請寫出12種連三次轉動之旋轉順序

xyz , xyx , xzx , xzy　　yxz , yxy , yzx , yzy　　zxy , zxz , zyx , zyz

請寫出12種Fixed angles之轉換矩陣

Fixed_angle_Rxyz =

$$\begin{bmatrix} \cos(A)\cos(B) & \cos(A)\sin(B)\sin(Y) - \sin(A)\cos(Y) & \sin(A)\sin(Y) + \cos(A)\sin(B)\cos(Y) \\ \cos(B)\sin(A) & \cos(A)\cos(Y) + \sin(A)\sin(B)\sin(Y) & \sin(A)\sin(B)\cos(Y) - \cos(A)\sin(Y) \\ -\sin(B) & \cos(B)\sin(Y) & \cos(B)\cos(Y) \end{bmatrix}$$

Fixed_angle_Rxyx =

$$\begin{bmatrix} \cos(B) & \sin(B)\sin(Y) & \sin(B)\cos(Y) \\ \sin(A)\sin(B) & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) & -\cos(A)\sin(Y) - \cos(B)\sin(A)\cos(Y) \\ -\cos(A)\sin(B) & \sin(A)\cos(Y) + \cos(A)\cos(B)\sin(Y) & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) \end{bmatrix}$$

Fixed_angle_Rxzx =

$$\begin{bmatrix} \cos(B) & -\sin(B)\cos(Y) & \sin(B)\sin(Y) \\ \cos(A)\sin(B) & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) & -\sin(A)\cos(Y) - \cos(A)\cos(B)\sin(Y) \\ \sin(A)\sin(B) & \cos(A)\sin(Y) + \cos(B)\sin(A)\cos(Y) & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) \end{bmatrix}$$

Fixed_angle_Rxzy =

$$\begin{bmatrix} \cos(A)\cos(B) & \sin(A)\sin(Y) - \cos(A)\sin(B)\cos(Y) & \sin(A)\cos(Y) + \cos(A)\sin(B)\sin(Y) \\ \sin(B) & \cos(B)\cos(Y) & -\cos(B)\sin(Y) \\ -\cos(B)\sin(A) & \cos(A)\sin(Y) + \sin(A)\sin(B)\cos(Y) & \cos(A)\cos(Y) - \sin(A)\sin(B)\sin(Y) \end{bmatrix}$$

Fixed_angle_Ryxz =

$$\begin{bmatrix} \cos(A)\cos(Y) - \sin(A)\sin(B)\sin(Y) & -\cos(B)\sin(A) & \cos(A)\sin(Y) + \sin(A)\sin(B)\cos(Y) \\ \sin(A)\cos(Y) + \cos(A)\sin(B)\sin(Y) & \cos(A)\cos(B) & \sin(A)\sin(Y) - \cos(A)\sin(B)\cos(Y) \\ -\cos(B)\sin(Y) & \sin(B) & \cos(B)\cos(Y) \end{bmatrix}$$

Fixed_angle_Ryxy =

$$\begin{bmatrix} \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) & \sin(A)\sin(B) & \cos(A)\sin(Y) + \cos(B)\sin(A)\cos(Y) \\ \sin(B)\sin(Y) & \cos(B) & -\sin(B)\cos(Y) \\ -\sin(A)\cos(Y) - \cos(A)\cos(B)\sin(Y) & \cos(A)\sin(B) & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) \end{bmatrix}$$

Fixed_angle_Ryzx =

$$\begin{bmatrix} \cos(B)\cos(Y) & -\sin(B) & \cos(B)\sin(Y) \\ \sin(A)\sin(Y) + \cos(A)\sin(B)\cos(Y) & \cos(A)\cos(B) & \cos(A)\sin(B)\sin(Y) - \sin(A)\cos(Y) \\ \sin(A)\sin(B)\cos(Y) - \cos(A)\sin(Y) & \cos(B)\sin(A) & \cos(A)\cos(Y) + \sin(A)\sin(B)\sin(Y) \end{bmatrix}$$

Fixed_angle_Ryzy =

$$\begin{bmatrix} \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) & -\cos(A)\sin(B) & \sin(A)\cos(Y) + \cos(A)\cos(B)\sin(Y) \\ \sin(B)\cos(Y) & \cos(B) & \sin(B)\sin(Y) \\ -\cos(A)\sin(Y) - \cos(B)\sin(A)\cos(Y) & \sin(A)\sin(B) & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) \end{bmatrix}$$

