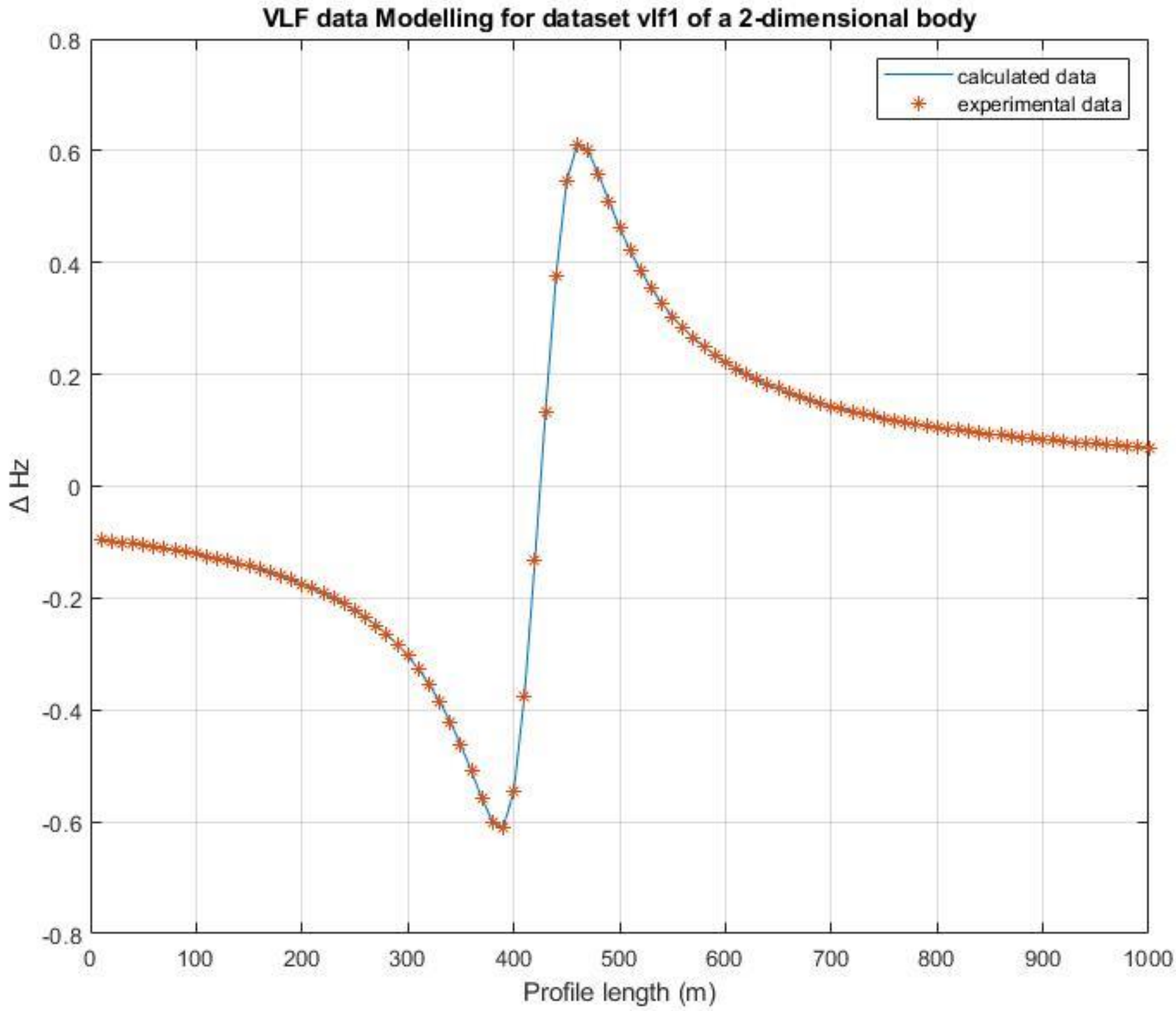


Fig: Modelling the dataset vlf1 with the model parameters**x1=400**; **x2=450**; **z1=20**; **z2=40**; with a **RMS error of $1.0233163 \times 10^{-16}$**

Data modelling for the dataset vlf2:

x1= 400m;   x2=450m;
z1=20m;   z2=40m;
The determined error from the vlf2 model was found to be RMS= 1.2221912 $\times 10^{-16}$

