Electrical Engineering

Quals Questions

2007

Stephen Boyd (1)

Ask me about anything that isn't clear.

$\mathbf{card}(x)$ denotes the number of nonzero entries in the vector $x \in \mathbf{R}^n$

suppose we have

$$y = Ax, \qquad \mathbf{card}(x) \leq k$$

you know $A \in \mathbf{R}^{m \times n}$, $y \in \mathbf{R}^m$, and k

how would you determine whether there is a **unique** x that satisfies these conditions, and if so, find x?

Discussion/solution.

Without the cardinality condition, there is a unique solution x only if A has zero nullspace. This requires that $m \geq n$ and that A have rank n. When we add the cardinality information, it can happen that we have a unique solution, even when $m < n$, or $\mathbf{Rank}(A) < n$. These ideas are central to a new research area called compressed sensing. But back to our problem ...

Let's consider *any* set of k indices. Form the matrix $\tilde{A} \in \mathbf{R}^{m \times k}$, taking only the associated columns of A. Now consider the equation $\tilde{A}z = y$. Any solution of this equation gives us a solution x of $Ax = y$, with $\mathbf{card}(x) \leq k$, just by inserting the entries of z into the positions of x associated with the indices, with zeros elsewhere. If the equation $\tilde{A}z = y$ has more than one solution, then the original x is not recoverable; there are at least two values of x that satisfy $Ax = y$ and $\mathbf{card}(x) \leq k$ (indeed, the two solutions have the same sparsity pattern). So the equation $\tilde{A}x = y$ can have only one or zero solutions. If $\tilde{A}x = y$ has one solution, then it is for sure a candidate for x.

Now, we carry out this analysis of the equation $\tilde{A}z = y$ for *all* $\begin{pmatrix} n \\ k \end{pmatrix}$ choices of k indices from $1, \ldots, n$. If for any choice of indices there is more than one solution, we can't recover x. We can just quit the whole process right there.

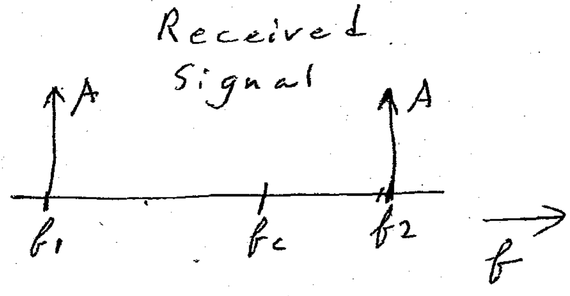
If for all choices that have a solution, the solution is the same, then that vector is x, and it is the unique solution.

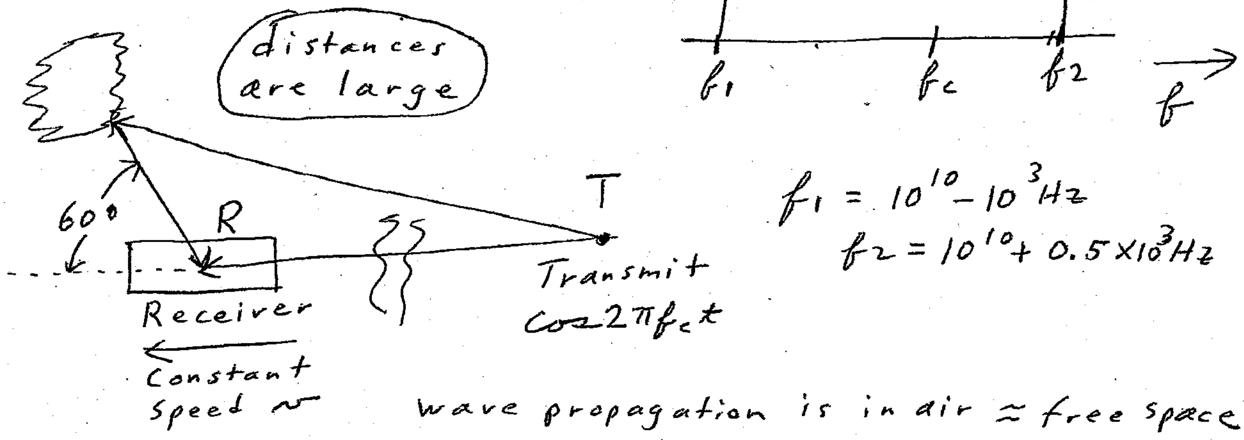
There are several ways to carry out this method. (There are also several incorrect ways to do it.) Here is one correct way: For each subset, check if $\tilde{A}z = y$ has a solution. If not, go on to the next subset. If it does, check the rank of $\tilde{A}$. If it is less than k, quit the entire algorithm, announcing

Now, this isn't really practical, since $\begin{pmatrix} n \\ k \end{pmatrix}$ is a really big number, unless k is very small. But I didn't ask for a practical method.

None of the following was needed, but you might find it interesting. It is likely there isn't a much better way to answer the question with certainty than to do an exhaustive search over subset of cardinality k. However, there are some very good heuristics for finding a sparse x that satisfies $Ax = y$. One way is to minimize $\|x\|_1$ subject to $y = Ax$. This can be done using linear programming. This is a heuristic — it can be wrong — but it very often does recover a sparse x from $y = Ax$.

Prof. Donald Cox
Ph.D. Quals Question
January 2007





Figures above were on the white board. The situation was explained: one attenuated direct path and one reflected path. The received signal spectrum is shown with two spectral lines at f_1 and f_2. There is no signal at f_c.

Questions for discussion:

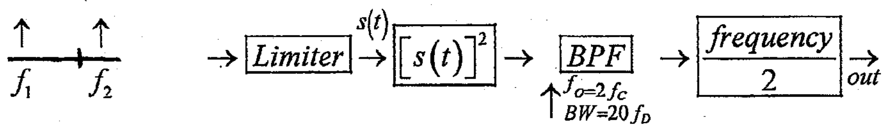
a) What is f_c?
(If student did not recognize or know Doppler relationship, he/she was coached to attempt to derive it from EM wave propagating as $\cos(2\pi f_c - kz)$.

b) What is Doppler shift frequency?
(resulted from work for a)

c) What is v?

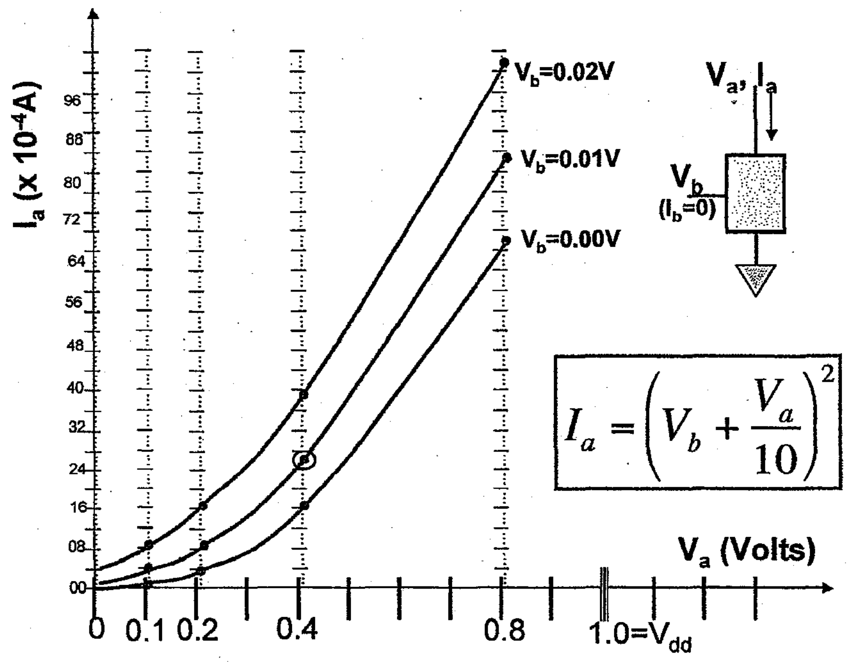
d) Is v reasonable speed for a car?

If student progressed this far, a block diagram on the white board was uncovered (it was covered by opaque paper at start of exam).



The block diagram was described and the question asked was what is the frequency or frequencies at the output?

Robert Dutton



$$\boxed{I_a = \left(V_b + \frac{V_a}{10}\right)^2}$$

Abbas El Gamal

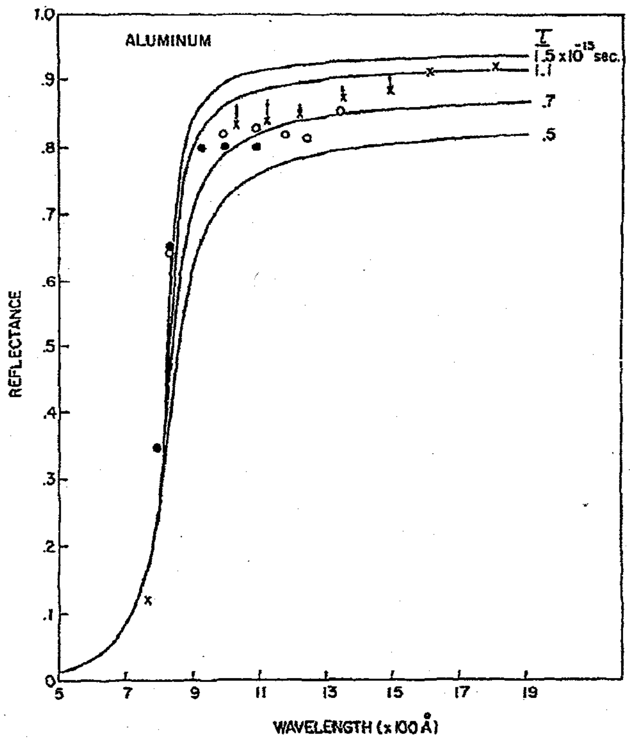
Uncorrelation versus Independence

Let $X \in \{a, b\}$ and $Y \in \{c, d\}$ be two uncorrelated random variables.

Are they necessarily independent? Justify your answer.

Shanhui Fan

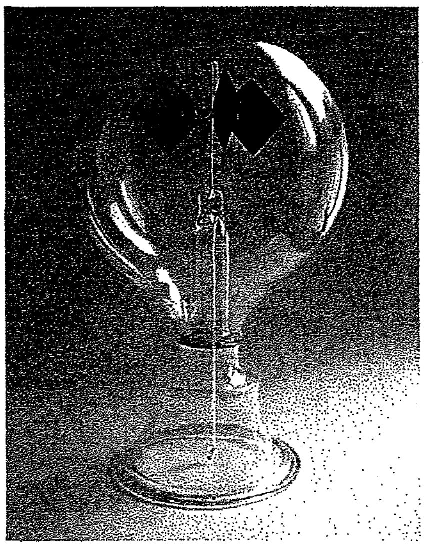
1. Why is metal highly reflecting for incident electromagnetic wave?
2. How do you explain the following measured reflectivity spectrum of aluminum, which shows a drastic reduction of reflectivity for aluminum at ultraviolet wavelength range? (Taken from R. C. Vehse, E. T. Arakawa, and J. L. Stanford, Journal of Optical Society of America, 57, 551, 1967)
3. Could you provide a simple microscopic model for this?



