18.06 Recitation 1

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1 Practical Information

1. Stellar is the main website for this class: `https://stellar.mit.edu/S/course/18/fa17/18.06`
   (a) Link to the github where lecture summaries, homework, etc. is posted.
   (b) Link to Piazza for asking questions.
   (c) Staff OH under "Staff" link. **My office hours are Tuesdays 8:30-9:30 pm in 2-239.**
   (d) Link to old OCW videos. *Warning: Prof. Johnson's lectures can differ substantially from Prof. Strang's! In particular he goes faster and emphasizes different things; so this is not a substitute for going to lecture!*
2. The syllabus is contained within the slides from the first lecture (on github)
   (a) Midterm 1: September 25
   (b) Midterm 2: October 30
   (c) Midterm 3: November 27
   (d) Final exam announced at the end of this month.
   (e) Homework is due every Wednesdays at 11am in my recitation box, *including on exam weeks!*
3. The Math Learning Center is a math-department-run drop-in tutoring service from 3-5pm and 7:30-9:30pm Monday-Thursday (16 hours per week!!) for the classes 18.01, 18.02, 18.03, and 18.06. This can be a great additional resource for you.

2 Pictures/Words Problems

1. Let m and n be natural numbers (you can think of $m = 3$ and $n = 2$ if you like). Let A be an $m \times n$ matrix, B be a $n \times n$ matrix, v be a $1 \times n$ vector, w be a $m \times 1$ vector, and u be a $1 \times m$ vector. Which of the following make sense (and if so, what is the result)
   (a) AB
   (b) BA
   (c) Av
   (d) Aw
   (e) vA

(f) vB

(g) wu

(h) wv

(i) uw

2. If you have two coupled systems of equations

$$\begin{aligned} Bx + Cy &= c \\ Dx + Ey &= d. \end{aligned}$$

where B, C, D, and E are 3×3 matrices and x, y, c and d are 3-component vectors, can you write this as a single system of equations,

$$Az = b,$$

where $z = \begin{bmatrix} x \\ y \end{bmatrix}$ is the 6-component vector of x on top of y. What are A and b?

3. Describe geometrically why the system of linear equations

$$\begin{aligned} 3x + y &= 2 \\ 6x + 2y &= 4 \end{aligned}$$

has infinitely many solutions. Describe geometrically why the system of linear equations

$$\begin{aligned} 3x + y &= 2 \\ 6x + 2y &= 10 \end{aligned}$$

has no solutions.

4. "Do" Gaussian elimination on the matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ -1 & 6 & 2 \end{bmatrix}.$$

5. What is the LU factorization of the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}?$$

3 Problems

1. Strang 2.2(22):

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}.$$

For which 3 numbers a will elimination on A fail to give 3 pivots (i.e. for which 3 a's is A singular)?

2. Strang 2.2(27): if you have a lower-triangular system, you can solve it quickly by "forward substitution". Solve the 3×3 problem $Lx = b$:

$$\begin{bmatrix} 3 & 0 & 0 \\ 6 & 2 & 0 \\ 9 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 9 \end{bmatrix}.$$

3. For the lower-triangular matrix L above, find matrices A and B such that

$$ALB = \begin{bmatrix} 1 & -2 & 9 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}.$$

4. Suppose that there are a billion Facebook users. List the billion people

$$1, 2, ..., \text{billion},$$

and consider the billion by billion matrix A which has ij-entry equal to 1 if people i and j are Facebook friends, and 0 otherwise (say by convention no person is friends with him/herself).

   (a) Can you think of a billion by 1 column vector x such that Ax records how many friends each person has, i.e. so that the ith entry of Ax is the number of friends person i has?

   (b) Can you think of a billion by 1 column vector y such that Ay records who person 5 is friends with?

   (c) Can you think of a billion by 1 column vector u and a 1 by billion row vector w so that the *scalar* wAu records how many friendships there are in the Facebook universe?

5. Consider the company Widgets-R-Us (WRU). WRU makes 100 kinds of widgets and sells them in all 50 US states. Consider the 100 by 50 matrix A with i, j-entry recording the number of widgets of type i sold in state j in the month of August 2017. What kind of 50 by 1 column vectors x could you cook up so that the 100 by 1 column vector Ax records

   (a) the number of each kind of widgets sold in Kansas,

   (b) the number of each kind of widget sold in all western states combined, and

   (c) the gross revenue obtained from each kind of widget (suppose all widgets have the same price in each state but vary state to state)?

   What kind of 1 by 100 row vectors y could you cook up so that the 1 by 50 row vector yA records

   (a) total widgets sold by state,

   (b) total revenue by state (suppose each kind of widget has the same price in all states, but that different widgets can be priced differently)?

6. (Strang 2.4.31, essentially) Consider a fixed complex number $A + Bi$, where A, B are real. View complex numbers $z = x + yi$ as column vectors

$$z = \begin{bmatrix} x \\ y \end{bmatrix}$$

   Can you think of a 2 by 2 matrix M such that left multiplication by M is the same as complex multiplication by $A + Bi$? When is this matrix singular?