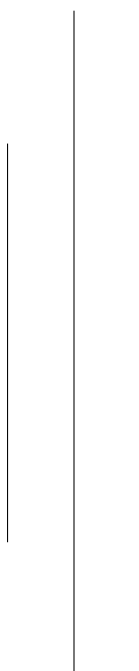




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INSTITUTE OF ENGINEERING  
PULCHOWK CAMPUS  
LAB REPORT**



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# **Half Adder, Half Subtractor, Full Adder, Full Subtractor**

## **1 Objectives**

1. To Study the half adder and full adder circuit and their truth table.
2. To study the half subtractor and full Subtractor circuits and their truth table.
3. To Realize half adder and half subtractor in a single circuit.
4. To Realize full adder and full subtractor in a single circuit.

## **2 Theory**

### **2.1 Combinational Logic Circuit**

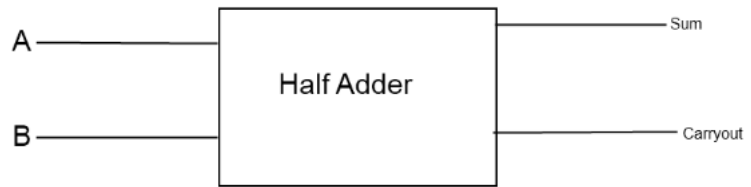
Combinational Logic Circuits are made up from basic logic NAND, NOR , NOT or basic gates that are combined or connected together to produce more complicated desired results. Some examples of Combinational logic circuits are encoder, decoders, adders, multiplexers, etc. As combinational logic circuits are made up from individual logic gates only they can be considered as "decision making circuits" and combinational logic is about coming logic gates together to process two or more signals in order to produce at least one output signal according to the logical function of each logic gate. Common combinational circuits made up from the individual logic gates that carry out a desired application include adders, subtractors, multiplexers etc.

## 2.2 Half Adder

Half adder is a type of adder circuit that performs binary addition on its two inputs, and provides a sum and a carry value. This computation can be represented by XOR (for sum) and AND (for carry).

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Table 1: Truth Table



Half Adder Block Diagram

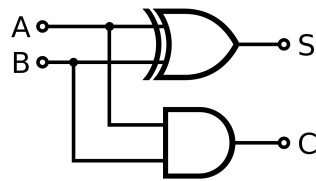


Figure 1: Circuit Diagram of Half Adder

### Boolean Expression

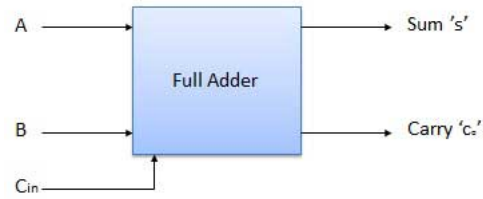
$$\begin{aligned} Sum &= A \oplus B \\ Carry &= A.B \end{aligned}$$

## 2.3 Full Adder

Full adder is a type of adder circuit that performs binary addition on its three inputs, and provides a sum and a carry value. Full adders can be made by using XOR, AND and OR gates. A full adder can also be made by using two half adders.

A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Table 2: Truth Table



Full Adder Block Diagram

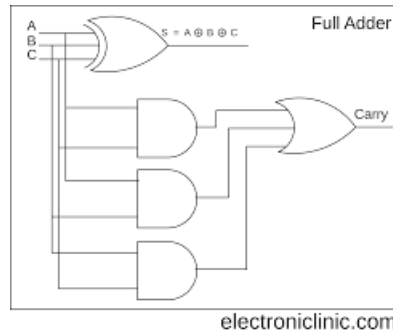


Figure 2: Circuit Diagram of Full Adder

### Boolean Expression

$$\begin{aligned}
 Sum &= A'B'C + A'BC' + AB'C' + ABC \\
 &= A'(B'C + BC') + A(B'C' + BC) \\
 &= A'(B \oplus C) + A(B \oplus C)' \\
 &= A \oplus B \oplus C
 \end{aligned} \tag{1}$$

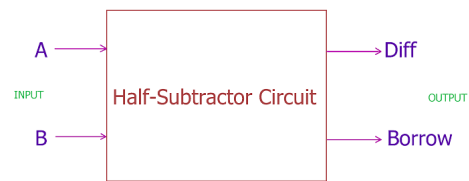
$$\begin{aligned}
 Carry &= A'BC + AB'C + ABC' + ABC \\
 &= BC(A + A') + A(B'C + BC') \\
 &= BC + AB'C + ABC'
 \end{aligned} \tag{2}$$

## 2.4 Half Subtractor

A half subtractor is a combinational circuit that subtracts two bits and produces their difference. It also has an output to specify if a 1 has been borrowed.