Ph.D. Quals Question
January 2007
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

Crookes Radiometer

The picture below shows the device that was placed on the table in front of each student being examined. It is usually considered merely a "conversation piece" nowadays, but when it was first invented in 1873 by a famous British experimental physicist, Sir William Crookes, it stimulated a number of scientific studies by eminent physicists, including James Clerk Maxwell, Osborne Reynolds, and Albert Einstein. This fact is drawn to the student's attention with the comment that "this kind of scientific interest probably indicates that there is more to this device than meets the eye." The students are asked if they have seen such a device before (most students have not) and they are asked if they know what it is called. The examiner prefers the name *Crookes radiometer* for it, but *light mill* and *solar engine* are also used.



Crookes Radiometer (from Wikipedia)

The most important part of the radiometer is a rotor to which four vanes are attached, each of which is blackened on one side and white on the other. The rotor is balanced on a vertical support and it is free to turn with very little friction. When a light is shone on the radiometer (or when it is placed in the Sun), the rotor and its four vanes begin to rotate, with the black surfaces moving away from the light and the white toward it. If the light is turned off the rotor soon begins to slow down and it is obvious that there is air or gas inside the clear glass bulb creating a drag on the vanes. At this stage the students are told that the air inside the bulb has been pumped out to create a partial vacuum but not a "perfect" vacuum (i.e., no gas at all).

Following this introduction, which only takes a short time, the student is asked how the device works.

The first and most obvious explanation is that the light pushes the black sides of the vanes away from the source of light – presumably because the light is absorbed on those sides but reflected from the white sides. Students considering this possibility are asked to work out the momentum imparted to the vanes by a single photon. It should immediately become apparent that a reflected photon (white side) imparts twice the momentum that is absorbed (black side). Thus the white sides should be pushed away from the light source, a conclusion clearly in conflict with observation. At this stage the student should dismiss this possible mechanism of operation.

The second explanation that should be looked into at this stage is based on the presence of gas in the bulb. Following up on the likelihood that the black sides of the vanes absorb more light than the white sides and are therefore warmer, the student here can draw upon the gas law, PV = nRT, to predict an increased PV in the gas close to the black sides of the vanes as the T of the gas increases. This increased PV should "push" the black sides away from the region of increased PV, thus causing the rotor to turn. This is not in fact the whole story, but it is good enough for some marks. The instructor will probably ask what the vanes are made of at this stage. After some period of puzzlement the student will possibly conclude that they must be made of insulating material, since materials that are good conductors of heat would soon lead to the white sides being the same temperature as the black sides and there would be no net force on the vanes.

A possible third mechanism is increased electromagnetic radiation away from the warm black sides of the vanes, as compared with the cooler white sides. This can lead to lengthy discussion but it is usually dismissed on the grounds that the bulb has to contain some gas for the rotor to rotate as demonstrated and thus this third possible mechanism must be secondary to the gas heating mechanism.

Scoring for this question consisted generally of 4 points for a scientifically-valid consideration of the first mechanism, with 4 more points for the second mechanism, and 2 discretionary points for such items as the third mechanism or questions relating to the composition of the vanes and its effect on the heating of the gas.

Hector Garcia-Molina
EE Quals Question 2007

The buffer manager for a file system keeps track of what disk blocks have been read into memory.
When a new block is to be read, the manager must select a buffer to hold the incoming data.
If no free space is available, the manage must first select a buffer to flush.

(a) What strategies can the buffer manager use to select a buffer to flush when space is needed?

(b) Consider a buffer manager that keeps track of its buffers using a doubly linked list.
Each record in the list represents a buffer and indicates what disk block is stored in that buffer.
The records are ordered by access time, where global pointer NEW points to the record that represents the most recently accessed buffer, and OLD points to the record that represents the least recently accessed buffer.

Write pseudo-code for a procedure REGISTER(P) that registers the fact that the buffer represented by record P is now being accessed. Thus, record P which is already in the list need to become the new NEW record at the top of the doubly-linked list.

2006-2007 Electrical Enginering Qualifying Examination

JOHN GILL

A *Bernoulli horse race* has m horses competing on a race course of length n steps.

At discrete times $k = 1, 2, 3, \ldots$ each horse flips an unbiased coin and advances one step if the coin comes up heads.

Consider just one horse. Let T be the number of coin flips that the horse takes to finish.

1. Find $\mathrm{E}(T)$, the average value of T.
2. Find $\mathrm{P}\{T \le 2n-1\}$, the probability that the horse finishes within $2n-1$ coin flips.
3. Find $p_T(r) = \mathrm{P}\{T = r\}$, the probability mass function.
4. Find the most probable value of T, that is, the largest value of $p_T(r)$.

SOLUTIONS

1. The random variable T can be written as the sum $T_1 + T_2 + \cdots + T_n$ where T_i is the number of coin flips needed to move from step $i-1$ to step i. Each T_i is an geometric random variable with success probability $p = 1/2$ and expected value $1/p = 2$. Therefore $\mathrm{E}(T) = \sum_{i=1}^{n} T_i = 2n$.
2. Consider all 2^{2n-1} sequences of $2n-1$ coin flips. A horse finishes in $2n-1$ flips if and only if at least n of the flips are heads. (If a horse finishes in less than $2n-1$ coin flips, then the remaining flips need not be looked at.) The probability of n heads in $2n-1$ flips of an unbiased coin is the probability that the majority of an odd number of flips is heads, namely, $1/2$.
3. If a horse finishes on the r-th coin flip, then the last flip is heads, and the first $r-1$ coin flips contain exactly $n-1$ heads. There are $\binom{r-1}{n-1}$ sequences satisfying these conditions. Each such sequence has probability 2^{-r}. Therefore $\mathrm{P}\{T = r\} = \binom{r-1}{n-1}2^{-r}$.

   (The sum of n geometric random variables has a *negative binomial probability distribution*.)
4. Because the pmf values are products, the easiest way to determine the maximum is to look at ratios of successive probabilities.
$$\frac{p_T(r)}{p_T(r+1)} = \frac{\binom{r-1}{n-1}2^{-r}}{\binom{r}{n-1}2^{-r-1}} = \frac{r-n+1}{\frac{1}{2}r} \lesseqgtr 1 \;\Leftrightarrow\; r-n+1 \lesseqgtr \tfrac{1}{2}r \;\Leftrightarrow\; r < 2n-2\,.$$
   This shows that if $r < 2n-2$ then $p_T(r) < p_T(r+1)$ and if $r > 2n-2$ then $p_T(r) > p_T(r+1)$. The maximum value occurs at $r = 2n-2$ and $r = 2n-1$.

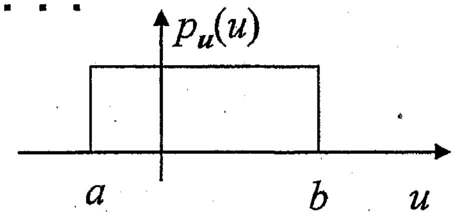
Bernd Girod

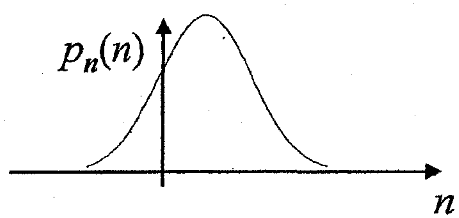
How to generate a population density map?

- Given: population density $d(\theta,\varphi)$ in people per km²
  where θ – longitude [0 . . . 360 degree]
  φ – latitude [-90 . . . 90 degree]

- **Devise an algorithm that generates random number pairs (θ,φ) in accordance with the population density $d(\theta,\varphi)$**

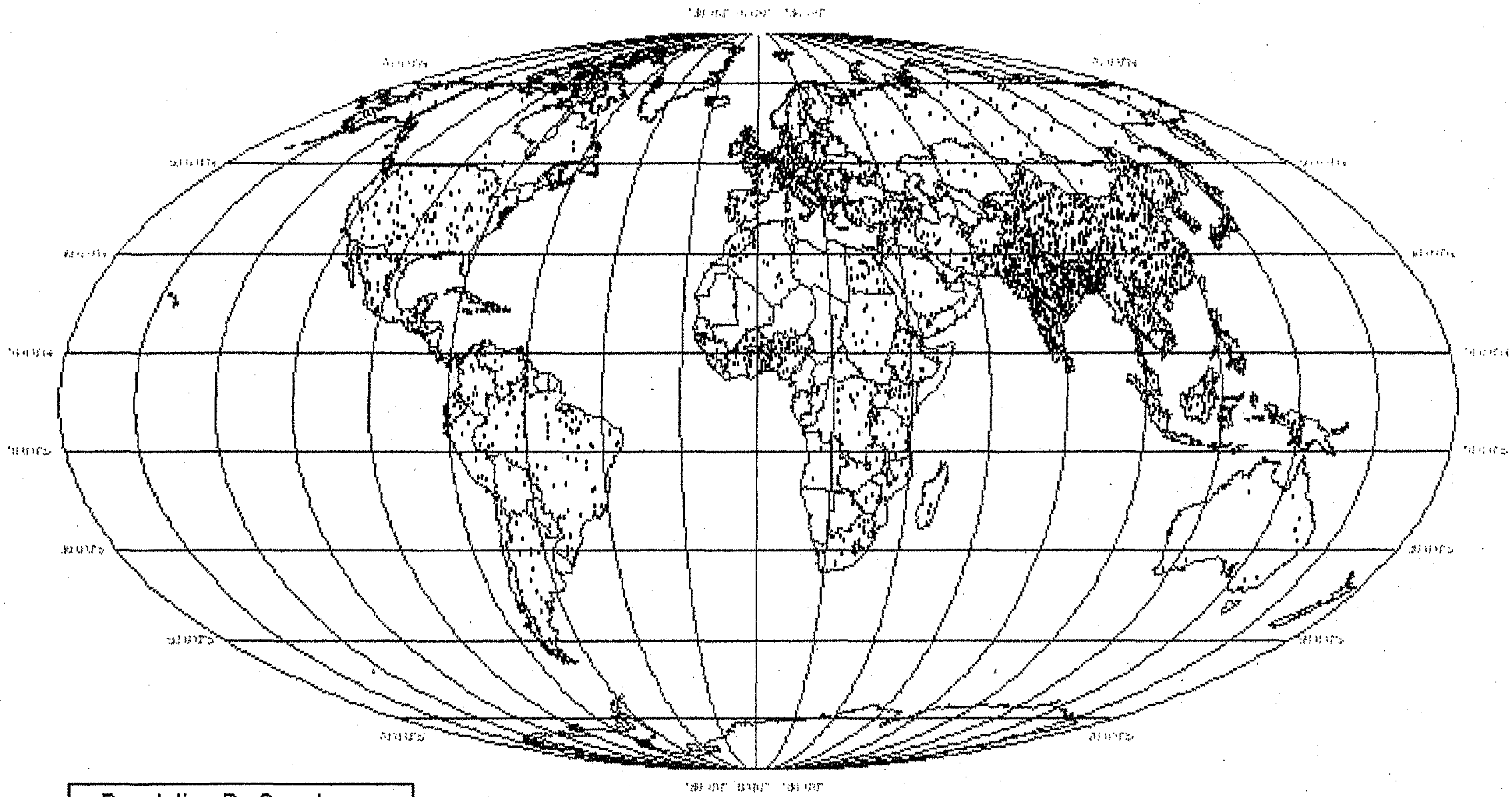
- You can generate random numbers by using . . .
  - . . . a function $u(a,b)$ that generates random numbers uniformly distributed between a and b
  - . . . a function $n(\mu,\sigma)$ that generates Gaussian random numbers with mean μ and variance σ^2





