Program Reasoning

14. Constraint-based Search

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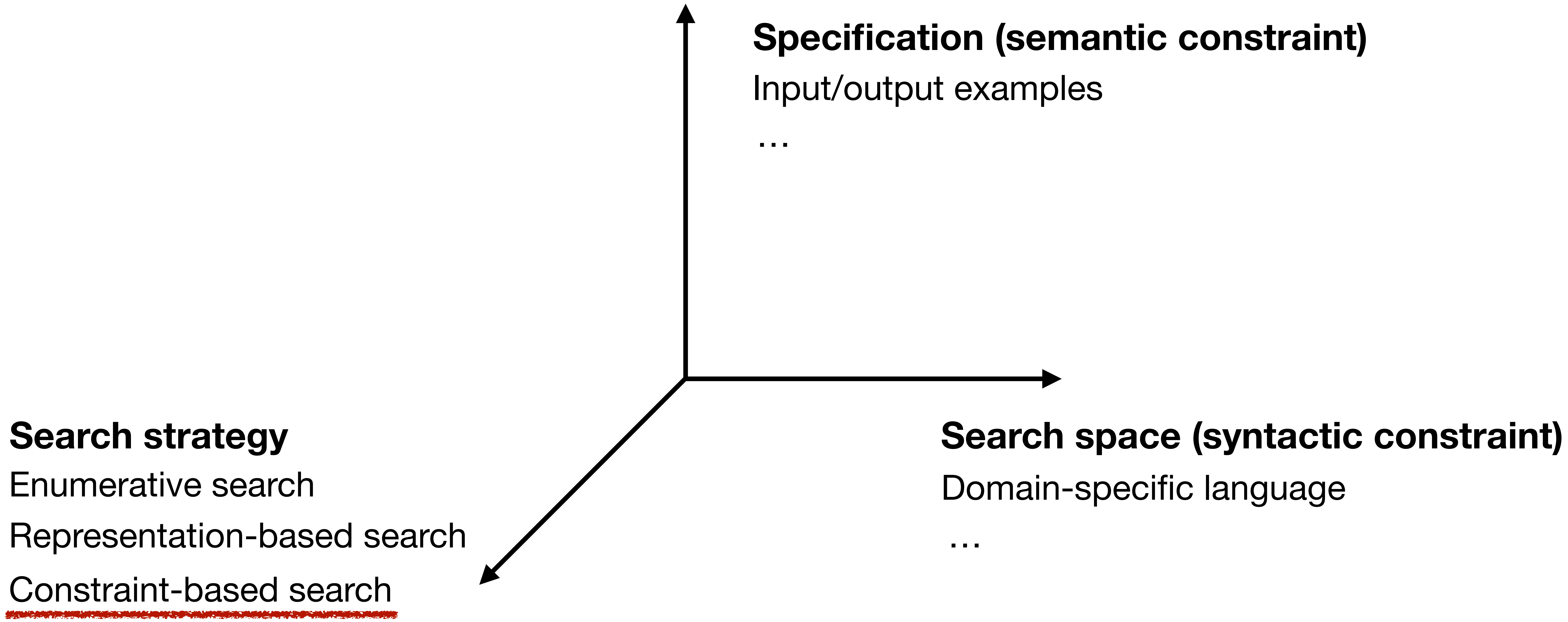
Dimensions in Program Synthesis



Application: Programming with APIs

- Synthesizing a program using a given set of APIs (e.g., java.awt.geom libraries)

