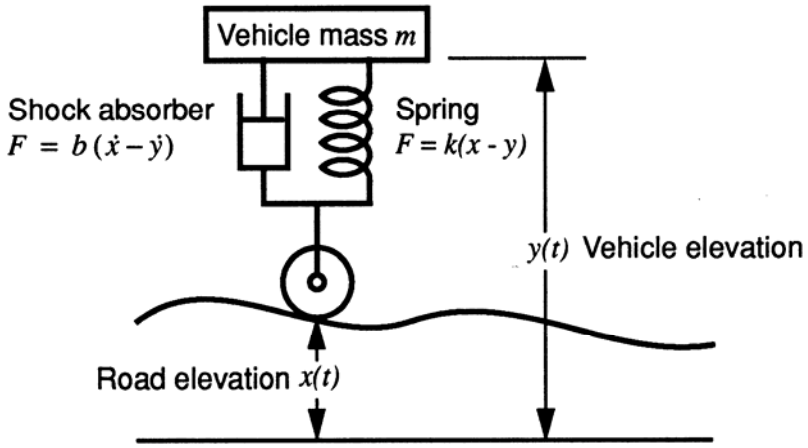
Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2010-11
Professor Joseph M. Kahn



A one-wheeled vehicle rolls along a road, supported by a suspension that includes a spring and a shock absorber. At time t, the road elevation is $x(t)$ and the vehicle elevation is $y(t)$. The suspension can be considered as a linear time-invariant system H with input $x(t)$ and output $y(t)$, $H\{x(t)\} = y(t)$, which is governed by the differential equation:

$$m\ddot{y} + b\dot{y} + ky = b\dot{x} + kx\,,$$

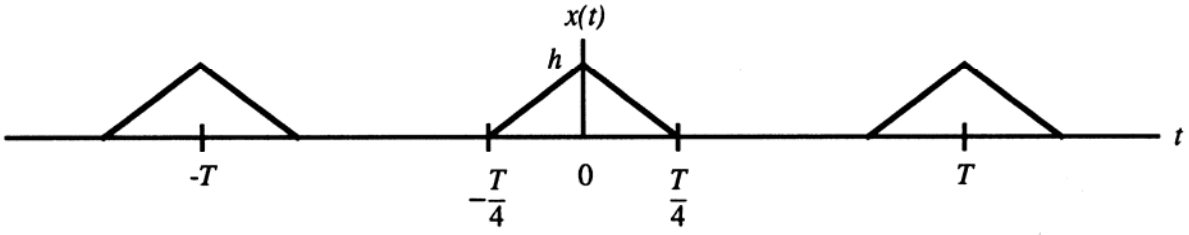
where m, b and k are positive real constants.

1. Find the frequency response $H(j\omega)$ of the system, which satisfies:

$$H\{e^{j\omega t}\} = H(j\omega)\cdot e^{j\omega t}\,.$$

2. Assume $m = 1$, $b = 1$ and $k = 1$. Make a sketch of the magnitude response of the system, $|H(j\omega)|$, and describe the system qualitatively.

3. The vehicle rolls along an infinitely long road with regularly spaced triangular speed bumps, so the road elevation $x(t)$ is the periodic signal indicated below. Give an exponential Fourier series representation of this $x(t)$.



4. Assuming general values of m, b and k, find an expression for the vehicle elevation $y(t)$ when the vehicle rolls over the road pictured above.