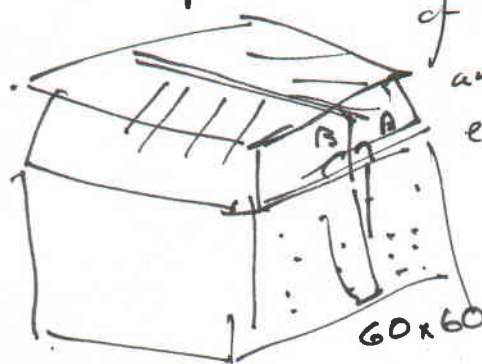


By letting the charge distribution be a series of point charges, we can represent it as a sum of points times delta functions and take an integral around them, to find an expression for total charge



Take a volume integral around a vertical line of delta charges, right below wire. Symmetric across half plane

$$\nabla \cdot \vec{F} = \alpha \rho \quad \rho = \sum_{ij} \sigma_{ij} \delta(\vec{r} - \vec{r}_{ij})$$

$$\alpha = -\frac{e^2}{\epsilon_0 \epsilon_R}$$

$$\oint_V \nabla \cdot \vec{F} d\tau = \oint_V \alpha \sum_{ij} \sigma_{ij} \delta(\vec{r} - \vec{r}_{ij}) d\tau \quad \sigma_{ij} = \sum_{kl} \lambda_{ij} \delta(\vec{r} - \vec{r}_{kl})$$

Large integral \rightarrow Select single sheet $\Rightarrow \lambda_{ij} = \sum_{kl} q_{kl} \delta(\vec{r} - \vec{r}_{kl})$ (closed)

$$\nabla \cdot \vec{F} = F_x + F_y + F_z \text{ (By definition)}$$

$$F_z = 0 \text{ (translational symmetry)}$$

$$\oint_C F_x + F_y d\tau = \int_{-B}^B F_x + F_y dx$$

$$= \int_{-B}^B F_x dx$$

Along top axis is cause edges cancel (reflect. and symmetry)

$\Rightarrow F_y$ also cancels

$$\propto \oint_C \sum_{ij} \sigma_{ij} \delta(\vec{r} - \vec{r}_{ij}) d\tau \Rightarrow \propto \oint_C \sum_{ij} \lambda_{ij} \delta(\vec{r} - \vec{r}_{ij}) d\tau$$

$$= \propto \lambda_{ij} = \alpha \cdot \text{Total charge in that line}$$

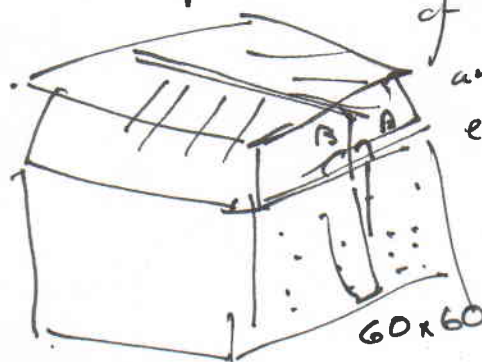
$$\int_{-B}^B F_x dx = -\frac{e^2}{\epsilon_0 \epsilon_R} \lambda_B$$

Let $y > B$ increasing/decreasing B without encompassing so by one unit cell more point charges won't change RHS so we can treat LHS as point forces above each line \rightarrow b/c line is centered btw unit cells, $\lambda_B \rightarrow 2\lambda_B$ and same reasoning applies

$$\int_{-y}^y F_x dx = \int_{-B}^B F_x dx + 2F_y = -\frac{e^2}{\epsilon_0 \epsilon_R} (\lambda_B + 2\lambda_y) \Rightarrow \boxed{F_y = -\frac{e^2}{\epsilon_0 \epsilon_R} \lambda_y}$$

Summing gives $\int F_x dx = -\frac{e^2}{\epsilon_0 \epsilon_R} \lambda_y$

By letting the charge distribution be a series of point charges, we can represent it as a sum of points times delta functions and take an integral around them, to find an expression for total charge



Take a volume integral around a vertical line of delta charges, right below wire. Symmetric across half plane



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$$\oint_C \nabla \cdot \vec{F} d\vec{r} = \oint_C \alpha \sum_{ij} \sigma_{ij} \delta(\vec{r} - \vec{r}_{ij}) d\vec{r} \quad \sigma_{ij} = \sum_{kl} \lambda_{kl} \delta(\vec{r} - \vec{r}_{kl})$$

