SOP for multiple antenna eavesdropper case

Now, we consider that Eve is equipped with $N_E (N_E \geq 1)$ antennas; the $N_E \times 1$ received signal vector at Eve is expressed as

$$\mathbf{y}_e = \left(\mathbf{G}_e \mathbf{\Theta} \mathbf{H} + \beta_l \mathbf{H}_e\right) \mathbf{w} s + \mathbf{n}_l = \left(\rho_e \widehat{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H} + \sqrt{1-\rho_e^2} \widetilde{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H} + \beta_l \mathbf{H}_e\right) \mathbf{w} s + \mathbf{n}_l \,, \tag{1}$$

where $\mathbf{G}_e \sim \mathcal{CN}_{N_E,M}(\mathbf{0}, \mathbf{I}_{N_E} \otimes \mathbf{I}_M)$ is the channel between IRS and Eve and $\mathbf{H}_e \sim \mathcal{CN}_{N_E,N_A}(\mathbf{0}, \mathbf{I}_{N_E} \otimes \mathbf{I}_{N_A})$ is the channel between Alice and Eve. The received signal-to-interference-plus-noise ratio (SINR) at Eve is given by

$$\gamma_e = \frac{\|\Phi_e \mathbf{f}\|^2}{\|\chi_e \mathbf{f}\|^2 + \frac{\sigma_l^2}{P_A}}, \tag{2}$$

where $\Phi_l = \rho_e \widehat{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H} + \beta_l \mathbf{H}_e$, $\chi_e = \sqrt{1-\rho_e^2} \widetilde{\mathbf{G}}_e \mathbf{\Theta} \mathbf{H}$, and $\mathbf{f} = \mathbf{w}/\sqrt{P_A}$. Following [17,Lemma1], for any $M \times 1$ vector $\boldsymbol{\nu} = \mathbf{\Theta} \mathbf{H} \mathbf{f}$ and $\beta_l \in [0,1]$, we have

$$X = \|\beta_e \mathbf{a}_e + \mathbf{C}_e \boldsymbol{\nu}\|^2, \tag{3}$$

where $a_e = \mathbf{H}_e \mathbf{f} \sim \mathcal{CN}_{N_E}(0, \mathbf{I}_{N_E})$ and $\mathbf{C}_e = \rho_e \widehat{\mathbf{G}}_e \sim \mathcal{CN}_{M,N_A}(0, \rho_e^2 \mathbf{I}_M \otimes \mathbf{I}_M)$. The CDF of X can be approximated by the gamma distribution, with the approximate CDF $F_X(x)$ given by

$$F_X(x) = 1 - \frac{\Gamma\left(k_1, \dfrac{x}{w_1}\right)}{\Gamma(k_1)}, \quad x \geq 0, \tag{4}$$

where

$$k_1 = \frac{[E(X)]^2}{\mathrm{var}(X)}\,, \quad w_1 = \frac{\mathrm{var}(X)}{E(X)}\,.$$

The mean $E(X)$ and variance $\mathrm{var}(X)$ are computed as

$$E(X) = N_E(\beta_e^2 + \rho_e^2 \|\boldsymbol{\nu}\|^2), \tag{5}$$

$$\mathrm{var}(X) = E\left(\|X\|^2\right) - \|E(X)\|^2 = N_E(\beta_e^4 + 2\beta_e^2 \rho_e^2 \|\boldsymbol{\nu}\|^2 + \rho_e^4 \|\boldsymbol{\nu}\|^4). \tag{6}$$

