Prof. Stephen Boyd

Ask me about anything that isn't clear.

Consider the autonomous linear dynamical system $\dot{x} = Ax$, with $x(t) \in \mathbf{R}^n$.

When is it true that $x_1(0) \geq 0$ implies $x_1(t) \geq 0$ for all $t \geq 0$?

Feel free to start with special cases.

Discussion/solution. The solution is very simple: This holds when (and only when)

$$A_{12} = \cdots = A_{1n} = 0,$$

$i.e.$, all entries in the first row of A, except the first one, are zero.

This condition implies that $\dot{x}_1 = A_{11}x_1$, so $x_1(t) = x_1(0)e^{tA_{11}}$, so we have $x_1(t) \geq 0$ for all t if $x_1(0) = 0$.

Conversely, suppose that $A_{1k} \neq 0$, where $k \geq 2$. Let's take $x(0) = -A_{1k}e_k$. Then $x_1(0) = 0$. We have

$$\frac{dx_1}{dt}(0) = e_1^T Ax(0) = -A_{1k}^2 < 0.$$

So for small $t > 0$, we have $x_1(t) < 0$. This shows the condition is also necessary.

A good warm-up would be to take $n = 1$. In this case you find that the condition *always* holds. The next thing to try is $n = 2$. And you'd find that the condition is $A_{12} = 0$.

EE Qualifying Exam 2016 – John Duchi

Question 1: You have two coins, both of which are fair (i.e. the probability of heads is $\frac{1}{2}$ for each of them), and they are independent.

(a) Describe how to use the two coins to generate an event that occurs with probability exactly 2/3.

(b) How many coin flips, on average, does your solution to part (a) require?

(c) Now you are given a value $p \in [0, 1]$, where p may be irrational. Generalize your solution to part (a) to use the coins to generate an event that occurs with probability exactly p. How many flips are required to generate your event (on average)?

(d) Now you wish to generate a series of independent random variables, $X_1, X_2, \ldots$, each with $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p$. Can you devise a procedure, similar to part (c), that uses fewer flips on average? What is this procedure, and how many coin flips does it require on average?

Answer:

(a) Let $E \in \{0, 1\}$ be the event. We use rejection sampling. Let HH denote the event that both coins flip to heads; HT that the first coin is heads and the second is tails; TH the first being tails, second heads; and TT the event that both coins are tails. We flip both coins. If we achieve HH, we set $E = 0$. If we have HT or TH, we set $E = 1$. If we see TT, we simply re-flip the coins and ignore the result. It is clear that the probability our procedure stops at HH, yielding $E = 0$, is 1/3.

(b) The probability of failing to stop in any round of the procedure is $\mathbb{P}(\mathsf{TT}) = \frac{1}{4}$, and on each round we perform two coin flips. Let R be the number of rounds. Then for $k \in \{1, 2, \ldots\}$, we have $\mathbb{P}(R \geq k) = (\frac{1}{4})^{k-1}$, because $R \geq k$ if and only if the first $k - 1$ rounds are failures. Thus

$$\begin{aligned}
\mathbb{E}[R] = \sum_{k=1}^{\infty} k\mathbb{P}(R = k) &= \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} \mathbf{1}\{l \leq k\}\, \mathbb{P}(R = k) \\
&= \sum_{l=1}^{\infty}\sum_{k=l}^{\infty} \mathbb{P}(R = k) = \sum_{l=1}^{\infty} \mathbb{P}(R \geq l) = \sum_{k=0}^{\infty} \frac{1}{4^k} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}.
\end{aligned}$$

We expect to perform 8/3 flips. (One could also note simply that this is the expectation of a geometric random variable.)

(c) Let $p = 0.b_1 b_2 b_3 \ldots$, where $b_j \in \{0, 1\}$, be the binary decimal expansion of p (so that $p = 1$ has expansion $1 = b_1 = b_2 = \cdots$, and $p = \frac{1}{2}$ has expansion $p = 0.10000\ldots$). The following procedure suffices. Let $C_1, C_2, \ldots$ be an infinite sequence of coin flips, where

$C_i = 1$ if the coin flip is heads and $C_i = 0$ if it is tails. Let $\widehat{p} = 0.C_1C_2C_3\ldots$. Then return

$$E = \begin{cases} 1 & \text{if } \widehat{p} \le p, \quad \text{i.e. } 0.C_1C_2C_3\ldots \le 0.b_1b_2b_3\ldots \\ 0 & \text{otherwise.} \end{cases}$$

As $\widehat{p}$ is uniform in $[0,1]$, we see that $\mathbb{P}(E=1) = p$ exactly.

An implementable version of this abstract procedure is as follows: repeat the following for iterations $k = 1, 2, \ldots$.

i. At iteration k, flip coin and get value C_k.

ii. If $C_k = 0$ and $b_k = 1$, then we know that $\widehat{p} \le p$, so return $E = 1$.

iii. If $C_k = 1$ and $b_k = 0$, we know that $\widehat{p} > p$, so return $E = 0$.

iv. If $C_k = b_k$, continue.

If as above we let R denote the number of rounds of this procedure, we see that $\mathbb{P}(R \ge k) = (\frac{1}{2})^{k-1}$, so that $\mathbb{E}[R] = \sum_{k=1}^{\infty} \mathbb{P}(R \ge k) = \sum_{k=0}^{\infty} 2^{-k} = 2$.

(d) We provide a more general solution to the problem, which allows us to generate events $E = 1, \ldots, E = m$ with probabilities $p_1 \ge \ldots \ge p_m$, and $\sum_{i=1}^m p_i = 1$. (In the case above, we fix some n, set $m = 2^n$, and let $E = 1$ correspond to $X_1 = 0, X_2 = 0, \ldots, X_n = 0$, $E = 2$ correspond to $X_1 = 0, \ldots, X_{n-1} = 0, X_n = 0$, and so on until $E = 2^n$ corresponds to $X_i = 1$ for all i.) Let $s_0 = 0, s_1 = p_1, s_2 = p_1 + p_2, \ldots, s_k = 1$, and give them binary expansions $s_i = 0.b_1^i b_2^i b_3^i \cdots$. Let $\widehat{p} = 0.C_1C_2C_3\ldots$ be the binary expansion of an i.i.d. sequence of coin flips. Then return

$$E = \{i \text{ such that } s_{i-1} \le \widehat{p} < s_i\}.$$

We have $\mathbb{P}(E = i) = p_i$, and we can use the ideas from part (c) to terminate the procedure.

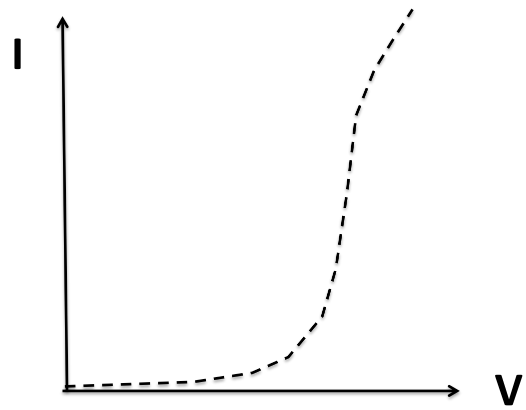
In particular, it is possible to show that $\mathbb{E}[R] \le -\sum_{i=1}^m p_i \log_2 p_i + 2$, though no one made it this far.

□

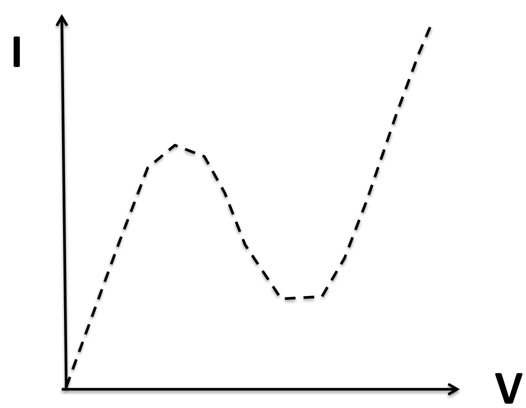
Prof. Robert Dutton

Initial (opening) Question*:
Semiconductor devices are mostly non-linear. So why do we want to linearize them?

Next part*:
Given the I-V curve for a non-linear (two terminal) device, how would you linearize it (particularly "piece-wise linear"), how would you bias it to operate at a given point (i.e. a real circuit) and generally about the behavior of both the non-linear device and the linearized model.



Final Question*:
Now consider that the non-linear device is as follows. Basically, repeat the analysis and questions from the simpler version from above.

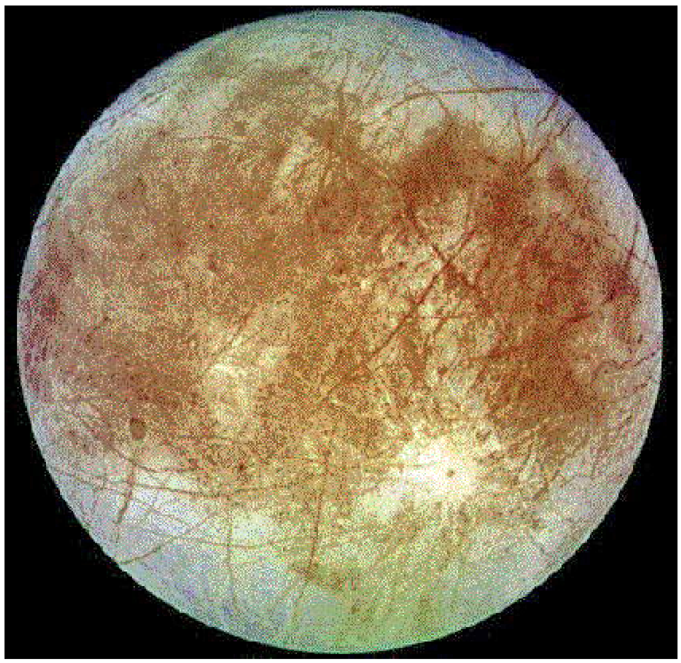


*Footnote: Answers are not given to my Quals Questions. These are for the students to ponder and to discuss, amongst themselves.

