

Type systems

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Possible
solutions

Static typing

Typing rules
and semantic
rules

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Hogeschool Rotterdam
Rotterdam, Netherlands

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Lecture topics

- Issues with Python
- Issues with Python and possible solutions
- Static typing

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Lack of...

- Lack of constraints: how can we specify that a function only takes integers as input
- Lack of structure: how can we specify that a variable will certainly support some methods
- Lack of assurances: how can we guarantee that programs with evident errors are not run

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What is wrong with this?

```
1 def f(x):  
2     return (x * 2)  
3 f("nonsense")
```

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What is wrong with this?

```
1 def f(x):  
2     return (x * 2)  
3 f("nonsense")
```

The function clearly works with integers, but is given a string

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What is wrong with this?

```
1 x = input()
2 if (x > 100):
3     print("dumb")
4 else:
5     print("dumber")
```


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What is wrong with this?

```
1 x = input()
2 if (x > 100):
3     print("dumb")
4 else:
5     print("dumber")
```

The comparison is nonsensical if x is not a number

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```
1 def g(car):  
2     return car.drive(2)  
3 g(-1)
```

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What is wrong with this?

```
1 def g(car):  
2     return car.drive(2)  
3 g(-1)
```

We expect something with a `drive` method, but get an integer instead

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Testing?

- Testing the program should be enough

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Testing?

- Testing the program should be enough
- Right?

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Testing?

- Testing the program should be enough
- Right?
- No. The number of possible execution paths is immense (order of billions), and each test only takes one.
- Testing can only guarantee presence of bugs, but not their absence!

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How many times would we need to test to be sure there is no error?

```
1 if (randint(0,100000) > 99999):  
2     g(-1)  
3 else:  
4     g(mercedesSL500)
```


How many times would we need to test to be sure there is no error?

```
1 if (randint(0,100000) > 99999):  
2     g(-1)  
3 else:  
4     g(mercedesSL500)
```

≥ 100000

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Testing?

- We want our programming languages to perform checks for us
- Clearly nonsensical programs should be rejected before we can even run them
- It is safer and easier to spend more time "talking" with the IDE than hoping to find all errors at runtime

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- The language verifies^a, before running code, that all variables are correctly used
- "Correctly used" means that they are guaranteed to support all operations used on them
- This is by far and large the most typical solution to increase safety and productivity

^aBy means of the **compiler**.

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What is static typing?

- When declaring a variable, we also specify what sort of data it will contain
- The **sort** of data contained is called **TYPE** of the variable
- Types can be either primitives (int, string, etc.), custom (classes), or compositions (functions, list of elements of a given type, etc.)

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What is static typing?

- Especially in mainstream languages, the specification of the type of a variable is done by hand by the programmer
- In other languages (mostly functional languages like F#, Haskell, etc.) the type of variables is automatically guessed by the compiler
- In our case our programs will become a bit more verbose but better specified
- Still, static typing is not necessarily connected with verbosity

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A variable declaration in C# or Java is prefixed by the type of the variable

- `int x;` declares an integer variable
- `string s;` declares a string variable
- `float f;` declares a floating point variable
- ...

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```
1 def f(x):  
2     return (x * 2)
```

Becomes, typed:

What has improved and why?

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```
1 def f(x):  
2     return (x * 2)
```

Becomes, typed:

What has improved and why?

The second definition encodes information about what goes in and what comes out of the function

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Is this still possible to write (as it was in Python)?

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Is this still possible to write (as it was in Python)?

No: we get a compiler error because a string cannot be used where a number is expected

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```
1 x = input()
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3     print("dumb")
4 else:
5     print("dumber")
```

Becomes, typed:

What has improved and why?

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```
1 x = input()
2 if (x > 100):
3     print("dumb")
4 else:
5     print("dumber")
```

Becomes, typed:

What has improved and why?

The variable declaration specifies what is allowed (and what is not) inside the variable.

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```
1 def g(car):  
2     return car.drive(2)  
3 g(-1)
```

Becomes, typed:

What has improved and why?

