MIT 18.06 Exam 2, Fall 2018
Johnson

Your name: ______________________________

Recitation: ______________________________

problem	**score**
1	/33
2	/34
3	/33
total	/100

Problem 1 (33 points):

The matrix A has a nullspace $N(A)$ spanned by

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

and a left nullspace $N(A^T)$ spanned by

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}.$$

(a) What is the **shape** of the matrix A and its **rank**?

(b) If we consider the vector

$$b = \begin{pmatrix} -1 \\ \alpha \\ 0 \\ \beta \end{pmatrix},$$

for **what value(s)** of α and β (if any) is $Ax = b$ solvable? Will the solution (if any) be **unique**?

(c) Give the orthogonal **projections** of

$$y = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

onto **two** of the four fundamental subspaces of A.

(blank page for your work if you need it)

Problem 2 (34 points):

You have a matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

(a) Give the **ranks** of A, A^T, and A^TA, and also give **bases** for $C(A)$, $N(A)$, and $N(A^TA)$. (**Look carefully at the columns** of A—very little calculation is needed!)

(b) Suppose we are looking for a least-square solution $\hat{x}$ that minimizes $\|b - Ax\|$ for $b = \begin{pmatrix} 0 \\ 2 \\ 1 \\ -1 \end{pmatrix}$. At this minimum, $p = A\hat{x}$ will be the projection of b onto ....? **Find** p. (**Hint**: your answer from (a) should help simplify the calculations.)

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Problem 3 (33 points):

Suppose that we apply Gram–Schmidt to the *rows* (in order from top to bottom) of a matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

in order to find three **orthonormal row vectors** q_1^T, q_2^T, q_3^T.

(a) What is q_2?

(b) Suppose that these orthonormal vectors are the **rows** of a matrix $U = \begin{pmatrix} q_1^T \\ q_2^T \\ q_3^T \end{pmatrix}$. Then:

(i) **Circle any** of the following that are **true**: $U^T U = I$ and/or $UU^T = I$?

(ii) **Circle any** of the following that are **true**: $C(A) = C(U)$, $N(A) = N(U)$, $C(A^T) = C(U^T)$, and/or $N(A^T) = N(U^T)$?

(iii) Which is **true**: $A = BU$ or $A = UB$? Is B upper or lower triangular?

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