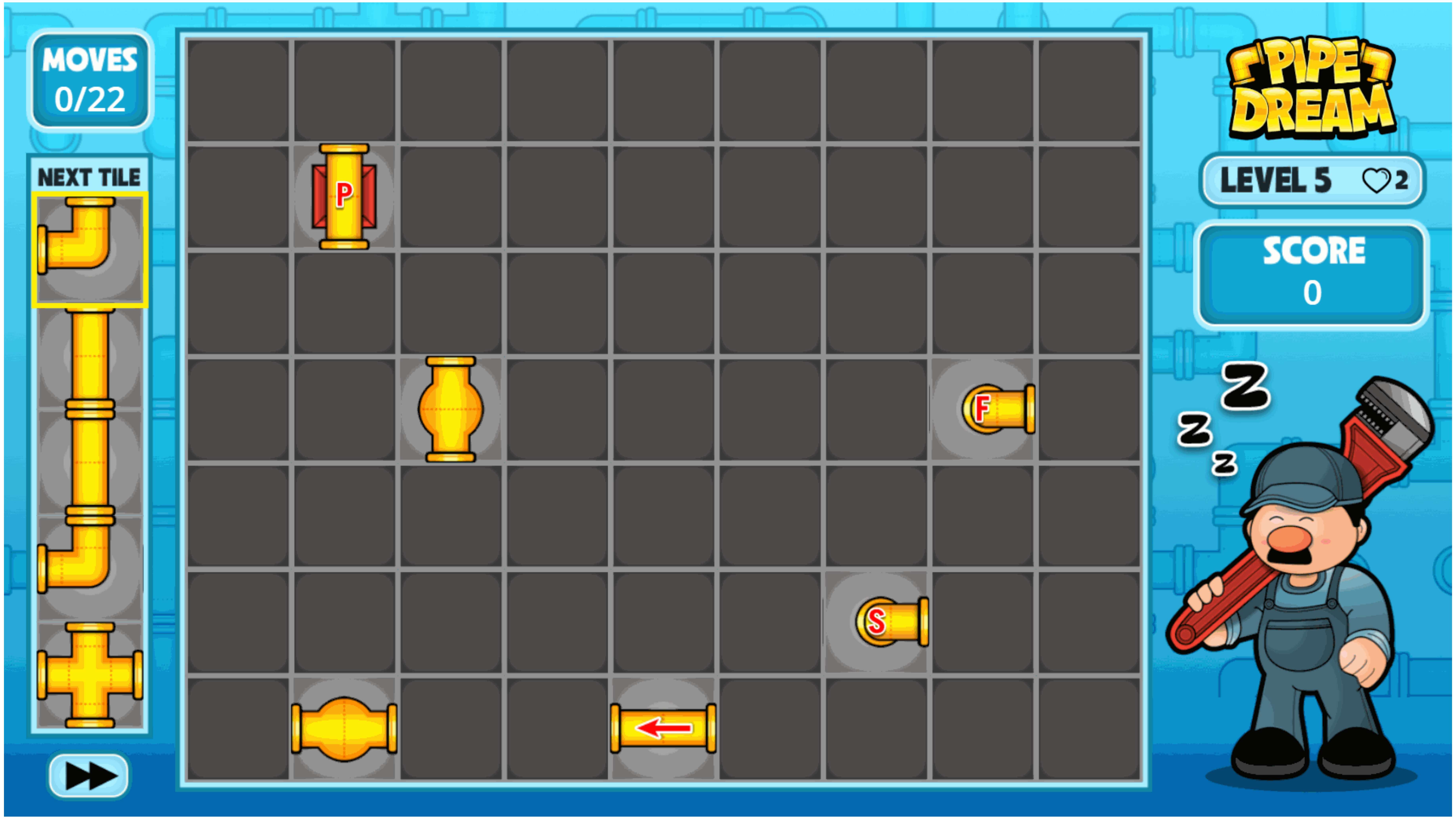
```
Area rotate(Area obj, Point2D pt, double angle) {
    AffineTransform at = new AffineTransform();
    double x = pt.getX();
    double y = pt.getY();
    at.setToRotation(angle, x, y);
    Area obj2 = obj.createTransformedArea(at);
    return obj2;
}
```

Application: Bit-twiddling

- Synthesizing a program using a given set of bitwise operators

```
// Round up to the next
// highest power of 2
fun f(x):
  o1 = bvsub(x, 1)
  o2 = bvshr(o1, 1)
  o3 = bvor(o1, o2)
  o4 = bvshr(o3, 2)
  o5 = bvor(o3, o4)
  o6 = bvshr(o5, 4)
  o7 = bvor(o5, o6)
  o8 = bvshr(o7, o8)
  o9 = bvor(o7, o8)
  o10 = bvshr(o9, 16)
  o11 = bvor(o9, o10)
  o12 = bvadd(o11, 1)
  return o12
```

Pipe Dream



Constraint-based Search

- Idea: encode the synthesis problem as a constraint-solving problem (SAT/SMT)
  - Constraints: syntactic and semantics constraints of the program
  - Program search: proof search by the solver
  - Solution program: directly derived from the solution of the SAT/SMT problem
- Target programs
  - Loop-free programs
  - Composition of functions that can be encoded as SMT formula

Target Program

- Given a finite multiset of components $\{\text{component}_1, \ldots, \text{component}_n\}$,

```
fun synthesize_program(inputs, …):
    temp_i = component_i(param_a, …)
    temp_j = component_j(param_b, …)
    …
    temp_k = component_k(param_k, …)
    return temp_k
```

- E.g., synthesize $f : A \times A \rightarrow C$ using components $\{f, f, g\}$ where $f : A \rightarrow B$ and $g : B \times B \rightarrow C$

```
fun f(x, y):
    tmp0 = f(x)
    tmp1 = g(tmp0, tmp0)
    return tmp1
```

✔

```
fun f(x, y):
    tmp0 = f(x)
    tmp1 = f(y)
    tmp2 = g(tmp0, tmp1)
    return tmp2
```

✔

```
fun f(x, y):
    tmp0 = f(x)
    tmp1 = g(tmp0, tmp0)
    tmp2 = g(tmp0, tmp0)
    return tmp2
```

✘

Example

- Bitvector manipulation program $f : BitVec \rightarrow BitVec$
- Specification: $f(01100) = 01000 \land f(10001) = 10000$
  - Intention: replace the rightmost 1 with 0
- Components:
  - $f_1(a) = a - 1$
  - $f_2(a, b) = a \,\&\, b$
- Solution: $f(x) = x \,\&\, (x - 1)$

