Homework 3

Due: **October 8th, 5pm** (late submission until October 11th, 5pm -- no submission possible afterwards)

Written assignment: 10 points

Coding assignment: 25 points

Project report: 15 points

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Link to the github repo: https://github.com/098pipi/data2060_hw3.git

Written assignment

Gradient Descent (10 points)

Consider using gradient descent to find the minimum of f, where,

- f is a convex function over the closed interval [-*b*,*b*], *b* > 0
- f' is the derivative of f
- α is some positive number which will represent a learning rate parameter

The steps of gradient descent are as follows:

- Start at $x_0 = 0$
- At each step, set $x_{t+1} = x_t - \alpha f'(x_t)$
- If x_{t+1} falls below -*b*, set it to -*b*, and if it goes above *b*, set it to *b*.

We say that an optimization algorithm (such as gradient descent) ϵ-*converges* if, at some point, x_t stays within ϵ of the true minimum. Formally, we have ϵ-*convergence* at time t if

$$|x_{t'} - x_{\min}| \leq \epsilon, \quad \text{where } x_{\min} = \underset{x \in [-b,b]}{argmin} f(x) \text{ for all } t' \geq t.$$

Question 1

For α = 0.1, b = 1, and ϵ = 0.001, find a convex function f so that running gradient descent does not ϵ-converge. Specifically, make it so that x_0 = 0, x_1 = b, x_2 = - b, x_3 = b, x_4 = - b, etc.

Solution: Parameters:
$\alpha = 0.1,\ b = 1,\ \epsilon = 0.001$

Iteration steps:

$$x_0 = 0$$
$$x_1 = x_0 - 0.1f'(x_0)$$
$$1 = 0 - 0.1f'(0)$$
$$f'(0) = -10$$

$$-1 = 1 - 0.1f'(1)$$
$$f'(1) = 20$$

$$1 = -1 - 0.1f'(-1)$$
$$f'(-1) = -20$$

Form of derivative:

$$f'(x) = ax^2 + bx + c$$

Using conditions:

$$f'(0) = c = -10$$
$$f'(1) = a + b - 10 = 20$$
$$f'(-1) = a - b - 10 = -20$$

Solve system:

$$a = 10,\ b = 20$$

So,

$$f'(x) = 10x^2 + 20x - 10 + K$$

since K is irrelevant for gradient descent, we set K = 0

Integrating:

$$f(x) = \frac{10}{3}x^3 + 10x^2 - 10x$$

Convexity on [-1,1]

f''(x) = 20x + 20 on [-1,1], f''(x) in a monotonically increasing function, min{f''(x)} = f''(-1) = 0, hence f''(x) >= 0 for any x in [-1, 1], f is convex on the closed interval [-1, 1]

Convergence check:

$$x_{\min} = 0, \quad |x_t - x_{\min}| = 1 > \epsilon = 0.001 \quad \forall\, t > 0$$

Thus, the method diverges.

Question 2

For α = 0.1, b = 1, and ϵ = 0.001, find a convex function f so that gradient descent does ϵ -*converge*, but only after at least 10,000 steps.

Solution

For simplicity, consider f a quadratic function

$$f(x) = \frac{\mu}{2}x^2, \quad \mu > 0$$
$$f'(x) = \mu x.$$
$$f''(x) = \mu > 0$$

so f(x) is convex

$$x_{n+1} = x_t - 0.1 f'(x_n) = (1 - 0.1\mu)\, x_n.$$

Define the convergence rate

$$r := 1 - 0.1\mu$$

By induction,

$$x_n = r^n x_0$$

Choose that

$$x_n = \epsilon, x_0 = 1, n = 10^4$$

$$r^n = \frac{\varepsilon}{|x_0|}.$$

$$r = \left(\frac{\varepsilon}{|x_0|}\right)^{1/n}, \quad \mu = \frac{1-r}{0.1}.$$

$$r = 10^{-3/10000}, \quad \mu \approx 0.00690537.$$

$$f(x) = 0.00345268\, x^2.$$

Coding Assignment (25 points)

Run the evironment test below, make sure you get all green checks. If not, you will lose 2 points for each red or missing sign.

