



Introduction to Cryptography

Alan Cao

Disclaimers

- Cryptography is rooted in both theoretical (math) and technical components.
 - **I AM NOT A MATH MAJOR, NOR AN EXPERT CRYPTOGRAPHER!!**
- What this talk **WILL**:
 - Help you make the best choices to make in selecting cryptographic primitives
 - Give you a whirlwind tour of the internals that make a cryptographic primitive work
 - Make you comfortable working with cryptographic software

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- What this talk **will NOT**:
 - Teach you how to roll your own crypto (please don't do this)
 - Cover post-quantum cryptography
 - Give in-depth looks into modern-day cryptography schemes and protocols

Cryptography Primer

- Cryptography is the study of protocols that can help effectively transmit information
 - Encryption and decryption occur with a **cipher**, which takes **plaintext** and turns it into **ciphertext** using a **secret key/password**.
 - We want to **take advantage** of the invertibility of certain mathematical properties to construct cryptographic schemes

Crypto and Security

- Our goal in secure modern cryptography: *what do we want to guarantee the users of cryptographic software?*
 - Kerckhoffs's Principle
 - A cryptographic scheme should rely on the *secrecy of the key*, rather than the secrecy of the cipher
 - If an attacker understands all the intricacies of a cipher, he/she should still not be able to get key K if it is kept secret.

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 - If an attacker understands all the intricacies of a cipher, he/she should still not be able to get key K if it is kept secret.
 - IND-CPA (semantic security)
 - Ciphertext should not leak information about plaintext if key is kept secure
 - Randomness is an important factor in adhering to IND-CPA



Objectives

Building Blocks

- OTPs
- RNGs
- Hashing

Symmetric Cryptography

Block Ciphers

Stream Ciphers

Authenticated
Encryption

Asymmetric Cryptography

RSA

Elliptic Curves

Diffie-Hellman

SSL/TLS

...and more!

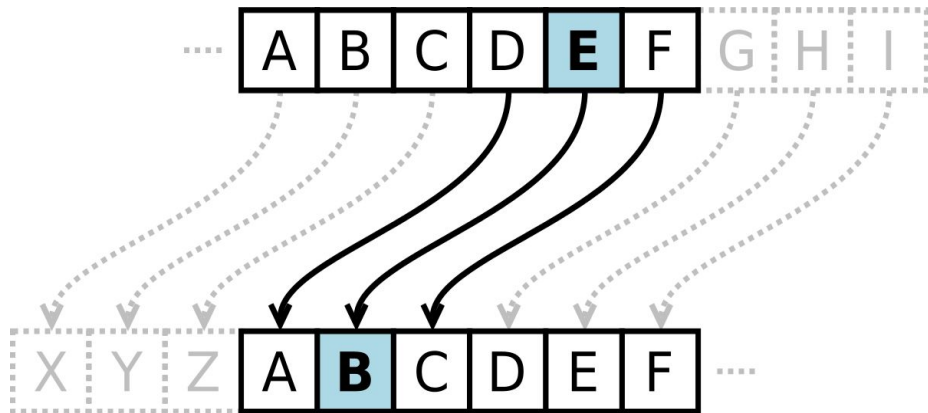
Classical Cryptography

- Substitution Ciphers
 - *Bijjective map* from each character to another
- Classic example: **Caesar's Cipher** (monoalphabetic)
- Later implementation: **Vigenere Cipher** (polyalphabetic)

- Efficient/sufficient when transmission of information wasn't digital
 - *We'll see why these all fail now, and how we've adapted from problems we've seen in them.*

Classical Crypto - Caesar's Cipher

- Simple family of shift ciphers
 - Each character maps to the character some number of positions down the alphabet
 - Key = index used for shifting
- “Modern” implementation
 - ROT13
- Brute force is relatively easy!
 - Search space: 2^{88} permutations
 - Demonstration



Caesar Cipher (fixed shift of 3)

Hello!

lfmmp!

Classical Crypto - Vigenère Cipher

- Slightly more complex, since it is **polyalphabetic**
 - Instead of a fixed shift value, a string is used instead
- More difficult to break, but still possible with **frequency analysis**

Vigenère Cipher (with key "LEMON")