Bernd Girod

World Population Density - 1994



Population By Country

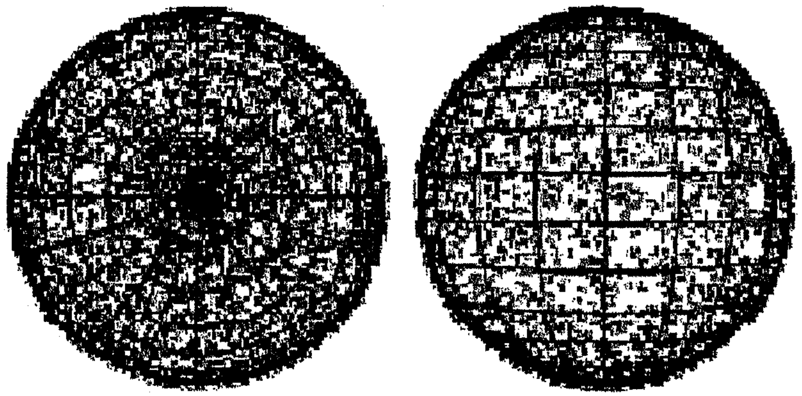
1 Dot = 2,000,000 people

Mapmaker: Mitch Comstock
Date Created: April 13, 2006
Source: ESRI, 1994
Projection: Mollweide

Bernd Girod

top view

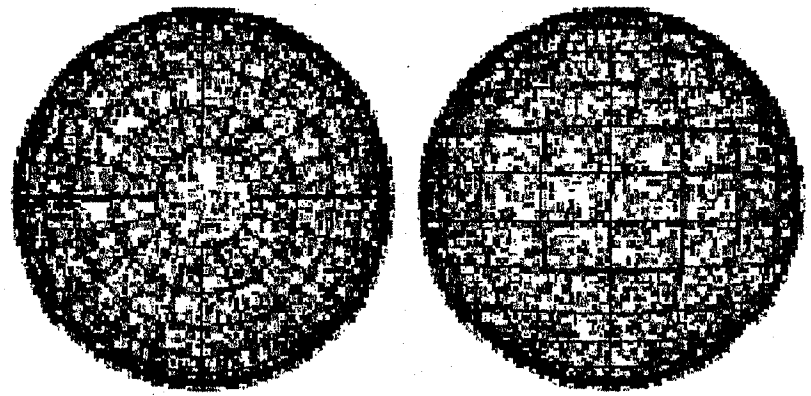
side view



incorrectly distributed points

top view

side view



correctly distributed points

Robert Gray

January 2007 Quals

Given

- Θ is a random variable described by a probability density function $f_\Theta(\theta) = 1/2\pi$ for $0 \le \theta < 2\pi$.
- $\{X_n;\ n = 0, 1, 2, \cdots\}$ is a discrete-time random process defined by $X_n = e^{jn\Theta}$.
- Fix a positive integer N and define

$$Y_k = \sum_{n=0}^{N-1} X_n e^{\frac{-j2\pi kn}{N}};\ k = 0, 1, \ldots, N-1.$$

---

1. Evaluate the mean $E(X_n)$ and autocorrelation function $R_X(n,k) = E(X_n X_k^*)$.
2. Is X_n stationary?
3. Evaluate or approximate the following sums assuming that N is very large:

$$\frac{1}{N}\sum_{n=0}^{N-1} X_n \qquad \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} X_n$$

$$\frac{1}{N}\sum_{k=0}^{N-1} Y_k \qquad \frac{1}{N}\sum_{k=0}^{N-1} |Y_k|^2 .$$

4. Suppose that we redefine X_n as $X_n = e^{jn\Theta_n}$ where now the Θ_n are independent identically distributed uniform random variables on $[0, 2\pi)$. Which of the above answers *change*?

Solutions

In all cases points are approximate, as more points could be awarded for clever solutions and fewer for meandering solutions or detours.

1. The first problem was intended to test basic probability skills. It counted for about 1.5 points out of the 10 as it is very elementary probability.

$$\begin{aligned} E(X_n) &= E(e^{jn\Theta}) \\ &= \int f_\Theta(\theta) e^{jn\theta}\, d\theta \\ &= \int_0^{2\pi} \frac{1}{2\pi} e^{jn\theta}\, d\theta = \begin{cases} 1 & n = 0 \\ 0 & n = 1, 2, \cdots \end{cases} \\ R_X(n,k) &= E(X_n X_k^*) \\ &= E(e^{jn\Theta} e^{-jk\Theta}) = E(e^{j(n-k)\Theta}) \\ &= \begin{cases} 1 & n = k \\ 0 & n \ne k \end{cases} \end{aligned}$$

Ideally the first integral was done by inspection since the integral is obviously 1 for $n = 0$ and the integral of a period of a complex exponential (or, equivalently, of a sine and cosine) is 0. Ideally the student would realize the second integral was identical to the first with $n - k$ replacing n and not redo the entire calculation.

2. The goal of the second problem was to find out what the student knew about stationarity. The problem counted for about 1/2 point, mainly for seeing this process is not stationary and why. The process cannot be stationary because the mean at time 0 is different from the mean at all other times. If the process were begun at time $n = 1$, however, this problem goes away. In that case, the fact that the autocorrelation depends only on the time difference would mean the process is weakly or wide-sense stationary.

3. Most students spent most of their time on this problem. The intent was that most students would spend the remaining time on this problem. The first sum counted for about 2.5 points and the remainder for about 1.5 points each or about 7 total.

   - $\frac{1}{N}\sum_{n=0}^{N-1} X_n$ if the law of large numbers holds, this should converge to the mean of the process, which is 0. Except for $n = 0$, the process is weakly stationary and it is an uncorrelated process, this is enough to ensure that the weak las of large numbers and hence the sum from $n = 1$ will converge in both mean-square and in probability to the common mean, which is 0. The $n = 0$ term does not effect the limit, so the answer is 0. Some students remembered this fact and a few derived the result as follows:

$$\begin{aligned} E\left[\left(\frac{1}{N}\sum_{n=0}^{N-1} X_n\right)^2\right] &= (\frac{1}{N})^2 \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} R_X(n,k) \\ &= (\frac{1}{N})^2 \sum_{n=0}^{N-1} R_X(n,n) = \frac{1}{N} \to 0. \end{aligned}$$

     Convergence in probability follows from this result and the Tchebychev inequality. Alternately and equally good was to use the method described in the next item and simply directly prove that the sum goes to zero by evaluating it using the geometric progression.

   - The $\sqrt{N}$ term instead of the N term in the denominator was intended to make the student think of the central limit theorem, but that does not work here because this process does not meet any of the conditions required for the central limit theorem to hold. So here something different is needed, and the trick is to try to find the sum exactly. Here the geometric progression can be used to write

$$\sum_{n=0}^{N-1} X_n = \sum_{n=0}^{N-1} e^{jn\Theta} = \frac{1 - e^{j\Theta(N+1)}}{1 - e^{j\Theta}}$$

     With probability 1 Θ will take on a sample value that is not 0, so the denominator has some fixed value independent of N and the numerator is a well behaved function of N that can never have magnitude greater than 2. Thus dividing by $\sqrt{N}$ in the denominator will drive $(1/\sqrt{N})\sum_{n=0}^{N-1} X_n$ to zero.

     If this method were used for the first sum, then this part was an obvious variation.

   - $\frac{1}{N}\sum_{k=0}^{N-1} Y_k$ The fast way to do this one is to realize that the Y_k constitute the DFT of the X_n, so this is just the inverse DFT for $n = 0$, which is 1.

     If the inverse DFT was not remembered, it could be derived as

$$\begin{aligned} \frac{1}{N}\sum_{k=0}^{N-1} Y_k e^{\frac{j2\pi kn}{N}} &= \frac{1}{N}\sum_{k=0}^{N-1}\left(\sum_{m=0}^{N-1} X_m e^{\frac{-j2\pi km}{N}}\right) e^{\frac{j2\pi kn}{N}} \\ &= \sum_{m=0}^{N-1} X_m \frac{1}{N}\sum_{k=0}^{N-1} e^{\frac{j2\pi k(n-m)}{N}} = X_n \end{aligned}$$

since

$$\frac{1}{N}\sum_{k=0}^{N-1} e^{\frac{j2\pi k(n-m)}{N}} = \begin{cases} 1 & n = m \text{ (modulo } N) \\ 0 & \text{otherwise} \end{cases}$$

- $\frac{1}{N}\sum_{k=0}^{N-1} |Y_k|^2$ This sum was intended to recall Parceval's equation for the DFT. So either from that relation directly or from

$$\begin{aligned} \frac{1}{N}\sum_{k=0}^{N-1} |Y_k|^2 &= \frac{1}{N}\sum_{k=0}^{N-1} Y_k Y_k^* \\ &= \frac{1}{N}\sum_{k=0}^{N-1} Y_k \left(\sum_{n=0}^{N-1} X_n e^{\frac{-j2\pi kn}{N}}\right)^* \\ &= \sum_{n=0}^{N-1} X_n^* \frac{1}{N}\sum_{k=0}^{N-1} Y_k e^{\frac{j2\pi kn}{N}} \\ &= \sum_{n=0}^{N-1} X_n^* X_n = \sum_{n=0}^{N-1} |X_n|^2 \end{aligned}$$

  Since $X_n = e^{jn\Theta}$ has magnitude 1 for all Θ, the answer is just N.

4. The last problem counted for about 1 point. If $X_n = e^{jn\Theta_n}$, then X_n becomes iid. The mean of X_0 changes to 0, but the remaining means and the autocorrelation are unchanged. The process is now strictly stationary since all iid processes are. All of the sums remain as before except the second sum, which now converges in distribution to a Gaussian random vector from the central limit theorem (both the real and imaginary parts converge to Gaussian random variables).

2007 PhD Quals Question, James Harris

1. Si MOSFET

A. Can you sketch the cross-section of a MOSFET?

B. Below your sketch, can you sketch the energy band diagram at thermal equilibrium with no gate or drain voltage?

C. Using a different color, can you show how the band diagram is changed if I apply a sufficient gate bias to reach inversion under the channel?

D. Can you now show how the band diagram is changed when I apply a drain bias?

E. Draw the drain I-V characteristic and briefly describe the 2 or 3 most important regions of the I-V characteristic and how these are related to the three band diagrams you've sketched.