In [2]:
```
from __future__ import print_function
from packaging.version import parse as Version
from platform import python_version

OK = '\x1b[42m[ OK ]\x1b[0m'
FAIL = "\x1b[41m[FAIL]\x1b[0m"

try:
    import importlib
except ImportError:
    print(FAIL, "Python version 3.12.11 is required,"
                " but %s is installed." % sys.version)

def import_version(pkg, min_ver, fail_msg=""):
    mod = None
    try:
        mod = importlib.import_module(pkg)
        if pkg in {'PIL'}:
            ver = mod.VERSION
        else:
            ver = mod.__version__
        if Version(ver) == Version(min_ver):
            print(OK, "%s version %s is installed."
                  % (lib, min_ver))
        else:
            print(FAIL, "%s version %s is required, but %s installed."
                  % (lib, min_ver, ver))
    except ImportError:
        print(FAIL, '%s not installed. %s' % (pkg, fail_msg))
    return mod


# first check the python version
pyversion = Version(python_version())
```

```
if pyversion >= Version("3.12.11"):
    print(OK, "Python version is %s" % pyversion)
elif pyversion < Version("3.12.11"):
    print(FAIL, "Python version 3.12.11 is required,"
                " but %s is installed." % pyversion)
else:
    print(FAIL, "Unknown Python version: %s" % pyversion)


print()
requirements = {'matplotlib': "3.10.5", 'numpy': "2.3.2",'sklearn': "1.7.1",
                'pandas': "2.3.2", 'pytest': "8.4.1", 'torch':"2.7.1"}

# now the dependencies
for lib, required_version in list(requirements.items()):
    import_version(lib, required_version)
```

```
[ OK ] Python version is 3.13.5

[ OK ] matplotlib version 3.10.5 is installed.
[ OK ] numpy version 2.3.2 is installed.
[ OK ] sklearn version 1.7.1 is installed.
[ OK ] pandas version 2.3.2 is installed.
[ OK ] pytest version 8.4.1 is installed.
[ OK ] torch version 2.7.1 is installed.
```

Introduction

In this assignment, you will be using a modified version of the UCI Census Income data set to predict the education levels of individuals based on certain attributes collected from the 1994 census database. You can read more about the dataset here: `https://archive.ics.uci.edu/ml/datasets/Census+Income`.

Stencil Code

We have provided the following stencil code within this file:

- `Model` contains the `LogisticRegression` model you will be implementing.
- `Check Model` contains a series of tests to ensure you are coding your model properly.
- `Main` is the entry point of program which will read in the dataset, run the model, and print the results.

You should not modify any code in `Check Model` and `Main`. If you do for debugging or other purposes, please make sure any additions are commented out in the final handin. All the functions you need to fill in reside in this notebook, marked by `TODO` s. You can see a full description of them in the section below.

The Assignment

In `Model`, there are a few functions you will implement. They are:

- `LogisticRegression`:
  - **train()** uses stochastic gradient descent to train the weights of the model.
  - **loss()** calculates the log loss of some dataset divided by the number of examples.
  - **predict()** predicts the labels of data points using the trained weights. For each data point, you should apply the softmax function to it and return the label with the highest assigned probability.
  - **accuracy()** computes the percentage of the correctly predicted labels over a dataset.

Note: You are not allowed to use any packages that have already implemented these models (e.g. scikit-learn). We have also included some code in `main` for you to test out the different random seeds and calculate the average accuracy of your model across those random seeds.

Logistic Regression

Logistic Regression, despite its name, is used in classification problems. It learns sigmoid functions of the inputs

$$h_{\mathbf{w}}(x)_j = \phi_{sig}(\langle \mathbf{w}_j, \mathbf{x} \rangle)$$

where $h_{\mathbf{w}}(x)_j$ is the probability that sample $\mathbf{x}$ is a member of class *j*.

