

機器學習概論作業

範圍： Backproagation Step by Step

銘傳大學電腦與通訊工程系

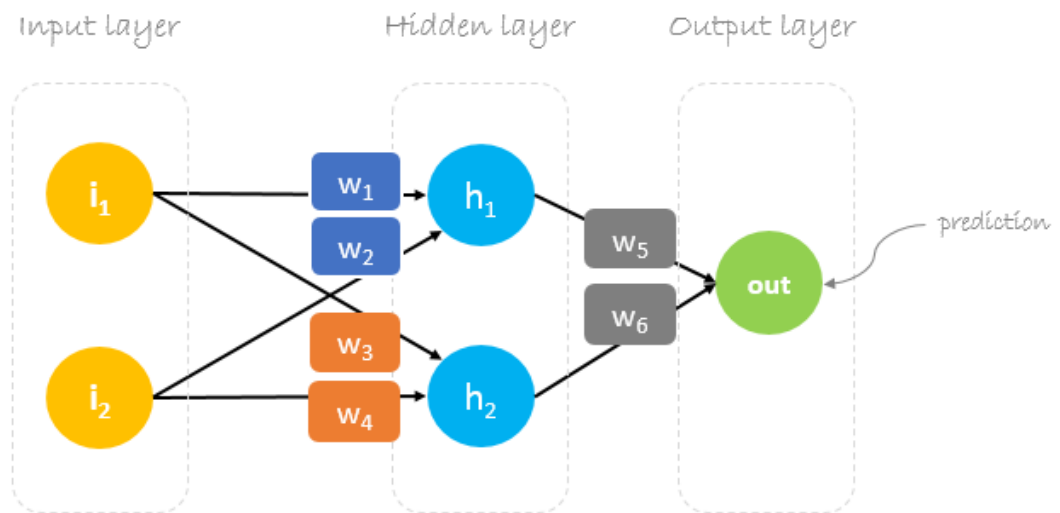
班 級	電通三乙
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作業成果	應繳作業共 <u>2</u> 題，每題 50 分 我共完成 <u>2</u> 題，應得 <u>100</u> 分
授課教師	陳慶逸

■ 請確實填寫自己寫完成題數，填寫不實者(如上傳與作業明顯無關的答案，或是計算題數有誤者)，本次作業先扣 50 分。

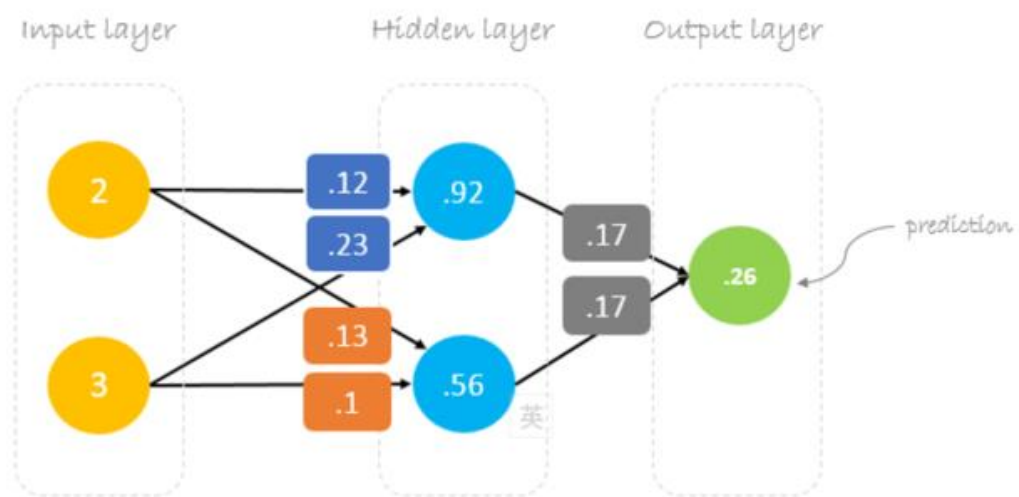
一、針對下面的 3 layer neural network，

其中 $h_1 = w_1*i_1 + w_2*i_2$, $h_2 = w_3*i_1 + w_4*i_2$,

$prediction = w_5*h_1 + w_6*h_2$



若權重 $w_1 = 0.12$, $w_2 = 0.23$, $w_3 = 0.13$, $w_4 = 0.1$, $w_5 = 0.17$, $w_6 = 0.17$;