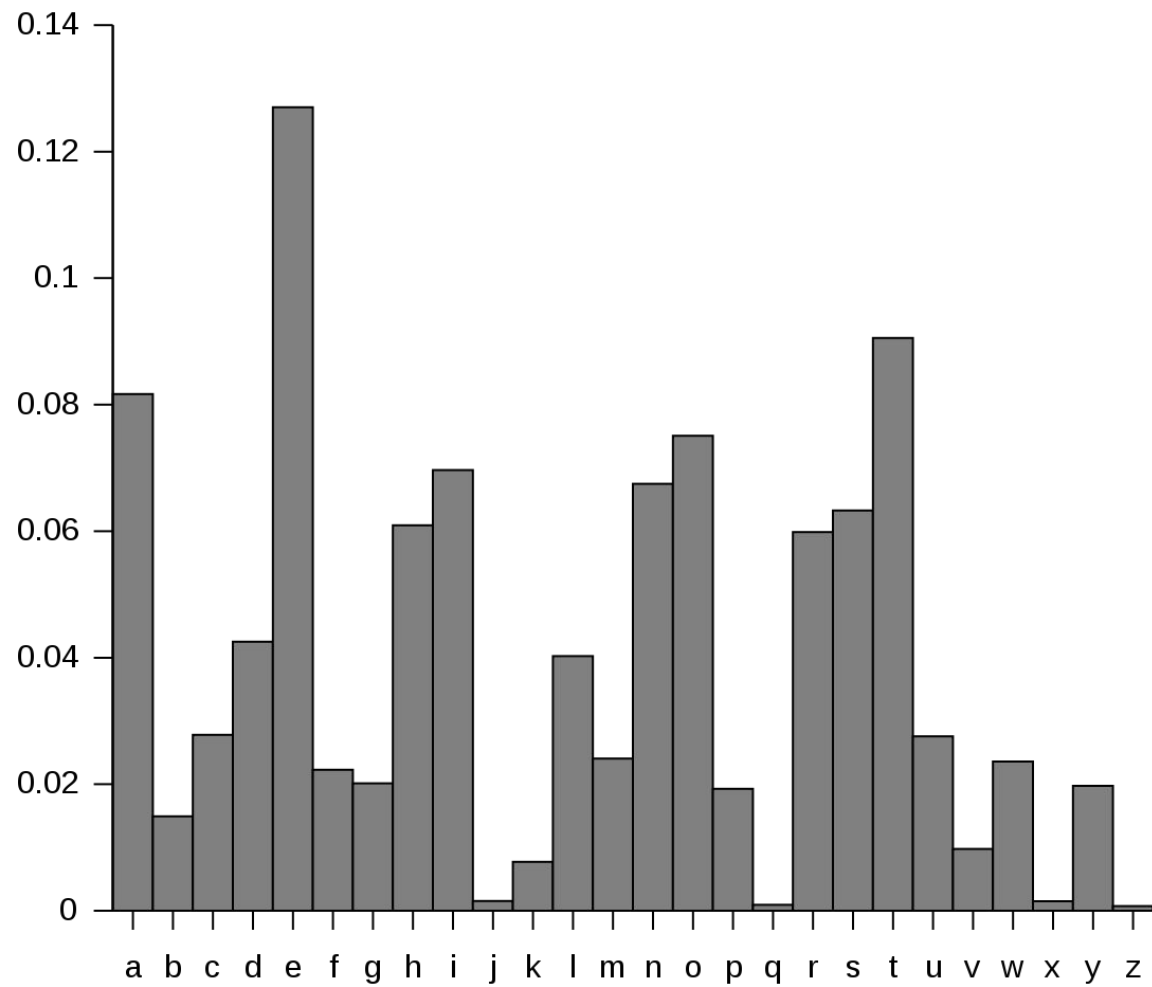
Plaintext	A	T	T	A	C	K	A	T	D	A	W	N
Key	L	E	M	O	N	L	E	M	O	N	L	E
Shift	+11	+4	+12	+14	+13	+11	+4	+12	+14	+13	+11	+4
Ciphertext	L	X	F	O	P	V	E	F	R	N	H	R

Vigenère square

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Frequency Analysis

- We know that some letters are used more than others. For example:
 - E is most commonly used
 - J and Z are almost never used
- For a large enough ciphertext, the **frequency distribution** should roughly match the general English distribution
- Extensions:
 - N-gram / bigram analysis - use pairs of letters rather than just one individual



Problems with Classical Cryptography

- Classical ciphers **FAIL** because
 - They don't bode well against modern-day computational power
 - Don't align with Kerckhoff's Principle!
- Two techniques:
 - Bruteforce is possible with very simple programs
 - Frequency analysis can be used in order to identify common patterns within large ciphertexts

Building Block: XOR

- Let's try solving this problem with the **eXclusive OR (XOR)** operation!
- Output is 1 if A or B is 1, but not if both are
 - Therefore: $A \oplus A = 0$
- Properties:
 - $A \oplus B \oplus A = B$
 - $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

INPUT		OUTPUT
A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

XOR Truth Table

Building Block: XOR and OTPs

- XOR Ciphers can be used as **One-Time Pads (OTPs)**
- One Time Pads introduce *perfect secrecy*
 - Impossible to learn anything about plaintext from ciphertext, even with immense computational power, other than the length.

Encryption: $C = P \oplus K$

Decryption: $P = C \oplus K$

Where a ciphertext C is produced by XORing plaintext P and random key K .

Building Blocks: OTPs

- Why are they secure? How do they uphold IND-CPA?
 - If K is *guaranteed* to be **random**, then the resultant C should also appear to be **random**
 - Therefore, the XOR cipher used **should not be** single-byte XOR. since it can be brute-forced and/or analyzed for character frequency!
 - (Demonstration)

Building Blocks: OTPs

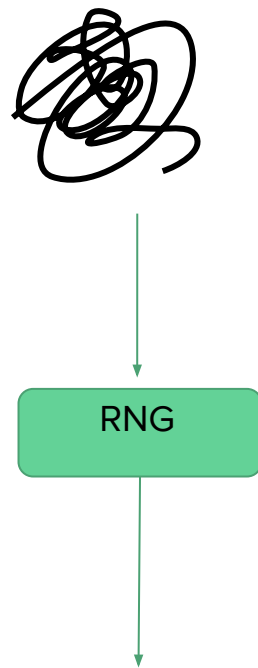
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 - Therefore, the XOR cipher used **should not be** single-byte XOR. since it can be brute-forced and/or analyzed for character frequency!
- Caveat:
 - One-time Pads should only be used once! If two ciphertexts are received by an attacker, the following can be done:

$$\mathbf{C1} \oplus \mathbf{C2} = (P1 \oplus K) \oplus (P2 \oplus K) = (P1 \oplus P2) \oplus (K \oplus K) = \mathbf{(P1 \oplus P2)}$$

Since $K \oplus K = 00000$.

