INFDEV026A - Algoritmiek
Week 3

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Practical examination and oral check

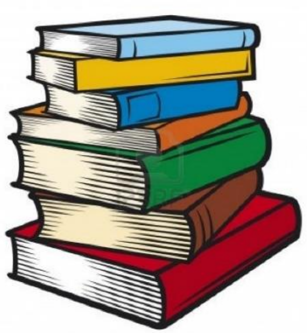
- Continue with the first exercise of the assignment!
- Oral check… how does it work?
  - We will remove a few lines of code from each of the exercises
  - You will be asked to make your program work again

Today

LIST

QUEUE

- ~~Why is my code slow?~~
  - ~~**Empirical and complexity analysis**~~
- ~~How do I order my data?~~
  - ~~**Sorting algorithms**~~
- How do I structure my data?
  - **Linear, tabular, recursive data structures**
- How do I represent relationship networks?
  - **Graphs**

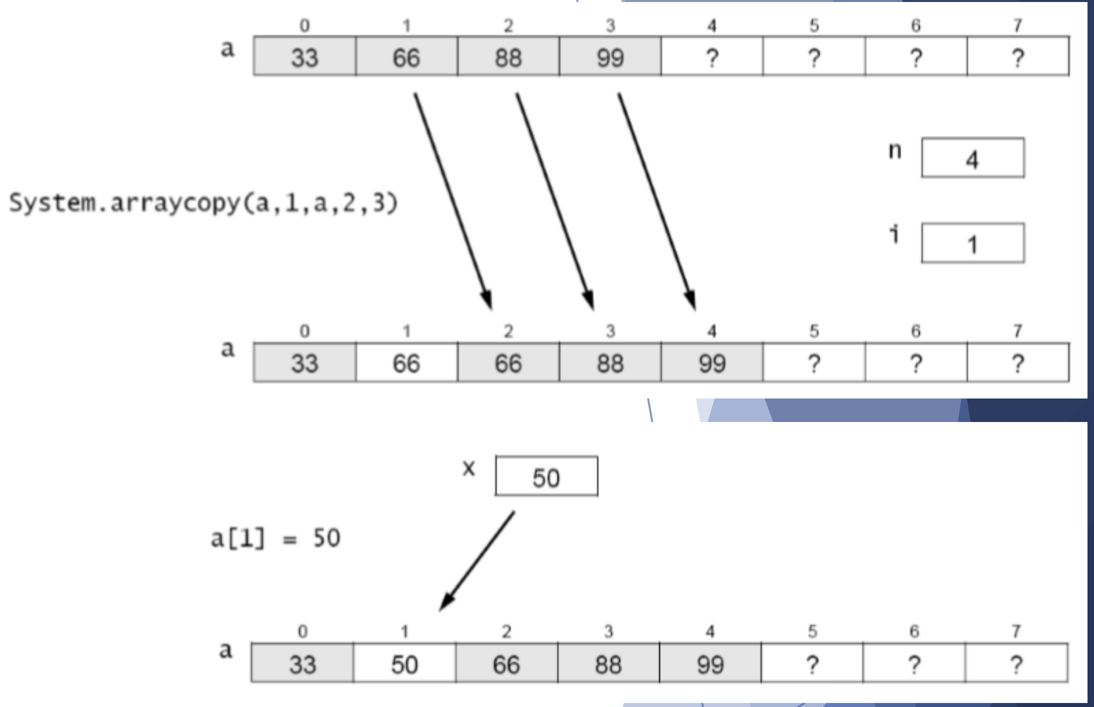


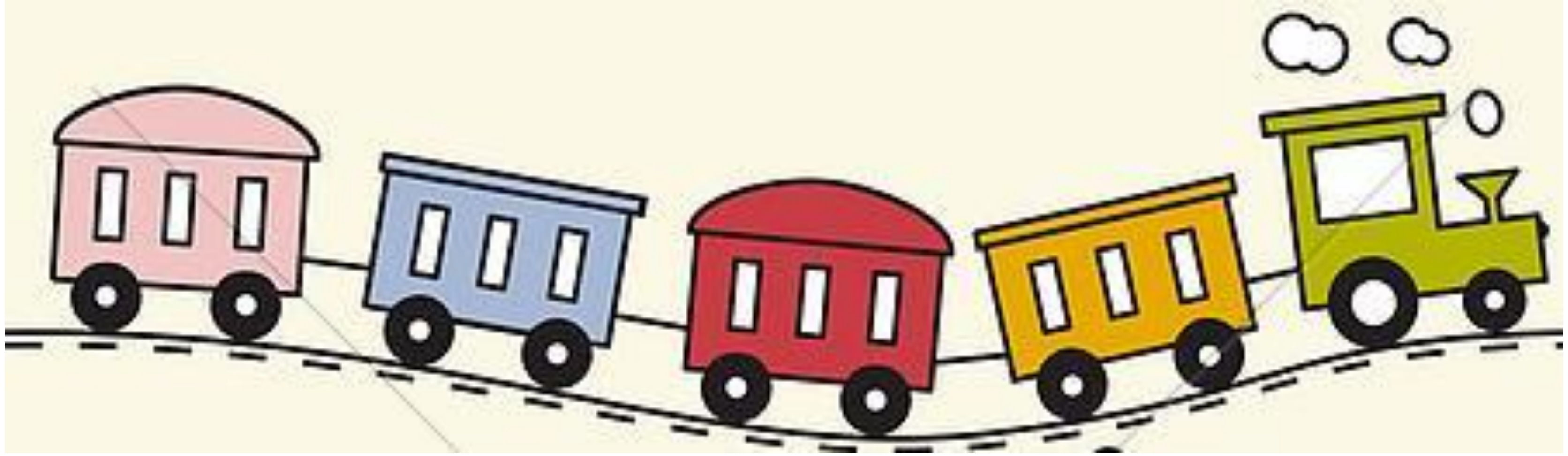
STACK

HASH TABLE

Why arrays are not enough?

- Arrays are good for…
  - Sequential access (cache)
- But not for…
  - Algorithmic stuff on dynamic data
- Why?
  - In an unsorted array, *searching* is slow
    - Linear search instead of binary search
  - But to maintain an array sorted, *inserting* & *deleting* elements is slow
    - Need to shift all elements bigger than the one to insert/delete



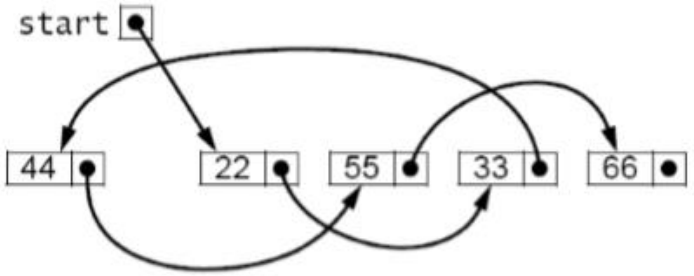


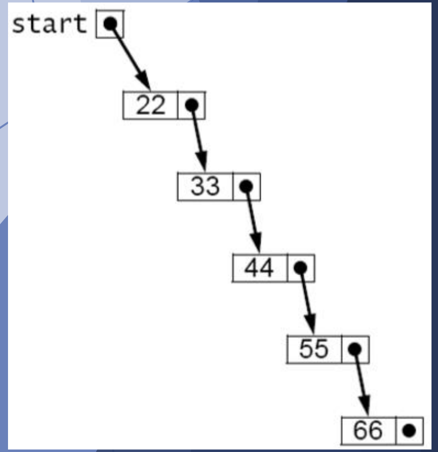
Linked lists

Linked list

- Simple and flexible representation
- Objects are arranged in linear order
  - Order is maintained through the use of *references* inside elements

- Each element (*node*) of a list is made by
  - Its value
  - A reference to the <u>*next*</u> element of the list
- A list is then defined by
  - The starting element
  - All other elements can be reached from there