1. 當 $inputs=[2, 3]$ ，期望輸出是 1 (但此時 neural network 的預測輸出是 0.26)



$$2. \text{Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

試手算再經過一次梯度修正後，得到所有權重及輸出為(learning rate = 0.05)：

$$w_1 = \underline{0.13258}, w_2 = \underline{0.24887}, w_3 = \underline{0.1428}, w_4 = \underline{0.11887},$$

$$w_5 = \underline{0.20404}, w_6 = \underline{0.19072}$$

$$h_1 = \underline{0.69356}, h_2 = \underline{0.85435},$$

$$\text{prediction} = \underline{0.30446}$$

$$\begin{aligned} & \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} \\ &= \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a * (\text{prediction} - \text{actual}) \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} [w_5 \ w_6] \\ &= \begin{bmatrix} 0.12 & 0.13 \\ 0.23 & 0.1 \end{bmatrix} - 0.05 * (-0.74) \begin{bmatrix} 2 \\ 3 \end{bmatrix} [0.17 \ 0.17] \\ &= \begin{bmatrix} 0.12 & 0.13 \\ 0.23 & 0.1 \end{bmatrix} - \begin{bmatrix} -0.01258 & -0.01258 \\ -0.01887 & -0.01887 \end{bmatrix} \\ &= \begin{bmatrix} 0.13258 & 0.1428 \\ 0.24887 & 0.11887 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} & \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} \\ &= \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a * (\text{prediction} - \text{actual}) \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \\ &= \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix} - 0.05 * (0.26 - 1) \begin{bmatrix} 0.92 \\ 0.56 \end{bmatrix} \\ &= \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix} - \begin{bmatrix} -0.03404 \\ -0.02072 \end{bmatrix} \\ &= \begin{bmatrix} 0.20404 \\ 0.19072 \end{bmatrix} \end{aligned}$$

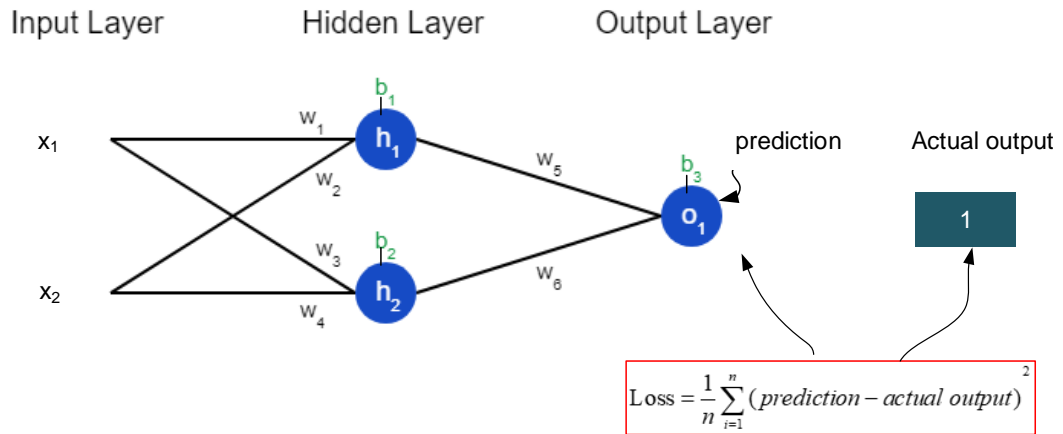
$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.13258 & 0.14280 \\ 0.24887 & 0.11887 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.69356 \\ 0.85435 \end{bmatrix}$$

$$prediction = [h_1 \ h_2] \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} = [0.69356 \ 0.85435] \begin{bmatrix} 0.20404 \\ 0.19072 \end{bmatrix} = 0.30446$$

二、針對下面的 3 layer neural network ,

其中 $h_1 = f(w_1 * x_1 + w_2 * x_2 + b_1)$, $h_2 = f(w_3 * x_1 + w_4 * x_2 + b_2)$,

$prediction = f(w_5 * h_1 + w_6 * h_2 + b_3)$ 。 f 為 sigmoid function 。



1. 當 $inputs = [-2, -1]$, 期望輸出是 1 (但此時 neural network 的預測輸出是?)
2. 誤差函數:

$$\begin{aligned}
 Loss &= \frac{1}{1} \sum_{i=1}^1 (prediction - actual\ output)^2 \\
 &= (actual\ output - prediction)^2 \\
 &= (1 - prediction)^2
 \end{aligned}$$