Building Block: Randomness

- Many secure ciphers utilize good sources of **randomness**.
- Getting random bits from a reliable source is an important problem!
 - Solution? Use *sources of entropy*
- **Random Number Generators (RNGs)**
 - General “umbrella term”
 - Harness sources of entropy to generate reliable random bits of information



92u8hdpu3dso12o1xj...

Building Block: Randomness

- What if the RNG itself is not always the most reliable?
 - ie what if the mouse / keyboard is not in use, but randomness is needed somewhere?
- **Pseudorandom Number Generators (PRNGs)**
 - Provide artificial bits from RNGs if analog source becomes unreliable
 - `/dev/urandom`

Building Block: Randomness

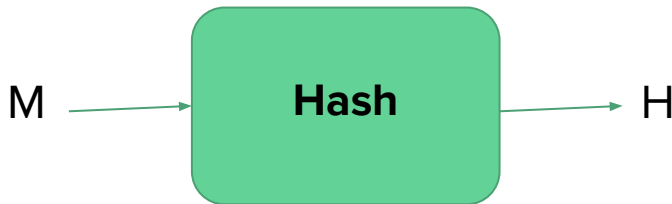
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 - ie what if the mouse / keyboard is not in use, but randomness is needed somewhere?
- **Pseudorandom Number Generators (PRNGs)**
 - Provide artificial bits from RNGs if analog source becomes unreliable
 - Seeds from *entropy pools* stored with bits from RNGs, and uses a DRBG (deterministic random bit generator) to expand the bits
 - DRBGs only help expand the bits, b/c same input == same output

Building Block: Randomness

- What do we ultimately want for cryptography?
 - Non-cryptographic PRNGs, like the *Mersenne Twister* may be predictable!
- **Cryptographically secure pseudorandom number generator (CSRNGs)**
 - “Crypto-reliable” PRNGs, meaning they are nearly unpredictable
 - Reliable source(s) of randomness available
 - Cryptographic algorithm to produce reliable bit
 - Should preserve both *backward* and *forward* secrecy
- Recommendations?
 - Stick with `/dev/urandom` or interfaces that rely on it!

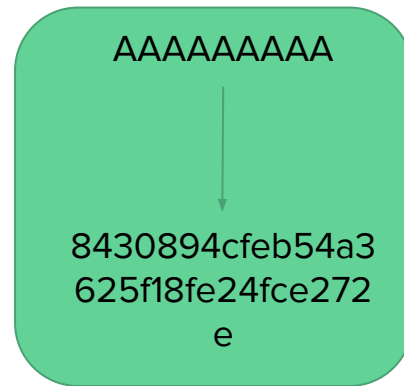
Building Block: Hashes

- “One-way functions”
 - Protect data *integrity* rather than guaranteeing *confidentiality*
 - Input given to hash function results in a random *hash*
 - Useful for
 - Validating files (MD5 checksums for integrity)
 - Persisting login credentials without storing cleartext
 - Digital signatures



Building Block: Hashes

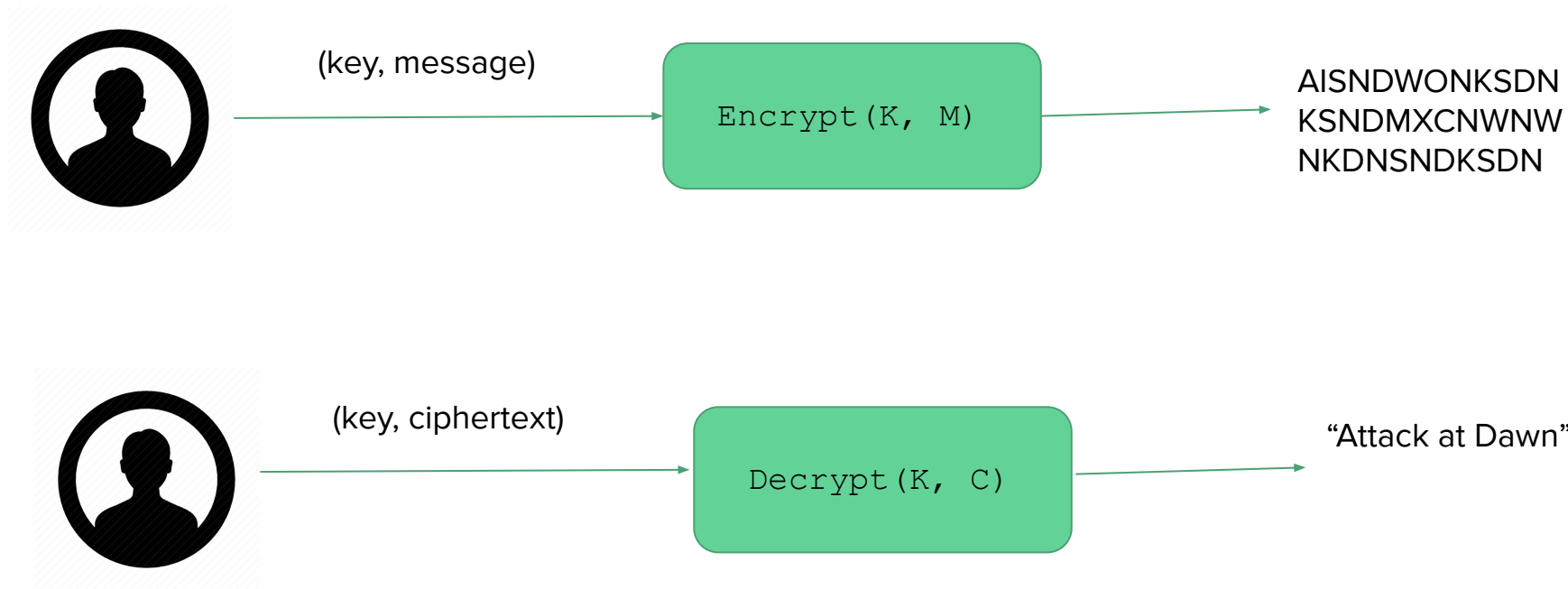
- **Secure** hash functions are
 - Avalanche Property - can't predict changes in hash from subtle changes in input
 - Preimage Resistant - encrypted output **cannot** be reverted back to original input!
 - Collision Resistant - highly unlikely to find hash collisions derived from different inputs



