Assessing 12-Bit Resolution in Oscilloscope Hardware Using a PCA-Enhanced FFT Approach

This project aims to rigorously determine whether the oscilloscope hardware under examination truly provides 12-bit vertical resolution. While the instrument's specifications may claim such capability, it is crucial to verify that the full 4,096 quantization levels (0 to $2^{12}-1$) are effectively utilized. Achieving and confirming 12-bit resolution is essential for high-precision applications—such as advanced communications, medical instrumentation, and scientific research—where the ability to discern subtle signal variations can significantly impact measurement fidelity.

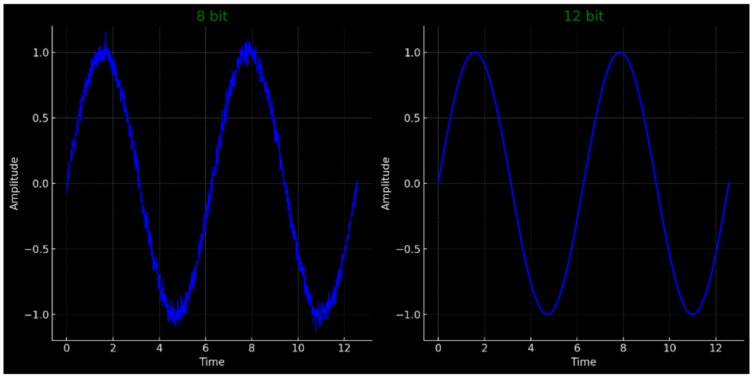

Fig 1. 8-bit vs. 12-bit resolution comparison

Key Approach: FFT and PCA

A central aspect of this study is the combined use of the Fast Fourier Transform (FFT) and Principal Component Analysis (PCA). FFT converts the time-domain signal into its frequency-domain representation, allowing us to identify and quantify dominant frequency components. PCA then helps reduce the complexity of the frequency data, highlighting the most critical contributors to signal variance. Together, these techniques provide a more nuanced view of the oscilloscope's performance than time-domain or bit-level analyses alone, enabling a comprehensive evaluation of the device's effective resolution.

The purpose of this project is to verify whether the oscilloscope hardware under evaluation genuinely supports a 12-bit vertical resolution. By analyzing and processing the raw data captured from the oscilloscope, we aim to assess its effective resolution and determine if it meets the stringent requirements of high-precision applications. Accurate data handling and thorough analysis are essential to ensure the oscilloscope's reliability and performance in industrial environments where precise measurements are critical.

Methodology

1. Data Acquisition

A high-fidelity sine wave signal was captured using the oscilloscope. Ensuring that the input fully utilized the instrument's vertical range was vital, as it allowed us to realistically test the scope's resolution limits. The time-domain data (100,000 points) was then exported for further processing.

2. Frequency Analysis Using FFT and PCA| 使用 FFT 和 PCA 进行频率分析

To verify the effective resolution and frequency content captured by the oscilloscope, we performed a frequency analysis using the **Fast Fourier Transform (FFT)** and **Principal Component Analysis (PCA)**. This approach allowed us to identify and quantify the dominant frequency components present in the signal and assess the oscilloscope's ability to capture detailed frequency information.

a. Fast Fourier Transform (FFT)

The FFT converts the time-domain signal into its frequency-domain representation, enabling us to analyze the frequency components within the signal.

1. **Signal Segmentation:** The captured signal, consisting of 100,000 data points, was divided into **10 equal-length segments**.

