Learning to Reconstruct Medical Images

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Why do we need machine learning?

Task: Identify a rabbit in an image

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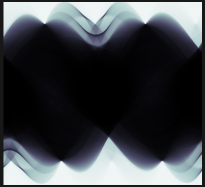
Inverse Problems

$$g = \mathcal{T}(f_{\text{true}}) + \delta g.$$

- $g \in Y$ *Data*
- $f_{\text{true}} \in X$ Image
- $\mathcal{T} : X \to Y$ Forward operator
- $\delta g \in Y$ Noise



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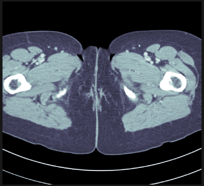

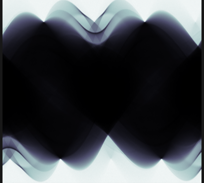

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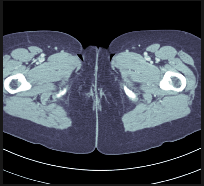
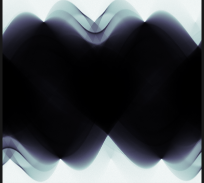


$$\xrightarrow{\mathcal{T}}$$



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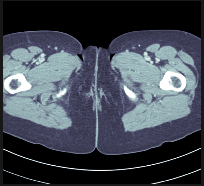

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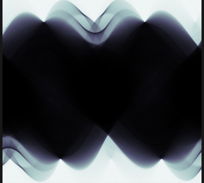

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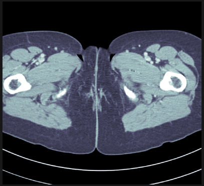

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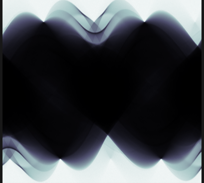

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- Strategy, solve an optimization problem:

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- Parameters (λ) need to be selected
- Large computational burden

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  $f = \mathcal{T}^{\dagger}(g)$
- Variational methods (TV, TGV, Huber)
  $f = \arg\min_{f} ||\mathcal{T}(f) - g||_Y^2 + \lambda||\nabla f||_1$
- *Machine learning*
  $f = \mathcal{T}_{\theta}^{\dagger}(g)$

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- We are given training data (f, g) which is a $X \times Y$ valued random variable such that $\mathcal{T}(\mathrm{f}) \approx \mathrm{g}$.

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- *Different from classification ($X \to \mathbb{R}^n$) and image processing ($X \to X$)*

Learned inversion methods

- *Fully learned*
- Learned post-processing
- Learned iterative schemes

Fully learned reconstruction

Goal: Learn ”the whole” mapping from data to signal

Tomographic image reconstruction based on artificial neural network (ANN) techniques
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Problem: $\mathcal{T}$ typically has symmetries, but the network has to learn them.
Example: 3D CBCT, data: 10^8 pixels and 10^8 voxels $\implies$ 10^{16} connections!

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Learned post-processing

Use deep learning to improve the result of another reconstruction

$$\mathcal{T}_\theta^\dagger = \Lambda_\theta \circ \mathcal{T}^\dagger$$

where $\mathcal{T}^\dagger$ is some reconstruction (FBP, TV, ...) and Λ_θ is a learned post-processing operator.

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Allows *separation of inversion and learning*, data can be seen as $(\underbrace{\mathcal{T}^\dagger(\mathrm{g})}_{\in X}, \underbrace{\ \mathrm{f}\ }_{\in X})$.

The problem becomes an image processing problem $\implies$ easy to solve.

Learned post-processing

Denoise in transform domain (Fourier, Wavelet, Shearlet, etc)

Won AAPM Low-Dose CT Grand Challenge:

A deep convolutional neural network using directional wavelets for low-dose X-ray CT reconstruction
Kang et. al. 2016

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We could do better by using the raw data!

Learned inversion methods

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- Learned post-processing
- *Learned iterative schemes*

Learned iterative reconstruction

- Problem: Data $g \in Y$, reconstruction $f \in X$
  How to include data in each iteration?

Learned iterative reconstruction

- Problem: Data $g \in Y$, reconstruction $f \in X$
  How to include data in each iteration?
- Inspiration from iterative optimization methods

$$f = \arg\min \frac{1}{2} || \mathcal{T}(f) - g ||_Y^2$$

Algorithm 1 Generic gradient based optimization algorithm

1: **for** $i = 1, \ldots$ **do**
2: $\quad f_{i+1} \leftarrow \text{Update}\big(f_i, \mathcal{T}^*(\mathcal{T}(f_i) - g)\big)$

Gradient descent:

$$\text{Update}\big(f_i, \mathcal{T}^*(\mathcal{T}(f_i) - g)\big) = f_i - \alpha \mathcal{T}^*(\mathcal{T}(f_i) - g)$$

Learned gradient descent

- Set a stopping criteria (fixed number of steps)
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Algorithm 2 Learned gradient descent

1: **for** $i = 1, \dots, I$ **do**
2: $\quad f_{i+1} \leftarrow \Lambda_\theta\big(f_i, \mathcal{T}^*(\mathcal{T}(f_i) - g)\big)$
3: $\mathcal{T}_\theta^\dagger(g) \leftarrow f_I$

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We separate problem dependent (and possibly global) components into $\mathcal{T}^*(\mathcal{T}(f_i) - g)$, and local into Λ_θ!

