# ECON312 Problem Set B3

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## Contents



```r
library(tidyverse)
library(knitr)

set.seed(112345)
```

# 2: Roy Model of College

$$Y_1 = \alpha + \bar{\beta} + U_1$$
$$Y_0 = \alpha + U_0$$

$$D = \mathbf{1}\{Y_1 - Y_0 - C \geq 0\}$$

$$\begin{pmatrix} U_0 \\ U_1 \\ V \end{pmatrix} \sim N(0, \begin{bmatrix} \sigma_0^2 & \sigma_{01} & \sigma_{0,V} \\ \sigma_{01} & \sigma_1^2 & \sigma_{1,V} \\ \sigma_{0,V} & \sigma_{1,V} & \sigma_V^2 \end{bmatrix})$$

Where $C = Z + V$, $Z$ is tuition and $V$ is psychic costs

$$Z \perp\!\!\!\perp (U_1, U_0, V)$$

$$U_D = \Phi(\frac{V}{\sigma_V})$$

Where $\alpha = 0.67$ and $\bar{\beta} = 0.2$

Assume $Z \sim U[-0.5, 0.5]$

## Monte Carlo simulation of model

```
roy_MC <- function(sigma_11, sigma_00, sigma_V, sigma_01, sigma_0V, sigma_1V,
                   alpha = 0.67, beta = 0.2, N = 10000,
                   zmin = -0.5, zmax = 0.5
                   ){

  Sigma <- matrix(c(sigma_00, sigma_01, sigma_0V, sigma_01, sigma_11, sigma_1V, sigma_0V, sigma_1V, sig

  Z <- runif(N, zmin, zmax)

  latent_vars <- MASS::mvrnorm(N, c(0,0,0), Sigma) %>%
          as_tibble()


  colnames(latent_vars) <- c("U_0", "U_1", "V")


  outcomes <- latent_vars %>%
    cbind(Z) %>%
    mutate(Y_1 = alpha + beta + U_1,
           Y_0 = alpha + U_0,
           beta_i = Y_1 - Y_0,
           C = Z + V,
           D = ifelse(Y_1 - Y_0 - C >= 0, 1, 0),
           U_D = pnorm(V/sigma_V),
           mu_d_sigma_check = pnorm((beta + U_1 - U_0 - Z)/sigma_V),
           D_check = ifelse(mu_d_sigma_check >= U_D, 1, 0),
           Y = D*Y_1 + (1-D)*Y_0)

  outcomes

}
```

