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TOPICWISE : DISCRETE MATHEMATICS-2 (GATE - 2019) - REPORTS

OVERALL ANALYSIS COMPARISON REPORT SOLUTION REPORT

ALL(17) CORRECT(6) INCORRECT(5) SKIPPED(6)

Q. 1

Which of the following is true?

Solution Video | Have any Doubt ?

A
If a graph is connected, then its complement must be disconnected.

B
Chromatic number of complete graph with n vertices is $n - 1$.

Your answer is **Wrong**

C
If two graph G_1 and G_2 are isomorphic, then their complements may or may not be isomorphic.

D
If any simple graph with n nodes with $n > 1$, there are atleast two vertices of same degree.

Correct Option

Solution :
(d)

- If a graph is connected, then its complement may or may not be disconnected.
  Example: cyclic graph on 5 vertices.
- Chromatic number of complete graph with n vertices is n.
- If two graph G_1 and G_2 are isomorphic, then their complements will always be isomorpl
- If any simple graph with n nodes with nodes > 1, there are atleast two vertices of s degree.

QUESTION ANALYTICS

Q. 2

What is the maximum number of edges present in a disconnected graph on $n \geq 3$ vertices?

Solution Video | Have any Doubt ?

A
$({}^{n}C_2 - 1)$

B
$n - 2$

C
${}^{(n-1)}C_2$

Your answer is **Correct**

Solution :
(c)

Maximum number of edges in connected graph:

$$ {}^{n}C_2 = \frac{n(n-1)}{2} $$

So, by disconnected one vertex from it, we get:

$$ = {}^{n-1}C_2 = \frac{(n-1)(n-2)}{2} $$

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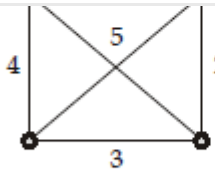
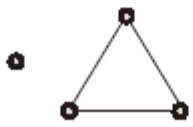


$= 6 \text{ edges} = {}^4C_2 \text{ edges}$

Disconnected graph on 4 vertices:



$= 3$

$= {}^{(4-1)}C_2 = {}^3C_2 = 3$

D

$(n-2)C_2$

QUESTION ANALYTICS

Q. 3

What is the number of partition of X = {*a*, *b*, *c*, *d*, *e*, *f*} where *a* and *c* are always in same block?

Solution Video | Have any Doubt ?

A

15

B

52

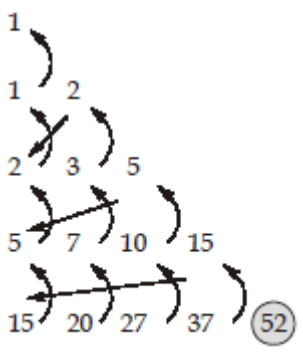
Correct Option

Solution :
(b)

Since "*a* and *c*" are present in same block

So, {*ac*, *b*, *d*, *e*, *f*}

Using Bell number:



C

203

D

None of these

QUESTION ANALYTICS

Q. 4

Consider the recurrance relation $a_k = -8a_{k-1} - 15a_{k-2}$ with initial conditions $a_0 = 0$ and $a_1 = 2$. Which of the following is an explicit solution to this recurrance relation?

Solution Video | Have any Doubt ?

A

$(-3)^k - (-5)^k$

Your answer is **Correct**

Solution :

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[Using characteristics equation method]

$$n^2 + 8n + 15 = 0$$
$$n^2 + 5n + 3n + 15 = 0$$
$$n(n+5) + 3(n+5) = 0$$
$$(n+3)(n+5) = 0$$
$$n = -3 \text{ and } -5$$

So, $a_k = (-3)^k C_1 + (-5)^k C_2$

$= (-3)^0 C_1 + (-5)^0 C_2 = 0$

$\Rightarrow C_1 + C_2 = 0$ ...(1)

$a_1 = (-3)^1 C_1 + (-5)^1 C_2 = 2$

$\Rightarrow -3C_1 + -5C_2 = 2$ ...(2)

Solving equation (1) and (2), we get $C_1 = 1$ and $C_2 = -1$

So, $a_n = (-3)^k - (-5)^k$

B

$k(-3)^k - k(-5)^k$

C

$(-5)^k - (-3)^k$

D

$k(-3)^k - (-5)^k$

QUESTION ANALYTICS

Q. 5

Consider the following statements:
S_1 : D_{85} is Boolean Algebra.
S_2 : Every finite lattice has a least element.
S_3 : Every Poset has a greatest element.
Which of the following is always true?

