5/3/19

Electromagnetic Theory

Vector Analysis:-

Any physical quantity may be represented either as a scalar or as a vector.

→ Scalar may be defined as a quantity which is completely characterised by its magnitude.

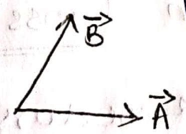
ex: mass, density, volume, area, time, temp etc...

→ Vector is defined as a quantity which is completely characterised by both magnitude & direction.

ex: force, velocity, displacement, acceleration, weight etc..

* vector quantities can be represented by a directed line segment (→).

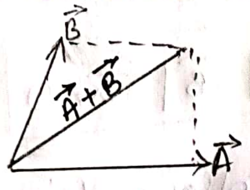
• The orientation depicts direction of the vector & length depicts the magnitude

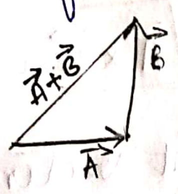


Addition of vectors:-

→ The sum of 2 vectors results into a vector. if $\vec{A}$ & $\vec{B}$ are two vectors then;

$$\boxed{\vec{A}+\vec{B} = \vec{B}+\vec{A} \Rightarrow \vec{C}}$$

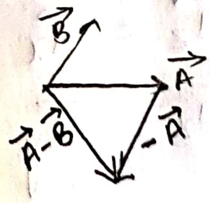




Subtraction of vectors:-

→ The difference b/w 2 vectors also results into a vector i.e

$$\boxed{\vec{A}-\vec{B} = \vec{C} \quad \text{but} \quad \vec{A}-\vec{B} \neq \vec{B}-\vec{A}}$$



Multiplication of vectors:

→ Vector multiplication & Scalar Multiplication

i) <u>Scalar product</u> or '<u>Dot product</u>" (·):-

By this multiplication of 2 vectors we get scalar value. If $\vec{A}$ & $\vec{B}$ are two vectors, then their dot product is given by:

$$\boxed{\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta}$$

⊙ Dot product of unit vectors follow this rule:

$$\boxed{\begin{array}{ll} \vec{a_i}\cdot\vec{a_j} = 1 & \Rightarrow \text{if } i=j \\ \vec{a_i}\cdot\vec{a_j} = 0 & \Rightarrow \text{if } i\neq j \end{array}}$$

$$\rightarrow \vec{A}\cdot\vec{B} = (A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z})\cdot(B_x\vec{a_x} + B_y\vec{a_y} + B_z\vec{a_z})$$

$$= A_xB_x + A_yB_y + A_zB_z$$

ii) <u>Vector product</u> or "<u>Cross product</u>" (×):-

→ cross product of 2 vectors always results into a vector quantity in the direction normal to the plane of vectors. It is given by:

$$\boxed{\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta}$$

⊙ Unit vectors follow this rule for × product:

$\vec{a_x}\times\vec{a_y} = \vec{a_z}$	$\vec{a_y}\times\vec{a_x} = -\vec{a_z}$
$\vec{a_y}\times\vec{a_z} = \vec{a_x}$	$\vec{a_z}\times\vec{a_y} = -\vec{a_x}$
$\vec{a_z}\times\vec{a_x} = \vec{a_y}$	$\vec{a_x}\times\vec{a_z} = -\vec{a_y}$

Coordinate Systems:

- To describe a vector accurately & to express a vector in terms of its components, it is necessary to have some reference directions. Such directions are represented by coordinate systems.
- Coordinate system are used to represent in three dimensional space.

There are 3 coordinate systems:

1) Cartesian system
2) Circular cylindrical system
3) Spherical coordinate system

(1) Cartesian System:-

- It has 3 axes, X, Y, Z mutually $\perp^r$ to each other
- Orientation of X, Y, Z follow right hand thumb rule

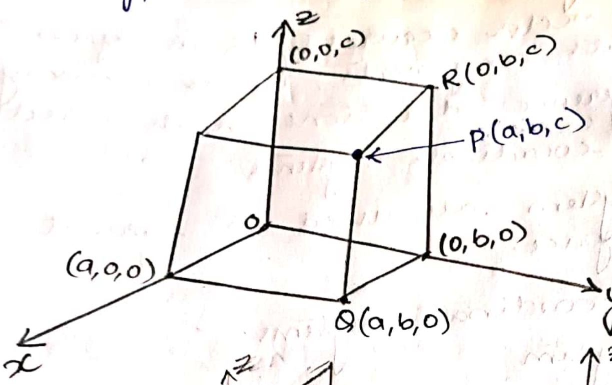
Range:-

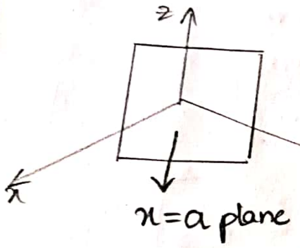
$$-\infty < X < \infty$$

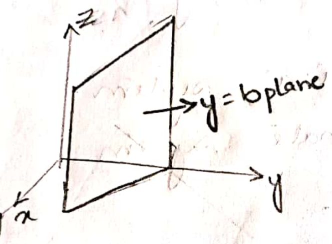
$$-\infty < Y < \infty$$

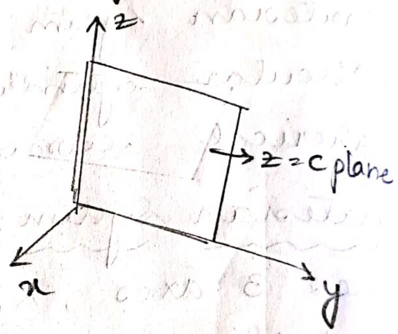
$$-\infty < Z < \infty$$

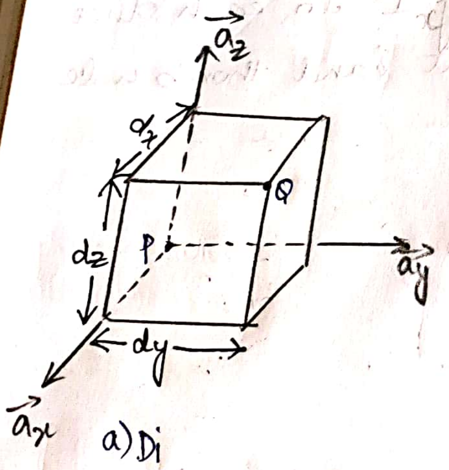
⊛ point $P(x, y, z)$ is the P.I of 3 planes.











a) Di

Differential length: $d\ell = dx\,a_x + dy\,a_y + dz\,a_z$

Differential surfaces: $d_{sx} = dy\,dz\,a_x$

$ds_y = dx\,dz\,a_y$

$ds_z = dx\,dy\,a_z$

Diff. volume: $dV = dx.dy.dz$

Cylindrical coordinate system :-

This is constructed by r = const plane ; ϕ = const plane & z = const plane.

A point $P(\rho, \phi, z)$ is the intersection of three 3 planes

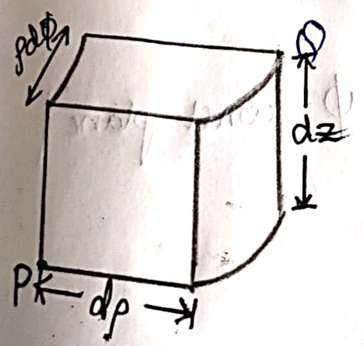
Range:-

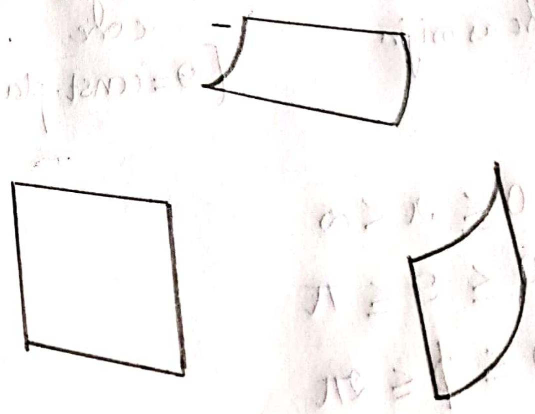
$$0 \le \rho < \infty$$

$$0 \le \phi \le 2\pi$$

$$-\infty < z < \infty$$

A vector is represented by: $OP = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$





Differential length: $dl = d\rho\, a_\rho + \rho d\phi\, a_\phi + dz\, a_z$

Differential surfaces: $ds_\rho = \rho\, d\phi\, dz \cdot a_\rho$ in ρ - direction

$ds_\phi = d\rho\, dz\, a_\phi$ in ϕ - direction

$ds_z = \rho\, d\rho\, d\phi \cdot a_z$ in z - direction

Differential volume: $dv = \rho \cdot d\rho\, d\phi\, dz$.

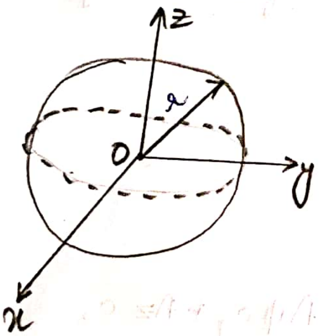
③ Spherical coordinate system :-

It is constructed by foll. 3 planes:

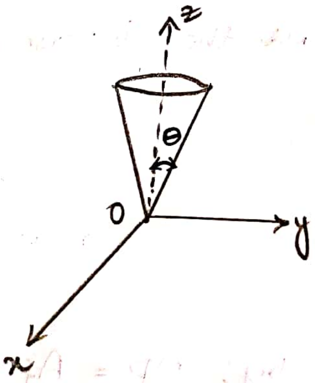
r = const plane

θ = const plane

ϕ = const plane.

