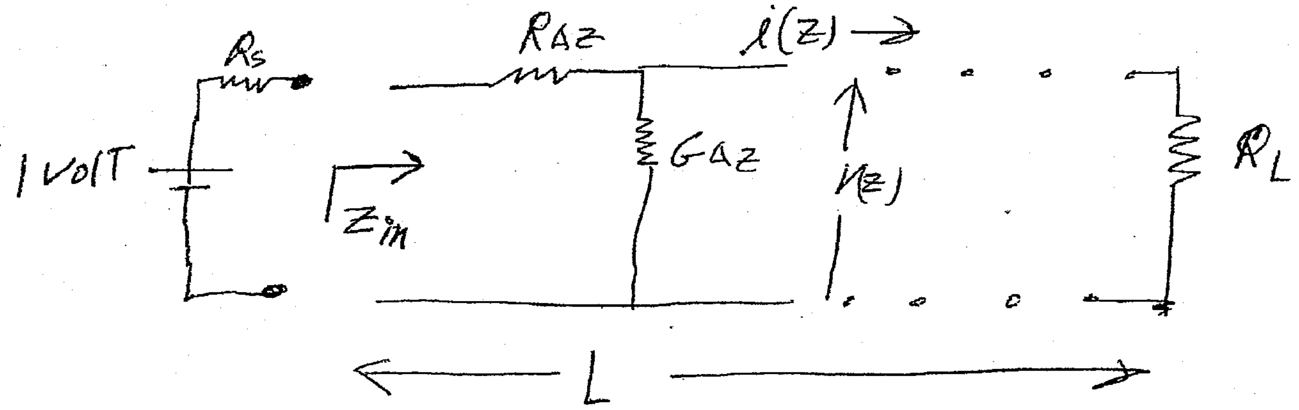
Quals (2007)
Prof. S.E. Harris

Consider a line with distributed resistance R and conductance G connected to a load R_L, with length L



With the source disconnected find the input impedance Z_{in}.

Note that the line is completely described by the differential Equations:

$$\frac{dV}{dz} = -Ri$$
$$\frac{di}{dz} = -GV \qquad (1)$$

where $V(z)$ and $i(z)$ are the voltage and current as a function of position.

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Solution

$$\frac{d^2V}{dz^2} = -R\frac{di}{dz} = +RGV$$

with $\alpha = \sqrt{RG}$

$$V = V_1 \exp -\alpha z + V_2 \exp +\alpha z$$

$$i = -\frac{1}{R}\frac{dV}{dz}\ ; \quad \text{with} \quad \frac{\alpha}{R} = \sqrt{\frac{G}{R}} \equiv \frac{1}{Z_0}$$

$$i = \frac{V_1}{Z_0}\exp -\alpha z - \frac{V_2}{Z_0}\exp +\alpha z$$

$$Z_{in} \equiv \frac{V(-L)}{i(-L)}$$

— Use the b.c. at $z=0$ $\frac{V(0)}{i(0)} = R_L$

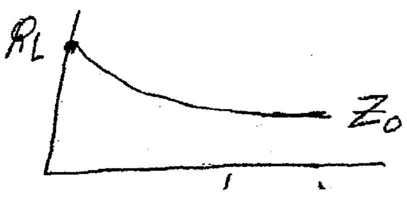
to find V_2/V_1 ;

with $R=G=1$ obtain

$$Z_{in} = \frac{R_L \cosh z - \sinh z}{\cosh z - R_L \sinh z}$$ ← Nobody got this far