Ph.D. Quals Question
January 25-289, 2016
A. C. Fraser-Smith
Department of Electrical Engineering
Stanford University

Europa's Ice and "Ocean"

The students enter the examiner's office and find a recent picture of Jupiter's moon Europa sitting on the table in front of them. Here it is:



The students were then given the following information concerning NASA's discoveries with respect to Europa (Jupiter's fourth largest moon): (1) images acquired during spaceprobe flybys show a surface covered with relatively-new water ice (discolored ice in some places) and the layer of ice appears to be quite thick, and (2) magnetic and gravity measurements suggest very strongly that there is a liquid ocean beneath the ice. Liquid oceans are very unusual in the solar system, and the existence of one on Europa suggests the possibility of life. NASA has therefore placed the highest priority on a mission to Europa to see what can be learned about life in the ocean. First, however, NASA has to access the water under the ice, and even before it can reach the water it has to determine the thickness of the ice.

Question: What methods might NASA use to determine the thickness of the Europan ice – and possibly also the depth of its ocean? Remember that neither the ice nor the water may be pure. Remember also that the ice may be very thick. And, finally, remember that NASA has limited financial resources and there is no possibility whatsoever of a manned expedition to Europa.

Answer. To get full marks for this question, the student was expected to, first, discuss some of the many possible methods that might be used to determine the thickness of the ice and depth of water and then, second, to discuss the most feasible appearing method in greater detail, as indicated below.

Possible methods could include (1) flying a drill rig to Europa and using it to drill through the ice. But it should have been decided that this was infeasible due to the weight and size of the rig; (2) measuring the attenuation of a radio signal passing between a satellite orbiting Europa and the Earth as the satellite becomes occulted by Europa (i.e., passes behind it); (3) beaming electromagnetic waves down to the surface from an orbiting satellite and receiving echoes back; (4) landing a probe on the surface of Europa and carrying out a seismic or acoustic sounding experiment; (5) landing a probe on the surface and carrying out an acoustic sounding experiment; (6) landing a probe on the surface and having it melt its way through the ice; (7) landing a probe on the surface and carrying out an electromagnetic sounding experiment; (9) landing several probes at different distances apart on the surface and transmitting various kinds of signals between them to probe the surface.

The above list covers many more possibilities than students were expected to discuss. In general, it should have been decided, quickly, and with some guidance from the examiner, that landing anything on the surface of Europa would involve a much heavier, and thus more expensive, spacecraft to Europa, leaving methods (2) and (3) as the most feasible. Ultimately method (2), if chosen, would be dismissed because the radio signals would have to propagate obliquely through the ice (and water) and thus subject to greater attenuation. This left (3) as the most likely method.

The student was expected to consider what kind of frequencies would be best for electromagnetic probing method (3). For this they would have their attention drawn to the discolorations in the ice and the likelihood that the ocean had salts dissolved in it – in other words, the ice and its underlying water are probably contaminated with salts and therefore electrically conducting. This should then lead into a discussion of the skin depth (definition?) and its inverse dependence on (the square root of) conductivity and frequency. Also looked for was some sensitivity to reflections/transmissions at interfaces.

Note: This Quals question was a repeat of the one given in 2001.

Binary Search

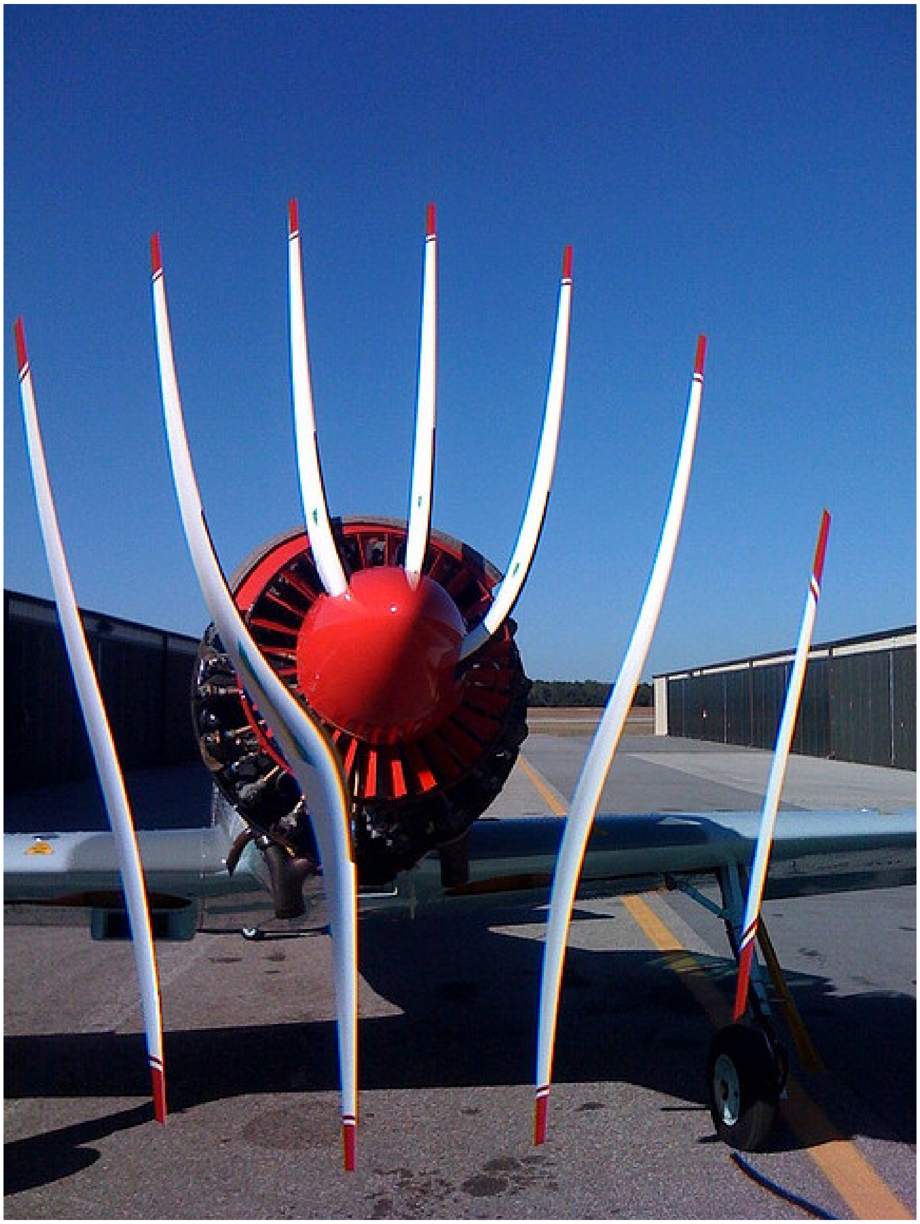
Prof. Hector
Garcia-Molina

- We are given an array A of sorted integers
  Example:

A												N=11
	3	7	12	13	21	43	44	45	62	69	75	

  - N is number of elements (N > 0)
  - A[1] is first element, A[N] is last
  - Variables A, N are global
  - There are no duplicates in array
- Your task: code following function
  Procedure FIND(V): returns INDEX
  - V is integer we are looking for
  - If V is in not in array, INDEX is set to -1
  - If V is in array, INDEX is location of V
  - Example:
    FIND(13) returns INDEX=4
    FIND(14) returns INDEX= -1
- Use binary search for your procedure

Prof. Bernd Girod



The picture shows the spinning propeller of a single-engine airplane captured by the camera of an iPhone 4. The artifacts are caused by the "rolling shutter" of the CMOS image sensor, which doesn't snap the entire image in one instance but instead captures it row-by-row over the time period of 28 ms.

1. Assuming clockwise rotation, in which direction does the rolling shutter sweep across the image?

2. How many blades does the propeller have?

3. How fast does the propeller spin?

Pat Hanrahan
2016 Quals Questions

What is a hash function? What are hash functions used for in computer systems? What properties should a good hash function have? How would you hash strings? And, finally, have you ever used hash functions in your work?

2016 PhD Quals Questions
J. S. Harris

1. Can you first tell me how a solar cell works? On an I-V diagram can you identify the different parameters by which we characterize solar cells under illumination?

2. There is a famous paper by Shockley and Queisser defining a limit efficiency for solar cells. Upon what parameter of the solar cell does the limit efficiency depend? Can you illustrate on an energy band diagram the bandgap-efficiency tradeoff and why there is a "limit efficiency"? How is this reflected in the quantum efficiency vs wavelength or energy for the solar cell. What happens to the energy that isn't converted? Would a solar cell illuminated with the same number of photons as from the sun, but from a LED produce the same, greater of lower output power? Would it have a higher conversion efficiency than when illuminated by the sun? Why?

3. What strategies might you suggest to overcome this limit and significantly increase solar cell efficiency?

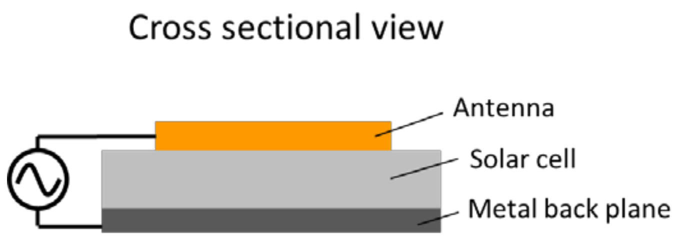
4. Can you sketch on an energy band diagram and explain how each of these strategies works? Which has proven successful and which unsuccessful and why?

Mark Horowitz
2016 Quals Questions

My quals question was about a device called a useless box. I had one on my desk that the student could play with, and I asked them how does it work. We then worked through how many states it has, the logical equations for the states, and then how to build it from switches, a battery and a motor.

Quals 2016, Electromagnetism, Jonathan Fan

Something that people are developing for the Internet of Things is a module that integrates a radio frequency (RF) antenna on top of a solar cell. Both devices take up a lot of space, and the combining of an “optically transparent” RF antenna with a solar cell could significantly reduce the footprint of the module.