Linked list operations: SEARCH

- Given a value k and a list L…
  - finds the first element with value k in the list L by a simple linear search
  - if no object with value k appears, the procedure returns NIL

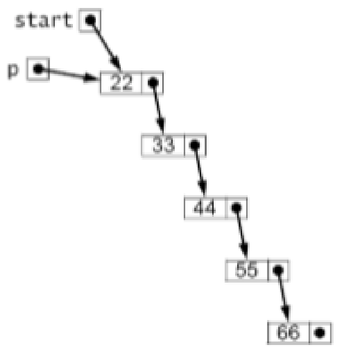
```
LIST-SEARCH(L,k)
  p = L.start
  while p ≠ NIL and p.data ≠ k
    p = p.next
  return p
```

- Complexity (worst case)?
  - $O(n)$ since it may have to search the entire list

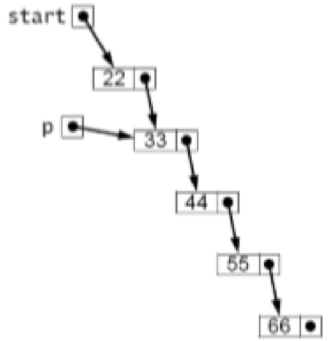
Linked list operations: SEARCH

```
LIST-SEARCH(L,k)
  p = L.start
  while p ≠ NIL and p.data ≠ k
    p = p.next
  return p
```

- Example: looking for $k = 44$
  - First iteration: p is the start node (containing 22)



  - Second iteration: p is the second node (containing 33)



  - Third (and last) iteration: p is the third node (containing $k = 44$)

Linked list operations: INSERT

- Given a value k and a (sorted) list L…
  - finds the right position in the list for k through a simple linear search

looking for the position

```
LIST-INSERT(L,k)
  p = L.start
  while p.next ≠ NIL and p.next.data ≤ k
    p = p.next
```

Linked list operations: INSERT

- Given a value k and a (sorted) list L…
  - finds the right position in the list for k through a simple linear search
  - inserts a new element with value k in such position

looking for the position

```
LIST-INSERT(L,k)
  p = L.start
  while p.next ≠ NIL and p.next.data ≤ k
    p = p.next
  p.next = new Node(k, p.next)
  return L.start
```

insertion of the new node

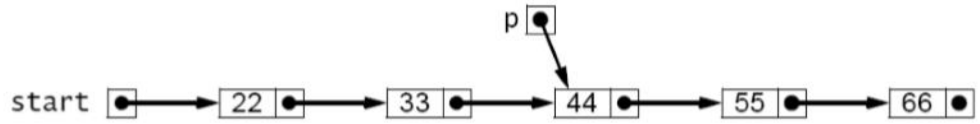
Linked list operations: INSERT

- Example: inserting 50

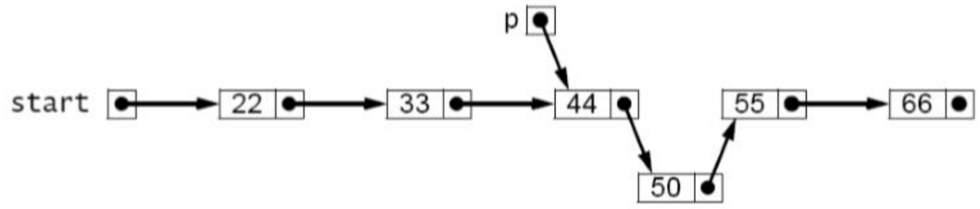
looking for the position

insertion of the new node

```
LIST-INSERT(L,k)
  p = L.start
  while p.next ≠ NIL and p.next.data ≤ k
    p = p.next
  p.next = new Node(k, p.next)
  return L.start
```

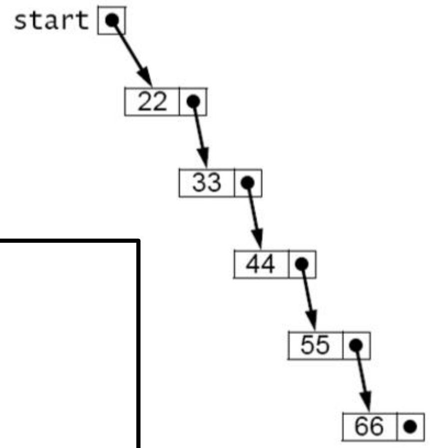


p.next = new Node(50,p.next)



Linked list operations: INSERT

- What if we tried to insert 20 in the previous example?



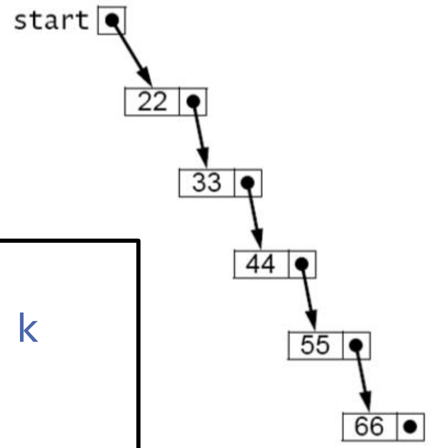

```
LIST-INSERT(L,k)



  p = L.start
  while p.next ≠ NIL and p.next.data ≤ k
    p = p.next
  p.next = new Node(k, p.next)
  return L.start
```

looking for the position

insertion of the new node

Linked list operations: INSERT

- What if we tried to insert 20 in the previous example?
  - Special case: insertion *AT THE FRONT* of the list
    - If the element to insert is smaller than the starting one



insertion at the front (if needed)

looking for the position

insertion of the new node

```
LIST-INSERT(L,k)
  if L.start == NIL or L.start.data > k
    L.start = new Node(k, L.start)
    return L.start

  p = L.start
  while p.next ≠ NIL and p.next.data ≤ k
    p = p.next
  p.next = new Node(k, p.next)
  return L.start
```

Linked list operations: INSERT

- Example:

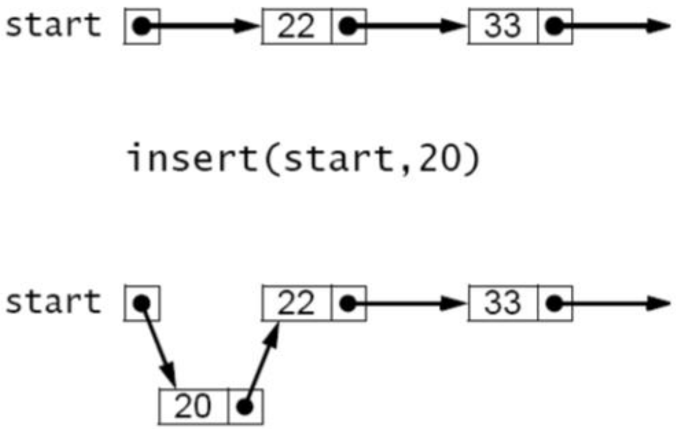
Inserting 20

insertion at the front (if needed)

looking for the position

insertion of the new node

```
LIST-INSERT(L,k)
  if L.start == NIL or L.start.data > k
    L.start = new Node(k, L.start)
    return L.start

  x = L.start
  while x.next ≠ NIL and x.next.data ≤ k
    x = x.next
  x.next = new Node(k, x.next)
  return L.start
```



Linked list operations: DELETE

- Given a value k and a (sorted) list L…
  - finds the first occurrence of the value k in the list through a simple linear search
  - deletes such element (if it exists!)