A	B	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Table 3: Truth Table



Half Subtractor Block Diagram

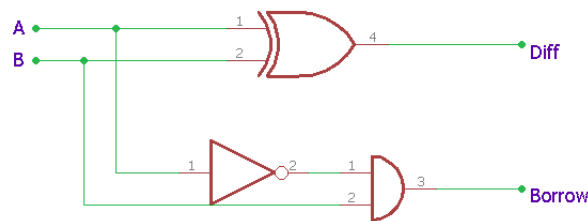


Figure 3: Circuit Diagram of Half Subtractor

### Boolean Expression

$$\begin{aligned} Sum &= A \oplus B \\ Carry &= A'.B \end{aligned}$$

A full subtractor is a combinational circuit that performs a subtraction between two bits, taking into account that 1 may have been borrowed by a lower significant stage. This circuit has three inputs and two outputs.

```

graph LR
    A[A] --> FSC[Full-Subtractor Circuit]
    B[B] --> FSC
    BI[Borrow In] --> FSC
    FSC --> Diff[Diff]
    FSC --> BO[Borrow Out]
  
```

Table 4: Truth Table


$$\begin{aligned}
Difference &= A'B'B_{in} + A'BB'_{in} + AB'B'_{in} + ABB_{in} \\
&= B_{in}(A'B' + AB) + B'_{in}(AB' + A'B) \\
&= B_{in}(A \oplus B)' + B'_{in}(A \oplus B) \\
&= B_{in} \oplus A \oplus B \\
&= A \oplus B \oplus B_{in}
\end{aligned} \tag{3}$$

5

## 2.6 Full adder using Half Adders

Full adder can be constructed by using half adders. It requires two half adders and a Or gate.

**Boolean Expression**

$$Sum = A \oplus B \oplus C \quad (5)$$

$$\begin{aligned} Carry &= A'BC + AB'C + ABC' + ABC \\ &= C(A'B + AB') + AB(C' + C) \\ &= C(A \oplus B) + AB \end{aligned} \quad (6)$$

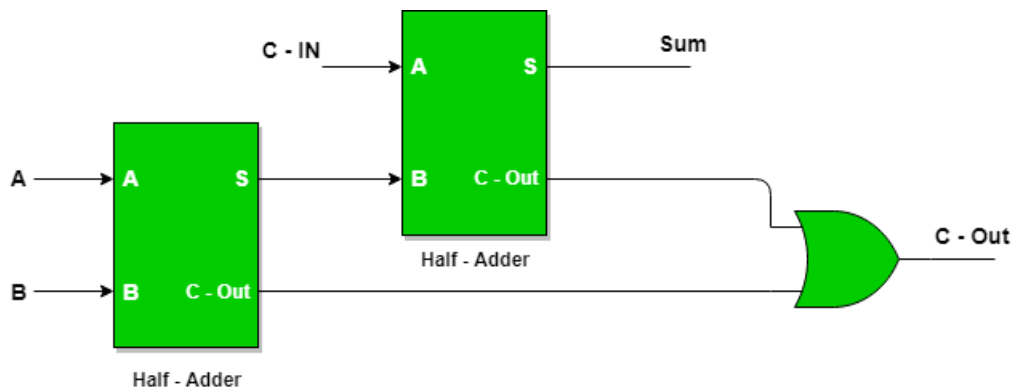


Figure 5: Block Diagram of Full Adder from Half Adders

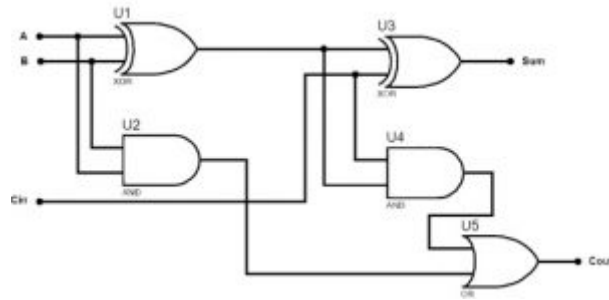


Figure 6: Circuit Diagram of Full Adder from half adders

## 2.7 Full Subtractor using Half Subtractor

Full subtractor can be constructed by using half subtractors. **Boolean Expression**

$$Sum = A \oplus B \oplus C \quad (7)$$

$$\begin{aligned} Carry &= A'B'B_{in} + A'BB_{in} + A'BB'_{in} + ABB_{in} \\ &= A'B(B_{in} + B'_{in}) + B_{in}(A'B' + AB) \\ &= A'B + C(A \oplus B)' \end{aligned} \quad (8)$$