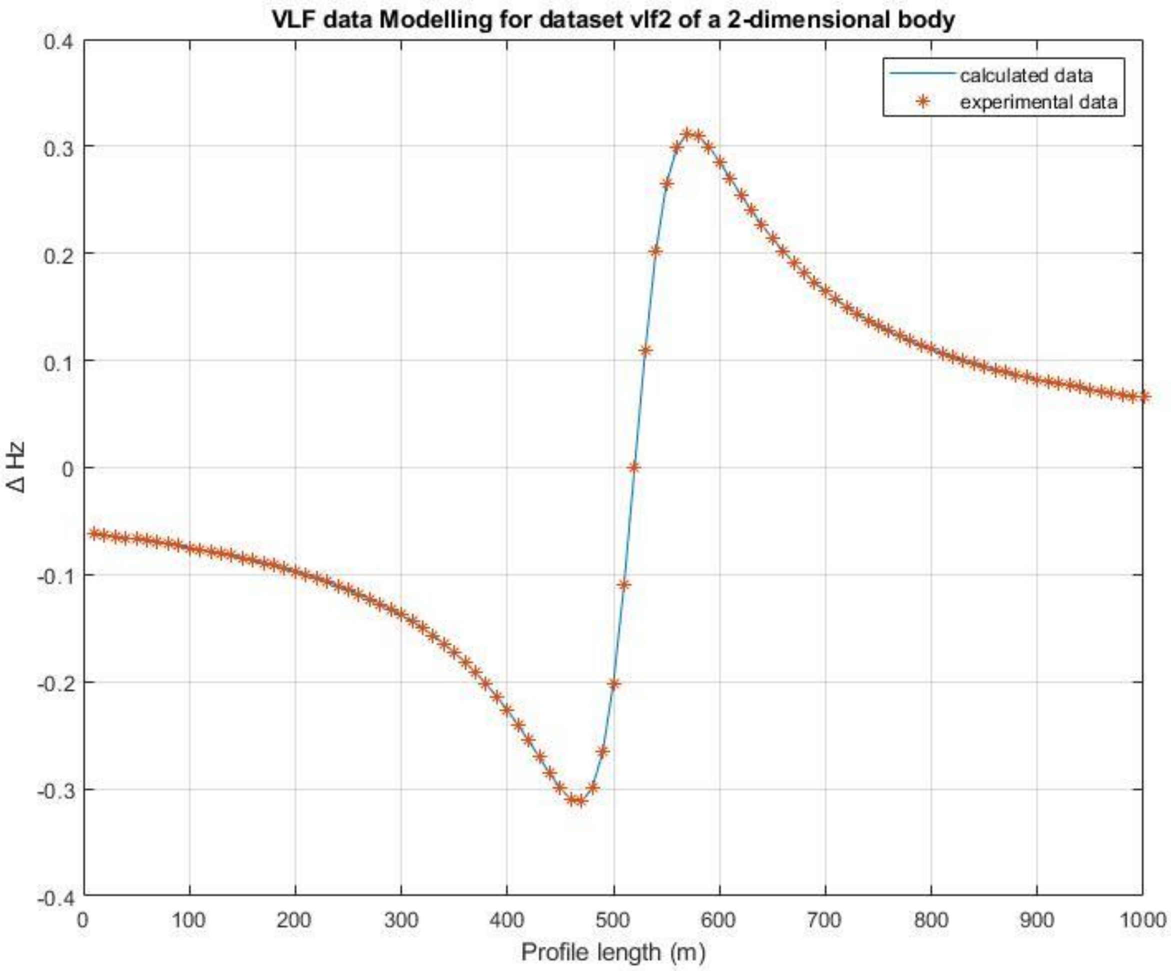


Fig: Modelling the dataset vlf1 with the model parameters **x1=500**; **x2=540**; **z1=40**; **z2=60**; with a **RMS error of 1.2221912 $\times 10^{-16}$**

Lab. 9

Rotation of Impedance Tensor in MT analysis

Documentations for Rotation of Impedance Tensor

Impedance tensor rotation is mathematical tool for field rotation in order to determine the direction of strike of the geological/geoelectric feature. This method is carried out with the motive to maximize the impedance values for off-component directions (z_{xy} and z_{yx}) and minimize the impedances for in-component directions (z_{xx} and z_{yy}).

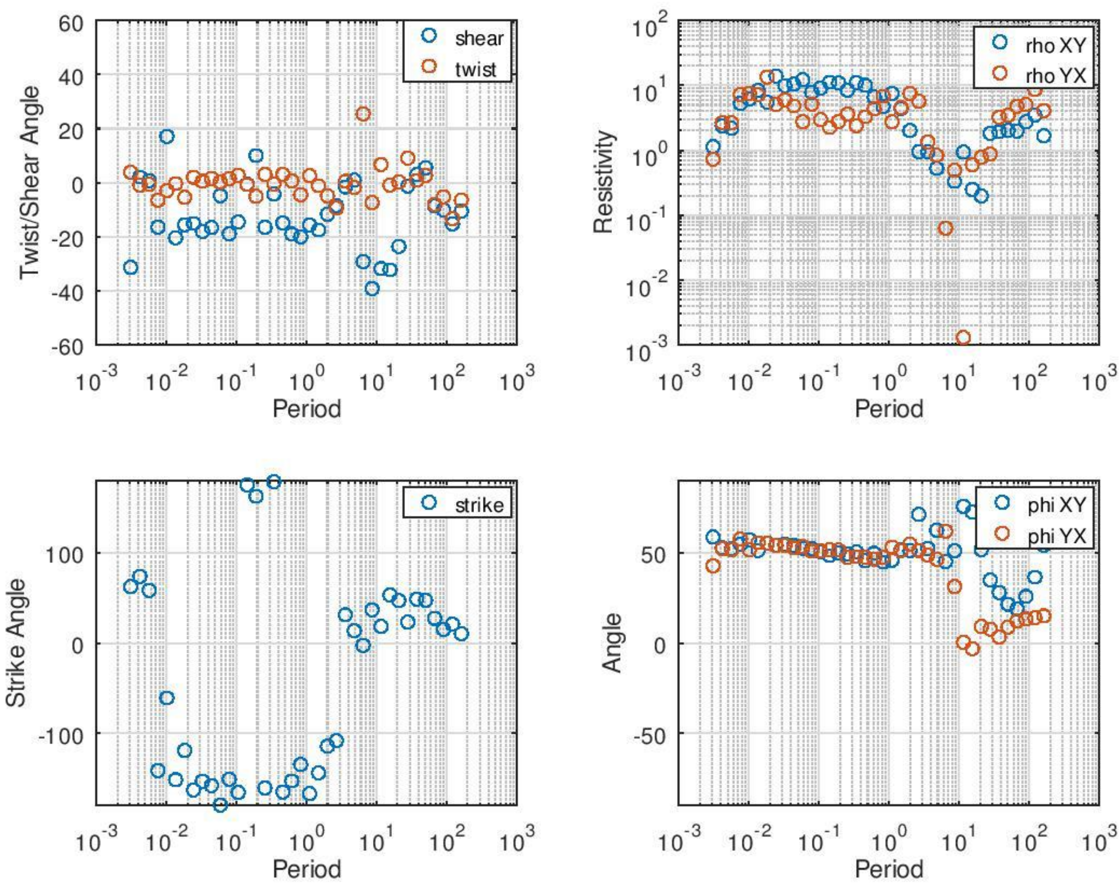
We here performing this by using the '**strike41.exe**' program by **Alan G Jones** & **Gary W McNeice** which takes in the input file in the .edi and .dat format. Using this program we get the processed data which is then converted to graphical format after processing it from the '**gbplot.exe**' program.