I am proposing to make an “optically transparent” radio frequency antenna by thinning its metal (for example, copper) to a thickness so thin, it becomes optically transparent.

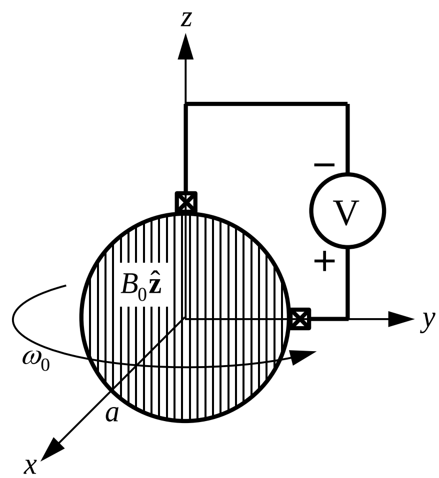
a) Is it possible to make an optically transparent structure in this way? And can we make an effective RF antenna in this way? Present rigorous arguments using Maxwell’s equations.

b) Propose alternative design schemes that will enable as much light to pass through the antenna into the solar cell

2016 EE Qualifying Examination
Electromagnetics
Professor Joseph M. Kahn

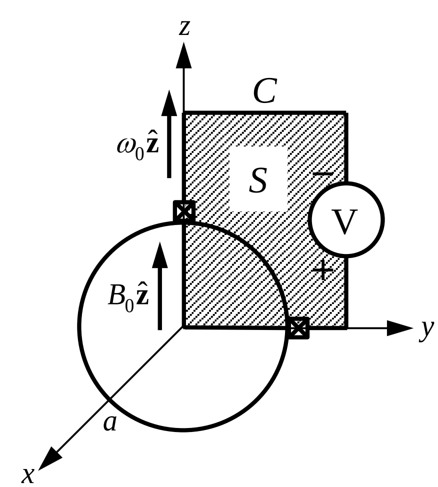
Question

A conducting sphere of radius a is uniformly magnetized along the z axis so that, inside the sphere, there is a uniform magnetic induction $\mathbf{B}_{in}(\mathbf{r}) = B_0\hat{\mathbf{z}}$. The sphere is rotated about its central axis at a constant angular velocity $\boldsymbol{\omega} = \omega_0\hat{\mathbf{z}}$. One sliding electrical contact ("brush") is touched against the top of the sphere (along the positive z axis), while another is touched to the sphere along the equator (in the x-y plane). Find an expression for the voltage measured between these two contacts. You may refer to the figure below.



Answer

Consider a contour C enclosing a surface S.



We apply Faraday's Law of Induction. The EMF around the contour C is

$$\oint_C \mathbf{E}' \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a} + \oint_C (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \, .$$

On the right-hand side, the first term is a "transformer EMF" that is present when there is a time-varying magnetic induction. The second terms is a reflection of the Lorentz force exerted on a unit test charge moving at velocity $\mathbf{v}$ in a magnetic induction. In this problem, there is no time-varying magnetic induction, $\partial \mathbf{B} / \partial t = 0$ everywhere, so the first term vanishes. Considering the second term, $\mathbf{v} \times \mathbf{B}$ is nonzero only along the *y* axis inside the sphere, where it has the value

$$\mathbf{v} \times \mathbf{B} = (\omega_0 r \hat{\boldsymbol{\varphi}}) \times (B_0 \hat{\mathbf{z}}) = \omega_0 r B_0 \hat{\mathbf{r}} \, ,$$

and where $d\mathbf{l} = dr\hat{\mathbf{r}}$ .The voltage measured is given by the integral

$$V = \int_0^a \omega_0 r B_0 \, dr = \frac{\omega_0 a^2 B_0}{2} \, .$$

Greg Kovacs
2016 Quals Question

Students were shown an analog circuit consisting of two op-amps and some resistors. They were asked to analyze it to explain its function and gain. Hints were provided on the paper. The goal was to demonstrate a clear and organized analytical approach, not necessarily to reach a particular [3]answer.[2]

Phil Levis
Quals Questions 2016

What year are you? Please tell me about the research you’re doing. The rest of questions were specific to student research. The goal of the questions was to gauge how well the student

1) understood their own work,
2) could explain their work,
3) could explain the work it builds on (prior work),
4) and discuss the overall research goals and future steps.

EE Quals 2016

Computer architecture and logic design

Nick McKeown

Question 1

You are to design the traffic-light controller for the intersection of two streets (A and B). Each street has a single input from a WALK button pushed by pedestrians wishing to cross the street (WALK_A and WALK_B). The lights are controlled by two outputs: RED_A and RED_B. When RED_A is 1, then the red light is on and the green light is off for street A. Same for RED_B.

If no pedestrians present and the WALK buttons are not pushed, the lights alternate between allowing traffic on A and B every 1 minute. If a WALK button is pushed, the current cycle is finished, and then both lights are RED for 1 minute, before returning to the previous alternating A/B cycle.

A: Create a state diagram, and show the state transition table.

B: Encode the states in the minimum number of bits.

C: Design the sequential circuit to implement the state machine.

Question 2

It's very important for a traffic-light controller to be reliable, and to fail safely. What techniques can we use to reduce the chances of a crash if the system fails?

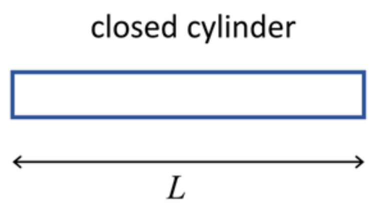
2016 EE Qualifying Exam Question – David Miller

Acoustic standing waves

Note: The goal of this set of questions is to see how you think about solving them, and that will be more important than whether the answers are "right" or "wrong". The answers are mostly qualitative, and little or no algebra should be required for them. If you finish the questions on this sheet, subsequent questions will be asked.

In this question, we will be considering the various possible resonant modes or standing wave patterns for sound waves (i.e., acoustic waves) inside various structures and volumes. We will presume any volumes are filled with air, and we can assume there is some sound velocity, which we will call v, for such acoustic waves in air.

(1) Suppose we have a closed cylinder of length L, shown in cross section below.



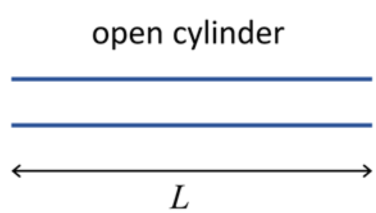
The diameter of the cylinder is presumed to be small compared to any sound wavelength of interest to us.

(i) Sketch the standing wave (the "mode") corresponding to the lowest resonant frequency for a sound wave within this volume. Specify what quantity it is you are sketching (e.g., pressure, particle velocity).

(ii) Give an expression for the frequency of this resonant mode in terms of the sound velocity v and the length L of the cylinder.

(iii) Give an expression for the frequencies of all of the resonant modes corresponding to such standing waves between left and right.

(2) Suppose now the cylinder is open at both ends, as shown in cross-section below.

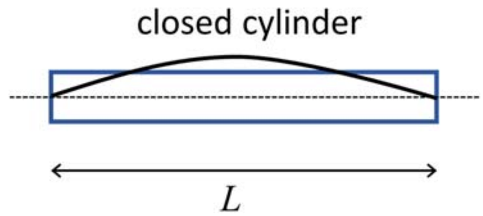


(i) Give an expression for the frequencies of all the resonant modes corresponding to standing waves between the left and the right.

(ii) What would happen to the resonant modes if we cut a small hole in the center of the cylinder in one side?

Solution

1) (i) The standing wave for the lowest resonant frequency corresponds to fitting half a wave between the left and the right. Most students would draw this as below, which is one form of a correct answer here. (The dashed line is the "zero". An upside-down version of this would also be correct.



Most students would incorrectly state this was the pressure in the wave. It is actually the velocity (the average velocity in the "longitudinal" direction along the pipe) for one "snapshot" of that velocity distribution. (That error of pressure rather than velocity was not in itself a problem, but it would lead to useful situation as the student tried later on to reason their way to the correct answer.) That velocity is zero at the two ends. The pressure wave actually has its largest amplitudes at the two ends and is zero in the middle.

(ii) In general, the relation between the velocity wave velocity v, the frequency f, and the wavelength λ, is

$$f = v / \lambda$$

so, since the wavelength here is $2L$, the frequency of this mode is

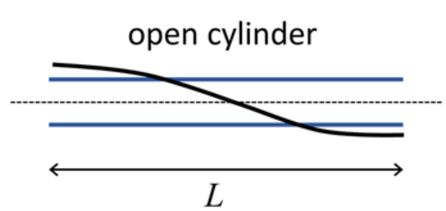
$$f_1 = v / 2L$$

(iii) For each successive mode, we fit in one more half wavelength in the mode, so the frequency rises accordingly, giving, for the frequency of the nth mode

$$f_n = nv / 2L$$

2) (i) This problem is essentially the "dual" of the problem of a cylinder with closed ends. Though at first sight it might not be obvious, there are wave reflections off the ends of the open pipe. These are not quite perfect reflections, but to a first approximation we can take them to be so. This situation is rather analogous to an electrical transmission line with open ends. In both cases there is some amount of radiation from the ends, though it can be relatively small.

Presuming then that the reflection from the ends is strong, the form of the velocity wave of the lowest resonance is as shown below.



(An upside down version of this would also be correct. Incidentally, this is the form of the pressure wave for (1) above, and the form of the pressure wave for this problem would look like the velocity wave for (1) above.) The velocity has maxima at the ends (and the pressure has minima).

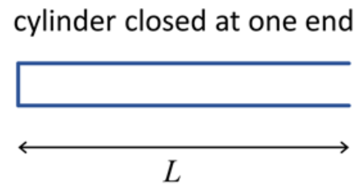
Note that this also corresponds to a complete half wave in the pipe. Higher modes correspond simply to more half waves in the pipe. The frequencies are therefore the same as in (1) above.

(ii) If we cut a hole in the side of this pipe in the middle, we are essentially killing all the odd numbered resonances; we cannot have a pressure maximum at the position of such a hole because that would leak power very strongly, so the "quality factor" of the resonances would be very low. This essentially corresponds to cutting the pipe into two sections, each of half the length of the original one. The frequencies of the resonances that remain are those of a pipe of half the length.