Building Block: Hashes

- Hash functions to use/not use:
 - Merkle-Damgard Constructions (MD*)
 - Very performant, but meaning attackers can bruteforce quickly
 - SHA family of hashing functions
 - SHA 1 is not collision-resistant
 - SHA 2 is vulnerable to length extension attacks
 - **SHA-3 / Keccak**
 - **BLAKE2 hash function**
 - Performance + security balanced
 - Modern day standard!!!

Symmetric Cryptography

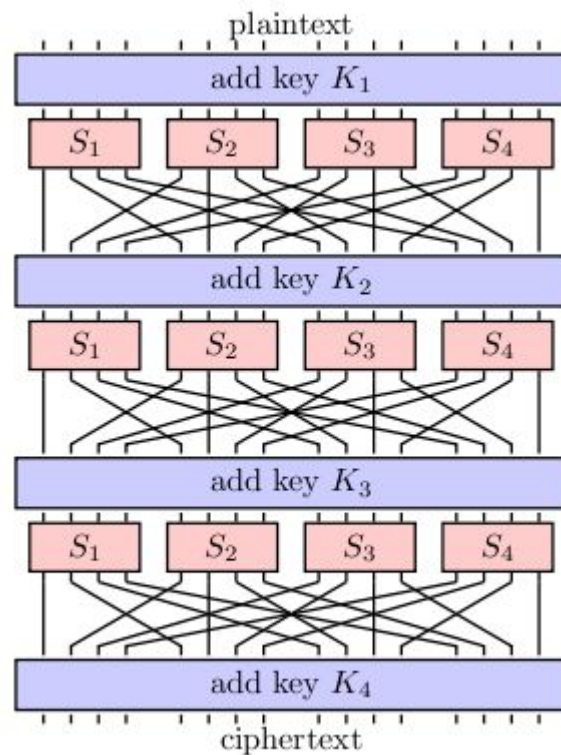


Symmetric Crypto - Block Ciphers

- XOR cipher was an example of a simple block cipher!
- **Block ciphers**
 - Encryption ($E(K, P)$) and decryption ($D(K, C)$) algorithm
 - *Pseudorandom permutations* - if key is secure, attacker should never be able to compute output from input (IND-CPA!!)

Symmetric Crypto - Block Ciphers

- How do they work?
 - Input is chunked into sizable blocks of 16, 24, or 32 bytes
 - Blocks are passed into several *rounds* of computation to transform the data
 - $C = R3(R2(R1(P)))$
 - **Substitution-Permutation Networks (S-Boxes)** are lookup tables that help mutate small chunks of data in some way during each round

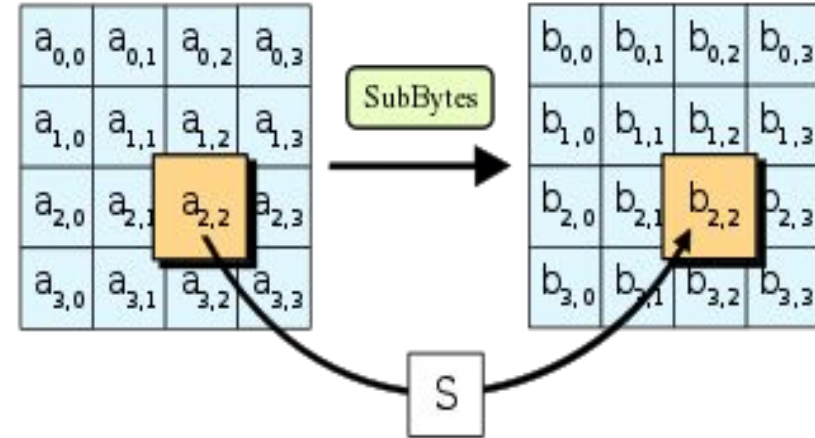


Symmetric Crypto - Block Ciphers

- Does not account for secure key-agreement between parties!
- Does not account for message integrity!
- Other common symmetric crypto schemes:
 - DES / 3DES
 - Blowfish
 - **AES**

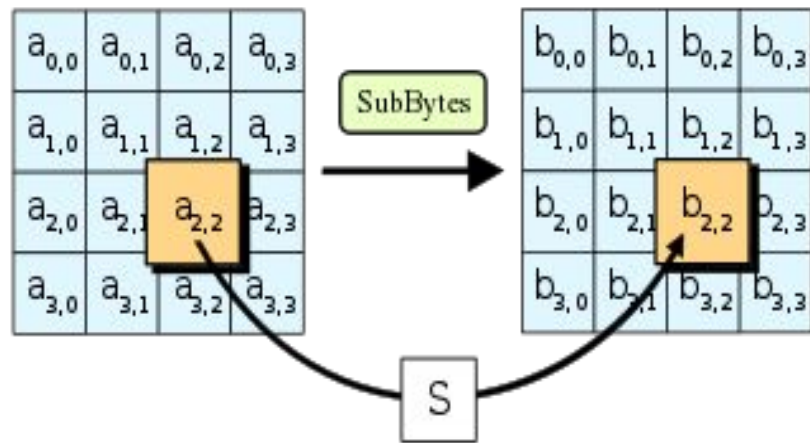
AES - Advanced Encryption Standard (Rijndael)