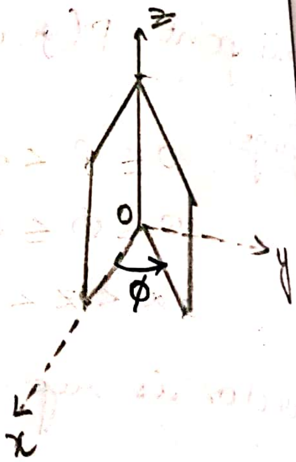


sphere of radius 'r' with centre as origin



Right circular cone

[θ = const. plane]



[ϕ = const. plane]

Range :-

$0 \leq r < \infty$

$0 \leq \theta \leq \pi$

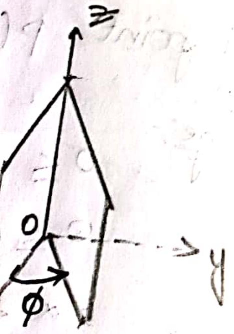
$0 \leq \phi \leq 2\pi$

→ vector is represented by: $OP = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$

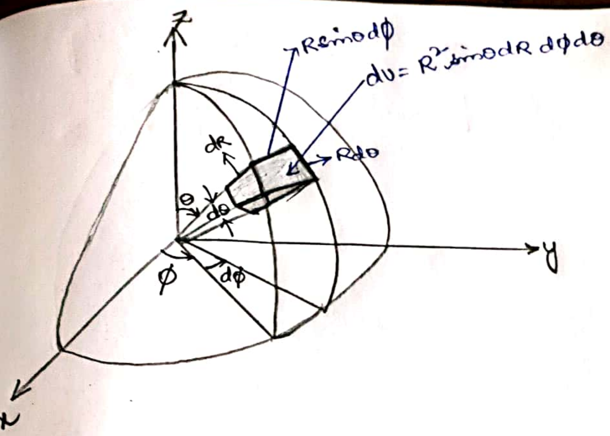
in ρ-direction

in ϕ-direction

in z-direction



...t-plane]



dr = differential change in r-direction

$r\,d\theta$ = " " " θ-direction

$r\sin\theta\,d\phi$ = " " " ϕ-direction

Differential length: $dl = dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$

Differential surfaces:

$$ds_r = r^2 \sin\theta\,d\theta\,d\phi\,a_r$$

$$ds_\theta = r\sin\theta\,dr\,d\theta\,a_\phi$$

$$ds_\phi = r\,dr\,d\theta\,a_\phi$$

Diff. volume: $dv = r^2\sin\theta\,dr\,d\theta\,d\phi$

* ELECTROSTATICS *

→ Electromagnetics deals with study of charges at rest & in motion

* principles of electromagnetics find its applications in disciplines such as microwaves, antennas, satellite comm., power generation, bio-electromagnetics, radar systems, EMI/EMC [electro-magnetic Interference, " " Compatability] electronic warfare, fibre optics etc....

→ A field produced by static charge is called "static electric field". It is an interesting phenomena with application in areas like electric power transmission, CRT, X-Ray machines, electric printers, lighting protection, solid state electronics, touchpads, capacitance, keyboards, LCD display, sorting machines in agriculture & mining & etc.....

→ static electric field is completely governed by two laws:

1> Coulomb's law

2> Gauss' law.

1. COULOMB'S LAW:

It was formulated by Charles Augestine De Coulomb in 1785.

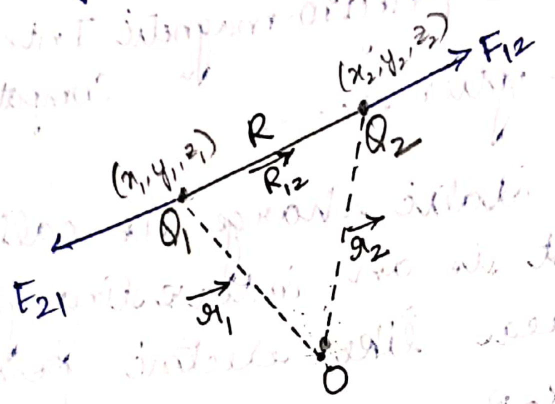
- It describes the force b/w two point charges
- Coulomb's law states that: "If two point charges Q_1 & Q_2 are seperated by large distance 'R' in a vaccum or free space; then the force b/w

two charges is:

1.) Directly proportional to the product of the charges Q_1 & Q_2

2.) Inversely proportional to the square of the distance b/w two charges.

3.) Along the line joining them. [Direction of Force]



$\therefore F \propto Q_1 Q_2$

$F \propto \frac{1}{R^2}$

$F = K \cdot \frac{Q_1 Q_2}{R^2}$

where $K = \frac{1}{4\pi \varepsilon}$

$\therefore \varepsilon = \varepsilon_0 \varepsilon_r$

for free space $\Rightarrow \boxed{\varepsilon_r = 1}$

$\Rightarrow K = \frac{1}{4\pi \varepsilon_0}$

$$\therefore \boxed{F = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q_1 Q_2}{R^2}}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{R^2} \vec{a}_{R_{12}}$$

$$\vec{a}_{12} = \frac{\vec{R}_{12}}{|R_{12}|} \Rightarrow \frac{\vec{R}_{12}}{R}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{R^3} \cdot \vec{R}_{12} \Rightarrow \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{(|\vec{r}_2 - \vec{r}_1|)^3} (\vec{r}_2 - \vec{r}_1)$$

||ly $$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^3} \cdot \vec{R}_{21} \Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{(|\vec{r}_1 - \vec{r}_2|)^3} (\vec{r}_1 - \vec{r}_2)$$

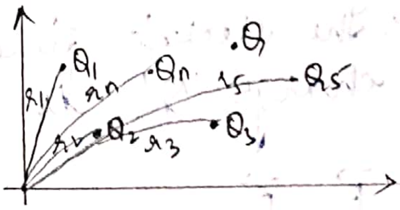
$$\therefore \boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

If a charge 'Q' is placed in 'N' no. of charges, then the ~~fo~~ total force experienced by charge 'Q' due to $Q_1, Q_2, Q_3 \cdots Q_N$ is the vector sum of individual forces by each charge i.e

$$F = F_1 + F_2 + F_3 + \cdots + F_N$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q}{R_1^3} \cdot \vec{R}_1 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q Q_2}{R_2^3} \cdot \vec{R}_2 + \cdots + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q Q_N}{R_N^3} \cdot \vec{R}_N$$

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q Q_k}{R_k^3} \cdot \vec{R}_k}$$



NOTE:

1. As $\vec{F}_{21} = -\vec{F}_{12}$, the force of Q_2 on Q_1 & the force of Q_1 on Q_2 is equal in magnitude but opposite in direction
2. Like charges repel each other and opposite charges attract.
3. The distance b/w the charges must be very large.
4. Q_1 & Q_2 must be static

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(5) The polarity of charges must be taken into consideration while calculating force.

(6) Force is measured in 'Newton'.

Electric Field Intensity:

- It is defined as the force per unit charge when placed in the electric field.
- It is also called as 'Electric field strength', denoted by $\vec{E}$ & measured in "Newton/coulomb" (or) "Volts/meter"

i.e. $\vec{E} = \frac{\vec{F}}{Q}$

- let 'Q' be any charge moved into the field of charge Q_1, then the force on Q due to Q_1 is given by:

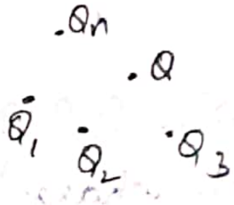
$$\vec{F_1} = \frac{1}{4\pi\epsilon_0} \cdot \frac{QQ_1}{R^3} \cdot \vec{R}$$

∴ The electric field intensity of Q_1 experienced by charge 'Q' is given by:

$$\vec{E} = \frac{\vec{F_1}}{Q} = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R^3} \cdot \vec{R}$$

- Electric field intensity is a vector with radially outward direction from the source
- As the force on the test charge 'Q' is dependent on the magnitude of test charge, if the test charge becomes a "zero" then, the force on it will also become "zero", But the force per unit charge remains 'same'

• If a unit charge 'Q' is placed among the 'N' no of charges, then the total electric field Intensity on the test charge is equal to the electric field intensity due to each charge.



$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \cdots\cdots + \vec{E_N}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{R_1^3} \cdot \vec{R_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2}{R_2^3} \cdot \vec{R_2} + \cdots\cdots$$

$$\cdots\cdots + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_N}{R_N^3} \vec{R_N}$$

$$\therefore \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{K=1}^{N} \frac{Q_K}{R_K^3} \cdot \vec{R_K}}$$

Electric field Intensity due to various charge distribution

There are basically 4 types of charge distribution

1) point charge
2) Line charge
3) Surface charge
4) Volume charge.

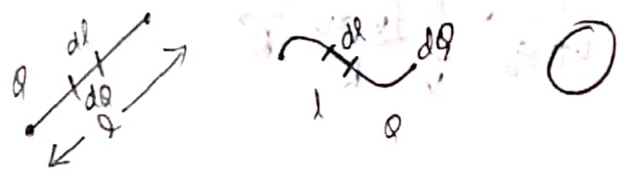
1) point charge:

A point charge distribution with no physical dimension is called point charge

.Q

2) Line charge:

A charge distribution with single dimension is called a line charge distribution.



