

Type systems

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Lecture topics

- Issues with Python
- Issues with Python and possible solutions
- Static typing



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Lack of...

- Lack of constraints: how can we specify that a function only takes integers as input
- Lack of structure: how can we specify that a variable will certainly support some methods
- Lack of assurances: how can we guarantee that programs with evident errors are not run

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What is wrong with this?

```
def f(x):
   return (x * 2)
f("nonsense")
```

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What is wrong with this?

```
def f(x):
   return (x * 2)
f("nonsense")
```

The function clearly works with integers, but is given a string

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What is wrong with this?

```
x = input()
if (x > 100):
  print("dumb")
else:
  print("dumber")
```

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What is wrong with this?

```
x = input()
if (x > 100):
   print("dumb")
else:
   print("dumber")
```

The comparison is nonsensical if x is not a number



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What is wrong with this?

```
def g(car):
   return car.drive(2)
g(-1)
```



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What is wrong with this?

```
def g(car):
   return car.drive(2)
g(-1)
```

We expect something with a drive method, but get an integer instead



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Testing?

Testing the program should be enough



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Testing?

- Testing the program should be enough
- Right?



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Testing?

- Testing the program should be enough
- Right?
- No. The number of possible execution paths is immense (order of billions), and each test only takes one.
- Testing can only guarantee presence of bugs, but not their absence!



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How many times would we need to test to be sure there is no error?

```
if (randint(0,100000) > 99999):
   g(-1)
else:
   g(mercedesSL500)
```



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How many times would we need to test to be sure there is no error?

```
if (randint(0,100000) > 999999):
    g(-1)
else:
    g(mercedesSL500)
```

```
> 100000
```



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Testing?

- We want our programming languages to perform checks for us
- Clearly nonsensical programs should be rejected before we can even run them
- It is safer and easier to spend more time "talking" with the IDE than hoping to find all errors with tests



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- The language verifies^a, before running code, that all variables are correctly used
- "Correctly used" means that they are guaranteed to support all operations used on them
- This is by far and large the most typical solution to increase safety and productivity

^aBy means of the compiler.



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What is static typing?

- When declaring a variable, we also specify what sort of data it will contain
- The sort of data contained is called TYPE of the variable
- Types can be either primitives (int, string, etc.), custom (classes), or compositions (functions, list of elements of a given type, etc.)



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What is static typing?

- Especially in mainstream languages, the specification of the type of a variable is done by hand by the programmer
- In other languages (mostly functional languages like F#, Haskell, etc.) the type of variables is automatically 'guessed' by the compiler
- In our case our programs will become a bit more verbose but better specified
- Still, static typing is not necessarily connected with verbosity



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A variable declaration in C# or Java is prefixed by the type of the variable

- int x; declares an integer variable
- string s; declares a string variable
- float f; declares a floating point variable
- ...

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```
def f(x):
   return (x * 2)
```

Becomes, typed:

```
int f(int x) {
  return (x * 2);
}
```

What has improved and why?



```
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```

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```
def f(x):
  return (x * 2)
```

Becomes, typed:

```
int f(int x) {
  return (x * 2);
}
```

What has improved and why?

The second definition encodes information about what goes in and what comes out of the function



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Is this still possible to write (as it was in Python)?

```
int f(int x) {
  return (x * 2);
}
f("nonsense");
```



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Is this still possible to write (as it was in Python)?

```
int f(int x) {
  return (x * 2);
}
f("nonsense");
```

No: we get a compiler error because a string cannot be used where a number is expected



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```
x = input()
if (x > 100):
  print("dumb")
else:
  print("dumber")
```

Becomes, typed:

```
int x = Int32.Parse(Console.ReadLine()):
if (x > 100) {
  Console.WriteLine("safe"):
 } else {
  Console.WriteLine("safer");
```

What has improved and why?



```
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```
1  x = input()
2  if (x > 100):
3    print("dumb")
4  else:
5    print("dumber")
```

Becomes, typed:

```
int x = Int32.Parse(Console.ReadLine());
if (x > 100) {
   Console.WriteLine("safe");
} else {
   Console.WriteLine("safer");
}
```

What has improved and why?