- Current modern standard (succeeded the DES / 3DES design)
- Processes blocks of **128** bits with a key of **128, 192, or 256** bits
 - Utilizes an internal 4x4 matrix array of 16 bytes



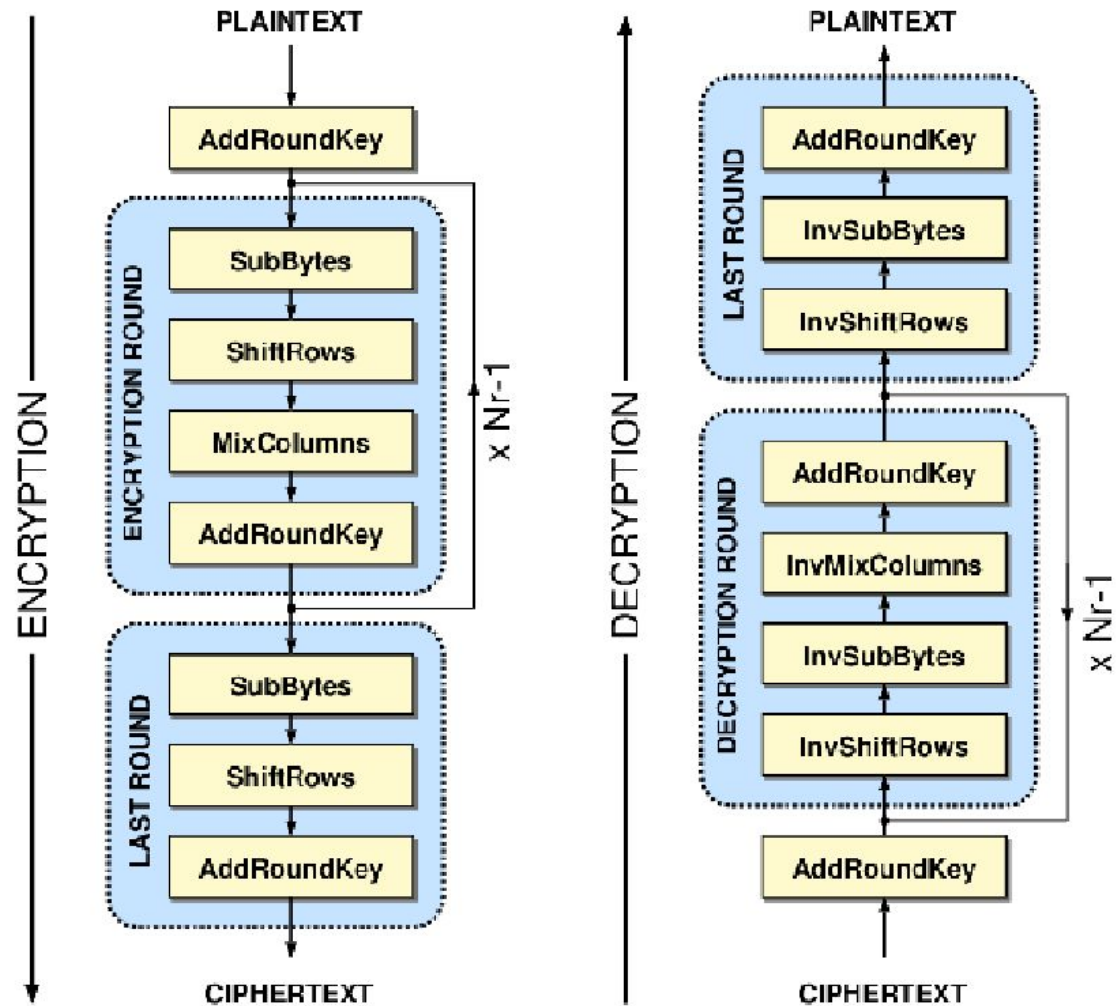
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- Inputs of arbitrary sizes are *padded* using the **PKCS#7** standard
 - | DD DD DD DD DD DD DD DD DD DD DD DD DD DD DD DD **01** |
 - | DD DD DD DD DD DD DD DD DD DD DD DD DD DD DD **02 02** |
 - | DD **0F 0F 0F 0F 0F 0F 0F 0F 0F 0F 0F 0F 0F 0F 0F** |

