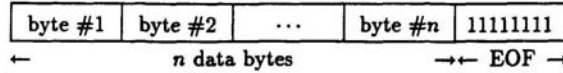


1994 Electrical Engineering Qualifying Examination Questions
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signal

A *packet* or *frame* of data to be transmitted over a network consists of 8-bit bytes followed by a single end of frame (EOF) byte consisting of all ones ($11111111_2 = FF_{16}$).



Question 1: Because arbitrary binary data can be transmitted over this network, there is a possibility that the EOF byte may occur within the data frame, thereby misleading the receiver into thinking that the frame has ended. Suppose that n data bytes are generated at random (all bits are independent and 0 or 1 occur with equal probability). Find the probability that the EOF byte occurs within the data.

Answers:

1. The probability that any particular byte equals the EOF byte is $p = 2^{-8} = 1/256$. The probability that any particular byte is *not* the EOF byte is $1 - p = 255/256$. The probability that all n bytes are *not* the EOF byte is $(1 - p)^n$ because the bytes are independent. Finally, the probability that at least one of the n bytes equals the EOF byte is $1 - (1 - p)^n$.
2. Using the binomial probability distribution, the probability of at least one occurrence of the EOF byte is given by

$$\sum_{i=1}^n \binom{n}{i} p^i (1-p)^{n-i} = 1 - \binom{n}{0} p^0 (1-p)^{n-0} = 1 - (1-p)^n.$$

3. If there is at least one occurrence of EOF, then the first occurrence is in either byte 1 or byte 2 ... or byte n . The probability that the *first* occurrence is in byte i is $(1-p)^{i-1}p$, which is the probability that the first $i-1$ bytes are not EOF but byte i does match. Therefore the probability of at least one occurrence of EOF is

$$\sum_{i=1}^n p(1-p)^{i-1} = p \sum_{i=0}^{n-1} (1-p)^i = p \cdot \frac{1 - (1-p)^n}{1 - (1-p)} = 1 - (1-p)^n.$$

Question 2: What happens to the above probability as n ranges from 1 to ∞ ?

Answers: The probability that the EOF byte occurs approaches 1 as n gets large.

Question 3: Suppose that data bytes are generated at random until the EOF byte is produced. What is the average number of *bytes* generated?