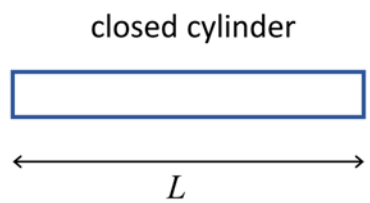
2016 EE Qualifying Exam Question – David Miller

Acoustic standing waves

Note: The goal of this set of questions is to see how you think about solving them, and that will be more important than whether the answers are "right" or "wrong". The answers are mostly qualitative, and little or no algebra should be required for them. If you finish the questions on this sheet, subsequent questions will be asked.

In this question, we will be considering the various possible resonant modes or standing wave patterns for sound waves (i.e., acoustic waves) inside various structures and volumes. We will presume any volumes are filled with air, and we can assume there is some sound velocity, which we will call v, for such acoustic waves in air.

(1) Suppose we have a closed cylinder of length L, shown in cross section below.



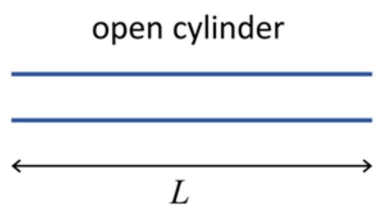
The diameter of the cylinder is presumed to be small compared to any sound wavelength of interest to us.

(i) Sketch the standing wave (the "mode") corresponding to the lowest resonant frequency for a sound wave within this volume. Specify what quantity it is you are sketching (e.g., pressure, particle velocity).

(ii) Give an expression for the frequency of this resonant mode in terms of the sound velocity v and the length L of the cylinder.

(iii) Give an expression for the frequencies of all of the resonant modes corresponding to such standing waves between left and right.

(2) Suppose now the cylinder is open at both ends, as shown in cross-section below.

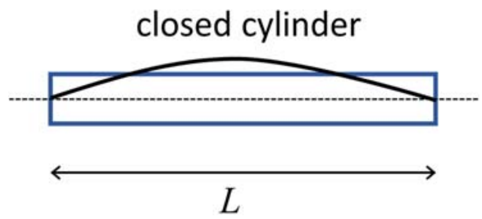


(i) Give an expression for the frequencies of all the resonant modes corresponding to standing waves between the left and the right.

(ii) What would happen to the resonant modes if we cut a small hole in the center of the cylinder in one side?

Solution

1) (i) The standing wave for the lowest resonant frequency corresponds to fitting half a wave between the left and the right. Most students would draw this as below, which is one form of a correct answer here. (The dashed line is the “zero”. An upside-down version of this would also be correct.



Most students would incorrectly state this was the pressure in the wave. It is actually the velocity (the average velocity in the “longitudinal” direction along the pipe) for one “snapshot” of that velocity distribution. (That error of pressure rather than velocity was not in itself a problem, but it would lead to useful situation as the student tried later on to reason their way to the correct answer.) That velocity is zero at the two ends. The pressure wave actually has its largest amplitudes at the two ends and is zero in the middle.

(ii) In general, the relation between the velocity wave velocity v, the frequency f, and the wavelength λ, is

$$f = v/\lambda$$

so, since the wavelength here is $2L$, the frequency of this mode is

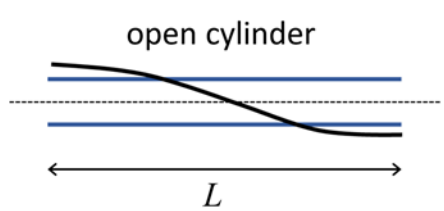
$$f_1 = v/2L$$

(iii) For each successive mode, we fit in one more half wavelength in the mode, so the frequency rises accordingly, giving, for the frequency of the nth mode

$$f_n = nv/2L$$

2) (i) This problem is essentially the “dual” of the problem of a cylinder with closed ends. Though at first sight it might not be obvious, there are wave reflections off the ends of the open pipe. These are not quite perfect reflections, but to a first approximation we can take them to be so. This situation is rather analogous to an electrical transmission line with open ends. In both cases there is some amount of radiation from the ends, though it can be relatively small.

