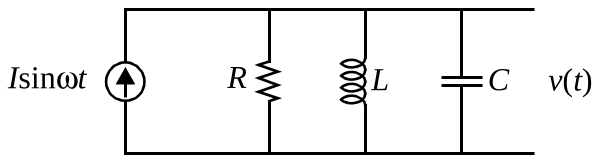
Consider the following circuit:

FIGURE 1. Parallel *RLC* circuit



Assume that the current source has an amplitude of 1A and a frequency of 1rad/s. Assume also that the inductance is 1H and that the resistance is 1kΩ.

a) Suppose that the capacitance is 1F. Sketch the steady-state voltage $v(t)$. Identify the amplitude and frequency (or period). What is the average power dissipated in R?

The component values are chosen to produce a resonance at 1rad/s, at which the inductive and capacitive impedances cancel to contribute a net infinite impedance. The 1A current thus flows through the 1kΩ resistor to produce a 1kV-amplitude sinusoidal $v(t)$. *The average dissipation in the resistor is just* $I^2R/2 = 500W$.

Common mistake**:** Many students immediately and mechanically started writing node equations and Laplace transformed them (or wrote differential equations and tried solving them directly), instead of thinking about the problem a bit first.

b) Now suppose that, starting from $t = 0$, the capacitance instantaneously decreases by 1% at the voltage extrema. Sketch the voltage $v(t)$, assuming that the capacitance returns to its nominal value at each zero crossing. Make reasonable assumptions/approximations.

Capacitive charge cannot change instantaneously if currents are finite (as they are here). Since Q=CV and Q is continuous, a discontinuous change in C must be balanced by a discontinuous change in V. Here, a drop in C implies a jump in V; the capacitor's voltage therefore increases by an amount Q/ΔC (or ~1%) at each extremum, growing without bound. This behavior is exploited in the parametric amplifier*, of which a child's swing is an example.*

At the zero-crossings of v(t), the capacitive charge is zero, so there is no change in capacitor voltage there.

Common mistake**:** Many students attempted to deduce the behavior using concepts of shifts in resonant frequency. Now, resonant phenomena are evident only over the order of Q cycles (where Q here is 1000), so deducing circuit responses to perturbations occurring on a per-cycle basis (and here, more often than that) is somewhat of an unnatural act for a resonance-based approach.