AO-MM:

The objective function of OP4 is given as

$$\frac{\varphi}{\beta_e^2 + \rho_e^2 \|\mathbf{\Theta}\mathbf{H}\mathbf{f}\|^2} = R \left[\frac{\|(\rho_b \hat{\mathbf{g}}_b \mathbf{\Theta}\mathbf{H} + \beta_b \mathbf{h}_b) \mathbf{f}\|^2}{\|\sqrt{1 - \rho_b^2} \tilde{\mathbf{g}}_b \mathbf{\Theta}\mathbf{H}\mathbf{f}\|^2 + \frac{\sigma_b^2}{P_A}} + \mathcal{B}_1 \right]. \tag{1}$$

where, $R = \frac{\alpha}{\beta_e^2 + \rho_e^2 ||\Theta \mathbf{H} \mathbf{f}||^2}$. Now, let us perform optimization over Θ by fixing \mathbf{f} . With (1), OP4 can be transformed as

P1:
$$\max_{\mathbf{\Theta}} \frac{\| (\rho_b \widehat{\mathbf{g}}_b \mathbf{\Theta} \mathbf{H} + \beta_b \mathbf{h}_b) \mathbf{f} \|^2}{\| \sqrt{1 - \rho_b^2} \widetilde{\mathbf{g}}_b \mathbf{\Theta} \mathbf{H} \mathbf{f} \|^2 + \frac{\sigma_b^2}{P_A}},$$

s.t. 12. (2)

Using (37) and (38), we have

P2:
$$\max_{\boldsymbol{v},|v_m|=1} \frac{\boldsymbol{v}^H \mathbf{y}_1 \boldsymbol{v} + |\alpha_A|^2 + 2\rho_b \Re\{\mathbf{e}_1^H \boldsymbol{v}^H \alpha_A^*\}}{\boldsymbol{v}^H \mathbf{y}_2 \boldsymbol{v} + \frac{\sigma_b^2}{P_A}},$$
 (3)

where $\mathbf{y}_1 = \rho_b^2 \mathbf{e}_1^H \mathbf{e}_1$, $\mathbf{y}_2 = (1 - \rho_b^2) \mathbf{e}_2^H \mathbf{e}_2$, and $\alpha_A = \beta_b \mathbf{h}_b \mathbf{f}$. Rewrite (3), we have

P3:
$$\min_{\boldsymbol{v},|v_m|=1} \frac{\boldsymbol{v}^H \mathbf{y}_2 \boldsymbol{v} + \frac{\sigma_b^2}{P_A}}{\boldsymbol{v}^H \mathbf{y}_1 \boldsymbol{v} + |\alpha_A|^2 + 2\rho_b \Re\{\mathbf{e}_1^H \boldsymbol{v}^H \alpha_A^*\}}.$$
 (4)

This problem belongs to fractional programming. Following [33], we consider the corresponding parametric program:

P4:
$$\min_{\boldsymbol{v},|v_m|=1} \boldsymbol{v}^H \boldsymbol{\theta} \boldsymbol{v} + \frac{\sigma_b^2}{P_A} - \mu |\alpha_A|^2 - 2\mu \rho_b \Re\{\mathbf{e}_1^H \boldsymbol{v}^H \alpha_A^*\},$$
 (5)

where $\mu > 0$ is an introduced parameter and $\boldsymbol{\theta} = \mathbf{y}_2 - \mu \rho_b^2 \mathbf{y}_1$. Here, we adopt MM algorithm to solve P4 due to its closed-form solution at each iteration [9],[11],[18].[30]. At iteration t, for any feasible point $\boldsymbol{v}(t)$,

$$\boldsymbol{v}^{H}\boldsymbol{\theta}\boldsymbol{v} + \frac{\sigma_{b}^{2}}{P_{A}} - \mu|\alpha_{A}|^{2} - 2\Re\{\mu\rho_{b}\mathbf{e}_{1}\boldsymbol{v}^{H}\alpha_{A}^{*}\}$$

$$\leq \lambda_{\max}(\boldsymbol{\theta})\|\boldsymbol{v}\|^{2} - \Re\{\boldsymbol{v}^{H}\boldsymbol{\varphi}\} + q_{1},$$
(6)

where $\varphi = (\lambda_{\max}(\boldsymbol{\theta})\mathbf{I} - \boldsymbol{\theta})\boldsymbol{v}(t) + \mu\rho_b\mathbf{e}_1\alpha_A^*$, $q_1 = (\lambda_{\max}(\boldsymbol{\theta})\mathbf{I} - \boldsymbol{\theta})\boldsymbol{v} + \frac{\sigma_b^2}{P_A} - \mu|\alpha_A|^2$. The simplified optimization problem becomes