## Derive and graph the MTE

The propensity score $P(Z)$ is

$$P(D=1|Z) = E[\mathbf{1}\{Y_1 - Y_0 - C \geq 0\}|Z]$$

plugging in for $Y_1 - Y_0 - C = \bar{\beta} + U_1 - U_0 - Z - V$, we have

$$P(Z) = P(\bar{\beta} + U_1 - U_0 - Z - V \geq 0)$$

re-arranging and dividing by $\sigma_V$ we get

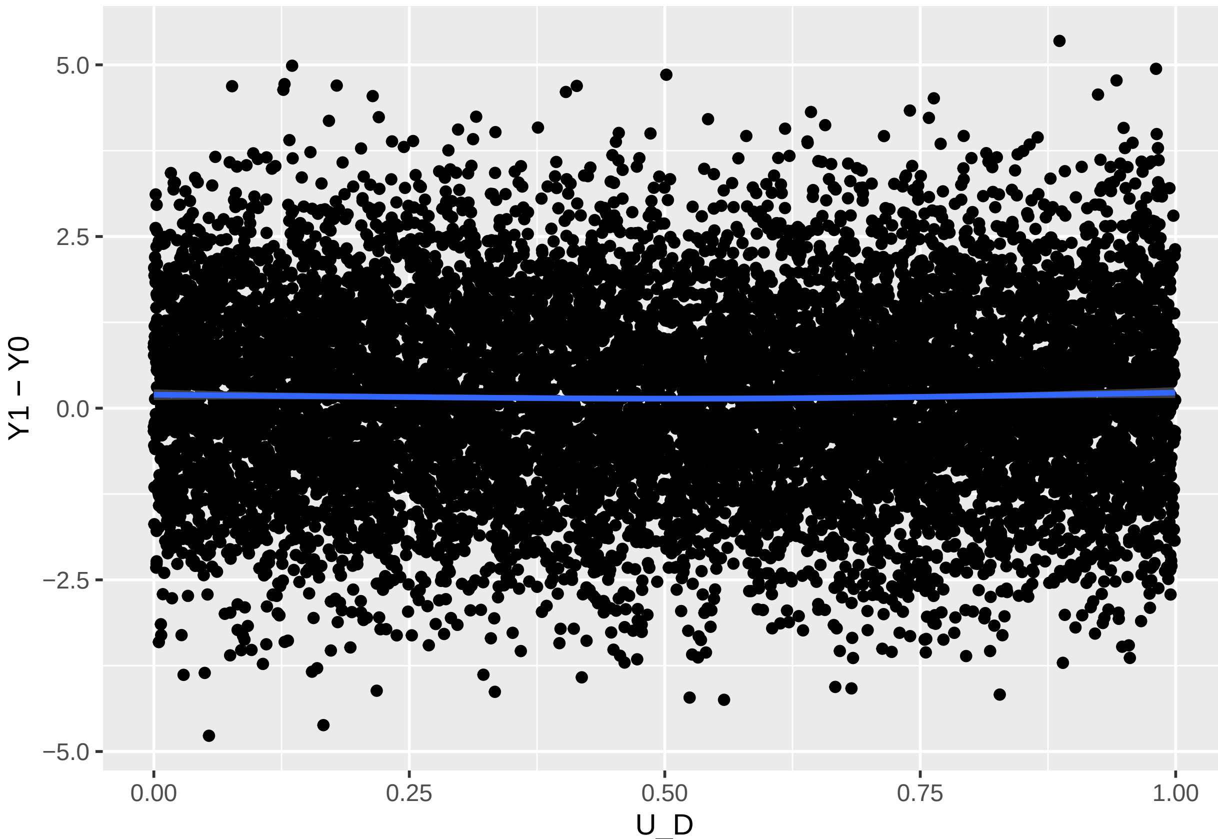
$$P(Z) = P(\frac{\bar{\beta} + U_1 - U_0 - Z}{\sigma_V} \geq \frac{V}{\sigma_V})$$