2. Single Electron Transistor

The above characteristics look pretty useful, so I'm going to SNF and fabricate some of these devices. I go down to the e-beam system to make very small gate length devices and find my line drawing skills are pretty good, but my alignment skills are somewhat lacking and so I have produced a series of devices in which the gate length is less than the source-drain spacing, so draw that on the cross-section schematic of your earlier device to make sure we are both clear what kind of device my inept skills have produced.

A. Would this device work by the normal processes by which we describe MOSFETs? Why not?

B. If I apply a gate bias, can I still create an inversion layer under the gate of this device?

C. Is there anything different about the inversion layer in this device and the normal MOSFET?

D. If I apply a drain bias to this device, would I expect to get any drain current? By what mechanism?

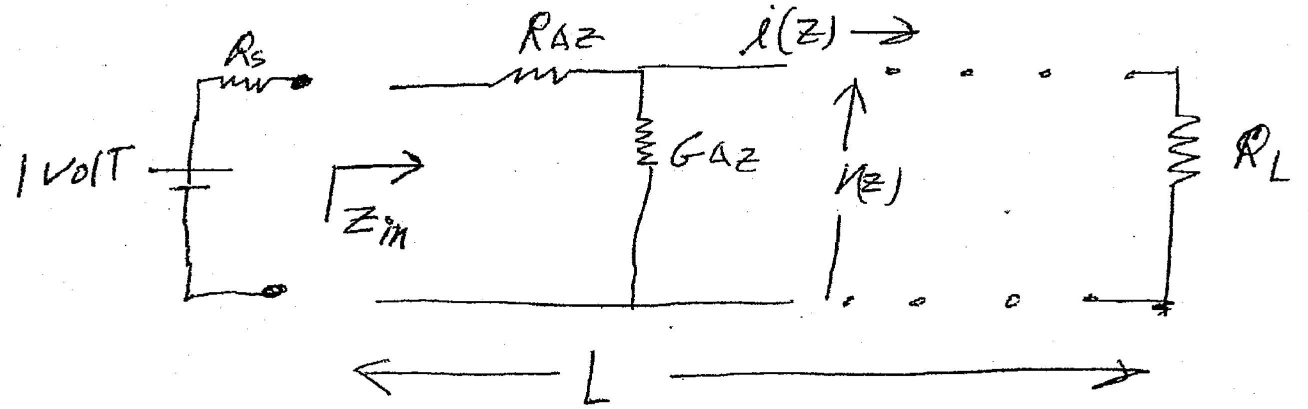
E. I made 25 of these devices in SNF with gate lengths from 100nm down to 5 nm and they all had the same degree of mis-alignment at source and drain, i.e., the barriers at each end were all exactly the same. As I measure the drain I-V characteristics of these devices, would I expect to see anything different about the characteristics as the gate length became shorter and shorter?

F. What do you imagine the characteristics might look like compared to the normal MOSFET characteristics and what would this be due to?

G. If I said that I measured a voltage step of 25mV for the tunneling of a SINGLE electron onto the inversion layer island of my device, how might you go about estimating how small such an island would have to be?

Quals (2007)
Prof. S.E. Harris

Consider a line with distributed resistance R and conductance G connected to a load R_L, with length L



With the source disconnected find the input impedance Z_{in}.

Note that the line is completely described by the differential Equations:

$$\frac{dV}{dz} = -Ri$$

$$\frac{di}{dz} = -GV \qquad (1)$$

where $V(z)$ and $i(z)$ are the voltage and current as a function of position.

Quals (2001)
S.E. Harris

Solution

$$\frac{d^2 V}{dz^2} = -R\frac{di}{dz} = +RGV$$

with $\alpha = \sqrt{RG}$

$$V = V_1 \exp -\alpha z + V_2 \exp +\alpha z$$

$$i = -\frac{1}{R}\frac{dV}{dz}; \quad \text{with} \quad \frac{\alpha}{R} = \sqrt{\frac{G}{R}} \equiv \frac{1}{Z_0}$$

$$i = \frac{V_1}{Z_0}\exp -\alpha z - \frac{V_2}{Z_0}\exp +\alpha z$$

$$Z_{in} \equiv \frac{V(-L)}{i(-L)}$$

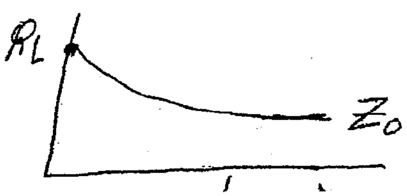
— Use the b.c. at $z = 0$ $\frac{V(0)}{i(0)} = R_L$ to find V_2/V_1;

with $R = G = 1$ obtain

$$Z_{in} = \frac{R_L \cosh z - \sinh z}{\cosh z - R_L \sinh z} \leftarrow$$ Nobody got this far

= Question: Plot Z_{in} as a function of L with attention to $L \to \infty$

Answer



$Z_{in} \to Z_0$ when

Quals Questions 2007

From: Mark Horowitz <horowitz@stanford.edu>

Here was my quals question:

Basic setup: I am asking people how to build the bypass network for a simple processor. This is looking for them to understand what operations are required, and then to explore how to build the comparison networks and then the priority encoder

If the value is available early, why isn't it written back when it is created

In a 5 stage pipeline, how many value need to be bypassed.

What does the input mux look like ? Does it depend on the register file design?

You need to create the control logic for the mux. What logic needs to be performed

Design both the match logic, and the priority encoder

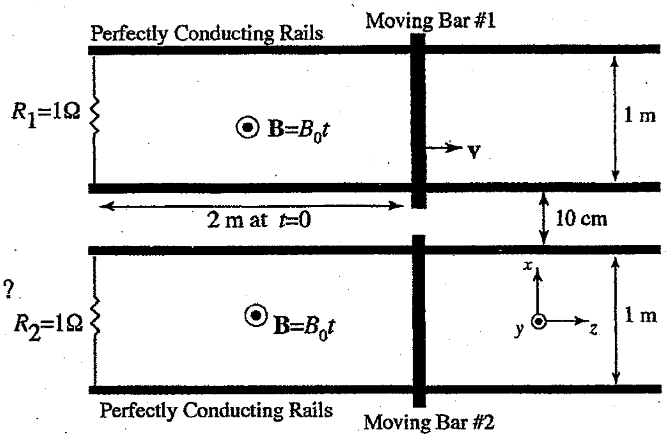
What happens if the machine has a fixed zero register

How does the complexity change with increasing issue width?

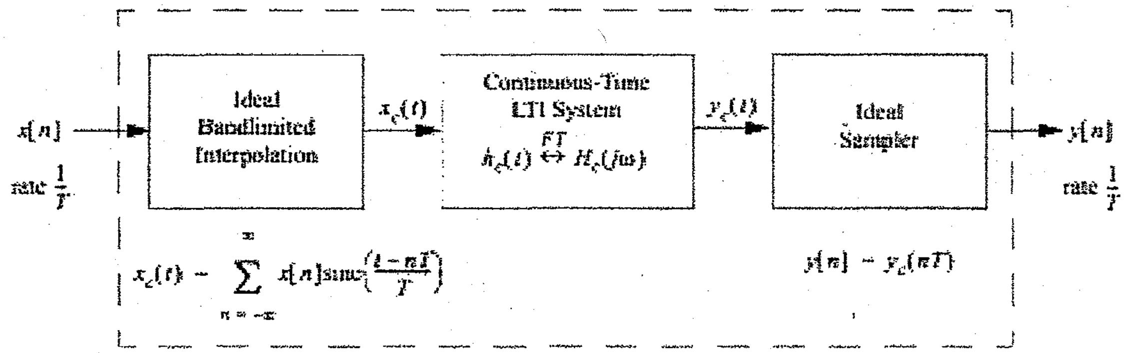
Inan/2007/Q2

Neglect friction and mass of bars.
State All Other Assumptions.
Bar#1 is moved at constant velocity v=1 m/s
The B-field is everywhere; linearly increases with time
B_0=1 wb/m^2

1) How much power is dissipated in R_1 at t=2 s ?
2) Does Bar#2 move as a result of the motion of Bar#1?
   If so, in what direction? What speed?



Stanford University
Department of Electrical Engineering
Qualifying Examination Winter 2006-07
Professor Joseph M. Kahn



The dashed box encloses a discrete-time system having input $x[n] \overset{DTFT}{\leftrightarrow} X(e^{j\Omega})$ and output $y[n] \overset{DTFT}{\leftrightarrow} Y(e^{j\Omega})$. Give an explicit relationship between $X(e^{j\Omega})$ and $Y(e^{j\Omega})$. Is this system linear and time-invariant?

Solution: Let $\Omega = \omega T$. Define $\Pi(x) = \begin{cases} 1 & |x| \le 1/2 \\ 0 & |x| > 1/2 \end{cases}$.

Then $x_c(t) \overset{FT}{\leftrightarrow} X_c(j\omega) = T \cdot \Pi\left(\frac{\omega T}{2\pi}\right) \cdot X(e^{j\omega T}) = \begin{cases} T \cdot X(e^{j\omega T}) & |\omega| \le \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$.

$$Y_c(j\omega) = H_c(j\omega) \cdot X_c(j\omega) = T \cdot \Pi\left(\frac{\omega T}{2\pi}\right) \cdot H_c(j\omega) \cdot X(e^{j\omega T}).$$

$$Y(e^{j\omega T}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} Y_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) = X(e^{j\omega T}) \cdot \sum_{k=-\infty}^{\infty} H_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) \cdot \Pi\left(\left(\omega - k\frac{2\pi}{T}\right)\frac{T}{2\pi}\right).$$

The system within the dashed box can be expressed in terms of the equivalent discrete-time frequency response:

$$H(e^{j\Omega}) = H(e^{j\omega T}) = \frac{Y(e^{j\omega T})}{X(e^{j\omega T})} = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) \cdot \Pi\left(\left(\omega - k\frac{2\pi}{T}\right)\frac{T}{2\pi}\right),$$ which is the periodic extension of $H_c(j\omega)$ bandlimited to $|\omega| \le \pi/T$. Since the system can be expressed in this way, it is linear and time-invariant.

Kahn Signals 06-07.pdf

NAME Christos Kozyrakis DATE ____________

- **What is the base idea behind the stored-program concept?**

The idea is to use a single storage (memory) in a computer to store both instructions and their data. It was introduced by von Neumann in the 40s,

- **What are the advantages of the stored-program concepts? Provide some examples.**

Better utilization of the storage

We can write programs that write programs (compilers, assemblers, JITs etc).

- **What are the disadvantages of the stored program concept? How can we address this problem?**

Security: a malicious (or buggy) program can produce data that are later treated as code and lead to crashes or other serious security issues for the application or the whole system. Buffer overflows that overwrite return addresses in stacks are a good example.

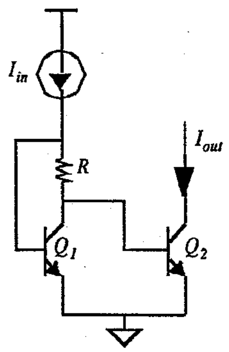
Various protection mechanisms could help: non-execute bits, virtual memory, etc.