For processing, we put a constraint to the parameters: 'twist' and 'shear' and to finally deduce an angle to which we constraint strike. For this we firstly constraint the values for twist component from the scatter plot of the given data, followed by the confinement of the values of shear component in the same. This done by checking the period range in which the twist and shear are most stable, then constraining them to a value (or a period range). At last, we determine a value for strike by following the same procedure for confining it. In case, we encounter the overlapping in scatters of twist and shear, we ensure the strike value by performing both FTC (First Twist Constraint), which we were previously performing, and FSC (First Shear Constraint).

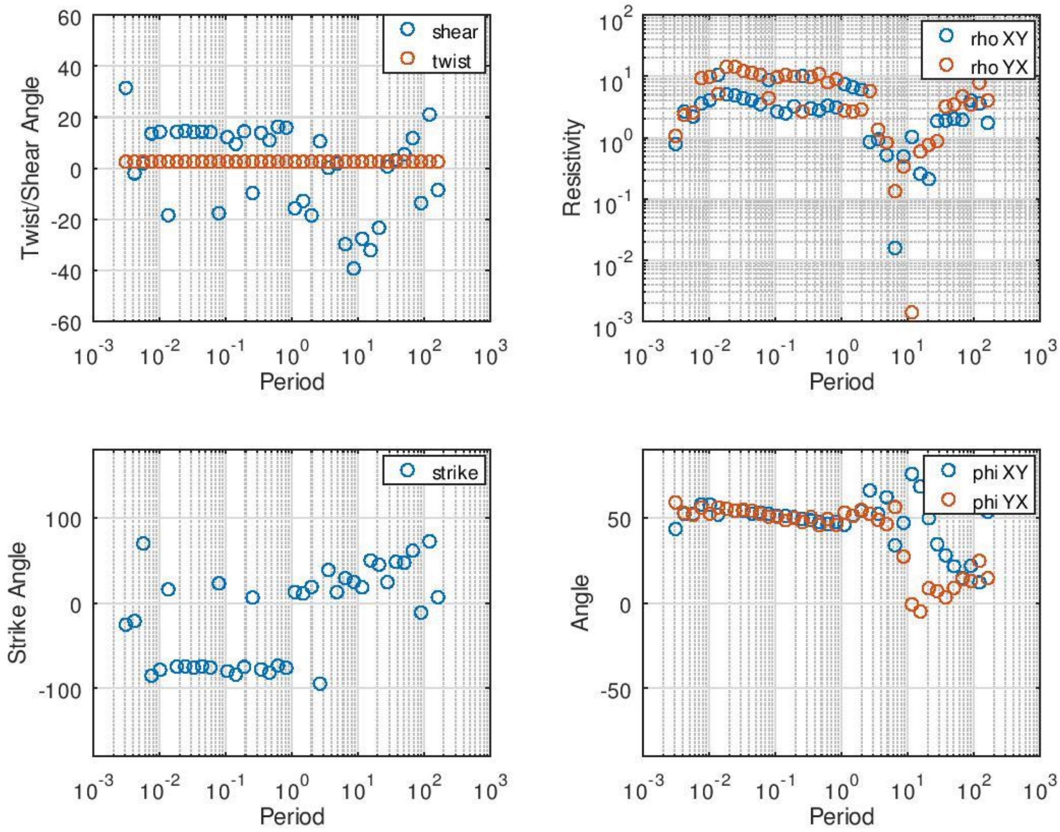
Overlapping TE and TM impedance characteristic gives the indication of 1-D structure behavior of the subsurface, and otherwise is the indication of non- 1-D (2-D or 3-D) structure behavior of the subsurface.

