Homotopy type theory (HoTT) is a new field of mathematics, which combines aspects of several different areas in a surprising way. It is based on a recently discovered connection between homotopy theory and type theory. Although the connections between the two are currently the focus of intense investigation, it is increasingly clear that they are just the beginning of a subject that will take more time and more hard work to fully understand. It touches on topics as seemingly distant as the homotopy groups of spheres, the decidability of type checking algorithms, and the definition of weak infinity-groupoids.

Homotopy type theory also brings new ideas into the very foundation of mathematics. On the one hand, there is Voevodsky's subtle and beautiful \emph{Univalence Axiom}. The univalence axiom implies, in particular, that isomorphic structures can be identified: a principle that mathematicians have been happily using on workdays, despite its incompatibility with the ``official'' doctrines of conventional foundations. On the other hand, we have Higher Inductive Types, which provide direct, logical descriptions of some of the basic spaces and constructions of homotopy theory: spheres, cylinders, truncations, localizations, etc. Both ideas are impossible in classical set-theoretic foundations, but when combined in homotopy type theory, they permit an entirely new kind of ``logic of homotopy types''.

This suggests a new conception of foundations of mathematics, with intrinsic homotopical content, an ``invariant'' conception of the objects of mathematics --- and convenient machine implementations, which can serve as a practical aid to the working mathematician. This is the univalent foundations program.

The present book is intended as a first systematic exposition of the basics of univalent foundations, and a collection of examples of this new style of reasoning --- but without requiring the reader to know or learn any formal logic, or to use any computer proof assistant. We contend that univalent foundations can replace set theory as the ``implicit foundation'' for the unformalized mathematics done by most mathematicians, independently of the success of its machine implementations.

Homotopy Type Theory: Univalent Foundations of Mathematics

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