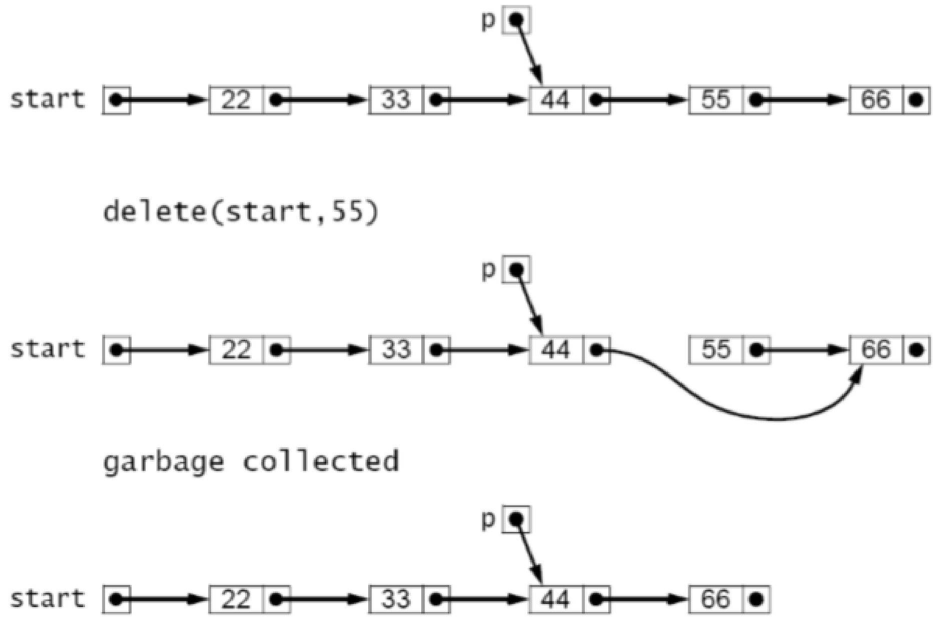
looking for the position

deletion of the node

```
LIST-DELETE(L,k)
  p = L.start
  while p.next ≠ NIL and p.next.data ≤ k
    if p.next.data == k
      p.next = p.next.next
      return L.start
    p = p.next

  return L.start
```

Linked list operations: DELETE

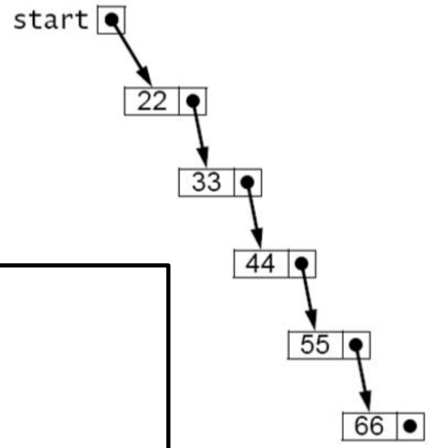
- Example: deleting 55




```
LIST-DELETE(L,k)
  p = L.start
  while p.next ≠ NIL and p.next.data ≤ k
    if p.next.data == k
      p.next = p.next.next
      return L.start
    p = p.next

  return L.start
```

Linked list operations: DELETE

- And if we wanted to delete 22?



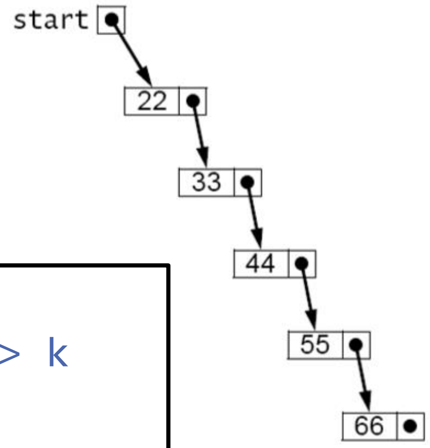

```
LIST-DELETE(L,k)




  x = L.start
  while x.next ≠ NIL and x.next.data ≤ k
    if x.next.data == k
      x.next = x.next.next
      return L.start
    x = x.next

  return L.start
```

looking for the position

deletion of the node

Linked list operations: DELETE

- And if we wanted to delete 22?
  - Special case: deleting *the first element* of the list




```
LIST-DELETE(L,k)
  if L.start == NIL or L.start.data > k
    return L.start
  else if L.start.data == k
    return L.start.next

  x = L.start
  while x.next ≠ NIL and x.next.data ≤ k
    if x.next.data == k
      x.next = x.next.next
      return L.start
    x = x.next

  return L.start
```

element not in the list

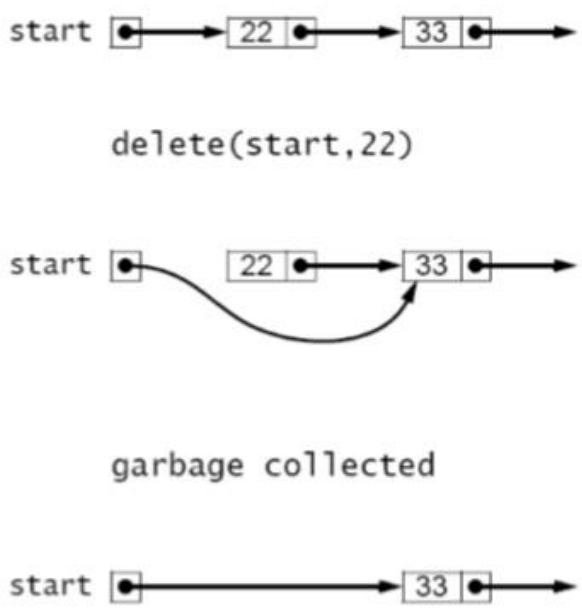
deleting the first element

looking for the position

deletion of the node

Linked list operations: DELETE

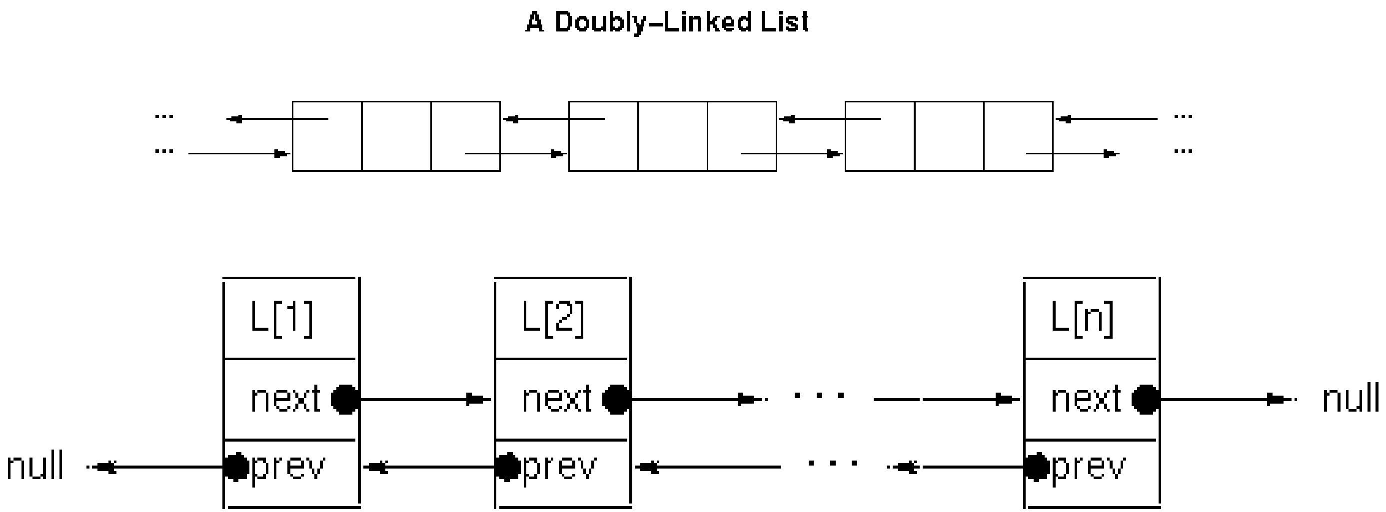
- Example: deleting 22




```
LIST-DELETE(L,k)
  if L.start == NIL or L.start.data > k
    return L.start
  else if L.start.data == k
    return L.start.next

  x = L.start
  while x.next ≠ NIL and x.next.data ≤ k
    if x.next.data == k
      x.next = x.next.next
      return L.start
    x = x.next

  return L.start
```