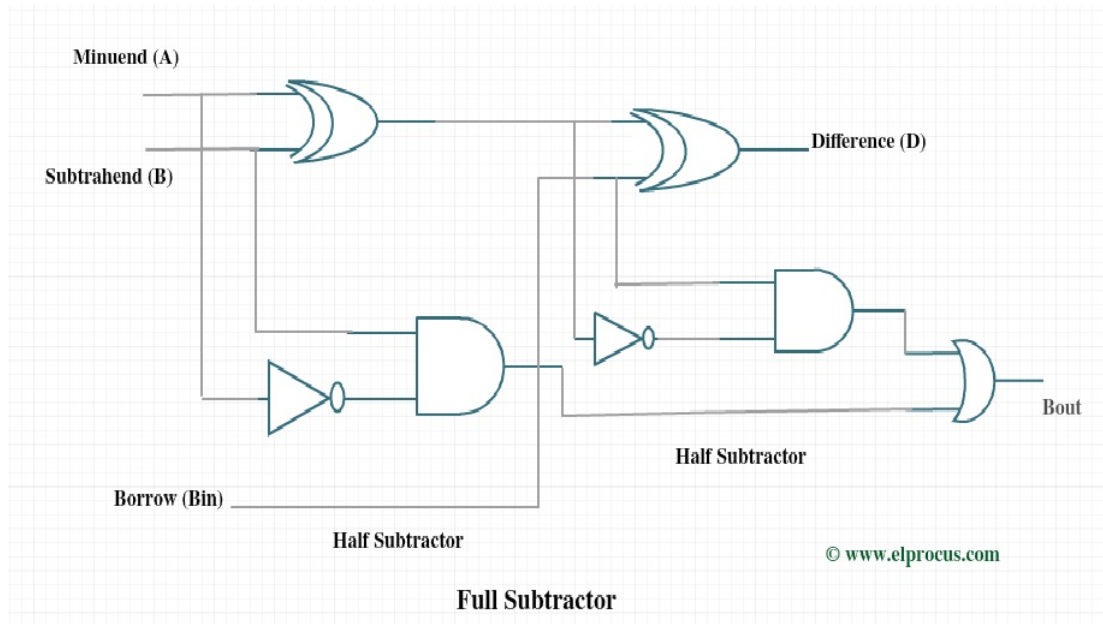


Figure 7: Circuit Diagram of Full subtractor from Half Subtractor



## 2.8 Realization of half adder and half subtractor in a single circuit

If we introduce a new input called switch or control state which chooses if the given two inputs will be added or subtracted. We can Realize a half adder and a half subtractor in a single circuit. We can make it so that, When S=0, addition occurs and when S=1, subtraction occurs.

$$Sum/Difference = A \oplus B \quad (9)$$

$$\begin{aligned} Borrow/Carry &= C'AB + CA'B \\ &= B(C'A + CA') \\ &= B(C \oplus A) \end{aligned} \quad (10)$$

S	A	B	Difference/Sum	Borrow/Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	0
1	1	1	0	0

Table 5: Truth Table

## 2.9 Realization of full adder and full subtractor in a single circuit

If we introduce a new input called switch or control state which chooses if the given two inputs will be added or subtracted. We can Realize a full adder and a full subtractor in a single circuit. We can make it so that, When S=0, addition occurs and when S=1, subtraction occurs.

$$\begin{aligned}
 \text{Sum/Difference} &= AB'C + A'B'C + ABC + A'BC' \\
 &= A'(B'C + BC') + A(B'C' + BC) \\
 &= A'(B \oplus C) + A(B \oplus C)' \\
 &= A \oplus B \oplus C
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 \text{Carry/Borrow} &= BC + S'AC + S'AB + SA'C + SA'B \\
 &= BC + S'AC + SA'C + S'AB + SA'B \\
 &= BC + C(S'A + SA') + (S'A + SA') \\
 &= BC + C(A \oplus S) + B(S \oplus A) \\
 &= BC + (A \oplus B)(B + C)
 \end{aligned} \tag{12}$$

