

Looping and iteration

Dr. Giuseppe Maggiore

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Repeating behaviors in Python

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Termination (or lack thereof)

Exponential explosion of potential control-paths

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Lecture topics

- Repeated behaviors
- while statements and their semantics
- Expressive power of while
- Termination and infinite iteration
- Explosion of states with while



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Repeated behaviors

- Sometimes running code just once is not enough
- We can *loop* execution of a block of code until some condition is met
- Extreme increase in expressive power



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Repeated behaviors

- While there are hostile aliens
- Do attack an alien



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Repeated behaviors

- Loops can solve very big problems
- Each step of the loop removes a part of the problem
- We typically stop when all parts of the problem have been removed



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Breaking problems up

Problem: kill all aliens

• Problem piece: a single alien to be killed

Solution piece: attack a single alien

Termination condition: there are no more aliens



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Breaking problems up

- Of course loops can be combined with each other
- This means that we can cascade repetition
- This is the building block of intelligent decisions in our programs



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Breaking problems up

- While there are hostile alien armies
 - Do pick an alien army
 - While there are aliens in the army
 - Do attack an alien



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- Python offers built-in facilities for repetition
- while statement
- We can repeat execution of a block of code

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- ullet The general form is while CONDITION: BODY (w_{CB})
- If the condition is true, then we jump to the beginning of BODY, otherwise we jump to the end of the whole while

$$\begin{cases} \left(PC,S\right) \overset{w}{\rightarrow} B \; \left(firstLine(B;w_{CB}),S\right) & when & \left(PC,S\right) \overset{C}{\rightarrow} TRUE \\ \left(PC,S\right) \overset{w}{\rightarrow} B \; \left(skipAfter(B),S\right) & when & \left(PC,S\right) \overset{C}{\rightarrow} FALSE \end{cases}$$



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- Remember that Python is indentation-based
- White-spaces go at the beginning of some lines
- A more indented line is within a less indented line above



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- Indentation specifies where the body begins and ends
- The general form of a while is thus:
 - while COND:
 - newline
 - indentation
 - code of body
 - de-indentation



A correct example

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```
n = 64
i = 0
while n > 1:
  n = n / 2
  i = i + 1
```



An incorrect example

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```
n = 64
i = 0
while n > 1:
n = n / 2
i = i + 1
```



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- while statements eventually terminate (hopefully)
- if the condition evaluates to False, then we skip after the end of the while

After a while

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```
n = 64
i = 0
while n > 1:
    n = n / 2
    i = i + 1
print(i)
```

After a while?

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Without indentation, this:

```
n = 64
i = 0
while n > 1:
n = n / 2
i = i + 1
print(i)
```

would be indistinguishable from both:

```
n = 64

i = 0

while n > 1:

n = n / 2

i = i + 1

print(i)

n = 64

i = 0

while n > 1:

n = n / 2

i = i + 1

print(i)
```



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- while effectively rewrites the code to become as long as the problem needs
- Until run-time, we are not really sure how long the code will become



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What values of n will we print?

```
i = 0
j = 1
n = 0
while i < 10:
    i = i + 1
n = n + j
print(n),</pre>
```



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What values of n will we print?

```
i = 0
j = 1
n = 0
while i < 10:
i = i + 1
n = n + j
print(n),</pre>
```

1 2 3 4 5 6 7 8 9 10



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What values of n will we print?

```
i = 0
j = 2
n = 0
while i < 10:
    i = i + 1
    n = n + j
print(n),</pre>
```



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What values of n will we print?

```
i = 0
j = 2
n = 0
while i < 10:
i = i + 1
n = n + j
print(n),</pre>
```

2 4 6 8 10 12 14 16 18 20



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- Each iteration produces new values of the variables
- These new values are then fed back into the next iteration
- Eventually these cause the condition to become false
 - Or the program runs forever and never produces a result



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- Each iteration produces new values of the variables
- These new values are then fed back into the next iteration
- Eventually these cause the condition to become false
 - Or the program runs forever and never produces a result
 - We'd rather not have this one



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```
i j n
0 2 0
```

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```

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i	j	n
0	2	0

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```

```
i j n
1 2 2
```



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```
i j n
1 2 2
```

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```

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i	j	n
1	2	2

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```

```
i j n
2 2 4
```



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```
i j n
2 2 4
```

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```

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i	j	n
2	2	4

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```

```
i j n
3 2 6
```



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After k iterations (for k < 10):

```
i j n
k 2 2 * k
```

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```



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After k iterations (for k < 10):

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```



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After k iterations (for k = 10):

```
i j n
10 2 20
```

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```



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After k iterations (for k = 10):

```
while i < 10:
    i = i + 1
    n = n + j
    print(n),</pre>
```

We jump to the first instruction **after** the while loop, and do not touch the state further.

i	j	n
10	2	20



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Readability and termination

- The loop above is well designed
- All iterations produce a new piece of a logical series
 - (The j-th row of the table of multiplication)



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Readability and termination

- After each iteration we know that we have i elements correctly computed
- After each iteration we know that we have 10-i elements still to compute
- When i=10 then we have 10-10 elements still to compute
 - This is the termination condition
 - Since i keeps growing, we know that eventually the termination condition will be met



