**OFFICE MEMORANDUM ◇ STAR LABORATORY**

January 26, 1996

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1996

**Question:** Suppose an earthquake occurs in the earth at a depth of $d = 10$ km (Figure 1). Although we have no idea what kind of electromagnetic signals, if any, are generated in the earth by earthquakes, let us assume that they do produce electromagnetic fields and that these fields have the same amplitude at all frequencies (Figure 2). Given the electrical conductivities ($\sigma$) shown in the figure, and given also that the skin depth $\delta$ at 1 Hz is 1.6 km for $\sigma = 0.1$ S/m, what frequencies are likely to be observed on the surface for earthquakes in California? Plot a figure equivalent to Figure 2 for the magnetic field measured on the surface. Are there any other ways in which the signal strength at the surface can be weakened in addition to absorption in the earth?

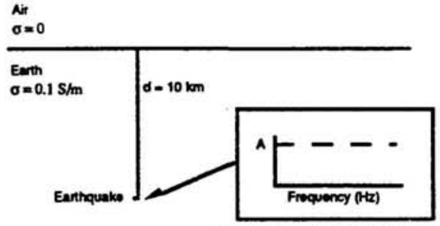


Figure 2: Frequency characteristic of the earthquake source.

**Figure 1.** Earthquakes in California typically occur at depths ($d$, in km) of around 10 km. Lacking information about the frequencies of the electromagnetic fields they may generate, we will assume they generate white noise (Figure 2; A is the amplitude of the magnetic component).

**Answer:** After a brief discussion of good conductors and poor conductors, leading to the conclusion that the above problem must treat the earth as a good conductor ($\sigma/\omega\epsilon \gg 1$, where $\omega$ is the angular frequency and $\epsilon$ is the permittivity), the student must either remember or derive an expression for the attenuation of electromagnetic fields in a conducting medium in terms of the skin depth, i.e. $\boxed{\delta = [2/(\omega\mu\sigma)]^{1/2}}$, where $\mu$ is the permeability. In addition, the student should demonstrate some knowledge of how (1) the wave propagates with phase velocity $v = \omega/\beta = \omega\delta$, and (2) it is exponentially attenuated with attenuation constant $\alpha = 1/\delta$. phase velocity has $\omega^{1/2}$ frequency dependence

Using the above information, the spectrum shown in Figure 3 can be derived by considering the attenuation at just a few specific frequencies around 1 Hz – say 0.01 Hz (skin depth 16 km) and 1 Hz. Obviously, the attenuation declines to zero at frequencies lower than about 0.03 Hz, at which the skin depth is just equal to 10 km, and increases to very high values at higher frequencies.