In multi-class classification, we need to apply the `softmax` function to normalize the probabilities of each class. The loss function of a Logistic Regression classifier over *k* classes on a *single* example (*x*,*y*) is the **log-loss**, sometimes called **cross-entropy loss**:

$$\ell(h_{\mathbf{w}}, (\mathbf{x}, y)) = -\sum_{j=1}^{k} \begin{Bmatrix} \log(h_{\mathbf{w}}(\mathbf{x})_j), & y = j \\ 0, & \text{otherwise} \end{Bmatrix}$$

Therefore, the ERM hypothesis of **w** on a dataset of *m* samples has weights

$$\mathbf{w} = \underset{\mathbf{w}}{argmin}\left(-\frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{k}\begin{cases}\log(h_{\mathbf{w}}(\mathbf{x}_i)_j), & y_i = j\\ 0, & \text{otherwise}\end{cases}\right)$$

To learn the ERM hypothesis, we need to perform gradient descent. The partial derivative of the loss function on a single data point

$$\frac{\partial l_S(h_{\mathbf{w}})}{\partial \mathbf{w}_{st}} = \begin{cases}h_{\mathbf{w}}(\mathbf{x})_s - 1, & y = s\\ h_{\mathbf{w}}(\mathbf{x})_s, & \text{otherwise}\end{cases}\mathbf{x}_t$$

With respect to a single row in the weights matrix, $\mathbf{w}_s$, the partial derivative of the loss is

$$\frac{\partial l_S(h_{\mathbf{w}})}{\partial \mathbf{w}_{s}} = \begin{cases}h_{\mathbf{w}}(\mathbf{x})_s - 1, & y = s\\ h_{\mathbf{w}}(\mathbf{x})_s, & \text{otherwise}\end{cases}\mathbf{x}$$

You will need to descend this gradient to update the weights of your Logistic Regression model.

Stochastic Gradient Descent

You will be using Stochastic Gradient Descent (SGD) to train your `LogisticRegression` model. Below, we have provided pseudocode for SGD on a sample *S*:

initialize parameters $\mathbf{w}$, learning rate α, and batch size b
  converge = False
  while not converge:
    epoch + 1
    shuffle training examples
    calculate last epoch loss
    for $i = 0, 1, \ldots, \lceil n_{examples}/b \rceil - 1$: -- iterate over batches:
      $X_{batch} = X[i \cdot b : (i+1) \cdot b]$ -- select the X in the current batch
      $\mathbf{y}_{batch} = \mathbf{y}[i \cdot b : (i+1) \cdot b]$ -- select the labels in the current batch
      initialize $\nabla L_{\mathbf{w}}$ to be a matrix of zeros
      for each pair of training data point $(\mathbf{x}, y) \in (X_{batch}, \mathbf{y}_{batch})$:
        for $j = 0, 1, \ldots, n_{classes} - 1$:
          -- calculate the partial derivative of the loss with respect to
          -- a single row in the weights matrix
          if $y = j$: $\nabla L_{\mathbf{w}_j} \mathrel{+}= (softmax(\langle \mathbf{w}_j, \mathbf{x}\rangle) - 1) \cdot \mathbf{x}$

else: $\nabla L_{\mathbf{w}_j} += (softmax(\langle \mathbf{w}_j, \mathbf{x} \rangle)) \cdot \mathbf{x}$

$\mathbf{w} = \mathbf{w} - \frac{\alpha \nabla L_{\mathbf{w}}}{len(X_{batch})}$ -- update the weights

calculate this epoch loss

if $|\text{Loss}(X, \mathbf{y})_{this-epoch} - Loss(X, \mathbf{y})_{last-epoch}| <$ CONV-THRESHOLD:

converge $=$ True -- break the loop if loss converged

Hints: Consistent with the notation in the lecture, $\mathbf{w}$ are initialized as a *k* x *d* matrix, where *k* is the number of classes and *d* is the number of features (with the bias term). With *n* as the number of examples, *X* is a *n* x *d* matrix, and $\mathbf{y}$ is a vector of length *n*.

Tuning Parameters

Convergence is achieved when the change in loss between iterations is some small value. Usually, this value will be very close to but not equal to zero, so it is up to you to tune this threshold value to best optimize your model's performance. Typically, this number will be some magnitude of 10^{-x}, where you experiment with *x*. Note that when calculating the loss for checking convergence, you should be calculating the loss for the entire dataset, not for a single batch (i.e., at the end of every epoch).

