## **ML Homework4**

Due Date: 2021/04/26 (MON.) 23:55

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## **Description:**

#### 1. Logistic regression

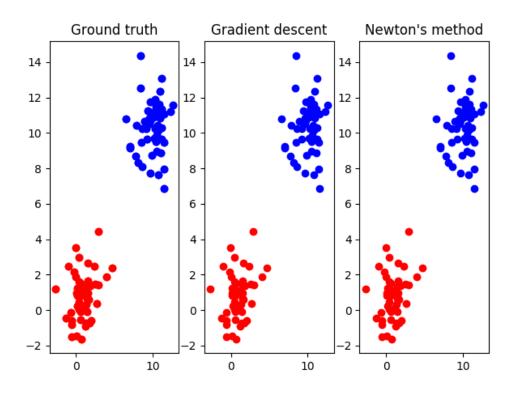
- Input:
  - 1. N (number of data points)
  - 2.  $mx_1, vx_1, my_1, vy_1, mx_2, vx_2, my_2, vy_2$  (m: mean, v: variance)
- Function:
  - 1. Generate n data point:  $D1=(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n)$ , where x and y are independently sampled from  $N(mx_1,vx_1)$  and  $N(my_1,vy_1)$  respectively.
  - 2. Generate n data point:  $D2=(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n)$ , where x and y are independently sampled from  $N(mx_2,vx_2)$  and  $N(my_2,vy_2)$  respectively.
  - 3. Use Logistic regression to separate D1 and D2. You should implement both Newton's and steepest gradient descent method during optimization.
    - In other words, when the Hessian is singular, use steepest descent for instead. You should come up with a reasonable rule to determine convergence.(a simple run out of the loop should be used as the ultimatum)
- Output:
  - 1. The confusion matrix and the **sensitivity** and **specificity** of the logistic regression applied to the training data *D*.
  - 2. Visualization
    - Plot the ground truth
    - Plot the predict result
      - Gradient descent
      - Newton's method

Use the Gaussian random number generator in homework 3.

- Sample input & output (for reference only)
  - $\circ$  Case 1:  $N=50, mx_1=my_1=1, mx_2=my_2=10, vx_1=vy_1=vx_2=vy_2=2$

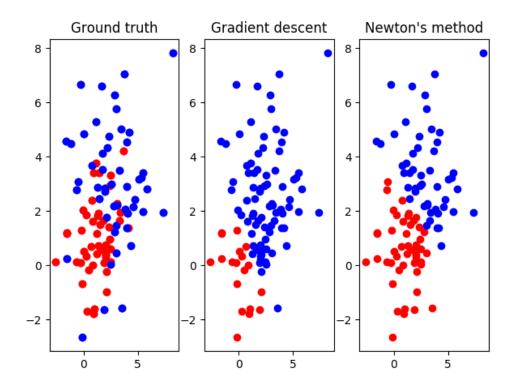
```
1 Gradient descent:
2
3 W:
```

```
-78.1766393662
 5
       6.7233419236
 6
      11.2430677919
 7
    Confusion Matrix:
 8
 9
                 Predict cluster 1 Predict cluster 2
10
    Is cluster 1
                        50
                                          0
                                          50
11
    Is cluster 2
                        0
12
    Sensitivity (Successfully predict cluster 1): 1.00000
13
    Specificity (Successfully predict cluster 2): 1.00000
14
15
16
17
    Newton's method:
18
19
    w:
    -118.3601516394
20
      8.7747332848
21
     10.1954120077
22
23
24
    Confusion Matrix:
25
                 Predict cluster 1 Predict cluster 2
26
    Is cluster 1
                        50
27
    Is cluster 2
                        0
                                          50
28
29
   Sensitivity (Successfully predict cluster 1): 1.00000
   Specificity (Successfully predict cluster 2): 1.00000
```