The MTE is defined as

$$E[Y_1 - Y_0|U_D = u_d]$$

### Configuration I

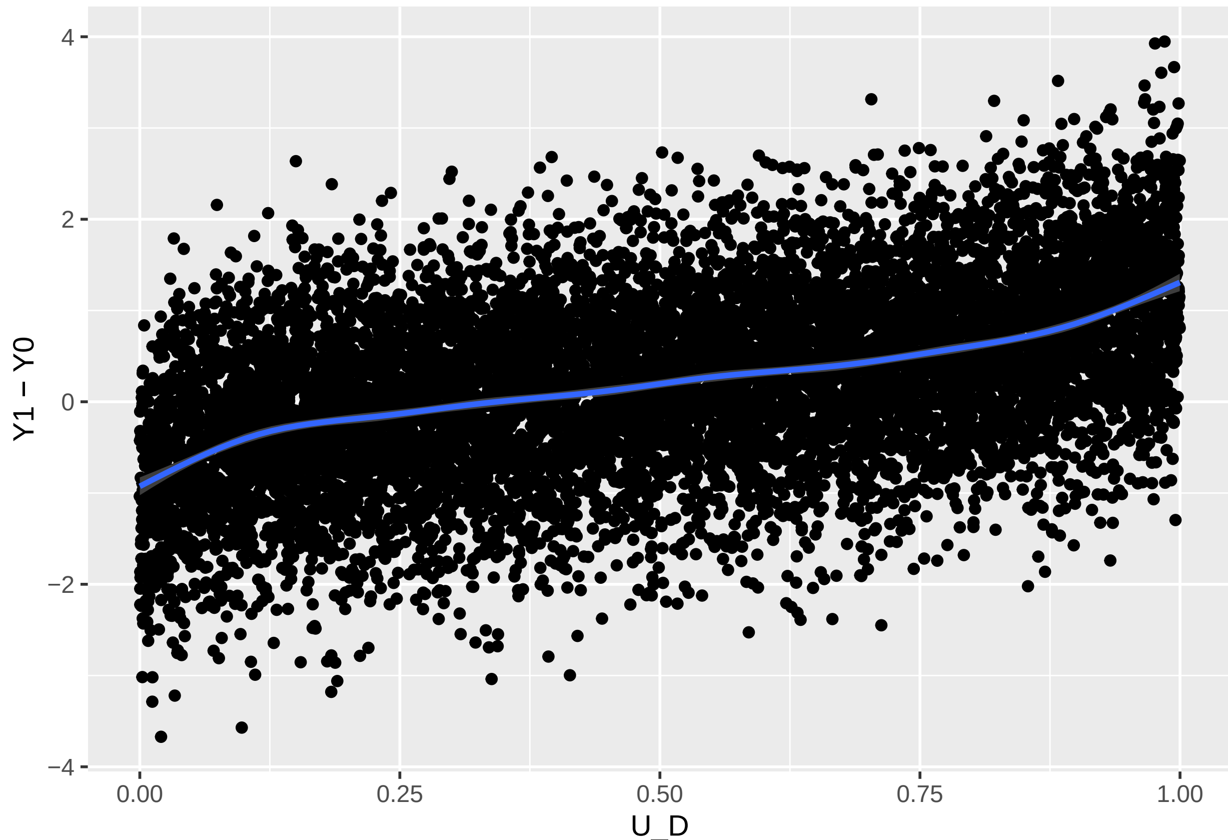
```
plot_MTE <- function(result){
  result %>%
  ggplot(aes(x = U_D, y = beta_i)) +
  geom_point() + geom_smooth() +
    labs(x = "U_D", y = "Y1 - Y0")