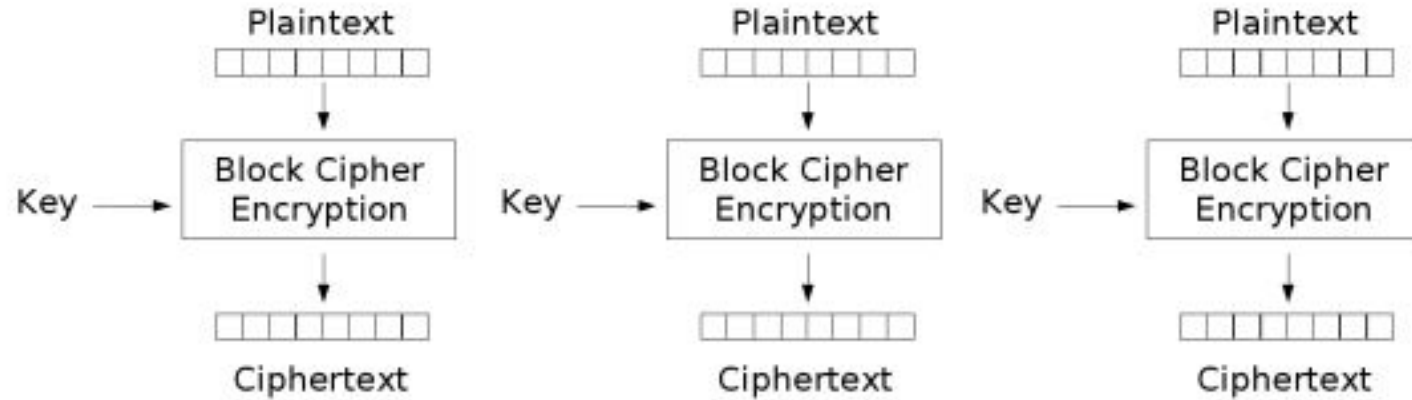
AES Internal Rounds



AES Block Cipher Modes

- Block ciphers harness different *modes of operations* that provide different security guarantees
 - **ECB** - Electronic Codebook Mode
 - **CBC** - Cipher Block Chaining Mode
 - CTR - Counter Mode (*stream cipher*)
 - GCM - Galois Counter Mode (*authenticated encryption*)

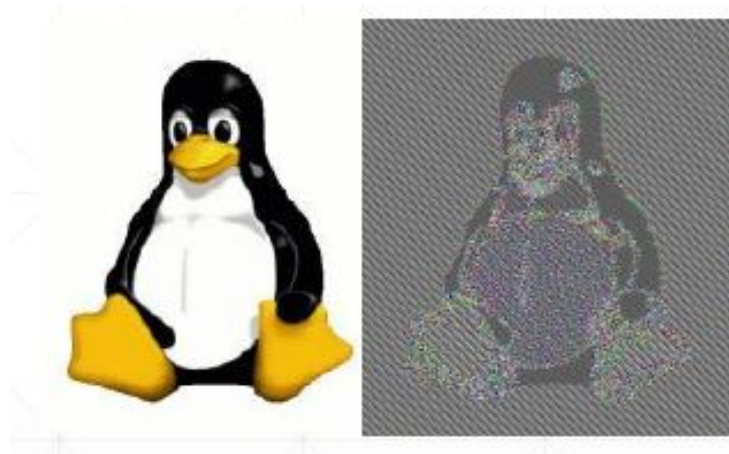
Modes of Operation: Electronic Codebook (ECB)



Electronic Codebook (ECB) mode encryption

ECB Mode

- Plaintext blocks are processed independently:
 - $C1 = E(K, P1)$, $C2 = E(K, P2)$...
- **KNOWN TO BE SEMANTICALLY INSECURE!!!**
 - Same input blocks in a plaintext result in same output ciphertext blocks
 - Chosen plaintext attacks



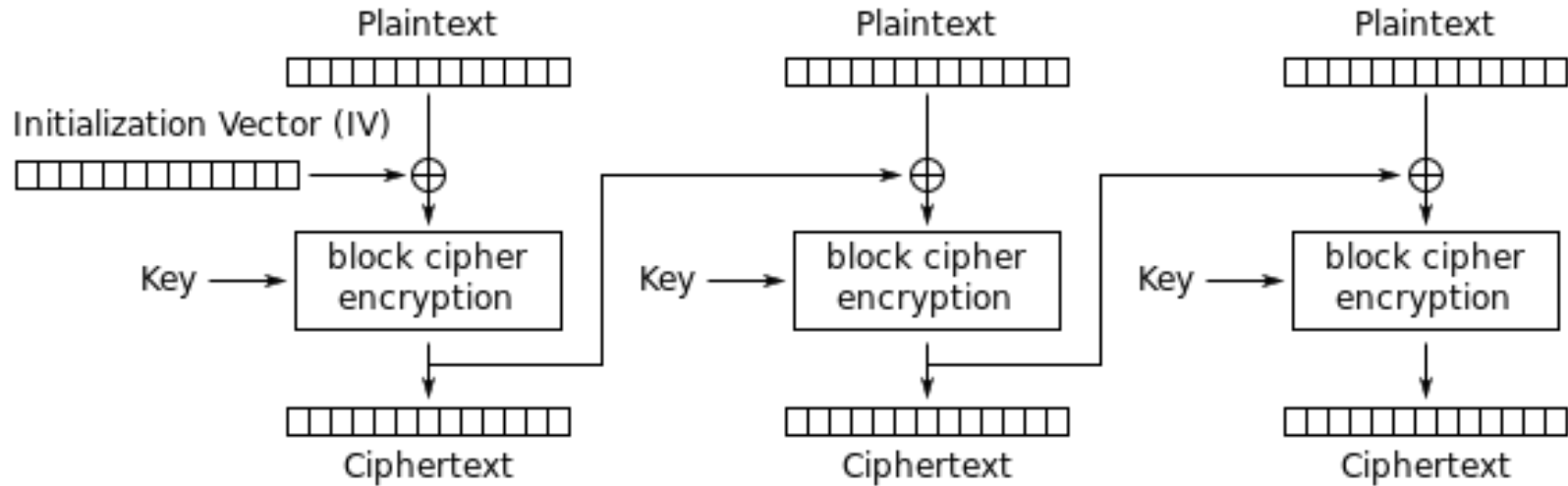
The Infamous ECB Penguin

ECB Mode - Chosen Plaintext Attack

- **Chosen plaintext attack**

- Observing patterns in ciphertexts can help recover plaintext
- Works if we have control over some part of the input being encrypted, with sensitive information also residing within some block
- Demonstration!