FAQ | Solution Video | Have any Doubt ?

A

S_1 and S_3 only

B

S_2 and S_3 only

C

S_1 and S_2 only

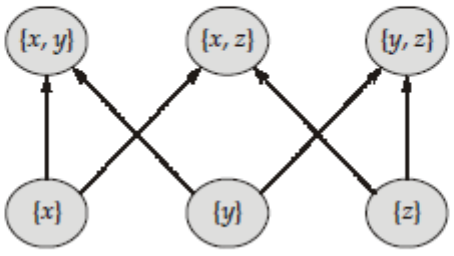
Correct Option

Solution :
(c)

S_1 : D_{85} : $85 = 5 \times 17$ i.e. product of distinct prime number, hence Boolean Algebra. So True

S_2 : Suppose elements of lattice are $a_1, a_2, a_3 \ldots\ldots a_n$, then $a_1 \wedge a_2 \wedge a_3 \ldots\ldots \wedge a_n$ is the least elemer
True

S_3 : Poset $(R, \leq)$ has no greatest element.



No greatest element as 3 sets are non comparable. So Not True

D

All of the above

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Q. 6

Consider an equivalence relation R on the positive integers A = {2, 3, 4, 5, 6, 7, ... 22} defined as mRn if the largest prime divisor of 'm' is the same as the largest prime divisor of 'n'. The number of equivalence classes of R is _____.

Solution Video | Have any Doubt ?

8

Correct Option

Solution :
8

"mRn" (largest prime divisor of m = largest prime divisor of 'n')
So, equivalence classes are
1. 2 R {2, 4, 8, 16}
2. 3 R {3, 6, 9, 12, 18, 21}
3. 5 R {5, 10, 15, 20}
4. 7 R {7, 14, 21}
5. 11 R {11, 22}
6. 13 R {13}
7. 17 R {17}
8. 19 R {19}

Number of equivalence classes are 8.

QUESTION ANALYTICS

Q. 7

Consider F be a family of all subsets of set {1, 2, 3, ... 100} that contain atleast 50 numbers, partially ordered with respect to containment. Then maximum size of chains in the Poset (F, ⊆) that cover F is _____.

Solution Video | Have any Doubt ?

51

Correct Option

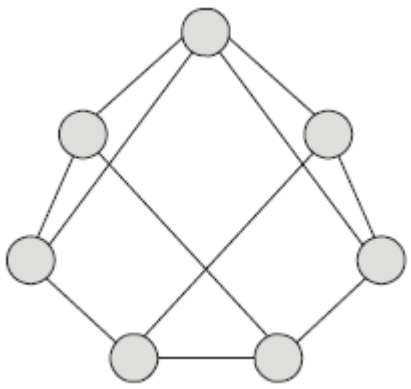
Solution :
51

The maximum size of chain will be:
{1, 2, 3, ... 50} ⊂ {1, 2, 3, ... 50, 51} ⊂ {1, 2, 3, ... 50, 51, 52} ..... {1, 2, 3, ... 100}
i.e. 51 (from 50 to 100)

QUESTION ANALYTICS

Q. 8

Consider the following graph:



The chromatic number of above graph is _____.

Solution Video | Have any Doubt ?

