Qualifying Exam 2011 Engineering Physics, Shanhui Fan

As a reminder, the time-dependent Schodinger equation of electrons:

$$i\hbar \frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \phi}{\partial x^2} + V(x)\phi$$

and the time-independent Schodinger equation of electrons:

$$E\phi = -\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + V(x)\phi$$

(a) Suppose an electron is confined in an infinite potential well

$$V(x) = \begin{cases} 0, \ 0 < x < a \\ \infty, \text{ everywhere else} \end{cases}$$

sketch the ground state $\phi_0(x)$ and the first excited state $\phi_1(x)$ for the electron in the potential well. Provide the eigen-energy of these two states.

(b) Suppose at t = 0, the electron has a wavefunction $\phi_0(x)$, what is the electron wavefunction at a time t later?