

SOP for multi-antenna Eavesdropper

Here, we consider that the Eve is equipped with $N_E (N_E \geq 1)$ antennas, the received signal at Eve is expressed as

$$y_e = (\mathbf{G}_e \mathbf{\Theta} \mathbf{H} + \beta_l \mathbf{H}_e) \mathbf{w} s + n_l = \left(\rho_e \hat{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H} + \sqrt{1 - \rho_e^2} \tilde{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H} + \beta_l \mathbf{H}_e \right) \mathbf{w} s + n_l, \quad (1)$$

where $\mathbf{G}_e \sim \mathcal{CN}_{N_E, M}(\mathbf{0}, \mathbf{I}_{N_E} \otimes \mathbf{I}_M)$ is the channel between IRS and Eve and $\mathbf{H}_e \sim \mathcal{CN}_{N_E, N_A}(\mathbf{0}, \mathbf{I}_{N_E} \otimes \mathbf{I}_{N_A})$ is the channel between Alice and Eve. Now, the The received signal-to-interference-plus-noise ratios (SINRs) at are given by

$$\gamma_e = \frac{\|\Phi_e \mathbf{f}\|^2}{\|\chi_e \mathbf{f}\|^2 + \frac{\sigma_e^2}{P_A}}, \quad (2)$$

where $\Phi_l = \rho_e \hat{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H} + \beta_l \mathbf{H}_e$, $\chi_e = \sqrt{1 - \rho_e^2} \tilde{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H}$, and $\mathbf{f} = \mathbf{w}/\sqrt{P_A}$. Following [17, Lemma1], for any $M \times 1$ vector $\boldsymbol{\nu} = \mathbf{\Theta} \mathbf{H} \mathbf{f}$ and $\beta_l \in [0, 1]$, we have

$$X = \|\beta_e \mathbf{a}_e + \mathbf{C}_e \boldsymbol{\nu}\|^2, \quad (3)$$

where $\mathbf{a}_e = \mathbf{H}_e \mathbf{f} \sim \mathcal{CN}_{N_E}(0, \mathbf{I}_{N_E})$ and $\mathbf{C}_e = \rho_e \hat{\mathbf{G}}_e \sim \mathcal{CN}_{M, N_A}(0, \rho_e^2 \mathbf{I}_M \otimes \mathbf{I}_M)$. The CDF of X can be approximated by the gamma distribution, with the approximate CDF $F_X(x)$ given by

$$F_X(x) = 1 - \frac{\Gamma\left(k_1, \frac{x}{w_1}\right)}{\Gamma(k_1)}, \quad x \geq 0, \quad (4)$$

where

$$k_1 = \frac{[E(X)]^2}{\text{var}(X)}, \quad w_1 = \frac{\text{var}(X)}{E(X)}.$$

The mean $E(X)$ and variance $\text{var}(X)$ are computed as

$$E(X) = N_E(\beta_e^2 + \rho_e^2 \|\boldsymbol{\nu}\|^2), \quad (5)$$

$$\text{var}(X) = E(\|X\|^2) - \|E(X)\|^2 = N_E(\beta_e^4 + 2\beta_e^2 \rho_e^2 \|\boldsymbol{\nu}\|^2 + \rho_e^4 \|\boldsymbol{\nu}\|^4). \quad (6)$$

Now let $Y = \|\chi_e \mathbf{f}\|^2 = \|\mathbf{D} \boldsymbol{\nu}\|^2$, where $\mathbf{D} = \sqrt{1 - \rho_e^2} \tilde{\mathbf{G}}_e \sim \mathcal{CN}_{N_E, M}(\mathbf{0}, (1 - \rho_e^2) \mathbf{I}_{N_E} \otimes \mathbf{I}_M)$. The CDF and PDF of $\|\chi_e \mathbf{f}\|^2$ can be approximated by the gamma distribution, with the approximate CDF $F_Y(y)$ and the approximate PDF $f_Y(y)$ given, respectively, as

$$F_Y(y) = 1 - \frac{\Gamma\left(k_2, \frac{y}{w_2}\right)}{\Gamma(k_2)}, \quad y \geq 0, \quad (7)$$

$$f_Y(y) = \frac{y^{k_2-1}}{w_2^{k_2} \Gamma(k_2)} \exp\left(-\frac{y}{w_2}\right), \quad y \geq 0, \quad (8)$$

where

$$k_2 = \frac{[E(Y)]^2}{\text{var}(Y)}, \quad w_2 = \frac{\text{var}(Y)}{E(Y)}.$$

The mean and variance of Y are calculated as $E(Y) = N_E(1 - \rho_e^2) \|\boldsymbol{\nu}\|^2$ and $\text{var}(Y) = N_E(1 - \rho_e^2)^2 \|\boldsymbol{\nu}\|^4$. The CDF of γ_e is given by

$$\begin{aligned} F_{\gamma_e}(x) &= \Pr(\gamma_e \leq x) \\ &= \Pr\left(\frac{\|\Phi_e \mathbf{f}\|^2}{\|\chi_e \mathbf{f}\|^2 + \frac{\sigma_e^2}{P_A}} \leq x\right) \\ &= \Pr\left(\|\Phi_e \mathbf{f}\|^2 \leq x \left(\|\chi_e \mathbf{f}\|^2 + \frac{\sigma_e^2}{P_A}\right)\right) \\ &= \int_0^\infty F_{\|\Phi_e \mathbf{f}\|^2}\left(x \left(y + \frac{\sigma_e^2}{P_A}\right)\right) f_{\|\chi_e \mathbf{f}\|^2}(y) dy. \end{aligned} \quad (9)$$