S	A	B	C	Difference/Sum	Borrow/Carry
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	1	1
1	0	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	1	1

Table 6: Truth Table

### 3 Lab

#### 4 Half Adder

The following half adder circuit was made in proteus

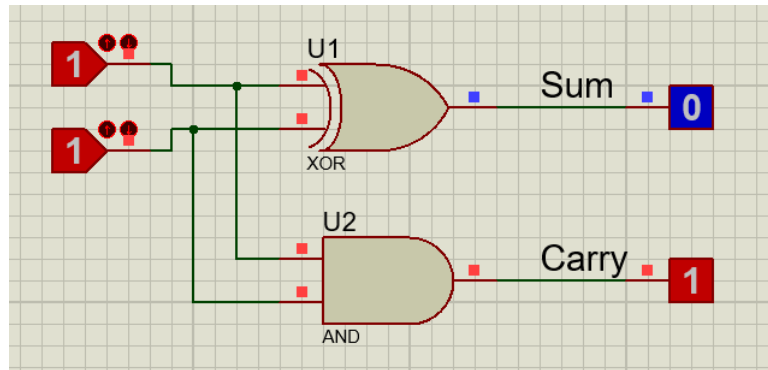


Figure 8: Half Adder circuit

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Table 7: Truth Table

#### 4.1 Full Adder

Full adder Circuit was simulated in proteus and truth table was verified.