You will also be tuning batch size (and one of the report questions addresses the impact of batch size on model performance). In order to reach the accuracy threshold, you will need to tune both parameters. α would typically be tuned during the training process, but we are fixing α = 0.03 for this assignment. **Please do not change α in your code**.

You can tune the batch size and convergence threshold in `Main`.

Extra: Numpy Shortcuts

While optional, there are many numpy shortcuts and functions that can make your code cleaner. We encourage you to look up numpy documentation and learn new functions.

Some useful shortcuts:

- `A @ B` is a shortcut for `np.matmul(A, B)`
- `X.T` is a shortcut for `np.transpose(X)`
- `X.shape` is a shortcut for `np.shape(X)`

Model

In [ ]:
```
import random
import math
import numpy as np

def softmax(x):
    '''
    Apply softmax to an array
    @params:
        x: the original 1D array
    @return:
        an 1D array with softmax applied elementwise.
    '''
    e = np.exp(x - np.max(x))   # avoid overflow of exp()
    return (e + 1e-6) / (np.sum(e) + 1e-6)  # avoid division by 0/log(0)

class LogisticRegression:
    '''
    Multiclass Logistic Regression that learns weights using
    stochastic gradient descent.
    '''
    def __init__(self, n_features, n_classes, batch_size, conv_threshold):
        '''
        Initializes a LogisticRegression classifer.
        @attrs:
            n_features: the number of features in the classification problem
            n_classes: the number of classes in the classification problem
            weights: The weights of the Logistic Regression model
            alpha: The learning rate used in stochastic gradient descent
        '''
        self.n_classes = n_classes
        self.n_features = n_features
        self.weights = np.zeros((n_classes, n_features + 1))  # An extra row
        self.alpha = 0.03  # DO NOT TUNE THIS PARAMETER
        self.batch_size = batch_size
        self.conv_threshold = conv_threshold

    def train(self, X, Y):
        '''
        Trains the model using stochastic gradient descent
        @params:
            X: a 2D Numpy array where each row contains an example, padded b
            Y: a 1D Numpy array containing the corresponding labels for each
        @return:
            num_epochs: integer representing the number of epochs taken to r
        '''
        # [TODO]
        converge = False
        num_epochs = 0
        # guarantee the diff is huge enough for the first step to diverge
        last_loss = float('inf')
```

```
        while not converge:
            num_epochs += 1
            # shuffle training examples, randomly permuting
            indices = np.arange(len(X))
            np.random.shuffle(indices)
            X, Y = X[indices], Y[indices]

            for i in range(math.ceil(len(X)/self.batch_size)):
                X_batch = X[i*self.batch_size : (i+1)*self.batch_size]
                Y_batch = Y[i*self.batch_size : (i+1)*self.batch_size]
                gLw = np.zeros((self.n_classes, self.n_features + 1))

                for x_i, y_i in zip(X_batch, Y_batch):
                    for j in range(self.n_classes):
                        # calculate the partial derivative of the loss w.r.t
                        z = self.weights @ x_i.T    # (k, d) * (d,1) --> (k,
                        prob = softmax(z.reshape(1,-1).flatten())  # --> (1,
                        if y_i == j:
                            gLw[j] += (prob[j] - 1) * x_i
                        else:
                            gLw[j] += prob[j] * x_i
                # update the weights
                self.weights -= (self.alpha * gLw)/len(X_batch)
            # calculate the epoch loss
            this_loss = self.loss(X, Y)
            if abs(this_loss - last_loss) < self.conv_threshold:
                converge = True
            last_loss = this_loss
        return num_epochs


    def loss(self, X, Y):
        '''
        Returns the total log loss on some dataset (X, Y), divided by the nu
        @params:
            X: 2D Numpy array where each row contains an example, padded by
            Y: 1D Numpy array containing the corresponding labels for each e
        @return:
            A float number which is the average loss of the model on the dat
        '''
        z = X @ self.weights.T # get(n,k)
        prob = np.array([softmax(z_i) for z_i in z])    # apply row_wise
        total_prob = prob[np.arange(len(Y)), Y]
        total_logLoss = - np.sum(np.log(total_prob))
        return total_logLoss/len(X)

    def predict(self, X):
        '''
        Compute predictions based on the learned weigths and examples X
        @params:
```

```
            X: a 2D Numpy array where each row contains an example, padded b
        @return:
            A 1D Numpy array with one element for each row in X containing t
        '''
        z = X @ self.weights.T  # get(n,k)
        prob = np.array([softmax(z_i) for z_i in z])
        predicted_class = np.argmax(prob, axis=1)
        return predicted_class

    def accuracy(self, X, Y):
        '''
        Outputs the accuracy of the trained model on a given testing dataset
        @params:
            X: a 2D Numpy array where each row contains an example, padded b
            Y: a 1D Numpy array containing the corresponding labels for each
        @return:
            a float number indicating accuracy (between 0 and 1)
        '''
        predicted_class = self.predict(X)
        correct = np.sum(predicted_class == Y)
        return correct/len(Y)
```