Now let $Y = \|\chi_e \mathbf{f}\|^2 = \|\mathbf{D}\boldsymbol{\nu}\|^2$, where $\mathbf{D} = \sqrt{1-\rho_e^2} \widetilde{\mathbf{G}}_e \sim \mathcal{CN}_{N_E,M}(\mathbf{0}, (1-\rho_e^2) \mathbf{I}_{N_E} \otimes \mathbf{I}_M)$. The CDF and PDF of $\|\chi_l \mathbf{f}\|^2$ can be approximated by the gamma distribution, with the approximate CDF $F_Y(y)$ and the approximate PDF $f_Y(y)$ given, respectively, as

$$F_Y(y) = 1 - \frac{\Gamma\left(k_2, \dfrac{y}{w_2}\right)}{\Gamma(k_2)}, \quad y \geq 0, \tag{7}$$

$$f_Y(y) = \frac{y^{k_2 - 1}}{w_2^{k_2} \Gamma(k_2)} \exp\left(-\frac{y}{w_2}\right), \quad y \geq 0, \tag{8}$$

where

$$k_2 = \frac{[E(Y)]^2}{\mathrm{var}(Y)}\,, \quad w_2 = \frac{\mathrm{var}(Y)}{E(Y)}\,.$$

The mean and variance of Y are calculated as $E(Y) = N_E(1-\rho_l^2)\|\boldsymbol{\nu}\|^2$ and $\mathrm{var}(Y) = N_E(1-\rho_l^2)^2 \|\boldsymbol{\nu}\|^4$. The CDF of γ_e is given by

$$\begin{aligned}
F_{\gamma_e}(x) &= \Pr\left(\gamma_e \leq x\right) \\
&= \Pr\left(\frac{\|\Phi_e \mathbf{f}\|^2}{\|\chi_e \mathbf{f}\|^2 + \frac{\sigma_l^2}{P_A}} \leq x\right) \\
&= \Pr\left(\|\Phi_e \mathbf{f}\|^2 \leq x\left(\|\chi_e \mathbf{f}\|^2 + \frac{\sigma_e^2}{P_A}\right)\right) \\
&= \int_0^\infty F_{\|\Phi_l \mathbf{f}\|^2}\left(x\left(y + \frac{\sigma_l^2}{P_A}\right)\right) f_{\|\chi_l \mathbf{f}\|^2}(y)\, dy.
\end{aligned} \tag{9}$$

Substituting the expressions for $F_{\|\Phi_e \mathbf{f}\|^2}$ and $f_{\|\chi_e \mathbf{f}\|^2}$ from (4) and (8) into (9), we have

$$F_{\gamma_e}(x) = 1 - \frac{1}{(\Gamma(N_E))^2} \int_0^\infty \Gamma\left(N_E, \frac{x\left(y + \frac{\sigma_e^2}{P_A}\right)}{\beta_e^2 + \rho_e^2 \|\boldsymbol{\nu}\|^2}\right) y^{N_E - 1} \exp\left(-\frac{y}{(1-\rho_e^2)\|\boldsymbol{\nu}\|^2}\right) dy. \tag{10}$$

A closed-form solution of is not possible. However, $P_A \to \infty$, we have

$$F_{\gamma_e}(x) \approx 1 - \frac{1}{(\Gamma(N_E))^2} \int_0^\infty \Gamma\left(N_E, \frac{xy}{\beta_e^2 + \rho_e^2 \|\boldsymbol{\nu}\|^2}\right) y^{N_E - 1} \exp\left(-\frac{y}{(1-\rho_e^2)\|\boldsymbol{\nu}\|^2}\right) dy. \tag{11}$$

Using [R1, eq. (6.455)], we have

$$F_{\gamma_e}(x) \approx 1 - \frac{1}{(\Gamma(N_E))^2} \left[\frac{\left(\frac{x}{\mathcal{A}}\right)^{N_E} \Gamma(2N_E)}{N_E \left(\frac{x}{\mathcal{A}} + \mathcal{B}\right)^{2N_E}}\right] {}_2F_1\left(1, 2N_E; N_E + 1; \frac{\mathcal{B}}{\frac{x}{\mathcal{A}} + \mathcal{B}}\right), \tag{12}$$

