The earthquake source is represented by the point force $\boldsymbol{f}$, which may be written in terms of a moment tensor $\boldsymbol{M}$ as (omitting the source time function)

$$\boldsymbol{f} = -\boldsymbol{M} \cdot \nabla \delta(\boldsymbol{x} - \boldsymbol{x}_s), \tag{1}$$

The point force $\boldsymbol{f}$ in 2D:

$$\begin{cases} f_x = -\left[M_{xx}\delta'(x-x_s)\delta(z-z_s) + M_{xz}\delta(x-x_s)\delta'(z-z_s)\right] \\ f_z = -\left[M_{xz}\delta'(x-x_s)\delta(z-z_s) + M_{zz}\delta(x-x_s)\delta'(z-z_s)\right] \end{cases}, \tag{2}$$

Multiplying equations (2) by the time-independent test functions, we can obtain:

$$\begin{cases} F_x(x,z) = -\int \left[M_{xx}\delta'(x-x_s)\delta(z-z_s) + M_{xz}\delta(x-x_s)\delta'(z-z_s)\right]\varphi_x(x,z)d\Omega \\ F_z(x,z) = -\int \left[M_{xz}\delta'(x-x_s)\delta(z-z_s) + M_{zz}\delta(x-x_s)\delta'(z-z_s)\right]\varphi_z(x,z)d\Omega \end{cases}, \tag{3}$$

where φ_x and φ_z are the test functions, respectively.

Applying the identity of the Dirac delta function

$$\begin{cases} F_x(x_s,z_s) = \left(M_{xx}\dfrac{\partial \varphi_x(x_s,z_s)}{\partial x} + M_{xz}\dfrac{\partial \varphi_x(x_s,z_s)}{\partial z}\right) \\ F_z(x_s,z_s) = \left(M_{xz}\dfrac{\partial \varphi_z(x_s,z_s)}{\partial x} + M_{zz}\dfrac{\partial \varphi_z(x_s,z_s)}{\partial z}\right) \end{cases}, \tag{4}$$

If we can find the dimensionless parameters (ξ_s, γ_s) satisfying

$$\begin{cases} x_s = \sum_{k=1}^{N}\sum_{m=1}^{N} \ell_k(\xi_s)\ell_m(\gamma_s)x_{km} \\ z_s = \sum_{k=1}^{N}\sum_{m=1}^{N} \ell_k(\xi_s)\ell_m(\gamma_s)z_{km} \end{cases}, \tag{5}$$

or,

$$\begin{cases} \xi_s = \sum_{k=1}^{N}\sum_{m=1}^{N} \ell_k(\xi_s)\ell_m(\gamma_s)\xi_{km} \\ \gamma_s = \sum_{k=1}^{N}\sum_{m=1}^{N} \ell_k(\xi_s)\ell_m(\gamma_s)\gamma_{km} \end{cases}, \tag{6}$$

Applying the chain rule, (4) can be written

$$\begin{cases} F_x(x_s,z_s) = \left[M_{xx}\left(\dfrac{\partial \varphi_x(\xi_s,\gamma_s)}{\partial \xi}\dfrac{\partial \xi_s}{\partial x} + \dfrac{\partial \varphi_x(\xi_s,\gamma_s)}{\partial \gamma}\dfrac{\partial \gamma_s}{\partial x}\right) + M_{xz}\left(\dfrac{\partial \varphi_x(\xi_s,\gamma_s)}{\partial \xi}\dfrac{\partial \xi_s}{\partial z} + \dfrac{\partial \varphi_x(\xi_s,\gamma_s)}{\partial \gamma}\dfrac{\partial \gamma_s}{\partial z}\right)\right] \\ F_z(x_s,z_s) = \left[M_{xz}\left(\dfrac{\partial \varphi_z(\xi_s,\gamma_s)}{\partial \xi}\dfrac{\partial \xi_s}{\partial x} + \dfrac{\partial \varphi_z(\xi_s,\gamma_s)}{\partial \gamma}\dfrac{\partial \gamma_s}{\partial x}\right) + M_{zz}\left(\dfrac{\partial \varphi_z(\xi_s,\gamma_s)}{\partial \xi}\dfrac{\partial \xi_s}{\partial z} + \dfrac{\partial \varphi_z(\xi_s,\gamma_s)}{\partial \gamma}\dfrac{\partial \gamma_s}{\partial z}\right)\right] \end{cases}, \tag{7}$$

where,

$$\begin{cases} \varphi_x(\xi_s,\gamma_s) = \sum_{i=1}^{N}\sum_{j=1}^{N}\ell_i(\xi_s)\ell_j(\gamma_s)\varphi_{xij} \\ \varphi_z(\xi_s,\gamma_s) = \sum_{i=1}^{N}\sum_{j=1}^{N}\ell_i(\xi_s)\ell_j(\gamma_s)\varphi_{zij} \end{cases}, \tag{8}$$

Substituting (6) and (8) into (7), and using the unit test function, we obtain

$$\begin{cases} F_x(x_s,z_s) = M_{xx}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial x} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial x}\right] \\ \qquad M_{xz}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial z} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial z}\right] \\ F_z(x_s,z_s) = M_{xz}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial x} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial x}\right] \\ \qquad M_{zz}\sum_{k=1}^{N}\sum_{l=1}^{N}\left[\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial z} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial z}\right] \end{cases}$$

(10)

Therefore, the source force at each GLL node can be written

$$\begin{cases} F_x(i,j) = M_{xx}\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial x} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial x} \\ \qquad M_{xz}\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial z} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial z} \\ F_z(i,j) = M_{xz}\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial x} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial x} \\ \qquad M_{zz}\ell'_i(\xi_s)\ell_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\xi_{km}}{\partial z} + \ell_i(\xi_s)\ell'_j(\gamma_s)\sum_{k=1}^{N}\sum_{k=1}^{N}\ell_k(\xi_s)\ell_l(\gamma_s)\frac{\partial\gamma_{km}}{\partial z} \end{cases}. \tag{11}$$