}

config_1 <- roy_MC(1,1,1,0,0,0)

plot_MTE(config_1)
```




The MTE is constant in configuration I

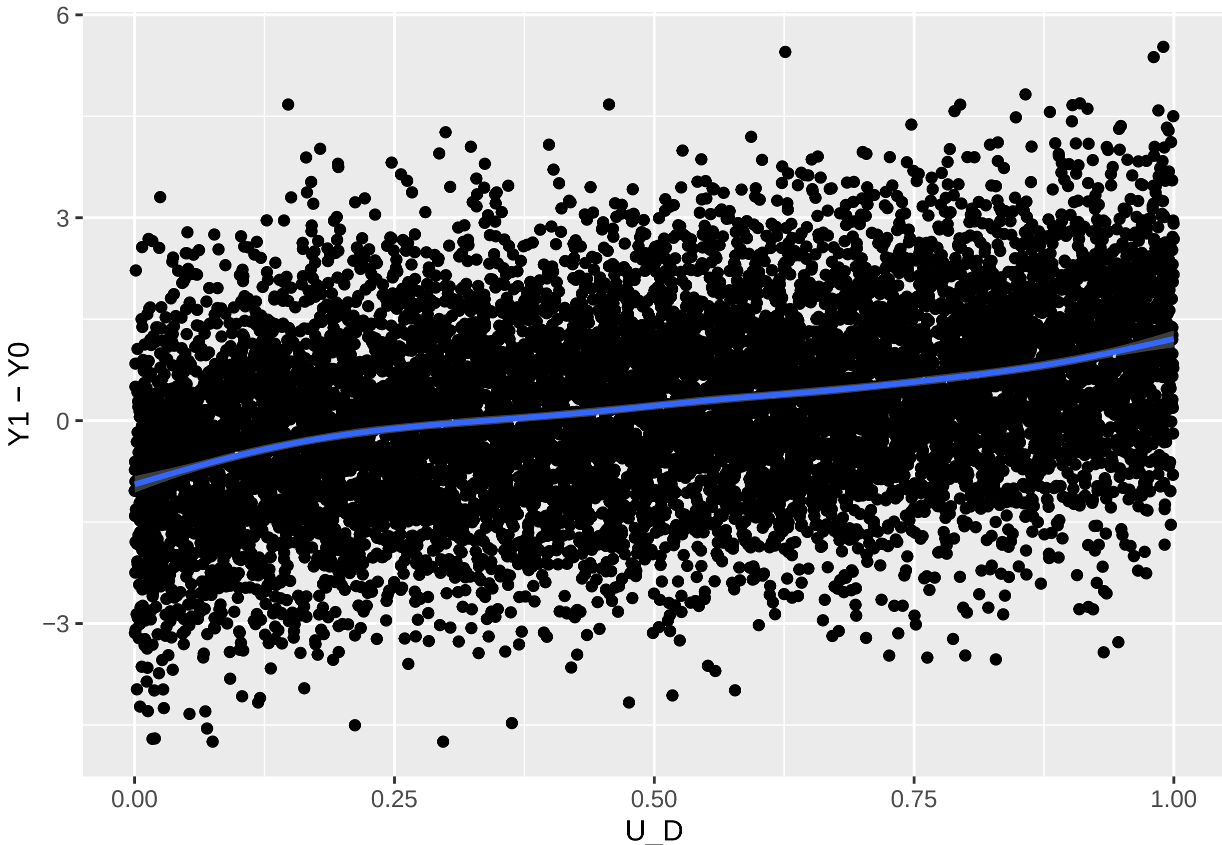
### Configuration II

```
config_2 <- roy_MC(1,1,1,0.5,-0.5,0)

plot_MTE(config_2)
```