Presuming then that the reflection from the ends is strong, the form of the velocity wave of the lowest resonance is as shown below.



(An upside down version of this would also be correct. Incidentally, this is the form of the pressure wave for (1) above, and the form of the pressure wave for this problem would look like the velocity wave for (1) above.) The velocity has maxima at the ends (and the pressure has minima).

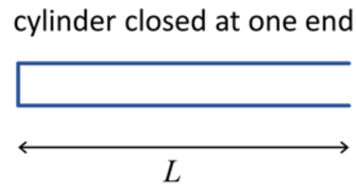
Note that this also corresponds to a complete half wave in the pipe. Higher modes correspond simply to more half waves in the pipe. The frequencies are therefore the same as in (1) above.

(ii) If we cut a hole in the side of this pipe in the middle, we are essentially killing all the odd numbered resonances; we cannot have a pressure maximum at the position of such a hole because that would leak power very strongly, so the “quality factor” of the resonances would be very low. This essentially corresponds to cutting the pipe into two sections, each of half the length of the original one. The frequencies of the resonances that remain are those of a pipe of half the length.

The acoustics of this double open-ended pipe are essentially those of the flute, which behaves as if it is a pipe that is open at both ends. (Obviously, the “far” end of the flute pipe is open. The hole that the player blows into at the other end is essentially the other open end of the pipe. A real Western flute also has a small closed “stub” of pipe to the left of this hole, but that is primarily there to give a minor correction to the tuning of some of the notes (it technically forms what is called a Helmholtz resonator.) If a flute player covers all the holes in the flute to play the lowest note, and then opens the hole nearest to the player’s mouth or the “blowing” end of the flute – a hole that is approximately half way down the pipe – that causes the frequency of the sound to double, changing the pitch by an octave musically. This is a standard part of flute playing technique.)

Supplementary question

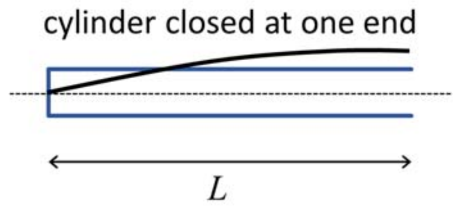
(i) What would be the frequencies of the resonant modes for a cylinder of length L that is closed at one end but open at the other, as shown in cross-section below?



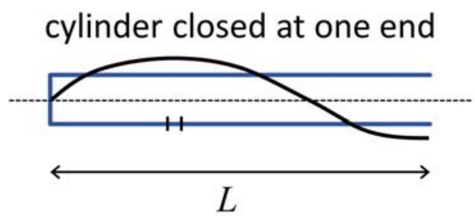
(ii) Where now would you put a hole in the side of the cylinder to select just a subset of the modes of this system?

Solution to supplementary question

(i) In this case, we will have a velocity minimum at the closed end and a velocity maximum essentially at the open end. The lowest frequency mode that will fit in here is one corresponding to a quarter wave, as sketched below. (Velocity is the quantity sketched, as before.)



Higher frequency modes require we add complete half waves to retain the velocity maximum. The sketch below shows the second mode of this cylinder.



The resonant frequencies of this cylinder are, therefore

$$f_n = (2n-1)v/4L$$

where $n=1$ is the first mode and so on.

(ii) In this case, cutting a hole 1/3 of the way along the pipe from the closed end will have essentially no effect on the second mode, but will suppress the first mode. This closed pipe represents the acoustics of the clarinet. The reed and mouthpiece correspond effectively to a closed end on the pipe. To change "registers" or modes on the clarinet, a register key is added, which opens a hole approximately 1/3 of the way along the pipe (at least for the "middle" note in the second "register" of the clarinet. Note also that the frequency of the lowest mode of the clarinet is a factor of two lower (an octave lower) than that of a flute of the same length. (This larger separation of the lowest register and the next register, basically a factor of 3 in frequency, is the reason why the clarinet needs extra holes opened by keys towards the top of the clarinet and extra holes closed by keys at the bottom of the clarinet – basically, we don't have enough fingers to play the instrument otherwise.)