

DSLsofMath 2017: Assignment 1

In this assignment the focus is on the following three learning outcomes:

- organize areas of mathematics in DSL terms
- develop adequate notation for mathematical concepts
- discuss and compare different software implementations of mathematical concepts

DSLs, sets and von Neumann

In this assignment you will build up a domain specific language (a DSL) for finite sets. The domain you should model is pure set theory where all members are sets.

Define a datatype SET v for the abstract syntax of set expressions with variables of type v and a datatype PRED for predicates over pure set expressions.

Part 1. SET should have constructors for

- the Empty set
- the one-element set constructor Singleton
- Union, and Intersection
 - you can also try Powerset
- set-valued variables ($\text{Var} :: v \rightarrow \text{SET } v$)

PRED should have constructors for

- the two predicates Elem, Subset
- the logical connectives And, Or, Implies, Not

Part 2. A possible semantic domain for pure sets is

```
newtype Set = S [Set]
```

Implement the evaluation functions

```
eval  :: Eq v => Env v Set -> SET v -> Set
check :: Eq v => Env v Set -> PRED v -> Bool

type Env var dom = [(var , dom)]
```

Note that the type parameter v to SET is for the type of variables in the set expressions, not the type of elements of the sets. (You can think of pure set theory as “untyped” or “untyped”.)

Part 3. The von Neumann encoding of natural numbers as sets is defined recursively as

```
vonNeumann 0      = Empty
vonNeumann (n + 1) = Union (vonNeumann n)
                      (Singleton (vonNeumann n))
```

Implement vonNeumann and explore, explain and implement the following “pseudocode” claims as functions in Haskell:

```
+ claim1 n1 n2 = "(n1 <= n2) implies (n1 n2)"
+ claim2 n     = "n = {0, 1, ..., n - 1}"
```

You need to insert some embeddings and types and you should use the eval and check functions. (For debugging it is useful to implement a show function for Set which uses numerals to show the von Neumann naturals.)

Admin:

- *Submission:* Assignments are to be submitted via Fire:
<https://dsls-lp3-17.frs.cse.chalmers.se/login>
- *Deadline:* Tuesday, 2017-01-31, 23:59.
- *Grading:* Discussions with each of the teams during the exercises session of Friday, 2017-02-03.

Note: The examination will be in English.