Line charge density $\rho_L = \frac{Q}{l} = \frac{dQ}{dl}$

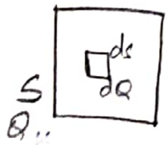
$\Rightarrow \boxed{dQ = \rho_L \, dl}$

Total charge $Q = \int_l dQ = \int_l \rho_L \, dl$

Electric field intensity; $E = \frac{1}{4\pi\varepsilon_0} \int_l \frac{\rho_L \, dl}{R^3} \cdot \vec{R}$

③ <u>Surface charge distribution</u>:

A 2D charge distribution that has some surface area is called surface charge distribution



→ surface charge density $\rho_s = \frac{Q}{S} = \frac{dQ}{ds}$

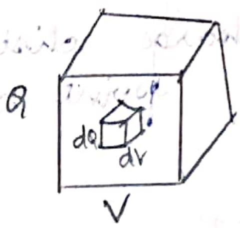
$$\boxed{dQ = \rho_s \, ds}$$

Total surface charge $Q = \int_S dQ = \iint \rho_s \, ds$

Electric Field Intensity; $E = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\rho_s \, ds}{R^3} \cdot \vec{R}$

④ <u>Volume charge distribution</u>:

A 3D charge distribution is called volume charge distribution



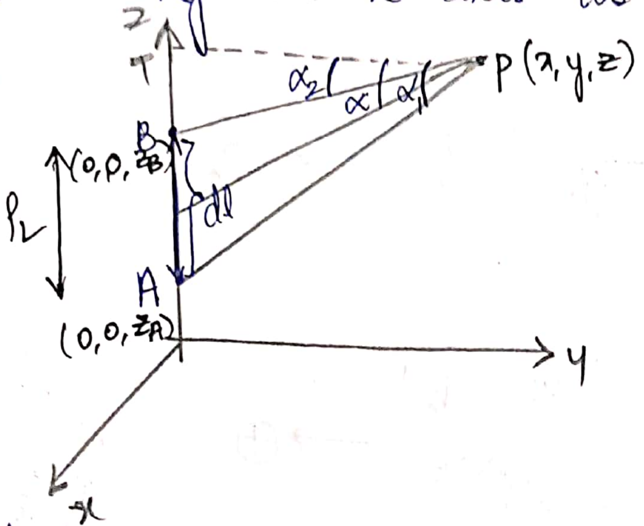
→ Volume charge density $\rho_v = \frac{Q}{V} = \frac{dQ}{dv}$

Total volume charge $Q = \int_V dQ = \iiint \rho_v \, dv$

Electric field Intensity; $E = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho_v \, dv}{R^3} \cdot \vec{R}$

Electric field Intensity due to a finite line Charge:

Consider a line charge of length 'l' & density 'ρ_L' placed along the Z-axis as shown in the fig.



To obtain the electric field intensity due to this line charge at a distant 'p' (x, y, z). Consider an infinitesimal length 'dl' with charge 'dQ'

$$\rho_L = \frac{dQ}{dl}$$

$$dQ = \rho_L \, dl = \rho_L \, dz'$$

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dQ}{R^3} \cdot \vec{R} \longrightarrow (1)$$

& $\vec{R} = x\vec{a}_x + y\vec{a}_y + (z-z')\vec{a}_z$

$$R = |\vec{R}| = \sqrt{x^2+y^2+(z-z')^2}$$

also $\vec{TP} = x\vec{a}_x + y\vec{a}_y$

$$\vec{TP} = \rho\vec{a}_\rho = \vec{\rho}$$

$$|\vec{\rho}| = \sqrt{\rho^2} = \rho$$

$\therefore \vec{R} = \rho\vec{a_\rho} + (z - z')\vec{a_z} \longrightarrow ②$

$R = \sqrt{\rho^2 + (z-z')^2} \longrightarrow ③$

$\therefore d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_L \, dz}{(\rho^2 + (z-z')^2)^{3/2}} \cdot [\rho\vec{a_\rho} + (z-z')\vec{a_z}]$

$$\vec{E} = \int_{(0,0,z_A)}^{(0,0,z_B)} \frac{\rho_L(\rho\vec{a_\rho} + (z-z')\vec{a_z})\cdot dz'}{4\pi\varepsilon_0(\rho^2 + (z-z')^2)^{3/2}} \longrightarrow ④$$

from $\triangle$le TPQ

$\sin\alpha = \frac{z - z'}{R}$

$\Rightarrow z - z' = R\sin\alpha \longrightarrow ✱$

Also $\cos\alpha = \frac{\rho}{R} \Rightarrow R = \frac{\rho}{\cos\alpha}$

sub 'R' in ✱

$z - z' = \rho\tan\alpha \longrightarrow ⑤$

diff w.r.t z'

$\Rightarrow 0 - 1 = \rho\sec^2\alpha \cdot \frac{d\alpha}{dz'}$

$\Rightarrow dz' = -\rho\sec^2\alpha \, d\alpha \longrightarrow ⑥$

✱ when $z' = z_A$; $\boxed{\alpha = \alpha_1}$

$z - z_A = \rho\tan\alpha_1$

$\therefore \alpha_1 = \tan^{-1}\left[\frac{z-z_A}{\rho}\right]$

✱ when $z' = z_B$; $\alpha =$

$\alpha_2 = \tan^{-1}[z-$

sub^n ⑤, ⑥, ⑦

$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{[\rho\vec{a_\rho} +}{(\rho^2+}$

$= \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}$

$= \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}$

$= \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}$

$= \frac{-\rho_L}{4\pi\varepsilon_0\rho}$

$= \frac{-\rho_L}{4\pi\varepsilon}$

if

the

$\therefore\ \alpha_1 = \tan^{-1}\left[\dfrac{z - z_A}{\rho}\right] \rightarrow (7)$

(4) when $z' = z_B$; $\boxed{\alpha = \alpha_2}$

$\alpha_2 = \tan^{-1}\left[\dfrac{z - z_B}{\rho}\right] \rightarrow (8)$

sub^n (5), (6), (7) & (8) in (4)

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{[\rho\vec{a}_\rho + \rho\tan\alpha\,\vec{a}_z]}{(\rho^2 + \rho^2\tan^2\alpha)^{3/2}}\cdot(-\rho\sec^2\alpha\, d\alpha)$$

$\rho^2(1+\tan^2\alpha)$

$(\rho^2\sec^2\alpha)^{3/2}$

$$= \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{\cancel{\rho}\,[\vec{a}_\rho + \tan\alpha\,\vec{a}_z]}{\rho^{\cancel{3}}\sec^{\cancel{3}}\alpha}\cdot\rho\,\cancel{\sec^2}\alpha\, d\alpha$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{(\vec{a}_\rho + \tan\alpha\,\vec{a}_z)}{\sec\alpha}\cdot d\alpha$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}(\cos\alpha\,\vec{a}_\rho + \sin\alpha\,\vec{a}_z)\, d\alpha$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[\left[\sin\alpha\,\vec{a}_\rho\right]_{\alpha_1}^{\alpha_2} - \left[\cos\alpha\,\vec{a}_z\right]_{\alpha_1}^{\alpha_2}\right]$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[(\sin\alpha_2 - \sin\alpha_1)\vec{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\vec{a}_z\right] \rightarrow (9)$$

if line charge is Infinite

then $A(0,0,-\infty)$ & $B(0,0,\infty)$

$\therefore z_A = -\infty \qquad z_B = \infty$

$\Rightarrow \alpha_1 = \tan^{-1}\left[\frac{z - z_A}{\rho}\right] = \tan^{-1}(\infty) = \pi/2$

$\Rightarrow \alpha_2 = \tan^{-1}\left[\frac{z - z_B}{\rho}\right] = \tan^{-1}(-\infty) = -\pi/2$

subn in ①

$\therefore \vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0 \rho}\left[(-1-1)\vec{a_\rho} - (0)\vec{a_z}\right]$

$= \frac{2\rho_L}{4\pi\varepsilon_0 \rho} \cdot \vec{a_\rho}$

$$\therefore \boxed{\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0 \rho} \cdot \vec{a_\rho}}$$

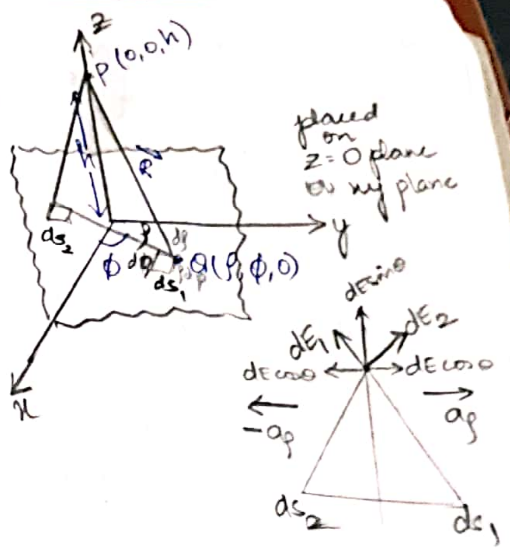
$\therefore$ The electric field intensity will always be towards radial direction.

Electric field Intensity due to Infinite Surface Charge:

Consider an infinite sheet of charge on X-Y plane with uniform charge density 'ρ_s'.

- To obtain the electric field intensity due to this arrangement, consider infinitesimal or incremental surface 'ds' with charge 'dQ' as shown in the fig.

$$ds = \rho \, d\rho \, d\phi \longrightarrow (1)$$



$$\vec{R} = (0-\rho)\vec{a_\rho} + (h-0)\vec{a_z}$$

$$= -\rho \vec{a_\rho} + h\vec{a_z} \longrightarrow (2)$$

$$R = |\vec{R}| = \sqrt{\rho^2+h^2} \longrightarrow (3)$$

$$\vec{dE} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{R^3} \cdot \vec{R} \longrightarrow (4)$$

$$\rho_s = \frac{dQ}{ds} \Rightarrow dQ = \rho_s \cdot ds \longrightarrow (5)$$

sub^n (1), (2), (3) & (5) in (4)

$$\Rightarrow \vec{dE} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_s \cdot \rho \, d\rho \, d\phi}{(\rho^2+h^2)^{3/2}} (-\rho\vec{a_\rho} + h\vec{a_z})$$

Total field due to infinite surface charge is

$$\vec{E} = \iint \vec{dE}$$

$$\vec{E} = \int_{\rho=0}^{\infty}\int_{\phi=0}^{2\pi} \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_s \, \rho \, d\rho \, d\phi}{(\rho^2+h^2)^{3/2}} (-\rho\vec{a_\rho} + h\vec{a_z})$$

As total electric field intensity due to infinite surface charge is the integration of the above eqn over the limits: $\rho \to 0$ to ∞ &

$\phi \to 0$ to 2π

Due to a symmetry of charge distribution, "a_ρ" component due to charge on surface '1' is equal to "a_ρ" component due to charge on surface '2'. But, in opposite direction resulting in "zero" 'a_ρ' component.