Fixed_angle_Rzxy =

$$\begin{bmatrix} \cos(A)\cos(Y) + \sin(A)\sin(B)\sin(Y) & \sin(A)\sin(B)\cos(Y) - \cos(A)\sin(Y) & \cos(B)\sin(A) \\ \cos(B)\sin(Y) & \cos(B)\cos(Y) & -\sin(B) \\ \cos(A)\sin(B)\sin(Y) - \sin(A)\cos(Y) & \sin(A)\sin(Y) + \cos(A)\sin(B)\cos(Y) & \cos(A)\cos(B) \end{bmatrix}$$

Fixed_angle_Rzxz =

$$\begin{bmatrix} \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) & -\cos(A)\sin(Y) - \cos(B)\sin(A)\cos(Y) & \sin(A)\sin(B) \\ \sin(A)\cos(Y) + \cos(A)\cos(B)\sin(Y) & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) & -\cos(A)\sin(B) \\ \sin(B)\sin(Y) & \sin(B)\cos(Y) & \cos(B) \end{bmatrix}$$

Fixed_angle_Rzyx =

$$\begin{bmatrix} \cos(B)\cos(Y) & -\cos(B)\sin(Y) & \sin(B) \\ \cos(A)\sin(Y) + \sin(A)\sin(B)\cos(Y) & \cos(A)\cos(Y) - \sin(A)\sin(B)\sin(Y) & -\cos(B)\sin(A) \\ \sin(A)\sin(Y) - \cos(A)\sin(B)\cos(Y) & \sin(A)\cos(Y) + \cos(A)\sin(B)\sin(Y) & \cos(A)\cos(B) \end{bmatrix}$$

Fixed_angle_Rzyz =

$$\begin{bmatrix} \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) & -\sin(A)\cos(Y) - \cos(A)\cos(B)\sin(Y) & \cos(A)\sin(B) \\ \cos(A)\sin(Y) + \cos(B)\sin(A)\cos(Y) & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) & \sin(A)\sin(B) \\ -\sin(B)\cos(Y) & \sin(B)\sin(Y) & \cos(B) \end{bmatrix}$$

請寫出12種Euler Angle之轉換矩陣

Euler_Angle_Rxyz =

$$\begin{bmatrix} \cos(B)\cos(Y) & -\cos(B)\sin(Y) & \sin(B) \\ \cos(A)\sin(Y) + \sin(A)\sin(B)\cos(Y) & \cos(A)\cos(Y) - \sin(A)\sin(B)\sin(Y) & -\cos(B)\sin(A) \\ \sin(A)\sin(Y) - \cos(A)\sin(B)\cos(Y) & \sin(A)\cos(Y) + \cos(A)\sin(B)\sin(Y) & \cos(A)\cos(B) \end{bmatrix}$$

Euler_Angle_Rxyx =

$$\begin{bmatrix} \cos(B) & \sin(B)\sin(Y) & \sin(B)\cos(Y) \\ \sin(A)\sin(B) & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) & -\cos(A)\sin(Y) - \cos(B)\sin(A)\cos(Y) \\ -\cos(A)\sin(B) & \sin(A)\cos(Y) + \cos(A)\cos(B)\sin(Y) & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) \end{bmatrix}$$

Euler_Angle_Rxzx =

$$\begin{bmatrix} \cos(B) & -\sin(B)\cos(Y) & \sin(B)\sin(Y) \\ \cos(A)\sin(B) & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) & -\sin(A)\cos(Y) - \cos(A)\cos(B)\sin(Y) \\ \sin(A)\sin(B) & \cos(A)\sin(Y) + \cos(B)\sin(A)\cos(Y) & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) \end{bmatrix}$$

Euler_Angle_Rxzy =

$$\begin{bmatrix} \cos(B)\cos(Y) & -\sin(B) & \cos(B)\sin(Y) \\ \sin(A)\sin(Y) + \cos(A)\sin(B)\cos(Y) & \cos(A)\cos(B) & \cos(A)\sin(B)\sin(Y) - \sin(A)\cos(Y) \\ \sin(A)\sin(B)\cos(Y) - \cos(A)\sin(Y) & \cos(B)\sin(A) & \cos(A)\cos(Y) + \sin(A)\sin(B)\sin(Y) \end{bmatrix}$$

