Lab Sheet - 4

Simulation of continuous system

Objectives:

- To develop the mathematical modeling of the (continuous system) mass spring damper system.
- To determine the state of the system i.e. x, distance moved at different point of time.

Theory

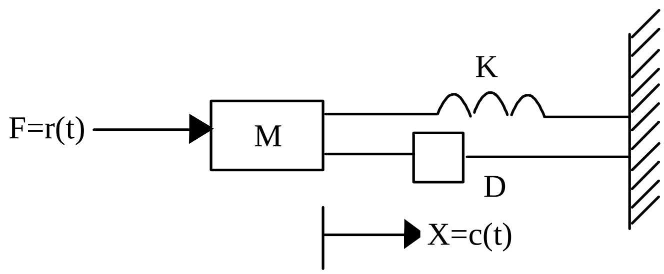
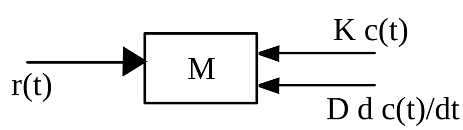


Figure 1: A mass-spring damper system

The free body diagram of the above system is



According to Newton's law of motion, we have

Force= Mass $\times$ Acceleration

The motion of the system is described by the following differential equation:

$$M \times \frac{d\ \ C(t)}{dt^2} = Kr(t) - \left(D\frac{dC(t)}{dt} + KC(t) \right)$$

$$M \times \frac{d^2C(t)}{dt^2} + D\frac{dC(t)}{dt} + KC(t) = Kr(t)$$

Therefore, the mathematical modeling of a mass spring damper system with mass M, force r(t), damper coefficient D, spring coefficient K and displacement C(t) is given by

$$M \times \frac{d^2C(t)}{dt^2} + D\frac{dC(t)}{dt} + KC(t) = Kr(t) \quad \text{---------[I]}$$

This equation can be re-written as

$$\frac{d^2C(t)}{dt^2} + \frac{D}{M}\frac{dC(t)}{dt} + \frac{K}{M}C(t) = \frac{1}{M}Kr(t) \qquad \text{---------[II]}$$

Equation [I] is the mathematical model of the given Mathematical system.
Comparing equation [II] with general second order partial equation.

$$\frac{d^2x(t)}{dt^2} + 2\xi.W\frac{dx(t)}{dt} + W^2x(t) = W^2F$$

Where,

Damping ration ξ = D/2.M.W

Angular frequency of oscillation W = $\sqrt{K/M}$

The response of this system with unit step function depends on the value of ξ.The condition for the motion to occur without oscillation requires that ξ>=1. Show how x varies in response to a steady force applied at time t=0 for the various values of ξ.