Substituting the expressions for $F_{\|\Phi_e \mathbf{f}\|^2}$ and $f_{\|\chi_e \mathbf{f}\|^2}$ from (4) and (8) into (9), we have

$$F_{\gamma_e}(x) = 1 - \frac{1}{(\Gamma(N_E))^2} \int_0^\infty \Gamma \left[N_E, \frac{x \left(y + \frac{\sigma_e^2}{P_A} \right)}{\beta_e^2 + \rho_e^2 \|\nu\|^2} \right] y^{N_E-1} \exp \left(-\frac{-y}{(1-\rho_e^2)\|\nu\|^2} \right) dy. \quad (10)$$

The closed-form solution of is not possible. However, $P_A \rightarrow \infty$, we have

$$F_{\gamma_e}(x) \approx 1 - \frac{1}{(\Gamma(N_E))^2} \int_0^\infty \Gamma \left[N_E, \frac{xy}{\beta_e^2 + \rho_e^2 \|\nu\|^2} \right] y^{N_E-1} \exp \left(-\frac{-y}{(1-\rho_e^2)\|\nu\|^2} \right) dy \quad (11)$$

Using [R1 , (Eq.6.455)], we have

$$F_{\gamma_e}(x) \approx 1 - \frac{1}{(\Gamma(N_E))^2} \cdot \frac{\left(\frac{x}{\mathcal{A}} \right)^{N_E} \Gamma(2N_E)}{N_E \left(\frac{x}{\mathcal{A}} + \mathcal{B} \right)^{2N_E}} \cdot {}_2F_1 \left(1, 2N_E; N_E + 1; \frac{\mathcal{B}}{\frac{x}{\mathcal{A}} + \mathcal{B}} \right) \quad (12)$$

where ${}_2F_1(\cdot)$ denotes the Gauss hypergeometric function, $\mathcal{A} = \beta_t^2 + \rho_t^2 \|\mathbf{\Theta H f}\|^2$, and $\mathcal{B} = (1 - \rho_t^2) \|\mathbf{\Theta H f}\|^2$. Now, SOP can be calculated as

$$\begin{aligned} P_{\text{out}}(R_s) &= P(C_e < C_b - R_s) \\ &= P(\gamma_e > 2^{C_b - R_s} - 1) \\ &= 1 - F_{\gamma_e}(2^{C_b - R_s} - 1), \end{aligned} \quad (13)$$

Substituting (12) in (13), we have

$$P_{\text{out}}(R_s) = \frac{1}{(\Gamma(N_E))^2} \cdot \frac{\left(\frac{\phi}{\mathcal{A}} \right)^{N_E} \Gamma(2N_E)}{N_E \left(\frac{\phi}{\mathcal{A}} + \mathcal{B} \right)^{2N_E}} \cdot {}_2F_1 \left(1, 2N_E; N_E + 1; \frac{\mathcal{B}}{\frac{\phi}{\mathcal{A}} + \mathcal{B}} \right), \quad (14)$$

where $\phi = 2^{C_b - R_s} - 1$.

R1. Gradshteyn, I. S., & Ryzhik, I. M. "Table of integrals, series, and products", Academic press, 2007.

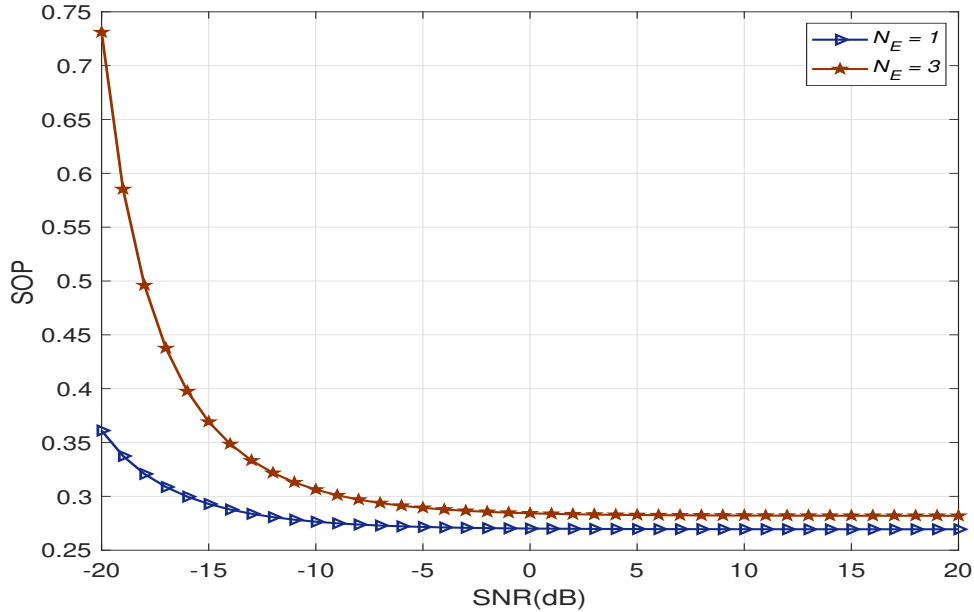


Figure 1: SOP versus SNR varying N_E when $M = 32$, $\rho_b = 1$, $\rho_e = .9$, $\alpha = 0.8$ and $\beta = 0.8$

Fig. 1 shows the impact of number of antennas at the eavesdropper (N_E) on the secrecy performance of IRS-assisted networks. As N_E increases, the SOP also increases. This trend occurs because a larger N_E enhances the eavesdropper's capacity, making Eve a more powerful candidate for intercepting information from the main channel. As a result, the eavesdropper becomes capable of decoding confidential information, leading to a higher SOP.