The variable declaration specifies what is allowed (and what is not) inside the variable.



```
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```

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```
def g(car):
return car.drive(2)
g(-1)
```

Becomes, typed:

```
int g(Car car) {
  return car.drive(2);
}
g(-1);
```

What has improved and why?



```
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```

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```
def g(car):
return car.drive(2)
g(-1)
```

Becomes, typed:

```
int g(Car car) {
  return car.drive(2);
}
g(-1);
```

What has improved and why?

The function declaration specifies that car is an instance of the Car class. We will thus get a compiler error.



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How do we describe them?

- How do we describe such relations clearly?
- We use the so-called typing rules, which specify what may be done and what not
- Typing rules are quite intuitive: they state that if one or more premises are true, then the conclusion is true as well



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$$\frac{A \wedge B}{C}$$

If A and B are true, then we can conclude C



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 $\frac{\text{I wish to buy a pretty car} \land \text{I have } 120000 \text{ euros}}{\text{I buy a Mercedes SL500}}$

How do we read this rule?



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 $\frac{\text{I wish to buy a pretty car} \land \text{ I have } 120000 \text{ euros}}{\text{I buy a Mercedes SL500}}$

How do we read this rule?

If I have 120000 euros and I wish to buy a pretty car, then I buy a Mercedes $\mathsf{SL}500$



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 $\frac{\text{It is raining} \wedge \text{ I have my umbrella with me}}{\text{I open my umbrella}}$

How do we read this rule?



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Conclusion

 $\frac{\text{It is raining} \wedge \text{ I have my umbrella with me}}{\text{I open my umbrella}}$

How do we read this rule?

If it is raining, and I have my umbrella with me, then I open my umbrella



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Reading typing rules

Let us apply this machinery to programming languages



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Reading typing rules

- Let us apply this machinery to programming languages
- We will effectively give the specification of a modern compiler
- This looks like a "broadly scoped" execution of the program, and it is indeed such
- This process is called type checking



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Reading typing rules

- We want to specify this in the typing rule notation
- The typing rules manipulate a stack of declarations which we will call D
- Each typing rule will add or remove variable declarations and return the type of the current expression
- Instead of coupling each variable with its value, we couple
 it with its type



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- The simplest typing rule is the one that finds a variable declaration
- A declaration adds to the declarations D the variable, connected with its type

$$\overline{\langle (\mathtt{T}\ \mathtt{v};), D \rangle \to \langle \mathtt{void}, D[v \mapsto T] \rangle}$$



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Declarations:

←ロト→個ト→差ト→差 990

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int x;

PC	
2	i

Declarations:

◆ロト ◆個 ト ◆ 恵 ト ◆ 恵 ・ 夕 Q ②



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- When we look the variable up, its type is whatever type was found connected to it in the declarations
- This does further nothing to the declarations
- Let's assume that x is a variable name

$$\overline{\langle \mathtt{x}, D \rangle \to \langle D[\mathtt{x}], D \rangle}$$



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```
1 int x = 10;
x = (x + 5);
```

```
Declarations: PC 1
```



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```
int x = 10;
x = (x + 5);
```

```
Declarations: \begin{array}{|c|c|c|c|}\hline PC & x \\ \hline 2 & int \\ \hline \end{array}
```



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rules

```
Declarations: PC x 3 int
```



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- Another simple typing rule is the one that types a constant value
- It does nothing to the declarations
- Let's assume that i is an integer constant

$$\overline{\langle \mathtt{i}, D \rangle o \langle \mathtt{int}, D \rangle}$$



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• Let's assume that f is a floating point constant

$$\overline{\langle \mathtt{f}, D \rangle \to \langle \mathtt{float}, D \rangle}$$



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• Let's assume that s is a string constant

$$\overline{\langle \mathtt{s}, D \rangle \to \langle \mathtt{string}, D \rangle}$$



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• Let's assume that b is a boolean constant

$$\overline{\langle \mathtt{b}, D \rangle \to \langle \mathtt{bool}, D \rangle}$$