3

Your answer is Correct3

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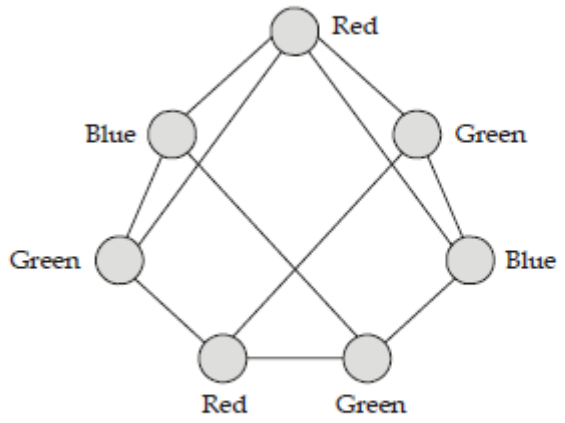
Here K_3 is subgraph present, so 2 cannot be the chromatic number.



So, 3 colors are needed to color the graph, hence chromatic number is 3.

QUESTION ANALYTICS

Q. 9

Suppose tree 'T' has 10 vertices of degree 4, 20 vertices of degree 3 and 30 vertices of degree 2. If all of the rest of vertices are of degree 1, then the number of vertices 'T' have is _____.

Solution Video | Have any Doubt ?

102

Your answer is **Correct** 102

Solution :
102

Consider number of vertices of degree 1 = x

$$\text{Total number of edges} = \frac{10\times4+20\times3+30\times2+x\times1}{2}$$

$$= \frac{(x+40+60+60)}{2} = \frac{x}{2}+\frac{160}{2}$$

$$= \frac{x}{2}+80$$

$$\text{Total number of vertices} = 10+20+30+x$$

$$= x+60$$

Since is tree, so number of edges must be = (Number of vertices) – 1

$$= (x+60)-1 = x+59$$

$$\text{Thus, } x+59 = \frac{x}{2}+80$$

$$2(x+59-80) = x$$

$$2x-2\times21 = x$$

$$x = 42$$

$$\text{Total number of vertices} = x+60$$

$$= 42+60 = 102$$

QUESTION ANALYTICS

Q. 10

Which of the following is true?

Have any Doubt ?

A
The edge uv in a simple graph G is a cut edge, if and only if $n(G) \geq d(u) + d(v)$.

B
Every graph with fewer edge than vertices has component of a tree.

Correct Option

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$n(G) \geq d(u) + d(v)$

$7 \geq 2 + 2$

$7 \geq 4$ satisfied but u/v is not cut edge. So false

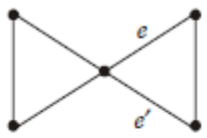
(b) Let, G be a graph such that $|E_G| < |V_G|$ further, suppose G_1, G_2, G_2 _____ G_k are con components of G, and if no connected component of G is a tree.

Hence, for each $1 \leq i \leq k$, $|E_{Gi}| \geq |V_{Gi}|$. Thus,

$$|E_G| = \sum_{i=1}^{k} |E_{Gi}| \geq \sum_{i=1}^{k} |V_{Gi}| \geq |V_G|$$

Which is a contradiction. Hence, there exists a component of G which is tree.

(c) Consider a graph:



Since graph is Eulerian graph but don't have eulerian circuit. So false

(d) Consider a graph:

Average degree $(G) = \frac{2e}{n}$ Average degree $(G) = \frac{2e}{n}$

Before removal of 'v' $= \frac{20}{7} = 2.857$ After removal of 'v' $= \frac{16}{6} = 2.66$

So false

C

If G is an Eulerian graph with edges e, e'sharing a vertex, then G has an Eulerian circuit in which e and e'appear consecutively.

D

In connected graph G with atleast 2 vertices and $\delta(G) < \Delta(G)$ deleting a vertex of $\delta(G)$ cannot reduce the average degree.

QUESTION ANALYTICS

Q. 11

Which one of the following is a solution for a_n, where $a_n = a_{n-1} + 3^{n-1}$ for $n = 0, 1, 2, 3, ...$ with $f(0) = 1$ and $f(1) = 2$?

Solution Video | Have any Doubt ?

