

You CANNOT consult any other person or online resource for solving the homework problems. You can definitely ask the instructor or TAs for hints and you are encourage to do so (in fact, you will get useful hints if you ask for help at least 1-2 days before the due date). If we find you guilty of academic dishonesty, penalty will be imposed as per institute guidelines.

Find an optimal edit sequence from GRADALGO to RAGA by following the space-optimal technique discussed in Lec 11. Show all the intermediate steps and the filled memos during the recursive calls.

Hint: Choose X and Y carefully to minimize your efforts.

Solution: The steps of the algorithm are described below and the tables are mentioned after the algorithm.

level 1 The algorithm will take as input “GRADALGO” and “RAGA”.

level 1 It will compute $h = \langle 4 \rangle$ using this table.

Edit Table

$Edit(\cdot, \cdot)$	X = Y = index	e	G	R	A	D	A	L	G	O
		0	1	2	3	4	5	6	7	8
e	0	0	1	2	3	4	5	6	7	8
R	1	1	1	1	2	3	4	5	6	7
A	2	2	2	2	1	2	3	4	5	6
G	3	3	2	3	2	2	3	4	4	5
A	4	4	3	3	3	3	2	3	4	5

Half Table

$h(\cdot, \cdot)$	X = Y = index	e	G	R	A	D	A	L	G	O
		0	1	2	3	4	5	6	7	8
e	0	∞	∞	∞	∞	∞	∞	∞	∞	∞
R	1	∞	∞	∞	∞	∞	∞	∞	∞	∞
A	2	0	1	2	3	4	5	6	7	8
G	3	0	0	2	3	4	4	4	6	6
A	4	0	0	0	2	4	4	4	4	4

level 1 It will recursively compute $\langle \text{“GRAD” and “RA”} \rangle$ and $\langle \text{“ALGO” and “GA”} \rangle$

level 2.1 Algorithm gets input “GRAD” and “RA”

level 2.1 It will compute $h = \langle 2 \rangle$ using this table.

$Edit(\cdot, \cdot)$	X = Y = index	e	G	R	A	D
		0	1	2	3	4
e	0	1	1	2	3	4
R	1	1	1	2	3	4
A	2	2	2	2	1	2

$h(\cdot, \cdot)$	X = Y = index	e	G	R	A	D
		1	2	3	4	5
e	0	∞	∞	∞	∞	∞
R	1	0	1	2	3	4
A	2	0	1	2	2	2

level 2.1 It will recursively compute $\langle \text{“GR” and “R”} \rangle$ and $\langle \text{“AD” and “A”} \rangle$

level 3.1 Algorithm gets input “GR” and “R”

Base Case: Just using Edit distance, standard DP can be used.

$Edit(\cdot, \cdot)$	X = Y = index	e	G	R
		0	1	2
e	0	0	1	2
R	1	1	1	1

level 3.1 It will output the sequence E3.1=“Delete ‘G’, Keep ‘R’ same.”

level 3.2 Algorithm gets input “AD” and “A”

Base Case: Just using Edit distance, standard DP can be used.

$Edit(\cdot, \cdot)$	$X =$	e	A	D
$Y =$	index	0	1	2
e	0	0	1	2
A	1	1	0	1

level 3.2 It will output the sequence E3.2=“Keep 'A' same, Delete 'D'”

level 2.1 It will output the sequence E2.1=(E3.1, E3.2)=“[Delete 'G', Keep 'R' same, Keep 'A' same, Delete 'D']”

level 2.2 Algorithm gets input “ALGO” and “GA”

level 2.2 It will compute $h = \langle 3 \rangle$ using this table.

$Edit(\cdot, \cdot)$	$X =$	e	A	L	G	O
$Y =$	index	0	1	2	3	4
e	0	1	1	2	3	4
G	1	1	1	2	2	3
A	2	2	1	2	3	3

$h(\cdot, \cdot)$	$X =$	e	A	L	G	O
$Y =$	index	1	2	3	4	5
e	0	∞	∞	∞	∞	∞
G	1	0	1	2	3	4
A	2	0	0	0	3	3

level 2.2 It will recursively compute $\langle \text{“ALG” and “G”} \rangle$ and $\langle \text{“O” and “A”} \rangle$

level 3.3 Algorithm gets input “ALG” and “G”

Base Case: Just using Edit distance, standard DP can be used.

$Edit(\cdot, \cdot)$	$X =$	e	A	L	G
$Y =$	index	0	1	2	3
e	0	0	1	2	3
G	1	1	1	2	2

level 3.3 It will output the sequence E3.3=“Delete 'A', delete 'L', Keep 'G' same”

level 3.4 Algorithm gets input “O” and “A”

Base Case: Just using Edit distance, standard DP can be used.

$Edit(\cdot, \cdot)$	$X =$	e	O
$Y =$	index	0	1
e	0	0	1
A	1	1	1

level 3.4 It will output the sequence E3.4=“replace 'O' with 'A'”

level 2.2 It will output the sequence E2.2=(E3.3, E3.4)=“Delete 'A', delete 'L', Keep 'G' same, replace 'O' with 'A'”

level 1 It will output the sequence (E2.1, E2.2)=“[Delete 'G', Keep 'R' same, Keep 'A' same, Delete 'D', Delete 'A', delete 'L', Keep 'G' same, replace 'O' with 'A']”

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