Check Model

In [ ]:

```
import pytest
# Sets random seed for testing purposes
random.seed(0)
np.random.seed(0)

# Creates Test Model with 2 predictors, 2 classes, a Batch Size of 5 and a T
test_model1 = LogisticRegression(2, 2, 5, 1e-2)

# Creates Test Data
x_bias = np.array([[0,4,1], [0,3,1], [5,0,1], [4,1,1], [0,5,1]])
y = np.array([0,0,1,1,0])
x_bias_test = np.array([[0,0,1], [-5,3,1], [9,0,1], [1,0,1], [6,-7,1]])
y_test = np.array([0,0,1,0,1])

# Creates Test Model with 2 predictors, 1 classes, a Batch Size of 1 and a T
test_model2 = LogisticRegression(2, 3, 1, 1e-2)

# Creates Test Data
x_bias2 = np.array([[0,0,1], [0,3,1], [4,0,1], [6,1,1], [0,1,1], [0,4,1]])
y2 = np.array([0,1,2,2,0,1])
x_bias_test2 = np.array([[0,0,1], [-5,3,1], [9,0,1], [1,0,1]])
```

```
y_test2 = np.array([0,1,2,0])


# Test Model Loss
assert test_model1.loss(x_bias, y) == pytest.approx(0.693, .001) # Checks if
assert test_model2.loss(x_bias2, y2) == pytest.approx(1.099, .001) # Checks

# Test Train Model and Checks Model Weights
assert test_model1.train(x_bias, y) == 14
assert test_model1.weights == pytest.approx(np.array([[-0.218, 0.231, 0.0174

assert test_model2.train(x_bias, y) == 9
assert test_model2.weights == pytest.approx(np.array([[-0.300,  0.560,  0.09

# Test Model Predict
assert (test_model1.predict(x_bias_test) == np.array([0., 0., 1., 1., 1.])).
assert (test_model2.predict(x_bias_test2) == np.array([0, 0, 1, 1])).all()

# Test Model Accuracy
assert test_model1.accuracy(x_bias_test, y_test) == .8-
assert test_model2.accuracy(x_bias_test2, y_test2) == .25
```

