Linear Decision Rule Approach

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1 Original Problem

Consider a network system consisting of m resources, with capacity levels $\mathbf{c} = (c_1, \ldots, c_m)^T$, and n products, with corresponding prices denoted by $\mathbf{v} = (v_1, \ldots, v_n)^T$. Each products needs at most one unit of each resource. Let $A = (a_{ij})$ be the resource coefficient matrix, where $a_{ij} = 1$ if product j uses one unit of resource i and $a_{ij} = 0$ otherwise. Define $\xi = (1, \xi_{1,1}, \xi_{1,2}, \ldots, \xi_{1,n}, \ldots, \xi_{t,1}, \xi_{t,2}, \ldots, \xi_{t,n})^T$ where $\xi_{t,j}$ is demand of product j in period t. Let ξ_t be the observed history demands and $\mathbf{x_t}(\xi^t)$ be the booking limits in period t,while $p_t(\xi) = (\xi_{t,1}, \xi_{t,2}, \ldots, \xi_{t,n})^T$. Realisation of ξ is limited to Ξ. The optimality equations can be expressed as

$$\begin{array}{ll} \max & \mathbb{E}_\xi \left(\sum_{t=1}^{T} \mathbf{v}^T \mathbf{x_t} \left(\xi^t\right) \right) \\ s.t. & \sum_{t=1}^{T} A \mathbf{x_t} \left(\xi^t\right) \leq \mathbf{c} \\ & \mathbf{x_t}(\xi^t) \leq \mathbf{p_t}(\xi) \\ & \forall \xi \in \Xi, t = 1, \ldots, T \end{array} \tag{1}$$

2 Linear Decision Rule Approach

2.1 Primal Problem

Approach original problem with $\mathbf{x_t}(\xi^t) = X_t P_t \xi$, $\Xi = \{\xi : W\xi \leq h\}$, and $\mathbf{p_t}(\xi) = p_t \xi$ where X_t, P_t , W and p_t are all matrix. Then we obtain the primal problem

$$
\begin{aligned}
\max \quad & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T}\mathbf{v}^{T}X_{t}P_{t}\xi\right) \\
s.t. \quad & \sum_{t=1}^{T}AX_{t}P_{t}\xi \leq \mathbf{c} \\
& X_{t}P_{t}\xi \leq p_{t}\xi \\
& \forall \xi \in \Xi = \left\{\xi : W\xi \leq h\right\}, t = 1, \ldots, T
\end{aligned}
\tag{2}
$$

where

$$
W = \begin{pmatrix}
1 & & & \\
-1 & & & \\
& 1 & & \\
& -1 & & \\
& & \vdots & \\
& & & 1 \\
& & & -1
\end{pmatrix}
$$

and

$$
h = \left(1, -1, q_{1,1,0.9}, -q_{1,1,0.1}, q_{1,2,0.9}, -q_{1,2,0.1}, \ldots, q_{t,n,0.9}, -q_{t,n,0.1}\right)^{T}
$$

in which $q_{t,j,p}$ is the p quantile of $\xi_{t,j}$.

There are $nmt\tau + 2nmt + n^2t^2$ variables and $m + 2nt$ constraints in total.

2.2 Duality

Firstly we transform equation(1) into a tighter formulation.

$$
\begin{aligned}
\max \quad & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T}\mathbf{v}^{T}\mathbf{x_t}\left(\xi^{t}\right)\right) \\
s.t. \quad & \sum_{t=1}^{T}\tilde{A}\mathbf{x_t}\left(\xi^{t}\right) \leq \tilde{\mathbf{c}}_{t}\left(\xi\right) \\
& \mathbf{x_t}\left(\xi^{t}\right) \geq 0 \\
& \forall \xi \in \Xi, t = 1, \ldots, T
\end{aligned}
\tag{3}
$$

Then it has a duality.

$$
\begin{aligned}
\min \quad & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T} \tilde{\mathbf{c}}_t\left(\xi\right)^T \mathbf{y_t}\left(\xi^t\right)\right) \\
s.t. \quad & \sum_{t=1}^{T} \tilde{A}^T \mathbf{y_t}\left(\xi^t\right) \geq \mathbf{v}^T \\
& \mathbf{y_t}\left(\xi^t\right) \geq 0 \\
& \forall \xi \in \Xi, t = 1, \ldots, T
\end{aligned} \tag{4}
$$

Also,we can use linear decision rule approach.

$$
\begin{aligned}
\min \quad & \mathbb{E}_{\xi}\left(\sum_{t=1}^{T} \left(\mathbf{c}^T, (p_t\xi)^T\right) \mathbf{y_t}\left(\xi^t\right)\right) \\
s.t. \quad & \sum_{t=1}^{T} \tilde{A}^T Y_t P_t \xi \geq \mathbf{v}^T \\
& Y_t P_t \xi \geq 0 \\
& \forall \xi \in \Xi \left\{\xi : W\xi \leq h\right\}, t = 1, \ldots, T
\end{aligned} \tag{5}
$$

3 Numerical Results

3.1 First Case in Re-solve

3.2 Second Case in Re-solve

3.3 Some Tricks