**Configuration III**

```
config_3 <- roy_MC(1,1,1,0,0,0.5)

plot_MTE(config_3)
```

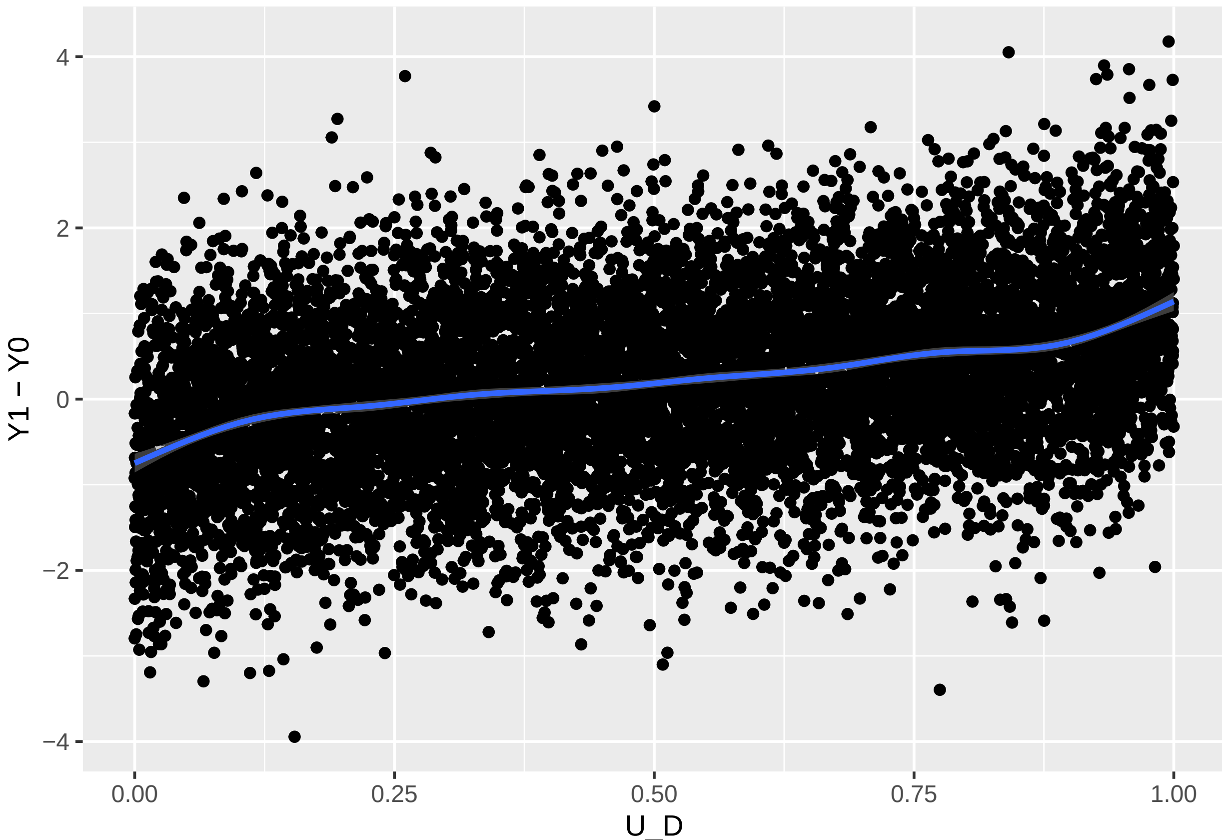



#### Configuration IV

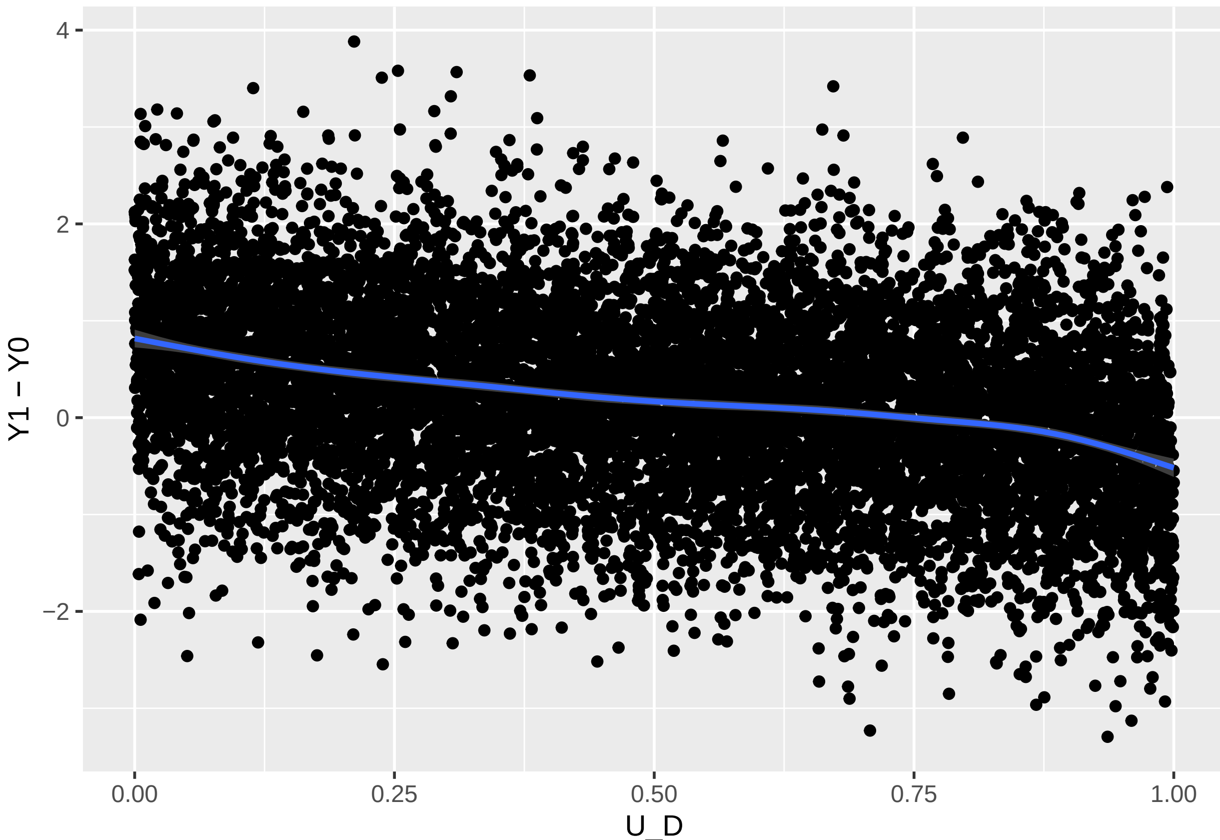
In configuration IV, $\sigma_V = 0$, so there is no latent variable that effects treatment choice.

