ECON312 Problem Set 1B: question 5

Futing Chen, Hongfan Chen, Will Parker

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Contents


```
library(tidyverse)
library(knitr)

library(readxl)
```

Load in data

```
sheets <- excel_sheets("PS1_Q5_Data.xlsx")


dataset_list <- list()

for (s in seq(1:length(sheets))) {
  dataset_list[[s]] <- readxl::read_excel("PS1_Q5_Data.xlsx", sheet = s) %>%
    mutate(dataset_num = s) %>%
    select(Y, X1, X2, dataset_num)
}
```

A: Pre-test estimator

```
df <- dataset_list[[1]]
sample_params <- function(df){

  m_1 <- lm(data = df, formula = formula(Y ~ X1 + X2))

  beta_1_hat <- m_1$coefficients[["X1"]]


  if (is.na(m_1$coefficients[["X2"]]) == FALSE){
    t_beta_2 <- summary(m_1)$coefficients[["X2", "t value"]]
```

```
  } else { t_beta_2 <- 0}


  m_2 <- lm(data = df, formula = formula(Y ~ X1))

  beta_1_tilda <- m_2$coefficients[["X1"]]


  if (abs(t_beta_2) < 1.964) {
    beta_1_star <- beta_1_hat
  }  else {
    beta_1_star <- beta_1_tilda
  }

  dataset_num <- filter(df, row_number() ==1)$dataset_num

  mu_1 <- mean(df$X1)
  mu_2 <- mean(df$X2)

  sigma2_1 <- var(df$X1)
  sigma2_2 <- var(df$X2)
  rho <- cov(df$X1, df$X2)/sqrt(sigma2_1*sigma2_2)

  m_1 <- lm(Y ~ X1 + X2, data = df)

  sigma2_epsilon <- mean(m_1$residuals^2)

  # Q_xx <- matrix(nrow =2, c(sigma2_1, sqrt(sigma2_1*sigma2_2)*rho, sqrt(sigma2_1*sigma2_2)*rho, sigma
  #
  # n <- df %>% nrow()
  #
  # std_err_beta_1_hat <- sqrt(sigma2_epsilon*solve(Q_xx)[[1,1]]/n)

  output <- tibble(dataset_num,
                   mu_1,
                   mu_2,
                   sigma2_1,
                   sigma2_2,
                   rho,
                   sigma2_epsilon,
                   t_beta_2,
                   beta_1_star) %>%
    mutate(empiric_bias = beta_1_star -1)

  return(output)
}
```

Test that function is working

```
summary(lm("Y~ X1 + X2", dataset_list[[19]]))
```

```
##
```

```
## Call:
## lm(formula = "Y~ X1 + X2", data = dataset_list[[19]])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.49686 -0.72767 -0.01812  0.72693  2.90517
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.4025     0.1149   3.503 0.000696 ***
## X1            1.0795     0.1060  10.183  < 2e-16 ***
## X2                NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.148 on 98 degrees of freedom
## Multiple R-squared:  0.5141, Adjusted R-squared:  0.5091
## F-statistic: 103.7 on 1 and 98 DF,  p-value: < 2.2e-16
```

```r
summary(lm("Y~ X1", dataset_list[[19]]))
```

```
##
## Call:
## lm(formula = "Y~ X1", data = dataset_list[[19]])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.49686 -0.72767 -0.01812  0.72693  2.90517
##
## Coefficients:
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##
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## Multiple R-squared:  0.5141, Adjusted R-squared:  0.5091
## F-statistic: 103.7 on 1 and 98 DF,  p-value: < 2.2e-16
```

```r
sample_params(dataset_list[[19]])
```

```
## # A tibble: 1 x 10
##   dataset_num    mu_1    mu_2 sigma2_1 sigma2_2   rho sigma2_epsilon t_beta_2
##         <int>   <dbl>   <dbl>    <dbl>    <dbl> <dbl>          <dbl>    <dbl>
## 1          19 -0.0567 -0.0567     1.18     1.18 1.000           1.29        0
## # ... with 2 more variables: beta_1_star <dbl>, empiric_bias <dbl>
```

```r
results <- map_dfr(dataset_list, sample_params)

results %>%
  kable(col.names = c("Dataset", "$\\mu_1$", "$\\mu_2$", "$\\sigma^2_1$", "$\\sigma_2^2$", "$\\rho$", "$
```

