Quals Question

Let the random variable X have CDF F. Suppose that X_i iid $\sim X$.

- (a) What is the CDF of $\max_{1 \le i \le n} X_i$?
- (b) What is the CDF of $\min_{1 \le i \le n} X_i$?

Suppose now that $X_{i,j}$ iid $\sim X$

- (c) What is the CDF of $f(n,m) = \max_{1 \le i \le n} \min_{1 \le j \le m} X_{i,j}$?
- (d) Suppose $X \sim \text{exponential}(\lambda)$. Let $Y_m = f\left(e^{\beta m}, m\right)$, for some parameter $\beta > 0$. What does Y_m converge to as $m \to \infty$? In what sense?
- (e) Repeat the previous part for the general case $X \sim F$ (can assume that F is continuous and strictly increasing).