The acoustics of this double open-ended pipe are essentially those of the flute, which behaves as if it is a pipe that is open at both ends. (Obviously, the "far" end of the flute pipe is open. The hole that the player blows into at the other end is essentially the other open end of the pipe. A real Western flute also has a small closed "stub" of pipe to the left of this hole, but that is primarily there to give a minor correction to the tuning of some of the notes (it technically forms what is called a Helmholtz resonator.) If a flute player covers all the holes in the flute to play the lowest note, and then opens the hole nearest to the player's mouth or the "blowing" end of the flute – a hole that is approximately half way down the pipe – that causes the frequency of the sound to double, changing the pitch by an octave musically. This is a standard part of flute playing technique.)

Supplementary question

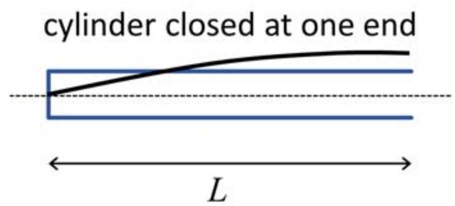
(i) What would be the frequencies of the resonant modes for a cylinder of length L that is closed at one end but open at the other, as shown in cross-section below?



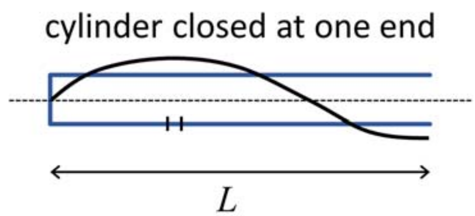
(ii) Where now would you put a hole in the side of the cylinder to select just a subset of the modes of this system?

Solution to supplementary question

(i) In this case, we will have a velocity minimum at the closed end and a velocity maximum essentially at the open end. The lowest frequency mode that will fit in here is one corresponding to a quarter wave, as sketched below. (Velocity is the quantity sketched, as before.)



Higher frequency modes require we add complete half waves to retain the velocity maximum. The sketch below shows the second mode of this cylinder.



The resonant frequencies of this cylinder are, therefore

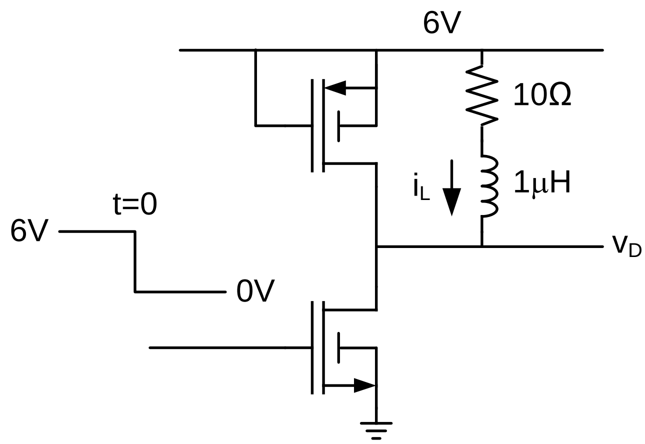
$$f_n = (2n-1)v/4L$$

where $n=1$ is the first mode and so on.

(ii) In this case, cutting a hole 1/3 of the way along the pipe from the closed end will have essentially no effect on the second mode, but will suppress the first mode. This closed pipe represents the acoustics of the clarinet. The reed and mouthpiece correspond effectively to a closed end on the pipe. To change "registers" or modes on the clarinet, a register key is added, which opens a hole approximately 1/3 of the way along the pipe (at least for the "middle" note in the second "register" of the clarinet. Note also that the frequency of the lowest mode of the clarinet is a factor of two lower (an octave lower) than that of a flute of the same length. (This larger separation of the lowest register and the next register, basically a factor of 3 in frequency, is the reason why the clarinet needs extra holes opened by keys towards the top of the clarinet and extra holes closed by keys at the bottom of the clarinet – basically, we don't have enough fingers to play the instrument otherwise.)

Name:	Stanford EE Quals 2016 Murmann

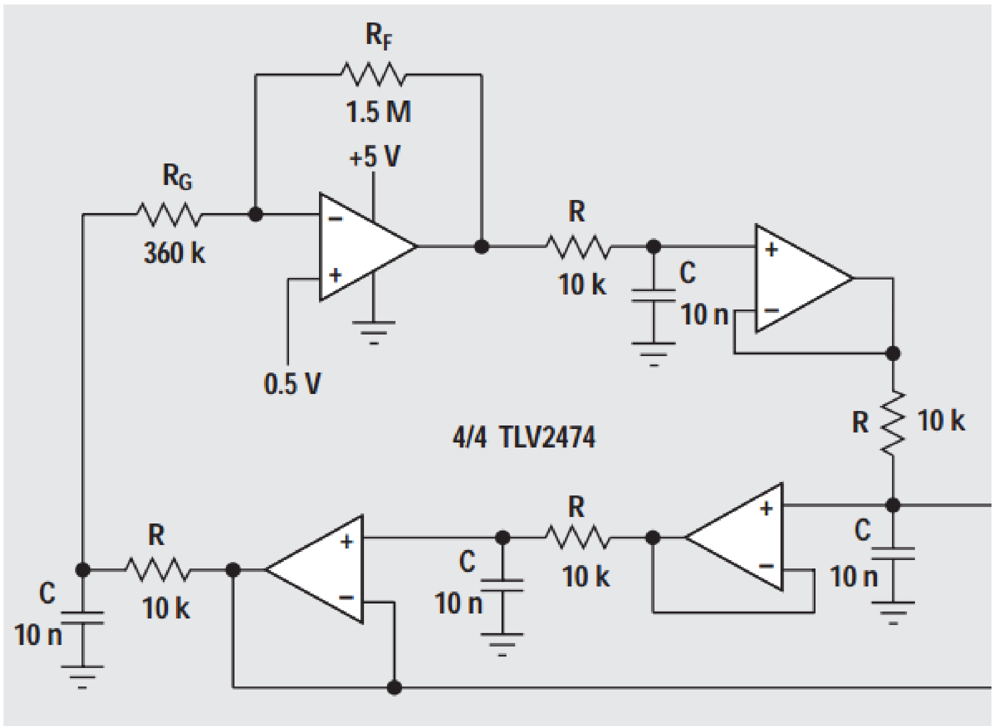
Sketch $i_L(t)$ and $v_D(t)$ versus time, starting at t=0. Ignore the capacitances of the MOSFETs and assume that they obey the ideal square law model. Assume that all transients before t=0 have settled.



$\mu C_{ox}W/L = 8mA/V^2$
$V_t = 1V$

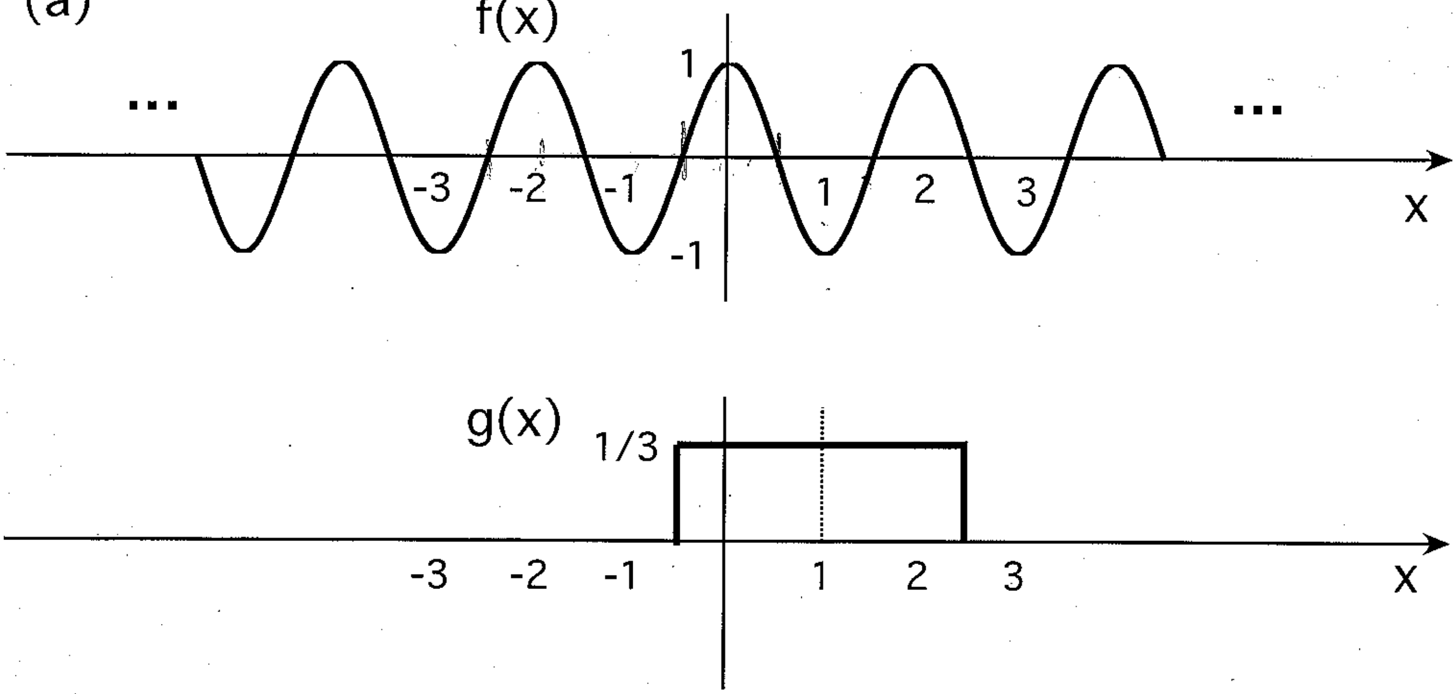
The op-amps in the circuit below are ideal.

a) Explain the function and operation of this circuit.
b) Explain the why the designer sized $R_F/R_G = 4.167$ in this circuit.