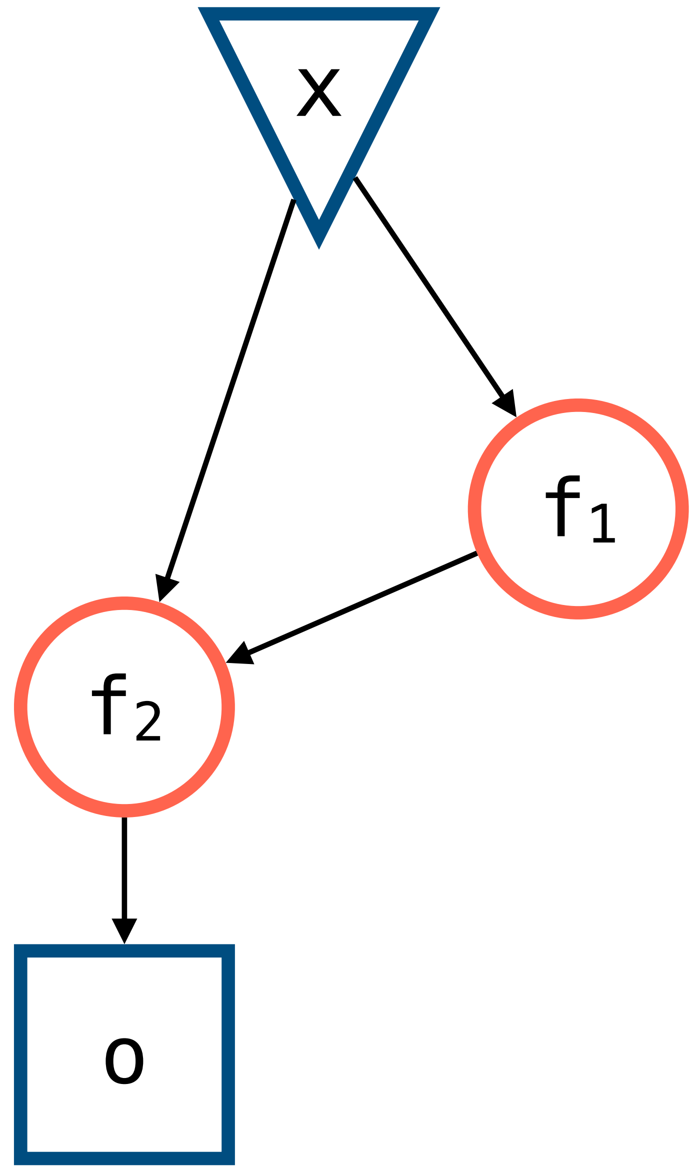
Program as DAG

Component library:

```
f1(a) = a - 1
f2(a, b) = a & b
```

Solution program:

```
fun f(x):
  o1 = f1(x)
  o2 = f2(x, o1)
  return o2
```



How to represent the correct edges between given nodes?

Program Location as Number

Component library:

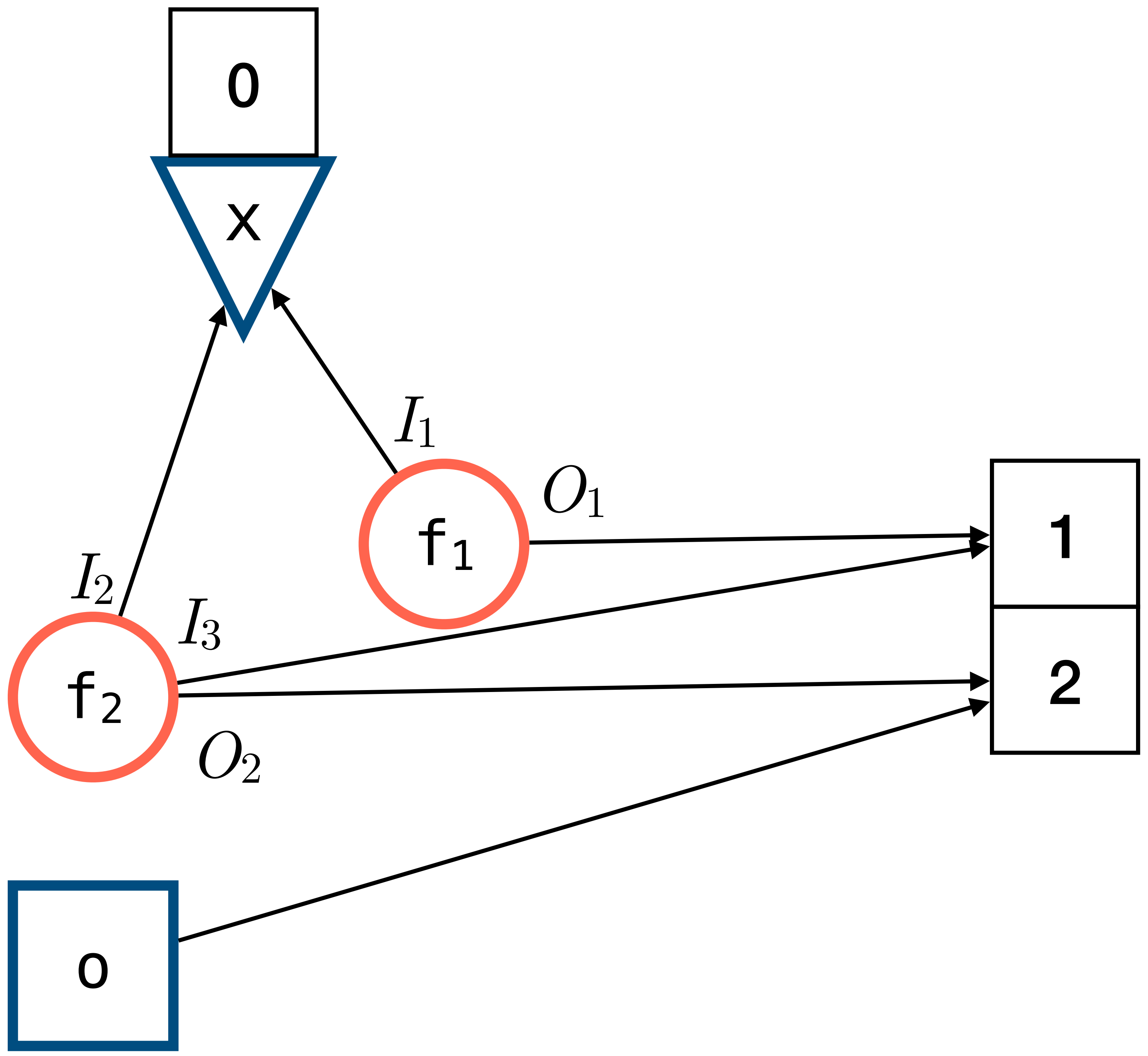
```
f₁(a) = a - 1
f₂(a, b) = a & b
```

Solution program:

```
0: fun f(x):
1:   o₁ = f₁(x)
2:   o₂ = f₂(x, o₁)
     return o₂
```

