

# Linear Decision Rule Approach

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## 1 Original Problem

$$\begin{aligned} \max \quad & \mathbb{E}_{\xi} \left( \sum_{t=1}^T \mathbf{v}^T \mathbf{x}_t(\xi^t) \right) \\ \text{s.t.} \quad & \sum_{t=1}^T A \mathbf{x}_t(\xi^t) \leq \mathbf{c} \\ & x_t(\xi^t) \leq p_t(\xi) \\ & \forall \xi \in \Xi, t = 1, \dots, T \end{aligned} \tag{1}$$

## 2 Linear Decision Rule Approach

### 2.1 Primal Problem

$$\begin{aligned} \max \quad & \mathbb{E}_{\xi} \left( \sum_{t=1}^T \mathbf{v}^T X_t P_t \xi \right) \\ \text{s.t.} \quad & \sum_{t=1}^T A X_t P_t \xi \leq \mathbf{c} \\ & X_t P_t \xi \leq p_t^T \xi \\ & \forall \xi \in \Xi = \{ \xi : W \xi \leq h \}, t = 1, \dots, T \end{aligned} \tag{2}$$

time complexity

### 2.2 Duality

Firstly we transform equation(1) into a tighter formulation.

$$\begin{aligned} \max \quad & \mathbb{E}_{\xi} \left( \sum_{t=1}^T \mathbf{v}^T \mathbf{x}_t(\xi^t) \right) \\ \text{s.t.} \quad & \sum_{t=1}^T \tilde{A} \mathbf{x}_t(\xi^t) \leq \tilde{\mathbf{c}}_t(\xi) \\ & \mathbf{x}_t(\xi^t) \geq 0 \\ & \forall \xi \in \Xi, t = 1, \dots, T \end{aligned} \tag{3}$$

Then it has a duality.

$$\begin{aligned}
\min \quad & \mathbb{E}_{\xi} \left( \sum_{t=1}^T \tilde{\mathbf{c}}_t(\xi) \mathbf{y}_t(\xi^t) \right) \\
s.t. \quad & \sum_{t=1}^T \tilde{A}^T \mathbf{y}_t(\xi^t) \geq \mathbf{v}^T \\
& \mathbf{y}_t(\xi^t) \geq 0 \\
& \forall \xi \in \Xi, t = 1, \dots, T
\end{aligned} \tag{4}$$

### 3 Numerical Results

#### 3.1 First Case in Re-solve

#### 3.2 Second Case in Re-solve

#### 3.3 Some Tricks