1996 Qualifying Exam Questions

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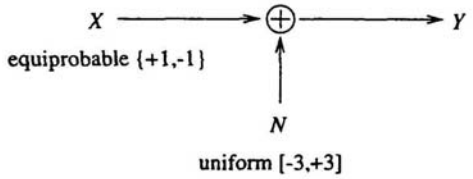
A communications system transmits binary data by sending one bit per unit time, representing binary values by the analog values $X = +1$ and $X = -1$.



The received signal is corrupted by additive noise; that is, $Y = X + N$. The noise N is uniformly distributed for $-3 \leq N \leq +3$. The two input values are equally probable.

1. Sketch the pdf (probability density function) of N.

2. What is the variance of N?

3. The noise power is defined to be its variance. What is the signal-to-noise ratio (SNR)?

4. Sketch the pdf of the received signal Y.

5. What is a good decision rule for estimating X given Y?

6. Suppose we use the simple decision rule:

$$\hat{X} = \begin{cases} +1 & \text{if } Y > 0 \\ -1 & \text{if } Y < 0 \end{cases}$$

   What is $\Pr(X = -1 \mid Y = +1)$, that is, the conditional error probability given $Y = +1$?

7. Find the overall error probability, $\Pr(\hat{X} \neq X)$.

8. To reduce the probability of error, we increase signal power by sending the same signal twice. The received signal Y is the sum of the two transmission:

$$Y = 2X + N_1 + N_2,$$

   where N_1 and N_2 are independent. What is the pdf of the received signal Y?

9. What is the optimum decision rule for estimating X given Y?