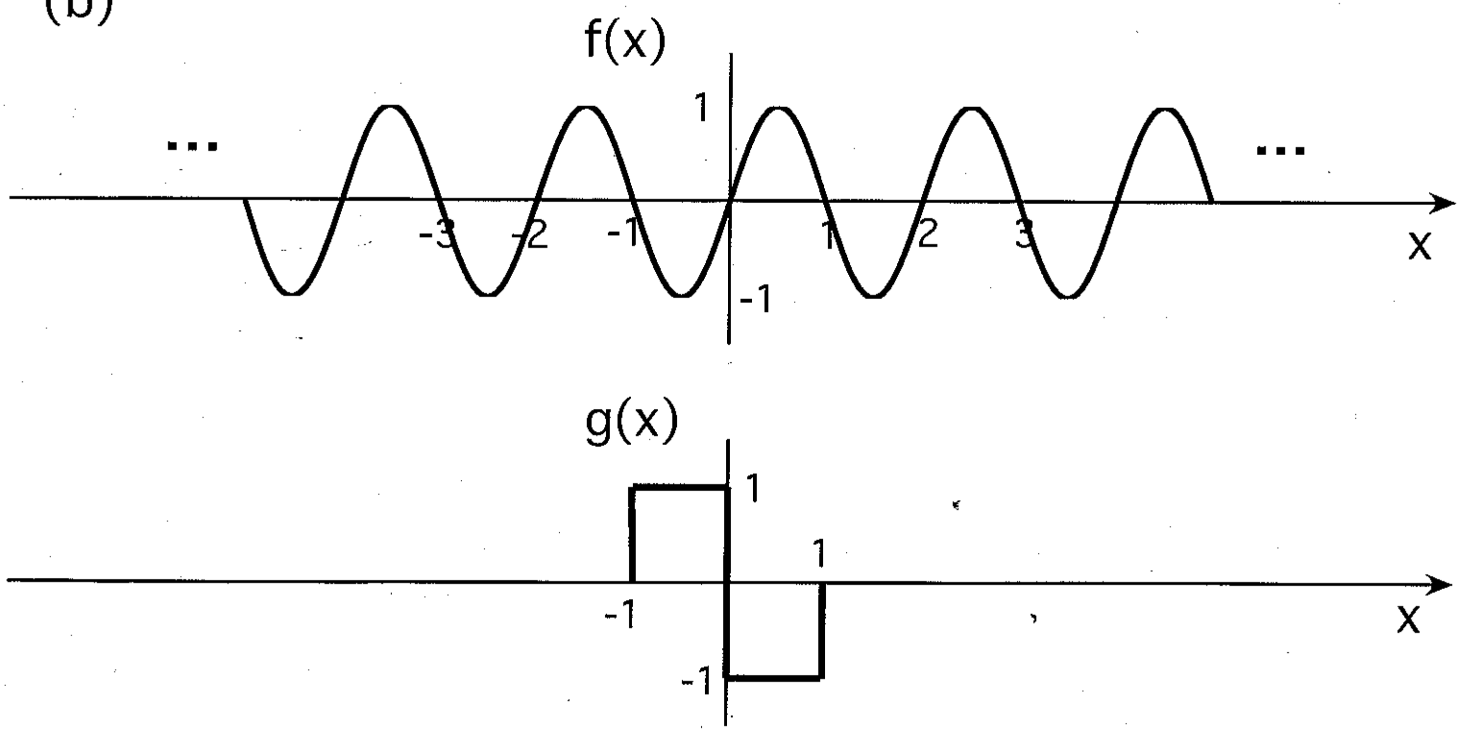


Sketch the following convolutions: f(x) * g(x)

(a)



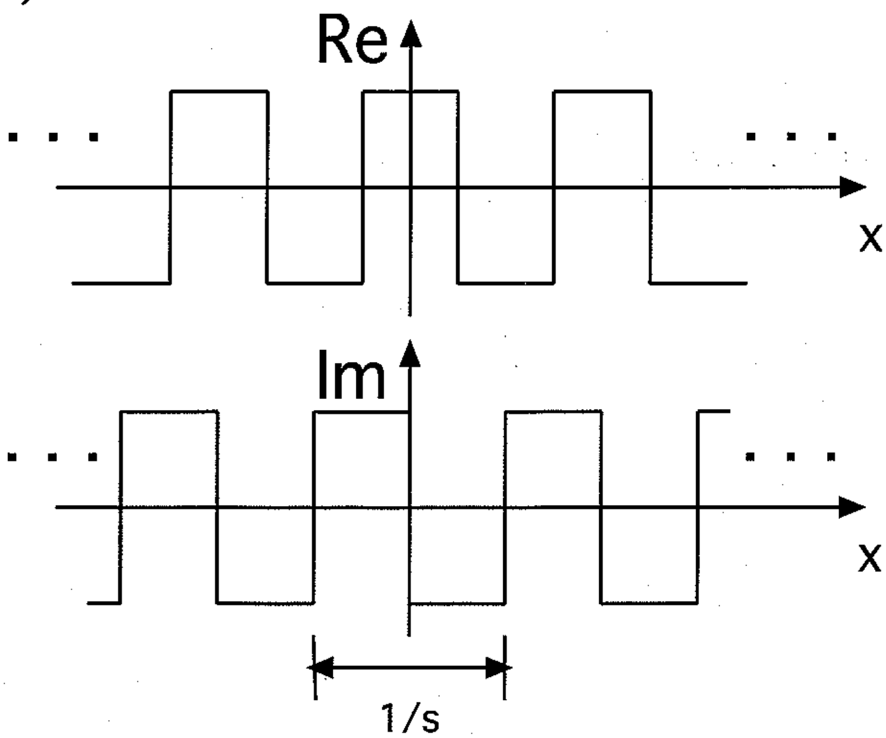
(b)



Consider the following transform: $g(x) \xrightarrow{\text{transforms to}} \tilde{G}(s)$

$$\tilde{G}(s) = \int g(x)\, w(x,s)\, dx$$

where w(x,s) is



Indicate if the following transform properties still apply.

a) $g(x - a) \longrightarrow \tilde{G}(s) \exp(-i2\pi as)$

b) $g(ax) \longrightarrow \dfrac{1}{|a|}\tilde{G}(s/a)$

c) $g(x) = f(x) * h(x) \longrightarrow \tilde{G}(s) = \tilde{F}(s)\, \tilde{H}(s)$

Prof. Brad Osgood

EE Qualifying Exam
January 2016

A population R of robins interacts with a population W of worms. An interaction between the two populations is good for the robins and bad for the worms. Find a system of differential equations of the form

$$\frac{dW}{dt} = f(R, W)$$
$$\frac{dR}{dt} = g(R, W)$$

that describes how the populations evolve. Hint: First ask what happens to the worm population if there are no robins, and what happens to the robin population if the are no worms, and then modify the model to take account of the interactions.

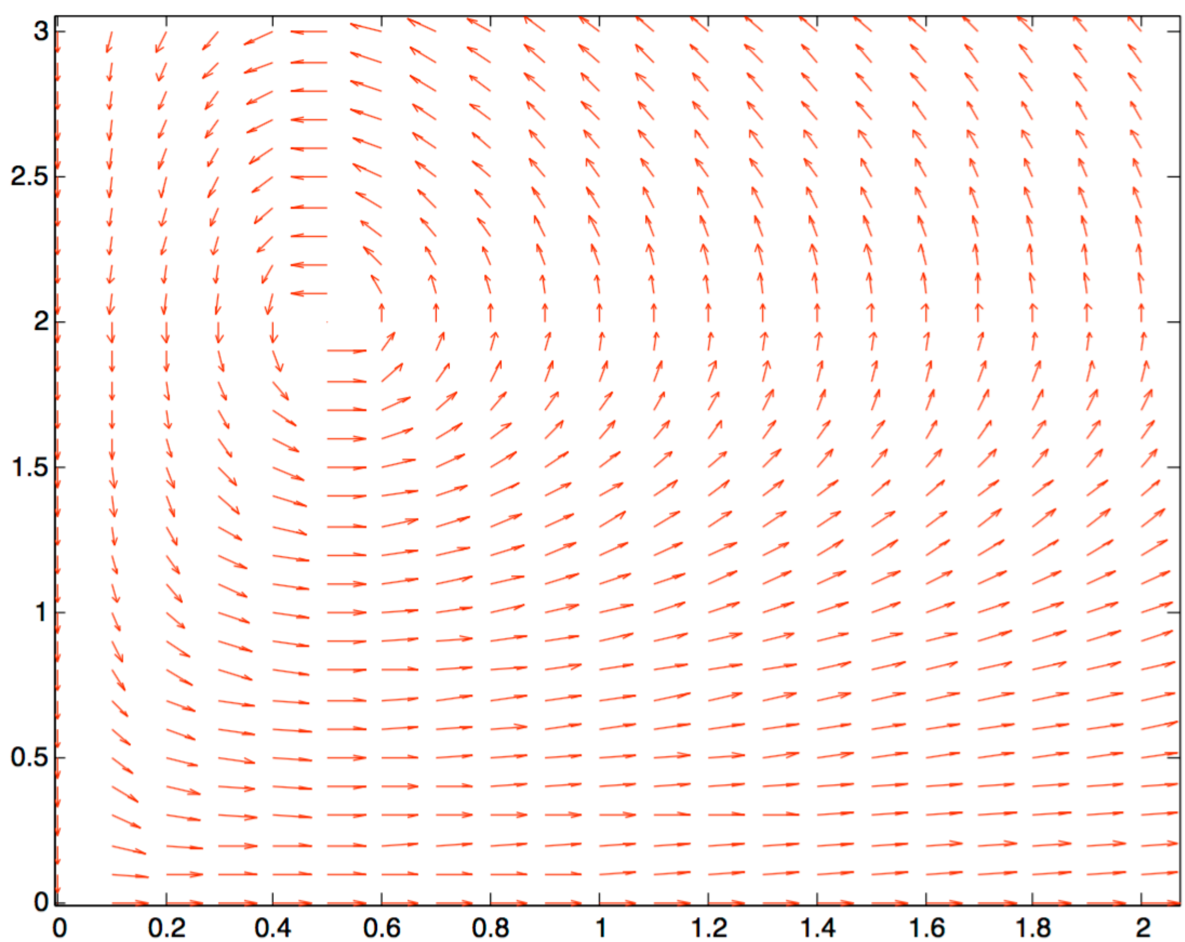
A system that models the interaction is

$$\frac{dW}{dt} = aW - bRW$$
$$\frac{dR}{dt} = -cR + dRW$$

where a, b, c, d are positive constants. Then

$$\frac{dW}{dR} = \frac{aW - bRW}{-cR + dRW}.$$

Here is a sketch of the slope field associated with the system in the (W, R)-plane for one choice of a, b, c, d. Sketch a few trajectories and interpret what you see in terms of how W and R are varying together, and be sure to indicate the direction of the trajectory. For what values of W and R do the populations remain constant?



