Islington College

MA4001NI

Logic and Problem Solving

12-hours assessment (25% Weighted)

Submitted By:

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1. If you are a flower lover, then you work in the garden. If you don't like roses, then you don't work in the garden. Therefore, if you are flower lover then you like roses.

Solution:

Let,

p = you are a flower lover.

q = you work in the garden.

r = you like roses.

Given,

$p \rightarrow q, \neg r \rightarrow \neg q \vdash p \rightarrow r$

Let,

$X = [(p \rightarrow q) \wedge (\neg r \rightarrow \neg q)] \rightarrow (p \rightarrow r)$

Truth Table:

p	q	r	$p \rightarrow q$	$\neg r$	$\neg q$	$\neg r \rightarrow \neg q$	$(p \rightarrow q) \wedge (\neg r \rightarrow \neg q)$	$p \rightarrow r$	X
T	T	T	T	F	F	T	T	T	T
T	T	F	T	T	F	F	F	F	T
T	F	T	F	F	T	T	F	T	T
T	F	F	F	T	T	T	F	F	T
F	T	T	T	F	F	T	T	T	T
F	T	F	T	T	F	F	F	T	T
F	F	T	T	F	T	T	T	T	T
F	F	F	T	T	T	T	T	T	T

Here, from the truth table it is proved that $X = [(p \rightarrow q) \wedge (\neg r \rightarrow \neg q)] \rightarrow (p \rightarrow r)$ is a tautology. Hence, the given argument is valid.

2.

De Morgan's Law:

$\neg(p \lor q) \equiv \neg p \land \neg q$

Truth Table:

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

This truth table verifies the De-Morgan's Law. Here, the column number 4 and 7 are identical. Hence, they are equivalent.

3.

a. Given,

$(p \rightarrow q) \leftrightarrow (\neg p \lor q)$

Truth Table:

p	q	$p \rightarrow q$	$\neg p$	$\neg p \lor q$	$(p \rightarrow q) \leftrightarrow (\neg p \lor q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Here, every value in the last column is true.

Hence, the given propositions are tautologies.

b. Given expressions,

i. $\neg(p \lor (\neg p \land q))$

ii. $(\neg p \land \neg q)$

To prove,

$$\neg(p \lor (\neg p \land q)) \equiv (\neg p \land \neg q)$$

L.H.S	$= \neg(p \lor (\neg p \land q))$	
	$= \neg p \land \neg(\neg p \land q)$	De Morgan's Law
	$= \neg p \land (\neg(\neg p) \lor \neg q)$	De Morgan's Law
	$= \neg p \land (p \lor \neg q)$	Double Negation Law
	$= (\neg p \land p) \lor (\neg p \land \neg q)$	Distributive Law
	$= (p \land \neg p) \lor (\neg p \land \neg q)$	Commutative Law
	$= \mathrm{False} \lor (\neg p \land \neg q)$	Complement Law
	$= (\neg p \land \neg q)$	Identity Law
	= R.H.S, proved	

Here, after simplification the expressions are identical.

Hence, the given propositions are logically equivalent.

4. Provided the output function:

$$X = A.B.C + A.B.\overline{C} + A.\overline{B}.C + \overline{A}.B.C$$

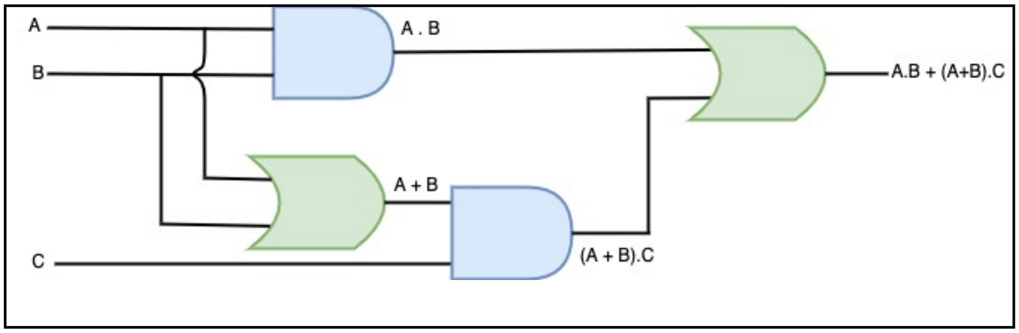
a. Using the laws, simplify the expression as much as possible.