Euler_Angle_Ryxz =

$$\begin{bmatrix} \cos(A)\cos(Y) + \sin(A)\sin(B)\sin(Y) & \sin(A)\sin(B)\cos(Y) - \cos(A)\sin(Y) & \cos(B)\sin(A) \\ \cos(B)\sin(Y) & \cos(B)\cos(Y) & -\sin(B) \\ \cos(A)\sin(B)\sin(Y) - \sin(A)\cos(Y) & \sin(A)\sin(Y) + \cos(A)\sin(B)\cos(Y) & \cos(A)\cos(B) \end{bmatrix}$$

Euler_Angle_Ryxy =

$$\begin{bmatrix} \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) & \sin(A)\sin(B) & \cos(A)\sin(Y) + \cos(B)\sin(A)\cos(Y) \\ \sin(B)\sin(Y & \cos(B) & -\sin(B)\cos(Y) \\ -\sin(A)\cos(Y) - \cos(A)\cos(B)\sin(Y) & \cos(A)\sin(B) & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y) \end{bmatrix}$$

Euler_Angle_Ryzx =

$$\begin{bmatrix} \cos(A)\cos(B), & \sin(A)\sin(Y) - \cos(A)\sin(B)\cos(Y), & \sin(A)\cos(Y) + \cos(A)\sin(B)\sin(Y) \\ \sin(B), & \cos(B)\cos(Y), & -\cos(B)\sin(Y) \\ -\cos(B)\sin(A), & \cos(A)\sin(Y) + \sin(A)\sin(B)\cos(Y), & \cos(A)\cos(Y) - \sin(A)\sin(B)\sin(Y) \end{bmatrix}$$

Euler_Angle_Ryzy =

$$\begin{bmatrix} \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y), & -\cos(A)\sin(B), & \sin(A)\cos(Y) + \cos(A)\cos(B)\sin(Y) \\ \sin(B)\cos(Y), & \cos(B), & \sin(B)\sin(Y) \\ -\cos(A)\sin(Y) - \cos(B)\sin(A)\cos(Y), & \sin(A)\sin(B), & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y) \end{bmatrix}$$

Euler_Angle_Rzxy =

$$\begin{bmatrix} \cos(A)\cos(Y) - \sin(A)\sin(B)\sin(Y), & -\cos(B)\sin(A), & \cos(A)\sin(Y) + \sin(A)\sin(B)\cos(Y) \\ \sin(A)\cos(Y) + \cos(A)\sin(B)\sin(Y), & \cos(A)\cos(B), & \sin(A)\sin(Y) - \cos(A)\sin(B)\cos(Y) \\ -\cos(B)\sin(Y), & \sin(B), & \cos(B)\cos(Y) \end{bmatrix}$$

Euler_Angle_Rzxz =

$$\begin{bmatrix} \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y), & -\cos(A)\sin(Y) - \cos(B)\sin(A)\cos(Y), & \sin(A)\sin(B) \\ \sin(A)\cos(Y) + \cos(A)\cos(B)\sin(Y), & \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y), & -\cos(A)\sin(B) \\ \sin(B)\sin(Y), & \sin(B)\cos(Y), & \cos(B) \end{bmatrix}$$

Euler_Angle_Rzyx =

$$\begin{bmatrix} \cos(A)\cos(B), & \cos(A)\sin(B)\sin(Y) - \sin(A)\cos(Y), & \sin(A)\sin(Y) + \cos(A)\sin(B)\cos(Y) \\ \cos(B)\sin(A), & \cos(A)\cos(Y) + \sin(A)\sin(B)\sin(Y), & \sin(A)\sin(B)\cos(Y) - \cos(A)\sin(Y) \\ -\sin(B), & \cos(B)\sin(Y), & \cos(B)\cos(Y) \end{bmatrix}$$

Euler_Angle_Rzyz =

$$\begin{bmatrix} \cos(A)\cos(B)\cos(Y) - \sin(A)\sin(Y), & -\sin(A)\cos(Y) - \cos(A)\cos(B)\sin(Y), & \cos(A)\sin(B) \\ \cos(A)\sin(Y) + \cos(B)\sin(A)\cos(Y), & \cos(A)\cos(Y) - \cos(B)\sin(A)\sin(Y), & \sin(A)\sin(B) \\ -\sin(B)\cos(Y), & \sin(B)\sin(Y), & \cos(B) \end{bmatrix}$$