= Question: Plot Z_{in} as a function of L with attention to $L \to \infty$

Answer

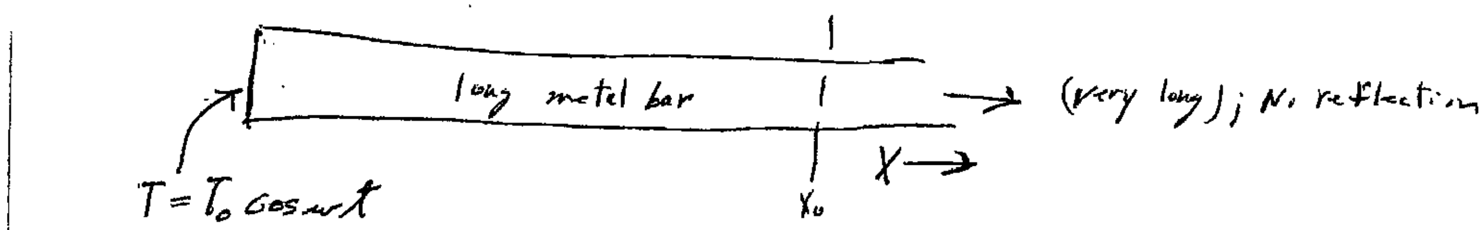


$Z_{in} \to Z_0$ when

EE Quals (2009)
S.E. Harris

The first portion of the exam was qualitative and took somewhat different directions with different students. The overall discusion was, in all cases, on properties of the one-dimensional wave equation as compared to the one dimensional heat flow (diffusion) equation.

We Then considered the following problem:



One end of a metal bar is held at a sinusoidally (steady state) temperature $T = T_o \cos \omega t$.

Making use of the heat equation

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial X^2}$$

Find The temperature as a function of time at The dashed line.

→ First recognize That any linear system driven by a sinusoid responds at The drive frequency. Therfore The waveform at $X = X_o$ will differ, at most, in amplitude and phase from That at the boundary.

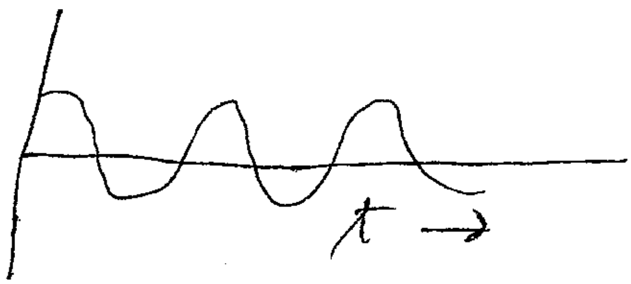
Assume $T \sim \exp j\omega t \, \exp -\gamma X$

$$j\omega T = K\gamma^2 T$$

$$\gamma^2 = j\omega / K$$

$$\gamma = \pm \frac{(1+j)}{\sqrt{2}} \left(\frac{\omega}{K}\right)^{1/2} = \alpha + j\beta$$

$$T = \mathrm{Re}\left[T_0 \exp j\omega t \, \exp -\alpha X \, \exp -j\beta X \right]$$



at $X = X_0$, the amplitude is reduced and there is a phase change.

Now assume that a rectangular temperature pulse is applied at $X = 0$;

how does it look at $X = X_0$

why ~~does it~~ does the pulse broaden

$$\alpha \sim \sqrt{\omega}$$

higher frequencies decay more rapidly with distance.

EE Qualifying Exam (2011)
S.E. Harris

1 Note The similarity of coulomb's law and The gravitational law; i.e. $\bar{E}=\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\Phi}{r^2}\,\bar{a}_r$ and $\bar{g}=-G\frac{m_1}{r^2}\,\bar{a}_r$

what is $\bar{g}$ at The center of a (spherical) earth? why?

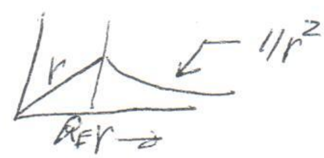
ans: $g=0$

2 Derive a Gauss's law equivalent for gravity

ans: $\bar{\nabla}\cdot\bar{g} = -4\pi G\rho_m$; ρ_m = mass density

3 Find The functional form of $\bar{g}$ versus distance from earth center

ans:



4 If the earth were an ellipsoid of revolution how would you do The problem

ans: direct (3D) vectorial integration

5 write the functional form for the escape velocity as a function of ~~pq~~ position above the earth's surface.

ans $V_{escape} \sim \sqrt{\frac{1}{r}}$

6 for a few very fast students, what law would you use To study a rocket where the mass is changing?

ans: conservation of momentum.