The solution program corresponds to the following assignment:

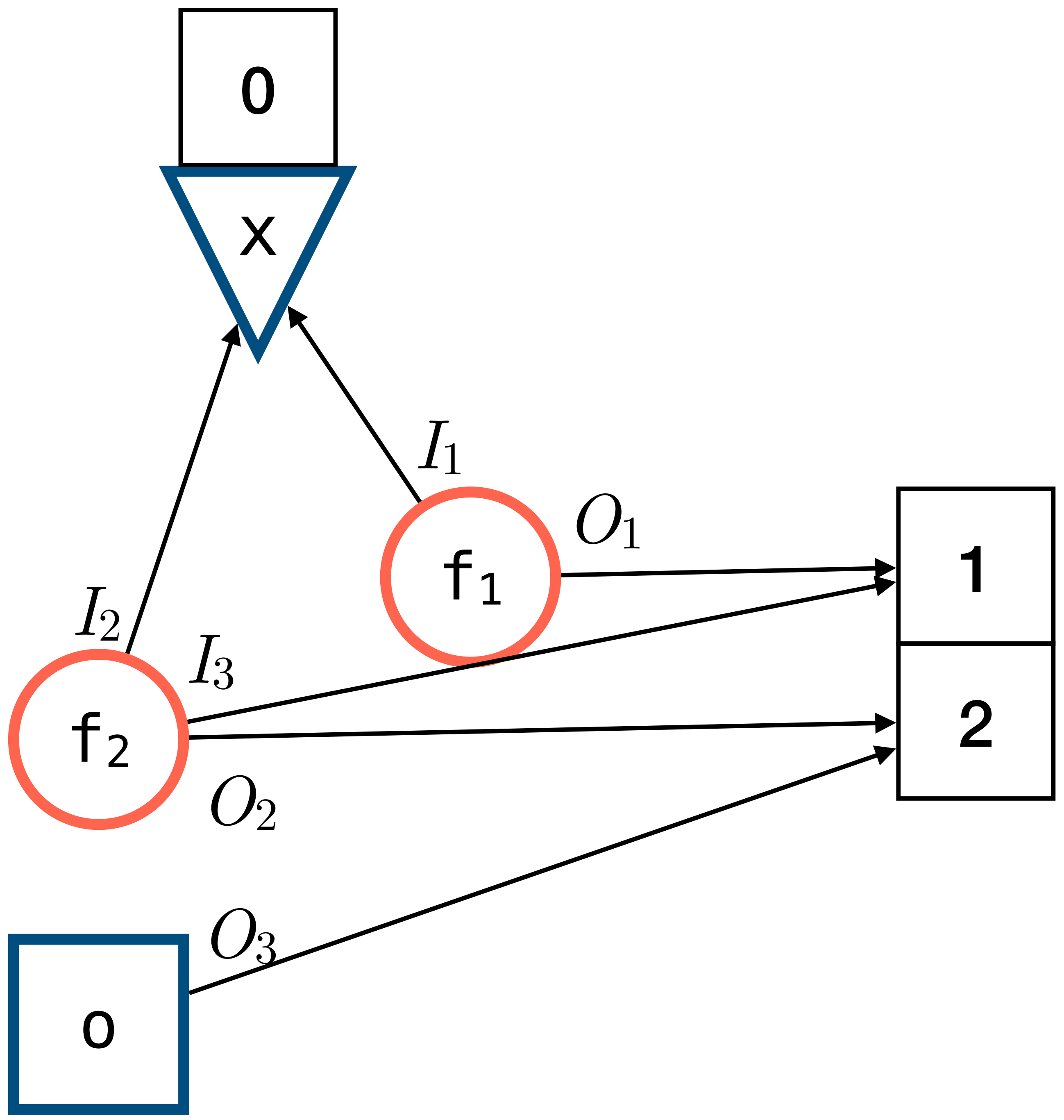
$$l_{I_1} = 0 \qquad l_{O_1} = 1$$
$$l_{I_2} = 0 \qquad l_{O_2} = 2$$
$$l_{I_3} = 1$$



How to make an SMT solver find this assignment?

