ECON312 Problem Set 1: question 2e

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Contents


```
library(tidyverse)
library(knitr)
```

Roy model simulator function

```
roy_model <- function(sigma, rho, N = 10000, means = c(0,0), seed = 12346){
  set.seed(seed)
  cov_matrix <- matrix(c(sigma^2, rho*sigma, rho*sigma, 1), nrow = 2)

  sample <- mvtnorm::rmvnorm(n = N, means, cov_matrix)

  colnames(sample) <- c("U0", "U1")

  sample <- as_tibble(sample) %>%
    mutate(D = ifelse(U1 > U0, 1, 0),
           beta = U1-U0,
           Y = D*U1 + (1-D)*U0)

  ATE <- sample$beta %>% mean()

  ATT <- filter(sample, D ==1)$beta %>% mean()

  ATUT <- filter(sample, D == 0)$beta %>% mean()

  # beta OLS with matrix calculation
  X <- rep(1, length(sample$D)) %>% cbind(sample$D)

  beta_OLS <- solve(t(X)%*%X)%*%t(X)%*%sample$Y

  beta_OLS <- beta_OLS[[2]]

  E_Y_D1 = filter(sample, D ==1)$Y %>% mean()

  E_Y_D0 = filter(sample, D ==0)$Y %>% mean()

  E_diff = E_Y_D1 - E_Y_D0
  #lm(Y ~ D, data = sample)

  results <- tibble(
    `quantity of interest` = c("ATE", "ATT", "ATUT", "Beta_OLS", "E[Y|D =1] - E[Y|D =0]"),
    estimate = c(ATE, ATT, ATUT, beta_OLS, E_diff),
    `theoretical result` = c(0,
                             2*sqrt(sigma^2 + 1 -2*rho*sigma)*dnorm(0),
                             -2*sqrt(sigma^2 + 1 -2*rho*sigma)*dnorm(0),
                             0,
                             0)
  ) %>%
    mutate(estimate = comma(estimate),
           `theoretical result` = comma(`theoretical result`))

  kable(results)
}
```

Simulation results

```
roy_model(2, 0.5)
```

quantity of interest	estimate	theoretical result
ATE	-0.019	0.0
ATT	1.401	1.4
ATUT	-1.356	-1.4
Beta_OLS	-1.357	0.0
E[Y|D =1] - E[Y|D =0]	-1.357	0.0

The simulation shows that

$$E[Y|D = 1] - E[Y|D = 0] = \beta_{OLS}$$

```
roy_model(2, 0)
```

quantity of interest	estimate	theoretical result
ATE	-0.022	0.0
ATT	1.810	1.8
ATUT	-1.749	-1.8
Beta_OLS	-1.033	0.0
E[Y|D =1] - E[Y|D =0]	-1.033	0.0

```
roy_model(2, -0.5)
```

quantity of interest	estimate	theoretical result
ATE	-0.024	0.0
ATT	2.139	2.1
ATUT	-2.073	-2.1
Beta_OLS	-0.861	0.0
E[Y|D =1] - E[Y|D =0]	-0.861	0.0

Fixing $\rho = 0.5$ and varying σ

```
roy_model(1, 0.5)
```

quantity of interest	estimate	theoretical result
ATE	-0.0054	0.0
ATT	0.8029	0.8
ATUT	-0.7898	-0.8
Beta_OLS	0.0082	0.0
E[Y|D =1] - E[Y|D =0]	0.0082	0.0

```
roy_model(2, 0.5)
```

quantity of interest	estimate	theoretical result
ATE	-0.019	0.0
ATT	1.401	1.4
ATUT	-1.356	-1.4
Beta_OLS	-1.357	0.0
E[Y|D =1] - E[Y|D =0]	-1.357	0.0

```
roy_model(4, 0.5)
```

quantity of interest	estimate	theoretical result
ATE	-0.047	0.0
ATT	2.893	2.9
ATUT	-2.838	-2.9
Beta_OLS	-3.262	0.0
E[Y|D =1] - E[Y|D =0]	-3.262	0.0

```
roy_model(10, 0.5)
```

quantity of interest	estimate	theoretical result
ATE	-0.13	0.0
ATT	7.61	7.6
ATUT	-7.53	-7.6
Beta_OLS	-8.18	0.0
E[Y|D =1] - E[Y|D =0]	-8.18	0.0