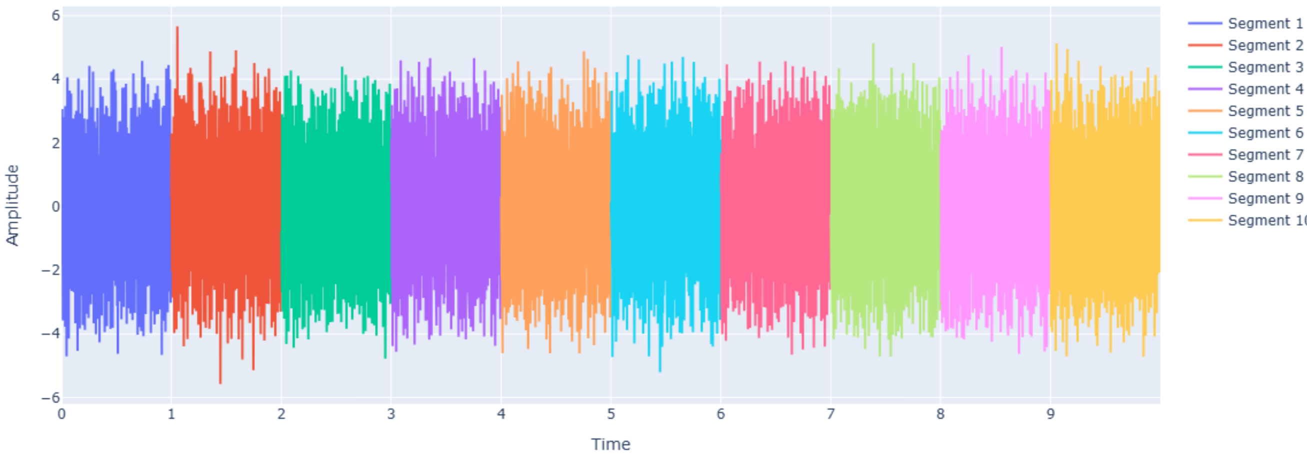

Fig 2. Division of the Original Signal into Ten Equal-Length Segments

   - Segmenting the signal provides multiple observations, which is essential for effective statistical analysis and improves the reliability of subsequent PCA.

2. **FFT Computation:**
   - For each segment, the FFT was computed to transform the time-domain data into the frequency domain.
   - We used an FFT size (N_{FFT}) of 1024 points for all segments.
     - **Zero-Padding:** If the segment length was shorter than N_{FFT}, zeros were added to extend the segment length. This increases the frequency resolution of the FFT without altering the original signal content.
   - The result is a set of **magnitude spectra**, each representing the amplitude of frequency components within that segment.

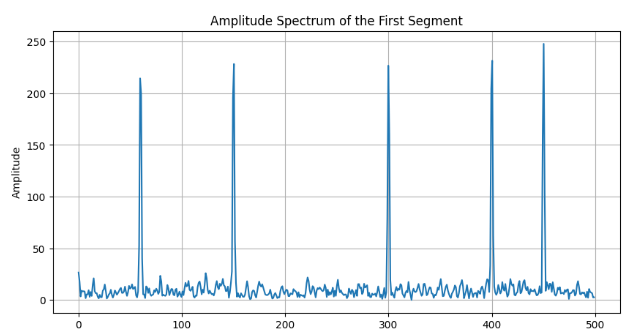

Fig 3. Example Amplitude Spectrum of the First Signal Segment

3. **Frequency Bins:**
    - The FFT output provides a set of frequency bins, each corresponding to a specific frequency.

b. Principal Component Analysis (PCA)

PCA was applied to the frequency-domain data to identify the most significant frequency components contributing to the signal's variance.

Why Use PCA:

- **Dimensionality Reduction:** The FFT produces many frequency bins (features). PCA reduces this high-dimensional data into a smaller set of uncorrelated variables called principal components.
- **Pattern Identification:** PCA helps to identify patterns in the data by focusing on the directions (principal components) that capture the most variance.

What PCA Represents:

- **Principal Components:** Each principal component is a linear combination of the original frequency bins and represents a direction in the feature space along which the data varies the most.
- **Eigenvectors and Eigenvalues:**
    - **Eigenvectors:** Represent the directions (frequency combinations) of maximum variance (principal components).
    - **Eigenvalues:** Indicate the amount of variance captured by each principal component.

