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Homework 5 Problem 1

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Solution:

def MakeListHelper($a = [0 \dots n-1], n$): ⟨⟨*Receives a list of (0,1, ... n-1) and the length of list 'n'*⟩⟩
  if $len(a) \leq 1$: return a
  even_list = []
  for ($i = 1 \dots len(a)$):
    if ($i\%2 = 0$): add i to even_list
  a_e ←MakeListHelper(even_list, n)
  a_o ←[add (+1) to all the elements of a_e]
⟨⟨*To check for the case if half of the total number of elements in the list are obtained,*
then do not perform the multiplication operation i.e. if 0, 4, 2, 6 are obtained when n is 8⟩⟩
  if $len(a\_e) < n/2$:
    a_e ←[multiply every element of the list by 2]
    a_o ←[multiply every element of the list by 2]
  return $[a_e, a_o]$

def bit_reversal($A[1 \dots n]$):
  final_indices ←MakeListHelper([i for(i = 0 ... n-1)], n)⟨⟨*Returns the indices of the final permutated list.*⟩⟩
  for (i in final_indices):
    print($A[i]$)

$T(n) = T(n/2) + O(n) = T(n/2^2) + O(n/2) + O(n)$
$T(n) = T(n/2^k) + O(n/2^{(k-1)} + \dots + n/2 + n)$
$T(n) = T(n/2^k) + n(1/2 + 1/2^2 + \dots + 1/2^{k-2} + 1/2^{k-1})$
Maximum no of levels $k = log n$ and Base Case $T(1) = 1$
$T(n) = 1 + n(\sum_{i=1}^{k}(1/2^i)) \leq O(n)$

Explanation: We see that second(odd) half of series is just 1 added to first(even) half of the series and the even half is double of the previous series.
Proof :
Hypothesis N=n : B(n) = [B(n/2)*2, B(n/2)*2+1]
Base Step: P(0) N=2^1 : $B(2) = [0]$
P(1): N=2^2 : $B(4) = [B(1)*2 \quad B(1)*2+1] = [0 \quad 1]$
Hence P(1) is True
Induction Step:
$Assume \quad P(x): \quad N = 2^x : B(x) = [B(x/2)*2 \quad B(x/2)*2+1]$ is true
P(x+1) N=$2^{(x+1)}$ $=> N = 2*2^x$
$B(2.2^x) = [(2B(x/2) \quad 2B(x/2)+1)*2 \quad (2B(x/2) \quad 2B(x/2)+1)*2+1]$
$=> B(2.2^x) = [B(x)*2 \quad B(x)*2+1]$
Since P(x) leads to P(x+1) hence proved

Iterating Over ABCDEFGH
We have an array of 8 elements so we will go to the base case of 1 and then expand
N=1 : B(1) = [0]
N=2 : B(2) = [2*B(2/2), 2*B(2/2)+1] =[2*B(1), 2*B(1)+1] = [0, 1]
N=4 : B(4) = [B(2)*2, B(2)*2+1] = [0 2, 1 3]
N=8 : B(8) = [B(4)*2, B(4)*2+1] = [0 4 2 6, 1 5 3 7]
Output: 'AECGBFDH'

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