Stanford University
Department of Electrical Engineering
Qualifying Examination Winter 2005-06
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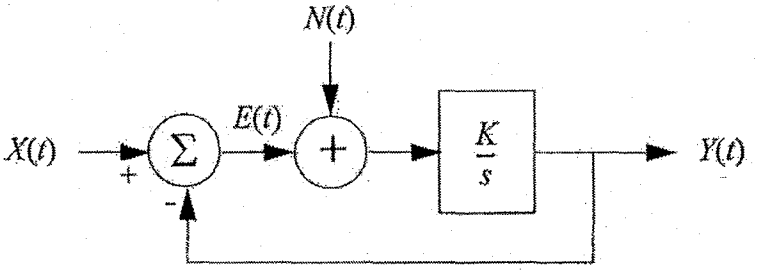

This system attempts to make the output $Y(t)$ follow the input $X(t)$. The input is a Wiener process with power spectral density:

$$S_X(\omega) = \frac{X_0}{\omega^2}.$$

The signal $E(t)$ represents the tracking error. The error signal is corrupted by addition of the noise $N(t)$, which is statistically independent of $X(t)$. $N(t)$ is a zero-mean Gaussian random process with power spectral density:

$$S_N(\omega) = N_0.$$

You are free to choose the parameter K.

Question: what value of K minimizes σ_E^2, the variance of the tracking error?

Hint: you may need to use the integral:

$$\int_{-\infty}^{\infty} \frac{d\omega}{\omega^2 + K^2} = \frac{\pi}{K}.$$

Answer:

By linearity of system,

$$E(t) = E_1(t) + E_2(t)$$

where $X(t) \rightarrow E_1(t)$ and $N(t) \rightarrow E_2(t)$

By independence of $X(t)$ and $N(t)$, $E_1(t)$ and $E_2(t)$ are independent, and:

$$\sigma_E^2 = \sigma_{E_1}^2 + \sigma_{E_2}^2$$