

## Problem

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### Towers of Hanoi

There are three posts,  $P_1$ ,  $P_2$ , and  $P_3$ . Post  $P_1$  starts with a tower of  $N$  disks on it,  $D_1, D_2, \dots, D_N$ , of strictly decreasing size from bottom to top. The goal is to move all  $N$  disks from post  $P_1$  to post  $P_2$ , possibly via post  $P_3$ , subject to:

- (1) At most one disk may be moved at a time.
- (2) No disk may ever be placed on top of a smaller disk.

Specifically, write a general procedure:

$\text{Move}(\{\text{disk}_1, \text{disk}_2, \dots, \text{disk}_M\}, \text{post}_1, \text{post}_2, \text{post}_3)$

that emits a sequence of instructions for moving  $\text{disk}_1, \text{disk}_2, \dots, \text{disk}_M$  from  $\text{post}_1$  to  $\text{post}_2$ , possibly via  $\text{post}_3$ . To solve the original problem we call:

$\text{Move}(\{D_1, D_2, \dots, D_N\}, P_1, P_2, P_3)$

Each emitted instruction is of the form "move  $D_i$  from  $P_j$  to  $P_k$ ".

Hint on request:

Use recursion-- note that  $\text{Move}()$  can be called with any set of disks and any parameter ordering of the three posts.

Additional/alternate problems:

(A1) Write a function that takes an argument  $N$  and returns the number of moves required to solve the problem with  $N$  disks.

(A2) What is the computational complexity of the problem (in #disks)?

### Solution

$\text{Move}(\{\text{disk}_1, \text{disk}_2, \dots, \text{disk}_M\}, \text{post}_1, \text{post}_2, \text{post}_3)$ :  
If  $M=1$  then emit "move  $[\text{disk}_1]$  from  $[\text{post}_1]$  to  $[\text{post}_2]$ "  
Else  
     $\text{Move}(\{\text{disk}_2, \text{disk}_3, \dots, \text{disk}_M\}, \text{post}_1, \text{post}_3, \text{post}_2)$   
     $\text{Move}(\{\text{disk}_1\}, \text{post}_1, \text{post}_2, \text{post}_3)$   
     $\text{Move}(\{\text{disk}_2, \text{disk}_3, \dots, \text{disk}_M\}, \text{post}_3, \text{post}_2, \text{post}_1)$

(A1)  $f(1) = 1$ ;  $f(N > 1) = 2 * f(N-1) + 1$

(A2) Exponential in  $N$ :  $O(2^N)$  Specifically,  $f(N) = 2^N - 1$

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