An ideal band-pass filter (BPF) with passband $\mathcal{B} = \{f : |f - f_c| \leq W\}$ is a linear system for which an input of the form $e^{j2\pi ft}$ produces an output of $Ae^{j2\pi f(t-t_0))}$ for $f \in \mathcal{B}$ and an output of 0 otherwise. f_c, W, A, and t_0 are system parameters.

Write a formula for the transfer function $H_{\mathrm{BP}}(f) = \int_{-\infty}^{\infty} h_{\mathrm{BP}}(t)e^{-j2\pi ft}$ where $h_{\mathrm{BP}}(t)$ is the impulse response of the ideal BPF filter.

Is the filter causal?

Solution The solution I was looking for was this: You are told the system is linear and what the input/output relation is for complex exponentials. You can either assume it is also time-invariant or argue it is time invariant since you are told the system has an impulse response that depends on only a single time argument. (Or you could ask.) Since a complex exponential is an eigenvalue of an LTI system, an input time signal $e^{j2\pi ft}$ yields an output $H(f)e^{j2\pi ft}$, which you are told is $Ae^{j2\pi f(t-t_0)}$ for f in the pass band and 0 outside. Hence

$$H_{\mathrm{BP}}(f) = \begin{cases} \frac{Ae^{j2\pi f(t-t_0))}}{e^{j2\pi ft}} = Ae^{-j2\pi ft_0} & f \in \mathcal{B} \\ 0 & \text{otherwise} \end{cases}$$

There were many more complicated ways to do this, but recognizing the input as a complex exponential and hence an eigenvalue of the system with $H(f)$ as the eigenvalue was the quickest. The inverse Fourier transform will give the impulse response, which is a modulated sinc function, which is not causal. (To be causal, the impulse response must be 0 for negative time.) I was not after detailed analysis here, rather I wanted to see what people could infer from the form of $H_{\mathrm{BP}}(f)$ without grinding through the computation. An even more direct answer was to observe that since the spectrum is bandlimited and symmetric, its inverse Fourier transform could not be 0 for all $t \leq 0$.