LPC analysis

From the source filter theory of speech production, the frequency response of the LP filter provides an approximation to the spectrum envelope of speech:

$$H(f) = \frac{g}{A(f)}$$

Where the system function for the LP filter is a complex polynomial of order p given by:

$$A(z) = 1 - \sum_{k=1}^{p} a_k z^{-k}$$

The frequencies of the formants can be estimated from $H(f)$ in different ways:

1. Estimate the frequencies of the peaks
2. Solve for the roots

We are going to use the 2nd one, solving the roots for the predictor polynomial $H(f)$.

The roots of $H(f)$, also called poles, always occur in pairs, one is the complex conjugate of the other (mirror image in the complex plane). Therefore, only half of the roots, conventionally between 0 and $\pi$ (i.e. have a positive imaginary part) are selected.

A polynomial of order p has p roots. Each root is a complex number defined as:

$r_k \exp\left(i2\pi \frac{f_k}{F_s}\right)$,

which represents a sinusoid of frequency $f_k$, and damping $r_k$.

The frequency $f_k$ of a root $\alpha_k$, can be computed as:

$$f_k = \frac{F_s}{2\pi} \tan^{-1}\left[\frac{Im(\alpha_k)}{Re(\alpha_k)}\right]$$

**The $\tan^{-1}$ can be implemented by np.angle, which if you look at the source code, internally calls the function np.arctan2()