18.06 Recitation 6

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1 Pictures/Words Problems

1. Explain geometrically the formula for projection onto a one-dimensional subspace spanned by a vector a.

2 Problems

1. (Problem 1, Exam 2, Spring 2017) You are given a 6×6 matrix

$$A = \begin{pmatrix} 1 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & -1 & 2 & -1 & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{pmatrix}.$$

   (a) Find the determinant of A.

   (b) What is the projection matrix onto $C(A)$?

   (c) If you perform Gram-Schmidt orthogonalization on the columns of A, what is the pattern of nonzero entries in the resulting orthogonal matrix Q?

2. (Problem 3(d), Exam 2, Spring 2017) If A and B are two matrices such that $A^T B = 0$ (the zero matrix), with QR factorizations

$$A = Q_A R_A, \qquad B = Q_B R_B$$

   write down the QR factorization of the matrix $C = \begin{pmatrix} A & B \end{pmatrix}$ in terms of Q_A, Q_B, R_A, R_B.

3. (Strang, Section 5.1, Problem 8) Prove that every orthogonal matrix (recall: a matrix Q so that $Q^T Q = I$) has determinant 1 or -1.

   (a) Use the product rule $|AB| = |A||B|$ and the transpose rule $|Q| = |Q^T|$.

   (b) Use only the product rule. If $|\det(Q)| > 1$ then $\det(Q^n) = (\det Q)^n$ blows up. How do you know that this can't happen to Q^n?

4. (essentially Strang, Section 4.4, Problem 33) Find all matrices that are both orthogonal and lower triangular in two parts:

    (a) What are all orthogonal and lower-triangular 2×2 matrices?

    (b) Can you generalize this to $m \times m$ matrices?

5. (Strang, Section 5.1, Problem 11) Suppose that $CD = -DC$ and find the flaw in this reasoning: Taking determinants gives

$$|C||D| = -|D||C|$$

   and so $|C||D| = 0$, so one of either C or D has determinant 0 and is singular. (This is not true).

6. (Strang, Section 5.1, Problem 18) Use row operations to show that the 3×3 "Vandermonde determinant" is

$$\det \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix} = (b-a)(c-a)(c-b).$$