Long integral \rightarrow Select single sheet $\circ \lambda_{ij} = \sum_{kl} q_{kl} \delta(\vec{r} - \vec{r}_{kl})$ (closed)

$$\nabla \cdot \vec{F} = F_x + F_y + F_z \text{ (By definition)}$$

$$F_z = 0 \text{ (translational symmetry)}$$

$$\oint_C F_x + F_y d\vec{r} = \int_{-B}^B F_x + F_y dx$$

$$= \int_{-B}^B F_x dx$$

Along top axis is cause edges cancel (reflect. and symmetry) $\Rightarrow F_y$ also cancels

$$\propto \oint_C \sum_{ij} \sigma_{ij} \delta(\vec{r} - \vec{r}_{ij}) d\vec{r} \Rightarrow \propto \oint_C \sum_{ij} \lambda_{ij} \delta(\vec{r} - \vec{r}_{ij}) d\vec{r}$$

$$\int_{-B}^B F_x dx = -\frac{e^2}{\epsilon_0 \epsilon_R} \lambda_B$$

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Summing gives $\int F_x dx = -\frac{e^2}{\epsilon_0 \epsilon_R} \lambda_y$

$$\frac{\lambda}{2\pi\epsilon_1\epsilon_0} r \cdot \frac{H}{r}$$

$$\int_{-\infty}^{\infty} E_N dy = \frac{\lambda}{2\pi\epsilon_1\epsilon_0} \arctan\left(\frac{\infty}{H}\right)$$

$$\oint_{\partial\Omega} \nabla \cdot \vec{E} dr = \int \frac{\sum \rho_{ij} \delta(\vec{r} - \vec{r}_{ij})}{\epsilon_2\epsilon_0} = \frac{e}{\epsilon_2\epsilon_0} \tilde{\rho}_2$$

$$= \int_{-\infty}^{\infty} E_N dy = \frac{\lambda}{\pi\epsilon_1\epsilon_0} \arctan\left(\frac{\infty}{H}\right)$$

P

treat it as $[\tilde{\rho}_2] = \frac{1}{m} \int$
cause integrating $\tilde{\rho}_2$ is a scalar multiple
electron charge

Both sides in volts

$$\tilde{\rho}_2 = \left(\frac{\epsilon_2}{\epsilon_1}\right) \left(\frac{\lambda}{e\pi}\right) \arctan\left(\frac{\infty}{H}\right)$$

Let $\tilde{\lambda} = \frac{\lambda}{e}$ be

Let $\tilde{\lambda} = \frac{\lambda}{e}$ be a multiple of
electron charge
ie $\tilde{\lambda} = 10^6$ electrons/meter

$$\tilde{\rho}_2 = \left(\frac{\epsilon_2}{\epsilon_1}\right) \left(\frac{1}{\pi}\right) \arctan\left(\frac{M}{2H}\right) \tilde{\lambda}$$

sweet line point

$$\nabla \cdot \nabla^2 V = \frac{\rho}{\epsilon_2\epsilon_0} = \frac{\sum \rho_{ij} \delta(\vec{r} - \vec{r}_{ij})}{\epsilon_2\epsilon_0} = \frac{e}{\epsilon_2\epsilon_0} \sum \tilde{\rho}_{ij} \delta(\vec{r} - \vec{r}_{ij})$$

I need V in volts, so U will be in eV $[\delta] = \frac{1}{m}$, $[e] = C$, $[\tilde{\rho}_{ij}] = \frac{F}{m}$
 $[e_0] = \frac{F}{m}$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} \rightarrow \frac{\partial^2}{\partial \left(\frac{x}{a}\right)^2} = \frac{\tilde{\nabla}^2}{a^2}$$

$$\frac{x}{a} = 1$$

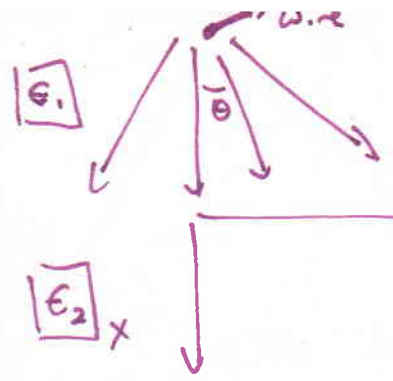
$$a = 8 \times 10^{-10} \text{ m}$$

$$\tilde{\nabla}^2 V = a^2$$

Just call a the base

$$\rightarrow \frac{V}{m^2} = \frac{C}{(F/m)} \cdot \frac{1}{m^3} = \frac{V}{m^2}$$