Ayfer Özgür

2016 Ph.D. Qualifying Examination

(There were three problems that I posed one after another. I did not expect anyone to do the last problem but one student did everything.)

1. Given a non-negative integer k, and a positive integer $n \geq k$, let $P = \{(x_1, \ldots, x_n) : 0 \leq x_i \leq 1, \sum_i x_i = k\}$. ($P$ is the subset of the n-dimensional unit cube on which the coordinates sum to k.) Suppose $c_1 \geq \cdots \geq c_n$ are real numbers. Find

$$\max_{x \in P} \sum_{i=1}^{n} c_i x_i.$$

2. Suppose $X \in \mathbb{C}^{n \times k}$, $k \leq n$, and $X^\dagger X = I_k$, that is, the columns of X are orthonormal in $\mathbb{C}^n$. Show that

$$\|X^\dagger y\|^2 \leq \|y\|^2 \quad \text{for all } y \in \mathbb{C}^n.$$

Conclude that $0 \leq (XX^\dagger)_{ii} \leq 1$ for all $1 \leq i \leq n$.

3. Suppose A is a Hermitian matrix. We know that such a matrix can be written as $A = U\Lambda U^\dagger$ where U is unitary and $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$, with $\lambda_i \in \mathbb{R}$. Without loss of generality, assume that we have permuted the rows and columns of A so that $a_{11} \geq a_{22} \geq \cdots \geq a_{nn}$, and we have indexed the eigenvalues so that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$.

(a) Show that $\max\limits_{x \in \mathbb{C}^n : x^\dagger x = 1} x^\dagger A x = \lambda_1$.

(b) Show that $a_{11} \leq \lambda_1$.

(c) Show that for any $k = 1, \ldots, n$,

$$\max_{X \in \mathbb{C}^{n \times k} : X^\dagger X = I_k} \mathrm{tr}(X^\dagger A X) = \max_{X \in \mathbb{C}^{n \times k} : X^\dagger X = I_k} \mathrm{tr}(X^\dagger \Lambda X) = \sum_{i=1}^{k} \lambda_i.$$

[Hint: $\mathrm{tr}(AB) = \mathrm{tr}(BA)$, and you may find the previous two problems to be useful.]

(d) Show that for any $k = 1, \ldots, n$, $\sum_{i=1}^{k} a_{ii} \leq \sum_{i=1}^{k} \lambda_i$, and that equality holds when $k = n$.

John Pauly
Quals Questions 2016

Question

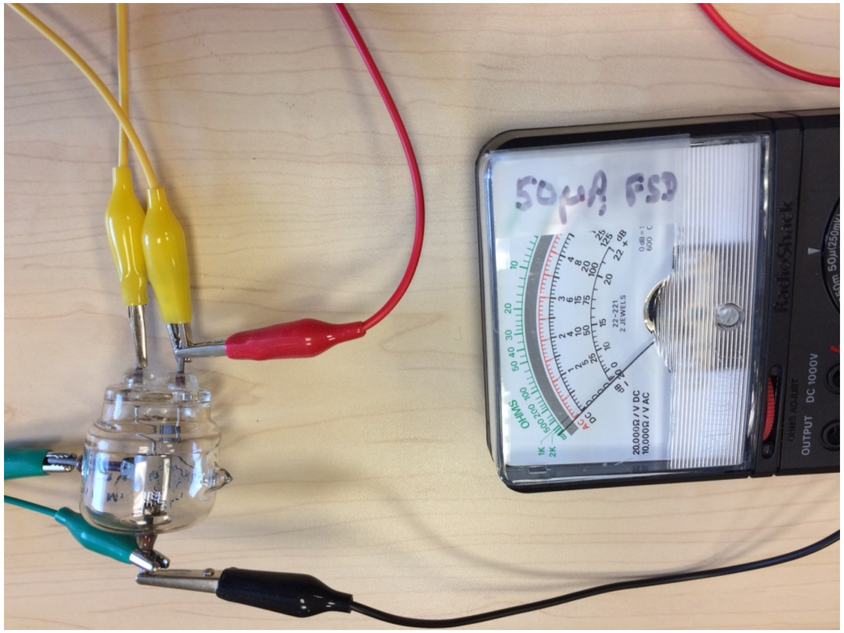
I have a real signal whose bandwidth goes from ± 500 kHz.

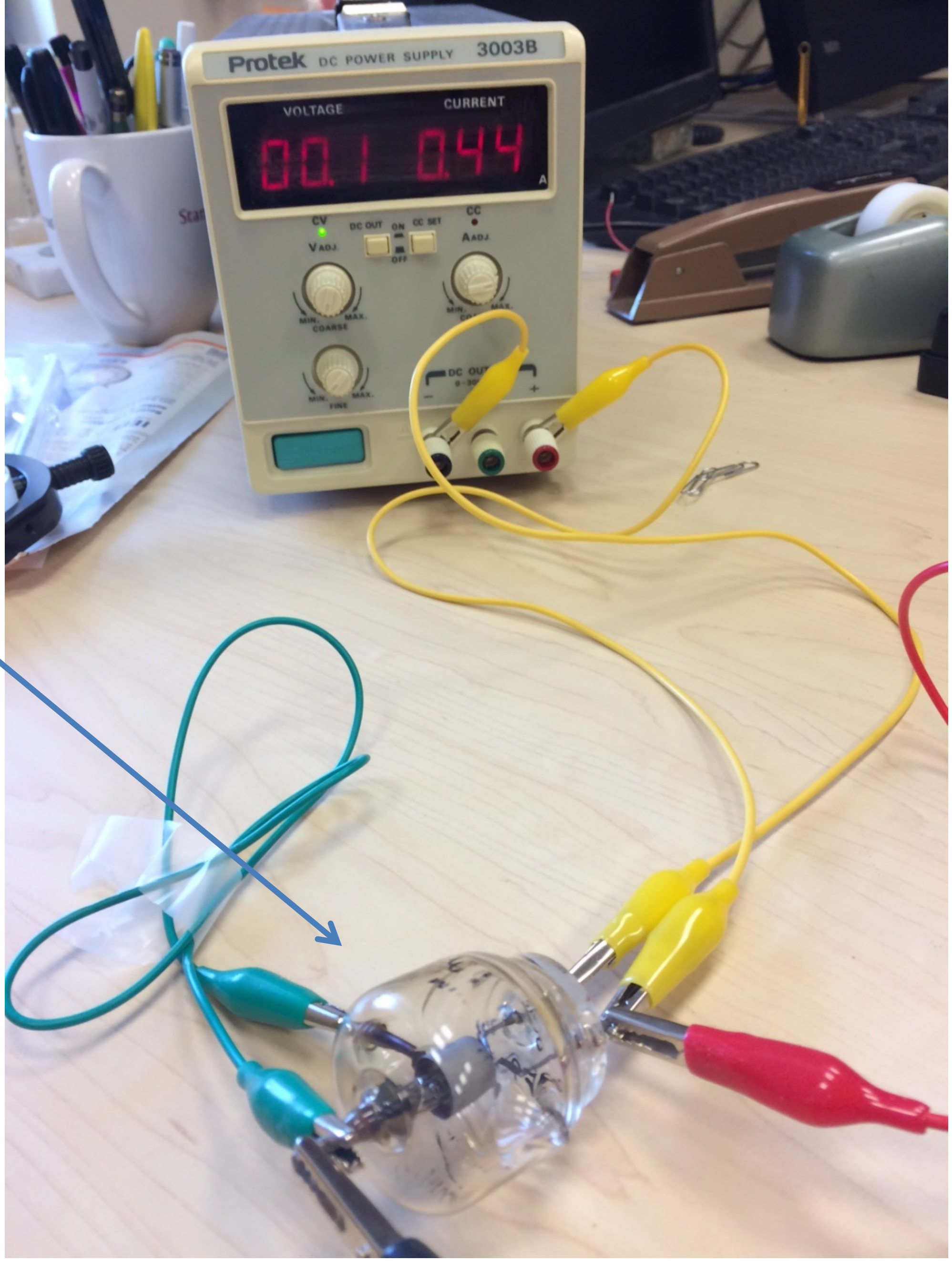
a) how fast do I need to sample to be able to reconstruct it perfectly?