$$\therefore \vec{E} = \frac{\rho_s}{4\pi\varepsilon_0} \int_{\rho=0}^{\infty} \int_{\phi=0}^{2\pi} \frac{\rho \, d\rho \, d\phi \, h \, \vec{a_z}}{(\rho^2+h^2)^{3/2}}$$

$$= \frac{\rho_s h}{4\pi\varepsilon_0} \int_{\rho=0}^{\infty} \frac{\rho \, d\rho \, \vec{a_z}}{(\rho^2+h^2)^{3/2}} \int_{\phi=0}^{2\pi} d\phi \longrightarrow \begin{array}{l}(2\pi - 0) \\ = 2\pi\end{array}$$

$$= \frac{\rho_s h}{4\pi\varepsilon_0} \cdot 2\pi \int_{\rho=0}^{\infty} \frac{\rho \, d\rho \, \vec{a_z}}{(\rho^2+h^2)^{3/2}}$$

$$= \frac{\rho_s h}{2\varepsilon_0} \vec{a_z} \int_{\rho=0}^{\infty} \frac{\rho \, d\rho}{(\rho^2+h^2)^{3/2}}$$

let $\rho^2 + h^2 = t^2$

diff w.r.t 'ρ'

$$\cancel{2}\rho = \cancel{2}t \frac{dt}{d\rho}$$

$$\rho \, d\rho = t \, dt$$

when $\rho \to 0$; $t = h$

$\rho \to \infty$; $t = \infty$

tribution,
urface '1' is
charge on
n resulting

$(2\pi - 0) = 2\pi$

$$\vec{E} = \frac{\rho_s h \vec{a}_z}{2\varepsilon_0} \int_h^\infty \frac{t\,dt}{t^3}$$

$$= \frac{\rho_s h \vec{a}_z}{2\varepsilon_0} \int_h^\infty t^{-2}\,dt$$

$$= \frac{\rho_s h \vec{a}_z}{2\varepsilon_0} \left[\frac{t^{-1}}{-1}\right]_h^\infty$$

$$= \frac{-\rho_s h \vec{a}_z}{2\varepsilon_0} \left[\frac{1}{t}\right]_h^\infty$$

$$= \frac{-\rho_s \cancel{h} \vec{a}_z}{2\varepsilon_0} \left[0 - \frac{1}{\cancel{h}}\right]$$

$$\boxed{\therefore \vec{E} = \frac{\rho_s \vec{a}_z}{2\varepsilon_0}}$$

---

problems:

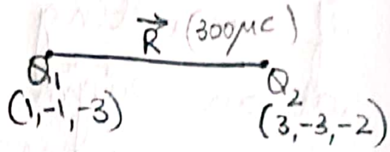
Q1.) Two point charges of magnitude '2' millicoulomb & '−7' millicoul are located at pt $(4, 7, -5)$ & $(-3, 2, -9)$ respectively. Evaluate

i) Distance b/w two points

ii) Vector force on charge at point 2

iii) Vector force on charge at point 1

Q: A point charge Q_1 is equal to 300 μ coulomb is located at (1,-1,-3) experiences a Force $F = 8\vec{a_x} - 8\vec{a_y} + 4\vec{a_z}$ at (3,-3,-2). Determine Q_2



Sol: $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 Q_2}{R^3} \cdot \vec{R}$ —— ①

where $\vec{R} = (1-3)\vec{a_x} + (-1+3)\vec{a_y} + (-3+2)\vec{a_z}$

$= -2\vec{a_x} + 2\vec{a_y} - 1\vec{a_z}$

$R = |\vec{R}| = \sqrt{4+4+1} = 3$

sub^n in ①

$$(8\vec{a_x} - 8\vec{a_y} + 4\vec{a_z}) = \frac{1}{4\pi\epsilon_0} \cdot \frac{300 \times 10^{-6} \times Q_2}{3^3}\left[-2\vec{a_x} + 2\vec{a_y} - 1\vec{a_z}\right]$$

$\epsilon_0 = 8.854 \times 10^{-12}$

$$-4\left(-2\vec{a_x} + \cancel{2\vec{a_y} - \vec{a_z}}\right) = \frac{1}{4\pi \times 8.854 \times 10^{-12}} \cdot \frac{300 \times 10^{-6} \times Q_2}{(27)}\left[-2\vec{a_x} \cancel{+ 2\vec{a_y} - \vec{a_z}}\right]$$

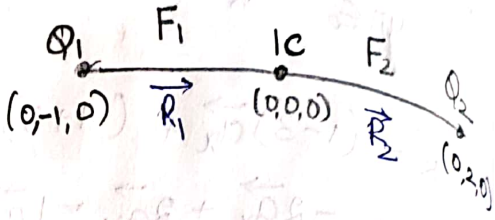
$$-4 = \frac{\cancel{9}^{\,9} \times 10^9 \times \cancel{300}^{\,100} \times 10^{-6} \times Q_2}{\cancel{27}\ 3}$$

$-4 = 100\, Q_2 \times 10^3$

$Q_2 = \frac{-4}{10^5} = -4 \times 10^{-5}$

$\therefore \boxed{Q_2 = -40\,\mu\text{coulomb}}$

Q.> A charge Q_1 at (0, -1, 0) & another charge Q_2 at (0, 2, 0) are located. Find the ratio Q_2/Q_1 if resulting force on a 1C test charge placed at an origin is 'zero'. All the charges are of same polarity.



Sol^n: $F = F_1 + F_2 = 0$ & $\boxed{Q = 1C}$

& $F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q}{R_1^3} \cdot \vec{R}$; $F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2 Q}{R_2^3} \cdot \vec{R}$

9×10^9

let $\vec{R_1} = \vec{a_y}$ $\qquad$ $\vec{R_2} = -2\vec{a_y}$

$R_1 = \sqrt{1} = 1$ $\qquad$ $R_2 = \sqrt{4} = 2$

$F_1 = 9\times10^9 \cdot \frac{Q_1}{1^3} \cdot \vec{a_y}$; $F_2 = 9\times10^9 \cdot \frac{Q_2}{\cancel{8}\,4} (-\cancel{2}\vec{a_y})$

$= \frac{-9\times10^9 Q_2 \vec{a_y}}{4}$

$\therefore \boxed{F_1 + F_2 = 0}$

$\Rightarrow 9\times10^9 Q_1 \vec{a_y} - \frac{9\times10^9 Q_2 \vec{a_y}}{4}$

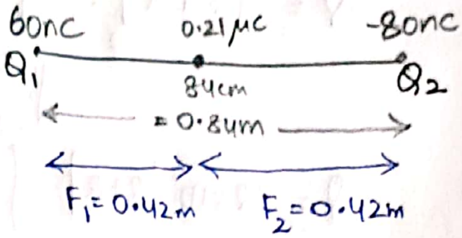
$9\times10^9 \left[Q_1 - \frac{Q_2}{4} \right] = 0$

$Q_1 - \frac{Q_2}{4} = 0$

$Q_1 = \frac{Q_2}{4}$

$\therefore \boxed{\frac{Q_2}{Q_1} = \frac{1}{4}}$

Q: A charge of 60nC is 84cm apart from a charge of -80n Coloumb. What force. What force will these charges of 0.21μC placed half way b/w them. [Take air as medium]



Sol: $F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1Q_2}{R^3} \cdot \vec{R}$

$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1Q_2}{R^2}$; $F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_2Q}{R_2^2}$

$F_1 = \frac{9\times10^9 \times 60\times10^{-9}\times 0.21\times10^{-6}}{(0.42)^2}$; $F_2 = \frac{9\times10^9\times(-80\times10^{-9})\times 0.21\times10^{-6}}{(0.42)^2}$

$F_1 = 0.642\times10^{-3}N$; $F_2 = -0.857\times10^{-3}N$

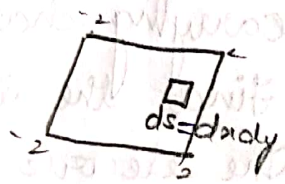
$\therefore |F| = |F_1| + |F_2|$

$= 0.642\times10^{-3} + 0.857\times10^{-3}$

$\boxed{F = 1.5mN \text{ away from } Q_1}$

Q: A square plane described by $-2 \le x \le 2$, $-2 \le y \le 2$ & $z=0$ carries a charge density $12|y|$ mc/m². Find the total charge on the plane surface

Sol: W.K.T $Q = \iint \rho_s \, ds$ where $\rho_s = 12|y|$ mc/m²



$\therefore Q = \int_{x=-2}^{2}\int_{y=-2}^{2} 12|y| \times 10^{-3} \, dx \cdot dy$

$$Q = 12\times10^{-3}\int_{x=-2}^{2} dx \int_{y=-2}^{2} |y|\,dy$$

$$Q = 12\times10^{-3}[x]_{-2}^{2} \cdot \int_{y=-2}^{2} |y|\,dy$$

$$Q = 12\times10^{-3}[2+2]\int_{y=-2}^{+2} |y|\,dy$$

$$Q = 48\times10^{-3}\int_{-2}^{+2} |y|\,dy$$

$$Q = 48\times10^{-3}\left[\int_{-2}^{0} -y\,dy + \int_{0}^{2} y\,dy\right]$$

$$Q = 48\times10^{-3}\left[-\left(\frac{y^2}{2}\right)_{-2}^{0} + \left(\frac{y^2}{2}\right)_{0}^{2}\right]$$

$$Q = 48\times10^{-3}\left[-\frac{(0-(-2)^2)}{2} + \frac{2^2}{2}\right]$$

$$= 48\times10^{-3}\left[\frac{4}{2}+\frac{4}{2}\right]$$

$$= 48\times10^{-3}\left[\cancel{2}\left(\frac{4}{\cancel{2}}\right)\right]$$

$$\boxed{Q = 192\,mC}$$

Q) A finite sheet $0 \le x \le 1$, $0 \le y \le 1$ lie on $\boxed{z=0}$ plane carrying charge density $\rho_s = xy(x^2+y^2+25)^{3/2}\ nC/m^2$.

i) Find the total charge on the sheet.