P5:
$$\max_{\boldsymbol{v}} \Re\{\boldsymbol{v}^H \boldsymbol{\varphi}\},$$

s.t. $|v_m| = 1, \forall \quad m = 1, \dots, M.$ (7)

The optimal solution of P5 at iteration t is

$$v(t+1) = \left[e^{j \arg(\varphi_1)}, \dots, e^{j \arg(\varphi_M)} \right]. \tag{8}$$

Algorithm A MM-Based Algorithm for Phase-shift Matrix

```
1: Input: \mathbf{H}, \widehat{\mathbf{g}}_b, \widetilde{\mathbf{g}}_b, \mathbf{h}_b, \mathbf{f}, \rho_b, \beta_b, \sigma_b^2, P_A, \epsilon, T_{\max}
 2: Initialize: Random unit-modulus vector v^{(0)}, set \mu_{\min}, \mu_{\max}, \mu \leftarrow (\mu_{\min} + \mu_{\max})/2
 4:
              Compute \theta = \mathbf{y}_2 - \mu \mathbf{y}_1
             t \leftarrow 0
 5:
             repeat
 6:
                    Update \varphi = (\lambda_{\max} \mathbf{I} - \boldsymbol{\theta}) \boldsymbol{v}^{(t)} + \mu \rho_b \mathbf{e}_1 \alpha_A^*, where
 7:
                                                                                              \lambda_{\max} = \lambda_{\max}(\boldsymbol{\theta});
                    Update each element: v_m^{(t+1)} = e^{j \arg(\varphi_m)}, \quad \forall m = 1, \dots, M
 8:
                    t \leftarrow t+1
 9:
              until \|\boldsymbol{v}^{(t)} - \boldsymbol{v}^{(t-1)}\| < \epsilon \text{ or } t \ge T_{\text{max}}
10:
              Compute numerator and denominator:
11:
                                                                   N(\mu) = \mathbf{v}^H \mathbf{y}_1 \mathbf{v} + |\alpha_A|^2 + 2\rho_b \Re\{\mathbf{e}_1^H \mathbf{v}^H \alpha_A^*\}
                                                                   D(\mu) = \boldsymbol{v}^H \mathbf{y}_2 \boldsymbol{v} + \frac{\sigma_b^2}{P_A}
             if N(\mu) - \mu D(\mu) \ge 0 then
12:
13:
                    \mu_{\min} \leftarrow \mu
14:
15:
                    \mu_{\text{max}} \leftarrow \mu
16:
              Update \mu \leftarrow (\mu_{\min} + \mu_{\max})/2
     until |\mu_{\text{max}} - \mu_{\text{min}}| < \epsilon_{\mu} or max iterations reached
      Output: Optimal oldsymbol{v}
```

According to [10], the optimal solution v can be obtained via an Algorithm A with a computational complexity of $\mathcal{O}\left(M^3 + T_{\max}M^2\right)$, where T_{\max} denotes the number of iterations required for convergence [10, D1].

Algorithm B Alternating Algorithm

```
Require: P_A, \rho_b, \rho_e, R_s, \sigma_b^2, \sigma_e^2, \mathbf{H}, \mathbf{h}_b, \mathbf{h}_e, N_A, M, \delta
Ensure: \mathbf{\Theta}^*, w^*

1: Initialize \mathbf{f}_o as a random unitary vector of size N_A \times 1.

2: Set iteration counter i=1 and maximum iteration limit i_{\text{max}}.

3: repeat

4: Obtain \mathbf{f}_i^* via generalized Rayleigh quotient.

5: Obtain \mathbf{v}^* using MM algorithm

6: Compute P_{\text{out}}(i).

7: Update iteration counter: i \leftarrow i+1.

8: until (i < i_{\text{max}}) and |P_{\text{out}}(i) - P_{\text{out}}(i-1)| \le \delta

9: Set \mathbf{v}^* \leftarrow \mathbf{v}_i^*, \mathbf{w}^* \leftarrow \mathbf{w}_i^*.

10: Compute \mathbf{\Theta}^* \leftarrow \text{diag}(\mathbf{v}^*).

11: return \mathbf{\Theta}^*, \mathbf{w}^*.
```

Algorithm B shows alternating algorithm for phase-shifter matrix and beamforming vector. The MM method is used to find optimal phase-shift matrix and generalized Rayleigh to find optimal beamforming vector.

D1: S. Hong, C. Pan, H. Ren, K. Wang and A. Nallanathan, "Artificial-Noise-Aided Secure MIMO Wireless Communications via Intelligent Reflecting Surface," in IEEE Transactions on Communications, vol. 68, no. 12, pp. 7851-7866, Dec. 2020.