Learned Primal-Dual

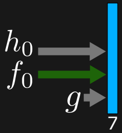
Algorithm 3 Learned Primal-Dual (conceptual)

1: **for** $i = 1, \dots, I$ **do**
2: $\quad h_i \leftarrow \Gamma_{\theta_i^d}\big(h_{i-1}, \mathcal{T}(f_{i-1}), g\big)$
3: $\quad f_i \leftarrow \Lambda_{\theta_i^p}\big(f_{i-1}, \mathcal{T}^*(h_i)\big)$
4: $\mathcal{T}_\theta^\dagger(g) \leftarrow f_I$

Learned Primal-Dual

- apply $\mathcal{T}$
- apply $\mathcal{T}^*$
- copy
- 3x3 conv + PReLU
- 3x3 conv

Learned Primal-Dual



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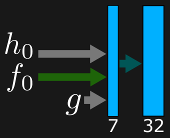

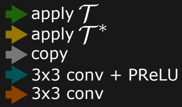

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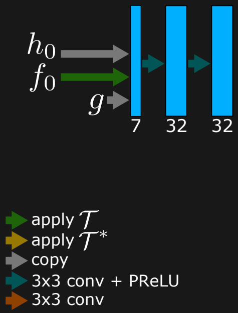

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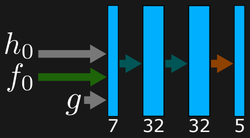

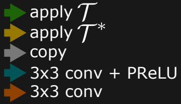

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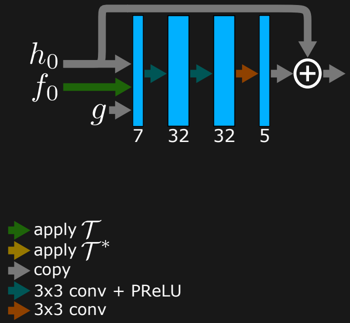

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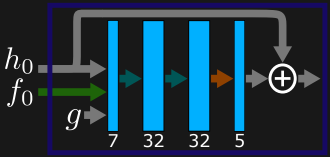

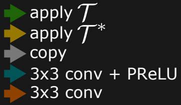

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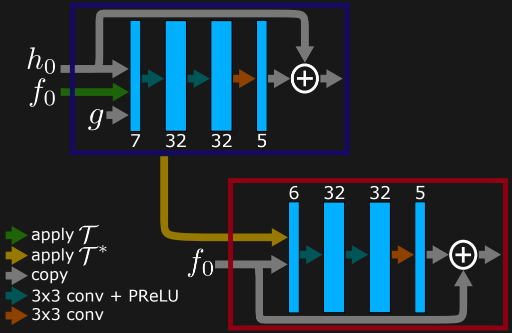

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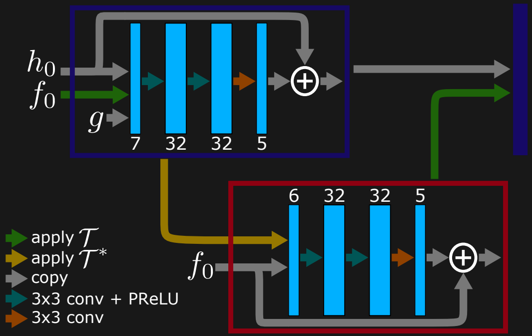

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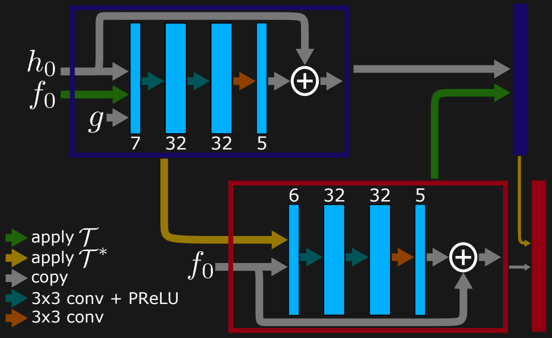