Name: Tom Lee **Solutions** Ph.D. Qualifying Examination 2007

STANFORD UNIVERSITY
Department of Electrical Engineering

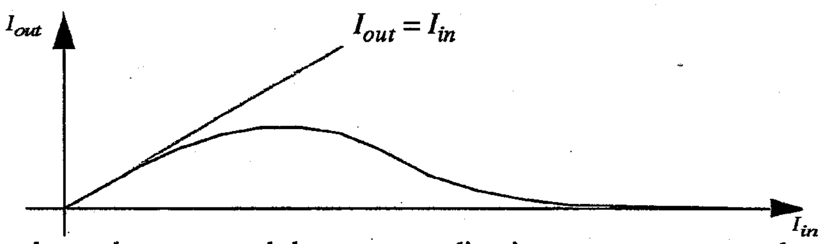
Bob Widlar is often considered to be the father of analog IC design. The following current source is but one of his many ingenious circuits (from the LM101A op-amp):

Assume that all components are ideal.



Plot the output current as a function of input current. Label any features of relevance.

Ans: A couple of quick observations enable a rough sketch. First, $V_{BE2} < V_{BE1}$ for any nonzero I_{in}. So, $I_{out} < I_{in}$ for all $I_{in} > 0$. There is a range of I_{in} (to be defined later) over which the drop across R is negligible, and thus over which I_{out} is approximately equal to I_{in}. But, the drop across R grows linearly, while V_{BE1} grows only logarithmically. So, V_{BE2} eventually decreases. At very high I_{in} the drop across R is large enough that V_{BE2} is essentially zero (V_{CEsat1} could be taken as ideally zero), and the output current heads to zero. The corresponding plot therefore looks roughly like this:



To compute the peak output, and the corresponding input current, we need an equation or two:

$$\left(I_{in}R = V_{BE1} - V_{BE2} = V_T \ln\frac{I_{in}}{I_{out}}\right) \Rightarrow I_{out} = I_{in}\exp\left(-\frac{I_{in}R}{V_T}\right). \qquad \text{(EQ 1)}$$

This equation is readily solved for the coordinates of the peak: $I_{in} = V_T/R$, $I_{out} = V_T/eR$. This circuit, known as a *peaking current source*, is useful because its output current is independent of input current (to first order), provided that the nominal input current is set to V_T/R. The circuit then produces an output current that is a factor of e smaller than that nominal value, even if the input current should deviate a bit from the nominal. Several may be combined to broaden the flatness, in a manner similar to filter design. Finally, the same topology functions for MOS implementations, although the numbers differ.

EE PhD Quals Question
January 2006
Professor Nick McKeown

1. What is *congestion control* and why do we use it in the Internet?

2. Flows in the Internet commonly use TCP (Transmission Control Protocol). TCP uses window-based flow control, in which a maximum number of packets, W, are allowed to be outstanding (i.e. transmitted but not yet acknowledged) at any one time. How many packets should a transmitter hold onto, just in case they need to be retransmitted?

3. To control congestion, TCP does not use a fixed value for W. Instead, W varies over time, depending on the current congestion in the network. Specifically, TCP follows two rules to control congestion:
   a. When a packet is successfully acknowledged: $W \rightarrow W+1$.
   b. When a packet is dropped: $W \rightarrow \frac{W}{2}$.

   Sketch the evolution of W as a function of time, assuming that exactly one packet is dropped every time W reaches $\hat{W}$.

4. Based on your sketch in (3), derive an approximate expression for the throughput of a TCP flow as function of p (the loss probability) and RTT (the round-trip-time, which we will assume is constant).

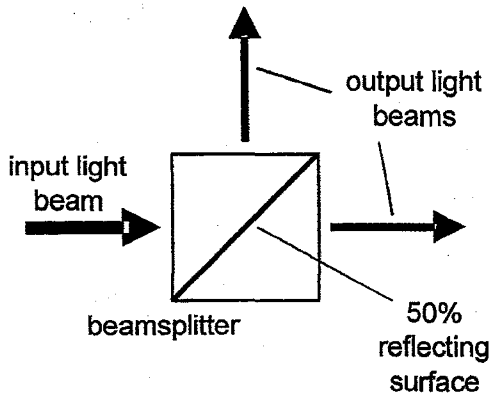
EE Ph.D. Qualifying Exam, January 2007 Question

David Miller

Backwards Beamsplitters

Note: if you finish the questions on this sheet, subsequent questions will be asked.

A mirror with 50% reflectivity can be used as a beamsplitter, turning one light beam into two, as shown in the figure. (A beamsplitter is often made in the shape of a cube of glass, with the reflecting surface in the middle of the cube, as sketched in the figure.)



Starter question (not part of the graded exam):

What happens if we shine the input light beam onto the bottom face of the beamsplitter, instead of the left face?

Question:

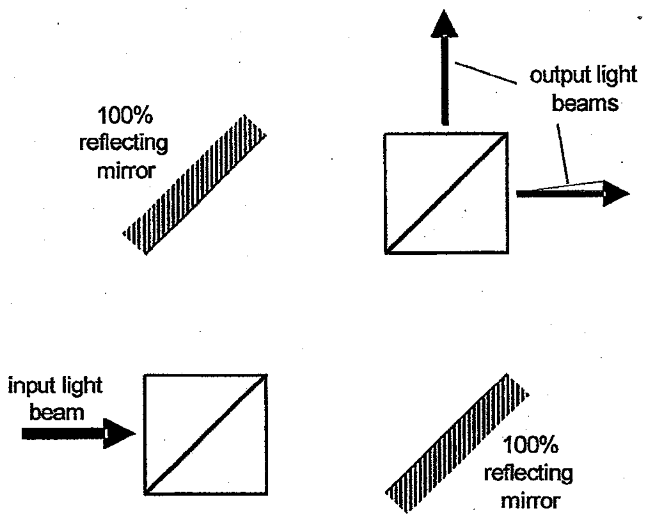
Can we run this beamsplitter backwards, using it to *combine* the power of two input light beams into one? (For example, we could have input light beams incident on the top and right faces, and want to combine them to give an output light beam only from the left face.) You may presume that the two light beams to be combined are monochromatic light beams (i.e., one color or frequency) of exactly the same frequency.

If we can combine them, what determines whether the combined beam comes out of the left face or out of the bottom face?

Supplementary questions (all on separate sheets)

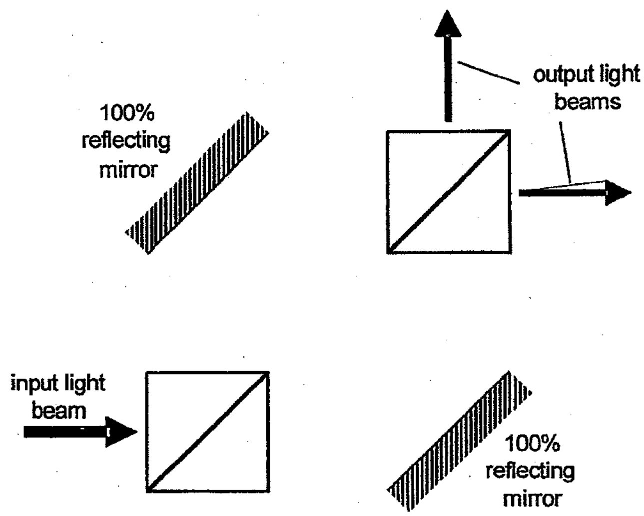
Supplementary question 1

In the apparatus below, in which there are two beamsplitters (each 50% reflecting) and two mirrors (each 100% reflecting), what would you do to the apparatus, or what would you add, that would allow you to control how much of the power came out in each of the output beams?



Supplementary question 2

In the apparatus below, what do you think will happen if we turn down the power in the input light beam so much that there is only ever one photon in the apparatus at a time? In that case, will we still be able to control the relative average powers in the two output light beams just as we did before?



Supplementary question 3

Suppose we had two hot bodies (e.g., like light-bulb filaments) emitting light. Do you think you could use a beamsplitter to combine the emitted light power from these two so that you could heat up another object (e.g., another light bulb filament) to a hotter temperature? If so, how would you do it? If not, why could you not do it?

Solution

Starter question:

If we shine the light beam onto the bottom of the beamsplitter, it will still come out of the top and right sides with equal powers.

(The reasons for asking this question are to make sure the examinee understands beamsplitters (so the exam is not a test of basic optics knowledge), and to set up the paradox in the examinee's mind that underlies the actual question – namely, if we are to run this beamsplitter backwards, what is it about the light beams that will make sure the light comes out of the left rather than out of the bottom of the beamsplitter.)

Question:

The answer is that we can run the beamsplitter backwards to get the light to come out of only one port.

Often students starting to answer this question conclude we cannot run the beamsplitter backwards because they think in powers or intensities, which leads them to conclude that the power will always be split between two outputs when we attempt to run the beam splitter backwards. To see that it can be run backwards, one needs to think in terms of waves, and wave equations can typically be time reversed, rather like taking a movie of the wave splitting and then running the movie backwards. That idea of reversibility is sufficient to allow the student to conclude that, at least for some conditions, the beamsplitter can be run backwards.

The key concept is to understand that the property that determines whether the beams come out of the bottom or the left of the beamsplitter is the relative phase of the two beams. If they have one particular relative phase, they will both come out of the left face, and add perfectlyFor that one particular phase, they add constructively out of the left face, and destructively out of the bottom face, and that is the relative phase required to run the beamsplitter backwards to get an output from the left. If the relative phase is changed by 180 degrees, the beam will come out of the bottom face.

It is also true that the two light beams have to be in the same spatial mode (beam shape and direction) on reflection off of, or transmission through, the mirror, and of the same polarization, though to keep the problem simple, I would not introduce these attributes unless the examinee brought them up. Without those two attributes being the same, the perfect cancellation of the waves in one output direction would not be possible.

Supplementary question 1:

The answer is that we need to do something to change the relative phase of one path light beam path within the apparatus compared to the other. We could do this by very slightly displacing one of the mirrors, or we could add something into one of the paths, such as a thin piece of glass, to otherwise change the relative phase of the two beams going into the upper beamsplitter. In this way, we can actually arbitrarily change the relative power in the two output beams.

This structure, by the way, is known as a Mach-Zehnder interferometer.

Supplementary question 2:

The answer is that it makes no difference to the relative average powers – we can still control the average power to come out in one or the other of the output beams, or any relative ratio between them that we wish, just as we could with higher powers. In this question, I am just interested to see how the examinee reasons on this one, and I do not expect that they know the quantum mechanics that might otherwise help with the answer.

Supplementary question 3:

The answer is that you could not combine the powers in such a way as to heat up another body to a hotter temperature. There are several ways of looking at the answer to this. One is to rely on the Second Law of Thermodynamics, which forbids such processes. A second way is to look at the microscopic physics. One argument would be that the phase of the light from the two different bodies is completely independent, and so on the average, one could not add the powers reliably into one output port or another. A third way is to rely on the Constant Brightness Theorem (or Constant Radiance Theorem) of optics, which also forbids such processes, though it is unlikely the examinee would have heard of this, and the most convincing way to prove that theorem is anyway by starting from the Second Law of Thermodynamics.