ii) The electric field at $(0,0,5)$

iii) The force experienced by a $-1mC$ charge located at $(0,0,5)$

sol^n :- $Q = \iint \rho_s \, ds$ where $ds = dx\,dy$

$$= \int_{x=0}^{1}\int_{y=0}^{1} xy\,(x^2+y^2+25)^{3/2} \times 10^{-9}\,dx\,dy$$

let $x^2 = t$, $y^2 = k$

$2x = \frac{dt}{dx}$; $2y = \frac{dk}{dy}$

$x\,dx = \frac{dt}{2}$; $y\,dy = \frac{dk}{2}$

if $x=0$; $t=0$ | if $y=0$; $k=0$

$x=1$; $t=1$ | $y=1$; $k=1$

$$\Rightarrow Q = \int_{t=0}^{1}\int_{k=0}^{1} \frac{(t+k+25)^{3/2}}{4}\,dt\cdot dk \times 10^{-9}$$

$$= \frac{10^{-9}}{4}\int_{k=0}^{1}\left[\frac{(t+k+25)^{5/2}}{5/2}\right]_0^1 dk$$

$$= \frac{10^{-9}}{10}\int_{k=0}^{1}\left[(k+26)^{5/2} - (k+25)^{5/2}\right] dk$$

$$= \frac{10^{-9}}{10}\left[\left[\frac{(k+26)^{7/2}}{7/2}\right]_0^1 - \left[\frac{(k+25)^{7/2}}{7/2}\right]_0^1\right]$$

$$= \frac{10^{-9}}{35}\left[(27)^{7/2} - (26)^{7/2} - (26)^{7/2} + (25)^{7/2}\right]$$

$$\boxed{Q = 33.14\,nC}$$

ii)

$$E = \iint \frac{1}{4\pi\varepsilon_0} \frac{\rho_s\, ds}{R^3} \cdot \vec{R}$$

$$\vec{R} = -x\vec{a_x} - y\vec{a_y} + 5\vec{a_z}$$

$$R = \sqrt{x^2+y^2+25}$$

$$\Rightarrow E = \int_{x=0}^{1}\int_{y=0}^{1} \frac{xy\cancel{(x^2+y^2+25)^{3/2}}(-x\vec{a_x} - y\vec{a_y} + 5\vec{a_z})}{4\pi\varepsilon_0 \cancel{(x^2+y^2+25)^{3/2}}}\, dx\, dy \times 10^{-9}$$

$$E = \frac{10^{-9}}{4\pi\varepsilon_0} \int_{x=0}^{1}\int_{y=0}^{1} -x^2y\vec{a_x} - xy^2\vec{a_y} + 5xy\vec{a_z}\, dx\, dy$$

$$= \frac{10^{-9}}{4\pi\varepsilon_0}\left[-\int_{x=0}^{1}\int_{y=0}^{1} x^2y\,\vec{a_x}\, dx\, dy - \int_{x=0}^{1}\int_{y=0}^{1} xy^2\,\vec{a_y}\, dx\, dy + 5\int_{x=0}^{1}\int_{y=0}^{1} xy\, dx\, dy\right]$$

$$= \frac{10^{-9}}{4\pi\varepsilon_0}\left[-\frac{1}{2}\int_{x=0}^{1} x^2 dx\,\vec{a_x} - \frac{1}{2}\int_{y=0}^{1} y^2 dy\,\vec{a_y} + \frac{5}{2}\int_{x=0}^{1} x\, dx\,\vec{a_z}\right]$$

$$= \frac{10^{-9}}{4\pi\varepsilon_0}\left[-\frac{1}{6}\vec{a_x} - \frac{1}{6}\vec{a_y} + \frac{5}{4}\vec{a_z}\right]$$

$$= 9\times\cancel{10^{9}}\times\cancel{10^{-9}}\left[-0.16\vec{a_x} - 0.16\vec{a_y} + 1.25\vec{a_z}\right]$$

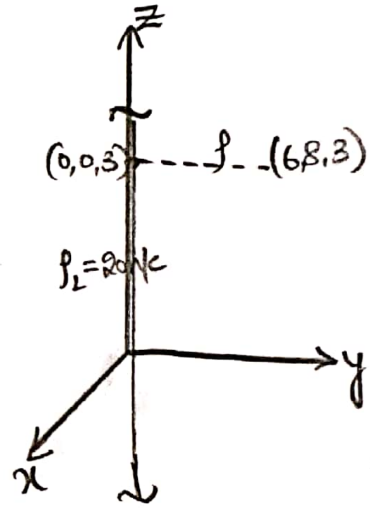
$$\boxed{\vec{E} = -1.54\vec{a_x} - 1.54\vec{a_y} + 11.25\vec{a_z}\ N/c \text{ or } V/m}$$

iii) $E = \frac{F}{Q}$

$F = EQ$

$$\boxed{F = (1.54\vec{a_x} + 1.54\vec{a_y} - 11.25\vec{a_z}) \times 10^{-3}\, N}$$

2) A uniform charge distribution infinite in extend lies along the x-axis with $\rho_L = 20$ n coloumb/m. Find the electric field at (6, 8, 3) m



Sol:- $\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho} \cdot \vec{a_\rho}$

$\vec{\rho} = 6\vec{a_x} + 8\vec{a_y}$

$\rho = \sqrt{36+64} = 10$

$\Rightarrow \vec{a_\rho} = \frac{\vec{\rho}}{\rho} = \frac{6\vec{a_x} + 8\vec{a_y}}{10}$

$= 0.6\vec{a_x} + 0.8\vec{a_y}$

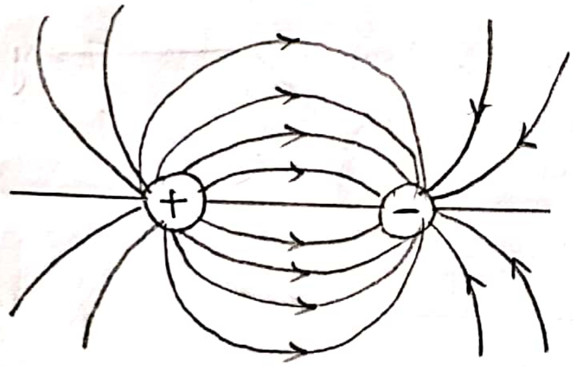
$\therefore \Rightarrow \vec{E} = \frac{20\times10^{-9}\,(0.6\vec{a_x} + 0.8\vec{a_y})}{2\times\pi\times 8.854\times10^{-12}\times10}$

$= 35.96\,(0.6\vec{a_x} + 0.8\vec{a_y})$

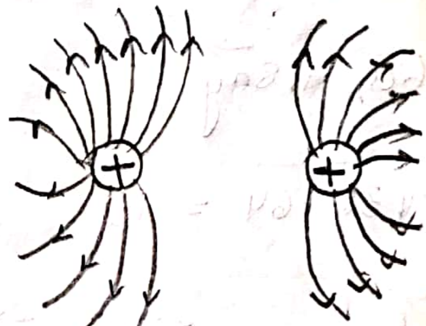
$$\boxed{\vec{E} = 21.57\vec{a_x} + 28.76\vec{a_y}\ N/c \text{ (or) } V/m}$$

Electric flux: [electric displacement]:

- Field lines emanate from a point charge symmetrically in all directions
- Field lines originate on positive charges and terminate on negative ones
- They cannot simply stop in midair, though they may extend out to infinity
- Field lines can never cross.



equal but opposite charges



equal charges

→ These lines of forces consituting an electric field are collectively termed as "electric flux". It is denoted by 'ψ'

→ Electric flux is found to be directly proportional to charge & in S.I units, the value of proportional constant is '1'

$\therefore \boxed{\psi = Q}$ & is measured in 'columbs'

Electric flux density:

As there are definite no. of lines of force i.e flux, passed through unit surface area which is normal to the direction of flux, Electric-flux density is defined as flux per unit normal area. Denoted by " $\vec{D}$ " measured in "coloumbs per m^2"

i.e $\vec{D} = \frac{\psi}{S}$ (or) $\vec{D} = \frac{d\psi}{ds}$

$$\therefore \psi = \oiint d\psi = \oiint \vec{D} \cdot \vec{ds}$$

GAUSS LAW:

"Gauss law states that total electric flux "ψ" through any closed surface is equal to the charge enclosed by that surface".

$$\psi = Q_{enclosed} \quad \text{—— ①}$$

i.e $\psi = \oint d\psi = \oint D \cdot ds$

$= \text{total charge enclosed } Q = \int \rho_v \, dv$ —— ②

(or) $Q = \oint D \cdot ds = \int \rho_v \, dv$ —— ③

From the divergence theorem;

$$\oint D \cdot ds = \int \nabla \cdot D \, dv \longrightarrow ④$$

$\boxed{\rho_v = \nabla \cdot D}$ ⇒ Maxwell's 1^{st} eqn

∇ = diff operator

→ GAUSS LAW is an alternative statement of Colomb's law

Procedure for Applying GAUSS's Law:

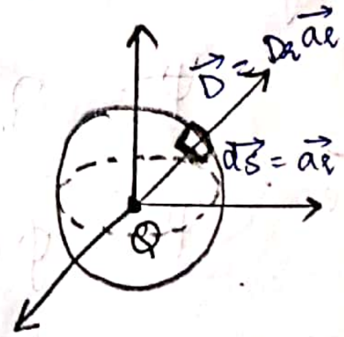
- To calculate the electric field using Gauss law 1st know the symmetric axis
- If the symmetric charge distribution exists, cons. a mathematical closed surface known as Gaussian surface consisting of field point
- Choose the surface such that displacement density is normal or tangential to Gaussian surface

→ If 'D' is normal to surface : $\vec{D}\,\vec{ds} = D\,ds$

→ If 'D' is tangent to surface ; $\vec{D}\,\vec{ds} = 0$

$\vec{E}$ [electric field Intensity] due to point charge:

W.K.T



$$\Psi = Q_{enclosed} \longrightarrow (1)$$

$$Q_{enclosed} = Q \longrightarrow (2)$$

$$\Psi = \oint\!\!\oint \vec{D} \cdot \vec{ds}$$

where $\vec{D} = D_r \cdot \vec{a_r}$ & $\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a_r}$

$$\Rightarrow \Psi = \oint\!\!\oint D_r \cdot \vec{a_r} \cdot r^2 \sin\theta\, d\theta\, d\phi\, \vec{a_r}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} D_r \cdot r^2 \sin\theta\, d\theta \cdot d\phi$$

$$= -2\pi D_r \cdot r^2 \left[\cos\theta\right]_0^{\pi}$$

$$= -2\pi D_r \cdot r^2 \left[-1-1\right]$$

$$\therefore \psi = 4\pi D_r r^2 \longrightarrow (3)$$

but from (1)

$$\Rightarrow 4\pi D_r \cdot r^2 = Q$$

$$D_r = \frac{Q}{4\pi r^2}$$

$$\vec{D} = \frac{Q}{4\pi r^2} \cdot \vec{a_r}$$

w.k.T

$$\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \vec{a_r}$$

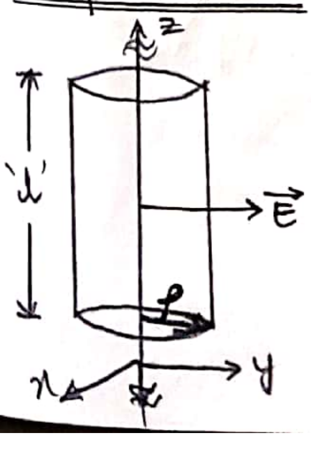
$$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

$$\boxed{\vec{D} = \varepsilon_0 \vec{E}}$$

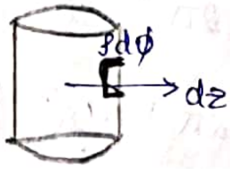
Applications of Gauss law:

Gauss law is particularly useful in computing E or D, where the charge distribution has some symmetry

AN INFINITE LINE CHARGE:



W.K.T



$$\Psi = Q_{enclosed} \longrightarrow (1)$$

$$Q_{enclosed} = \rho_l \cdot l$$

$$\Psi = \oint\oint \vec{D} \cdot \vec{ds} \longrightarrow (2)$$

& $\vec{D} = D_\rho \cdot \vec{a_\rho}$ & $\vec{ds} = \rho \, d\phi \, dz \, \vec{a_\rho}$

sub in (2)

$$\Psi = \oint\oint D_\rho \vec{a_\rho} \cdot \rho \, d\phi \, dz \cdot \vec{a_\rho}$$

$$= \int_{z=0}^{l} \int_{\phi=0}^{2\pi} D_\rho \cdot \rho \, d\phi \, dz$$

$$= 2\pi D_\rho \cdot \rho \, [z]_0^l$$

$$\Psi = 2\pi D_\rho \rho \cdot l$$

From (1):

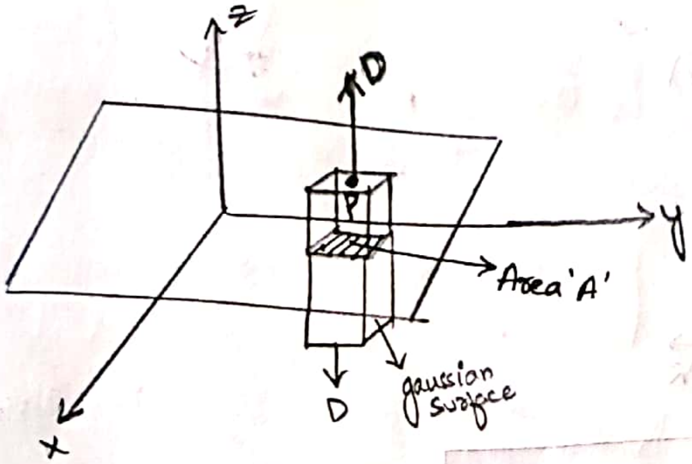
$$2\pi D_\rho \cdot \rho \cancel{l} = \rho_l \cancel{l}$$

$$D_\rho = \frac{\rho_l}{2\pi\rho}$$

$$\vec{D} = \frac{\rho_l}{2\pi\rho} \vec{a_\rho}$$

$$\therefore \boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_l}{2\pi\varepsilon_0 \rho} \cdot \vec{a_\rho}}$$

Electric field intensity due to Infinite Surface Charge:



w.k.T

$$\Psi = Q_{enclosed} \longrightarrow (1)$$

$$Q_{enclosed} = \rho_s \cdot A \longrightarrow (2)$$

$$\Psi = \oiint \vec{D} \cdot \vec{ds} \longrightarrow (3)$$

where $\vec{D} = D_z \cdot \vec{a_z}$ & $\vec{ds} = dx\, dy\, \vec{a_z} \Rightarrow ds\, \vec{a_z}$

sub in (3)

$$\Psi = \oiint D_z \vec{a_z} \cdot ds\, \vec{a_z}$$

$$= \oiint D_z\, ds$$

$$= D_z \oiint ds$$

$$= D_z \left[\oiint_{top} ds + \oiint_{bottom} ds \right]$$

$$\therefore \Psi = D_z [A + A]$$

$$\Psi = 2 A D_z \longrightarrow (4)$$

From ①

$$2\cancel{A} D_z = \rho_s \cancel{A}$$

$$D_z = \frac{\rho_s}{2}$$

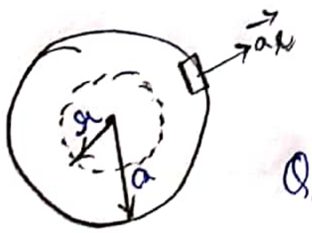
$$\vec{D} = \frac{\rho_s}{2} \cdot \vec{a_z}$$

$$\therefore \boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_s}{2\varepsilon_0} \cdot \vec{a_z}}$$

Electric field Intensity due to Uniformly Charged sphere:

consider a volume charge density 'ρ_v' enclosed in a gaussian surface of radius 'r', then its electric field intensity is given by the following cases:

case-i : $0 \le r < a$:



$$\psi = Q_{enclosed} \longrightarrow ①$$

$$Q_{enc} = \iiint \rho_v \cdot dv$$

$$= \rho_v \int_{r=0}^{r} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= 2\pi \rho_v \int_{r=0}^{r} \left[-\cos\theta\right]_0^{\pi}$$

$$\Rightarrow Q_{enc} = 4\pi \rho_v \int_{r=0}^{r} r^2 \, dr$$

$$Q_{enc} = \frac{4}{3}\pi \rho_v r^3 \longrightarrow (2)$$

&

$$\Psi = \iint \vec{D} \, \vec{ds}$$

Now, $\vec{D} = D_r \vec{a_r}$

$$\vec{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r}$$

sub^n in $\Psi = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} D_r \cdot r^2 \sin\theta \, d\theta \, d\phi$

$$= 2\pi D_r \cdot r^2 \int_{\theta=0}^{\pi} \sin\theta \, d\theta$$

$$\Psi = 4\pi D_r r^2 \longrightarrow (3)$$

$\therefore$ (2) = (3)

$$\frac{4\pi \rho_v r^3}{3} = 4\pi D_r \cdot r^2$$

$$D_r = \frac{\rho_v r}{3}$$

$$\vec{D} = \frac{\rho_v r}{3} \vec{a_r}$$

$$\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_v r}{3\varepsilon_0} \vec{a_r}}$$

case-ii: $r \geq a$

$$Q_{enc} = \iiint \rho_v \, dv$$

$$= \int_{r=0}^{a} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_v \, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$\Rightarrow \therefore Q_{enc} = \frac{4}{3}\pi \rho_v a^3$$

$$\Rightarrow \quad \Psi = \iint \vec{D} \cdot \vec{ds}$$

$$= 4\pi D_r r^2$$

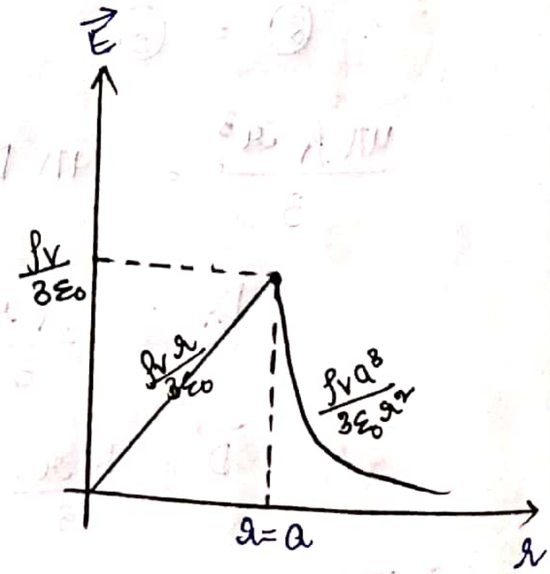
Now, $\Psi = Q_{enclosed}$

$$4\pi D_r \cdot r^2 = \frac{4}{3}\pi \rho_v a^3$$

$$D_r = \frac{\rho_v}{3} \cdot \frac{a^3}{r^2}$$

$$\vec{D} = \frac{\rho_v a^3}{3r^2} \cdot \vec{a_r}$$

$$\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_v a^3}{3\varepsilon_0 r^2} \cdot \vec{a_r}}$$



Q) If $\vec{D} = (2y^2+z)\vec{a_x} + 4xy\vec{a_y} + x\vec{a_z}$ c/m². Find

i) The volume charge density at (-1, 0, 3)

ii) The flux through the cube defined by $0 \le y \le 1$; $0 \le z \le 1$ & $0 \le x \le 1$

iii) The total charge enclosed by 'Q'

Sol: i) $\rho_v = ?$

From Gauss law; we have $\boxed{\rho_v = \nabla \cdot D}$

where $\nabla = \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$

$\therefore \nabla \cdot D = \frac{\partial}{\partial x}(2y^2+z) + \frac{\partial}{\partial y}(4xy) + \frac{\partial}{\partial z}(x)$

$= 0 + 4x + 0$

$\boxed{\nabla \cdot D = \rho_v = 4x \text{ c/m}^3}$ ρ_v @ (-1, 0, 3) is $\Rightarrow \boxed{\rho_v = -4 \text{ c/m}^3}$

ii) $\psi = \iint \vec{D} \cdot \vec{ds}$

$\psi = Q_{enc} = \iiint \rho_v \cdot dv$

$= \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} 4x \, dx\, dy\, dz$

$= 4\left[\frac{x^2}{2}\right]_0^1$

$= \cancel{4}^{2}\left[\frac{1}{\cancel{2}}\right]$

$\therefore \boxed{\psi = 2 \text{Coloumbs}}$

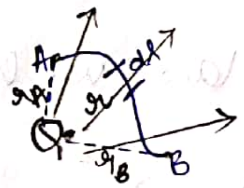
iii) $\therefore$ The total charge enclosed $\Rightarrow \boxed{Q_{enc} = 2C}$

ELECTRIC POTENTIAL:

It is defined as the work done in moving a point charge 'Q' from one point to another point in an electric field $\vec{E}$.