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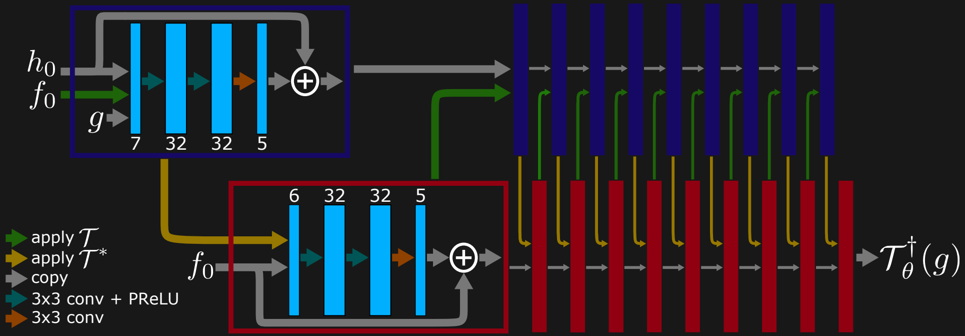

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Results

Results for CT with *Human data*

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$$g = \mathcal{P}(f) + \delta g$$
- Geometry: fan beam 1000 angles
- Noise: Poisson noise (low dose CT)
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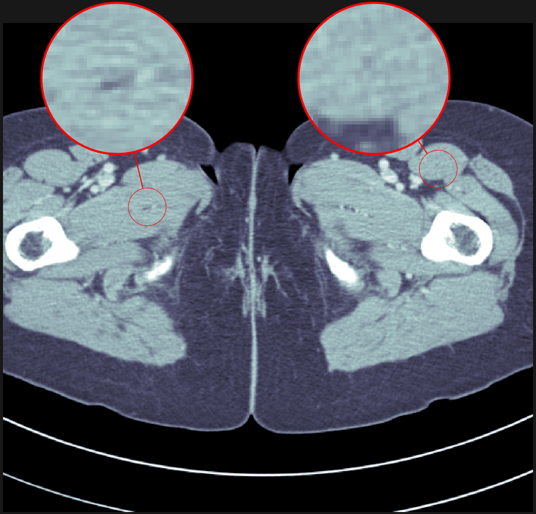
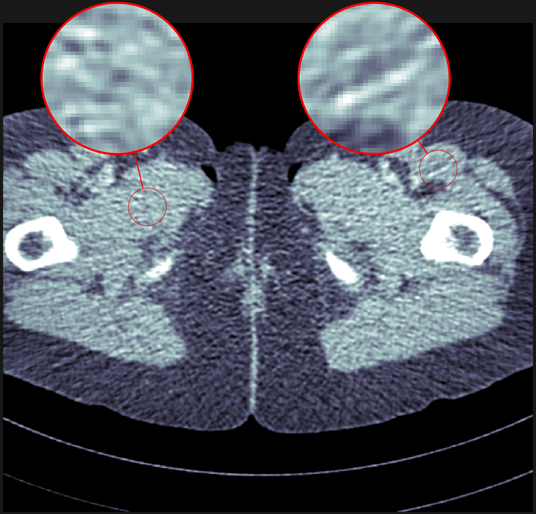
Compare to:

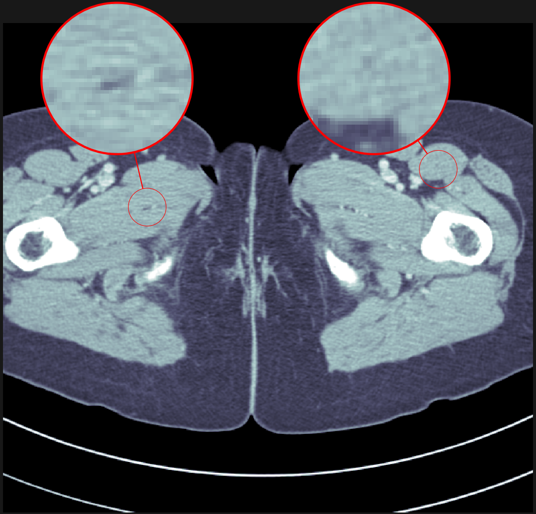
- Analytic Pseudo-Inverse (FBP)
- Variational methods (TV-regularization)
- Post-processing deep learning by U-Net