where ${}_2F_1(\cdot)$ denotes the Gauss hypergeometric function, $\mathcal{A} = \beta_l^2 + \rho_l^2 \|\boldsymbol{\Theta}\mathbf{Hf}\|^2$, and $\mathcal{B} = (1-\rho_l^2)\|\boldsymbol{\Theta}\mathbf{Hf}\|^2$. The SOP can be calculated as

$$\begin{aligned} P_{\text{out}}(R_s) &= P(C_e < C_b - R_s) \\ &= P\left(\gamma_e > 2^{C_b - R_s} - 1\right) \\ &= 1 - F_{\gamma_e}\left(2^{C_b - R_s} - 1\right). \end{aligned} \tag{13}$$

Substituting (12) in (13), we have

$$P_{\text{out}}(R_s) = \frac{1}{(\Gamma(N_E))^2} \left[\frac{\left(\frac{\phi}{\mathcal{A}}\right)^{N_E} \Gamma(2N_E)}{N_E \left(\frac{\phi}{\mathcal{A}} + \mathcal{B}\right)^{2N_E}}\right] {}_2F_1\left(1, 2N_E; N_E + 1; \frac{\mathcal{B}}{\frac{\phi}{\mathcal{A}} + \mathcal{B}}\right), \tag{14}$$

where $\phi = 2^{C_b - R_s} - 1$.

R1. Gradshteyn, I. S., & Ryzhik, I. M. *"Table of integrals, series, and products"*, Academic press, 2007.

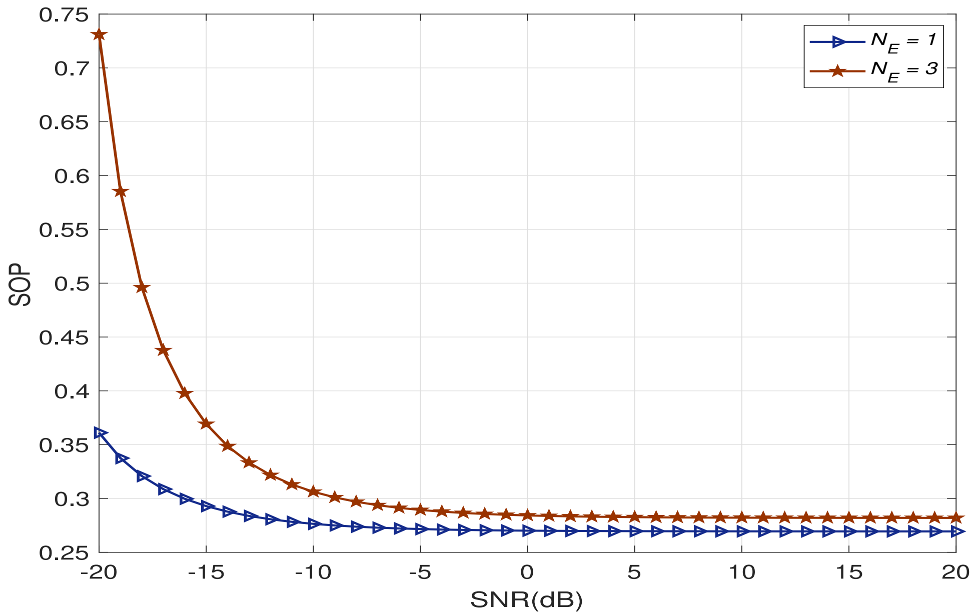

Figure 1: SOP versus SNR varying N_E when $M = 32$, $\rho_b = 1$, $\rho_e = .9$ $\alpha = 0.8$ and $\beta = 0.8$

Fig. 1 shows the impact of number of antennas at the eavesdropper (N_E) on the secrecy performance of IRS-assisted networks. As N_E increases, the SOP also increases. This trend occurs because a larger N_E enhances the eavesdropper's capacity, making Eve a more powerful candidate for intercepting information from the main channel. As a result, the eavesdropper becomes capable of decoding confidential information, leading to a higher SOP.