#### Configuration V

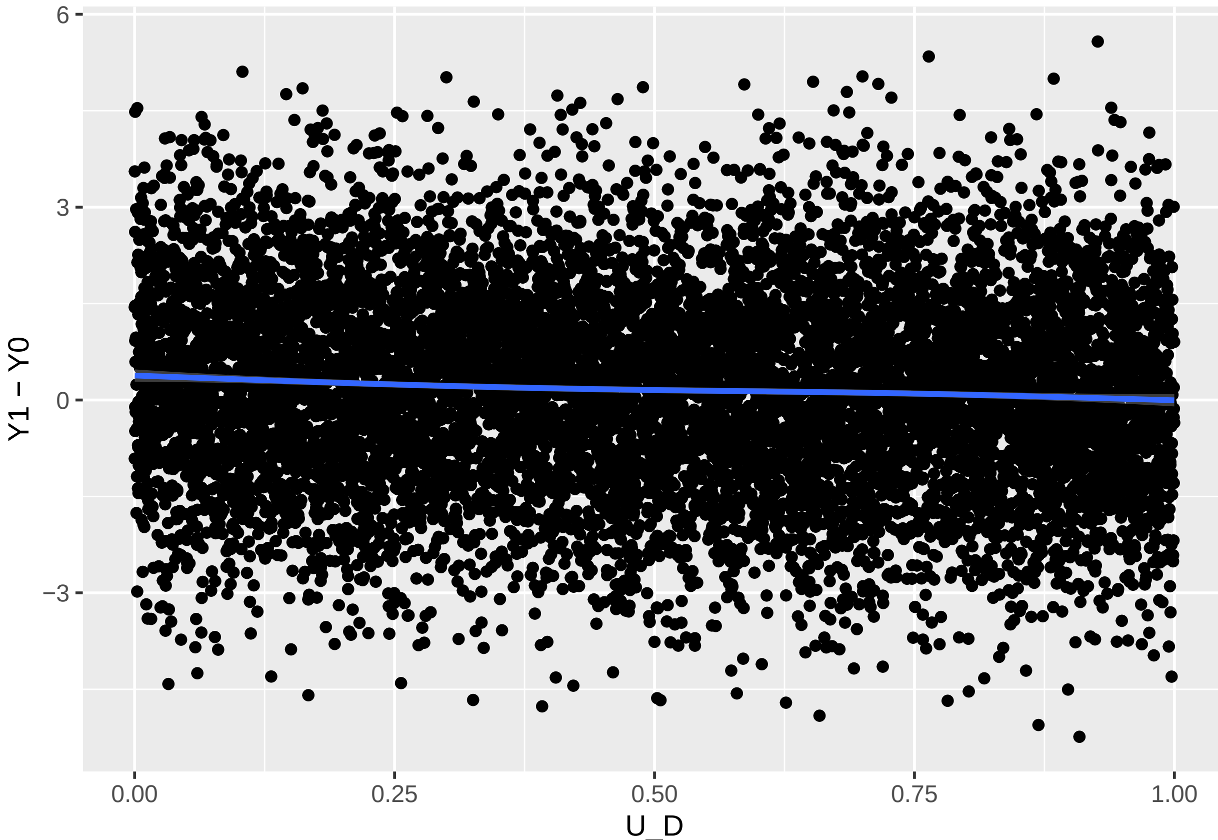
```
config_5 <- roy_MC(1,0.25,1,0.1,-0.2,0.2)

plot_MTE(config_5)
```




```
config_6 <- roy_MC(0.25,0.25,1,-0.2,0.1,-0.2)

plot_MTE(config_6)
```




```
config_7 <- roy_MC(1,1,1,-0.2,0.6,0.5)

plot_MTE(config_7)
```



6
3
0
−3
Y1 − Y0
0.00
0.25
0.50
0.75
1.00
U_D