ECON312 Problem Set 1: question 2e

Futing Chen, Will Parker

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Contents


```
library(tidyverse)
library(knitr)
```

Roy model simulator function

```
roy_model <- function(sigma, rho, N = 10000, means = c(0,0), seed = 12346){
  set.seed(seed)
  cov_matrix <- matrix(c(sigma^2, rho*sigma, rho*sigma, sigma^2), nrow = 2)

  sample <- mvtnorm::rmvnorm(n = N, means, cov_matrix)

  colnames(sample) <- c("U0", "U1")

  sample <- as_tibble(sample) %>%
    mutate(D = ifelse(U1 > U0, 1, 0),
           beta = U1-U0,
           Y = D*U1 + (1-D)*U0)

  ATE <- sample$beta %>% mean()

  ATT <- filter(sample, D ==1)$beta %>% mean()

  ATUT <- filter(sample, D == 0)$beta %>% mean()

  # beta OLS with matrix calculation
  X <- rep(1, length(sample$D)) %>% cbind(sample$D)

  beta_OLS <- solve(t(X)%*%X)%*%t(X)%*%sample$Y

  beta_OLS <- beta_OLS[[2]]

  E_Y_D1 = filter(sample, D ==1)$Y %>% mean()

  E_Y_D0 = filter(sample, D ==0)$Y %>% mean()

  E_diff = E_Y_D1 - E_Y_D0
  #lm(Y ~ D, data = sample)

  results <- tibble(
    `quantity of interest` = c("ATE", "ATT", "ATUT", "Beta_OLS", "E[Y|D =1] - E[Y|D =0]"),
    estimate = c(ATE, ATT, ATUT, beta_OLS, E_diff)
  ) %>%
    mutate(estimate = comma(estimate))

  kable(results)
}
```

Simulation results

```
roy_model(2, 0.5)
```

quantity of interest	estimate
ATE	-0.013

quantity of interest	estimate
ATT	1.967
ATUT	-1.935
Beta_OLS	0.019
E[Y|D =1] - E[Y|D =0]	0.019

The simulation shows that

$$E[Y|D = 1] - E[Y|D = 0] = \beta_{OLS}$$

```
roy_model(2, 0)
```

quantity of interest	estimate
ATE	-0.015
ATT	2.271
ATUT	-2.234
Beta_OLS	0.021
E[Y|D =1] - E[Y|D =0]	0.021

```
roy_model(2, -0.5)
```

quantity of interest	estimate
ATE	-0.017
ATT	2.539
ATUT	-2.497
Beta_OLS	0.023
E[Y|D =1] - E[Y|D =0]	0.023

Fixing $\rho = 0.5$ and varying σ

```
roy_model(1, 0.5)
```

quantity of interest	estimate
ATE	-0.0054
ATT	0.8029
ATUT	-0.7898
Beta_OLS	0.0082
E[Y|D =1] - E[Y|D =0]	0.0082

```
roy_model(2, 0.5)
```

quantity of interest	estimate
ATE	-0.013
ATT	1.967
ATUT	-1.935

quantity of interest	estimate
Beta_OLS	0.019
E[Y|D =1] - E[Y|D =0]	0.019

```
roy_model(4, 0.5)
```

quantity of interest	estimate
ATE	-0.029
ATT	4.248
ATUT	-4.179
Beta_OLS	0.040
E[Y|D =1] - E[Y|D =0]	0.040

```
roy_model(10, 0.5)
```

quantity of interest	estimate
ATE	-0.074
ATT	11
ATUT	-11
Beta_OLS	0.104
E[Y|D =1] - E[Y|D =0]	0.104