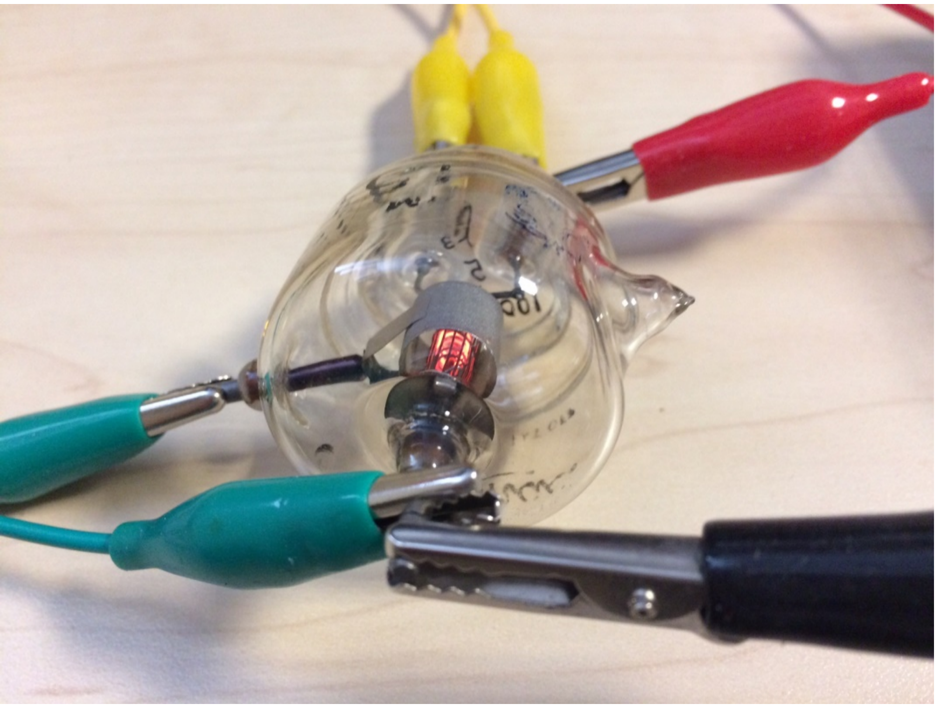
b) You have 2 A/D’s that sample at 500 kHz. How can you use these to sample the signal? You can use additional components if you need them.

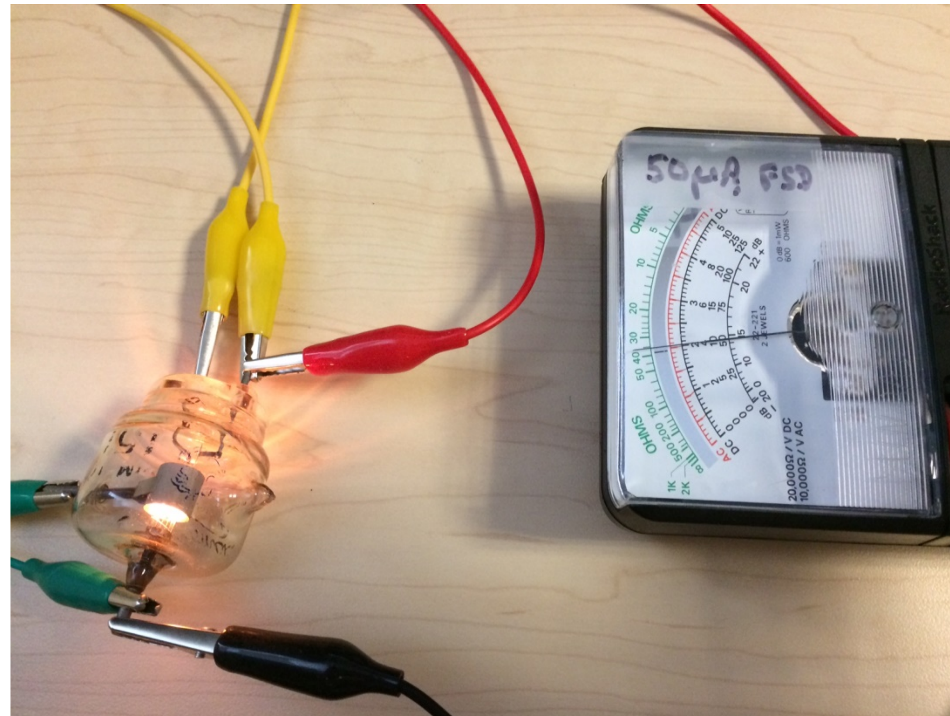
Prof. Fabian Pease

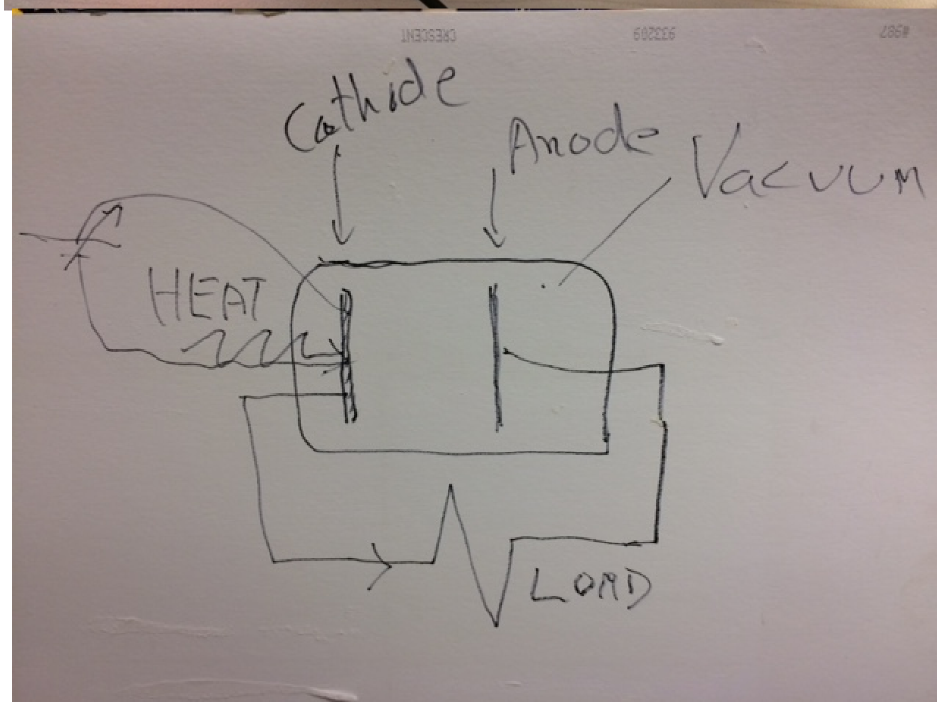
What can you tell me about this electron device?











The diagram illustrates a device that can convert heat (at the cathode) into electricity. How would you choose the metals for the cathode and the anode?

How would you choose the cathode-to-anode distance?

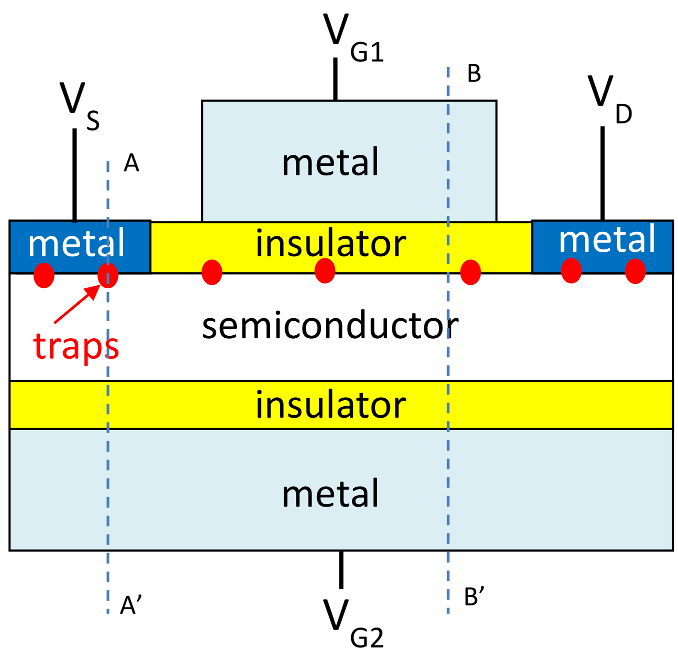
Does the Carnot limit to efficiency apply to this device? (*1 person got this far*)

2016 Qualifying Exam Questions

Prof. H.-S. Philip Wong

Consider the device below (first without the traps):

1. Draw the band diagram across the metal semiconductor junction (section A-A')
2. Draw the band diagram across the metal/insulator/semiconductor/insulator/metal stack (section B-B')
3. Now, add the traps in the picture. The traps can hold an electron if filled.
4. Sketch the I_D-V_{G1} curve of the device, for a finite V_D, with $V_S = V_{G2} = 0$ V
5. Sketch the I_D-V_{G2} curve of the device, for a finite V_D, with $V_S = V_{G1} = 0$ V
6. Obviously, the answer depends on whether the semiconductor is thick or thin. How would you determine whether the semiconductor is "thick" or "thin"? What are the signatures you would look for in the band diagram or in the measured characteristics?
7. Now, add the traps in the picture. The traps can hold an electron and become negatively charged if filled.
8. What do the I-V curves look like if the traps are filled with electrons?



Piero Pianetta
Quals Questions 2016

Consider a device structure (known as a vacuum tube) that consists of two parallel plates in a vacuum. The first plate is grounded and emits electrons at near zero energy and at a constant rate. You can apply a voltage to the second plate and measure the current. Using simple reasoning, tell me how you would determine the characteristics of this device and how you could use this a device in a circuit.

Answer: Vary the voltage on the second plate from negative to positive; realizing that a positive voltage draws current and a negative voltage draws zero current should lead to the conclusion that this device works like a diode. Additional items that would add points to the score would be a discussion of how and why the current varied with voltage, would expect the current to saturate, what would be the affect of different work functions on the two plates.

Now, let's put a grid between the two plates that is attached to its own voltage source. Again, given the measurements you could make on this device, how would you figure out how you could use it in a circuit?

Answer: The grid controls the current and so could be used in an amplifier.

Prof. Juan Rivas

Name:

Consider the design of a power MOSFET gate drive circuit switching at $f_s = 10$ MHz. The MOSFET has a gate capacitance $C_g = 1$ nF, gate resistance $R_g = 100$ mΩ, and lead/package gate inductance $L_g = 250$ pH. The device has negligible gate-drain capacitance C_{gd} and neglect the effects the drain voltage/current may have on the gate-drive circuit. To prevent damage in the device, the gate-source voltage must remain between ± 15 V. The threshold voltage for this part is $V_{th} = 2$ V and the channel can be considered fully enhanced when the gate voltage is greater than 8 V.
Consider the following gate driving circuit schemes:

1 Simple gate drive

In the gate drive circuit of figure 1, the amplitude of the driving square voltage is $V_{sq,max} = 10$ V. Sketch the effective internal gate voltage $v_g(t)$, and label the salient features. Comment on any issues that can arise by driving the circuit in this way. How much power is lost in this gate drive circuit? Propose a way to use a single passive component (what value?) to improve the performance of the driving circuit. How much power is lost in the gate drive circuit after the improvement?