unit, convert ϵ_0 to a



$\vec{E} = \langle \frac{\lambda H}{2\pi\epsilon_0 r^2}, \frac{\lambda y}{2\pi\epsilon_0 r^2} \rangle$
 $E_x = \frac{\lambda}{2\pi\epsilon_0} \frac{H}{r^2}$
 $E_y = \frac{\lambda}{2\pi\epsilon_0} \frac{y}{r^2}$
 $x = H$ is fixed

$r = \sqrt{H^2 + y^2}$
 $\cos \theta = \frac{H}{r}$
 $\sin \theta = \frac{y}{r}$
 $8\pi k = \frac{e^2}{\epsilon_0 a_0}$

$a = \text{Unit cell} = 3.904 \text{ \AA}$
 $H = 12 \text{ \AA} = 3.074 a$
 $\epsilon_1 = 10$
 $\epsilon_2 = \epsilon_3 = 5$

$\int_{-\infty}^{\infty} \vec{E} dy = \left\langle \frac{\lambda H}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{1}{H^2 + y^2} dy, 0 \right\rangle$
 $= \frac{\lambda}{2\pi\epsilon_0} \left(\arctan\left(\frac{y}{H}\right) - \arctan\left(-\frac{y}{H}\right) \right)$
 $= \frac{\lambda}{\pi\epsilon_0} \arctan\left(\frac{y}{H}\right)$

$MM = y \text{ axis} = 20$
 $NN = x \text{ axis} = 10$
 $\alpha = \frac{MM}{2} \cdot a = 10 a$
 $\lambda = \frac{q}{\text{Unit cell length}}$ (for convenience)

$\oint \nabla \cdot \vec{F} = -\frac{e^2}{\epsilon_0 \epsilon_k} \lambda_{ij} = e \int_{-\infty}^{\infty} \vec{E} dy$ (integer multiple of electron charge)
 $\int_{-\infty}^{\infty} \vec{E} dy = -\frac{e^2}{\epsilon_0 \epsilon_k} p_{ij} = \frac{e\lambda}{\pi\epsilon_0 \epsilon_1} \arctan\left(\frac{\alpha}{H}\right)$
 $\epsilon_k = \epsilon_2$

$\int \vec{E} dy = -\frac{e}{\epsilon_0 \epsilon_k} p_z$
 $p_z = -\frac{\epsilon_0 \epsilon_k}{e} \int \vec{E} dy = -\left(\frac{e}{e}\right) \left(\frac{\epsilon_k}{\epsilon_1}\right) \left(\frac{\lambda}{\pi}\right) \arctan\left(\frac{\alpha}{H}\right)$
 $p_z = -\frac{1}{2} \arctan\left(\frac{39.04}{12}\right) (0.215748) \cdot \left(\frac{\pi}{0.137(\pi)}\right)$
 $= -0.137$

$\frac{\epsilon_0}{e} = 55 \cdot 263 \cdot 496 \frac{F}{m \cdot C}$
 $= 55 \dots \frac{1}{V \cdot m}$
 $= 55 \cdot \frac{1}{V \cdot m} \left(\frac{m}{10^{10} \text{ \AA}} \right) \left(\frac{3.9}{\text{Unit cell length}} \right)$
 $= 2.15748 \cdot 10^{-2} \frac{1}{V \cdot m}$

$[\lambda] = \frac{C}{m}$
 $\left[\frac{\lambda}{\epsilon_1}\right] = \frac{V}{m}$

$\frac{e}{\epsilon_0} = 46.3504 \frac{1}{a \cdot V}$

$\nabla \cdot \vec{E} = -\frac{e}{\epsilon_0 \epsilon_k} p_{ij} = -46.3504 \left(\frac{1}{5} \right)$

$\sum p_{ij} = p_z = 1$

$\epsilon_N = \frac{\pi}{(0.137)} \cdot \frac{1}{2\pi(10)} \cdot \frac{3.074}{(3.074^2 + (\frac{1}{2} - \frac{1}{2})^2)}$