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Reading typing rules

More complex typing rules compose together the types of different statements



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- The typing rule for operators such as + requires the operands to be compatible
- The type of both operands is often the same, for example int or float
- The resulting type is then the type of both operands
- Operands do not modify the current declarations

$$\frac{\langle \mathbf{a}, D \rangle \to \langle \mathbf{int}, D \rangle \wedge \langle \mathbf{b}, D \rangle \to \langle \mathbf{int}, D \rangle}{\langle (\mathbf{a} + \mathbf{b}), D \rangle \to \langle \mathbf{int}, D \rangle}$$

$$\frac{\langle \mathbf{a}, D \rangle \to \langle \mathbf{float}, D \rangle \wedge \langle \mathbf{b}, D \rangle \to \langle \mathbf{float}, D \rangle}{\langle (\mathbf{a} + \mathbf{b}), D \rangle \to \langle \mathbf{float}, D \rangle}$$

$$\frac{\langle \mathbf{a}, D \rangle \to \langle \mathbf{string}, D \rangle \wedge \langle \mathbf{b}, D \rangle \to \langle \mathbf{string}, D \rangle}{\langle (\mathbf{a} + \mathbf{b}), D \rangle \to \langle \mathbf{string}, D \rangle}$$



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- The type of both operands could differ, but still be compatible (for example adding an int and a float)
- The resulting type is then the most generic type of the operands
- Operands do not modify the current declarations

$$\frac{\langle \mathtt{a}, D \rangle \to \langle \mathtt{int}, D \rangle \wedge \langle \mathtt{b}, D \rangle \to \langle \mathtt{float}, D \rangle}{\langle (\mathtt{a} + \mathtt{b}), D \rangle \to \langle \mathtt{float}, D \rangle}$$

$$\frac{\langle \mathbf{a}, D \rangle \to \langle \mathbf{float}, D \rangle \land \langle \mathbf{b}, D \rangle \to \langle \mathbf{int}, D \rangle}{\langle (\mathbf{a} + \mathbf{b}), D \rangle \to \langle \mathbf{float}, D \rangle}$$



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- Statements in a sequence both modify, top-to-bottom, the declarations
- Usually we expect the statements to simply return nothing, that is void
- Further we cannot say anything about what they each do

$$\frac{\langle \mathbf{a}, D \rangle \to \langle \mathrm{void}, D_1 \rangle \wedge \langle \mathbf{b}, D_1 \rangle \to \langle \mathrm{void}, D_2 \rangle}{\langle (\mathbf{a}; \mathbf{b}), D \rangle \to \langle \mathrm{void}, D_2 \rangle}$$



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Tunina mulaa

Typing rules and semantic rules

```
int x = 10;
int y = 20;
x = (x + y);
```

```
Declarations: PC 1
```

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```
int x = 10;
int y = 20;
x = (x + y);
```

PC	×
2	int

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```
int x = 10;
int y = 20;
x = (x + y);
```

PC	×	у
3	int	int

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```
int x = 10;
int y = 20;
x = (x + y);
```

PC	×	у
4	int	int



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- The typing rule for an if-then-else requires the condition to be a boolean expression, and assumes the type of both the then and the else bodies
- The type of both the then and the else bodies must be the same (usually void, something else in case of function returns)
- It does not add anything to the declarations, even though the bodies of the then and the else might declare local variables

$$\frac{\langle \mathtt{c}, D \rangle \to \langle \mathtt{bool}, D \rangle \wedge \ \langle \mathtt{A}, D \rangle \to \langle \mathtt{T}, D' \rangle \wedge \ \langle \mathtt{B}, D \rangle \to \langle \mathtt{U}, D' \rangle \wedge \ T = U}{\langle (\mathtt{if} \ \mathtt{c} \ \{ \ \mathtt{A} \ \} \mathtt{else} \{ \ \mathtt{B} \ \}), D \rangle \to \langle \mathtt{T}, D \rangle}$$



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

Declarations: PC 1



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

Declarations: PC x 2 int



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

Declarations: PC

PC	×	у
3	int	int



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

X	у	PC
int	int	4



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

×	у	PC	z
int	int	5	string



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

PC	×	У
6	int	int



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

×	у	PC
int	int	7



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

×	у	PC	z
int	int	8	string



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```
int x = 10;
int y = 20;
if (x > y) {
    string z = "x";
    Console.WriteLine(z);
} else {
    string z = "y";
    Console.WriteLine(z);
}
```

PC	×	У
9	int	int



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- The typing rule for a while loop requires the condition to be a boolean expression, and assumes the type of the body
- The type of the body can be anything (usually void, something else in case of function returns)
- It does not add anything to the declarations, even though the body might declare local variables

$$\frac{\langle \mathtt{c}, D \rangle \to \langle \mathtt{bool}, D \rangle \wedge \langle \mathtt{B}, D \rangle \to \langle \mathtt{T}, D_1 \rangle}{\langle (\mathtt{while c} \ \{ \ \mathtt{A} \ \}), D \rangle \to \langle \mathtt{T}, D \rangle}$$