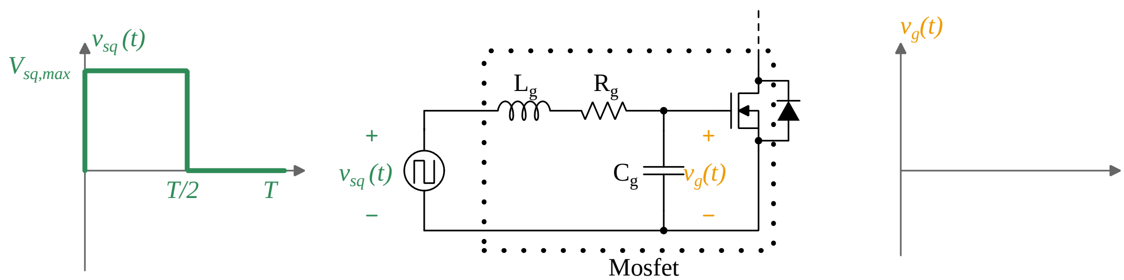


Figure 1: Simple Gate drive

2 Sinusoidal Gate drive circuit

In the gate drive circuit of figure 2, the circuit is driven by a voltage source $v_{ac}(t)$ that results in a Sinusoidal internal gate voltage $v_g(t)$ of amplitude $V_{g,ac-max} = 12.5$ V. Sketch the driving voltage $v_{ac}(t)$ and label the salient features. What are the maximum and minimum voltages of $v_{ac}(t)$. How much power is lost in this gate drive circuit?

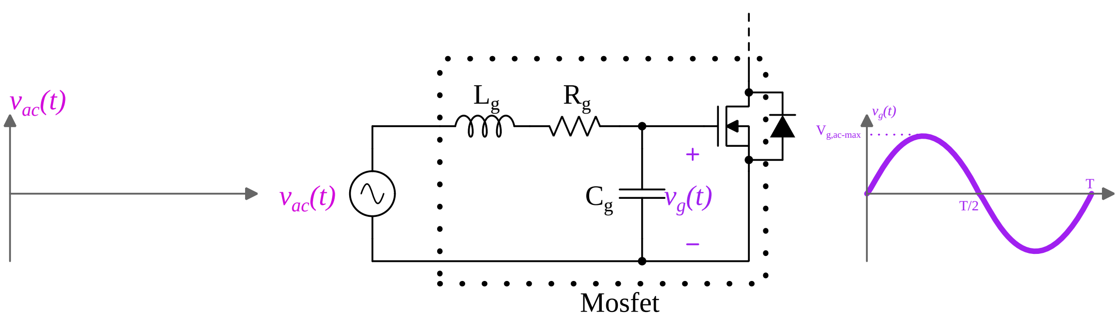


Figure 2: Sinusoidal Gate drive circuit

3 Trapezoidal Gate drive circuit

In the gate drive circuit of figure 3, the circuit is driven by a current source $i_t(t)$ that results in a trapezoidal gate voltage $v_g(t)$ with $V_{g,t-max} = 10$ V and rise time and fall times $t_r = t_f = \frac{1}{10 f_s} = 10$ ns . Sketch the driving current $i_t(t)$ and label the salient features. What are the maximum and minimum current of $i_t(t)$. How much power is lost in this gate drive circuit?

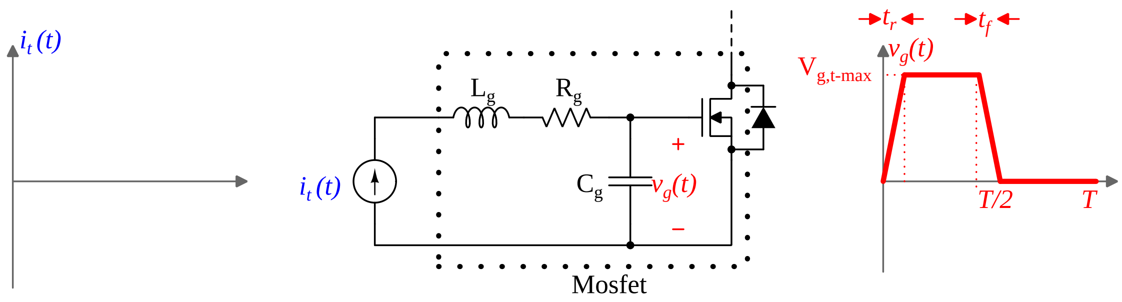


Figure 3: Trapezoidal Gate drive circuit

EE Quals 16, Engineering Physics
Shan Wang

a) If you can arrange a set of bar permanent magnets in any manner, could you make the magnetic field confined to one side?

The answer is yes, but how? Most people will be a bit surprised initially by this question for its apparent difficulty or even impossibility. It is meant to be an open ended question to see if one can think of a way to cancel the magnetic field in one side of a plane by arranging magnets along a straight line. Hints are given verbally. The answer only needs to be correct qualitatively.

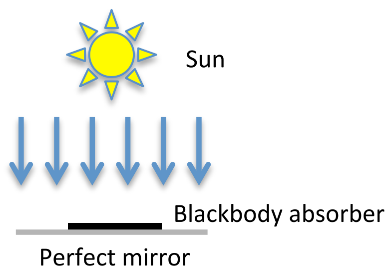
b) Do you know how refrigerator magnets work?

This part is only for people completely lost in a). A few points are given if one describes refrigerator magnets correctly.

EE Ph. D. Qualifying Exam. Jan. 25-29, 2016. Shanhui Fan. Engineering Physics

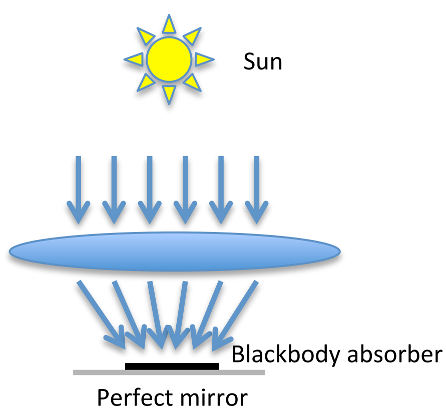
(a) Suppose the sun can be approximated as having a temperature of 6000K. Sketch the emission spectrum of the sun.

(b) Consider the following solar heating experiment, where the direct sunlight is normally incident upon a piece of blackbody backed by mirror. The blackbody is placed somewhere near earth. What is the maximum temperature that the blackbody can reach?



(c) Consider instead the experiment where one places a lens in front of the blackbody absorber to focus the sun light onto the blackbody. Could you sketch how would you predict the temperature of the blackbody?

(d) If one can use a lens that can have arbitrarily large aperture, what is the maximum temperature that the blackbody can reach?



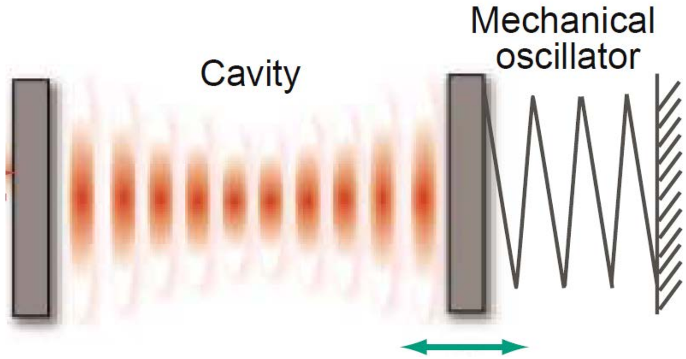
Simon Wong
2016 Quals Questions

Given a common source amplifier with PMOS active load,
1. What is the small signal voltage gain?
2. Show the DC biasing circuits.

Clearly state any assumptions you make while solving the problems. Good luck!

1. Optical resonator

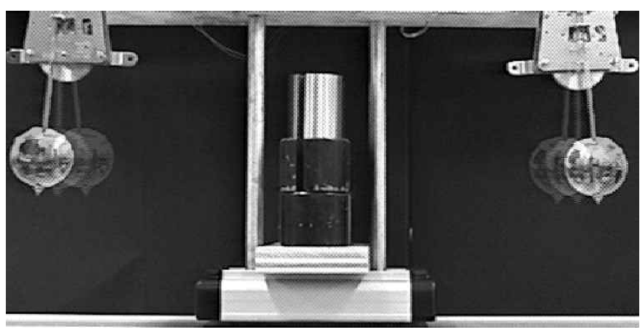
An optical resonator has one of its mirrors attached on a spring, as shown in the figure. How would a spectrum of such a resonator look like (and would it be any different than a spectrum of a resonator with two fixed mirrors)?



Kippenberg and Vahala, *Science* (2008)

2. Spontaneous synchronization

In 1665, Dutch scientist Christiaan Huygens discovered that two pendulum clocks suspended side by side on the same rod eventually synchronize. Why does this synchronization happen?



Bennett et al, *Proceedings of the Royal Society* (January 2015)

Prof. Tsachy Weissman

Quals Question - 2016

Consider 100 ants walking on a one dimensional horizontal stick of length one yard, each walking either to the left or to the right at the same constant speed of one yard per minute. When an ant reaches an edge of the stick it falls off. When 2 ants collide, they each turn to the respective opposite directions and continue to walk at the same speed.

(a) How much time will it take, at the most, for all the ants to fall off the stick?

Suppose initially the ants are placed independently of each other, uniformly at random on the stick, with a random initial direction.

(b) What is the expected number of ants that remain on the stick after half a minute?

(c) What is the expected amount of time until all the ants fall off the stick?

(d) What is the probability that exactly k ants remain on the stick t minutes after the initialization?

(e) What is the CLT approximation of the probability that, half a minute after initialization, there are 60 or more ants remaining on the stick?