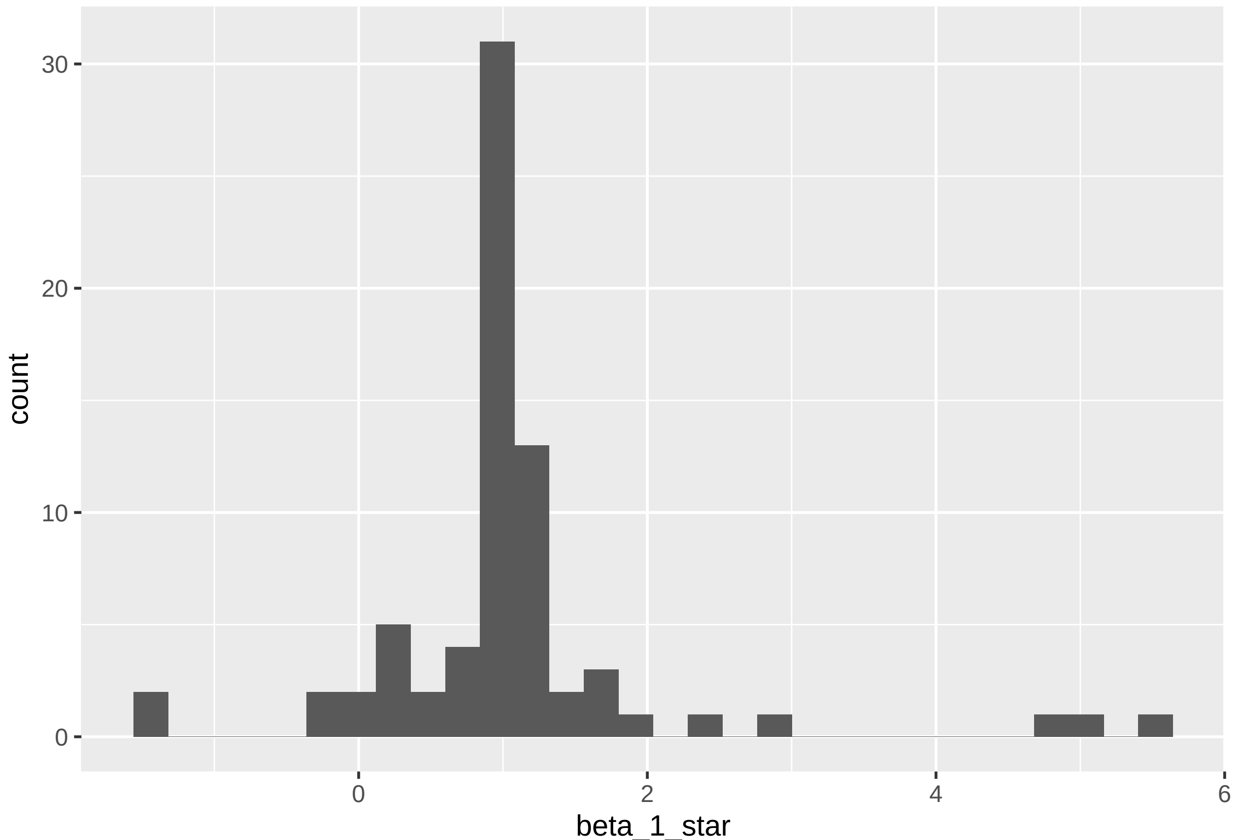
Dataset	μ_1	μ_2	σ_1^2	σ_2^2	ρ	σ_e^2	t_{β_2}	β_1*	empiric bias
1	0.10	-0.16	1.18	1.00	-0.14	0.75	0.38	0.92	-0.08
2	10.02	0.04	1.02	1.21	0.04	0.76	2.19	0.97	-0.03