Impedance tensor rotation for Site04 data

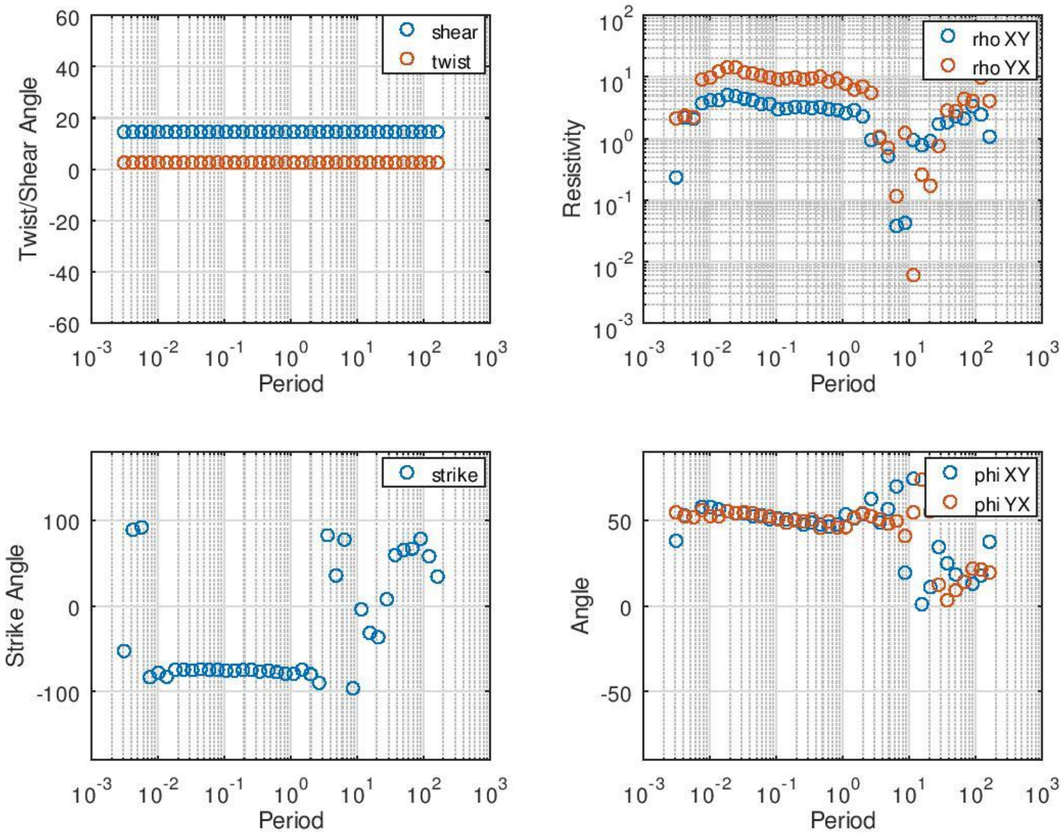
i) Unconstrained data



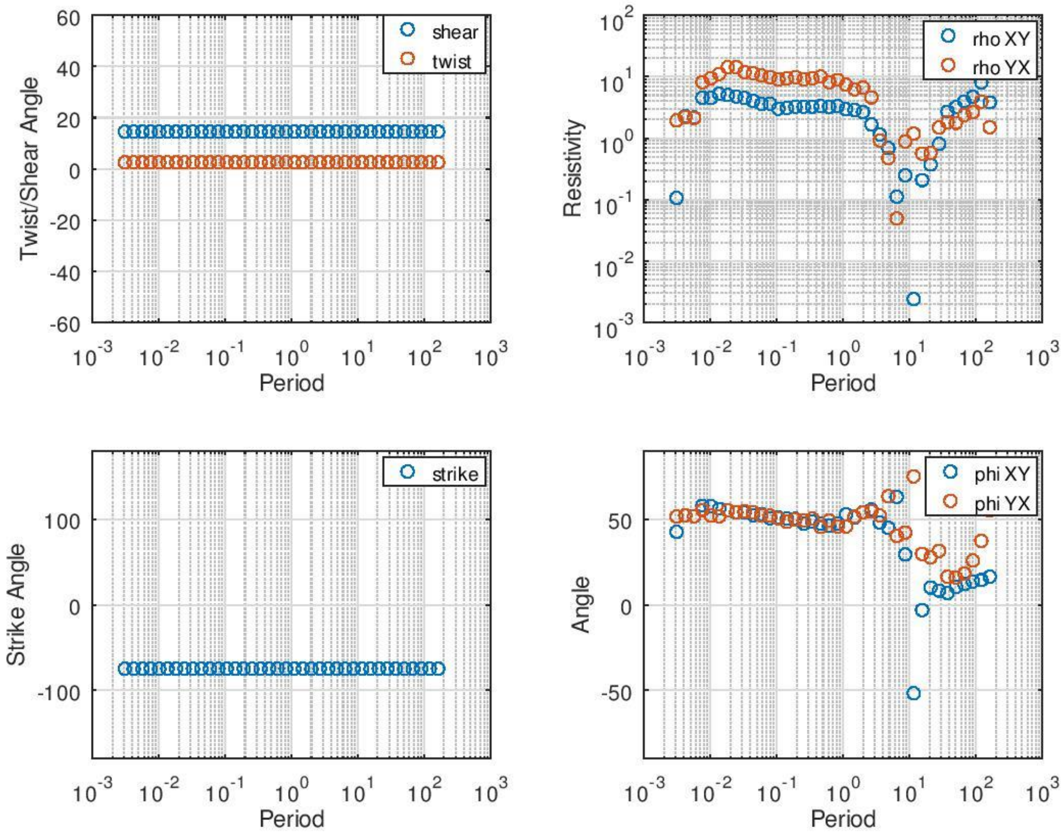
ii) Constraining the twist= 2.5^0 for the data