Here are my Quals 07 questions.

Thanks

We use a number system S where the positions represent
1, -2, 4, -8, and so on, and each entry can be 0 or 1.
1. How will you convert a number in S to binary without converting to decimal?
2. How about binary to S without converting to decimal?
3. How will you add two numbers in the S system without going
through the binary system?
4. Is there a problem with the S system?
5. Are all numbers in the S system unique?

--
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Andrea Montanari

Here is the question I asked
(it is a bit difficult to type it in plain text. I could do it in Tex but don't know if this is ok with you).

A

****
*

Let H be a matrix with n columns, m rows and entries in {0,1}. Assume that H has 3 ones per row. In the following x denotes a binary vector of length n.

1)
Consider the linear system Hx=0 mod 2.
Let Z(H) be the number of solutions of such a system.
What is the maximum and minimum of Z(H) over all the matrices H.

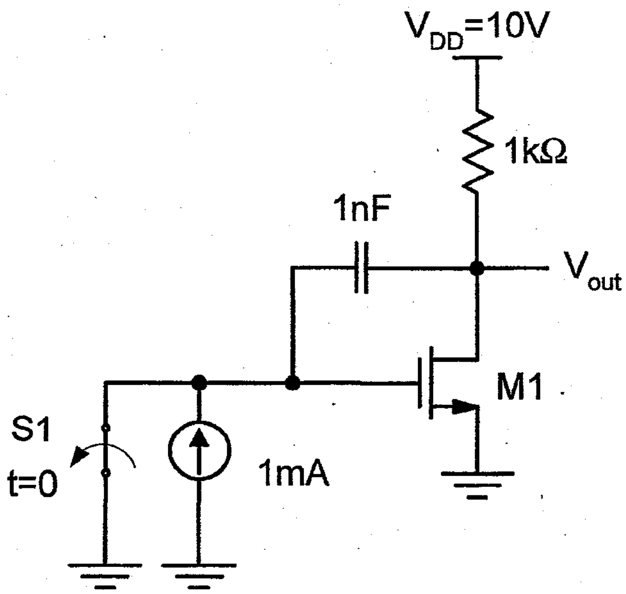
2)
Assume H to be a uniformly random matrix as above.
(The tree non-vanishing positions in each row are chosen uniformly at random among the n choose 3 possible ones). Let b be a uniformly random binary vector of length m, and Z(H,b) be the number of solutions of the linear system Hx=b mod 2. What is the expectation of Z(H,b)?

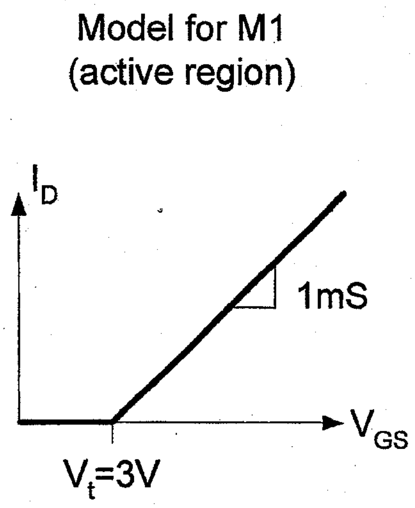
3) How does the calculation change if yu want to compute the expectation of Z(H)?

Name:	Stanford EE Quals 2007 Murmann

Neglect all capacitances, except the explicitly drawn 1nF capacitor. S1 is an ideal switch; it is closed for $-\infty < t < 0$ and open for $t \geq 0$.

Sketch $V_{out}(t)$, calculate and mark pertinent breakpoints.





2006-2007 PhD Qualifying Examination

Professor Yoshio Nishi

1. Please describe basic operation of Si n-channel MOSFET by drawing potential diagram along the channel direction, while the surface orientation is (100) and the MOSFET is built on SOI, Silicon on Insulator, substrate.

2. What would happen if the thickness of silicon in the question 1 is decreased below the spread of electron wave function, in terms of drain current, gate leakage current and the subthreshold leakage current, and why?

3. What if silicon is replaced by a material in which ions can move around in stead of electrons and holes? Describe possible behavior of such devices where source, drain and gate electrode exist.

Nishimura 2007

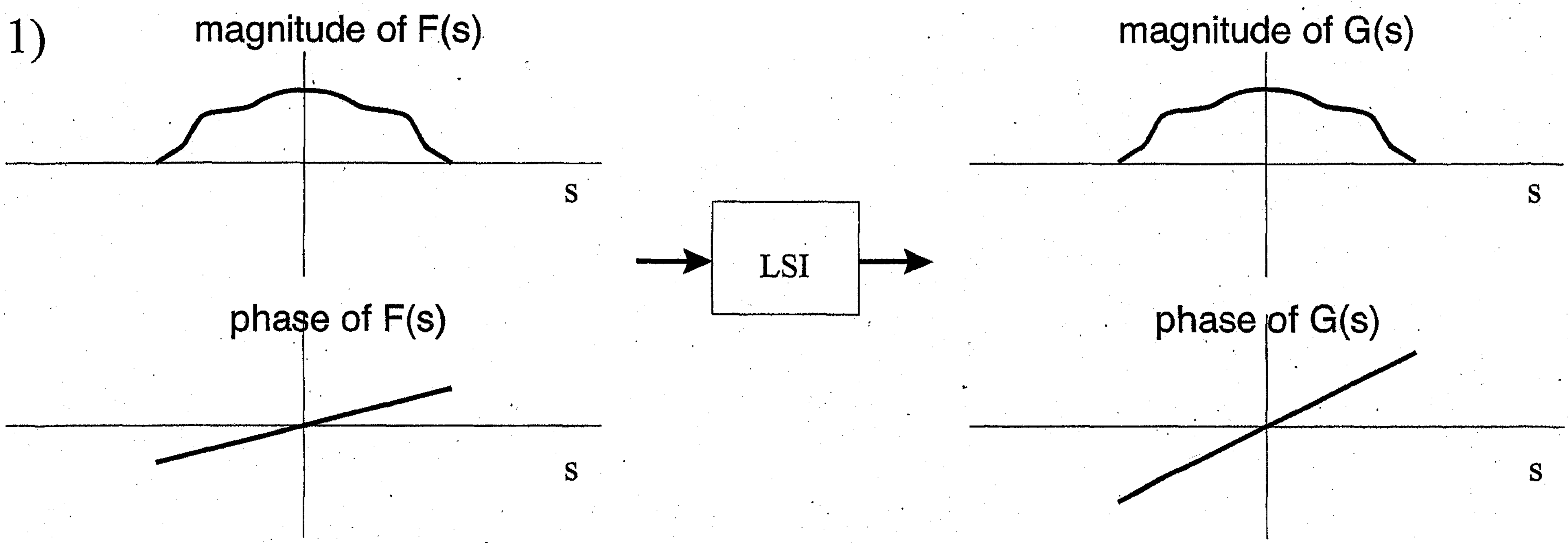
Consider

complex-valued input $f(x) \longrightarrow$ $\boxed{\text{system}}$ $\longrightarrow$ output $g(x)$

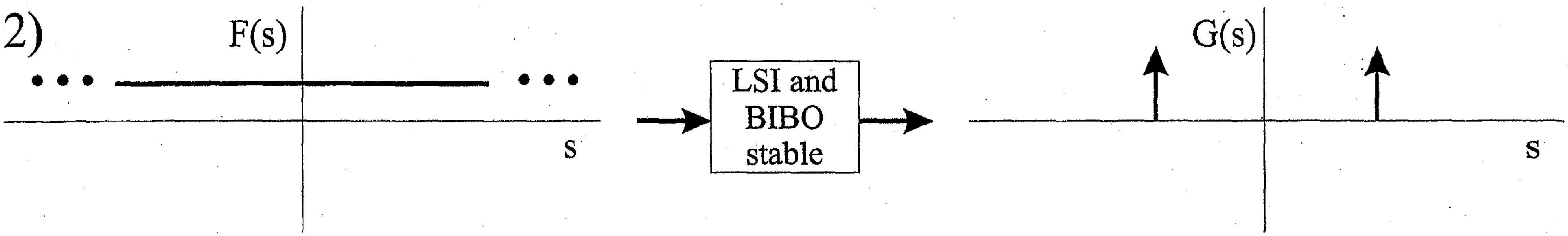
Fourier transform$\{f(x)\} = F(s)$
Fourier transform$\{g(x)\} = G(s)$

For each case below, is it possible to achieve that output? Explain.

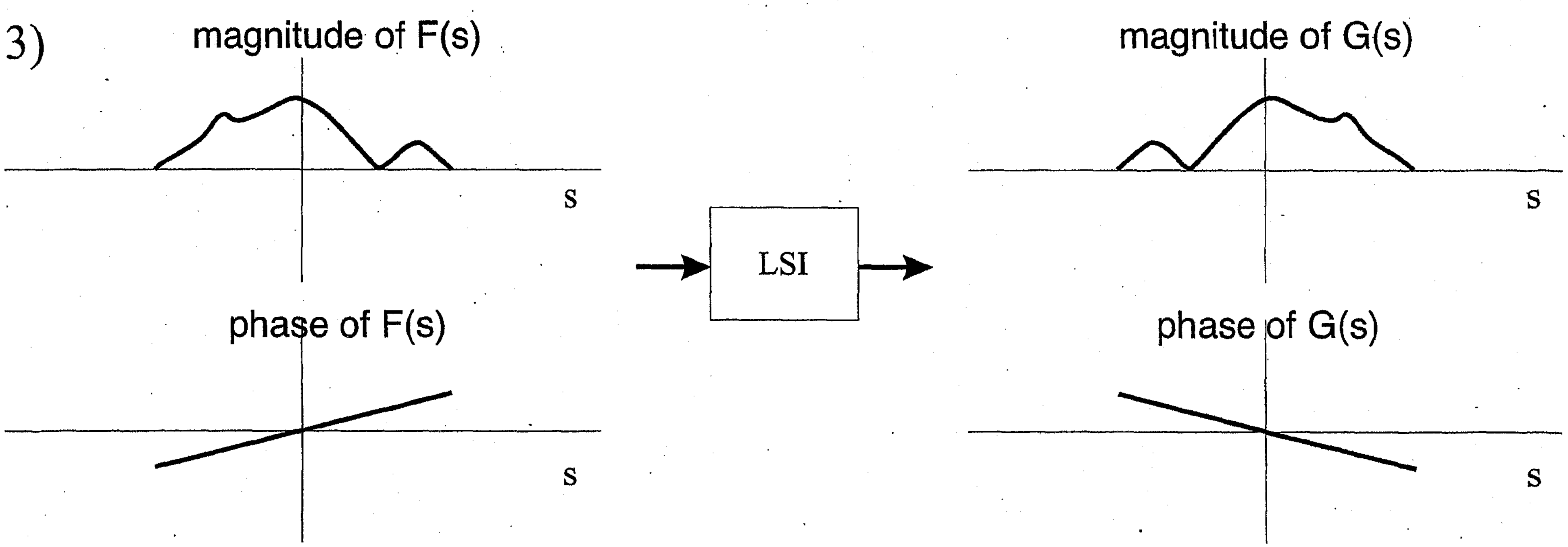
1)