SMT Encoding (1): Variables

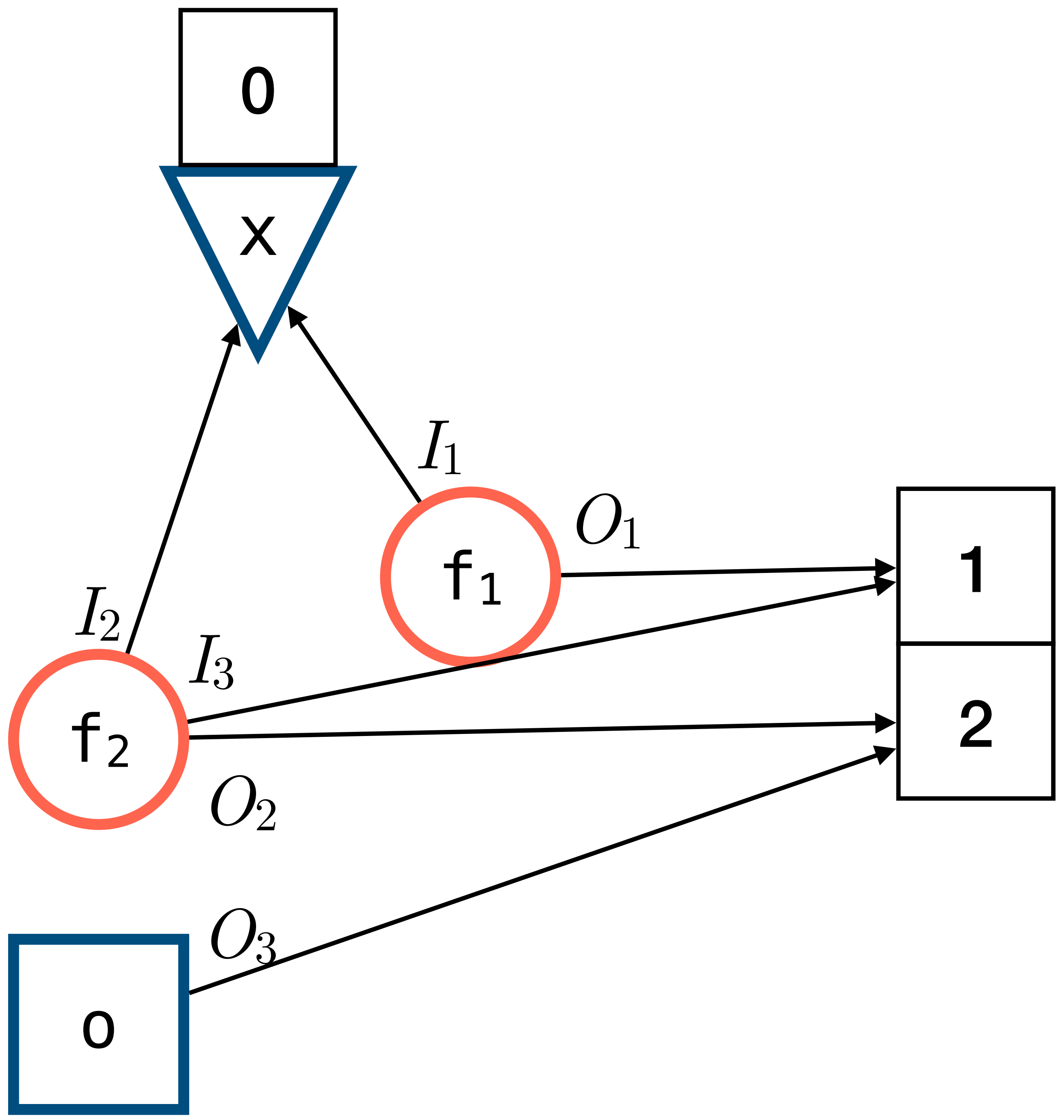
- Input variables: represent the inputs of each component
  - $P = \{I_1, I_2, I_3\}$
- Output variables: represent the outputs of each component
  - $R = \{O_1, O_2\}$
- Location variables: represent the location of each input and output
  - $L = \{l_x \mid x \in P \cup R\}$



SMT Encoding (2): Well-formedness

- A program is well-formed if and only if

$$\psi_{\text{wfp}} = \underbrace{\bigwedge_{x \in P} (0 \le l_x \le 2)}_{\text{Locations of the inputs of each component}} \wedge \underbrace{\bigwedge_{x \in R} (1 \le l_x \le 2)}_{\text{Locations of the outputs of each component}} \wedge \psi_{\text{cons}} \wedge \psi_{\text{acyc}}$$



Consistency constraint

$$\underbrace{\psi_{\text{cons}} = l_{O_1} \neq l_{O_2}}_{\text{At most one component per each line}}$$

Acyclicity constraint

$$\underbrace{\psi_{\text{acyc}} = l_{I_1} < l_{O_1} \wedge l_{I_2} < l_{O_2} \wedge l_{I_2} < l_{O_2}}_{\text{No uninitialized variable}}$$

