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076BCT082

1. Determine whether each of the follwing is true or false.

   (a) $\emptyset \subseteq \emptyset$
   True. Everyset is a subset of itself

   (b) $\emptyset \in \emptyset$
   False. As an empty set cannot contain another set, in this case $\emptyset$.

   (c) $\emptyset \in \{\emptyset\}$
   True. As the set has the element $\emptyset$

   (d) $\emptyset \subseteq \{\emptyset\}$
   True. Emptyset is the subset of every set.

   (e) $\{a, b\} \in \{a, b, c, \{a, b\}\}$
   True. The set contains both elements a and b.

   (f) $\{a, b\} \subseteq \{a, b, \{a, b\}\}$
   True. As the set $\{a, b\}$ has elements a, b from $\{a, b, \{a, b\}\}$ making it a subset.

   (g) $\{a, b\} \subseteq 2^{\{a,b,\{a,b\}\}}$
   False. Power set is a set of sets and it doesn't have elemtns a and b.

   (h) $\{\{a, b\}\} \in 2^{\{a,b,\{a,b\}\}}$
   True. The power set does have a element that is a set of set containing elemetns a and b.

   (i) $\{a, b, \{a, b\}\} - \{a, b\} = \{a, b\}$
   False. After the operation on the left hand side we are left with $\{\{a, b\}\}$ which is not equal to $\{a, b\}$

2. What are these sets? Write them using brackets, commas and numerals only.

   (a) $(\{1, 3, 5\} \cup \{3, 1\}) \cap \{3, 5, 7\} = \{3, 5\}$

   (b) $\cup\{\{3\}, \{3, 5\}, \cap\{\{5, 7\}, \{7, 9\}\}\} = \{3, 5, 7\}$

   (c) $(\{1, 2, 5\} - \{5, 7, 9\}) \cup (\{5, 7, 9\} - \{1, 2, 5\}) = \{1, 2, 7, 9\}$

   (d) $2^{\{7,8,9\}} - 2^{\{7,9\}} = \{\{8\}, \{7, 8\}, \{8, 9\}, \{7, 8, 9\}\}$

   (e) $2^{\emptyset} = \{\emptyset\}$

3. Prove each of the following

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   Let,
   $L = A \cup (B \cap C)$ and $R = (A \cup B) \cap (A \cup C)$
   We are to show that $L = R$. We do this by showing (i) $L \subseteq R$ and (ii) $R \subseteq L$

      i. Let x be an element of L. Then either, $x \in A$ or $x \in (B \cap C)$
      if $x \in A$, then this emplies, $x \in (A \cup B)$ and $x \in (A \cup C)$. And therefore, $x \in ((A \cup B) \cap (A \cup C))$. Similarly, if $x \in (B \cap C)$, this emplies that $x \in B$ and $x \in C$.Thus, $x \in (A \cup B)$ and $x \in (A \cup C)$. And hence, $x \in ((A \cup B) \cap (A \cup C))$.
      Therefore $L \subseteq R$.

      ii. Let $x \in R$, then x in an element of both $A \cup B$ or $A \cup C$.
      If, $x \in A$, then $x \in L$. Similarly, if $x \notin A$, then x must be in both B and C. i.e $x \in B$ and $x \in C$. Then $xin(B \cap C)$ and thus $x \in L$.
      Therefore $R \subseteq L$.
      Hence we established $L = R$

   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   Let,
   $L = A \cap (B \cup C)$ and $R = (A \cap B) \cup (A \cap C)$
   We are to show that $L = R$. We do this by showing (i) $L \subseteq R$ and (ii) $R \subseteq L$

      i. Let x is an element of L; then $x \in A$ and $x \in (B \cup C)$. x must be in A but can be a member of either B or C.
      Let $x \in B$, then $xin(A \cap B)$ and thus $x \in (A \cap B) \cup (A \cap C)$. Similarly, let $x \in C$, then $xin(A \cap C)$ and thus, $x \in (A \cap B) \cup (A \cap C)$. Therefore $L \subseteq R$.

      ii. Let $x \in R$. Then $x \in (A \cap B)$ or $x \in (A \cap C)$.
      If, $x \in (A \cap B)$, then $x \in A$ and $x \in B$. With this, $x \in A \cap (B \cup C)$. Similarly, it $x \in (A \cap C)$ then $x \in A$ and $x \in C$. This emplies $x \in A \cap (B \cup C)$. Therefore $R \subseteq L$.
      Hence we established $L = R$

   (c) $A \cap (A \cup B) = A$
   Let, x in $A \cap (A \cup B)$, then because of intersection $x \in A$.
   Similarly, let $x \in A$ then, $x \in (A \cup C)$ and thus, $x \in A \cap (B \cup C)$.

   (d) $A \cup (A \cap B) = A$
   Let $x \in A \cup (A \cap B)$, with this we can say, $x \in A$.
   Similarly, let $x \in A$, then by $x \in A \cup (A \cap B)$.

(e) $A - (B \cap C) = (A - B) \cup (A - C)$
Let, $L = A - (B \cap C)$ and $R = (A - B) \cup (A - C)$
Let, $x \in L$, then $x \in A$ and $x \notin (B \cap C)$. Thus x cannot be in B and C both. When $x \in B$ implies, $x \in (A - C)$ and thus $x \in R$. Similarly, when $x \in C$ implies $x \in (A - B)$ and thus $x \in R$. When x is not in both B and C, then too $x \in R$. Thus $L \subseteq R$.
Similarly, let $x \in R$, then $x \in A$, similarly $x \notin (B \cap C)$. Thus, $x \in L$. And $R \subseteq L$. From this we have $L = R$.

4. Let $S = \{a, b, c, d\}$.

(a) What partitions of S has the fewest numbers? The most members?
The partition of S that has the fewest members is {{a, b, c, d}}, a set containing S. Similarly, the partition of S has the most members when it is a set of subsets of S such that each has one element of S, that is {{a}, {b}, {c}, {d}}.

(b) List all the partitions of S with exactly two members.
They are:
{{a, b}, {c, d}}, {{a, c}, {b, d}}, {{a, d}, {b, c}}, {{a}, {b, c, d}}, {{b}, {a, c, d}}, {{c}, {a, b, d}}, {{d}, {a, b, c}}