Solution:

$X = A.B.C + A.B.\overline{C} + A.\overline{B}.C + \overline{A}.B.C$	
$= A.B(C + \overline{C}) + A.\overline{B}.C + \overline{A}.B.C$	Distributive Law
$= (A.B).1 + A.\overline{B}.C + \overline{A}.B.C$	Complement Law
$= A.B + A.\overline{B}.C + \overline{A}.B.C$	Identity Law
$= A(B + \overline{B}C).(\overline{A}.B.C)$	Distributive Law
$= A\big((B + \overline{B}).(B + C)\big) + \overline{A}.B.C$	Distributive Law
$= A\big(1.(B + C)\big) + \overline{A}.B.C$	Complement Law
$= A(B + C) + \overline{A}.B.C$	Identity Law
$= A.B + A.C + \overline{A}.B.C$	Distributive Law
$= A.B + C(A + \overline{A}.B)$	Distributive Law
$= A.B + C\big((A + \overline{A}).(A + B)\big)$	Distributive Law
$= A.B + C\big(1 . (A + B)\big)$	Complement Law
$= A.B + C(A + B)$	Identity law

b. Construct the logic circuit of the simplified expression.

Logic Circuit



c. Truth Table

A	B	C	$A.B$	$A+B$	$(A+B).C$	$A.B+(A+B).C$
1	1	1	1	1	1	1
1	1	0	1	1	0	1
1	0	1	0	1	1	1
1	0	0	0	1	0	0
0	1	1	0	1	1	1
0	1	0	0	1	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0

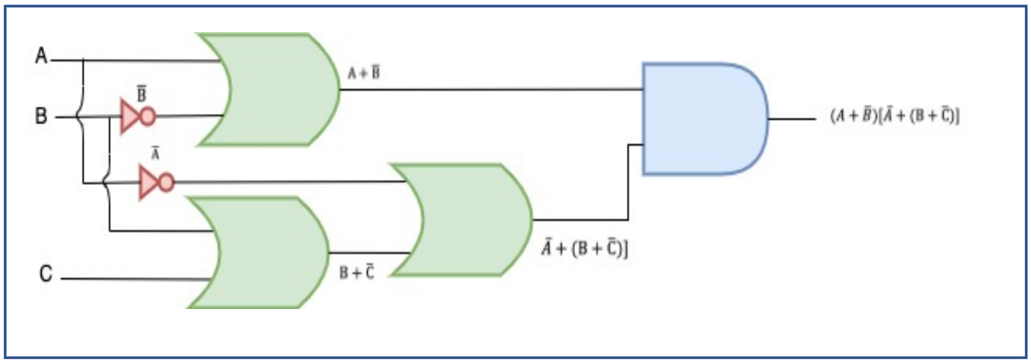
5. Build a digital circuit that produces the output $(\mathrm{A}+\overline{\mathrm{B}})[\overline{\mathrm{A}}+(\mathrm{B}+\ \overline{\mathrm{C}})]$ when given input bits A, B, and C. Also, construct the truth table.

