2015 EE PhD Quals

Prof. Daniel Spielman

1. A post office has two clerks who are currently serving customers A and B. Customer C walks into the office and is told she will be served as soon as one of the clerks is free. Assuming the time it takes a clerk to serve a customer is given by an exponential random variable with mean λ, what is the probability that customer C is NOT the last to leave the office?

2. Is this a good model for a post office?

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Answers

1. The probability that customer C is NOT the last to leave = 0.5. The reasoning is as follows…

An exponential random variable has the pdf

$$\Pr\{X = x\} = f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

from which it is easily shown that mean(X)=λ, Var(X)=$\lambda^2/2$, and X is memoryless, i.e.

$$\Pr\{X > s + t \,|\, X > t\} = \Pr\{X > s\}$$

Based on the memoryless property, once customer A or B leaves, customer C has equal probability of finishing after the other remaining customer.

2. Whether or not this is a good model for service at a post office is the heart of this quals question. Re-assessing the pdf is a good place to start. Perhaps considering sums of exponential variables? I was looking for a discussion of how one develops a mathematical model for a physical process. Taking data is almost always a good thing to do. What are the assumptions and are they reasonable? What are the sources of error? How is a model tested and refined?