- Suppose 'Q_t' is the test charge moved in electric field of point charge 'Q' from point A to B. Then,

$$V = \frac{W}{Q_t}$$



- since, to move point charge from A to B, work is done externally,

$$\therefore W = -F \cdot l$$

- If 'dW' is the workdone to move charge at distance 'dl' then,

$$dW = -F \cdot dl$$

$$W = -\int_A^B F \cdot dl$$

- Electric force on test charge 'Q_t' is given by

$$\Rightarrow F = EQ_t$$

$$\Rightarrow W = -\int_A^B EQ_t \, dl$$

$$W = -Q_t \int_A^B \vec{E} \, dl$$

$$\frac{W}{Q_t} = -\int_A^B \vec{E} \cdot dl$$

$$V = -\int_A^B \vec{E} \cdot dl$$

we have electric field intensity due to point charge

is, $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot \vec{a_r}$

$$\therefore V = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2} \cdot \vec{a_r} \cdot dl$$

dl in spherical coordinates is:

$$dl = dr\,\vec{a_r} + r\,d\theta\,\vec{a_\theta} + r\sin\theta\,d\phi\,\vec{a_\phi}$$

$$V_{AB} = \int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$V_{AB} = \frac{-Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_A}^{r_B}$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0 r_B} - \frac{Q}{4\pi\varepsilon_0 r_A}$$

$$\therefore V_{AB} = V_B - V_A$$

If point 'A' is at infinity then $r_A \to \infty$

⇒ If potential at any particular point is measured with reference as point at infinity, then the measured potential is called "absolute potential".

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0 r_B} - 0$$

$$\boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0 r_B}}$$

If source charge 'Q' point is not at origin but somewhere with position vector 'r''

then $V = \dfrac{Q}{4\pi\varepsilon_0 |(r - r')|}$

if there are 'N' no. of source charges then

$$V = V_1 + V_2 + V_3 + \cdots + V_N$$

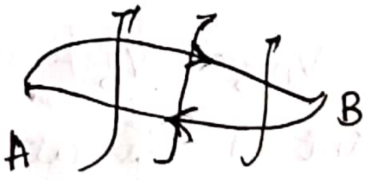
$$V = \frac{Q}{4\pi\varepsilon_0 |(r - r_1')|} + \frac{Q}{4\pi\varepsilon_0 |(r - r_2)|} + \cdots + \frac{Q}{4\pi\varepsilon_0 |(r - r_N)|}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q}{|r - r_k|}}$$

NOTE:
Electric potential is a scalar quantity
units: volts or J/coulomb.

Relation b/w E & V - Maxwell's Equation:

The potential difference b/w points A & B is independent of path taken.



$$\therefore V_{AB} = -V_{BA}$$

$$\Rightarrow V_{AB} + V_{BA} = 0$$

$$\therefore -\int_A^B \vec{E}\cdot dl - \int_B^A \vec{E}\cdot dl = 0$$

$$-\oint \vec{E}\cdot dl = 0$$

This shows that the integral of $\vec{E}$ along a closed path is '0'

→ On applying "Stokes" theorem:

$$\oint \vec{E} \cdot dl = \iint \nabla \times \vec{E} \cdot ds = 0$$

$$\Rightarrow \nabla \times \vec{E} = 0$$

. Any vector field that satisfy the above eqn is said to be conservative or irrotanial.

∴ Static electric field is a conservative field or irrotational.

→ The above eqn is also referred to as "Maxwell's Eqn" for static electric field.

$$\therefore V = -\int_A^B \vec{E} \cdot dl$$

$$dV = -\vec{E} \cdot dl$$

$$dV = -\left[E_x dx + E_y dy + E_z dz\right]$$

$$L.H.S = dV$$

$$= \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$= \left[\frac{\partial V}{\partial x} \vec{a_x} + \frac{\partial V}{\partial y} \vec{a_y} + \frac{\partial V}{\partial z} \vec{a_z}\right] \cdot \left(dx \vec{a_x} + dy \vec{a_y} + dz \vec{a_z}\right)$$

Equating LHS & RHS:

$$\therefore \nabla V \cdot dl = -\vec{E} \cdot dl$$

$$\boxed{\vec{E} = -\nabla \cdot V}$$

Energy in Static electric field:

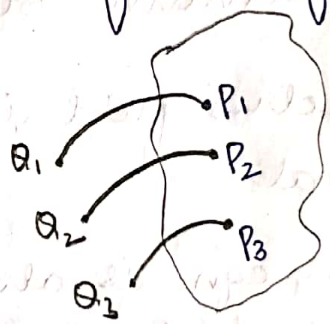
To determine the energy present in the assembly of charges, 1st determine the amount of workdone to assemble them if Q_1, Q_2 & Q_3 are charges placed in empty space at points P_1, P_2 & P_3 respectively. If W_1, W_2 & W_3 are the works done to transfer these charges from infinity then

$$W_1 = 0$$

$$W_2 = Q_2 V_{21}$$

$$W_3 = Q_3 (V_{31} + V_{32})$$

$$W = W_1 + W_2 + W_3 \longrightarrow (1)$$



→ If the charges are brought to the region in Reverse Order

$$W_3 = 0$$

$$W_2 = Q_2 V_{23}$$

$$W_1 = Q_1 (V_{13} + V_{12})$$

$$W = W_1 + W_2 + W_3 \longrightarrow (2)$$

(1) + (2)

$$\Rightarrow 2W = 0 + Q_2 V_{21} + Q_3 [V_{31} + V_{32}] + 0 + Q_2 V_{23} + Q_1 [V_{13} + V_{12}]$$

$$W = \frac{1}{2} \left[Q_1 (V_{13} + V_{12}) + Q_2 (V_{21} + V_{23}) + Q_3 (V_{31} + V_{32}) \right]$$

$$W = \frac{1}{2} [Q_1 V_1 + Q_2 V_2 + Q_3 V_3]$$

if 'N' no. of charges are moved from ∞ to region

$$W = \frac{1}{2}\left[Q_1V_1 + Q_2V_2 + \cdots\cdots + Q_NV_N\right]$$

$$W = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k$$

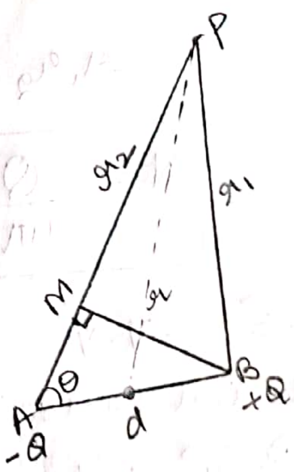
from the definition, energy present = Work done

$$\therefore \boxed{W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k}$$

Electric Dipole:

Electric dipole is an arrangement of the two charges equal in magnitude but of opposite polarity seperated by a small distance.

Consider '+Q' charge at pt. B
'-Q' charge at pt. A
seperated by a distance 'd'



potential due to charge at pt. B at a distant point 'P' is given by $V_B = \frac{Q}{4\pi\varepsilon_0 r_1}$

potential due to charge at pt. A at a distant pt 'P' is given by

$$V_A = \frac{-Q}{4\pi\varepsilon_0 r_2}$$

∴ Total potential due to charge @ pt 'P' is $V = V_A + V_B$

$$V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

from Δ^{le} AMB

$$\cos\theta = \frac{AM}{AB}$$

$$AM = AB\cos\theta$$

$$AM = r_2 - r_1 = d\cos\theta$$

if 'd' is small & 'p' is large distance then

$$r_1 \cong r_2 \cong r$$

$$r_1 r_2 = r^2$$

$$\therefore \boxed{V = \frac{Q}{4\pi\epsilon_0}\frac{d\cos\theta}{r^2}}$$

W.K.T $\vec{E} = -\nabla \cdot V$

In spherical coordinates

$$\nabla = \frac{\partial}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial}{\partial\theta}\cdot\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\vec{a_\phi}$$

$$\Rightarrow \vec{E} = -\frac{\partial V}{\partial r}\vec{a_r} - \frac{1}{r}\frac{\partial V}{\partial\theta}\vec{a_\theta} - \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\vec{a_\phi}$$

$$\Rightarrow \vec{E} = \frac{Qd}{4\pi\epsilon_0}\left[\frac{2\cos\theta}{r^3}\vec{a_r} + \frac{\sin\theta}{r^3}\vec{a_\theta}\right]$$

$$\therefore \boxed{\vec{E} = \frac{Qd}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\vec{a_r} + \sin\theta\,\vec{a_\theta}\right]}$$

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Current & Current density:

The current through a given area is the electric charge passing through the area per unit time

i.e $I = \frac{dQ}{dt}$

(OR)

Current is defined as the rate of flow of charge.

If the "dI" is the current flowing through a planar differential surface "ds", then current density is defined as "Current passing through unit normal surface area" i.e

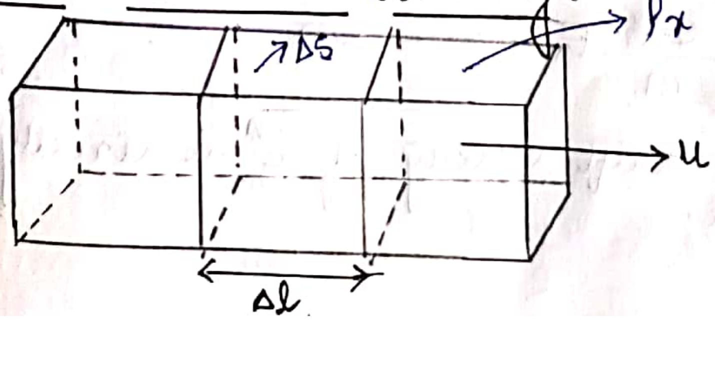
$$\vec{J_n} = \frac{I}{S}$$

$$J_n = \frac{dI}{ds} \Rightarrow I = \int_S dI = \int_S \vec{J_n}\, \vec{ds}$$

Depending upon the medium through which current pass or current produced, There are 3 types of current densities.