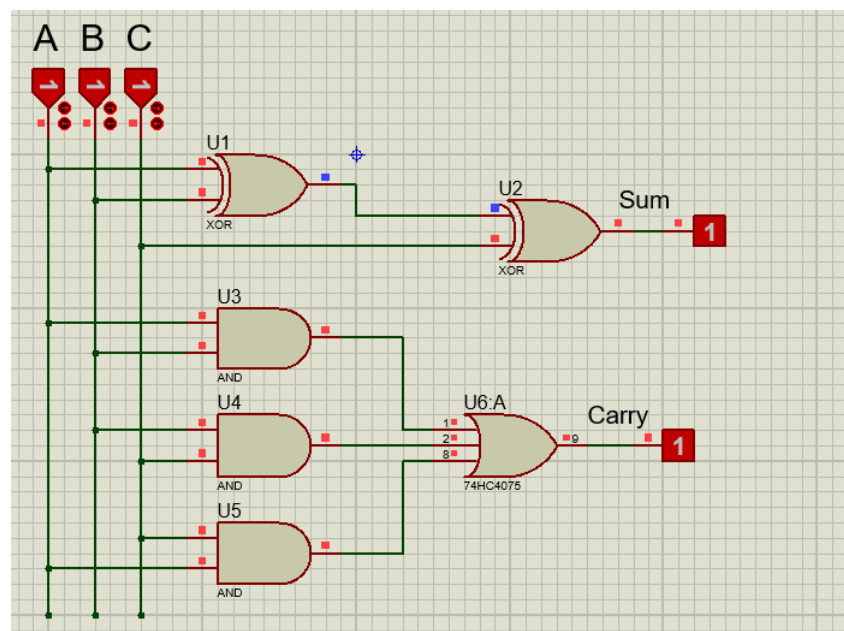


Figure 9: full adder circuit

A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Table 8: Truth Table

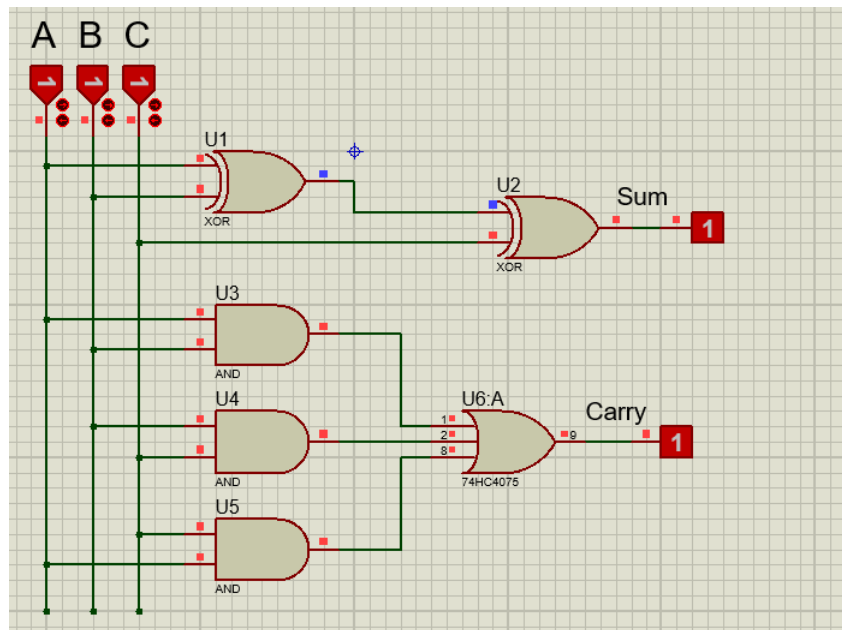


Figure 10: Full adder from half adders

## 4.2 Half Subtractor

Half Subtractor Circuit was built, and simulated in proteus and truth table was obtained.

A	B	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Table 9: Truth Table

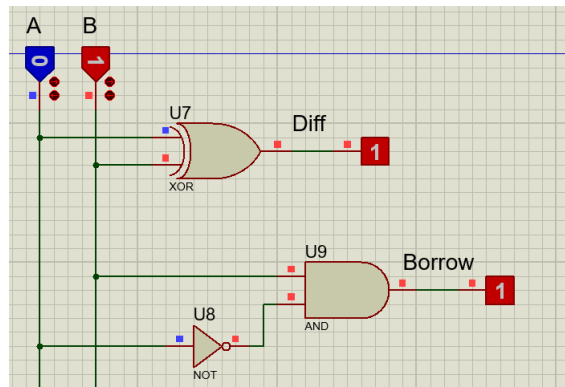


Figure 11: Half Subtractor

### 4.3 Full Subtractor

Half Subtractor Circuit was built, and simulated in proteus and truth table was obtained.

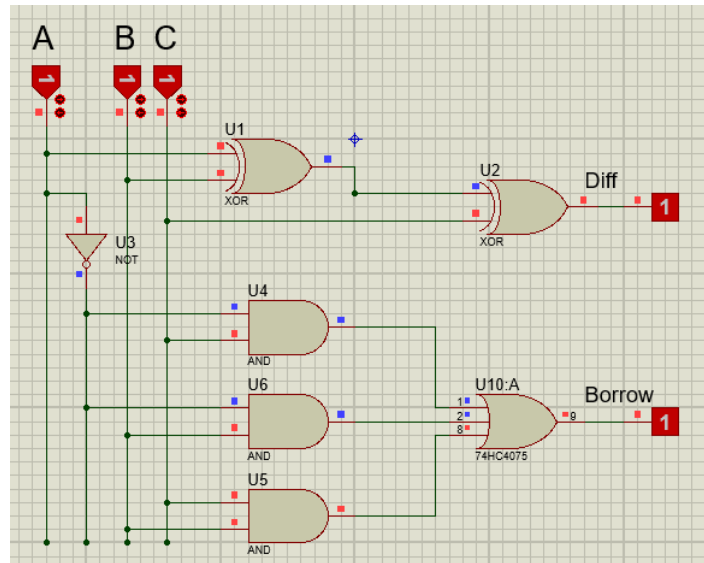


Figure 12: Full Subtractor

A	B	$B_{in}$	Difference	$B_{out}$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Table 10: Truth Table

#### 4.4 Realization of half adder and half subtractor in a single circuit

If we introduce a new input called switch or control state which chooses if the given two inputs will be added or subtracted. We can Realize a half adder and a half subtractor in a single circuit. We can make it so that, When S=0, addition occurs and when S=1, subtraction occurs.