1. $\epsilon_1 = 5, 10$ $\epsilon_2 = 5$
2. Linear dielectric
3. Constant ϵ (capacitor)

$$\nabla^2 V = \nabla \cdot \vec{\epsilon} = \frac{\rho_{ij}}{\epsilon_2 \epsilon_0}$$

$$[\rho] = \frac{C}{m^3}$$

$$[\nabla^2 V] = \frac{V}{m^2}$$

$$[\epsilon_0] = \frac{C}{V}$$

$$[\epsilon_0 \nabla^2 V] = \frac{V}{m^2} \frac{C}{V} = \frac{C}{m^2}$$

$$Q = CV$$

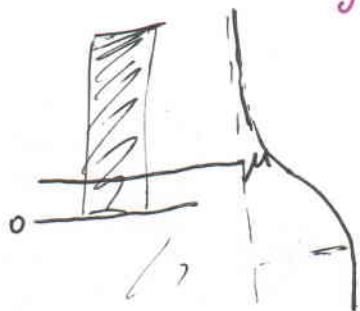
$$C = \frac{V}{Q}$$

$$C = FV$$

$$F = \frac{C}{V}$$

$$\rho_{ij} = \lambda_{ij} \delta(x-x_0) \delta(y-y_0)$$

$$[\lambda_{ij}] = \frac{C}{m^2}$$



$$\int \frac{\rho_{ij}}{|\epsilon|} dx dy = \frac{\lambda_{ij}}{|\epsilon|} \rightarrow \frac{1}{L} \frac{\epsilon_2 \lambda}{2\epsilon_1}$$

$$V \rightarrow \frac{e}{L} \frac{V_{km}}{C} \checkmark$$

$$[\nabla^2 V] = \frac{V}{L^2}$$

$$\Rightarrow [\epsilon_0 \nabla^2 V] = \frac{V}{L^2} \frac{C}{V_{km}} = \frac{C}{L^3}$$

$$[\epsilon_0] = \frac{C}{V_{km}}$$

$$\Rightarrow [\rho] = \frac{C}{L^3}$$

Define $\rho(x,y) = \sum_{ij} \epsilon \lambda_{ij} \delta(x-x_0) \delta(y-y_0)$

$$[\lambda_{ij}] = \frac{1}{L} \Rightarrow [\rho] = \frac{e}{L^3}$$

$$(a \nabla)^2 \rightarrow \tilde{\nabla}^2 \Rightarrow \tilde{\nabla}^2 V = \frac{a^2 \rho(x,y)}{\epsilon_0 \epsilon_r} = \left(\frac{e}{\epsilon_0 \epsilon_r a_0} \right) \left(\frac{a_0}{a} \right) \underbrace{a^3 \rho(x,y)}_{\tilde{\rho}}$$

$$\tilde{\rho} = \frac{a^3 \rho(x,y)}{|\epsilon|} = \sum_{ij} \underbrace{a^3 \lambda_{ij} \delta(x-x_0) \delta(y-y_0)}_{\tilde{\lambda} a \delta(x-x_0) \delta(y-y_0)} \Rightarrow \tilde{\lambda} \frac{\delta(\tilde{x}-\tilde{x}_0)}{\delta(\tilde{y}-\tilde{y}_0)}$$

$$\oint_C \nabla \cdot \vec{F} \, d\tau$$

$$\nabla \cdot \vec{F} = -\frac{e^2 \rho}{\epsilon_0 \epsilon_n}$$

$$\rho = \sum_{ij} \sigma_{ij} \cdot \delta(r - r_{ij}) = \sigma_{xy} \cdot \delta(r - r_{xy}) \quad \text{Single sheet}$$

$$\sigma_{xy} = \sum_{ij} \lambda_{ij} \delta(r - r_{ij})$$

$$\oint_C \nabla \cdot \vec{F} \, d\tau = \oint_C -\frac{e^2 \rho}{\epsilon_0 \epsilon_n} \, d\tau$$

$$\oint_C (F_x + F_y) \, d\tau$$

$$\oint_C \epsilon(x) \, dx$$

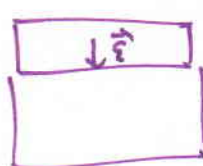
$$\int \epsilon(x) \, dx = -\frac{e^2}{\epsilon_0 \epsilon_n} \oint_C \rho \, d\tau = \frac{e^2}{\epsilon_0 \epsilon_n} \left(\sum \lambda_{ij} \delta(r - r_{ij}) \, d\tau \right)$$

$$= \frac{e^2}{\epsilon_0 \epsilon_n} \sum \lambda_{ij} = \frac{e^2}{\epsilon_0 \epsilon_n} \cdot \text{TOTAL CHARGE}$$