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- The typing rule for a class declaration adds the class declaration to the declarations with all its attributes and methods
- When adding the declaration of the class, we have to check that the types of the method bodies match their declarations
- Assume that C is the class name, a_i is the i-th attribute in the class (of type A_i), and \mathbf{m}_j is the j-th method in the class (with type M_j)

$$\begin{array}{c} D_1 := D[C \mapsto [...,a_i \mapsto A_i,...,m_j \mapsto M_j,...]] \wedge \ \langle \mathbf{M_j} \ \mathbf{m_j}, D_1[\mathtt{this} \mapsto C] \rangle \rightarrow \langle M_j^{'}, D_2 \rangle \wedge \ M_j = \\ & \langle (\mathtt{class} \ \mathtt{C} \ \{..., \mathbf{A_i} \ \mathbf{a_i},..., \mathbf{M_j} \ \mathbf{m_j},...\}), D \rangle \rightarrow \langle \mathtt{T}, D_1 \rangle \end{array}$$



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 When type checking a method declaration (within a class declaration we type check its body and compare the result with the type of the declaration

- Assume that C is the class name, p_i is the i-th parameter of the method (of type P_i), and b is the method body
- The type of a method is of the form $P_1 \times P_2 \times \cdots \times P_n \to R$, where P_i is the type of the i-th parameter and R is the return type

$$\frac{\langle (\mathbf{b}),D[...,p_i\mapsto P_i,...]\rangle \to \langle \mathbf{R},D_1\rangle}{\langle (\mathbf{R}\ \mathbf{m}(...,\mathbf{P}_1\ \mathbf{p}_1,...)\mathbf{b}),D\rangle \to \langle (P_1\times P_2\times \cdots \times P_n\to R),D\rangle}$$



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
```

Declarations: PC



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```
class Counter {
   private int cnt;
   public Counter() {
      this.cnt = 0;
   }
   public void incr(int diff) {
      this.cnt = (this.cnt + diff);
   }
}
```



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```
class Counter {
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```



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
```

```
Declarations: PC diff this 7 int Counter
```

Classes:

```
\begin{array}{c} \mathsf{Counter} \\ \mathsf{Counter} {=} \mathsf{Counter} \to \mathsf{Counter} \\ \mathsf{cnt} {=} \mathsf{int} \\ \mathsf{incr} {=} (\mathsf{Counter} {\times} \mathsf{int}) \to \mathsf{void} \end{array}
```



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
```

```
Declarations: PC diff this 8 int Counter
```

Classes:

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\begin{array}{c} \mathsf{Counter} \\ \mathsf{Counter} {=} \mathsf{Counter} \to \mathsf{Counter} \\ \mathsf{cnt} {=} \mathsf{int} \\ \mathsf{incr} {=} (\mathsf{Counter} {\times} \mathsf{int}) \to \mathsf{void} \end{array}
```



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
```

```
\begin{tabular}{c|c} Declarations: & \hline PC \\ \hline 10 \\ \hline \\ Classes: & \hline & Counter \\ \hline & Counter=Counter \rightarrow Counter \\ \hline & cnt=int \\ incr=(Counter\times int) \rightarrow void \\ \hline \end{tabular}
```



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- When type checking a return statement, we typecheck its argument
- The type of the argument is also the type of the return statement
- There is no change to the declarations

$$\frac{\langle \mathtt{x}, D \rangle \to \langle \mathtt{T}, D \rangle}{\langle (\mathtt{return} \ \mathtt{x}), D \rangle \to \langle T, D \rangle}$$



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- Statements in a sequence might contain return statements
- In this case one of them might not return void
- Their sequence will assume the non-void type

$$\frac{\langle \mathtt{a}, D \rangle \to \langle \mathtt{T}, D_1 \rangle \wedge \langle \mathtt{b}, D_1 \rangle \to \langle \mathtt{void}, D_2 \rangle}{\langle (\mathtt{a}; \mathtt{b}), D \rangle \to \langle \mathtt{T}, D_2 \rangle}$$

$$\langle \mathtt{a}, D \rangle \to \langle \mathtt{void}, D_1 \rangle \wedge \langle \mathtt{b}, D_1 \rangle \to \langle \mathtt{T}, D_2 \rangle$$

