Linear Decision Rule Approach

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1 Original Problem

Consider a network system consisting of m resources, with capacity levels $\mathbf{c} = (c_1, \ldots, c_m)^T$, and n products, with corresponding prices denoted by $\mathbf{v} = (v_1, \ldots, v_n)^T$. Each products needs at most one unit of each resource. Let $A = (a_{ij})$ be the resource coefficient matrix, where $a_{ij} = 1$ if product j uses one unit of resource i and $a_{ij} = 0$ otherwise. Define $\xi = (1, \xi_{1,1}, \xi_{1,2}, \ldots, \xi_{1,n}, \ldots, \xi_{t,1}, \xi_{t,2}, \ldots, \xi_{t,n})^T$ where $\xi_{t,j}$ is demand of product j in period t. Let ξ^t be the observed history demands and $\mathbf{x_t}(\xi^t)$ be the booking limits in period t,while $p_t(\xi) = (\xi_{t,1}, \xi_{t,2}, \ldots, \xi_{t,n})^T$. Realisation of ξ is limited to Ξ. The optimality equations can be expressed as

$$\begin{array}{ll} \max & \mathbb{E}_\xi \left(\sum_{t=1}^T \mathbf{v}^T \mathbf{x_t} \left(\xi^t\right) \right) \\ s.t. & \sum_{t=1}^T A \mathbf{x_t} \left(\xi^t\right) \leq \mathbf{c} \\ & \mathbf{x_t}(\xi^t) \leq \mathbf{p_t}(\xi) \\ & \forall \xi \in \Xi, t = 1, \ldots, T \end{array} \tag{1}$$

2 Linear Decision Rule Approach

2.1 Primal Problem

Approach original problem with $\mathbf{x_t}(\xi^t) = X_t P_t \xi$, $\Xi = \{\xi : W\xi \leq h\}$, and $\mathbf{p_t}(\xi) = p_t \xi$ where X_t, P_t , W and p_t are all matrix. Then we obtain the primal problem

$$
\begin{array}{ll}
\max & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T}\mathbf{v}^{T}X_tP_t\xi\right)\\
s.t. & \sum_{t=1}^{T}AX_tP_t\xi\leq\mathbf{c}\\
 & X_tP_t\xi\leq p_t\xi\\
 & \forall\xi\in\Xi=\left\{\xi:W\xi\leq h\right\},t=1,\ldots,T
\end{array}
\tag{2}
$$

where

$$
W=\begin{pmatrix}
1 & & & \\
-1 & & & \\
 & 1 & & \\
 & -1 & & \\
 & & \vdots & \\
 & & & 1\\
 & & & -1
\end{pmatrix}
$$

and

$$
h=\left(1,-1,q_{1,1,sup},-q_{1,1,inf},q_{1,2,sup},-q_{1,2,inf},\ldots,q_{t,n,sup},-q_{t,n,inf}\right)^{T}
$$

in which $q_{t,j,p}$ is the p quantile of $\xi_{t,j}$.

There are $nmt\tau+2nmt+n^2t^2$ variables and $m+2nt$ constraints in total.

2.2 Duality

Firstly we transform equation(1) into a tighter formulation.

$$
\begin{array}{ll}
\max & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T}\mathbf{v}^{T}\mathbf{x_t}\left(\xi^{t}\right)\right)\\
s.t. & \sum_{t=1}^{T}\tilde{A}\mathbf{x_t}\left(\xi^{t}\right)\leq\tilde{\mathbf{c}}_t\left(\xi\right)\\
 & \mathbf{x_t}\left(\xi^{t}\right)\geq 0\\
 & \forall\xi\in\Xi,t=1,\ldots,T
\end{array}
\tag{3}
$$

Then it has a duality.

$$
\begin{aligned}
\min \quad & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T} \tilde{\mathbf{c}}_t\left(\xi\right)^T \mathbf{y_t}\left(\xi^t\right)\right) \\
s.t. \quad & \sum_{t=1}^{T} \tilde{A}^T \mathbf{y_t}\left(\xi^t\right) \geq \mathbf{v}^T \\
& \mathbf{y_t}\left(\xi^t\right) \geq 0 \\
& \forall \xi \in \Xi, t = 1, \ldots, T
\end{aligned} \tag{4}
$$

We can apply same approach and obtain

$$
\begin{aligned}
\min \quad & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T} \left(\mathbf{c}^T, (p_t\xi)^T\right) \mathbf{y_t}\left(\xi^t\right)\right) \\
s.t. \quad & \sum_{t=1}^{T} \tilde{A}^T Y_t P_t \xi \geq \mathbf{v}^T \\
& Y_t P_t \xi \geq 0 \\
& \forall \xi \in \Xi \left\{\xi : W\xi \leq h\right\}, t = 1, \ldots, T
\end{aligned} \tag{5}
$$

3 Numerical Results

Rounding can matter a lot to benefits. Therefore, I tried genetic algorithm to get a not bad policy. However, it sometimes offer a worse policy than simply ceil. Also, when I adjust parameters, t and quantile seem play an important role in optimizing. It suggests that if we optimize the problem with changes of t and quantile, higher benefits should be possible. What's more, please notice that all results obtained now are under constant policies.

3.1 First Case in Re-solving

The highest benefit achieved is 418538 with $t = 10$, $sup = 0.9$ and $inf = 0.65$, better than values obtained without re-solving in the paper,that is, 401980(DLP-alloc), 415410(SLP-alloc), 347690(DLP-bid) and 347990(SLP-bid).

3.2 Second Case in Re-solving

The highest benefit achieved is 601788 with $t = 10$, $sup = 0.9$ and $inf = 0.7$, better than values obtained without re-solving in the paper,that is, 583630(DLP-alloc), 595620(SLP-alloc), 518470(DLP-bid) and 520210(SLP-bid).

3.3 Data from Approximate Linear Programming

As far as I am concerned,their data scale is limited and not suitable for decision rule approach. Maybe we shouldn't make use of it?

3.4 Computational Ability

Amazon EC2 provides cheap cloud computing resources, which should be e-nough to satisfy our needs.