(Since $F = \epsilon$
except for units) \times

Write ρ as a multiple of electron charge?

with a constant boundary electric field


 $|\vec{E}| = \gamma, \quad \oint_{\partial\Omega} \nabla \cdot \vec{E} \, dr = \int_{-\infty}^{\infty} \gamma \, dy = \gamma \cdot y \Big|_{-\infty}^{\infty} = 2\gamma a = 2\gamma \cdot MM$

$$A_{max} = \frac{MM}{2} \cdot a$$

$$P_2 = - \frac{\epsilon_0 \epsilon_2}{e} \oint_{\partial\Omega} \nabla \cdot \vec{E} \, dr = - \left(\frac{\epsilon_0}{e} \right) \epsilon_2 \cdot \gamma MM$$

$$\nabla \cdot \vec{E} = - \frac{e}{\epsilon_0 \epsilon_2} \rho_{ij}$$

$$[\epsilon_0] = \frac{F}{m}$$

$$C = FV$$

$$|\vec{E}_N| = \frac{\lambda H}{2\pi \epsilon_1 \epsilon_0 r}$$

$$[\epsilon_N] = \frac{F}{m} \cdot \frac{m}{m^2} \cdot \frac{C}{m} = \frac{V}{m}$$

$$P = - \frac{1}{e} \left(\frac{\epsilon_2}{\epsilon_1} \right) \left(\frac{\lambda}{\pi} \right) \arctan\left(\frac{\alpha}{H}\right) = \frac{\epsilon_0 \epsilon_2}{e} \left(\frac{\lambda}{\pi} \right) \arctan\left(\frac{\alpha}{H}\right)$$

$$[P] = \frac{F}{m} \cdot \frac{C}{m} \cdot \frac{1}{m \cdot C} \cdot V = \frac{V}{m}$$

$$= \frac{1}{m}$$

$\int \Sigma q = \frac{\lambda}{2\pi \epsilon_1 \epsilon_0} \arctan\left(\frac{\alpha}{H}\right)$ for sure, units = V

$$\int \Sigma q = - \frac{e}{\epsilon_2 \epsilon_0} \rho_{\Sigma}$$

P_2 is a multiple of electron charge

$$\vec{E} = \frac{e}{\epsilon_2 \epsilon_0} \tilde{n}$$

$$[\tilde{n}] = \phi$$

$$[\rho_{\Sigma}] = \phi \quad \left[\frac{\epsilon_0}{e} \right] = \frac{V}{m}$$

$$V = \frac{m}{V}$$

$$\int \Sigma q = V = \left| \frac{e}{\epsilon_0 \epsilon_2} \sum \lambda_{ij} \delta(\vec{r} - \vec{r}_{ij}) \right| \delta r = \left(\frac{e}{\epsilon_0 \epsilon_2} \right) \cdot \frac{1}{m} = \frac{1}{m}$$

$$= \frac{m}{F} \cdot \frac{C}{m^2} = \frac{V}{m}$$

$$\int \Sigma q = \frac{1}{\epsilon_0 \epsilon_2} \sum \lambda_{ij}$$

no integrate to get only the sum

$$\int \Sigma q = \frac{1}{\epsilon_0 \epsilon_2} \cdot \rho_{\Sigma} = \frac{e}{\epsilon_0 \epsilon_2} \tilde{\rho} \quad \tilde{\rho} \in \mathbb{Z}^+$$

$$= \frac{\lambda}{\pi \epsilon_1 \epsilon_0} \arctan\left(\frac{\alpha}{H}\right) \Rightarrow \tilde{\rho} = \left(\frac{\epsilon_2}{\epsilon_1} \right) \cdot \frac{1}{\pi} \left(\frac{\lambda}{e} \right) \arctan\left(\frac{\alpha}{H}\right)$$