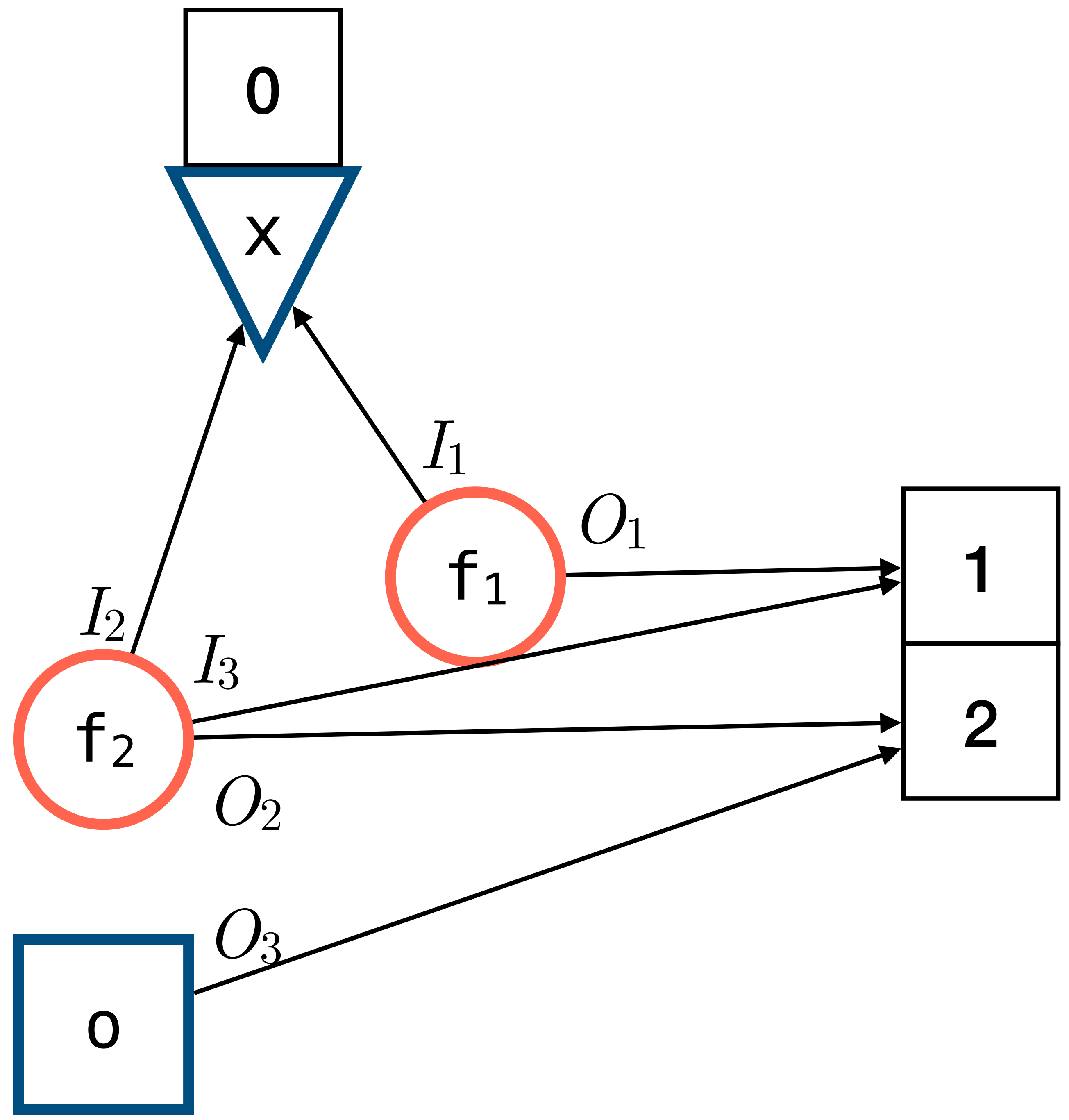
SMT Encoding (3): Semantics of Components

- A program consists of a given set of components

Component library:

```
f₁(a) = a - 1
f₂(a, b) = a & b
```

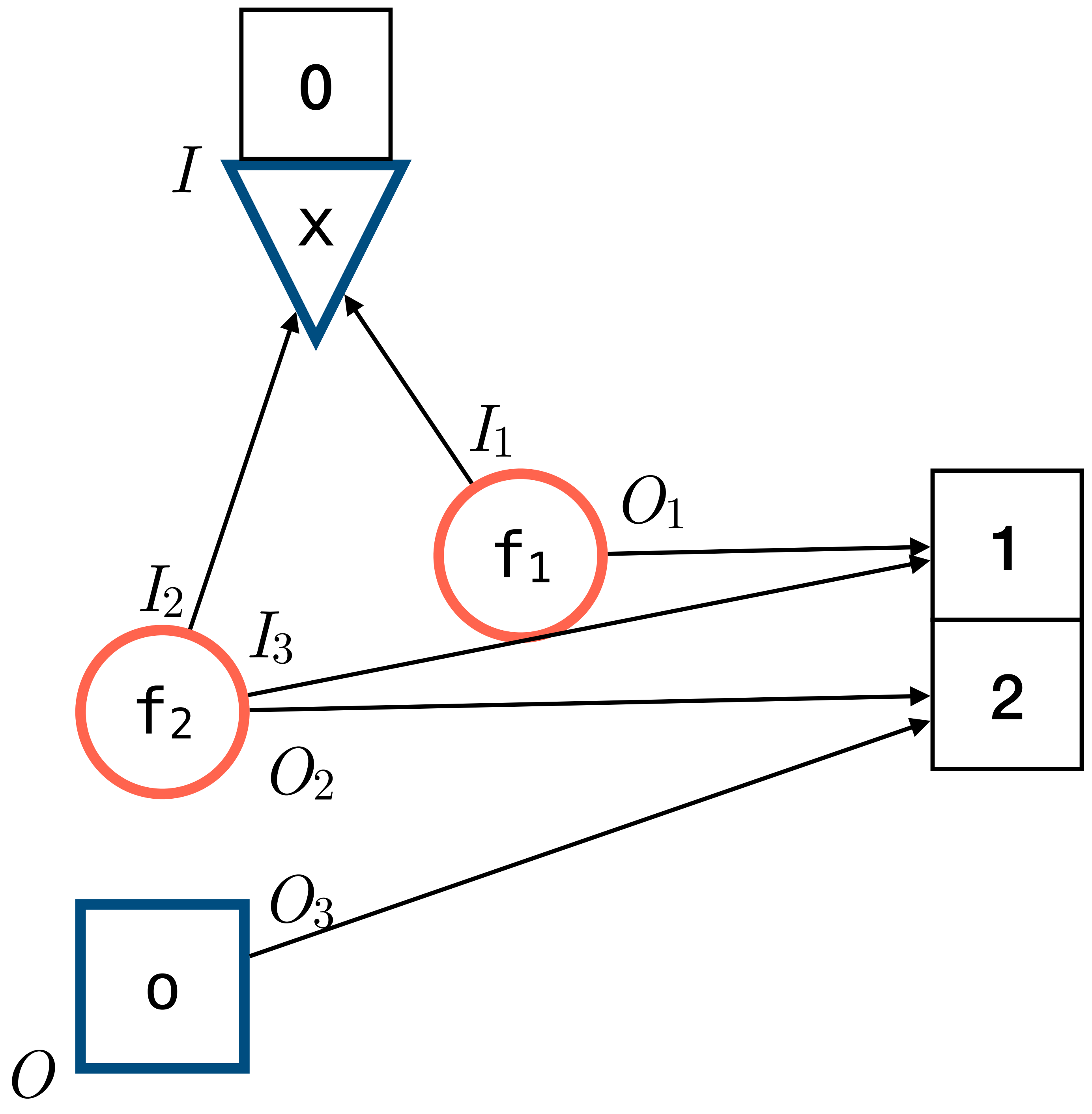
$$\psi_{\text{lib}} = (O_1 = I_1 - 1) \wedge (O_2 = I_2 \,\&\, I_3)$$