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Nesting

- Loops can be nested
- A loop can be inside a loop (which can further be inside other loops)
- This makes it slightly harder to reason



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```
j = 1
while j <= 10:
    i = 0
    n = 0
while i < 10:
    i = i + 1
    n = n + j
    print(n),
    print("\t"),
    j = j + 1
print("")</pre>
```



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We now know that the semantics of the inner loop is **print the j-th row of the table of multiplication**, so instead of reasoning on:

```
j = 1
while j <= 10:
    i = 0
    n = 0
while i < 10:
    i = i + 1
    n = n + j
    print(n),
    print("\t"),
    j = j + 1
print("")</pre>
```

we reason on:

```
j = 1
while j <= 10:
print the j-th row of the table of multiplication
j = j + 1
print("")</pre>
```



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```
j output
1 nothing
```

```
while j <= 10: print the j-th row of the table of multiplication j = j + 1 print("")
```



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```
j output
1 nothing
```

```
while j <= 10: print the j-th row of the table of multiplication j = j + 1 print("")
```

```
j output
2 1st row of table
```



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```
j output2 1st row of table
```

```
while j <= 10:
   print the j-th row of the table of multiplication
   j = j + 1
   print("")</pre>
```



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```
j output
2 1st row of table
```

```
while j <= 10:
   print the j-th row of the table of multiplication
   j = j + 1
   print("")</pre>
```

```
j output
3 1st and 2nd rows of table
```



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...



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Thus for all k's such that $k \le 10$:

```
j output
k first k-1 rows of table
```

```
while j <= 10: print the j-th row of the table of multiplication j = j + 1 print("")
```



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Thus for all k's such that $k \le 10$:

```
j output
k first k-1 rows of table
```

```
while j <= 10:
   print the j-th row of the table of multiplication
   j = j + 1
   print("")</pre>
```

```
j output
k+1 first k rows of table
```

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thereof)

Eventually we get j = 11:

```
j output PC
11 first 10 rows of table 1
```

```
while j <= 10:
    print the j-th row of the table of multiplication
    j = j + 1
    print("")
...</pre>
```

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Eventually we get j = 11:

j	output					PC
11	first	10	rows	of	table	1

```
while j <= 10: print the j-th row of the table of multiplication j = j + 1 print("") ...
```

```
j output PC
11 first 10 rows of table 5
```



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Wait! It gets worse!

- It is not guaranteed that a loop will terminate
- A loop that does not terminate gets the program stuck forever
- It is a bit sad for the machine
- Care when designing loops is needed to prevent this



Termination (or lack thereof)

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Care in the design

- A loop changes the state of the program many times
- A good loop changes the state in one direction
- Every step should bring us closer to the **final state**
- The condition defines the aspects of the final state



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```
i = 1
while i > 0:
    i = i + 1
    print(i)
```

What is the issue here?



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```
i = 1
while i > 0:
    i = i + 1
print(i)
```

What is the issue here?

Iterations do not go towards the condition, but away from it.



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This is not a duplicated slide.

```
i = 1
while i < 10:
   i = i + 1
print(i)</pre>
```

What is the issue here?



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This is not a duplicated slide.

```
i = 1
while i < 10:
   i = i + 1
print(i)</pre>
```

What is the issue here?

Iterations are **orthogonal (unrelated)** to the condition.

No iteration changes elements tested in the condition.



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- How many things can happen in a while loop?
- Depends on its content



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The more we nest loops and conditionals within a loop...



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- The more we nest loops and conditionals within a loop...
- ...the more things may happen at run-time



Loops with nested if's

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```
while CO:
    if C1:
        A1
    else:
        B1
```

How many execution paths per N iterations of the loop?



Loops with nested if's

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```
while CO:
if C1:
A1
else:
B1
```

How many execution paths per N iterations of the loop?

 $\begin{tabular}{ll} \bf 2 & {\rm execution \ paths \ per \ iteration} \\ 2^N & {\rm execution \ paths \ per \ N \ iterations} \\ \end{tabular}$



Loops with nested if's

```
Looping and iteration
```

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```
while CO:
if C1:
A1
else:
B1
```

Example: execution paths per **3** iterations of the loop.

A1 A1 A1

A1 A1 B1

A1 B1 A1

A1 B1 B1

B1 A1 A1

B1 A1 B1

B1 B1 A1

B1 B1 B1



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- Consider a loop that performs m iterations
- With n if's inside
- ullet Each iteration can have 2^n possible execution paths
- The whole loop can have $2^{n \times m}$ possible execution paths^a

$$^{\rm a}2^{2\times 10}=1,79e308$$



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- Each path can alter the state in a different way
- After a while with many billions possible paths
 - We have many billions possible resulting states



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- The more nested the code inside a while
- The more complex its behavior
- The harder it is to reason about it!



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The value of reasoning

- Always keep in mind:
- You have the power to make your own life a living Hell...



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The value of reasoning

- Always keep in mind:
- You have the power to make your own life a living Hell...
- ...unless you reason first and then structure code logically



This is it!

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The best of luck, and thanks for the attention!