$$Sum/Difference = A \oplus B \quad (13)$$

$$Carry/Borrow = B(S \oplus A) \quad (14)$$

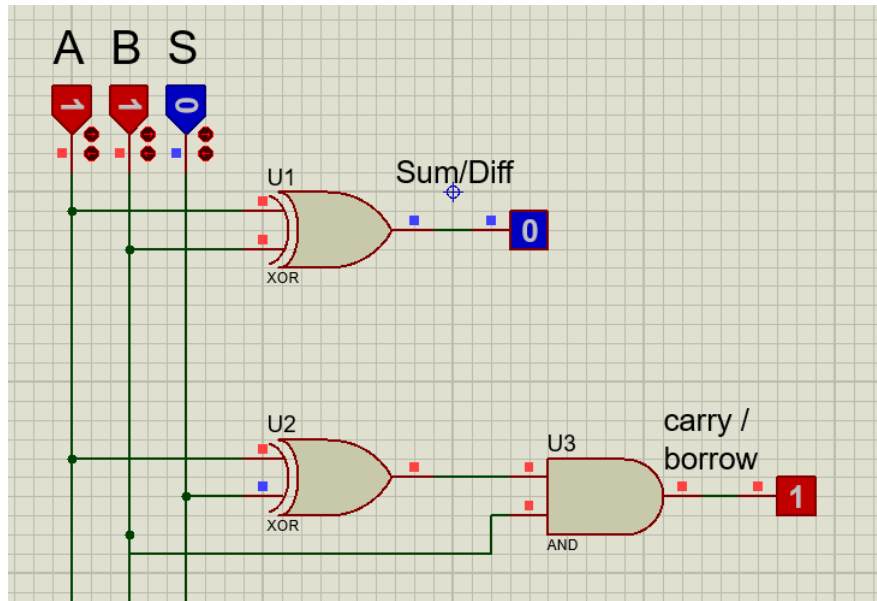


Figure 13: Half adder and Half subtractor in a single circuit

S	A	B	Difference/Sum	Borrow/Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	0
1	1	1	0	0

Table 11: Truth Table

## 4.5 Realization of full adder and full subtractor in a single circuit

If we introduce a new input called switch or control state which chooses if the given two inputs will be added or subtracted. We can Realize a full adder and a full subtractor in a single circuit. We can make it so that, When S=0, addition occurs and when S=1, subtraction occurs.

$$Sum/Difference = A \oplus B \oplus C \quad (15)$$

$$Carry/Borrow = BC + (A \oplus S).(B + C) \quad (16)$$

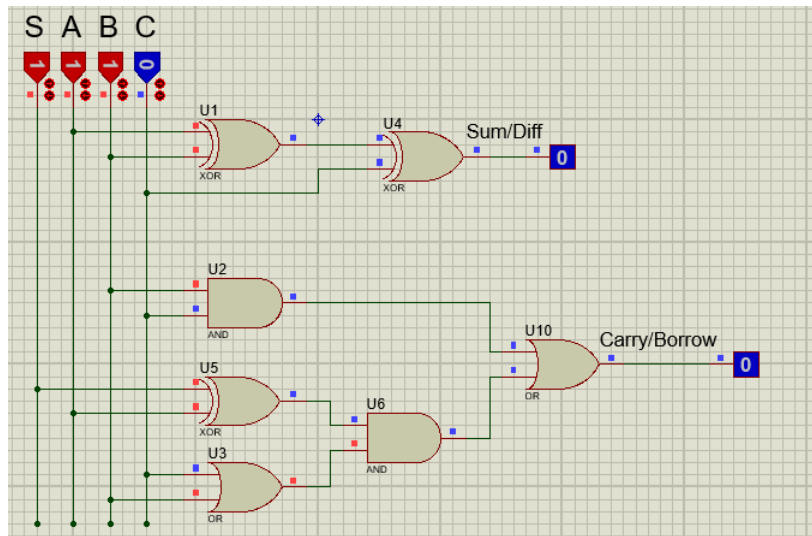


Figure 14: Full adder and Full subtractor in a single circuit

S	A	B	C	Difference/Sum	Borrow/Carry
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	1	1
1	0	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	1	1

Table 12: Truth Table



## 5 Discussion

Various digital circuits were constructed in the practical their truth tables were obtained. The truth table of the circuits and Kmap was used to obtain a boolean expression for the circuit. With this their circuits were constructed. We got a better understanding of the Combinational logic circuits. We were able to construct half adder, half subtractor, full adder, full subtractor circuits. We also learned that full adder circuit can be constructed using 2 half adders. Similarly a full subtractor circuit can be constructed using 2 half subtractors. We were also able to construct half adder and half subtractor in a single circuit, and similarly full adder and full subtractor in a single circuit. This was possible with the help of a switch(S) signal. The circuit would perform addition when  $S=0$  and subtraction when  $S=1$ .

## 6 Conclusion

Thus, a thorough understanding of various combinational logic circuits was obtained. Kmap was used to obtain the boolean expression for half adder, half subtractor, full adder, full subtractor, half adder and half subtractor in single circuit, full adder and full subtractor in single circuit. These circuits were constructed and their behaviour was deeply studied.