

# 18.06 Recitation 7

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## 1 Foundational Problems

1. What assumptions on the dimensions of  $A$  do we have when finding eigenvalues/eigenvectors of  $A$ ?
2. Find/describe as many eigenvalues and corresponding eigenvectors as you can (without doing any serious calculation) for the following matrices:

(a)  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$

(b)  $B = \begin{pmatrix} 4 & 3 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix}.$

- (c) A projection matrix  $P$  some subspace. If you want to be concrete, think about projection to the column space of

$$C = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- (d) The permutation matrix

$$M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

- (e) A rank one matrix  $uv^T$ . If you want to be concrete, think about

$$u = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

3. Are there any real eigenvalues of a rotation matrix

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix},$$

for any possible  $\theta$ ?

4. Suppose that  $A, B, C$  are  $m \times m$  matrices with eigenbases that you know.
- (a) What do you know about the eigenvectors and eigenvalues of  $A^{2017}$ ?
  - (b) What do you know about the eigenvectors and eigenvalues of  $A^{-1}$ ?
  - (c) What do you know about the eigenvectors and eigenvalues of  $A^T$ ?
  - (d) What do you know about the eigenvectors and eigenvalues of  $AB$ ?
  - (e) What do you know about the eigenvectors and eigenvalues of  $A + B$ ?
  - (f) What do you know about the eigenvectors and eigenvalues of  $\begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$ ?

## 2 Problems

1. Suppose that  $A$  is the matrix

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

- (a) What is the pattern when you multiply  $A$  repeatedly by some vector? After \_\_\_\_\_ multiplications, you get back the same vector, so

$$A^{\text{---}} = \text{---}.$$

- (b) What are eigenvalues and eigenvectors of  $A$ ? Is this consistent with the previous part?
- (c) Write the vector  $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  in the basis of the eigenvectors and give a formula for  $A^n x$ .
- (d) What are the eigenvectors and eigenvalues of  $B = 2A + I$ ?
- (e) What do you know about  $B^n x$  as  $n \rightarrow \infty$  and  $n \rightarrow -\infty$ ?

2. Suppose that  $A$  is any  $m \times m$  matrix with entries in  $\mathbb{R}$  and

$$Ax = \lambda x$$

for some  $\lambda \in \mathbb{C}, \lambda \notin \mathbb{R}$ .

- (a) What must be another eigenvector and eigenvalue?
  - (b) How can you tell if  $A^n x$  blows up as  $n \rightarrow \infty$ ?
3. (Strang, Section 6.1, Problem 19) A  $3 \times 3$  matrix  $B$  is known to have eigenvalues 0, 1, 2. This is enough information to determine 3 of the following. Which are they and what are the answers:
- (a) The rank of  $B$ .
  - (b) The determinant of  $B^T B$ .
  - (c) The eigenvalues of  $B^T B$ .
  - (d) The eigenvalues of  $(B^2 + I)^{-1}$ .