 $\circ$  Case 2:  $N=50, mx_1=my_1=1, mx_2=my_2=3, vx_1=vy_1=2, vx_2=vy_2=4$ 

```
Gradient descent:
2
3
   w:
4
   -71.1902536008
    46.0123814025
5
    54.6803199701
6
7
8
   Confusion Matrix:
9
              Predict cluster 1 Predict cluster 2
10
   Is cluster 1 16
                                     34
   Is cluster 2
                     3
                                     47
11
12
   Sensitivity (Successfully predict cluster 1): 0.32000
13
14
   Specificity (Successfully predict cluster 2): 0.94000
15
16
17
   Newton's method:
18
19
   w:
20
   -1.9045831451
21
     0.3940876974
22
     0.5695243849
23
24
  Confusion Matrix:
25
              Predict cluster 1 Predict cluster 2
26 Is cluster 1 40
27
   Is cluster 2
                 10
                                     40
28
29 Sensitivity (Successfully predict cluster 1): 0.80000
30 Specificity (Successfully predict cluster 2): 0.80000
```



#### 2. EM algorithm

- Input: MNIST training data and label sets. (Same as HW02)
- Function:
  - 1. Binning the gray level value into **two bins**. Treating all pixels as random variables following Bernoulli distributions. Note that each pixel follows a different Binomial distribution independent to others.
  - 2. Use EM algorithm to cluster each image into ten groups. You should come up with a reasonable rule to determine convergence. (a simple run out of the loop should be used as the ultimatum)
- Output:
  - 1. For each digit, output a confusion matrix and the **sensitivity** and **specificity** of the clustering applied to the training data.
  - 2. Print out the imagination of numbers in your classifier
    - Just like before, about the details please refer to HW02
- Hint: The algorithm is a kind of unsupervised learning, so the labels are not used during training. But you can use these labels to help you to figure out which class belongs to which number.

In other words, you should find a way to assign label to each class which you classified **before you compute the confusion matrix** 

• Sample input & output (for reference only)

```
53
54
56
57
58
59
60
... all other unlabeled imagination of numbers goes here ...
61
62
63
class 9:
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
8.3
84
85
86
87
88
89
90
91
92
93
No. of Iteration: 1, Difference: 3176.579389514846
94
95
96
97
class 0:
98
99
100
101
```

```
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
... all other iterations goes here ...
128
129
class 9:
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
```

```
151
152
153
154
155
156
157
158
No. of Iteration: 10, Difference: 19.89546432548733
159
160
161
______
162
163
164
labeled class 0:
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
labeled class 1:
195
196
197
198
199
```

```
200
2.01
202
203
2.04
205
206
207
2.08
209
210
211
2.12
213
214
2.15
216
217
218
219
220
221
222
2.2.3
224
... all other labeled imagination of numbers goes here ...
225
226
labeled class 9:
227
228
229
2.30
231
232
233
234
235
236
237
238
239
2.40
2.41
242
243
2.44
245
246
247
2.48
```

```
249 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
   250
251
   252
   253
   254
255
256
257
258
   Confusion Matrix 0:
259
              Predict number 0 Predict not number 0
              3023
260
   Is number 0
                                2900
   Isn't number 0 113
261
                                 53964
262
263
   Sensitivity (Successfully predict number 0) : 0.51038
264
   Specificity (Successfully predict not number 0): 0.99791
265
266
267
268
   Confusion Matrix 1:
269
              Predict number 1 Predict not number 1
             5986
270 Is number 1
                                756
271
   Isn't number 1
                 800
                                52458
272
   Sensitivity (Successfully predict number 1) : 0.88787
273
274
   Specificity (Successfully predict not number 1): 0.98498
275
276
277
278
   ... all other confusion matrix goes here ...
279
280
281
282
   Confusion Matrix 9:
              Predict number 9 Predict not number 9
283
284
   Is number 9
              2718
                                3231
   Isn't number 9
                 5147
                                48904
285
286
287
   Sensitivity (Successfully predict number 9) : 0.45688
288
   Specificity (Successfully predict not number 9): 0.90478
289
290 Total iteration to converge: 10
291 | Total error rate: 0.5081666666666667
```