Solution:

Given,

$(\mathrm{A}+\overline{\mathrm{B}})[\overline{\mathrm{A}}+(\mathrm{B}+\ \overline{\mathrm{C}})]$

i. Logic Circuit



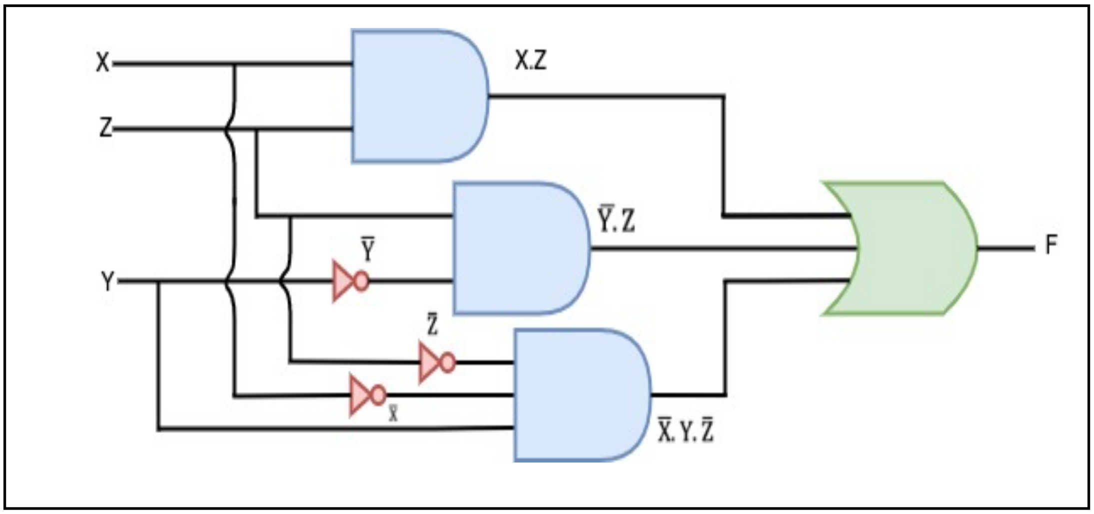
ii. Truth Table

A	B	C	$\overline{\mathrm{A}}$	$\overline{\mathrm{B}}$	$\overline{\mathrm{C}}$	$\mathrm{A}+\overline{\mathrm{B}}$	B +$\overline{\mathrm{C}}$	$\overline{\mathrm{A}}+(\mathrm{B}+\overline{\mathrm{C}})$	$(\mathrm{A}+\overline{\mathrm{B}})[\overline{\mathrm{A}}+(\mathrm{B}+\overline{\mathrm{C}})]$
1	1	1	0	0	0	1	1	1	1
1	1	0	0	0	1	1	1	1	1
1	0	1	0	1	0	1	0	0	0
1	0	0	0	1	1	1	1	1	1
0	1	1	1	0	0	0	1	1	0
0	1	0	1	0	1	0	1	1	0
0	0	1	1	1	0	1	0	1	1
0	0	0	1	1	1	1	1	1	1

6.

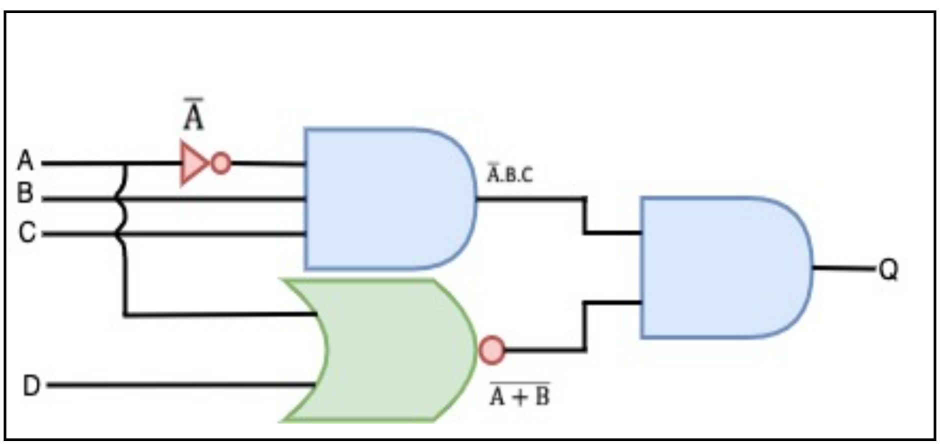
i. $F = XZ + \overline{Y}Z + \overline{X}Y\overline{Z}$