Main

In [81]:
```
from sklearn.model_selection import train_test_split

DATA_FILE_NAME = 'normalized_data.csv'
# DATA_FILE_NAME = 'unnormalized_data.csv'
# DATA_FILE_NAME = 'normalized_data_nosens.csv'

CENSUS_FILE_PATH = DATA_FILE_NAME

NUM_CLASSES = 3
BATCH_SIZE = 2  # [TODO]: tune this parameter
CONV_THRESHOLD = 1e-4 # [TODO]: tune this parameter

def import_census(file_path):
    '''
        Helper function to import the census dataset
        @param:
            train_path: path to census train data + labels
            test_path: path to census test data + labels
        @return:
            X_train: training data inputs
            Y_train: training data labels
            X_test: testing data inputs
            Y_test: testing data labels
    '''
```

```
    data = np.genfromtxt(file_path, delimiter=',', skip_header=False)
    X = data[:, :-1]
    Y = data[:, -1].astype(int)
    X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size=0.3,
    return X_train, Y_train, X_test, Y_test

def test_logreg():
    X_train, Y_train, X_test, Y_test = import_census(CENSUS_FILE_PATH)
    num_features = X_train.shape[1]

    # Add a bias
    X_train_b = np.append(X_train, np.ones((len(X_train), 1)), axis=1)
    X_test_b = np.append(X_test, np.ones((len(X_test), 1)), axis=1)

    ### Logistic Regression ###
    model = LogisticRegression(num_features, NUM_CLASSES, BATCH_SIZE, CONV_T
    num_epochs = model.train(X_train_b, Y_train)
    acc = model.accuracy(X_test_b, Y_test) * 100
    print("Test Accuracy: {:.1f}%".format(acc))
    print("Number of Epochs: " + str(num_epochs))

# Set random seeds. DO NOT CHANGE THIS IN YOUR FINAL SUBMISSION.
random.seed(0)
np.random.seed(0)
test_logreg()
```

```
Test Accuracy: 98.4%
Number of Epochs: 103
```

Check Model (Cont'd)

In [82]:

```
### test your model on the census dataset
X_train, Y_train, X_test, Y_test = import_census(CENSUS_FILE_PATH)
num_features = X_train.shape[1]

# Add a bias
X_train_b = np.append(X_train, np.ones((len(X_train), 1)), axis=1)
X_test_b = np.append(X_test, np.ones((len(X_test), 1)), axis=1)

# Logistic Regression, average accross 10 random states
random.seed(0)
num_states = 10
num_epochs, test_accuracies = [], []

for _ in range(num_states):
    random_state = random.randint(1, 1000)
    random.seed(random_state)
    np.random.seed(random_state)
```

```
    model = LogisticRegression(num_features, n_classes=3, batch_size=1, conv
    num_epochs.append(model.train(X_train_b, Y_train))
    test_accuracies.append(model.accuracy(X_test_b, Y_test) * 100)

avg_test_accuracy = sum(test_accuracies) / num_states
avg_num_epochs = sum(num_epochs) / num_states
print("Average Test Accuracy: {:.1f}%".format(avg_test_accuracy))
print("Average Number of Epochs: " + str(avg_num_epochs))

assert 1.5 < avg_num_epochs < 2.5
assert 75 < avg_test_accuracy < 80
```

```
Average Test Accuracy: 77.8%
Average Number of Epochs: 2.2
```

Report Questions (15 points)

Question 1

Make sure that you have implemented a variable batch size using the constructor given for `LogisticRegression`. Try different batch sizes ([1, 8, 64, 512, 4096] - there are ~5700 points in the dataset), and try different convergence thresholds ([1e-1, 1e-2, 1e-3]) in the cell below. Visualize the accuracy and number of epochs taken to converge.

Answer the following questions:

- What tradeoffs exist between good accuracy and quick convergence?
- Why do you think the batch size led to the results you received?

Fill in the `generate_array()` and `generate_heatmap()` functions so you can visualize how accuracy and number of epochs taken changes as we change batch size and convergence threshold. Fill out BATCH_SIZE_ARR and CONV_THRESHOLD_ARR with the values described above.

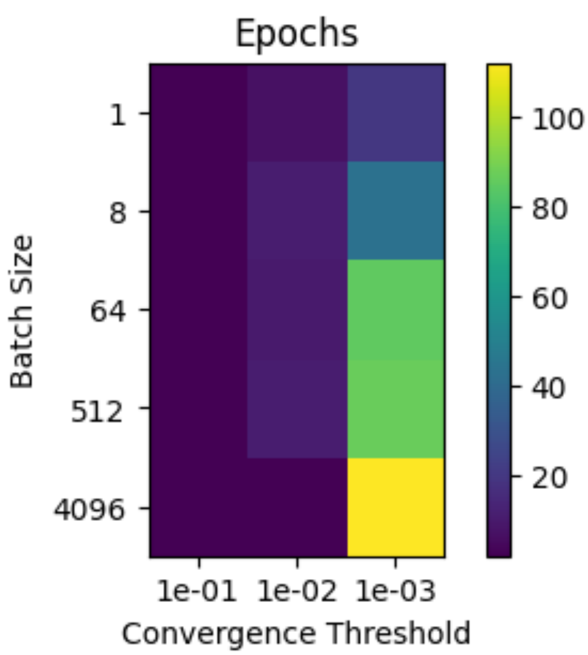
- **generate_array()** should loop through both BATCH_SIZE_ARR and CONV_THRESHOLD_ARR to populate `epoch_arr` and `acc_arr`. Make sure to round `acc_arr` to 2 decimal places before returning (Hint: `np.round`).