試推導

$$\frac{\partial Loss}{\partial w_6} =$$

$$\frac{\partial Loss}{\partial w_2} =$$

$$\frac{\partial Loss}{\partial w_3} =$$

$$\frac{\partial Loss}{\partial w_4} =$$

$$\frac{\partial Loss}{\partial b_1} =$$

$$\frac{\partial Loss}{\partial b_2} =$$

$$\frac{\partial Loss}{\partial b_3} =$$

$$\begin{aligned} & \frac{\partial Loss}{\partial w_6} \\ &= \frac{\partial Loss}{\partial prediction} * \frac{\partial prediction}{\partial w_6} \\ &= \frac{\partial (1 - prediction)^2}{\partial prediction} * \frac{\partial f(w_5 * h_1 + w_6 * h_2 + b_3)}{\partial w_6} \\ &= -2(1 - prediction) * h_2 * f'(w_5 h_1 + w_6 h_2 + b_3) \end{aligned}$$

$$\begin{aligned} & \frac{\partial Loss}{\partial w_2} \\ &= \frac{\partial Loss}{\partial prediction} * \frac{\partial prediction}{\partial h_1} * \frac{\partial h_1}{\partial w_2} \\ &= \frac{\partial (1 - prediction)^2}{\partial prediction} * \frac{\partial f(h_1 w_5 + h_2 w_6 + b_3)}{\partial h_1} * \frac{\partial f(w_1 x_1 + w_2 x_2 + b_1)}{\partial w_2} \\ &= -2(1 - prediction) * w_5 * f'(h_1 w_5 + h_2 w_6 + b_3) * x_2 * f'(w_1 x_1 + w_2 x_2 + b_1) \end{aligned}$$

$$\begin{aligned} & \frac{\partial Loss}{\partial w_3} \\ &= \frac{\partial Loss}{\partial prediction} * \frac{\partial prediction}{\partial h_2} * \frac{\partial h_2}{\partial w_3} \\ &= \frac{\partial (1 - prediction)^2}{\partial prediction} * \frac{\partial f(w_5 h_1 + w_6 h_2 + b_3)}{\partial h_2} * \frac{\partial f(w_3 x_1 + w_4 x_2 + b_2)}{\partial w_3} \\ &= -2(1 - prediction) * w_6 * f'(w_5 h_1 + w_6 h_2 + b_3) * x_1 * f'(w_3 x_1 + w_4 x_2 + b_2) \end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{Loss}}{\partial w_4} \\
&= \frac{\partial \text{Loss}}{\partial \text{prediction}} * \frac{\partial \text{prediction}}{\partial h_2} * \frac{\partial h_2}{\partial w_4} \\
&= \frac{\partial(1 - \text{prediction})^2}{\partial \text{prediction}} * \frac{\partial f(w_5 h_1 + w_6 h_2 + b_3)}{\partial h_2} * \frac{\partial f(w_3 x_1 + w_4 x_2 + b_2)}{\partial w_4} \\
&= -2(1 - \text{prediction}) * w_6 * f'(w_5 h_1 + w_6 h_2 + b_3) * x_2 * f'(w_3 x_1 + w_4 x_2 + b_2)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{Loss}}{\partial b_1} = \\
&= \frac{\partial \text{Loss}}{\partial \text{prediction}} * \frac{\partial \text{prediction}}{\partial h_1} * \frac{\partial h_1}{\partial b_1} \\
&= \frac{\partial(1 - \text{prediction})^2}{\partial \text{prediction}} * \frac{\partial f(w_5 h_1 + w_6 h_2 + b_3)}{\partial h_1} * \frac{\partial f(w_1 x_1 + w_2 x_2 + b_1)}{\partial b_1} \\
&= -2(1 - \text{prediction}) * w_5 * f'(w_5 h_1 + w_6 h_2 + b_3) * 1 * f'(w_1 x_1 + w_2 x_2 + b_1)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{Loss}}{\partial b_2} \\
&= \frac{\partial \text{Loss}}{\partial \text{prediction}} * \frac{\partial \text{prediction}}{\partial h_2} * \frac{\partial h_2}{\partial b_2} \\
&= \frac{\partial(1 - \text{prediction})^2}{\partial \text{prediction}} * \frac{\partial f(h_1 w_5 + h_2 w_6 + b_3)}{\partial h_2} * \frac{\partial f(w_3 x_1 + w_4 x_2 + b_2)}{\partial b_2} \\
&= -2(1 - \text{prediction}) * w_6 * f'(h_1 w_5 + h_2 w_6 + b_3) * 1 * f'(w_3 x_1 + w_4 x_2 + b_2)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{Loss}}{\partial b_3} \\
&= \frac{\partial \text{Loss}}{\partial \text{prediction}} * \frac{\partial \text{prediction}}{\partial b_3} \\
&= \frac{\partial(1 - \text{prediction})^2}{\partial \text{prediction}} * \frac{\partial f(w_5 h_1 + w_6 h_2 + b_3)}{\partial b_3} \\
&= -2(1 - \text{prediction}) * 1 * f'(w_5 h_1 + w_6 h_2 + b_3)
\end{aligned}$$