Logic Circuit



ii. $Q = (\overline{A}BC)(\overline{A + D})$

Logic Circuit



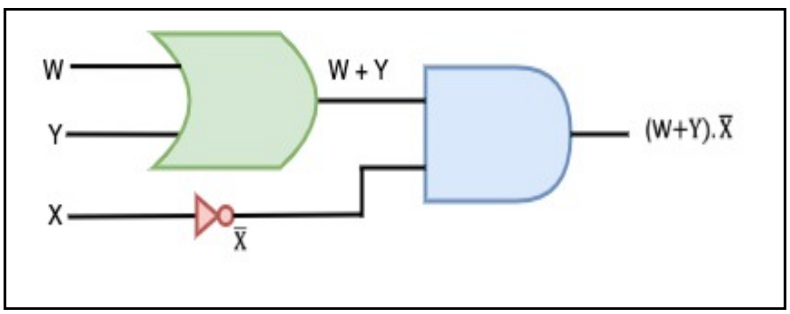
7. A system uses 3 switches W, X and Y ; a combination of switches determines whether an alarm, Z, sounds. If switch W or switch Y are in the ON position and switch X is in the OFF position then a signal to sound an alarm, Z, is produced. Design the logic of the circuit using the appropriate logic gates and construct the truth table to show all possible output.

Solution:

From the question,

$(W + Y).\overline{X}$

a. Logic Circuit



b. Truth Table

W	Y	X	$W + Y$	$\overline{X}$	$(W + Y).\overline{X}$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	0	0
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	0

8.