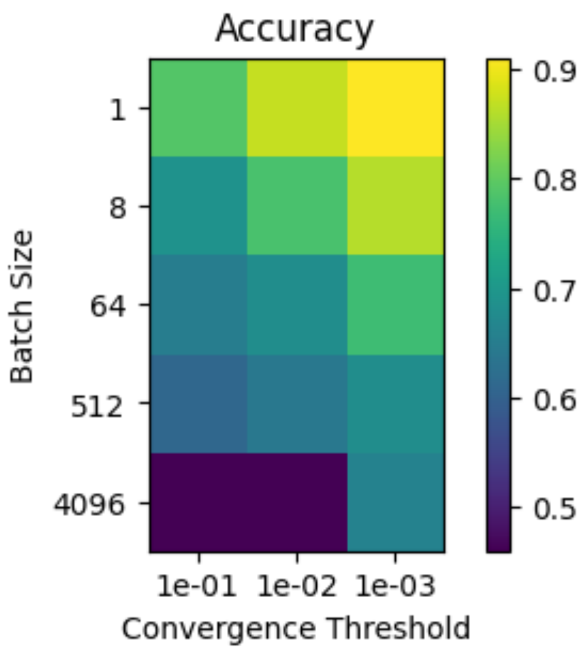
- **generate_heatmap()** should create a matplotlib heatmap of the arrays. You should label the axis and title of each plot using BATCH_SIZE_ARR and CONV_THRESHOLD_ARR. It might be helpful to look at Matplotlib's guide for heatmaps: https://matplotlib.org/stable/gallery/images_contours_and_fields/image_annotated_heat

Hint: Runs with large batch sizes and low convergence thresholds might take several minutes to half an hour to complete. We recommend that you develop the code below with a small subset of the parameters (e.g., batch size of [1,2,4] and conv_threshold of [1e-1, 1e-2]). Once your code works and your figures look good, rerun everything with the batch size and conv_threshold values described above.

