ODL

A Python framework for rapid prototyping in inverse problems

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Conclusion: Need a *common software framework* to exchange implementations of concepts and methods.

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Initial situation: No existing framework fit our purpose.

Operator Discretization Library

Main components:

Functional analysis module

Handling of *vector spaces*, *operators*, *discretizations* – generally with a *continuous* point of view

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Tomography module

Acquisition *geometries* and *forward operators* for tomographic applications.

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Main components:

Library of atomic mathematical components

– Deformation operators
– Function transforms: wavelet, Fourier, shearlet, ...
– Differential operators: partial derivative, gradient, Laplacian, ...
– Discretization-related: (re-)sampling, interpolation, domain extension, ...

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Utility functions

– Visualization: Slice viewer, real time plotting, …
– Phantoms: Shepp-Logan, FORBILD, Defrise, …
– Data I/O: MRC2014, Mayo Clinic, …

Operator Discretization Library

Main components:

User-contributed modules

“Fast track” for experimental or slightly exotic code

– Figures of Merit (FOMs) for image quality assessment

– Handlers for specific data formats or geometries

– Functionality to download and import public datasets

– Wrappers for Deep Learning frameworks: Tensorflow, Theano, Pytorch, …

Design principle: modularity

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- $\rightsquigarrow$ Mathematics as strong guideline for software design
- $\rightsquigarrow$ Makes the software “feel” natural

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Landweber's method: Determine f from given data $g = \mathcal{T}(f)$ and initial guess f_0 by

$$f_{k+1} = f_k + \omega[\partial\mathcal{T}(f_k)]^*\big(g - \mathcal{T}(f_k)\big), \quad k = 0, 1, \dots, K-1$$

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def landweber(T, f, g, omega, K):
    for i in range(K):
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Uses abstract properties of operators in the iteration:

$\rightsquigarrow$ `T(f)` $\longleftrightarrow \mathcal{T}(f)$ (operator evaluation)

$\rightsquigarrow$ `T.derivative(f)` $\longleftrightarrow \partial\mathcal{T}(f)$ (derivative *operator* at f)

$\rightsquigarrow$ `T.derivative(f).adjoint` $\longleftrightarrow [\partial\mathcal{T}(f)]^*$ (adjoint of the derivative at f)

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There are *many* readily implement operators in ODL, all implementing the above interface

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Separates the “what” (abstract interface) of an object class from the “how” (concrete implementation)

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- Code becomes more maintainable, low risk for "god objects" or scope glide

Further design considerations

ODL is a *prototyping* framework, not a black-box solution

$\rightsquigarrow$ Give users freedom to experiment and tinker, do "unorthodox" things

$\rightsquigarrow$ Very little "intelligence" that guesses what a user wants

$\rightsquigarrow$ Instead: make things "just work" that a typical user would expect to work

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Don’t sacrifice performance!
$\leadsto$ Use libraries in the most efficient way possible (avoid copies, operate in-place, work with low-dimensional arrays, vectorization, broadcasting, …)
$\leadsto$ Compute on the GPU whenever possible (new fast back-end coming soon)

Example: Tomography

Inverse Problem: Determine attenuation coefficient $\mu\colon \Omega \to \mathbb{R}$ from its ray transform $\mathcal{P}\colon L^2(\Omega) \to Y$ defined as

$$\mathcal{P}(\mu)(\ell) := \int_\ell \mu(x)\mathrm{d}x$$

for all lines ℓ.

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Regularization: Conjugate gradient (CGLS) with early termination

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- Solve inverse problem using CGLS
- Display the results

Example: Tomography

```python
# Create reconstruction space and ray transform
space = odl.uniform_discr([-20, -20], [20, 20], shape=(256, 256))
geometry = odl.tomo.parallel_beam_geometry(space, num_angles=1000)
ray_transform = odl.tomo.RayTransform(space, geometry)

# Create artificial data with around 5 % noise (data max = 10)
phantom = odl.phantom.shepp_logan(space, modified=True)
g = ray_transform(phantom)
g_noisy = g + 0.5 * odl.phantom.white_noise(ray_transform.range)

# Solve inverse problem
x = space.zero()
odl.solvers.conjugate_gradient_normal(ray_transform, x, g_noisy, niter=20)

# Display results
phantom.show('Phantom')
g_noisy.show('Noisy data')
x.show('CGLS after 20 iterations')
```

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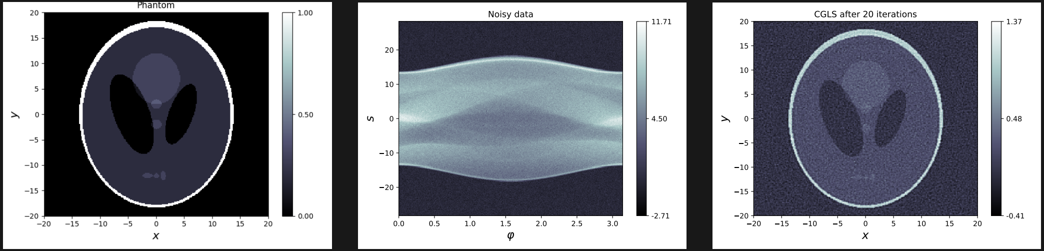

Conclusions and Outlook

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Anyone is welcome to use and/or contribute!

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