Modes of Operation: Cipher Block Chaining



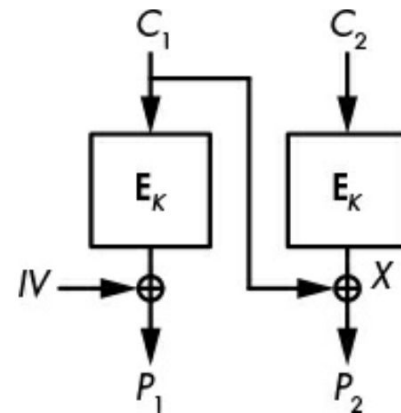
Cipher Block Chaining (CBC) mode encryption

CBC Mode

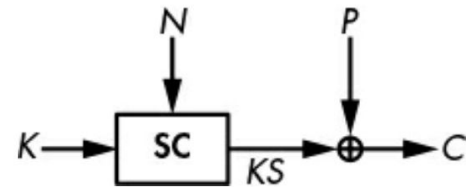
- Alleviates problems with ECB by:
 - Incorporating an *Initialization Vector (IV)* that can be publicly known
 - Instead of independently encrypting blocks, the result of one encryption is chained with the input of the next encryption
 - $C_i = E_k(P_i \oplus C_{i-1})$, and $C_0 = IV$
 - $P_i = D_k(C_i) \oplus C_{i-1}$, and $C_0 = IV$
 - Encrypting same inputs = different outputs now!
- **PROBLEMS!**
 - IVs that are constant and non-random will make CBC like ECB
 - Padding Oracle Attacks

CBC Mode - Padding Oracle Attack

- Think of program doing crypto as an **black-box oracle** that gives results that dictate success/fail based on inputs
 - Given a padding oracle, we want to see which inputs we throw have *valid padding* and *which don't* in order to try to figure out plaintext
- CBC modes use padding oracles to check to ensure that padding during encryption is valid
 - Pass in a C_1 where $C_1[15] \oplus X[15] = 01$, and so on with each byte from the end in order to determine C_2 's decryption



Symmetric Crypto - Stream Ciphers



- **Stream ciphers**

- Pseudorandom bits are generated from a keystream, and encrypted against plaintext through XOR
- Similar to the earlier concept of DRBGs!

- **How it works:**

- A keystream is generated with a key and a one-time 64-128 bit nonce.
- Keystream XORed against clear/ciphertext for encryption/decryption
- Allow repeating inputs, as long as nonce is uniquely generated each time!

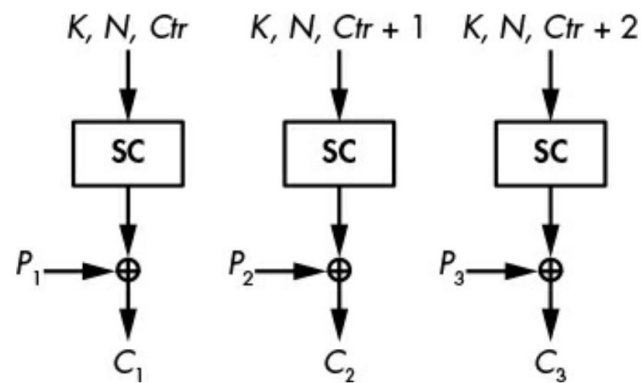
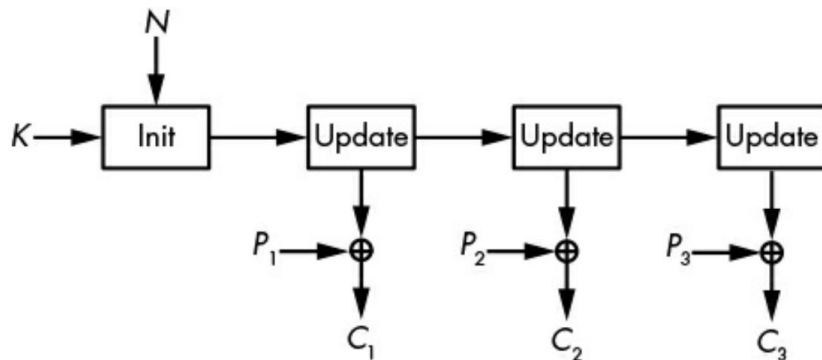
$$KS = SC(K, N)$$

$$C = P \oplus KS$$

$$P = C \oplus KS$$

Symmetric Crypto - Stream Ciphers

- Software stream ciphers
 - Alleviate padding oracle attacks
 - Work with 32/64-bit words
- Stateful stream ciphers
 - Internal state that changes throughout keystream generation
- Counter-based stream ciphers
 - Involves a counter that increments rather than an internal state



