18.06 Recitation 8

Isabel Vogt

October 1, 2018

1. The Fibonacci sequence is defined recursively by specifying initial values $F_0 = 1, F_1 = 1$, and the relation
$$F_{n+1} = F_n + F_{n-1}.$$
   (a) Given the input vector $v_n = \begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix}$, what is the matrix A, so that $Av_n = v_{n+1}$?

   Give an expression for v_{n+1} in terms of A and $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   (b) What are the eigenvectors and eigenvalues of A?

   (c) Using the eigenbasis, give an exact formula for F_n. What do you know about $|F_n|$ as $n \to \infty$?

2. Let A be an $m \times m$ matrix and let A' be the matrix obtained from A be reversing the rows and columns:
$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ & & \ddots & \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{pmatrix}, \qquad A' = \begin{pmatrix} a_{mm} & a_{m(m-1)} & \dots & a_{m1} \\ a_{(m-1)m} & a_{(m-1)(m-1)} & \dots & a_{(m-1)1} \\ & & \ddots & \\ a_{1m} & a_{1(m-1)} & \dots & a_{11} \end{pmatrix}.$$
   (a) When $m = 2$, what do you notices about the eigenvalues of A'?

   (b) What is true in general and why?

3. (Strang, Section 10.3, Problem 6)

   (a) For a Markov matrix, show that the sum of the components of x equals the sum of the components of Ax.

   (b) If $Ax = \lambda x$ with $\lambda \neq 1$, prove that the components of this non-steady eigenvector x add to zero.

4. (a) Show that every square matrix is similar to its transpose. That is,
$$A^T = SAS^{-1},$$
   for some invertible matrix S.

   (b) Assuming A is diagonalizable, give a formula for S in terms of the matrix X of eigenvectors of A and the matrix Y of eigenvectors of A^T (which is also diagonalizable).