Dataset	μ_1	μ_2	σ_1^2	σ_2^2	ρ	σ_e^2	t_{β_2}	β_1*	empiric bias
3	0.00	-0.10	0.10	0.93	-0.05	0.84	-0.44	1.58	0.58
4	10.02	-0.04	0.09	1.01	-0.04	0.80	1.09	0.98	-0.02
5	-0.57	-0.07	9.47	1.09	0.12	0.83	1.00	0.94	-0.06
6	9.27	-0.02	8.14	1.60	-0.02	1.01	0.32	1.02	0.02
7	0.03	-0.19	0.92	0.88	0.01	9.51	-0.63	1.18	0.18
8	9.93	-0.01	0.95	0.98	0.01	9.90	-2.00	1.24	0.24
9	-0.02	0.11	0.10	1.08	-0.04	10.86	-1.37	0.72	-0.28
10	10.01	-0.09	0.12	0.86	0.09	9.52	0.59	0.13	-0.87
11	0.08	0.09	9.61	1.05	-0.01	9.38	-0.03	1.00	0.00
12	9.89	0.05	11.27	0.82	-0.02	9.89	1.47	1.06	0.06
13	0.08	0.02	1.11	1.10	0.02	102.48	-0.78	1.00	0.00
14	10.15	0.13	1.10	1.25	0.04	134.33	-0.26	2.44	1.44
15	0.04	0.03	0.10	1.07	-0.03	92.24	0.58	5.45	4.45
16	9.99	-0.12	0.09	1.38	-0.05	99.28	-0.67	0.25	-0.75
17	0.63	-0.08	10.44	0.89	0.00	110.50	-0.88	1.08	0.08
18	10.69	0.02	9.17	1.35	-0.12	105.48	0.11	0.52	-0.48
19	-0.06	-0.06	1.18	1.18	1.00	1.29	0.00	1.08	0.08
20	9.92	-0.08	1.14	1.14	1.00	1.48	0.00	0.88	-0.12
21	0.04	0.14	0.09	0.95	1.00	0.98	0.00	0.73	-0.27
22	10.03	0.09	0.08	0.85	1.00	0.98	0.00	1.09	0.09
23	0.11	0.04	11.04	1.10	1.00	1.09	0.00	0.97	-0.03
24	9.60	-0.13	11.22	1.12	1.00	0.86	0.00	0.98	-0.02
25	0.01	0.01	1.03	1.03	1.00	9.34	0.00	1.15	0.15
26	9.97	-0.03	1.14	1.14	1.00	9.13	0.00	1.03	0.03
27	-0.01	-0.04	0.12	1.19	1.00	11.85	0.00	-0.02	-1.02
28	9.94	-0.19	0.13	1.32	1.00	10.15	0.00	0.87	-0.13
29	0.08	0.02	9.13	0.91	1.00	8.70	0.00	0.94	-0.06
30	9.75	-0.08	6.65	0.66	1.00	7.76	0.00	1.06	0.06
31	0.08	0.08	0.92	0.92	1.00	107.84	0.00	1.01	0.01
32	10.10	0.10	0.75	0.75	1.00	96.66	0.00	-1.39	-2.39
33	-0.05	-0.17	0.09	0.86	1.00	104.87	0.00	1.12	0.12
34	9.96	-0.12	0.10	1.00	1.00	102.50	0.00	0.06	-0.94
35	-0.80	-0.25	9.99	1.00	1.00	102.70	0.00	1.03	0.03
36	10.06	0.02	9.56	0.96	1.00	82.33	0.00	0.82	-0.18
37	0.08	0.13	1.21	1.05	0.54	1.09	0.71	0.97	-0.03
38	10.10	0.11	1.17	1.15	0.65	0.92	-1.41	1.12	0.12
39	0.02	0.08	0.13	1.23	0.64	1.01	0.81	0.52	-0.48
40	10.01	0.03	0.10	1.02	0.59	0.82	-0.27	0.90	-0.10
41	0.32	-0.18	9.49	0.82	0.47	1.06	0.44	0.95	-0.05
42	10.24	-0.02	9.36	0.79	0.39	1.14	-1.04	1.01	0.01
43	0.14	0.03	0.95	0.97	0.47	9.38	-1.03	1.30	0.30
44	9.91	0.06	1.10	0.96	0.55	6.45	-1.47	1.53	0.53
45	0.03	0.15	0.10	1.02	0.47	9.51	-0.29	1.43	0.43
46	10.01	0.03	0.10	0.99	0.62	9.48	0.01	0.20	-0.80
47	0.06	-0.04	9.65	0.84	0.49	11.00	1.17	0.98	-0.02
48	9.55	-0.10	11.89	1.32	0.64	10.18	0.39	1.05	0.05
49	0.00	0.16	0.89	0.96	0.51	112.69	-0.15	0.13	-0.87
50	10.13	0.02	1.04	0.69	0.45	79.51	0.71	-0.29	-1.29
51	0.04	0.01	0.09	1.16	0.37	80.31	-0.40	1.92	0.92
52	10.00	-0.16	0.09	1.00	0.33	134.49	-0.50	4.77	3.77
53	-0.31	-0.12	7.32	0.91	0.39	109.05	-1.17	0.88	-0.12
54	10.38	0.09	11.73	1.06	0.47	104.18	0.07	1.23	0.23