SMT Encoding (4): Dataflow

- A program consists of data-flows

$$\psi_{\text{conn}}(I, O) = \bigwedge_{x,y \in P \cup R \cup \{I,O\}} (l_x = l_y \implies x = y)$$



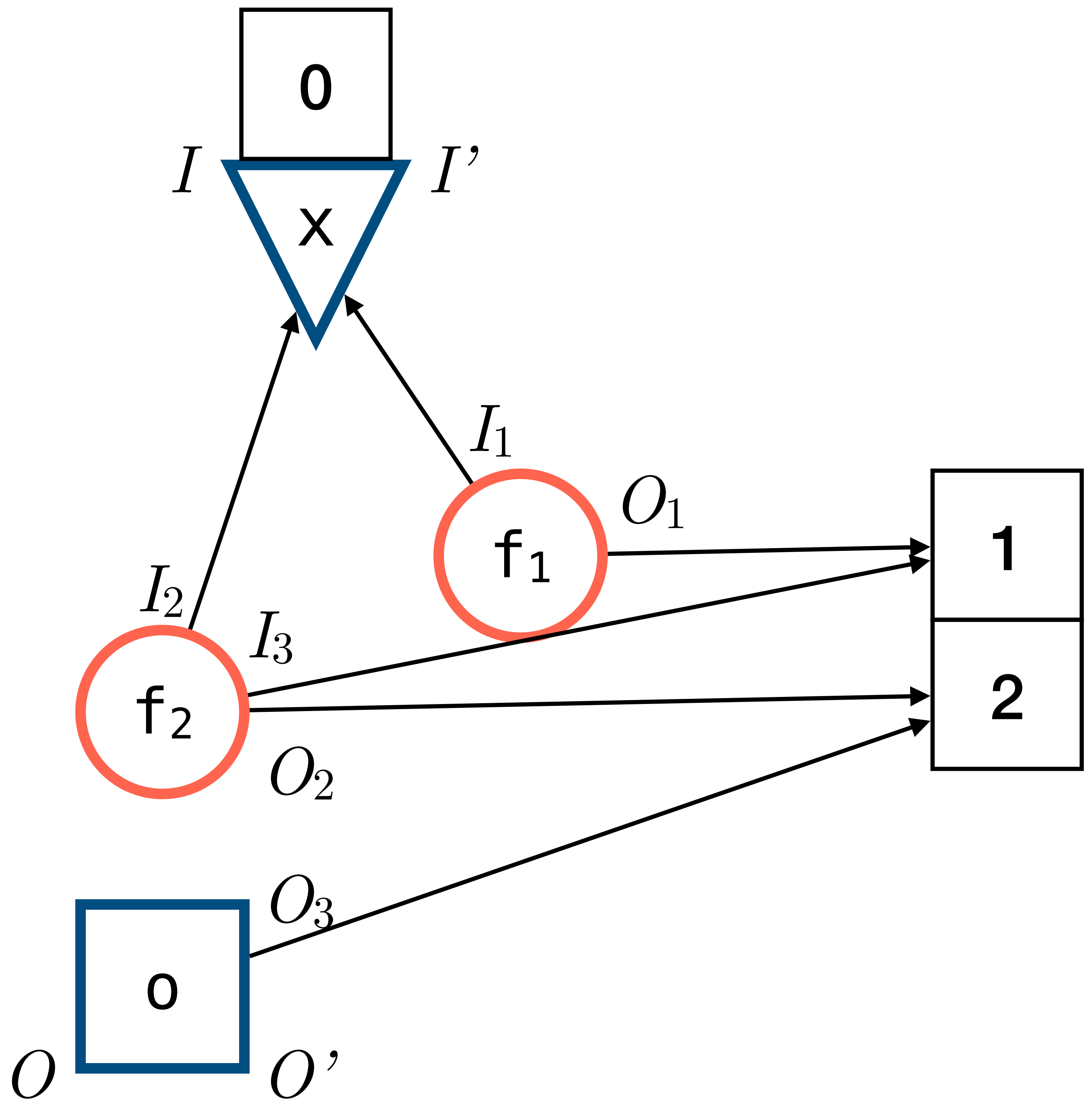
Putting All Together

- Specification: $f(01100) = 01000 \wedge f(10001) = 10000$
  - Intention: replace the rightmost 1 with 0
- Components: $f_1(a) = a - 1$ and $f_2(a, b) = a \,\&\, b$
- Solution: assignments of variables l_x satisfying formula F