Process:

1. **Data Matrix Construction:**
    - The magnitude spectra from the FFT of each segment was organized into a **data matrix** X.
        - **Rows (N)**: Each row corresponds to a segment.
        - **Columns (M)**: Each column corresponds to a frequency bin from the FFT.
2. **Standardization:**
    - The data matrix was standardized to have **zero mean** and **unit variance** across each frequency bin.
    - **Purpose:** Standardization ensures that all frequency bins contribute equally to the PCA, preventing bias due to differing scales.
3. **PCA Computation:**
    - PCA was performed on the standardized data matrix X_{std}.
    - **Computation Steps:**
        - Calculate the covariance matrix of X_{std}, called it C
        - Compute the eigenvalues and eigenvectors of the covariance matrix C.
        - Sort the principal components based on the eigenvalues in descending order.
4. **Selection of Top Frequencies:**
    - We analyzed the **top five principal components** that explained the most variance in the data.
    - **Identifying Dominant Frequencies:**
        - For each principal component (eigenvector), we identified the frequency bin with the highest absolute value (loading).
        - The frequency bins with the highest loadings are the ones that contribute most to that principal component.

- By mapping these frequency bins back to actual frequency values, we identified the dominant frequencies in the signal.

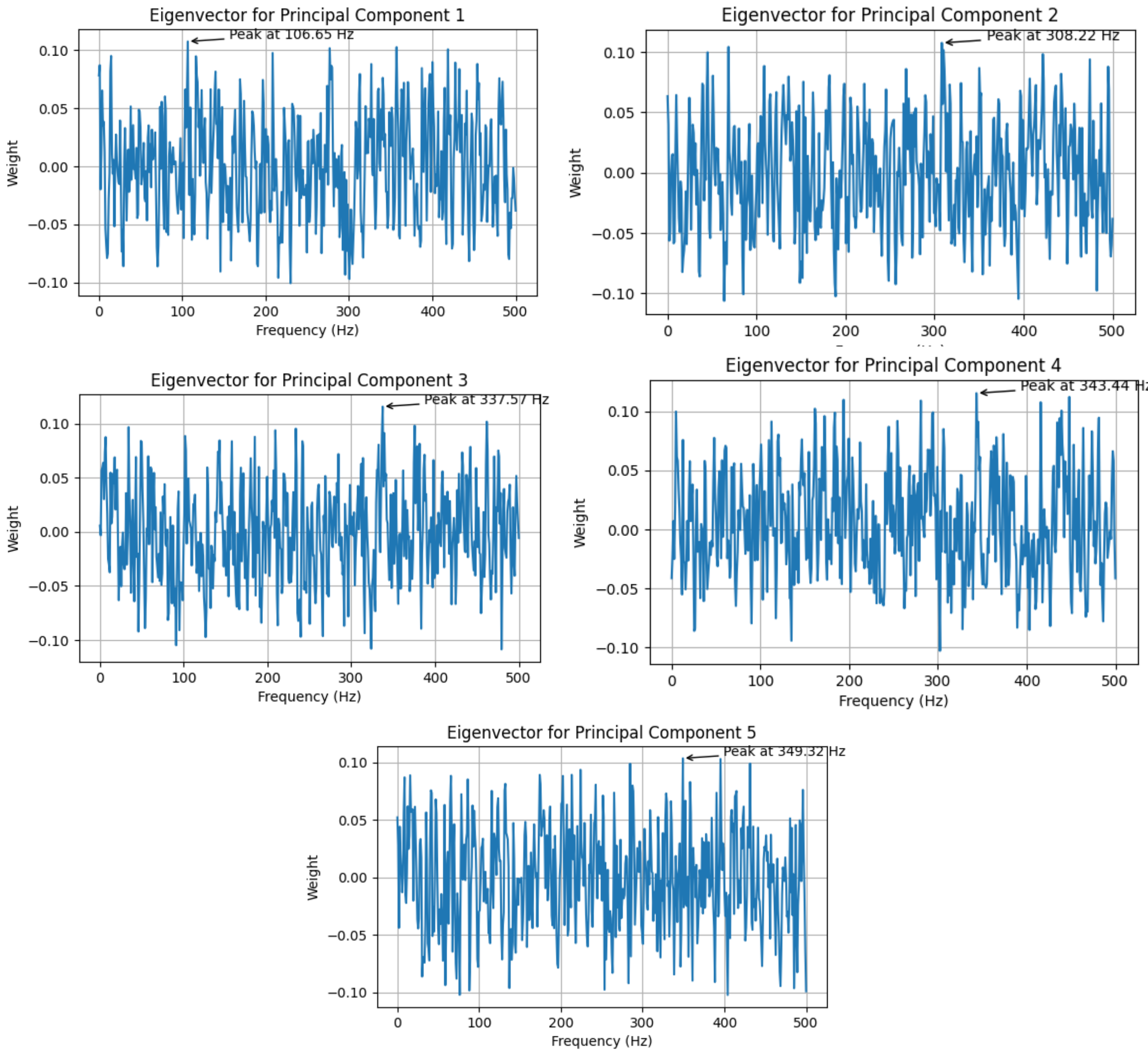

Fig 4. Eigenvectors Corresponding to the Top Five Principal Components

Dominant Frequencies:

These frequencies correspond to the principal peaks in the eigenvectors of the top principal components. The elevated loadings at these values indicate that they play a dominant role in the signal's overall variance. Specifically, the dominant frequencies identified are 106.65 Hz, 308.22 Hz, 337.57 Hz, 343.44 Hz, and 349.32 Hz.

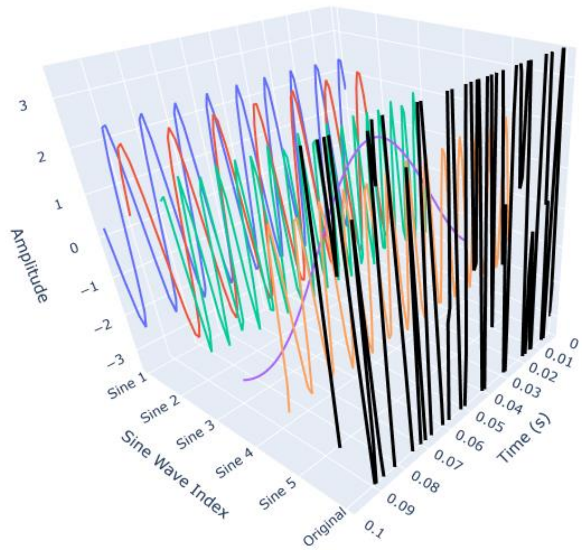

Fig 5. 3D Representation of FFT-Based Signal Decomposition into Top 5 Components

3. 数据转换 & 覆盖范围分析 | Data Conversion & Coverage Analysis