iii) Constraining shear= 14.5^0, after the constraining of twist for the data

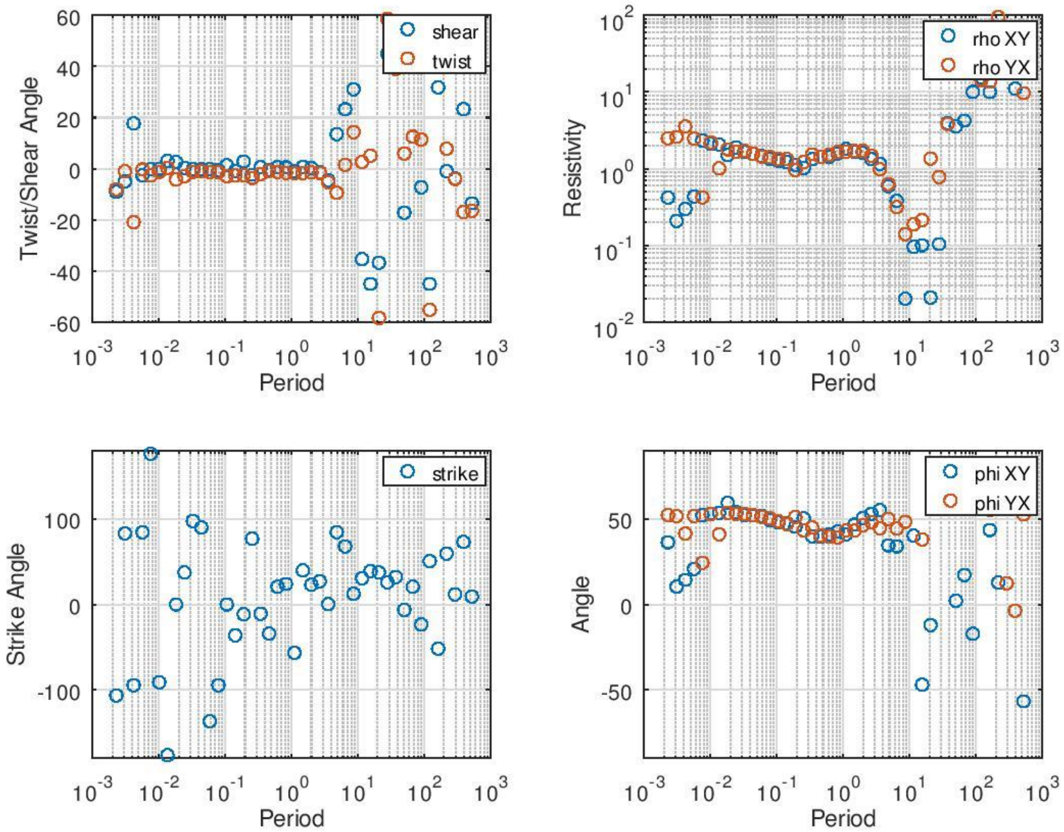


iv) Finally constraining the strike= -75^{0}, after the constraining the twist and shear