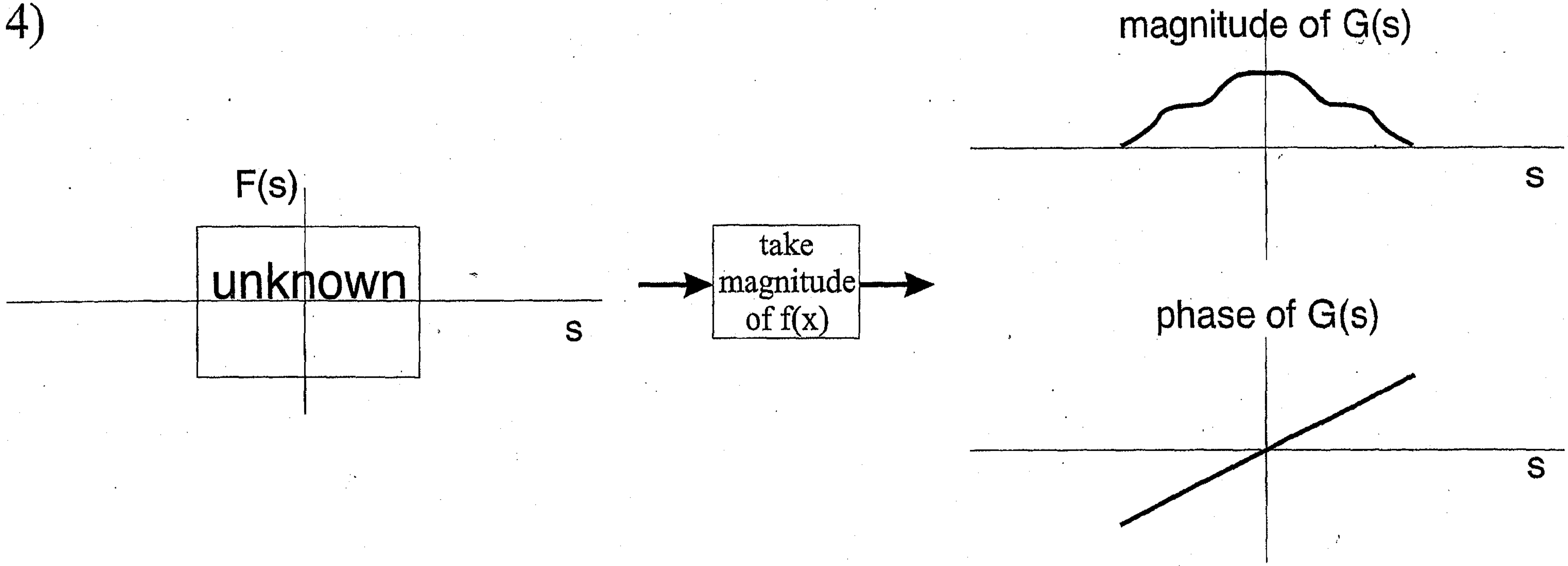
2)


F(s)
s
LSI and
BIBO
stable
G(s)
s

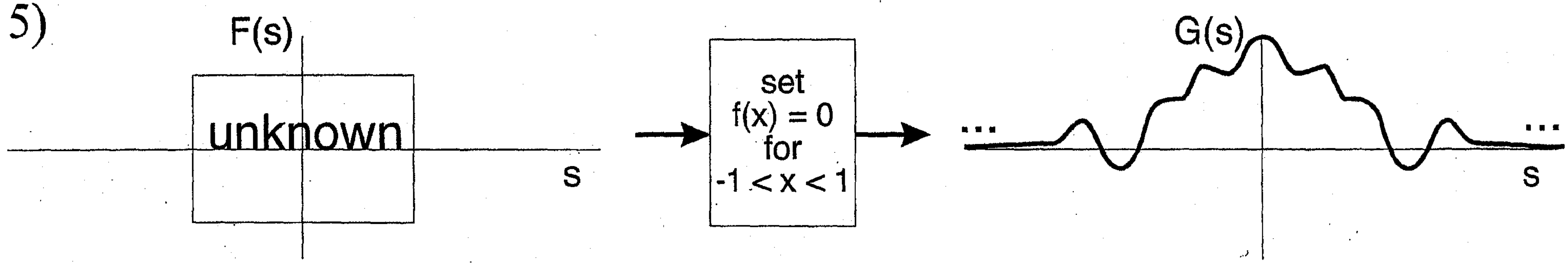
3)


magnitude of F(s)
s
phase of F(s)
s
LSI
magnitude of G(s)
s
phase of G(s)
s

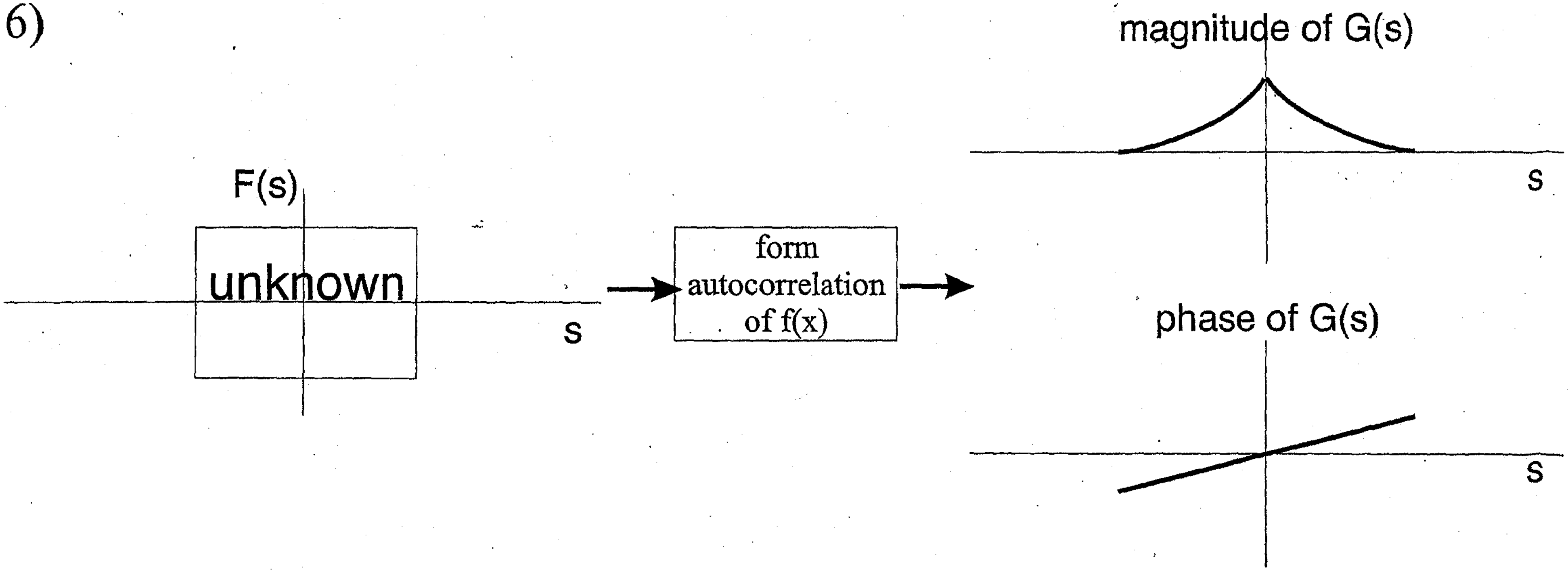
4)


F(s)
unknown
s
take magnitude of f(x)
magnitude of G(s)
s
phase of G(s)
s

5)


F(s)
unknown
s
set f(x) = 0 for -1 < x < 1
G(s)
...
...
s

6)


F(s)
unknown
s
form
autocorrelation
of f(x)
magnitude of G(s)
s
phase of G(s)
s

Answers: Nishimura 2007

1) Yes

2) LSI = Yes   BIBO : No

3) No   (almost Yes in degenerate case)

4) Yes

5) No

6) No

Kunle Olukotun

Hash Table Code

Assumptions

1. 1 KB F. A. data cache
2. 16 byte cache line
3. 100 cycle miss penalty
4. sizeof (Element) = 16 bytes
5. *Element = 8 bytes
6. N_ELEMENTS > 1024

```
Element element[N_ELEMENTS], *bucket[1024]
for (i = 0; i < N_ELEMENTS; i++)
  {
    Element *ptrCurr, **ptrUpdate;
    int hash_index;

    /* Find the location at which the new element is to be inserted. */
    hash_index = element[i].value & 1023;
    ptrUpdate = &bucket[hash_index];
    ptrCurr = bucket[hash_index];
    /* Find the place in the chain to insert the new element. */
    while (ptrCurr && ptrCurr->value <= element[i].value)
    {
      ptrUpdate = &ptrCurr->next;
      ptrCurr = ptrCurr->next;
    }
    /* Update pointers to insert the new element into the chain. */
    element[i].next = *ptrUpdate;
    *ptrUpdate = &element[i];
  }
```

Explain how you would use architectural/software techniques to run this code as fast as possible

Brad Osgood

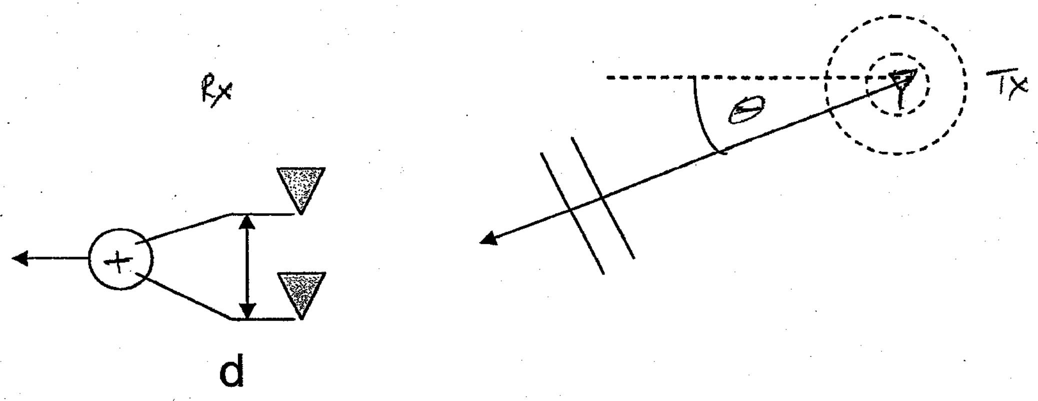
EE Qualifying Exam
January 2007

An important property of linear, time-invariant systems is that complex exponentials are eigenfunctions.

What about the converse? That is, suppose L is a linear system and the complex exponentials $e^{2\pi i\nu x}$ are eigenfunctions of L for all $\nu \in \mathbf{R}$. Is L necessarily a time-invariant system?