A

$\frac{1}{2}(1 + 3^{n-1})$

Your answer is **Wrong**

B

$\frac{1}{2}(1 + n3^{n-1})$

C

$\frac{1}{2}(1 + 3^{n})$

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$$a_n = a_{n-1} + 3^{n-1}$$

$$a_n = \sum_{i=0}^{n} a_i x^i$$

$$= 1 + \sum_{i=1}^{n} a_i x^i$$

$$= 1 + \sum_{i=1}^{n} (a_{i-1} + 3^{i-1}) x^i$$

$$= 1 + \sum_{i=1}^{n} (a_{i-1} + x^i) + \sum_{i=1}^{n} (3^{i-1} x^i)$$

$$= 1 + x\left[\sum_{i=0}^{n} a_i x^i\right] + x\left(\sum_{i=0}^{n} 3^i x^i\right)$$

$$a_n = 1 + x a_n + \frac{x}{1-3x}$$

$$a_n(1-x) = 1 + \frac{x}{1-3x}$$

$$a_n = \frac{1-3x+x}{(1-3x)(1-x)} = \frac{1-2x}{(1-x)(1-3x)}$$

$$= \frac{A}{1-x} + \frac{B}{1-3x} = \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1-3x}$$

$$a_n = \frac{1}{2}(1 + x + x^2 + x^3 .....) + \frac{1}{2}(1 + 3x + (3x^2) + ...)$$

$$= \frac{1}{2}(1 + 3^n)$$

D

$\frac{1}{2}(1 + 3^{n+1})$

QUESTION ANALYTICS

Q. 12

Consider x, y, z and w be elements of a group G:

P_1 : If given that $xyz^{-1}w = 1$, then y must be equal to $x^{-1}\, w^{-1}z$.

P_2 : If $xyz = 1$, then $yxz = 1$.

Which of the following is true?

FAQ | Solution Video | Have any Doubt ?

A

Only P_1

Your answer is **Correct**

Solution :

(a)

P_1 : $xyz^{-1}w = 1$, then $y = x^{-1}\, w^{-1}z$

Put $y = x^{-1}\, w^{-1}\, z$ in $xyz^{-1}w = 1$

$x(x^{-1}\, w^{-1}z)z^{-1}w = 1$

$w^{-1}z\, z^{-1}w = 1$

$w^{-1}w = 1$

$1 = 1$ Hence true

P_2 : $xyz = 1$, then $xyz = 1$

Assume, $x = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, z = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \end{bmatrix}$

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And $xyz \neq yxz$ Hence false

B
Only P_2

C
Both P_1 and P_2

D
Neither P_1 nor P_2

QUESTION ANALYTICS

Q. 13

Which of the following is false?

FAQ | Solution Video | Have any Doubt ?

A
Every cyclic group is Abelien group.

B
Every Abelien group is cyclic group.

Correct Option

Solution :
(b)
• Every cyclic group is Abelien group but every Abelien group is not cyclic group.
• Every group of prime order is cyclic group and we know that every cyclic group is Abelien group hence, every group of prime order is Abelien group.
• If $(G, *)$ be a cyclic group of even order, then there exist atleast one elements other than identity element such that $a^{-1} = a$.

C
Every group of prime order is Abelien group.

D
If $(G, *)$ be a cyclic group of even order, then there exist atleast one elements other than identity element such that $a^{-1} = a$.

Your answer is **Wrong**

QUESTION ANALYTICS

Q. 14

Consider a_n represent the number of bit string of length 'n' containing even member of 0's. What is the recurrence relation?

FAQ | Solution Video | Have any Doubt ?

A
$a_{n-2} + (2^{n-1} - a_{n-1})$

Your answer is **Wrong**

B
$a_{n-1} + a_{n-2} + 2^{n-1}$

C
$2a_{n-1} - a_{n-1} - a_{n-2}$

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Solution :
(d)

$a_1 = 1$ [$\therefore$ strings = 1]
$a_2 = 2$ [$\therefore$ strings are : 00, 11]
$a_3 = 4$ [$\therefore$ strings are : 001, 100, 111, 010]
$a_4 = 8$ [$\therefore$ strings are : 1111, 1001, 0011, 1100, 0101, 1010, 0110, (

Option (a):

$a_n = a_{n-2} + (2^{n-1} - a_{n-1})$
$\Rightarrow \quad a_4 = a_{4-2} + 2^{4-1} - a_{4-1}$
$= a_2 + 2^3 - a_3 = 2 + 8 - 1 = 9$ which is false.