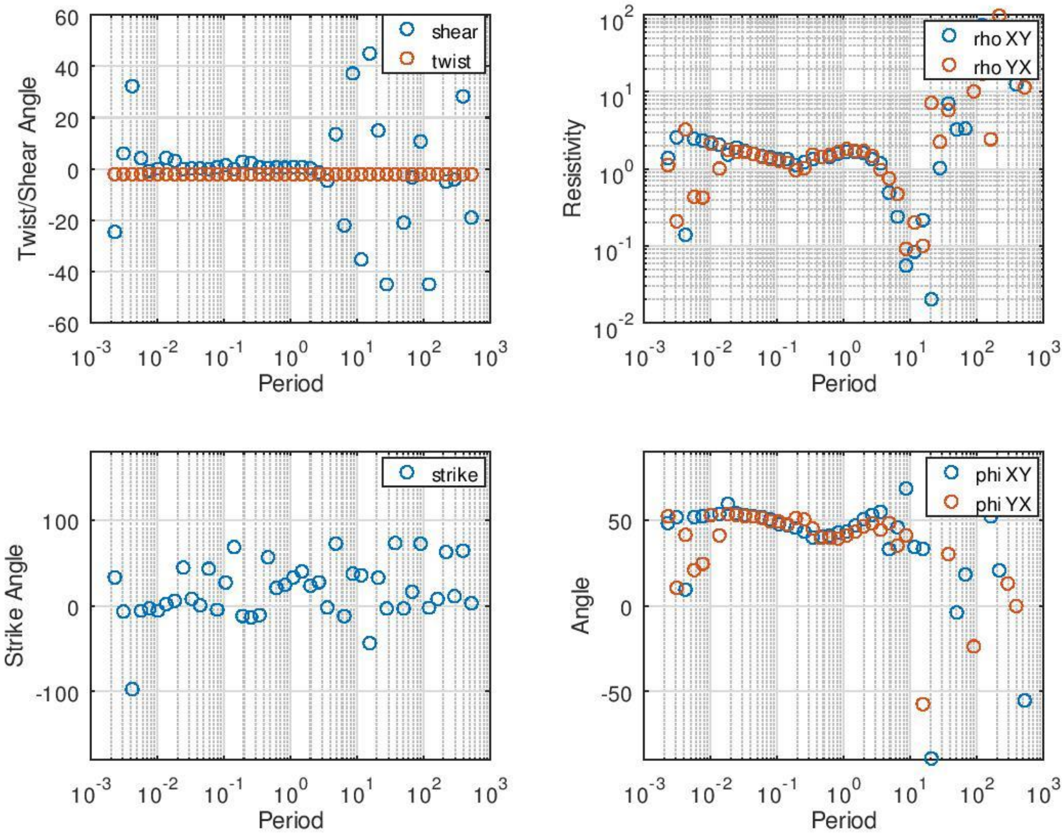


<u>Impedance tensor rotation for Site02 data</u>: Constraining the Twist first

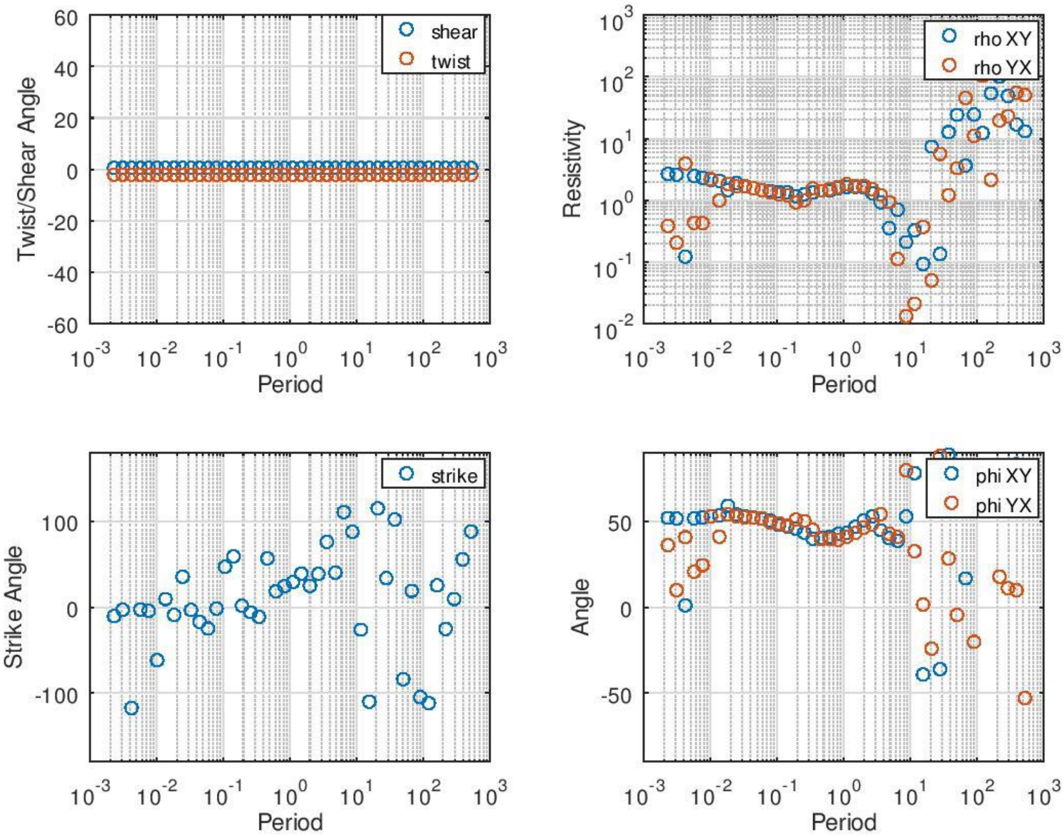
i) Unconstrained data



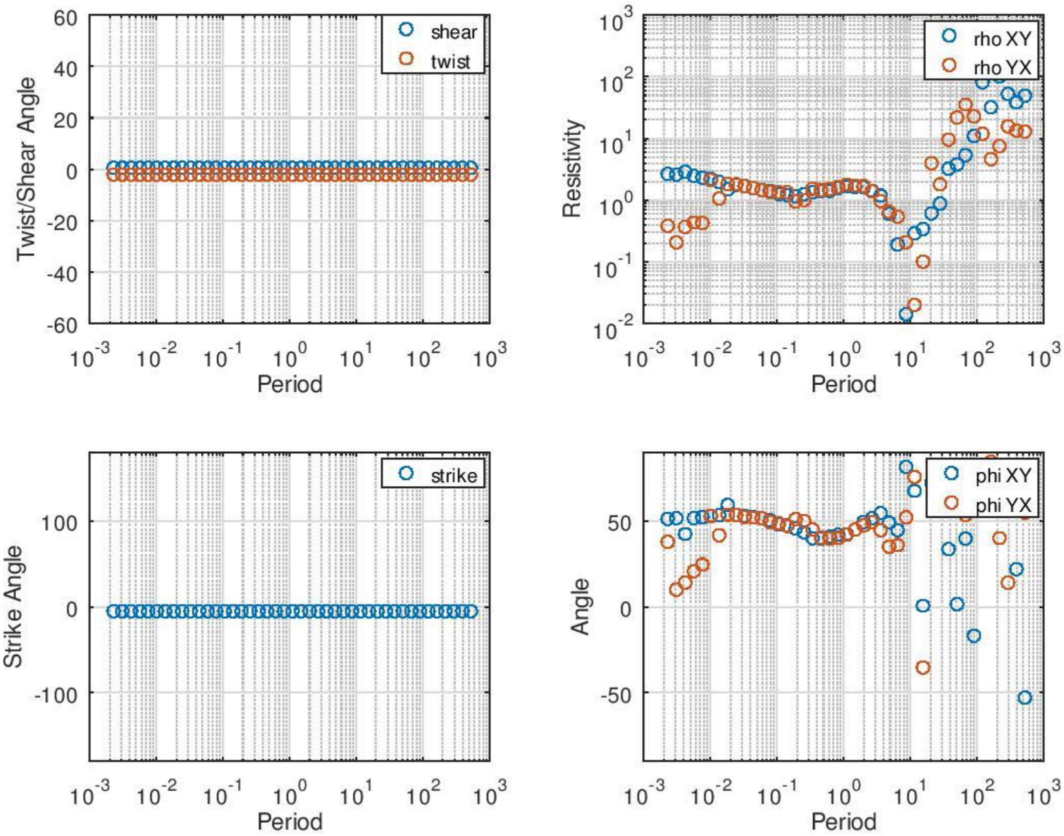
ii) Constraining the twist= -2^0 for the data