Doubly linked list

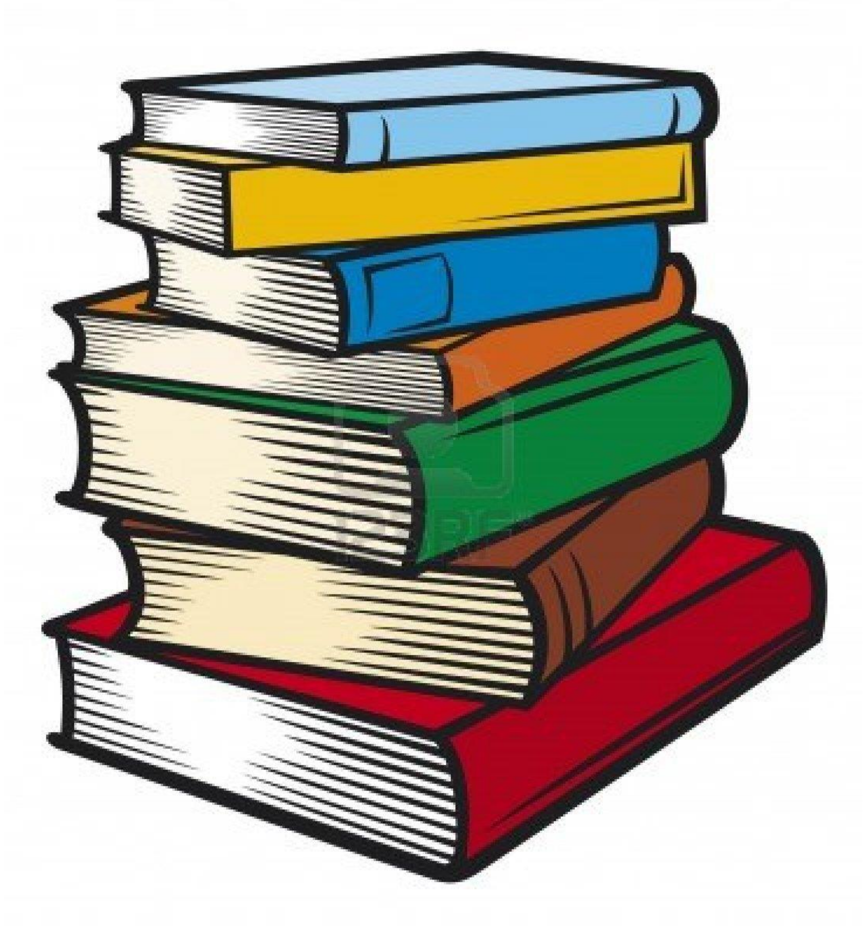
- What if we want to move both forward and backward?
  - Add another reference to the node: the *previous* element in the list



Suggested exercise

- Write the code to...
  - Insert a new node after/before a certain node in a doubly linked list
    - `function **insertAfter**(List list, Node node, Node newNode)`
    - `function **insertBefore**(List list, Node node, Node newNode)`
  - Insert a new node at the beginning of a doubly linked list
    - `function **insertBeginning**(List list, Node newNode)`
  - Delete a certain node in a doubly linked list
    - `function **remove**(Lis list, Node node)`

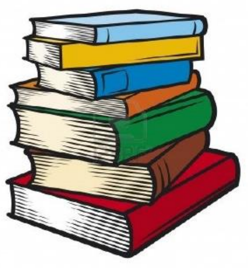
```
DoublyLinkedNode {
    prev // A reference to the previous node
    next // A reference to the next node
    data // Data or a reference to data
}
DoublyLinkedList {
     DoublyLinkedNode firstNode   // points to first node of list
     DoublyLinkedNode lastNode    // points to last node of list
}
```



Stack

Stack – Definition

- Collection implementing the LIFO protocol
  - LIFO = **L**ast **I**n **F**irst **O**ut
  - Only accessible object: last one inserted



- Operations allowed
  - Adding an element onto the top of the stack (**PUSH**)
  - Accessing the current element on the top of the stack (**PEEK**)
  - Removing the current element on the top of the stack (**POP**)

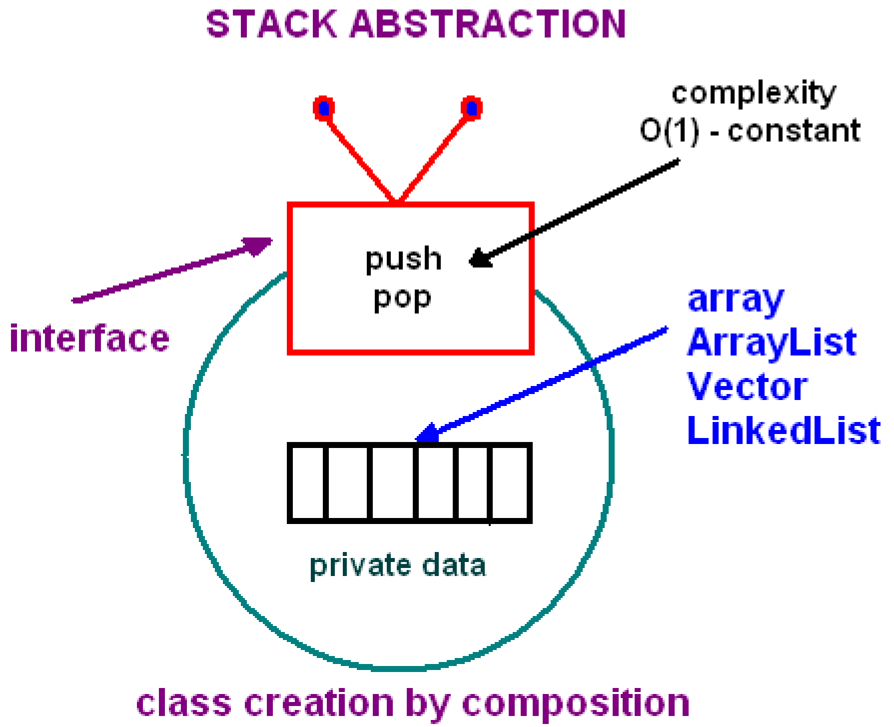
Stack – Implementation

- Built on top of other data structures
  - array, linked list, …
- However, it implements always the same functionality
  - defined by the following interface

```
public interface StackInterface<AnyType>
{
  public void push(AnyType e);
  public AnyType pop();
  public AnyType peek();
  public boolean isEmpty();
}
```

Stack – Implementation

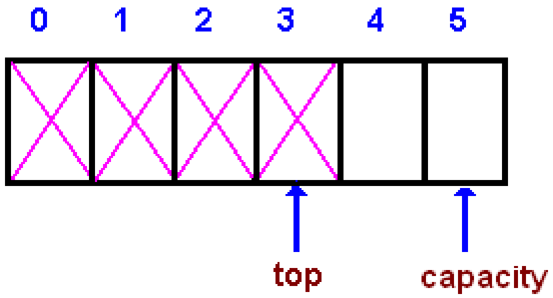
- Built on top of other data structures, but implementing always the same functionality



