Find $\sigma_{\epsilon_1}^2$:

If $X(t)$ and $E_1(t)$ were deterministic, we would have:

$$\frac{E_1(s)}{X(s)} = \frac{X(s)-Y(s)}{X(s)} = 1 - \frac{Y(s)}{X(s)} = 1 - \frac{K/s}{1+K/s} = \frac{s}{s+K}$$

Now consider stochastic $X(t)$:

$$\sigma_{E_1}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\left|\frac{s}{s+K}\Big|_{s=j\omega}\right|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{X_0}{\omega^2}\,\frac{\omega^2}{\omega^2+K^2}\,d\omega$$

$$= \frac{X_0}{2\pi}\int_{-\infty}^{\infty} \frac{d\omega}{\omega^2+K^2} = \frac{X_0}{2K}$$

Find $\sigma_{E_2}^2$:

If $N(t)$ and $E_2(t)$ were deterministic, we would have:

$$\frac{E_2(s)}{N(s)} = \frac{-K/s}{1+K/s} = \frac{-K}{s+K}$$

Now consider stochastic $N(t)$:

$$\sigma_{E_2}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_N(\omega)\left|\frac{-K}{s+K}\Big|_{s=j\omega}\right|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} N_0\,\frac{K^2}{\omega^2+K^2}\,d\omega$$

$$= \frac{N_0K^2}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega^2+K^2} = \frac{N_0K}{2}$$

Total:

$$\sigma^2 = \sigma_{E_1}^2 + \sigma_{E_2}^2 = \frac{X_0}{2K} + \frac{N_0K}{2}$$

$$\frac{d\sigma^2}{dK} = -\frac{X_0}{2K^2} + \frac{N_0}{2} = 0$$

$$K = \sqrt{\frac{X_0}{N_0}}$$