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```
1 def g(car):  
2     return car.drive(2)  
3 g(-1)
```

Becomes, typed:

What has improved and why?

The function declaration specifies that `car` is an instance of the `Car` class. We will thus get a compiler error.

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How do we describe them?

- How do we describe such relations clearly?
- We use the so-called **typing rules**, which specify what may be done and what not
- Typing rules are quite intuitive: they state that if one or more premises are true, then the conclusion is true as well

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$$\frac{A \wedge B}{C}$$

If A and B are true, then we can conclude C

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$$\frac{\text{I wish to buy a pretty car} \wedge \text{I have 120000 euros}}{\text{I buy a Mercedes SL500}}$$

How do we read this rule?

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$$\frac{\text{I wish to buy a pretty car} \wedge \text{I have 120000 euros}}{\text{I buy a Mercedes SL500}}$$

How do we read this rule?

If I have 120000 euros and I wish to buy a pretty car, then I buy a Mercedes SL500

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It is raining \wedge I have my umbrella with me
I open my umbrella

How do we read this rule?

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It is raining \wedge I have my umbrella with me
I open my umbrella

How do we read this rule?

If I have my umbrella with me, and it is raining, then I open my umbrella

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Reading typing rules

Let us apply this machinery to programming languages

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Reading typing rules

- Let us apply this machinery to programming languages
- We will effectively give the specification of a modern compiler
- This looks like a “broadly scoped” execution of the program, and it is indeed such
- This process is called type checking

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- We want to specify this in the typing rule notation
- The typing rules manipulate a stack of declarations which we will call D
- Each typing rule will add or remove variable declarations and return the type of the current expression
- Instead of coupling each variable with its value, we couple it with its type

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- The simplest typing rule is the one that finds a variable declaration
- A declaration adds to the declarations D the variable, connected with its type

$$\overline{\langle (T \ v;), D \rangle \rightarrow \langle \text{void}, D[v \mapsto T] \rangle}$$

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- When we look the variable up, its type is whatever type was found connected to it in the declarations
- This does further nothing to the declarations
- Let's assume that x is a variable name

$$\overline{\langle x, D \rangle} \rightarrow \langle D[x], D \rangle$$

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- Another simple typing rule is the one that types a constant value
- It does nothing to the declarations
- Let's assume that `i` is an integer constant

$$\overline{\langle i, D \rangle} \rightarrow \overline{\langle \text{int}, D \rangle}$$

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- Let's assume that `f` is a floating point constant

$$\overline{\langle f, D \rangle} \rightarrow \langle \text{float}, D \rangle$$

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- Let's assume that s is a string constant

$$\overline{\langle s, D \rangle} \rightarrow \langle \text{string}, D \rangle$$

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- Let's assume that b is a boolean constant

$$\overline{\langle b, D \rangle} \rightarrow \overline{\langle \text{bool}, D \rangle}$$

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Reading typing rules

More complex typing rules compose together the types of different statements

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- The typing rule for operators such as `+` requires the operands to be compatible
- The type of both operands is often the same, for example `int` or `float`
- The resulting type is then the type of both operands
- Operands do not modify the current declarations

$$\frac{\langle a, D \rangle \rightarrow \langle \text{int}, D \rangle \wedge \langle b, D \rangle \rightarrow \langle \text{int}, D \rangle}{\langle (a + b), D \rangle \rightarrow \langle \text{int}, D \rangle}$$

$$\frac{\langle a, D \rangle \rightarrow \langle \text{float}, D \rangle \wedge \langle b, D \rangle \rightarrow \langle \text{float}, D \rangle}{\langle (a + b), D \rangle \rightarrow \langle \text{float}, D \rangle}$$

$$\frac{\langle a, D \rangle \rightarrow \langle \text{string}, D \rangle \wedge \langle b, D \rangle \rightarrow \langle \text{string}, D \rangle}{\langle (a + b), D \rangle \rightarrow \langle \text{string}, D \rangle}$$

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- The type of both operands could differ, but still be compatible (for example adding an `int` and a `float`)
- The resulting type is then the most generic type of the operands
- Operands do not modify the current declarations

$$\frac{\langle a, D \rangle \rightarrow \langle \text{int}, D \rangle \wedge \langle b, D \rangle \rightarrow \langle \text{float}, D \rangle}{\langle (a + b), D \rangle \rightarrow \langle \text{float}, D \rangle}$$
$$\frac{\langle a, D \rangle \rightarrow \langle \text{float}, D \rangle \wedge \langle b, D \rangle \rightarrow \langle \text{int}, D \rangle}{\langle (a + b), D \rangle \rightarrow \langle \text{float}, D \rangle}$$

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- Statements in a sequence both modify, top-to-bottom, the declarations
- Usually we expect the statements to simply return nothing, that is `void`
- Further we cannot say anything about what they each do

$$\frac{\langle a, D \rangle \rightarrow \langle \text{void}, D_1 \rangle \wedge \langle b, D_1 \rangle \rightarrow \langle \text{void}, D_2 \rangle}{\langle (a; b), D \rangle \rightarrow \langle \text{void}, D_2 \rangle}$$

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- The typing rule for an `if-then-else` requires the condition to be a boolean expression, and assumes the type of both the then and the else bodies
