

$$\Lambda\left(\frac{t}{\tau}\right) \leftrightarrow \tau \text{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$$

to obtain

$$q(t) \leftrightarrow Q(j\omega) = \frac{Th}{4} \text{sinc}^2\left(\frac{\omega T}{8\pi}\right).$$

Representing $x(t)$ by a Fourier series:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t},$$

where the fundamental frequency is $\omega_0 = \frac{2\pi}{T}$, the Fourier series coefficients are given by:

$$a_n = \frac{1}{T} Q(j\omega) \Big|_{\omega=n\omega_0} = \frac{h}{4} \text{sinc}^2\left(\frac{n\omega_0 T}{8\pi}\right) = \frac{h}{4} \text{sinc}^2\left(\frac{n}{4}\right).$$

4. Given an input $x(t)$, the output is:

$$y(t) = H\{x(t)\}.$$

Representing $x(t)$ as a Fourier series, and using the fact that complex exponentials are eigenfunctions of any LTI system, so that $H\{e^{jn\omega_0 t}\} = H(jn\omega_0) e^{jn\omega_0 t}$, the output is:

$$\begin{aligned} y(t) &= H\left\{\frac{h}{4} \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n}{4}\right) e^{jn\omega_0 t}\right\} \\ &= \frac{h}{4} \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n}{4}\right) H\{e^{jn\omega_0 t}\} \\ &= \frac{h}{4} \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n}{4}\right) H(jn\omega_0) e^{jn\omega_0 t} \\ &= \frac{h}{4} \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n}{4}\right) \frac{b(jn\omega_0) + k}{m(jn\omega_0)^2 + b(jn\omega_0) + k} e^{jn\omega_0 t} \end{aligned}$$