

# Reasoning on programs

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Reasoning on  
programs

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Introduction

Conditional  
expressions

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## Lecture topics

- We introduce conditional expressions
- We show how to verify properties on complex expressions

## Conditional expressions

- Sometimes we can make decisions within an expression
- The general form is `VALUE if CONDITION else VALUE'` ( $if_{VCV'}$ )
- If the condition is true, then we return `VALUE`, otherwise `VALUE'`

$$\left\{ \begin{array}{ll} (PC, S) \stackrel{if_{VCV'}}{\rightarrow} V & \text{when } (PC, S) \stackrel{C}{\rightarrow} TRUE \\ (PC, S) \stackrel{if_{VCV'}}{\rightarrow} V' & \text{when } (PC, S) \stackrel{C}{\rightarrow} FALSE \end{array} \right.$$

## Conditional expressions

- "adult" if `age >= 18` else "minor" = ?
- **Can you guess the results for `age = 18` and `age = 16`?**

## Conditional expressions

- `"adult" if age >= 18 else "minor" = ?`
- **Can you guess the results for `age = 18` and `age = 16`?**
- `age = 16: "minor"`
- `age = 18: "adult"`

## Reasoning on programs

- Sometimes we do not know exactly the values of all variables at all times
- The program may be too complex to allow it

Consider a throttle control system.

**The throttle may never go under 1000RPM**, or the engine stops and everybody dies.

**The temperature must be kept under control**, or the engine blows up and everybody dies.

```
throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

The question thus is: **could the code above cause everyone to die?**

throttle	temp
1000..10000	-20.0..400.0

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```



throttle	temp
1000..10000	-20.0..400.0

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

throttle	temp
?!?!?	-20.0..400.0

## Reasoning on programs

- We cannot list all possible combinations of variable values
- We cannot just “hope it works”

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- We cannot list all possible combinations of variable values
- We cannot just “hope it works”
- We can reason in terms of conditions on variables

## Reasoning on programs

- We partition the state based on the conditional
- $(temp > 350.0) \ \& \ (throttle > 2500)$  generates four states
  - $temp > 350$  and  $throttle > 2500$
  - $temp \leq 350$  and  $throttle > 2500$
  - $temp > 350$  and  $throttle \leq 2500$
  - $temp \leq 350$  and  $throttle \leq 2500$
- We study the semantics on each of these four states

`temp > 350 and throttle > 2500`

throttle	temp
<code>&gt;2500.0 ..10000</code>	<code>&gt;350.0 ..400.0</code>

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

`temp > 350 and throttle > 2500`

throttle	temp
<code>&gt;2500.0 ..10000</code>	<code>&gt;350.0 ..400.0</code>

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

throttle	temp
<code>&gt;1500.0..9000.0</code>	<code>&gt;350.0..400.0</code>

`temp <= 350 and throttle > 2500`

throttle	temp
<code>&gt;2500.0..10000</code>	<code>-20.0..350.0</code>

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

`temp <= 350 and throttle > 2500`

throttle	temp
>2500.0..10000	-20.0..350.0

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

throttle	temp
>2500.0..10000.0	-20.0..350.0



`temp > 350 and throttle <= 2500`

throttle	temp
<code>1000..<code>&gt;2500.0</code></code>	<code>&gt;350..<code>400.0</code></code>

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

`temp > 350 and throttle <= 2500`

throttle	temp
1000.. <b>&gt;2500.0</b>	<b>&gt;350..400.0</b>

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

throttle	temp
1000.. <b>&gt;2500.0</b>	<b>&gt;350..400.0</b>

`temp <= 350 and throttle <= 2500`

throttle	temp
<code>1000..<code>&gt;2500.0</code></code>	<code>-20.0..<code>350.0</code></code>

```
1 throttle = throttle - 1000 if (temp > 350.0) & (  
    throttle > 2500) else throttle
```

`temp <= 350 and throttle <= 2500`

throttle	temp
<code>1000..&gt;2500.0</code>	<code>-20.0..350.0</code>

1 `throttle = throttle - 1000 if (temp > 350.0) & (  
throttle > 2500) else throttle`

throttle	temp
<code>1000..&gt;2500.0</code>	<code>-20.0..350.0</code>

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## Reasoning on programs

- Each of the four states has a result
- We now merge the results

We now merge these states, knowing that each of them may actually happen:

throttle	temp
>1500.0..9000.0	>350.0..400.0
>2500.0..10000.0	-20.0..350.0
1000.0..>2500.0	>350..400.0
1000.0..>2500.0	-20.0..350.0

We now merge these states, knowing that each of them may actually happen:

throttle	temp
>1500.0..9000.0	>350.0..400.0
>2500.0..10000.0	-20.0..350.0
1000.0..>2500.0	>350..400.0
1000.0..>2500.0	-20.0..350.0

throttle	temp
1000.0..10000.0	-20.0..400.0

We know that the throttle will never go below 1500RPM, and we also know that if the temperature is above 350 degrees then maximum throttle is never above 9000RPM.

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Nobody dies :)



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The best of luck, and thanks for the  
attention!