Prof. Tsachy Weissman

Quals Question - 2016

Consider 100 ants walking on a one dimensional horizontal stick of length one yard, each walking either to the left or to the right at the same constant speed of one yard per minute. When an ant reaches an edge of the stick it falls off. When 2 ants collide, they each turn to the respective opposite directions and continue to walk at the same speed.

(a) How much time will it take, at the most, for all the ants to fall off the stick?

Suppose initially the ants are placed independently of each other, uniformly at random on the stick, with a random initial direction.

(b) What is the expected number of ants that remain on the stick after half a minute?

(c) What is the expected amount of time until all the ants fall off the stick?

(d) What is the probability that exactly k ants remain on the stick t minutes after the initialization?

(e) What is the CLT approximation of the probability that, half a minute after initialization, there are 60 or more ants remaining on the stick?