# Domain Specific Languages of Mathematics (DAT325)

Re-Exam 2016–08–23, 14:00–18:00

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**Results** Announced within at most 19 days (by Monday 2016-09-16)

**Re-Exam check** Thursday and Friday 2016-08-25 and 26. Both at 12.30-12.55 in EDIT 5468.

**Aids** One textbook of your choice (e.g., Adams and Essex, or Rudin). No printouts, no lecture notes, no notebooks, etc.

**Grades** 3: 40p, 4: 60p, 5: 80p, max: 100p

Remember to write legibly. Good luck!

1. [30pts] An *abelian monoid* is a set `M` together with a constant (nullary operation) `0` $\in$ `M` and a binary operation $\oplus$ : `M` $\to$ `M` $\to$ `M` such that:

   - `0` is a unit of $\oplus$

     $\forall$ x $\in$ M  x $\oplus$ 0 = x  **and** 0 $\oplus$ x = x

   - $\oplus$ is associative

     $\forall$ x, y, z $\in$ M  x $\oplus$ (y $\oplus$ z) = (x $\oplus$ y) $\oplus$ z

   - $\oplus$ is commutative

     $\forall$ x, y $\in$ M  x $\oplus$ y = y $\oplus$ x

   i. Define a type class `AbMonoid` that corresponds to the abelian monoid structure.

   ii. Define a datatype `AbMonoidExp` for the language of abelian monoid expressions and define an `AbMonoid` instance for it. (These are expressions formed from applying the monoid operations to the appropriate number of arguments, e.g., all the left hand sides and right hand sides of the above equations.)

   iii. Find one other instance of the `AbMonoid` class and give an example which is *not* an instance of `AbMonoid`.

   iv. Define a general evaluator for `AbMonoidExp` expressions on the basis of an assignment function.

   v. Specialise the evaluator to the `AbMonoid` instance defined at point iii. Take three `AbMonoidExp` expressions, give the appropriate assignments and compute the results of evaluating the three expressions.

   Each question carries 6pts.

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2. [20pts] In the simplest case of probability theory, we start with a *finite*, non-empty set Ω of *elementary events*. *Events* are subsets of Ω, i.e. elements of the powerset of Ω, (that is, $\mathcal{P}$ Ω). A *probability function* `P` associates to each event a real number between 0 and 1, such that

   i. `P` ∅ `= 0`, `P` Ω `= 1`

   ii. If events `A` and `B` are disjoint (i.e., `A` ∩ `B` = ∅), then: `P A + P B = P (A` ∪ `B)`.

   Conditional probabilities are defined as follows (*Elementary Probability 2nd Edition*, Stirzaker 2003):

   > Let `A` and `B` be events with `P B > 0`. Given that `B` occurs, the *conditional probability* that `A` occurs is denoted by `P(A | B)` and defined by

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P(A | B) = P(A ∩ B) / P B
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a)[10pts] What are the types of the elements involved in the definition of conditional probability? (`P`, ∩, `/`, `|`)

b)[10pts] In the 1933 monograph that set the foundations of contemporary probability theory, Kolmogorov used, instead of `P(A | B)`, the expression $P_A\, B$. Type this expression. Which notation do you prefer (provide a *brief* explanation).

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3. [25pts] Consider the following differential equation:

$$f''\, t - 5 * f'\, t + 6 * f\, t = e^t, \quad f\, 0 = 1, \quad f'\, 0 = 4$$

   i. [5pts] Write an expression to solve the equation assuming that `f` can be expressed by a power series `fs`, that is, use `deriv` and `integ` to compute `fs`.

   ii. [20pts] Solve the equation using the Laplace transform. You should need only one formula (and linearity):

$$\mathcal{L}(\lambda t.e^{\alpha * t})\, s = 1/(s - \alpha)$$

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4. [25pts] Consider the classical definition of continuity:

   > *Definition:* Let $X \subseteq \mathbb{R}$, and $c \in X$. A function $f : X \to \mathbb{R}$ is *continuous at* $c$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that, for every $x$ in the domain of $f$, if $|x - c| < \delta$, then $|f x - f c| < \varepsilon$.

   i. [5pts] Write the definition formally, using logical connectives and quantifiers.

   ii. [10pts] Introduce functions and types to simplify the definition.

   iii. [10pts] Prove the following proposition: If `f` and `g` are continuous at `c`, `f + g` is continuous at `c`.