 $\langle (a;b), D \rangle \rightarrow \langle T, D_2 \rangle$



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- Statements in a sequence might contain return statements
- They might both return a non-void type
- Their sequence will assume the non-void type of both, which must be the same

$$\frac{\langle \mathbf{a}, D \rangle \to \langle \mathbf{T}, D_1 \rangle \wedge \ \langle \mathbf{b}, D_1 \rangle \to \langle \mathbf{U}, D_2 \rangle \wedge \ T = U}{\langle (\mathbf{a}; \mathbf{b}), D \rangle \to \langle \mathbf{T}, D_2 \rangle}$$



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public int incr(int diff) {
    this.cnt = (this.cnt + diff);
    return this.cnt;
  }
}
```

Declarations: PC 1



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```
class Counter {
  private int cnt;
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    this.cnt = (this.cnt + diff);
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  }
}
```



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```
class Counter {
  private int cnt;
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```
class Counter {
  private int cnt;
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```
class Counter {
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```
class Counter {
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class Counter {
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    this.cnt = (this.cnt + diff);
    return this.cnt;
  }
}
```



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- ullet Sometimes we may look a attribute a up from an instance c of a class
- This assumes the type of the attribute, which needs to be looked up in the class descriptor found in the declarations
- No declaration is further modified

$$\frac{\langle \mathtt{c}, D \rangle \to \langle \mathtt{C}, D \rangle \wedge \langle \mathtt{a}, C \rangle \to \langle \mathtt{A}, C \rangle \wedge T = U}{\langle (\mathtt{c}.\mathtt{a}), D \rangle \to \langle \mathtt{A}, D \rangle}$$



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```
class Counter {
  public int cnt;
  public Counter() {
    this.cnt = 0;
  }
}
Counter c = new Counter();
int a = c.cnt;
```

Declarations: PC 1



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```
class Counter {
  public int cnt;
  public Counter() {
    this.cnt = 0;
  }
}
Counter c = new Counter();
int a = c.cnt;
```



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```
class Counter {
  public int cnt;
  public Counter() {
    this.cnt = 0;
  }
}
Counter c = new Counter();
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```



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```
class Counter {
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  }
}
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int a = c.cnt;
```



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```
class Counter {
  public int cnt;
  public Counter() {
    this.cnt = 0;
  }
}
Counter c = new Counter();
int a = c.cnt;
```



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- Sometimes we may call a method m up from an instance c of a class and with parameters p_i
- This assumes the return type of the method, provided that all parameter types match the types expected by the method
- No declaration is further modified

$$\frac{\langle \mathtt{c}, D \rangle \to \langle \mathtt{C}, D \rangle \wedge \ \langle \mathtt{m}, C \rangle \to \langle (\mathtt{P}_1 \times \mathtt{P}_2 \times \dots \times \mathtt{P}_n \to \mathtt{R}), C \rangle \wedge \ \langle \mathtt{p}_i, D \rangle \to \langle \mathtt{P}_i', D \rangle \wedge \ \mathtt{P}_i = \mathtt{P}_i'}{\langle (\mathtt{c}.\mathtt{m}(...\mathtt{p}_i..)), D \rangle \to \langle \mathtt{R}, D \rangle}$$



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```

Declarations:

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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```



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class Counter {
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  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```

```
 \begin{array}{c|c} \hline \text{Declarations:} & \hline PC & c \\ \hline 11 & Counter \\ \hline \hline \\ Classes: & \hline \\ Classes: & \hline \\ Counter=Counter \rightarrow Counter \\ Incr=(Counter \times int) \rightarrow void \\ cnt=int \\ \hline \end{array}
```



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```

Declarations:		С		PC	ret	arg ₁	this
		Counter		11	null	int	Counter
Classes:		Co]				
	(Counter=Cou					
		Incr=(Count					
		cn					



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```

Declarations:		С	PC	ret	arg ₁	this
		Counter	11	void	int	Counter
Classes:		Co]			
	(Counter=Cou	1			
		Incr=(Count				
		cn				



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```
class Counter {
  private int cnt;
  public Counter() {
    this.cnt = 0;
  }
  public void Incr(int diff) {
    this.cnt = (this.cnt + diff);
  }
}
Counter c = new Counter();
c.Incr(5);
```

```
 \begin{array}{c|cccc} \textbf{Declarations:} & \hline {PC} & c \\ \hline 12 & Counter \\ \hline \\ \textbf{Classes:} & \hline \\ \textbf{Counter=Counter} \rightarrow \textbf{Counter} \\ \hline \textbf{Incr=(Counter\times int)} \rightarrow \textbf{void} \\ \textbf{cnt=int} \\ \end{array}
```