Hexadecimal representation offers a concise and human-readable format for binary data. Each hexadecimal digit represents four binary bits, simplifying the visualization and analysis of high-resolution data.

Data Processing Steps:

- **Conversion to Hexadecimal:** Each raw data point was converted to its corresponding hexadecimal value using Python scripts. This step ensures that the data is in a standardized format for bit-level analysis.

```
def convert_to_hex(num):
    if num < 0:
        return hex((1 << 32) + num)[2:]
    else:
        return format(num, '04x')
```

- **Extraction of Relevant Bits:** To focus on the 12-bit resolution, the 12 least significant bits (LSBs) were extracted from each data point. This extraction isolates the portion of the data that directly corresponds to the oscilloscope's vertical resolution.

```
def extract_12_bits(hex_str):
    binary_str = bin(int(hex_str, 16))[2:].zfill(16)
    return binary_str[-12:]
```

- **Conversion to Binary Representation:** The extracted 12-bit data was converted to binary format for detailed bit-level analysis, enabling us to inspect the distribution and utilization of the quantization levels.

A thorough analysis was conducted to determine whether all possible 12-bit values are represented in the dataset, which is essential for verifying the oscilloscope's true resolution. We generated a complete list of all possible 12-bit hexadecimal values from 000 to FFF, totaling 4,096 possible values. The analysis revealed that the oscilloscope did not utilize the full range of 12-bit values in the captured data:

- **Total Possible Quantization Levels:** 4,096
- **Unique Quantization Levels Observed: 3,005**
- **Missing Quantization Levels: 1,091**, indicating gaps in the resolution.

Implications for Precision Measurements: The absence of 1,091 quantization levels suggests limitations in the oscilloscope's ability to resolve fine signal details. In high-precision applications, this could lead to measurement inaccuracies and a failure to detect subtle signal variations essential for accurate diagnostics and analysis.

Reference | 引用

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