iii) Constraining shear= 0.5^0, after the constraining of twist for the data

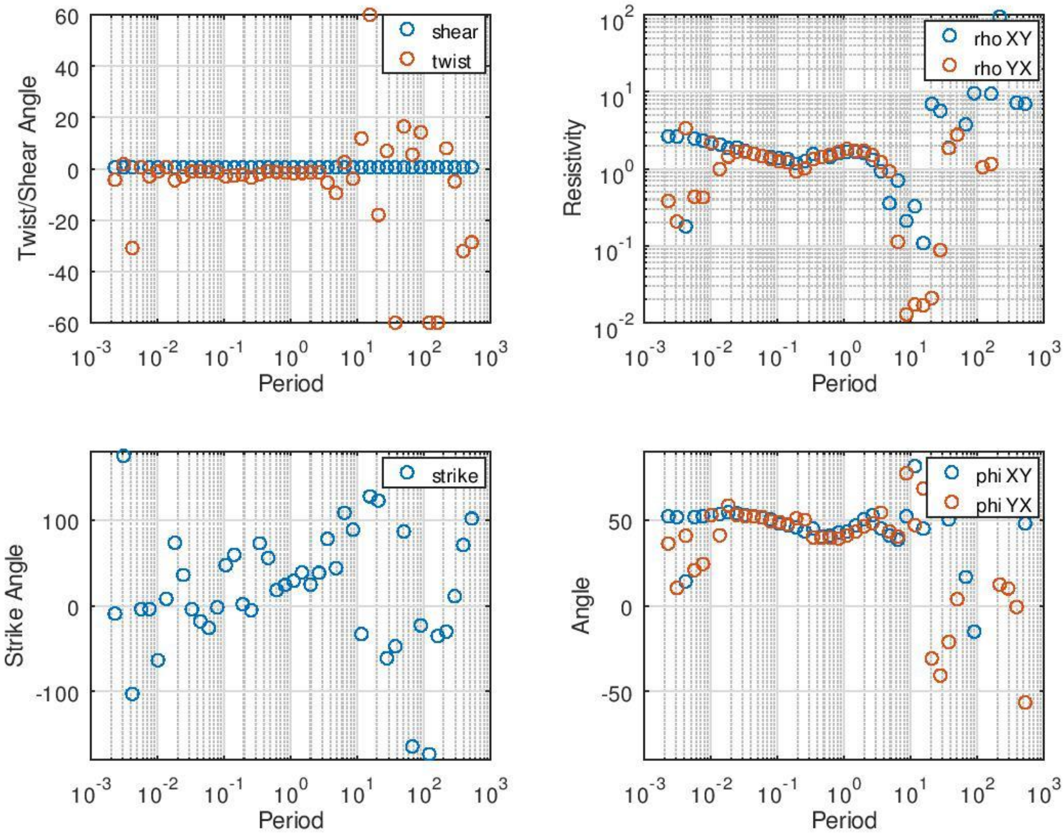


iv) At last constraining the strike= -5^0, after the constraining the twist and shear