Phantom



FBP
PSNR 33.65 dB, SSIM 0.830, 423 ms



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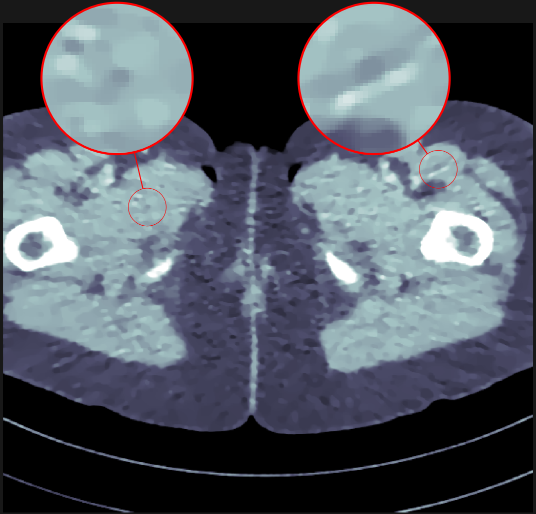
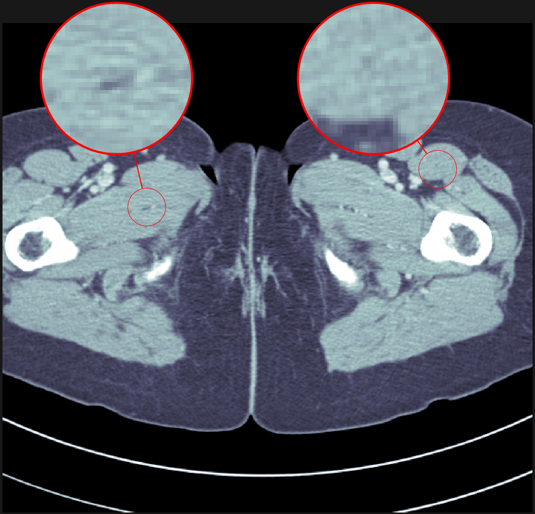


TV
PSNR 37.48 dB, SSIM 0.946, $64\,371$ ms



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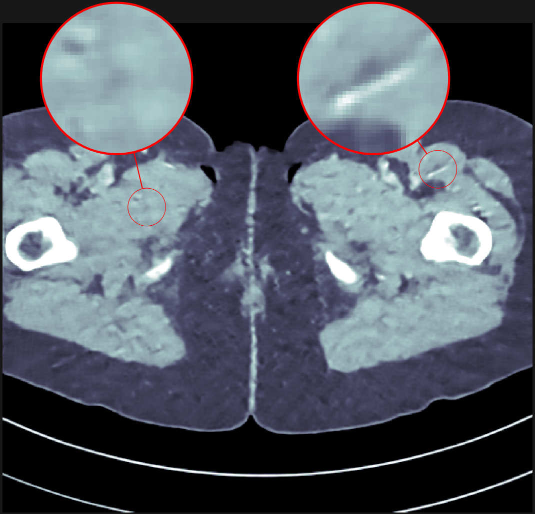
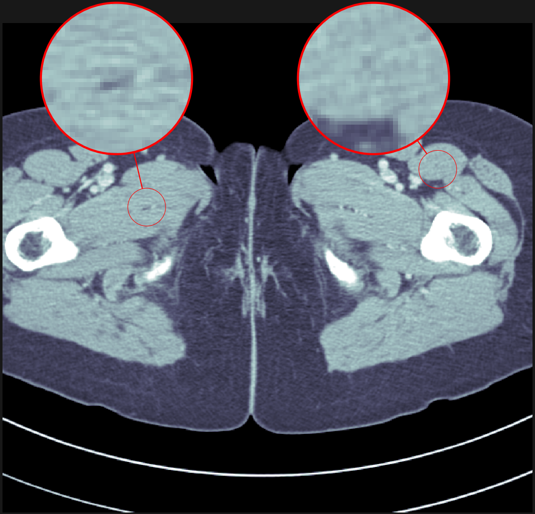


FBP + U-Net denoising
PSNR 41.92 dB, SSIM 0.941, 463 ms



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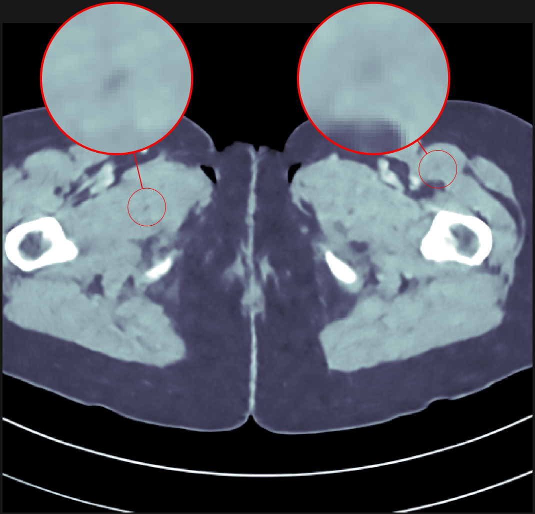

Learned Primal-Dual
PSNR 44.11 dB, SSIM 0.969, 620 ms

Conclusions

- Machine learning allows us to handle complicated priors

Source:
`github.com/adler-j`
Contact:
`jonasadl@kth.se`

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