4. $s(t) = m(t)\cos w_c t$

$$S(jw) = \frac{1}{2}\left[\underbrace{M(j(w-w_c))}_{\text{nonzero}} + \underbrace{M(j(w+w_c))}_{\text{nonzero}}\right]$$

nonzero: $|w-w_c| \le w_m$

nonzero: $|w+w_c| \le w_m$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(jw)\, e^{jwt}\, dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S(jw)\, H(jw)\, e^{jwt}\, dw$$

$$= \frac{1}{4\pi}\left[\int_{-\infty}^{\infty} M(j(w-w_c))\, H(jw)\, e^{jwt}\, dw + \int_{-\infty}^{\infty} M(j(w+w_c))\, H(jw)\, e^{jwt}\, dw\right]$$

Near $w = w_c$:

$$H(jw) \simeq |H(jw_c)|\, e^{j[\phi(w_c) + \phi'(w_c)(w-w_c)]}$$

Near $w = -w_c$:

$$H(jw) \simeq |H(-jw_c)|\, e^{j[\phi(-w_c) + \phi'(-w_c)(w+w_c)]}$$

$$|H(-jw_c)| = |H(jw_c)|$$

$$\phi(-w) = -\phi(w) \Rightarrow \phi(-w_c) = -\phi(w_c)$$

$$\phi'(-w_c) = \phi'(w_c)$$