Dataset	μ_1	μ_2	σ_1^2	σ_2^2	ρ	σ_e^2	t_{β_2}	β_1*	empiric bias
55	0.02	-0.04	0.76	0.86	-0.52	0.90	1.92	1.18	0.18
56	10.01	-0.06	1.00	0.82	-0.48	1.04	-0.67	1.11	0.11
57	0.04	-0.08	0.11	1.12	-0.53	0.92	2.05	1.20	0.20
58	10.02	-0.13	0.11	1.00	-0.52	0.76	0.22	1.58	0.58
59	0.05	-0.08	11.28	1.16	-0.44	0.91	-0.51	0.98	-0.02
60	10.13	0.00	10.28	0.88	-0.45	1.07	-0.30	1.00	0.00
61	-0.13	0.10	0.86	0.74	-0.37	10.73	-0.18	0.21	-0.79
62	10.05	-0.11	0.87	0.82	-0.48	10.22	-0.09	0.75	-0.25
63	0.08	-0.11	0.11	0.80	-0.45	9.32	-1.68	2.94	1.94
64	9.99	-0.02	0.10	1.03	-0.39	13.47	-0.44	1.74	0.74
65	0.43	-0.05	8.14	0.93	-0.47	8.02	-0.32	0.91	-0.09
66	10.34	0.06	9.43	0.98	-0.52	9.48	-0.99	1.01	0.01
67	-0.12	0.10	1.30	1.08	-0.66	99.34	-0.25	1.16	0.16
68	10.12	0.00	1.12	0.87	-0.48	86.29	-1.89	-0.24	-1.24
69	-0.06	0.15	0.10	0.87	-0.54	96.46	0.10	-1.51	-2.51
70	10.01	-0.14	0.11	1.02	-0.47	89.17	1.33	5.16	4.16
71	-0.31	0.21	8.23	0.99	-0.55	101.82	0.60	1.28	0.28
72	10.10	-0.03	10.83	0.98	-0.59	89.17	0.48	1.02	0.02

Distribution of β_1^* across the 72 samples

```
results %>%
  ggplot(aes(x  = beta_1_star)) +
  geom_histogram()
```



‘

Sampling distribution for the pre-test estimator

If $|t|_{\hat{\beta}_2} > 1.96$, then β_1^* has the typical OLS asymptomic variance, i.e. for $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2)$

$$\sqrt{n}(\hat{\beta}_n - \beta) \xrightarrow{d} N(0, \sigma_\epsilon^2 * E[X'X]^{-1})$$

In terms of the model parameters, we can write

$$E[X'X] = (\begin{matrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{matrix})$$

And calculate the limiting distribution in this case

If $|t|_{\hat{\beta}_2} \leq 1.96$, then the variance is more complex

...

B

Analytic Bias of β_1^*

If $|t|_{\hat{\beta}_2} > 1.96$, then $\beta_1^* = \hat{\beta}_1$ which is unbiased, i.e.

$$E[\hat{\beta}_1] = \beta_1$$

If $|t|_{\hat{\beta_2}} \leq 1.96$, then $\beta_1^* = \tilde{\beta}_1$ which has the standard missing variable bias