Given,

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

A = $\{x : x \in U, x \geq 8\}$

= {8, 9, 10, 11, 12, 13, 14, 15}

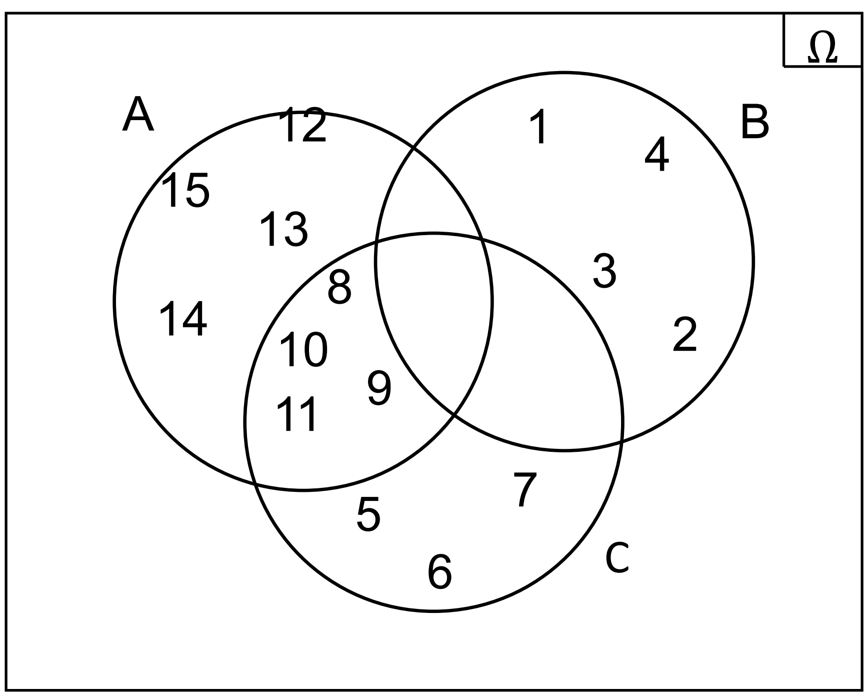
B = $\{x : x \in U, x \leq 4\}$

= {1, 2, 3, 4}

C = $\{x : x \in U, 4 < x < 12\}$

={5, 6, 7, 8, 9, 10, 11}

a. Venn-Diagram



b. $A \cap C$

= {8, 9, 10, 11, 12, 13, 14, 15} ∩ {5, 6, 7, 8, 9, 10, 11}

= {8, 9, 10, 11}

c. $B \cup C$

= {1, 2, 3, 4} ∪ {5, 6, 7, 8, 9, 10, 11}

= {1, 2, 3, 4, 5, 6, 7, 8, 9,10 ,11}

d. $(A \cup C) - B$

= {{8, 9, 10, 11, 12, 13, 14, 15} ∪ {5, 6, 7, 8, 9, 10, 11}} – B

= {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} – {1, 2, 3, 4}

= {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

e. $(A \cup B) - (A \cap B)$

= {{8, 9, 10, 11, 12, 13, 14, 15} ∪ {1, 2, 3, 4}} – {{8, 9, 10, 11, 12, 13, 14, 15} ∩ $\{1, 2, 3, 4\}$}

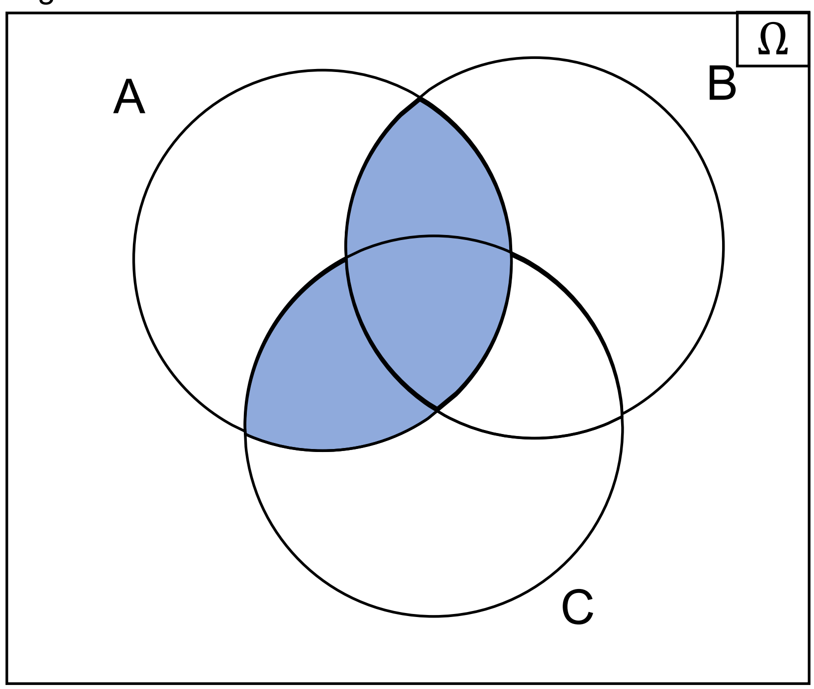
= {1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15 } – {}

= {1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15 }

9.