1) Convection current density
2) Conduction " "
3) Displacement " "

1) Convection Current density:



Total current flowing through surface 'S'

$$I = \int J \cdot ds$$

$$\Delta I = \frac{\Delta Q}{\Delta t}$$

$$\boxed{\begin{aligned} dQ &= \rho_v \, dv \\ &= \rho_v \, ds \, dl \\ &= dl \Rightarrow dy \\ \Rightarrow dQ &= \rho_v \, ds \, dy \end{aligned}}$$

$$\Rightarrow \Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \cdot \Delta s \frac{\Delta y}{\Delta t}$$

$$\boxed{\Delta I = \rho_v \, \Delta s \cdot u_y}$$

The current density at a given pt is the current through a unit normal area at that pt.

Conduction current density:

when an electric field 'E' is applied, the force on an electron with charge $-e$ is

$$F = -eE \longrightarrow ① \qquad \left(\because F = QE \text{ or } E = \frac{F}{Q}\right)$$

If an $\bar{e}^n$ which mass 'm' is moving in an electric field 'E' with an average drift velocity (u), acc to Newton's law; the average change in momentum of free $e^{\ominus}$ must match the applied force.

$$\frac{mu}{\tau} = -eE$$

(or)

$$u = -\frac{e\tau}{m} E \longrightarrow ②$$

from above eqn, the drift velocity of $\bar{e}^n$ is directly proportion to applied field.

If there are 'n' e^{-ns} per unit volume, the electric charge density given by

$$\rho_v = -ne \longrightarrow (3)$$

$$J = \rho_v \cdot u = \frac{\tau n e^2}{m} E = \sigma E \quad \text{or} \quad J = \sigma E \longrightarrow (4)$$

where $\sigma = \frac{\tau n e^2}{m}$ is the conductivity of the conductor

$$\therefore \boxed{J = \sigma E}$$

Continuity Equation:

The principle of charge conservation; the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the surface of the volume.

Current I_{out} coming out of closed surface is:

$$I_{out} = \oint J \cdot ds = -\frac{dQ_{in}}{dt} \longrightarrow (1)$$

where Q_{in} is the total charge enclosed by the surface.

& J - conduction current density.

→ By divergence theorem:

$$\oint J \cdot ds = \int \nabla \cdot J \, dv \longrightarrow (2)$$

but $$-\frac{dQ_{in}}{dt} = -\frac{d}{dt} \int \rho_v \, dv = -\int \frac{\partial \rho_v}{\partial t} \, dv \longrightarrow (3)$$

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subⁿ ② & ③ in ①

$$\Rightarrow \int \nabla \cdot J \, dv = -\int \frac{\partial \rho_v}{\partial t} dv$$

(or)

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

This is called "Continuity Equation"

Relaxation Time:

The Maxwell's first eqn is given by:

$$\boxed{\rho_v = \nabla \cdot D}$$

W.K.T $J = \sigma E$

$$\rho_v = \nabla \cdot \varepsilon E \quad [\because D = \varepsilon E]$$

$$\frac{\rho_v}{\varepsilon} = \nabla \cdot E$$

$$\frac{\sigma \rho_v}{\varepsilon} = \nabla \cdot \sigma E \Rightarrow \frac{\sigma \rho_v}{\varepsilon} = \nabla \cdot J$$

From continuity eqn

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} = -\frac{\sigma \rho_v}{\varepsilon} \Rightarrow \frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\varepsilon} \partial t$$

$$\therefore \boxed{\rho_v = \rho_{v_0} e^{-\frac{t}{T_r}}}$$

where $\boxed{T_r = \frac{\varepsilon}{\sigma}}$ is known as "Relaxation Time" (or) "Rearrangement Time"

- It is the time taken by the charge placed in interior of a material to drop to "$\frac{1}{e}$" of its initial value.

POISSONS & LAPLACE Equations:

Maxwell's 1^{st} eqn is given by : $\boxed{\rho_v = \nabla \cdot D}$

$$\rho_v = \nabla \cdot \varepsilon E \quad (\because D = \varepsilon E)$$

$$\rho_v = \varepsilon \nabla \cdot E$$

$$\rho_v = \varepsilon \nabla \cdot (-\nabla \cdot V)$$

$$\boxed{\frac{\rho_v}{\varepsilon} = -\nabla^2 \cdot V} \quad \text{(or)} \quad \boxed{\nabla^2 \cdot V = -\frac{\rho_v}{\varepsilon}}$$

which is known as "POISSON'S LAW".

* A special case of above eqn occurs when $\boxed{\rho_v = 0}$ (i.e for a free charge region)

$$\therefore \boxed{\nabla^2 \cdot V = 0}$$ → which is known as "LAPLACE Eqn".

• LAPLACE eqn in cartesian coordinate system is:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 .$$

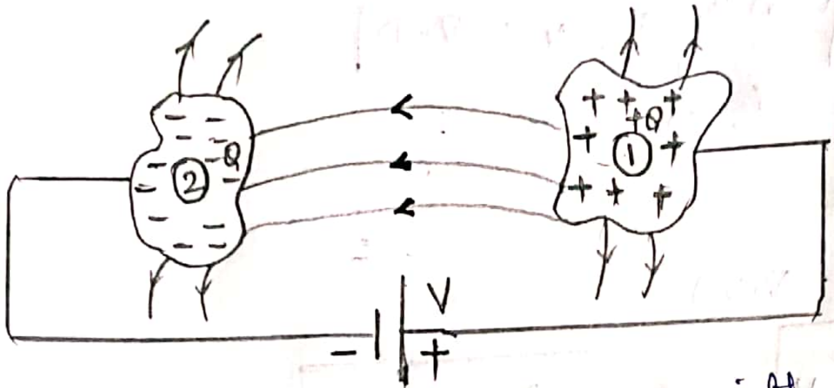
CAPACITOR & CAPACITANCE :-

Capacitor is an arrangement of two or more conductors carrying equal but opposite charge, hence flux lines start from conductor with positive charge & terminate at conductor with negative charge.

→ It stores energy in the form of Electric field denoted by 'C'

& measured in "Farads" or "Coloumbs/volts".

procedure to Calculate Capacitance:



consider two conductors with +ve charge on conductor-1 & -ve charge on conductor 2

- The potential diff b/w two conductors is:

$$V = V_1 - V_2$$

$$V = -\int_2^1 E \cdot dl$$

The capacitance 'C' of the capacitor is defined as the ratio of magnitude of charge on one of the plates to the potential diff b/w them

i.e $C = \frac{Q}{V}$

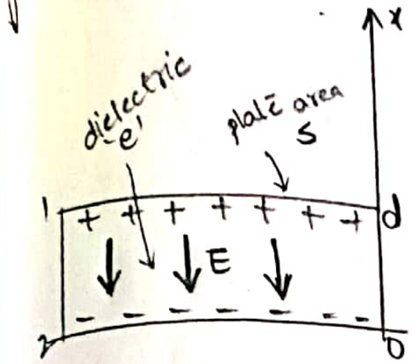
$$\boxed{C = \frac{\varepsilon \oint E \cdot ds}{\int E \cdot dl}}$$

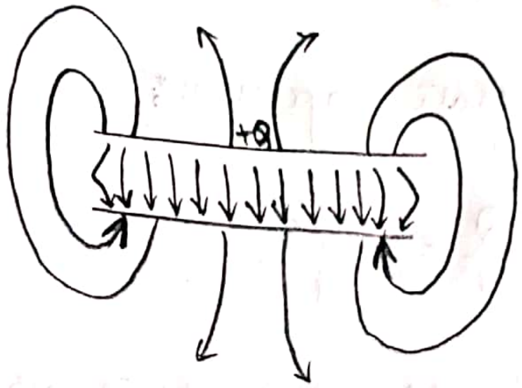
Capacitance is calculated with two approaches:

① Assume 'Q' & calculate 'E' using Coulomb's or Gauss law & then calculate 'V' using $\boxed{V = -\int \vec{E} \cdot dl}$

② Assume 'V' & calculate 'Q' in terms of 'V' using laplace eqn: $\boxed{\nabla^2 V = 0}$

parallel plate Capacitor:





Assuming +Q & −Q charges on plates 1 & 2 respectively such that

$$\rho_s = \frac{Q}{S} \longrightarrow (1)$$

The electric field intensity $\vec{E}$ due to 1^{st} plate in −ve x-direction is

$$E_1 = \frac{-\rho_s}{2\varepsilon_0}\vec{a_x}$$

$||^{ly}$ E_2 in +ve x-direction $E_2 = \frac{-\rho_s}{2\varepsilon_0} \cdot \vec{a_x}$

∴ Total electric field Intensity $\vec{E}$ due to 1 & 2 is

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E} = \frac{-\rho_s}{2\varepsilon_0}\vec{a_x} - \frac{\rho_s}{2\varepsilon_0}\vec{a_x}$$

$$\vec{E} = \frac{-\rho_s}{\varepsilon_0}\vec{a_x}$$

$$\therefore \vec{E} = \frac{-Q}{\varepsilon_0 S}\vec{a_x} \quad \left(\text{From } (1)\ \rho_s = \frac{Q}{S}\right)$$

$$V = -\int_2^1 \vec{E}\cdot dl = \int_0^d \left(-\frac{Q}{\varepsilon_0 S}\vec{a_x}\right) dx\,\vec{a_x}$$

$$V = \frac{Qd}{\varepsilon_0 S}$$

For a ll^el plate capacitor:

$$C = \frac{Q}{V} = \frac{\varepsilon_0 S}{d}$$

If dielectric b/w the plates is not free space then capacitance is given by:

$$C = \frac{\varepsilon S}{d}$$