Stack – Indexed implementation

Fields of the implementation

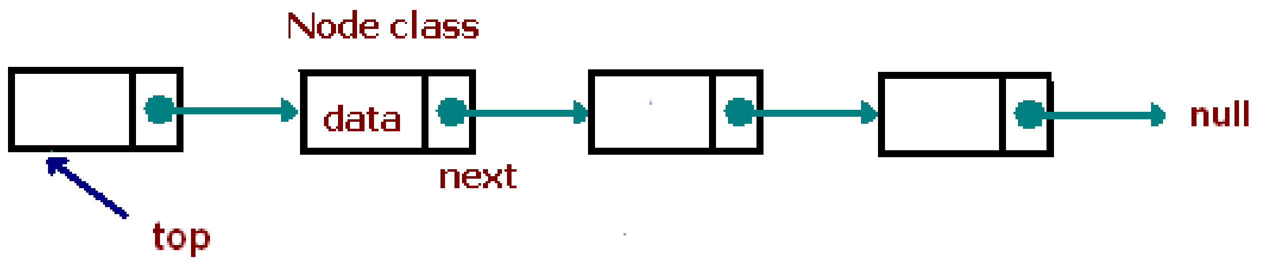
- Array A of a default size
- Variable top (reference to the top element)
- Variable $capacity$ (last index of the array)



- Stack empty $\Leftrightarrow top = -1$
- Stack full $\Leftrightarrow top = capacity$
  - Static implementation → adding another element throws exception
  - Dynamic implementation → double the size of the stack

Stack – Linked implementation

- Best (in efficiency) dynamic stack implementation
  - Be careful at the special case of empty stack
- Top?
  - starting element of the list
- Access (peek)?
  - Read the content of the top
- Push?
  - Create a new node and add it at the beginning of the list
- Pop?
  - Move the beginning of the list at the second element



Queue

Queue – Definition

- Collection implementing the FIFO protocol
  - FIFO = **F**irst **I**n **F**irst **O**ut
  - Only accessible object: <u>first one</u> inserted
    - In the stack it's the opposite (last one inserted)

- Operations allowed
  - Adding an element to the back of the queue (**ENQUEUE**)
  - Accessing the current element at the front of the queue (PEEK)
  - Removing the current element at the front of the queue (**DEQUEUE**)

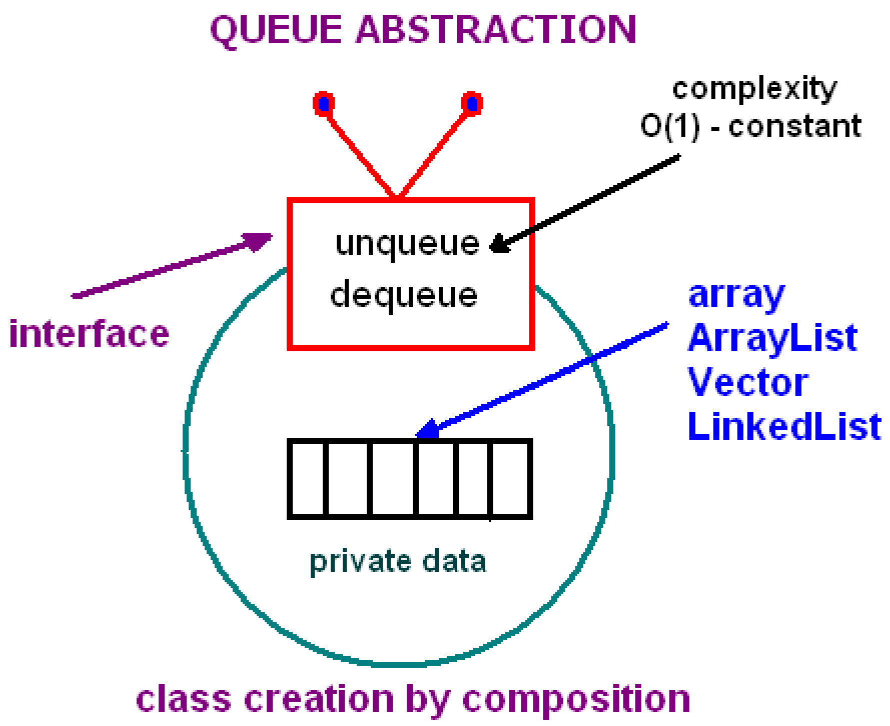
Queue – Implementation

- Built on top of other data structures
  - array, linked list, …
- However, it implements always the same functionality
  - defined by the following interface

```
interface QueueInterface<AnyType>
{
  public void enqueue(AnyType e);
  public AnyType peek();
  public AnyType dequeue();
  public boolean isEmpty();
}
```

Queue – Implementation

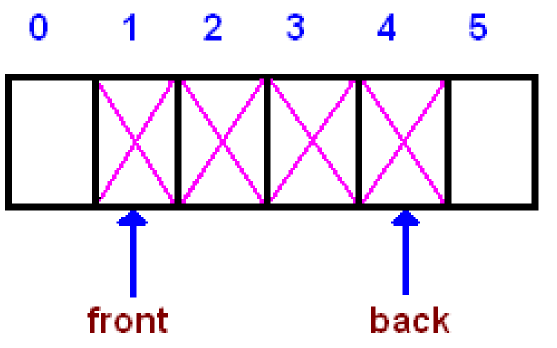
- Built on top of other data structures, but implementing always the same functionality



Queue – Indexed implementation

Fields

- Array A
- Variable $front$ (reference to the front of the queue)
- Variable $back$ (reference to the back of the queue)

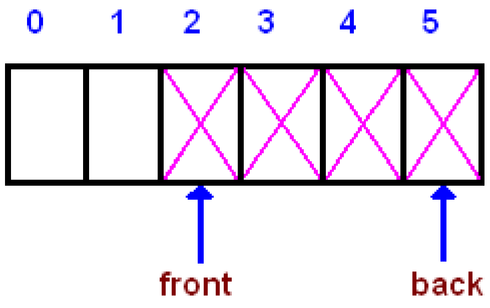


The queue moves in the array from left to right

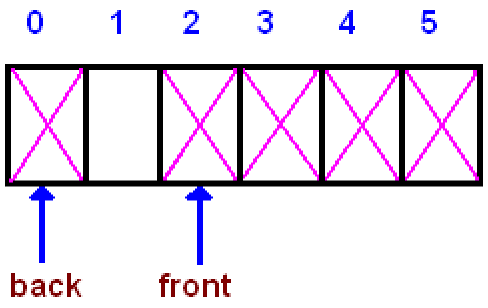
- Inserting a new item (enqueue) → increase the back index
- Removing an item (dequeue) → increase the front index

Queue – Indexed implementation

- What happens when $back$ reaches the end of the array?



- We can use the free space before the front index to store new items
  - *Wrap around queue* or *Circular queue*



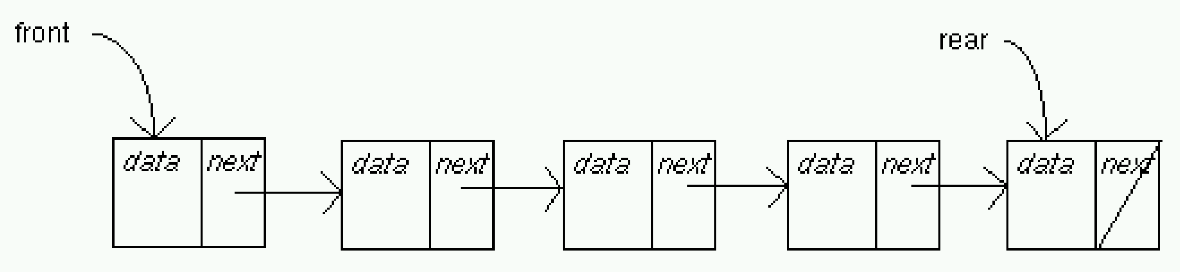
Queue – Indexed implementation

- And what happens when $back$ reaches $front$?
  - The queue is completely full
  - Two choices to handle this situation (as with the stack)
    - Throw exception
    - Double the array size

Queue – Linked implementation

- Almost the same as the stack linked implementation
  - Here we maintain also a pointer to the last element