Option (b):

$a_n = a_{n-1} + a_{n-2} + 2^{n-1}$
$\Rightarrow \quad a_4 = a_3 + a_2 + 2^3$
$= 4 + 2 + 8 = 14$ which is False.

Option (c):

$a_n = 2a_{n-1} - a_{n-1} - a_{n-2}$
$\Rightarrow \quad a_4 = 2a_3 + a_3 - a_2$
$= 2 \times 4 - 4 + 2 = 6$ which is False.

Option (d):

$a_n = 2a_{n-1}$
$\Rightarrow \quad a_4 = 2a_3$
$= 2 \times 4 = 8$ which is true.

$\therefore$ Option (d): $a_n = 2a_{n-1}$ is correct.

QUESTION ANALYTICS

Q. 15

The number of ways to roll 5 six sided dice to get sum of 25 is _____.

FAQ | Solution Video | Have any Doubt ?

126

Correct Option

Solution :
126

Number of possible values on top of dice:

$$= x + x^2 + x^3 + x^4 + x^5 + x^6$$

$$= \frac{x(1-x^6)}{1-x}$$

We need to find coefficient of x^{25}:

$$\left(\frac{x(1-x^6)}{1-x}\right)^5 = x^5(1-x^6)^5 \cdot \frac{1}{(1-x)^5} = x^{25}$$

$\Rightarrow$ Coefficient of x^{20} in $(1-x^6)^5 \cdot \frac{1}{(1-x)^5}$

$\Rightarrow$ Coefficient of x^{20} in $(1 - 5x^6 + 10x^{12} - 10x^{18} + 5x^{24} - x^{30}) \left(\sum_{n=0}^{\infty} \binom{n+4}{4} x^n\right)$

$\Rightarrow$ Coefficient of x^{20} in $[({}^{20+4}C_4) - 5 \times ({}^{14+4}C_4) + 10 \times ({}^{8+4}C_4) - 10 \times ({}^{2+4}C_4)]x^{20}$

$\Rightarrow \quad = ({}^{24}C_4) - 5 \times ({}^{18}C_4) + 10 \times ({}^{12}C_4) - 10 \times ({}^{6}C_4)$
$= 10626 - 5 \times (3060) + 10 \times (495) - 10 \times (15)$
$= 126$

Your Answer is 45

QUESTION ANALYTICS

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Consider the simple graph with degree sequence {7, 3, 3, 3, 3, 3, 3, 3}. If x be the cardinality of largest independence set and *y* be cardinality of the minimum vertex cover, then the $x \times y$ is _____.

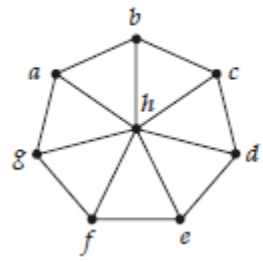
Solution Video | Have any Doubt ?

15

Correct Option

Solution :
15

With given degree sequence, simple graph will be:



Independence set or stable set is a set of vertices in a graph, no two of which are adjacent.
So, largest independence set is $|\{a, d, f\}| = 3$
We know that,

$$\text{Total vertex} = \text{Largest independence set} + \text{Minimal vertex cover}$$
$$8 = 3 + y$$
$$y = 5$$
$$\text{So,} \quad x \times y = 5 \times 3 = 15$$

QUESTION ANALYTICS

Q. 17

The number of non-negative integral solutions to the equation:
$x_1 + x_2 + x_3 + x_4 \le 10$
where $x_1, x_2, x_3, x_4 \ge 0$ is _____.

FAQ | Have any Doubt ?

1001

Your answer is **Correct** 1001

Solution :
1001

$$x_1 + x_2 + x_3 + x_4 \le 10$$
$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 + x_5 = 10 \text{ [Box method]}$$
$$\text{Number of solutions} = {}^{5-1+10}C_{10}$$
$$= ({}^{14}C_{10}) = 1001$$

QUESTION ANALYTICS