In [86]:
```
import matplotlib.pyplot as plt

random.seed(0)
np.random.seed(0)

BATCH_SIZE_ARR = [1, 8, 64, 512, 4096] # [TODO]: try different values
CONV_THRESHOLD_ARR = [1e-1, 1e-2, 1e-3]  # [TODO]: try different values

def generate_array():
    '''
        Runs the logistic regression model on different batch sizes and
        convergence thresholds to populate arrays for accuracy and number of
        @return:
            epoch_arr: 2D array of epochs taken, for each batch size and cor
            acc_arr: 2D array of accuracies, for each batch size and conv th
    '''
    X_train, Y_train, X_test, Y_test = import_census(CENSUS_FILE_PATH)
    num_features = X_train.shape[1]

    # Add a bias
    X_train_b = np.append(X_train, np.ones((len(X_train), 1)), axis=1)
    X_test_b = np.append(X_test, np.ones((len(X_test), 1)), axis=1)

    # Initializes the accuracy and epoch arrays
    acc_arr = np.zeros((len(BATCH_SIZE_ARR), len(CONV_THRESHOLD_ARR)))
    epoch_arr = np.zeros((len(BATCH_SIZE_ARR), len(CONV_THRESHOLD_ARR)))

    ### Populate arrays ###
    for b in range(len(BATCH_SIZE_ARR)):
        for c in range(len(CONV_THRESHOLD_ARR)):
            model = LogisticRegression(n_features = num_features,
                                       n_classes = 3,
                                       batch_size = BATCH_SIZE_ARR[b],
                                       conv_threshold = CONV_THRESHOLD_ARR[c
            epoch_arr[b][c] = model.train(X = X_train_b, Y = Y_train)
            acc_arr[b][c] = np.round(model.accuracy(X = X_test_b, Y = Y_test
    return epoch_arr, acc_arr


def generate_heatmap(arr, name):
    '''
        Generates a matplotlib heatmap for an array
```

```
        convergence thresholds to populate arrays for accuracy and number of
        @param:
            arr: 2D array to generate heatmap of
            name: title of the plot (Hint: use plt.title)
        @return:
            None
    '''
    fig, ax = plt.subplots(figsize=(5,3))
    im = ax.imshow(arr, cmap = "viridis")

    ax.set_xticks(np.arange(len(CONV_THRESHOLD_ARR)))
    ax.set_yticks(np.arange(len(BATCH_SIZE_ARR)))

    ax.set_xticklabels([f'{c:.0e}' for c in CONV_THRESHOLD_ARR])
    ax.set_yticklabels(BATCH_SIZE_ARR)

    ax.set_xlabel('Convergence Threshold')
    ax.set_ylabel('Batch Size')
    ax.set_title(name)

    cbar = ax.figure.colorbar(im, ax=ax)

epoch_arr, acc_arr = generate_array()
generate_heatmap(epoch_arr, "Epochs")
generate_heatmap(acc_arr, "Accuracy")
```





Solution:

Q1: What tradeoffs exist between good accuracy and quick convergence?

A smaller convergence threshold (e.g., 1e-3) forces the model to train for more epochs before stopping, which results in more stable weight updates and higher accuracy. In contrast, a larger threshold (e.g., 1e-1) allows the model to stop earlier, saving computation time but often at the cost of underfitting and lower accuracy. Batch size also plays a critical role: smaller batches lead to noisier gradients and slower convergence, but they sometimes encourage better generalization. Larger batches, on the other hand, provide smoother gradient estimates and converge in fewer epochs, but they can plateau at lower accuracy. Thus, achieving higher accuracy typically requires smaller thresholds and moderate batch sizes, whereas quick convergence is achieved with larger thresholds and larger batches, though with weaker performance.

Q2: Why do you think the batch size led to the results you received?

From the heatmap, we can see that very small batch sizes (like 1 or 8) lead to higher accuracy, but they also take more epochs because the updates are noisy and slower to settle down. As the batch size grows (64, 512), training becomes faster and smoother, but accuracy starts to drop compared to the small batches. For the very large batch size (4096), the model converges very quickly, but accuracy is the worst, since the updates are too “smooth” and the model stops learning useful patterns.

Question 2

Try to run the model with `unnormalized_data.csv` instead of `normalized_data.csv`. Report your findings when running the model on the unnormalized data. In a few short sentences, explain what normalizing the data does and why it affected your model's performance.

When I ran the model with unnormalized data, the runtime was shorter (fewer epochs), but the accuracy was much lower compared to the normalized dataset. This happens because without normalization, features on very different scales distort gradient updates, leading the model to converge poorly. Normalization balances the feature scales, allowing gradient descent to train more effectively and achieve much higher accuracy.

Question 3

Try the model with `normalized_data_nosens.csv`; in this data file, we have removed the `race` and `sex` attributes. Report your findings on the accuracy of your model on this dataset (averaging over many random seeds here may be useful). Can we make any conclusion based on these accuracy results about whether there is a correlation between sex/race and education level? Why or why not?

Solution:

When training on normalized_data_nosens.csv (with sex and race removed), the model's accuracy was actually slightly higher than with the full dataset. This suggests that sex and race were not strong independent predictors once other features such as education, occupation, and hours worked were included. Their removal may have reduced noise or collinearity, allowing the model to generalize better. Therefore, we cannot conclude that sex and race have a strong correlation with education or other predictors just based on accuracy results. The increase in accuracy is more likely due to feature redundancy than a lack of real-world correlation.