$$E[\tilde{\beta}_1] = \beta_1 + \beta_2 \frac{Cov(X_1, X_2)}{Var(X_1)}$$

based on the data geneterating process we know

$$Cov(X_1, X_2) = \rho \sigma_1 \sigma_2$$
$$Var(X_1) = \sigma_1^2$$

So by solving we have the bias

$$E[\tilde{\beta}_1] = \beta_1 + \beta_2 \frac{\rho \sigma_2}{\sigma_1}$$

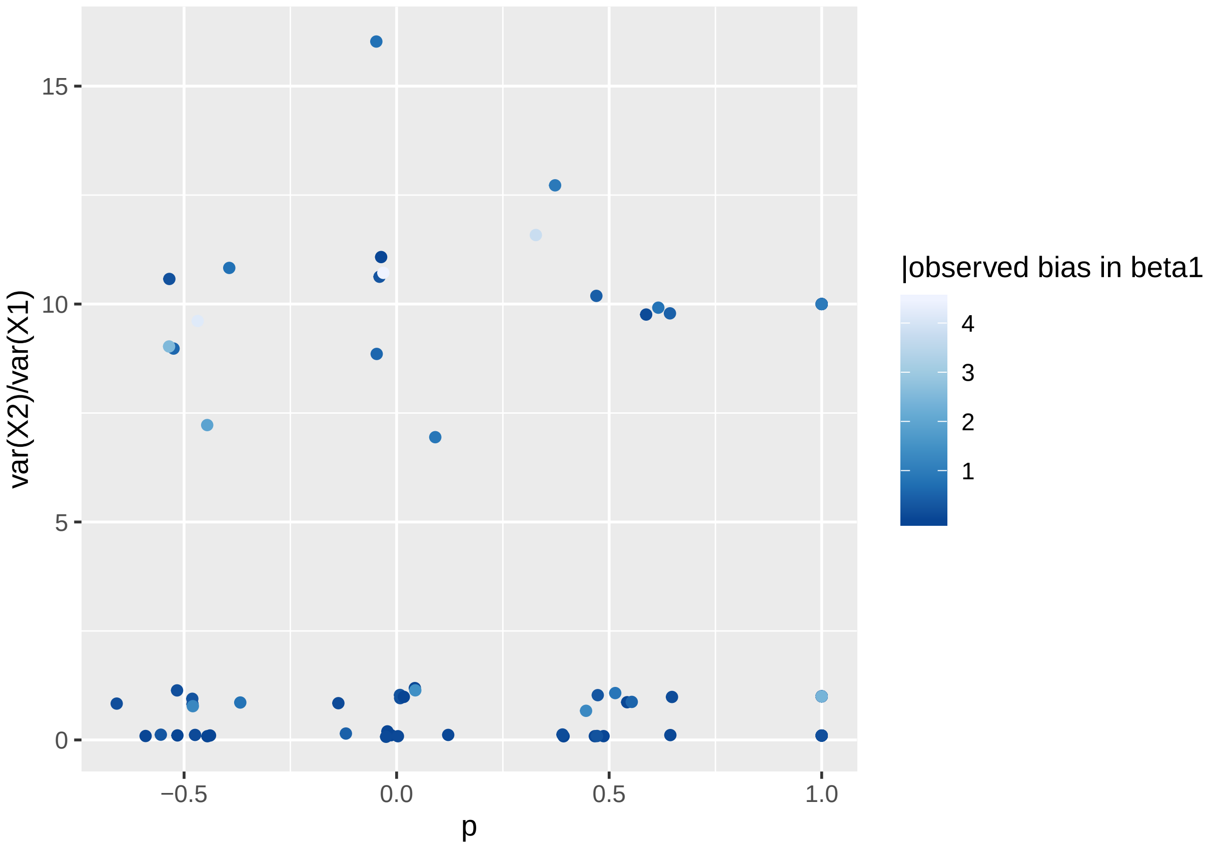
So then the $E[\beta_1^*]$

$$E[\beta_1^*] = P(|t|_{\hat{\beta_2}} > 1.96) * \beta_1 + P(|t|_{\hat{\beta_2}} \leq 1.96) * (\beta_1 + \beta_2 \frac{\rho \sigma_2}{\sigma_1})$$

…plug in for P(t>1.96)…

Relationship of parameters to observed bias

```
results %>%
  ggplot(aes(x = rho, y = sigma2_2/sigma2_1,  color = abs(empiric_bias))) +
  geom_point() + labs(x = "p", y = "var(X2)/var(X1)", color = "|observed bias in beta1|") +
  scale_color_distiller(palette = )
```


var(X2)/var(X1)
15
10
5
0
−0.5
0.0
0.5
1.0
p
|observed bias in beta1|
4
3
2
1