Symmetric Crypto - Stream Ciphers

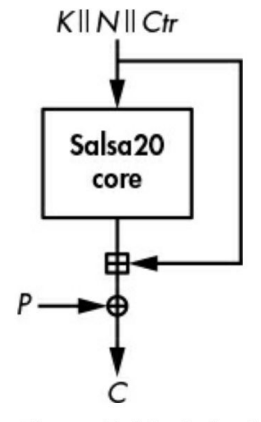
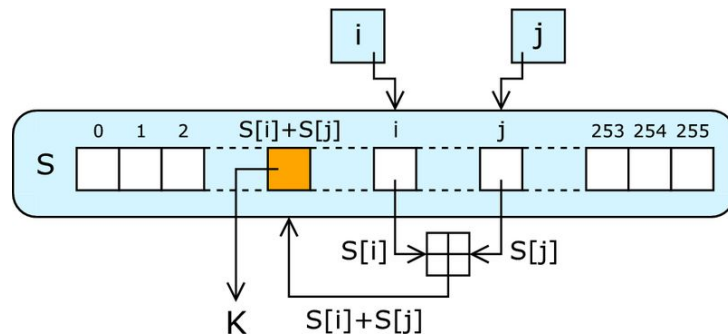
- RC4

- State-based
- Used in WEP and SSL/TLS
- Internal state involves byte swaps as part of its key scheduling algorithm
- Broken, but because of various implementation-level reasons

- Demonstration!

- Salsa20

- Counter-based
- Modern standard!!
- Security and performance balanced

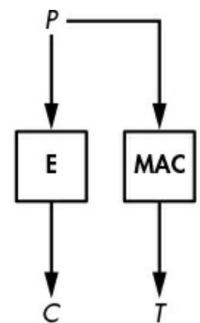


Symmetric Crypto - Authenticated Encryption

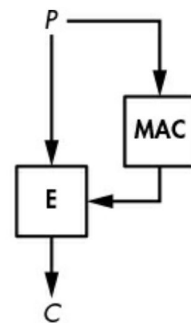
- If symmetric ciphers help preserve *confidentiality*, and hash functions help preserve *integrity*, can we combine both?
- **Authenticated Encryption** (or Authenticated Encryption with associated data, AE, AEAD) combine BOTH!
 - Utilize **MACs (Message Authentication Codes)** in order protect integrity of encrypted data in transmission plus a strong block/stream cipher
 - Aka “keyed hashing”
 - An authentication tag is generated with K and M: $T = \text{MAC}(K, M)$
 - Sent along with message, and if tampered, recomputed tag won't be the same!
 - Hash-based MACs (HMACs) are used in crypto-schemes

Symmetric Crypto - Authenticated Encryption

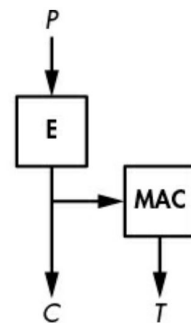
- Choices of HMACs
 - Poly1305
 - SipHash
- Performing AEAD with Cipher + HMAC
 - Encrypt-and-MAC
 - MAC-then-encrypt
 - Encrypt-then-MAC



Encrypt-and-MAC



MAC-then-encrypt



Encrypt-then-MAC

Symmetric Crypto - Authenticated Encryption

- Choices of AEAD Schemes

- AES-GCM
- OCB
- **Salsa20-Poly1305**
- **ChaCha20-Poly1305**

```
import os
from cryptography.hazmat.primitives.ciphers.aead import ChaCha20Poly1305

data = b"a secret message"
aad = b"authenticated but unencrypted data"

# generate a random key and object instance
key = ChaCha20Poly1305.generate_key()
chacha = ChaCha20Poly1305(key)

# create a random nonce
nonce = os.urandom(12)
ct = chacha.encrypt(nonce, data, aad)

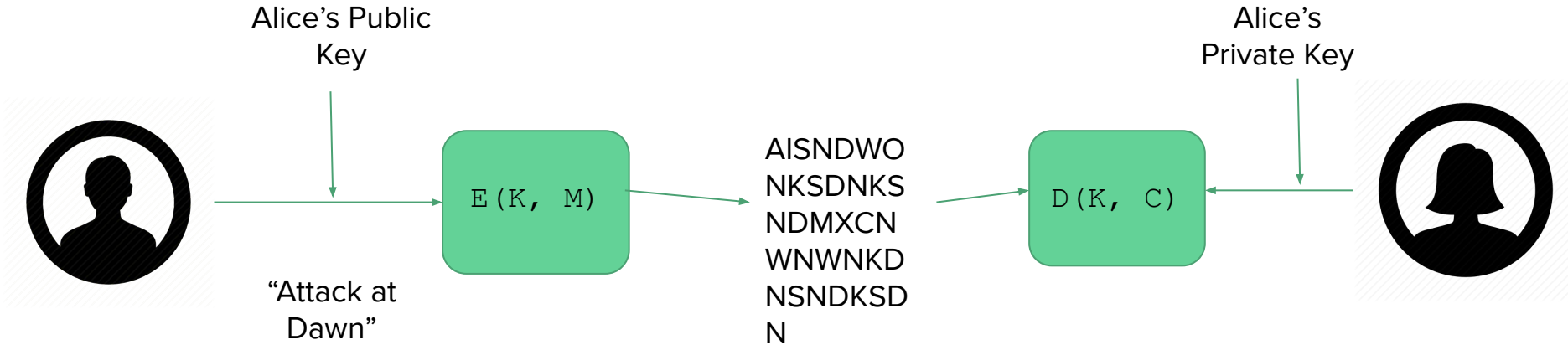
print(chacha.decrypt(nonce, ct, aad))
~
```

Quick Look: Asymmetric Cryptography

Quick Look: Asymmetric Cryptography

“What if we want to share our keys with others to be able to decrypt transmitted info?”

Public-Key Cryptography



Closing Thoughts

- Cryptography is well-developed and studied
 - Attacks have been existing for very long, so the choices we make today are currently the best standards!
- Never implement your own crypto!
 - Understand the best design choices in different implementations and how they fit your needs



Questions?