Quals Question – 2007
Prof. A. Paulraj

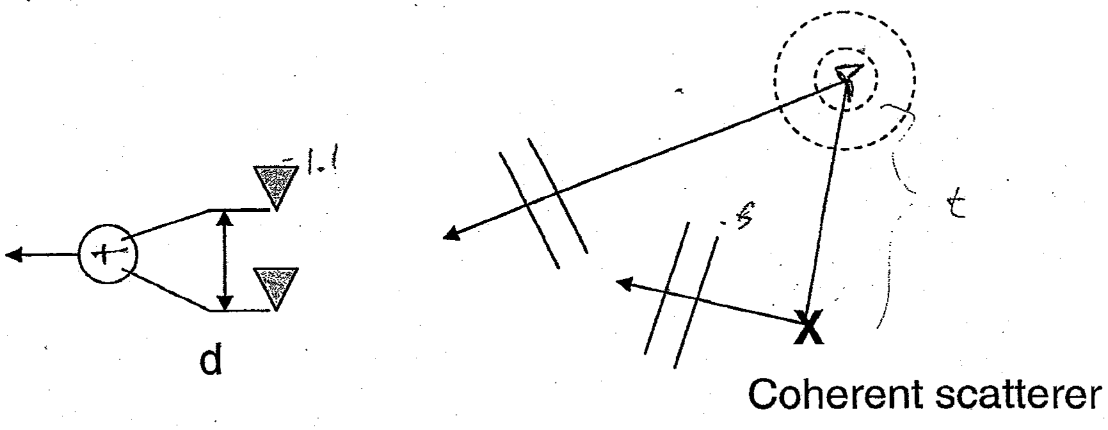
Consider a receiving antenna array with 2 omni directional elements and a beamformer that combines the antenna outputs (equi-phase: ie no phase shifts). Let a planar CW wave front impinge on the antenna array at angle θ



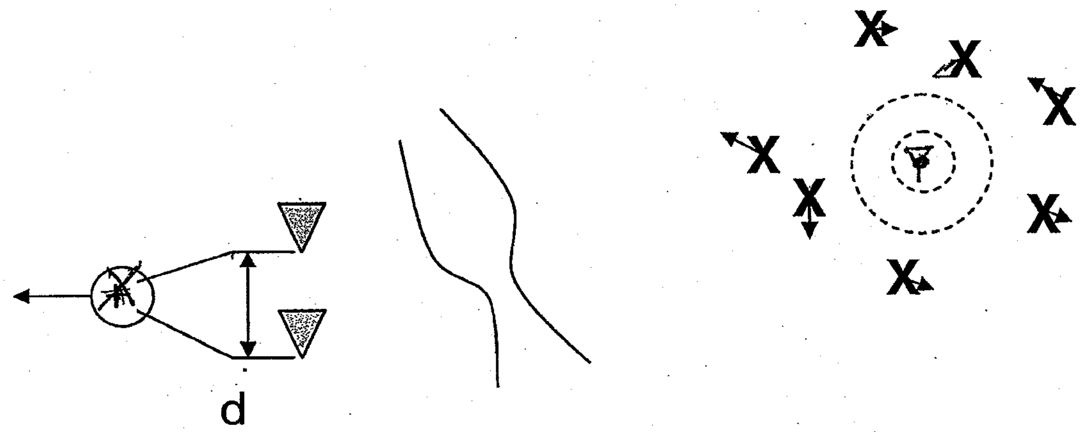
Questions:

1. If $d = \lambda/2$, what is the beamformer response as a function of the angle of arrival

2. If $d = 5\lambda$, what will be the new beamformer response



3. How will the beamformer response be different now ?



4. If the wave front is scattered by random and incoherent scatterers, what will the beamformer response be like.

5. What will the beamformer output waveform look like

6. What is the concept of antenna correlation and how does it depend on the angle spread of the scatterers

John Pauly
Quals Questions 2007

You have a band limited signal that is sampled at known but apparently random times.

Can you recover the signal?

What conditions do you need?

How would you go about the reconstruction?

Fabian Pease

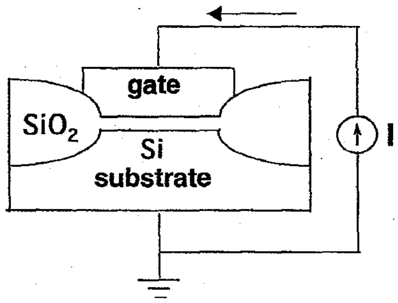
A cathode-ray oscilloscope (CRO) operates by focusing and deflecting a beam of free electrons onto a screen. Outline the limitations to the sensitivity, resolution and speed. You may assume the electron lens is free of aberrations.

NAME: Peter Peumans

Consider a pn-junction of an ideal material (only recombination mechanism is radiative recombination). When you slightly forward bias this diode (say V<bandgap/2), will you see light emission? If so, why and how?

If you now consider this pn-junction configures as above, perhaps slightly forward biased, in close proximity to another pn junction that is operated as a solar cell. Now suppose I heat up the LED (first pn junction) using sunlight and use the light generated by this LED to power the solar cell. Is this a good idea for an energy conversion device from sunlight to electrical energy?

PhD Quals Question 2006-07
Krishna Saraswat



If the device shown in the figure is connected to a current source what kind of current voltage characteristics would result?

Integrated Circuits Lab
Department of Electrical Engineering
Stanford University

Qualifying exam
January 2007
Prof. K.V. Shenoy

Problem 1

You are stranded on a melting glacier with (1) a hand-full of batteries, (2) a hand-full of resistors, (3) an n-channel MOSFET and (4) a length of wire. You need to somehow build an amplifier to try to send for help.

(a) With just these parts, can you build an amplifier? Please "chisel circuit in the ice" & explain its operation.

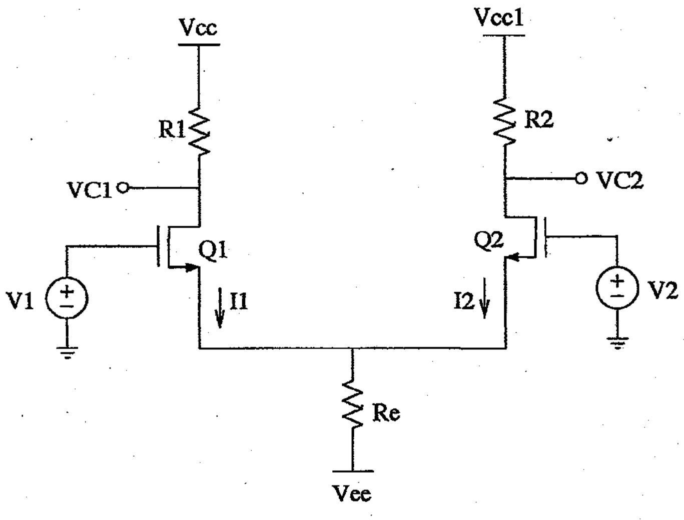
(b) What is your goal with the battery(batteries)?

(c) What is your goal with the resistor(s)?

(d) You now find another MOSFET in your coat pocket, and its a p-FET. Should you modify your circuit? Why?

(e) You suspect that your amplifier's high-frequency cuttoff is too low to be useful. What should you look for (and change if you could) in the circuit and MOSFETs?

Problem 2

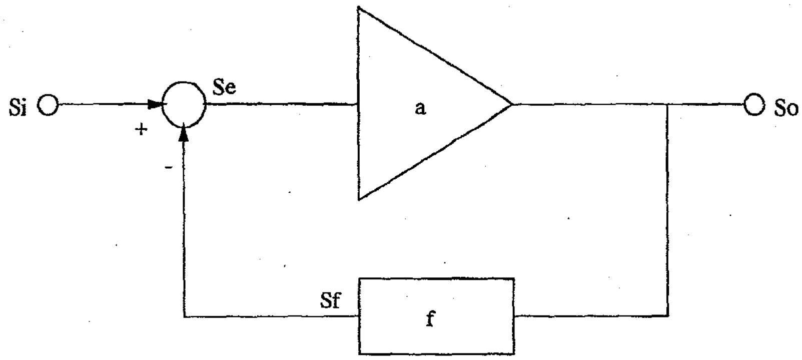


Consider this symmetric differential amplifier. Please explain how the following change when a common-mode voltage is ramped up ($V_{ee} \rightarrow V_{cc}$) at the inputs:

(a) The FETs' mode of operation

(b) The FETs' source voltage

Problem 3



Consider this idealized feedback configuration.

(a) Please derive an expression for the closed-loop gain (A).

(b) Please derive an expression for the closed-loop 3dB frequency ($\omega_{3dB,CL}$) when the basic amplifier (a) is now modeled as a one pole system:

$$a(s) = \frac{a_o}{1 + \frac{s}{\omega_{3dB}}}$$

where a_o is the open-loop DC gain and ω_{3dB} is the 3dB open-loop frequency.

(c) Does the addition of feedback increase the gain-bandwidth product? Why or why not?

Problem 4

Please describe the major sub-circuits of an op-amp, and the critical design considerations of each.

EE Quals Problem
January 22-26, 2007
Julius Smith

Consider the following second-order system:

$$\begin{aligned} x_1(n+1) &= x_1(n) - \epsilon\, x_2(n) + u(n) \\ x_2(n+1) &= \epsilon\, x_1(n+1) + x_2(n) \\ y(n) &= x_1(n) + x_2(n) \end{aligned}$$

where $u(n)$ and $y(n)$ are the input and output signals at time n, respectively, and $x_1(n)$ and $x_2(n)$ denote the two *state variables* at time n. Assume all signals and states are zero for $n < 0$, and that typically $|\epsilon| \ll 1$.

1. Derive the *state-space description* for this system. That is, find matrices (A,B,C,D) such that $\underline{x}(n+1) = A\underline{x}(n) + Bu(n)$ and $y(n) = C\underline{x}(n)$, where $\underline{x}(n) = [x_1(n), x_2(n)]^T$ denotes the *state vector* at time n.
2. Write down or derive an expression for the system *transfer function* in terms of the state-space description.
3. Give a formula that can be solved to find the *poles* of the system.
4. Find the product of the poles of the system.
5. Give a formula for *diagonalizing* this state-space description, if possible.
6. Write an expression for the *maximum decay time-constant* in the impulse response, assuming the system is stable.
7. If time remains, work to find the impulse response in closed form. Otherwise, state how this should be carried out.

Qualifying Exam for the Electrical Engineering PhD program, Stanford University, January 2007

Olav Solgaard

Questions:

1) What is this? (Showing the student a co-axial cable.)

2) How is it constructed?

3) How is information transferred along the cable?

4) What is the speed of information transfer?

5) How can you model the information transfer on a co-axial cable using lumped-circuit elements?

6) What is the bandwidth of the ideal loss-less transmission line?

7) What effects will degrade the signal quality as it propagates on real transmission lines?

8) How is signal propagation on an optical fiber different from signal propagation on a co-axial cable?

9) What effects degrade signals that propagate on optical fibers?

10) How can you mitigate dispersion on single-mode optical fibers?