MIT 18.06 Exam 1, Fall 2018
Johnson

Your name: ____

Recitation: ____

problem	**score**
1	/30
2	/30
3	/10
4	/30
total	/100

Problem 1 (30 points):

You have the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 2 & -1 & 0 & 1 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix}$$

(a) Find matrices P, U, L for a $PA = LU$ factorization of A. Hint: look at A carefully first: if you find the right permutation P (a matrix to *re-order the rows*) it will be simple.

(b) Compute $x = A^{-1}b$ where $b = \begin{pmatrix} -2 \\ 2 \\ 2 \\ 1 \\ -4 \end{pmatrix}$.

(blank page for your work if you need it)

Problem 2 (30 points):

A is a 3×5 matrix. One of your Harvard friends performed row operations on A to convert it to rref form, but did something weird—instead of getting the usual $R = \begin{pmatrix} I & F \end{pmatrix}$, they reduced it to a matrix in the form $\begin{pmatrix} F & I \end{pmatrix}$ instead. In particular, their row operations gave:

$$A \rightsquigarrow \begin{pmatrix} 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 0 & 1 & 0 \\ 6 & 7 & 0 & 0 & 1 \end{pmatrix}.$$

(a) Find a basis for $N(A)$.

(b) Give a matrix M so that if you multiply A by M (on the **left or right?**) then the **same** row operations as the ones used by your Harvard friend will give a matrix in the usual rref form:

$$\text{either } MA \text{ or } AM \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 4 & 5 \\ 0 & 0 & 1 & 6 & 7 \end{pmatrix}.$$

(blank page for your work if you need it)

Problem 3 (10 points):

In class, when we derived the LU factorization, we initially found L by multiplying a sequence of elementary elimination matrices, *one to eliminate below each pivot*. (We later found a more clever way to get L just by writing down the multipliers from the elimination steps, no arithmetic required.)

If A is a non-singular $m \times m$ matrix and we compute L in the **"naive" way**, by directly multiplying the elementary elimination matrices (by the usual rows $\times$ columns method, no tricks), how would the cost to compute L (the number of scalar-arithmetic operations) scale with m? (That is, roughly proportional to m, m^2, m^3, m^4, m^5, 2^m, or...?)

Problem 4 (30 points):

Here are some miscellaneous questions that require little calculation:

(a) Is V a vector space or not? (For multiplication by real scalars and the usual $\pm$ operations.) If **false**, give a rule of vector spaces that is violated:

(i) A is a 3×6 matrix. $V =$ *all* solutions x to $Ax = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

(ii) A is a 3×6 matrix. $V =$ *all* 6×2 matrices X where $AX = 0$ (the 3×2 zero matrix).

(iii) $V =$ *all* 3×3 singular matrices A.

(iv) $V =$ *all* 3×3 matries whose diagonal entries average to zero.

(v) $V =$ *all* differentiable functions $f(x)$ with $f'(0) = 2f(0)$. (f' is the derivative.)

(vi) $V =$ *all* functions $f(x)$ with $f(x+y) = f(x)f(y)$.

(b) Give a matrix A whose null space is spanned by $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

(c) Give a nonzero matrix A whose column space is in $\mathbb{R}^3$ but does *not* include $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

(blank page for your work if you need it)