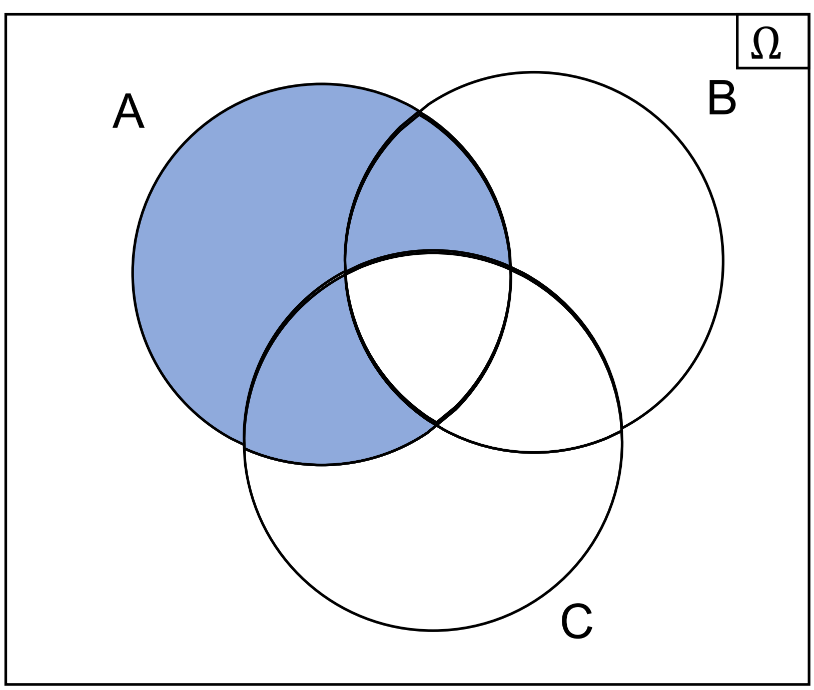
a) $(A \cap B) \cup (A \cap C)$

Venn-Diagram



b) $(A \cap B') \cup (A \cap C')$

Venn-Diagram



10. In a group of students 18 read Books, 19 read Magazines and 16 read Novels. 6 read Books only, 9 read Magazines only, 5 read Books and Magazines only and 2 read Magazines and Novels only.

Solution:

Let , B, M and N represent the number of students who read Books, Magazines and novels, respectively. Then,

Given,

n(B) = 18

n(M) = 19

n(N) = 16

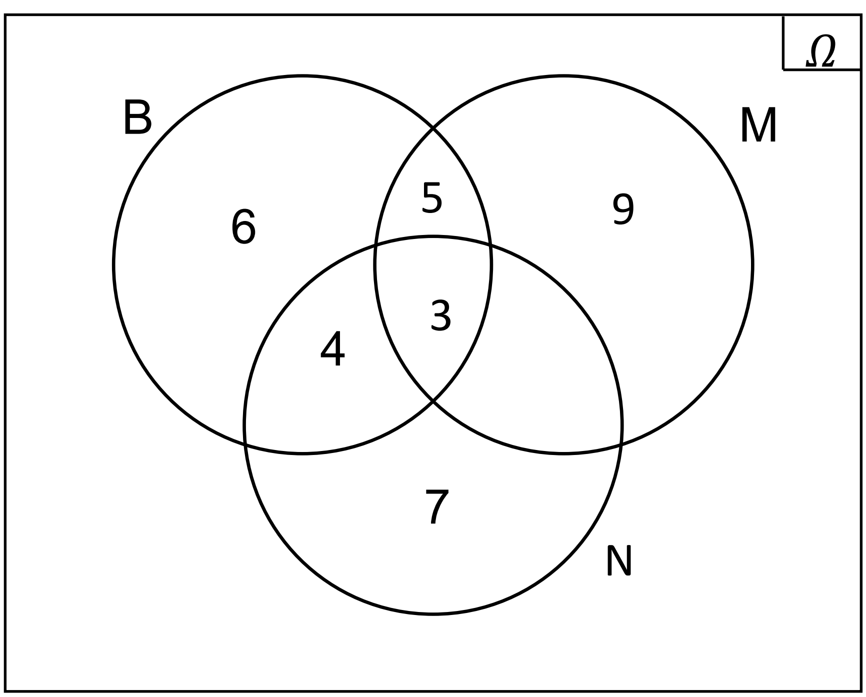
$n_o(B) = 6$

$n_o(M) = 9$

$n_o(B \cap M) = 5$

$n_o(M \cap N) = 2$

a. Venn-Diagram



b. How many students read all three ?

= Number of students who reads all three books,

$n(B \cap M \cap N) = 3$

c. How many read Books and Novels only?

= Number of students who reads Books and Novels only,

$n_o(B \cap N) = 4$

d. How many read Novels only?

= Number of students who reads Novel only,

$n_o(N) = 7$

e. How many students are there all together?

= There are all together 36 students in the group.