252.3

$$Z = \frac{\tilde{\lambda}}{2\pi\epsilon_1\epsilon_0} \frac{H}{r^2} = \frac{e\tilde{\lambda}}{2\pi\epsilon_1\epsilon_0} \left(\frac{H}{a}\right) \cdot \frac{1}{r^2}$$

$$= \left(\frac{e}{a_0\epsilon_0}\right) \left(\frac{1}{2\pi\epsilon_1}\right) \left(\frac{a_0}{a}\right) \tilde{\lambda} \cdot \frac{H}{r^2}$$

$$\begin{matrix} \uparrow \\ 8\pi R \end{matrix} \cdot \tilde{\lambda}^4 \cdot \boxed{\frac{H}{r^2}}$$

Want $\left[\frac{H}{r^2}\right] = \frac{1}{m}$

$$\downarrow \frac{12A^0}{(12A^{0^2} + ja^2A^0)} = \frac{1}{A^0} \frac{12}{(12^2 + j^2a^2)} \frac{10^0 A^0}{10^{-10} m} = \frac{10^0 \cdot 12}{12^2 j^2 a^2}$$

$$R = 13.6 \text{ eV}$$

$$k = \frac{e^2}{8\pi\epsilon_0 a_0}$$

$$\frac{e^2}{\epsilon_0 a_0} = 8\pi R \text{ eV} \Rightarrow \frac{e}{\epsilon_0 a_0} = 8\pi R \text{ V}$$

$$\frac{e\tilde{\lambda}}{a} = \lambda \quad \frac{\lambda}{\pi\epsilon_1} \arctan\left(\frac{L}{2H}\right) = \frac{1}{\epsilon_2} \left(\frac{e}{\epsilon_0 a_0}\right) \left(\frac{a_0}{a}\right) \sum_{ij} \int \frac{\partial x}{a} \frac{\partial y}{a} a^3 \tilde{\rho}_{ij}$$

$$a^3 \tilde{\rho}_{ij} = \left(\frac{\epsilon_2}{\epsilon_1}\right) \left(\frac{a_0}{e}\right) \cdot \frac{1}{\pi} \arctan\left(\frac{L}{2H}\right) \lambda$$

$$\boxed{a^3 \tilde{\rho}_z = \left(\frac{\epsilon_2}{\epsilon_1}\right) \frac{1}{\pi} \arctan\left(\frac{L}{2H}\right) \tilde{\lambda}}$$

$\tilde{\lambda}$ is electrons per unit cell

$$\nabla^2 V = \frac{1}{\epsilon_2} \left(\frac{e}{\epsilon_0 a_0}\right) \left(\frac{a_0}{a}\right) \sum_{ij} a^3 \tilde{\rho}_{ij}$$

$$\frac{e}{\epsilon_0 a_0} \approx \frac{1.6 \times 10^{-19}}{(8 \times 10^{-12})(.5 \times 10^{-10})} = \frac{.53 \text{ A}}{3.9 \text{ A}} \approx .4 \times 10^3 \text{ V} \approx 400 \text{ V} = 8\pi R$$

$$a = 3.904 \text{ \AA}$$

$$H = 12 \text{ \AA}$$

$$a_0 = .529 \text{ \AA}$$

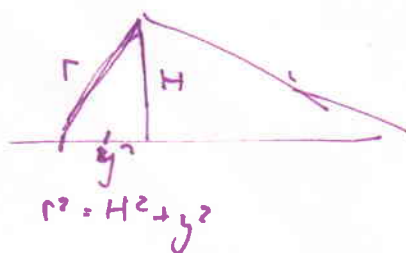
$$C = FV$$

$$\frac{e}{\epsilon_0} = \frac{1.6 \times 10^{-19}}{8.8 \times 10^{-12}} = 1.8 \times 10^{-8}$$

$$\epsilon = \frac{\lambda}{2\pi\epsilon_1\epsilon_0} \frac{H}{r^2}$$

$$= \frac{e\tilde{\lambda}}{2\pi\epsilon_1\epsilon_0} \frac{H}{r^2}$$

$$= \frac{e\tilde{\lambda}}{2\pi\epsilon_1\epsilon_0} \frac{H}{r^2}$$



$$r^2 = H^2 + y^2$$

$$\epsilon_0 = 8.854 \times 10^{-12}$$

$$a = 3.9 \times 10^{-10} \text{ m}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$H = 12 \times 10^{-10} \text{ m}$$

$$r^2 = H^2 + y^2$$

steps

$$\frac{1}{r^2} \text{ Let } \Rightarrow j = y$$

$$r^2 = \left(\frac{H}{a}\right)^2 + j^2 \quad \frac{H}{a} = 3.09 \text{ unit cell}$$