- Front → starting element of the list
- Rear → last element of the list



- Enqueue
  - Create a new node and add it at the end of the list
- Dequeue
  - Move the beginning of the list at the second element

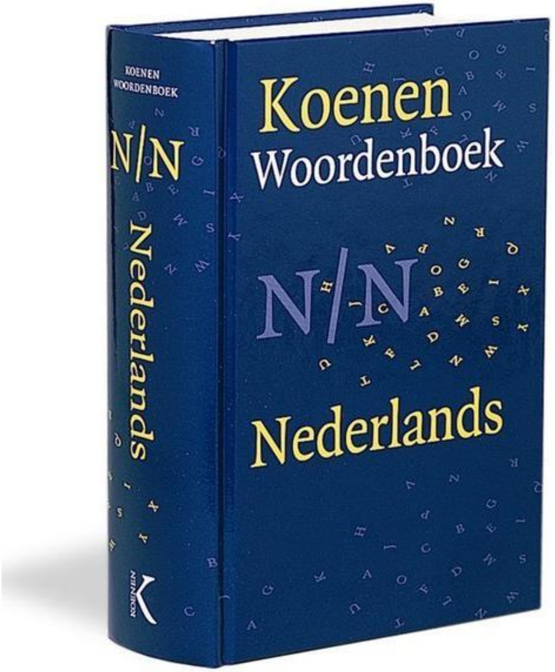
Lists, stacks, queues in .NET

- http://msdn.microsoft.com/en-US/library/ms379570(v=vs.80).aspx
- http://msdn.microsoft.com/en-us/library/ms379571(v=vs.80).aspx

- http://www.dotnetperls.com/list
- http://www.dotnetperls.com/stack
- http://www.dotnetperls.com/queue

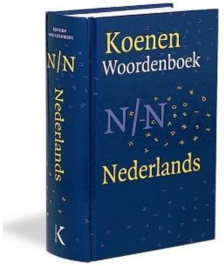
Suggested exercise

- Implement by yourself the generic data structures
  - $Queue < T >$
  - $Stack < T >$
  - $DoublyLinkedList < T >$



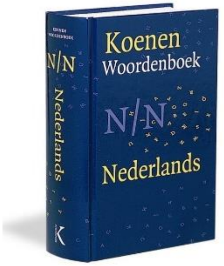
Hash table

Hash table - Definition



- Hash table → data structure used to implement an *associative array*, a structure that can map keys to values
- Hash table → container that allows direct access by any index type: it works like an array or vector except that the index variable need not be an integer
  - Also called: hash map, lookup table, associative array, dictionary
  - Analogy with the dictionary: index = word to look up; value indexed = dictionary definition

Hash table - Definition

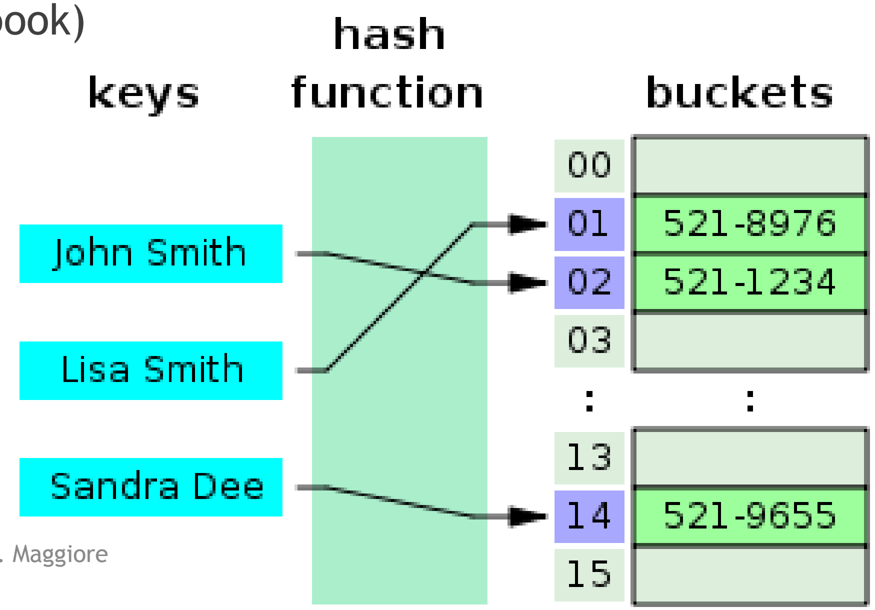


- Hash table → data structure used to implement an *associative array*, a structure that can map keys to values
- Hash table → container that allows direct access by any index type: it works like an array or vector except that the index variable need not be an integer
  - Also called: hash map, lookup table, associative array, dictionary
  - Analogy with the dictionary: index = word to look up; value indexed = dictionary definition

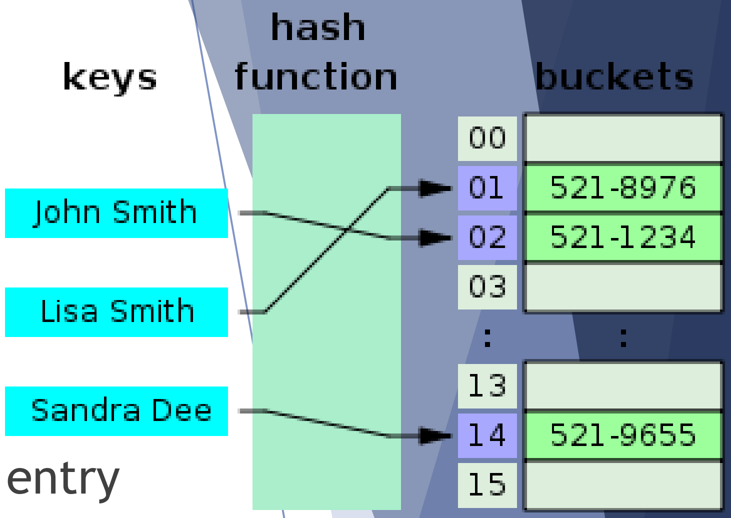
- Entries of a hash table are called "**key-value**" pairs
  - *Key* → index into the table
  - *Value* → information being looked up

Hash table - Definition

- Hashing idea → distribute the entries (key/value pairs) across an array of *buckets* (also called *slots*)
- A **hash function** is used to compute the index in the buckets array, from which the correct value can be found
  - Example (phone book)



Hash table - Definition



- Given a **key**, the algorithm computes an **index** that suggests where the entry can be found:

```
hash = hashfunc(key)
index = hash % array_size
```

- The hash is independent of the array size; it is then reduced to an index (a number between 0 and $array\_size - 1$) using the modulo operator (%)

Hash table - Definition

- In Java and .NET, every object is associated to a hash code (computed from the actual hard data stored in the object), accessible through the methods:
  - [Java] Object.hashCode()
  - [.NET] Object.GetHashCode()
- Example: hash codes of some strings made by three characters

```
Rad: 81909
Uhr: 85023
Ohr: 79257
Tor: 84279
Hut: 72935
Tag: 83834
```

```
hash = hashfunc(key)
```

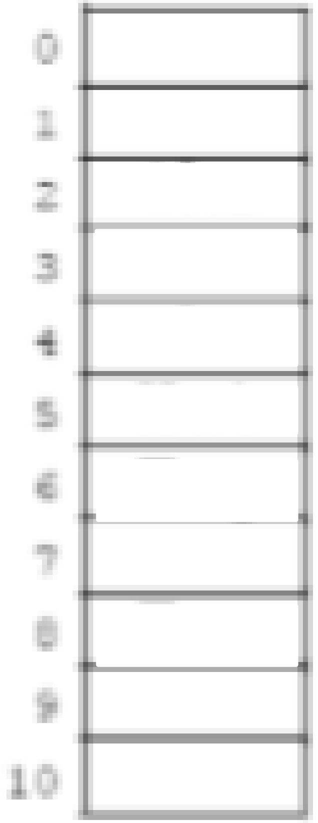
Hash table - Definition

- After computing the hash code, we must compute the index inside the array
  - Suppose that array_size = 11

```
index = hash % array_size
```