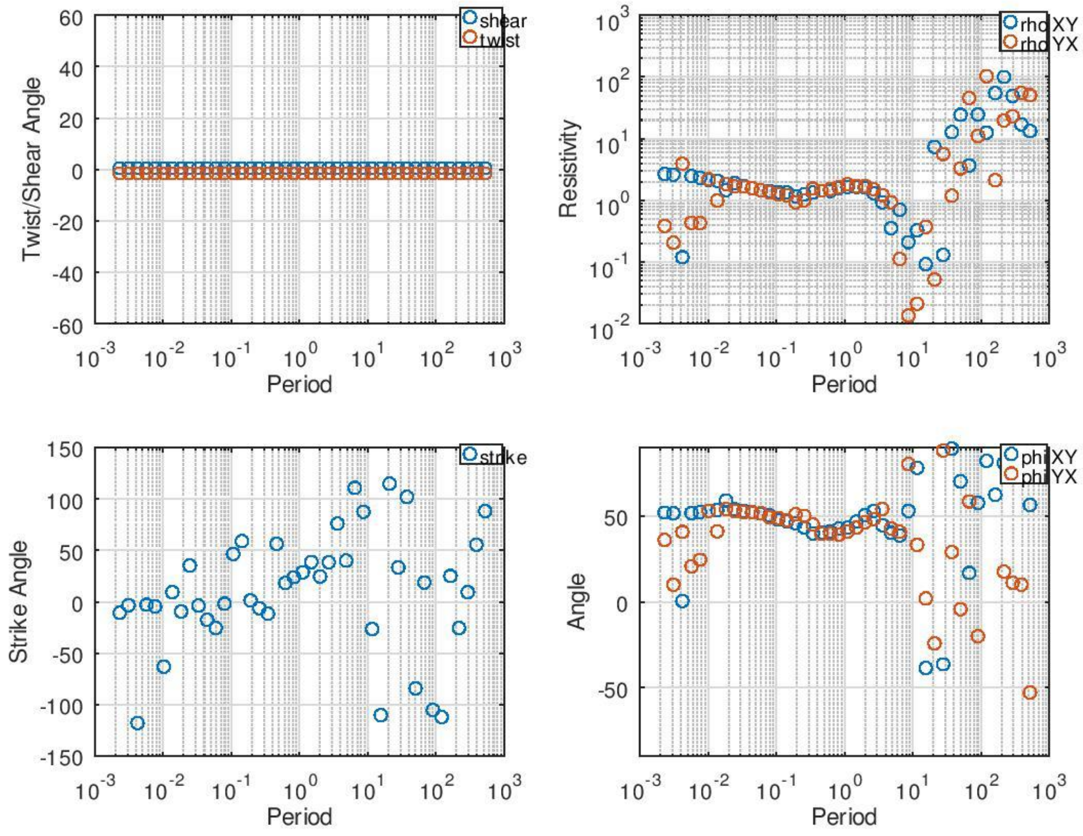


<u>Impedance tensor rotation for Site02 data</u>: Constraining the Shear first

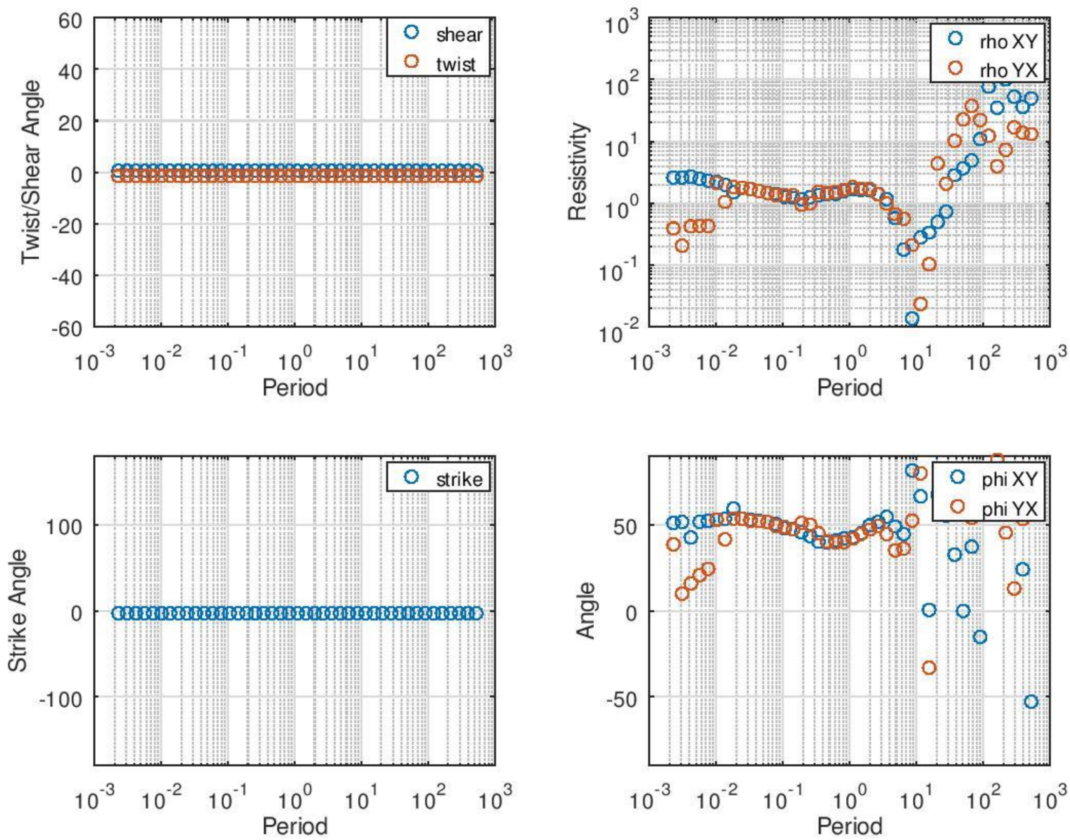
i) Constraining the shear= 0.5^0 for the data



iii) Constraining twist= -1.5^0, after the constraining of shear for the data



iv) At last constraining the strike= -3^0, after the constraining the shear and twist



Results / Implications:

- The earth model as implied from the Impedance Tensor Rotation for the site04 can be 2-D or 3-D or may be, there could be a static shift in the 1-D model, since the scatter plots for resistivities are parallel yet non-coinciding, but their phase curves are almost overlapping.

  The constraining parameters for the site04 was determined to be:
  Twist = 2.5^0
  Shear = 14.5^0
  Strike = -75^0

- The earth model as implied from the Impedance Tensor Rotation for the site02 can be a 1-D model, since the scatter plots for resistivities are almost parallel and coinciding, as well as their phase curves are almost overlapping.

  But since the twist and shear were overlapping in the scatter plot, we need to determine the model for both the cases, where, in one case we constraint the twist first, followed by the shear (FTC) then finally constraining the strike; and in other case we first constraint the shear, then twist (FSC) and lastly the strike. If the modelling is correct, the value of strike derived in both cases be almost same. Also their resistivity and phase characteristics should not be showing much differences.

  The constraining parameters for site02, determined by **constraining twist first** are:
  Twist = -2^0
  Shear = 0.5^0
  Strike = -5^0

  The constraining parameters for site02, determined by **constraining shear first** are:
  Twist = -1.5^0
  Shear = 0.5^0
  Strike = -3^0

  Since here we see that the constraining parameters for both the cases are almost same, which ensures that the determined parameters are independent of the order of choice, which one to constraint first. Hence, we can be sure about our estimated geological/geoelectric strike angle value.