Alternatively, if a signal $e^{j2\pi k f_0 n}$; $n = 0, 1, 2, \ldots$ is put into the system, the output must be $H(kf_0)e^{j2\pi k f_0 n}$, where $H(kf_0)$ is the discrete-time Fourier transform of the Kronecker delta (discrete-time impulse) response $h_k = r^k$, $k = 0, 1, 2, \ldots$:

$$H(kf) = \sum_{n=0}^{\infty} r^k e^{-j2\pi k f_0} = \frac{1}{1 - r e^{-j2\pi f_0}},$$

which yields the same answer.