  - Index of Rad: 81901 % 11 = 3
  - Index of Uhr: 85023 % 11 = 4
  - Index of Ohr: 79257 % 11 = 2
  - Index of Tor: 84279 % 11 = 8
  - Index of Hut: 72935 % 11 = 5
  - Index of Tag: 83834 % 11 = 3 … same index as for the first string!!!


```
Rad: 81909
Uhr: 85023
Ohr: 79257
Tor: 84279
Hut: 72935
Tag: 83834
```



Hash table – Hash function

- You can also implement your *own hash function*
  - A good hash function and implementation algorithm are **essential** for good hash table performance, but may be difficult to achieve.
  - If all keys are known ahead of time, a *perfect hash function* can be used to create a perfect hash table that has no collisions.
- Basic requirement → the function should provide a *uniform* distribution of hash values (to avoid collisions as much as possible)
  - The hash function should also avoid *clustering* (= the mapping of two or more keys to consecutive slots) if the open addressing method is used to resolve collisions

Hash table – Load factor

- Load factor is a critical statistics for a hash table
  - Good performance depends a lot on it

$$\boldsymbol{loadFactor} = \frac{\#entries}{\#buckets}$$

  - *Entries* = actual number of elements inside the table
  - *Buckets* = capacity of the table (number of total available slots)
    - Example: 6 elements stored in a table with 101 slots → load factor = $\frac{6}{101} = 0.0594 \Rightarrow 5.9\%$

- If the load factor is too large, the hash table becomes slow
  - Possible way to solve the problem: resize the table when the load factor reaches a threshold (usually 75%)

Hash table - Collision resolution

- **Collision** → different keys are assigned by the hash function to the same bucket
  - Ideally, the hash function will assign each key to a unique bucket, but this situation is rarely achievable in practice → collisions are <u>practically unavoidable</u> when hashing a random subset of a large set of possible keys

- Most hash table implementations have some collision resolution strategy to handle such events (all requiring to store the key together with the value inside the table):
  - Separate chaining
  - Open addressing (linear probing, quadratic probing)
  - …

Hash table - Collision resolution with *open addressing*

- Open addressing → when a new entry has to be inserted, the buckets are examined, starting with the hashed-to slot and proceeding in some probe sequence, until an unoccupied slot is found
  - The location ("address") of the item is not determined by its hash value (that's why is called *open addressing*)

- Probing sequences
  - Linear probing
  - Quadratic probing
  - Double hashing
  - …

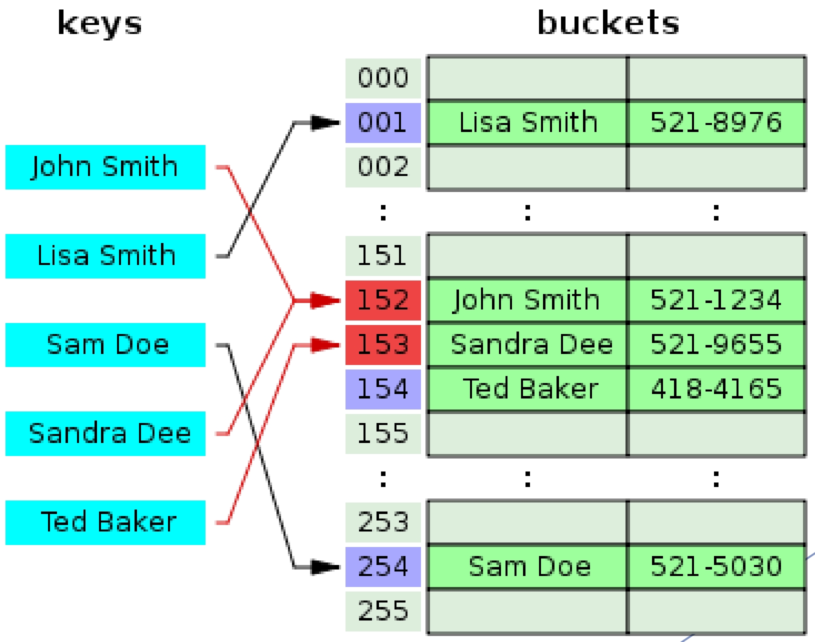
Hash table - Collision resolution with open addressing and *linear probing*

- **Linear probing** → when a new item hashes to a table component that is already in use, the algorithm specifies to *increment the index* until an empty component is found

- Given the hash code H, the probing sequence is
$$H + 1 \rightarrow H + 2 \rightarrow H + 3 \rightarrow H + 4 \rightarrow \cdots$$
  - NB: this may require a “wraparound” back to the beginning of the hash table

Hash table - Collision resolution with open addressing and *linear probing*

- Linear probing examples
  - Tag & Rad from a few slides earlier
  - Sandra Dee; Ted Baker in the phonebook



Hash table - Collision resolution with open addressing and *quadratic probing*

- **Quadratic probing** → taking the original hash index and adding successive values of an arbitrary *quadratic polynomial* until an open slot is found
  - Instead of searching linearly, it uses a squared increment
  - NB: this also may require a “wraparound” back to the beginning of the hash table

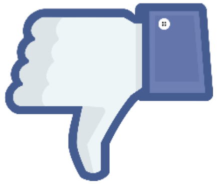
- Given the hash code H, a possible quadratic probing sequence is:

$$H + 1^2 \rightarrow H + 2^2 \rightarrow H + 3^2 \rightarrow H + 4^2 \rightarrow \cdots$$

- Improved performance with respect to linear probing, but it is also more likely to result in an infinite loop…

Hash table - Collision resolution with open addressing

- Open addressing methods drawbacks



1. the number of stored entries cannot exceed the number of slots in the bucket array
   - performance dramatically degrades when the load factor grows beyond 0.7 → dynamic resizing is mandatory
2. more stringent requirements on the hash function
   - besides distributing the keys more uniformly over the buckets, the function must also minimize the clustering of hash values that are consecutive in the probe order

Hash table - Collision resolution with ***separate chaining***

- Instead of *resolving* collisions, we can **avoid** them… how?
  - Allowing more than one item per bucket!
  - Method called "separate chaining" because it uses linked lists ("chains") to hold the multiple items

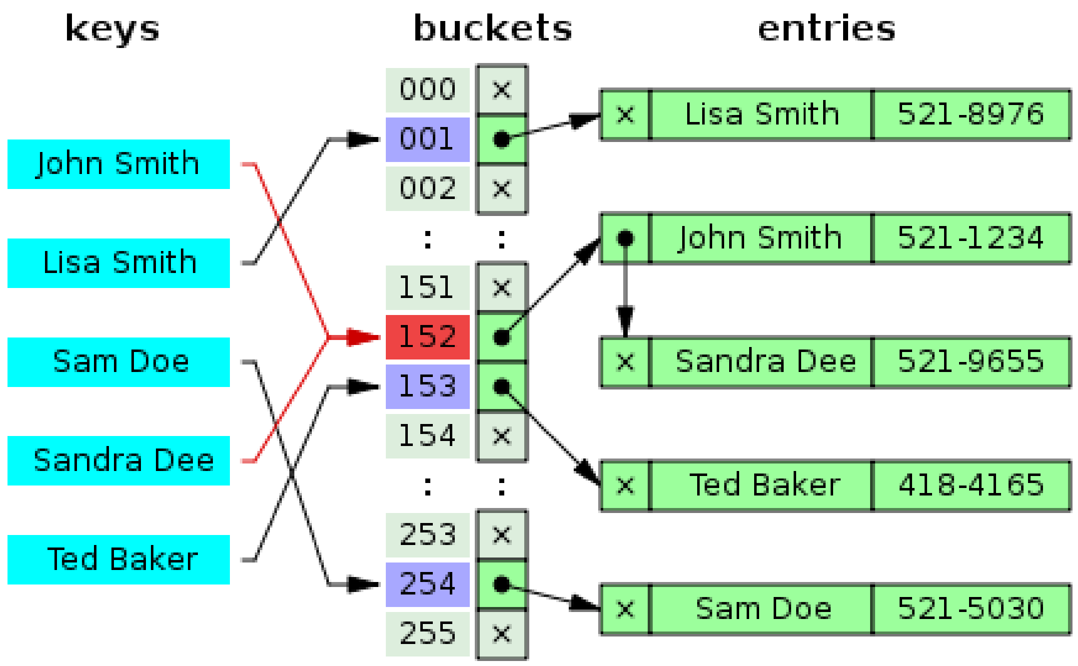
Hash table - Collision resolution with ***separate chaining***

