CSE525 Monsoon 2020 | Pramil Panjawani (PhD19008)
Homework 20 Problem 1 | Ashima Garg (PhD19003)

You CANNOT consult any other person or online resource for solving the homework problems. You can definitely ask the instructor or TAs for hints and you are encourage to do so (in fact, you will get useful hints if you ask for help at least 1-2 days before the due date). If we find you guilty of academic dishonesty, penalty will be imposed as per institute guidelines.

(a) Prove that for any set of jobs, the makespan of the greedy assignment is at most $(2-1/m)$ times the makespan of the optimal assignment, where m is the number. of machines.

Solution: Input: Set $T = [t_1, t_2, \ldots, t_n]$ of n elements where n is the number of jobs and integer M where M is the number of machines.
Makespan of the assignment is the maximum time a machine is busy. Our challenge is to compute the assignment which minimizes the makespan.
Idea: Assign the new job to the machine with minimum load.

Proof: Let $OPT(I)$ be the optimum assignment. We can say 2 things about OPT that is
1) it is greater than duration of longest job and
2) it is greater than the total time divided number of machines.

$$OPT(I) \geq \max_i t_i \tag{1}$$

$$OPT(I) \geq 1/m \sum_{i=1}^{n} (t_i) \tag{2}$$

Let L be the makespan of our greedy algorithm, and L(M_i) be the load on i^{th} machine,
M_i^* be the machine with largest load and J_j* be the last job assigned to M_i* and t_j* be the time required for the job J_j*.
L*(M_i*) be the load on M_i^* before t_j* was added. Since our algorithm assigns job to the machine with smallest load, we can say each machine at least greater than L-t_j*
Let Total time before J_j* job be T , and average time be A $(L - t_j*) \leq A$ because the machine with lowest load will at most be equal to average load by machines)

$$\begin{aligned} T &= \sum_{i=1}^{n} t_i - t_j* \\ A = T/m &= 1/m \left(\sum_{i=1}^{n} t_i - t_j* \right) \\ &= 1/m \sum_{i=1}^{n} t_i - t_j*/m \\ &= OPT(I) - t_j*/m \end{aligned} \tag{3}$$

$$\begin{aligned} L - t_j* &\leq A \\ L - t_j* &\leq OPT(I) - t_j*/m \\ L &\leq OPT(I) + t_j*(1-1/m) \\ &\leq OPT(I) + OPT(I)*(1-1/m) \\ &= (2-1/m)*OPT(I) \end{aligned}$$

Reference: Tutorial 12, and JE-J □

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