CSE525 Monsoon 2020 — Pramil Panjwani (PhD19008)
Homework 15 Problem 1 — Ashima Garg (PhD19003)

You CANNOT consult any other person or online resource for solving the homework problems. You can definitely ask the instructor or TAs for hints and you are encourage to do so (in fact, you will get useful hints if you ask for help at least 1-2 days before the due date). If we find you guilty of academic dishonesty, penalty will be imposed as per institute guidelines.

Solution: SCHEDULE is problem with parameters $(t[1...n], m[1...n], d[1...n], M)$ where t_i is time required solving the i^{th} question, m_i is the maximum marks obtainable for that problem, d_i is the deadline and M is the minimum marks required to pass. SCHEDULE checks if instance <t,m,d,M> is valid or not.
SUBSETSUM is the problem that takes as input an array A and a target integer T and returns true if A has a subset whose sum is t.

To reduce SUBSETSUM to the SCHEDULE :
Given a SUBSET instance <A,T> of order n. We construct an instance of SCHEDULE <t,m,d,M> such that $t = m = A[1...n]$ and $\forall(i = 1 \text{ to } n) d_i = T$

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def SUBSETSUM-TO-Schedule(A,T):
    return S=(t,m,d,M) : t = A and m = A and M = T and d_i = T ∀(i = 1...n)
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Lemma1: The instance <A,T> of SUBSETSUM produces "Yes" iff instance <t,m,d,M> of SCHEDULE produces "Yes".

For SUBSETSUM instance <A,T> to be yes, there should exit a subset A' whose sum equals T. let its and order be k. Then time required to complete all the assignments is $\sum_{i=1}^{k} t = T$.Hence the deadlines are met. According to this the total marks obtained $\sum_{i=1}^{k} m = T = M$. Therefore the instance of SCHEDULE <t,m,d,M> would produce "YES".

Lemma2: Conversely, Let <t,m,d,M> be a "YES" instance of SCHEDULE, then <A, T> be "YES" instance of SUBSET-SUM.
Let k be subset of n tasks that get scheduled, before the deadline d. The time taken to complete these tasks $\sum_{i=1}^{k} t_i$. Since, it is "YES" instance, it should complete before their deadlines, i.e. T. $\sum_{i=1}^{k} t_i \leq T$.
The marks obtained using these questions $\sum_{i=1}^{k} m_i$. For this instance to be "YES" instance of SCHEDULE, the summation of marks should be greater than M which implies, it is greater than equal to T.$\sum_{i=1}^{k} m_i \geq T$. Thus, $\sum_{i=1}^{k} a_i = T$, where $a_i = t_i = m_i$.

$\therefore$ there exists a subset of A of elements a_i which has sum equal to target sum T.

Time Complexity: Assignment of timeline and marks will take O(n) time. Similarly assignment of deadline will take $O(n)$ time. So total time is $O(n)$. $\therefore$ this is linear time reduction.

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