請找出Euler/Fixed angles 之對偶性組合

Fix_angle_Rxyz = Euler_angle_Rzyx

Fix_angle_Rxyx = Euler_angle_Rxyx

Fix_angle_Rxzx = Euler_angle_Rxzx

Fix_angle_Rxyy = Euler_angle_Ryyx

Fix_angle_Ryxz = Euler_angle_Rzxy

Fix_angle_Ryxy = Euler_angle_Ryxy

Fix_angle_Ryzx = Euler_angle_Rxzy

Fix_angle_Ryzy = Euler_angle_Ryzy

Fix_angle_Rzxy = Euler_angle_Ryxz

Fix_angle_Rzxz = Euler_angle_Rzxz

Fix_angle_Rzyx = Euler_angle_Rxyz

Fix_angle_Rzyz = Euler_angle_Rzyz

請將所有Fixed angles及Euler Angle之轉換矩陣寫成程式

https://github.com/07050862/Control_Robot/blob/main/%E7%A9%BA%E9%96%93%E5%BA%A7%E6%A8%99%E8%BD%89%E6%8F%9B/rotate.m

逆推　　以數值模擬驗證對偶性

```
>> my_test = [0.5 0.75 0.433;0 0.5 -0.866;-0.866 0.433 0.25]

my_test =

    0.5000    0.7500    0.4330
         0    0.5000   -0.8660
   -0.8660    0.4330    0.2500

>> B = atan(-(my_test(3,1))/(my_test(1,1)^2+my_test(2,1)^2)^0.5) /(pi/180)

B =

   59.9993

>> A = atan((my_test(2,1)/cos(B))/(my_test(1,1)/cos(B))) /(pi/180)

A =

     0

>> Y = atan((my_test(3,2)/cos(B))/(my_test(3,3)/cos(B)))/(pi/180)

Y =

   59.9993
```

Fixed Angle - X-Y-Z-求角度

- X-Y-Z Fixed Angles – 由R求角度
- $${}^{W}_{B}R_{xyz} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$
- If $\beta \neq 90°, -90° < \beta < 90°$, 唯一解
  - $\beta = \tan^{-1}\frac{-r_{31}}{\sqrt{r_{11}^2+r_{21}^2}}, \quad \alpha = \tan^{-1}\frac{r_{21}/\cos\beta}{r_{11}/\cos\beta}, \quad \gamma = \tan^{-1}\frac{r_{32}/\cos\beta}{r_{33}/\cos\beta}$
- If $\beta = 90°$, 不唯一解
  - $\alpha = 0, \gamma = \tan^{-1}\frac{r_{12}}{r_{22}}$
- If $\beta = -90°$, 不唯一解
  - $\alpha = 0, \gamma = -\tan^{-1}\frac{r_{12}}{r_{22}}$

Ex8. Fixed Angle 旋轉

- 已知${}^{W}_{B}R_{xyz}$(如下)，以X-Y-Z Fixed angle 方法，求角度。

$${}^{W}_{B}R_{xyz} = \begin{bmatrix} 0.5 & 0.75 & 0.433 \\ 0 & 0.5 & -0.866 \\ -0.866 & 0.433 & 0.25 \end{bmatrix}$$

- $\beta = \tan^{-1}\frac{-(-0.866)}{\sqrt{0.5^2+0}} = 60°$
- $\alpha = \tan^{-1}\frac{0/\cos 60°}{0.5/\cos 60°} = 0°$
- $\gamma = \tan^{-1}\frac{0.433/\cos 60°}{0.25/\cos 60°} = 60°$
- 先對x轉60度，再對y轉60度

```
>> my_test = [0.6124 -0.0474 0.7891;0.6124 0.6597 -0.4356;-0.5 0.75 0.433]

my_test =

    0.6124   -0.0474    0.7891
    0.6124    0.6597   -0.4356
   -0.5000    0.7500    0.4330

>> B = atan(-(my_test(3,1))/(my_test(1,1)^2+my_test(2,1)^2)^0.5) /(pi/180)

B =

   29.9989

>> A = atan((my_test(2,1)/cos(B))/(my_test(1,1)/cos(B))) /(pi/180)

A =

    45

>> Y = atan((my_test(3,2)/cos(B))/(my_test(3,3)/cos(B)))/(pi/180)

Y =

   60.0007
```