- The type of both the then and the else bodies must be the same (usually `void`, something else in case of function returns)
- It does not add anything to the declarations, even though the bodies of the then and the else might declare local variables

$$\frac{\langle c, D \rangle \rightarrow \langle \text{bool}, D \rangle \wedge \langle A, D \rangle \rightarrow \langle T, D' \rangle \wedge \langle B, D \rangle \rightarrow \langle U, D' \rangle \wedge T = U}{\langle (\text{if } c \{ A \} \text{else} \{ B \}), D \rangle \rightarrow \langle T, D \rangle}$$

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- The typing rule for a `while` loop requires the condition to be a boolean expression, and assumes the type of the body
- The type of the body can be anything (usually `void`, something else in case of function returns)
- It does not add anything to the declarations, even though the body might declare local variables

$$\frac{\langle c, D \rangle \rightarrow \langle \text{bool}, D \rangle \wedge \langle B, D \rangle \rightarrow \langle T, D_1 \rangle}{\langle (\text{while } c \{ A \}), D \rangle \rightarrow \langle T, D \rangle}$$

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- The typing rule for a class declaration adds the class declaration to the declarations with all its fields and methods
- When adding the declaration of the class, we have to check that the types of the method bodies match their declarations
- Assume that C is the class name, f_i is the i -th field in the class (of type F_i), and m_j is the j -th method in the class (with type M_j)

$$\frac{D_1 := D[C \mapsto [\dots f_i \mapsto F_i \dots m_j \mapsto M_j \dots]] \wedge \langle M_j \ m_j, D_1[\text{this} \mapsto C] \rangle \rightarrow \langle M_j^1, D_2 \rangle \wedge M_j = M_j^1}{\langle (\text{class } C \{ \dots F_i \ f_i \dots \dots M_j \ m_j \dots \}), D \rangle \rightarrow \langle T, D_1 \rangle}$$

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- When type checking a method declaration (within a class declaration we type check its body and compare the result with the type of the declaration
- Assume that C is the class name, p_i is the i -th parameter of the method (of type P_i), and b is the method body
- The type of a method is of the form $P_1 \times P_2 \times \dots \times P_n \rightarrow R$, where P_l is the type of the l -th parameter and R is the return type

$$\frac{\langle\langle b \rangle, D[.p_l \mapsto P_l.]\rangle \rightarrow \langle R, D_1 \rangle}{\langle\langle R \text{ m}(\dots P_1 \text{ } p_1 \dots) b \rangle, D \rangle \rightarrow \langle\langle P_1 \times P_2 \times \dots \times P_n \rightarrow R \rangle, D \rangle}$$

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- When type checking a `return` statement, we typecheck its argument
- The type of the argument is also the type of the `return` statement
- There is no change to the declarations

$$\frac{\langle x, D \rangle \rightarrow \langle T, D \rangle}{\langle (\text{return } x), D \rangle \rightarrow \langle T, D \rangle}$$

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- Statements in a sequence might contain return statements
- In this case one of them might not return void
- Their sequence will assume the non-void type

$$\frac{\langle a, D \rangle \rightarrow \langle T, D_1 \rangle \wedge \langle b, D_1 \rangle \rightarrow \langle \text{void}, D_2 \rangle}{\langle (a; b), D \rangle \rightarrow \langle T, D_2 \rangle}$$
$$\frac{\langle a, D \rangle \rightarrow \langle \text{void}, D_1 \rangle \wedge \langle b, D_1 \rangle \rightarrow \langle T, D_2 \rangle}{\langle (a; b), D \rangle \rightarrow \langle T, D_2 \rangle}$$

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- Statements in a sequence might contain return statements
- They might both return a non-void type
- Their sequence will assume the non-void type of both, which must be the same

$$\frac{\langle a, D \rangle \rightarrow \langle T, D_1 \rangle \wedge \langle b, D_1 \rangle \rightarrow \langle U, D_2 \rangle \wedge T = U}{\langle (a; b), D \rangle \rightarrow \langle T, D_2 \rangle}$$

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- Sometimes we may look a field f up from an instance x of a class
- This assumes the type of the field, which needs to be looked up in the class descriptor found in the declarations
- No declaration is further modified

$$\frac{\langle x, D \rangle \rightarrow \langle C, D \rangle \wedge \langle f, C \rangle \rightarrow \langle F, C \rangle \wedge T = U}{\langle (x.f), D \rangle \rightarrow \langle F, D \rangle}$$

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- Sometimes we may call a method m up from an instance x of a class and with parameters p_i
- This assumes the return type of the method, provided that all parameter types match the types expected by the method
- No declaration is further modified

$$\frac{\langle x, D \rangle \rightarrow \langle C, D \rangle \wedge \langle m, C \rangle \rightarrow \langle (P_1 \times P_2 \times \dots \times P_n \rightarrow R), C \rangle \wedge \langle p_i, D \rangle \rightarrow \langle P'_i, D \rangle \wedge P_i = P'_i}{\langle (x.f(..p_i..)), D \rangle \rightarrow \langle R, D \rangle}$$

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- We may call a static method m from class C and with parameters p_i
- This assumes the return type of the method, provided that all parameter types match the types expected by the method
- We do not need to look up the class because it is already specified in the call
- No declaration is further modified

$$\frac{\langle \mathfrak{m}, C \rangle \rightarrow \langle (P_1 \times P_2 \times \dots \times P_n \rightarrow R), C \rangle \wedge \langle p_i, D \rangle \rightarrow \langle P'_i, D \rangle \wedge P_i = P'_i}{\langle (C.f(..p_i..)), D \rangle \rightarrow \langle R, D \rangle}$$

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Reading typing rules

- The constructor of a class is simply a specially named static method
- It has no further typing rules

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Looking back

- Issues with Python
- Static typing as a way to run a coarse simulation of the program

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The best of luck, and thanks for the
attention!