$F = \psi_{\text{wfp}} \wedge \psi_{\text{lib}} \wedge F_1 \wedge F_2$ **where**

$F_1 : I = 01100 \wedge O = 01000 \wedge \psi_{\text{conn}}(I, O)$

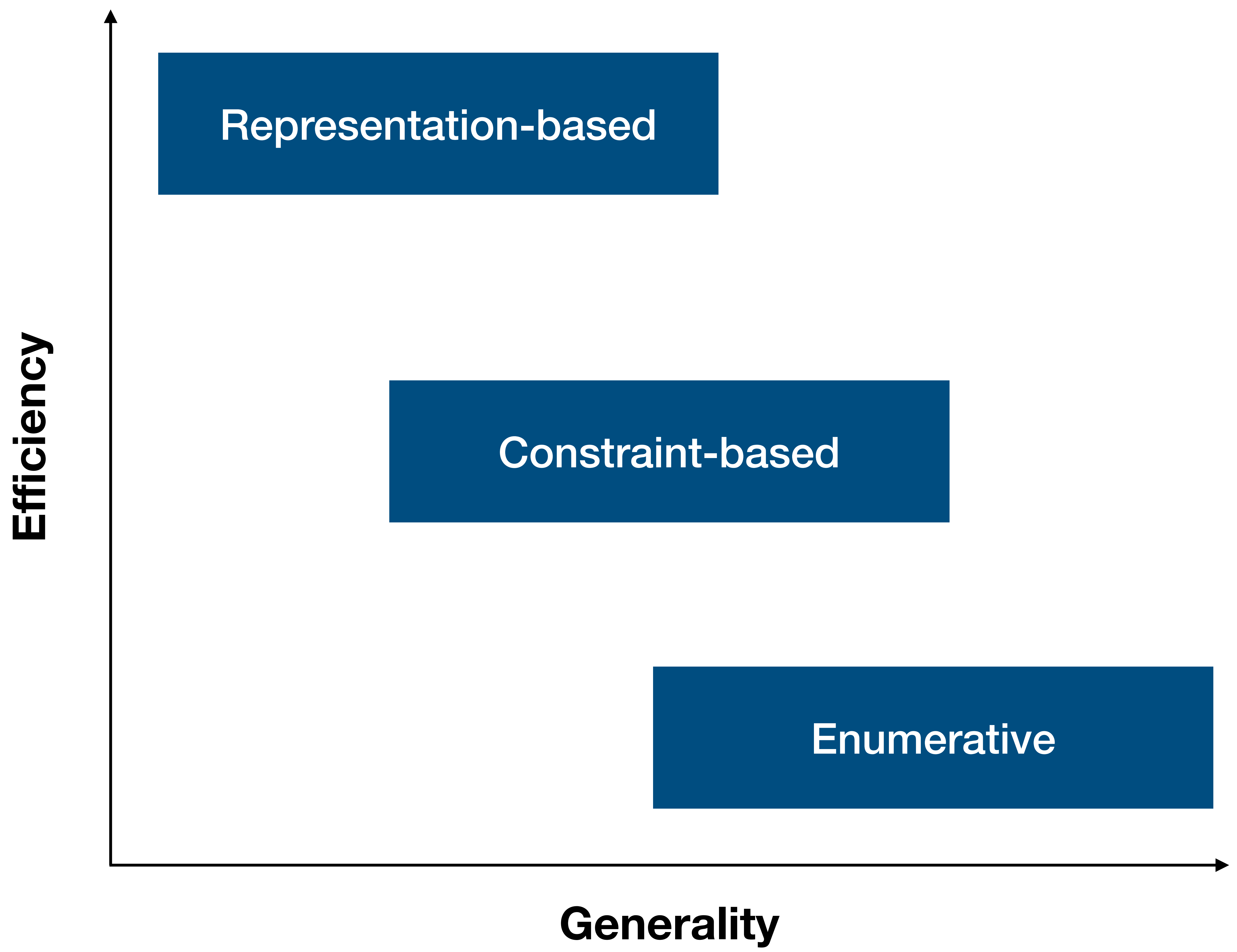
$F_2 : I' = 010001 \wedge O' = 10000 \wedge \psi_{\text{conn}}(I', O')$



Properties

- Decisive performance factor: # components
- Depending on the performance of SMT solvers (gradually improving)
- Multiplicity constraints: "must use some operator $\leq$ n times"
  - Not easy to specify such syntactic constraints using CFG
  - Applicable to apply to synthesize resource-sensitive programs
    (e.g., use multiplication operator up to 3 times in homomorphic encryption scheme)

Comparison of Search Strategies



Summary

- Constraint-based search: encode synthesis problems as constraint-solving problems
- SMT encoding: syntactic constraints $\wedge$ semantic constraints
- Application: API programming, bit-twiddling, resource-sensitive programming