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Typing rules and semantic rules

- We may call a static method m from class C and with parameters p_i
- This assumes the return type of the method, provided that all parameter types match the types expected by the method
- We do not need to look up the class because it is already specified in the call
- No declaration is further modified

$$\frac{\langle \mathtt{m},C\rangle \rightarrow \langle (\mathtt{P}_1 \times \mathtt{P}_2 \times \cdots \times \mathtt{P}_n \rightarrow \mathtt{R}),C\rangle \wedge \ \langle \mathtt{p}_i,D\rangle \rightarrow \langle \mathtt{P}'_i,D\rangle \wedge \ \mathtt{P}_i = \mathtt{P}'_i}{\langle (\mathtt{C.f}(...\mathtt{p}_i..)),D\rangle \rightarrow \langle \mathtt{R},D\rangle}$$



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```
class Utils {
  static public int AddThree(int a,int b,int c) {
    return ((a + b) + c);
  }
int x = Utils.AddThree(10,20,30);
```

Declarations:

PC 1



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```
class Utils {
  static public int AddThree(int a,int b,int c) {
    return ((a + b) + c);
  }
}
int x = Utils.AddThree(10,20,30);
```

```
        PC
        a
        b
        c

        3
        int
        int
        int

        ...
        Utils
```

Classes:

 $\mathsf{AddThree} = (\mathsf{int} \times \mathsf{int} \times \mathsf{int}) \to \mathsf{int}$



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```
class Utils {
   static public int AddThree(int a,int b,int c) {
    return ((a + b) + c);
   }
}
int x = Utils.AddThree(10,20,30);
```

```
 \begin{array}{c} \text{Declarations:} & \hline PC \\ \hline 6 \\ \\ \text{Classes:} & \hline \\ & \text{AddThree=}(\text{int}\times\text{int}\times\text{int}) \rightarrow \text{int} \\ \end{array}
```



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```
class Utils {
  static public int AddThree(int a,int b,int c) {
    return ((a + b) + c);
  }
}
int x = Utils.AddThree(10,20,30);
```

```
 \begin{array}{c|c} \text{Declarations:} & \hline PC & \times \\ \hline 7 & \text{int} \\ \hline \\ \text{Classes:} & \hline \\ & AddThree=(\text{int}\times\text{int}\times\text{int}) \rightarrow \text{int} \\ \end{array}
```



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Reading typing rules

- The constructor of a class is simply a specially named static method
- It has no further typing rules



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```
class CounterFrom {
  public int cnt;
  public CounterFrom(int cnt0) {
    this.cnt = cnt0;
  }
}
CounterFrom c = new CounterFrom(100);
```

```
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```



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```
class CounterFrom {
  public int cnt;
  public CounterFrom(int cnt0) {
    this.cnt = cnt0;
  }
}
CounterFrom c = new CounterFrom(100);
```



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```
class CounterFrom {
  public int cnt;
  public CounterFrom(int cnt0) {
    this.cnt = cnt0;
  }
}
CounterFrom c = new CounterFrom(100);
```



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```
class CounterFrom {
  public int cnt;
  public CounterFrom(int cnt0) {
    this.cnt = cnt0;
  }
}
CounterFrom c = new CounterFrom(100);
```



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```
class CounterFrom {
  public int cnt;
  public CounterFrom(int cnt0) {
    this.cnt = cnt0;
  }
}
CounterFrom c = new CounterFrom(100);
```



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Looking back

- Python is brittle and breaks easily
- Static typing is a way to run a coarse simulation of the program
- If type checking fails, then the program cannot be guaranteed to run correctly, and we get a compiler error
- Safer programming, but at the cost of being able to run less programs that might still be valid



This is it!

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The best of luck, and thanks for the attention!