- Instead of *resolving* collisions, we can **avoid** them… how?
  - Allowing more than one item per bucket!
  - Method called “separate chaining” because it uses linked lists (“chains”) to hold the multiple items

- In a good hash table, each bucket has zero or one entries, and sometimes two or three, but rarely more than that
  - Otherwise performance in hash table operations decreases because we have to add the time for the list operation

Hash table - Collision resolution with ***separate chaining***

- Example

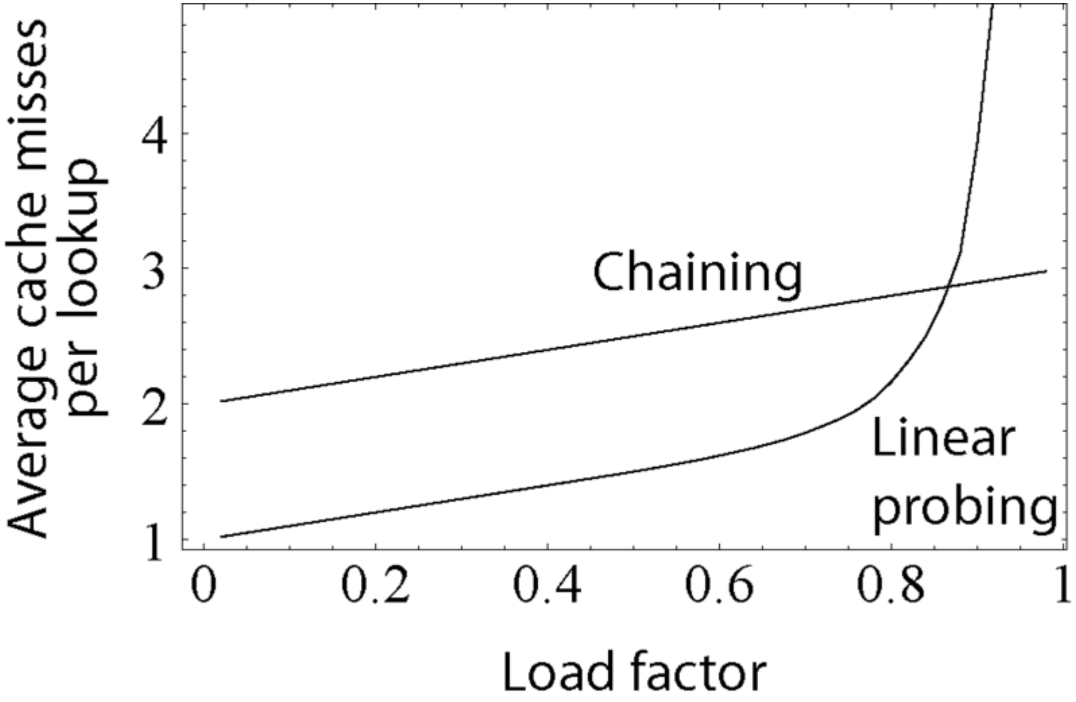


Hash table - Collision resolution with separate chaining

- Which data structure should we use to store the multiple items in each bucket?
  - **Linked lists**
    - Popular because it requires only basic data structures with simple algorithms
    - When storing small keys and values, the space overhead of the next pointer in each entry record can be significant
    - Traversing a linked list has poor cache performance, making the processor cache ineffective
  - Ordered lists, sorted by key field
  - Self-balancing search trees
    - Only worth the trouble and extra memory cost if long delays must be avoided at all costs (e.g. in a real-time application) or if one must guard against many entries hashed to the same slot (e.g. if one expects extremely non-uniform distributions, or in the case of web sites or other publicly accessible services, which are vulnerable to malicious key distributions in requests)
  - Dynamic arrays

Hash table - Collision resolution

- Comparison between the “performance” (seen as the average number of cache misses required to look up elements in tables) with separate chaining and linear probing
  - Linear probing's performance drastically degrades for load factors > 0.8



Hash table – Dynamic resizing

- A hash table functions well when the table size is proportional to the number of entries
- Practical problem: usually the number of entries is not known in advance
  - Very important to provide some method to resize the table in order to prevent the hash table from becoming too full
  - Resizing happens only when the load factor becomes too large
    - In Java the default load factor threshold for table expansion is 0.75; in Python's *dict* 2/3
  - Resizing is accompanied by a *full* or *incremental* table **rehash** whereby existing items are mapped to new bucket locations

Hash table – Dynamic resizing

- *Resizing by copying all entries*
  - Common approach → automatically trigger a complete resizing when the load factor exceeds some threshold
  - All the entries of the old table are removed and inserted into the new table

- *Incremental resizing*
  - Some hash table implementations (especially real-time systems), cannot pay the price of enlarging the hash table all at once: it may interrupt time-critical operations
  - Keep both the old and the new table; do lookups and deletions in both tables; new insertions only in the new one; at each insertion move some elements from the old to the new table until they are all removed (and then deallocate the old table)

Hash table – Performance analysis

- Average case
  - In a well-dimensioned hash table, the average cost (number of instructions) for each lookup is independent of the number of elements stored in the table
  - If the load factor is kept below some bound, the access functions are immediate, running in constant time → direct access, just like an array
- Worst case
  - Worst choice of hash function → every insertion causes a collision → hash tables degenerate to linear search

Operation	Average case	Worst case
Searching	$O(1)$	$O(n)$
Insertion	$O(1)$	$O(n)$
Deletion	$O(1)$	$O(n)$

Hash table – Pros & Cons

- Main advantage
  - **Speed** → particularly efficient when the maximum number of entries can be predicted in advance (no resize)
- Disadvantages
  - The cost of a good hash function can be significantly higher than the inner loop of the lookup algorithm for a sequential list or search tree
    - hash tables not effective when the number of entries is very small
  - Entries can be enumerated only in pseudo-random order
    - no efficient way to locate an entry whose key is *nearest* to a given key → separate sorting step needed
  - With dynamic resizing, an insertion or deletion operation may occasionally take time proportional to the number of entries → problem in real-time or interactive applications
  - Quite inefficient when there are many collisions

Hash table – Applications

- In many situations, hash tables turn out to be more efficient than search trees or any other table lookup structure → widely used in many kinds of computer software
  - systems programming
  - primary building blocks of relational databases
  - associative arrays
  - caches
  - sets
  - …

Hash tables in C#

- **Dictionary** class
  - Generic with respect to the types of keys and values
  - http://www.dotnetperls.com/dictionary
  - http://msdn.microsoft.com/en-us/library/xfhwa508%28v=vs.110%29.aspx

- Live demo?

Summary

- **Array** and **Hash tables**
  - *random access* data structure → each element can be accessed directly and in constant time
- **Linked list**
  - *sequential access* data structure → each element can be accessed only in a particular order
- **Stack & Queue**
  - *limited access* data structures (subcase of sequential data structures)

That’s it

- See you next week ☺