Ex9. Fixed Angle 旋轉

- 已知${}^{W}_{B}R_{xyz}$(如下)，以X-Y-Z Fixed angle 方法，求角度。

$$ {}^{W}_{B}R_{xyz} = \begin{bmatrix} 0.6124 & -0.0474 & 0.7891 \\ 0.6124 & 0.6597 & -0.4356 \\ -0.5 & 0.75 & 0.433 \end{bmatrix} $$

- $\beta = \tan^{-1}\frac{-(-0.5)}{\sqrt{0.6124^2+0.6124^2}} = 30°$
- $\alpha = \tan^{-1}\frac{0.6124/\cos 30°}{0.6124/\cos 30°} = 45°$ $\quad\quad$ $\gamma = \tan^{-1}\frac{0.75/\cos 30°}{0.433/\cos 30°} = 60°$
- 旋轉依序x轉60度，y轉30度，z轉45度
- 驗證： ${}^{W}_{B}R_{xyz} = R_z(45°)R_y(30°)R_x(60°)$

$$ = \begin{bmatrix} c45 & -s45 & 0 \\ s45 & c45 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c30 & 0 & s30 \\ 0 & 1 & 0 \\ -s30 & 0 & c30 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c60 & -s60 \\ 0 & s60 & c60 \end{bmatrix} $$

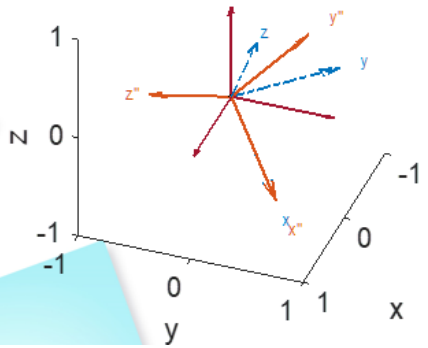
$$ = \begin{bmatrix} 0.6124 & -0.0474 & 0.7891 \\ 0.6124 & 0.6597 & -0.4356 \\ -0.5 & 0.75 & 0.433 \end{bmatrix} $$

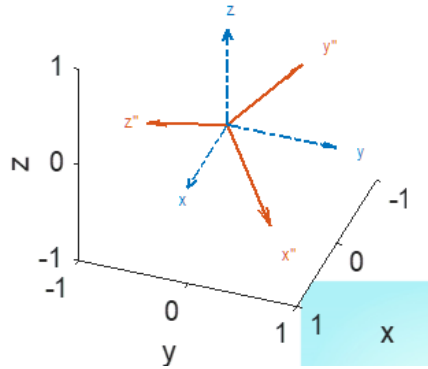
```
>> A = 45*(pi/180);
>> B = 30*(pi/180);
>> Y = 60*(pi/180);

>> double(subs(Euler_Angle_Rzyx))

ans =

    0.6124   -0.0474    0.7891
    0.6124    0.6597   -0.4356
   -0.5000    0.7500    0.4330
```

Ex10. Euler Angle 旋轉

- 以Euler Angle旋轉：旋轉依序z轉45度、y轉30度、x轉60度
- ${}^{W}_{B}R_{ZYX}(45°, 30°, 60°) = R_z(45°)R_y(30°)R_x(60°)$

$$= \begin{bmatrix} 0.6124 & -0.0474 & 0.7891 \\ 0.6124 & 0.6597 & -0.4356 \\ -0.5 & 0.75 & 0.433 \end{bmatrix}$$





```
>> A = -28.9*(pi/180);
>> B = 64.34*(pi/180);
>> Y = 56.31*(pi/180);
>> double(subs(Euler_Angle_Rzyz))

ans =

    0.6124   -0.0474    0.7891
    0.6123    0.6597   -0.4356
   -0.5000    0.7500    0.4330
```

Ex11. Euler Angle Z-Y-Z

- 以Euler Angle旋轉：旋轉依序z轉-28.9度、y轉64.34度、 z轉56.31度，求 ${}^{W}_{B}R_{ZYX}$
- ${}^{W}_{B}R_{ZYZ}(-28.9°, 64.34°, 56.31°) = R_z(-28.9°)R_y(64.34°)R_x(56.31°)$

$$= \begin{bmatrix} 0.6124 & -0.0474 & 0.7891 \\ 0.6124 & 0.6597 & -0.4356 \\ -0.5 & 0.75 & 0.433 \end{bmatrix}$$

- 比較Ex10(Z-Y-X)及Ex11(Z-Y-Z)，有相同的 ${}^{W}_{B}R$