Isachy Weissman

Quals Question

Suppose I give you a random variable $U \sim \text{Unif}[0,1]$.

Can you use it (and only it) to generate

1. An unbiased bit $B \sim \text{Bernoulli}(1/2)$?

2. Two independent unbiased bits B_1, B_2

3. An infinite stream of independent unbiased bits $B_1, B_2, B_3, \ldots$?

4. Two independent Unif[0, 1] random variables U_1, U_2 ?

5. An infinite sequence of i.i.d.$\sim$ Unif[0, 1] random variables $U_1, U_2, U_3, \ldots$?

WEISSMAN

Quals Question

Let the random variable X have CDF F. Suppose that X_i iid$\sim X$.

(a) What is the CDF of $\max_{1\leq i\leq n} X_i$?

(b) What is the CDF of $\min_{1\leq i\leq n} X_i$?

Suppose now that $X_{i,j}$ iid$\sim X$

(c) What is the CDF of $f(n,m) = \max_{1\leq i\leq n} \min_{1\leq j\leq m} X_{i,j}$?

(d) Suppose $X \sim$ exponential(λ). Let $Y_m = f\left(e^{\beta m}, m\right)$, for some parameter $\beta > 0$. What does Y_m converge to as $m \to \infty$? In what sense?

(e) Repeat the previous part for the general case $X \sim F$ (can assume that F is continuous and strictly increasing).

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Suppose you are required to guess the value of a discrete random variable X, of a known PMF, via a sequence of questions of the form "is $X = x$?".

- What questioning strategy is optimal in the sense of minimizing the expected number of trials until guessing correctly?
- What is the value of this minimum expected number of guesses until correctly guessing (inclusive) for $X \sim \text{Poisson}(1)$?
- Repeat the question for $X \sim \text{Poisson}(2)$.

Prof. Tsachy Weissman

Quals Question

You are drawing variables $U_0, U_1, \cdots$ iid$\sim U[0,1]$.

(a) T_1 is the number of attempts until you surpass your first draw (i.e., break your first record):

$$T_1 = \min\{n \geq 1 : U_n > U_0\}$$

What is the PMF of T_1?

(b) What is the PDF of U_{T_1}?

(c) T_2 is the number of attempts it takes you to set your next record:

$$T_2 = \min\{n \geq 1 : U_{T_1+n} > U_{T_1}\}$$

What is the PMF of T_2 (bring to as explicit a form as you can)?

(d) Repeat parts (a) and (c) for $U_0, U_1, \cdots$ iid$\sim N(0,1)$.