CSE525 Monsoon 2020 | Pramil Panjawani (PhD19008)
Homework 12 Problem 1 | Ashima Garg (PhD19003)

You CANNOT consult any other person or online resource for solving the homework problems. You can definitely ask the instructor or TAs for hints and you are encourage to do so (in fact, you will get useful hints if you ask for help at least 1-2 days before the due date). If we find you guilty of academic dishonesty, penalty will be imposed as per institute guidelines.

Solution: To find a walk between vertices s and t of a directed graph G=(V,E) such that it's length is divisible by 3 with repeated edges and vertices.

A) Check if directed graph G=(V,E) is cyclic or acyclic. If acyclic simply use Deapth first search (DFS) between source vertix s and target vertex t to check if a path exists. If the path does not exists return path does not exist. If the path exist, trace back the parent pointer to s and calculate number of edges. If length of the walk divisible by 3 the return walk of length divisible by 3 possible, else return not possible.

B) If the graph is cyclic we will have to modify the graph
1) Create 3 new graphs G0=(V0,E0),G1=(V1,E1),G2=(V2,E2) all equal to G=(V,E)
2) Create G' = (V',E') by stacking G0,G1,G2, on top of itself thrice.(We do it 3 times because (3n+K)mod3= (k)mod3 where n is length of cycle and k length of path between s and t. So if we don't find the path after 2 cycles, it doesn't exists).

Proof:
Let Length of walk between vertices s and t of graph G be $\delta(s,t) = k$
Number edges in a cycle =n
and remainder of k(mod 3)=r
Then remainder after 1st cycle =(k+n)(mod 3)=K(mod 3) + n(mod 3)=r+n(mod 3)
Then remainder after 2nd cycle =(k+2n)(mod 3)=K(mod 3) + 2n(mod 3)=r+2n(mod 3)
Then remainder after 3rd cycle =(k+3n)(mod 3)=K(mod 3) + 3n(mod 3)=r
Hence we need to check it only with 3 cycles total.

To merge the graphs we will start from s(source) and if we encounter a cycle we will we will merge the node to next graph's if source.(In the example given in book we will go s0 -> w0 -> y0-> x0 -> s1 -> w1 -> t1 and s0 -> w0 -> y0-> x0 -> s1 -> w1 -> y1 -> x1 -> s2-> w2 -> t2), i.e. as soon as a cycle is found instead of going on a point where we already have been we go to same node in next graph.
V total vertices V' = V0 + V1 + V2 = 3V = O(V).
E total edges E' = E0 + E1 + E2 +1 (from G0(x0)-> G1(y1)) + (from G1(x1)-> G2(y2)) = 3E+2=O(E).
(Note y represents the first node from which we enter the cycle and x represents last node of the cycle)

3) Add the edges such that any edge from s to t always follows the same path in all the layers.
4) This changes the graph from cyclic to acyclic
5) Now we can perform DFS on the new graph G' starting from source vertex s0 to target vertices t0, t1 and t2. If DFS does not reach any of the target vertices return no path exists
6) Otherwise path exists take the parent pointer back to source counting the number of edges encountered. If length of any of them is divisible by 3 the return walk of length divisible by 3 possible, else return not possible.

Complexity analysis:
Constructing of graph G'=(V',E') will take O(V+E) time by copying the graph twice and connecting using accurate edges.
Again using DFS will take O(V+E) time.
T(n)=O(V+E)+O(V+E)=O(V+E)
Since we are constructing a new graph space complexity will be $\Theta(V+E)$

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