Discussion/solution.

The system is constant norm if and only if

$$\begin{aligned} 0 &= \frac{d}{dt}\|x(t)\|^2 \\ &= 2x(t)^T\dot{x}(t) \\ &= 2x(t)^T Ax(t) \\ &= x(t)^T(A + A^T)x(t) \end{aligned}$$

for all $x(t)$, which occurs if and only $A + A^T = 0$, which is the same as $A^T = -A$, *i.e.*, A is skew-symmetric. There are many other ways to see this. For example, the norm of the state will be constant provided the velocity vector is always orthogonal to the position vector, *i.e.*, $\dot{x}(t)^T x(t) = 0$. This also leads us to $A + A^T = 0$.

Another approach uses the state transition matrix e^{tA}. The system is constant norm provided e^{tA} is orthogonal for all $t \geq 0$. From here, you'd have to argue that A must be skew-symmetric.

The system is constant speed if and only if

$$\begin{aligned} 0 &= \frac{d}{dt}\|\dot{x}(t)\|^2 \\ &= \frac{d}{dt}\|Ax(t)\|^2 \\ &= 2(Ax(t))^T A\dot{x}(t) \\ &= 2x(t)^T A^T A^2 x(t) \\ &= x(t)^T A^T(A + A^T)Ax(t) \end{aligned}$$

for all $x(t)$, which occurs if and only $A^T(A + A^T)A = 0$. In other words, the matrix $A^T A^2$ is skew-symmetric.

We see that if a system is constant norm, then it must be constant speed, since $A + A^T = 0$ implies that $A^T(A + A^T)A = 0$.

But the converse is false, as the simple system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x,$$

which is a double integrator, shows. This system has trajectories of the form

$$x(t) = \begin{bmatrix} x_1(0) + tx_2(0) \\ x_2(0) \end{bmatrix}.$$

It doesn't have constant norm, but it does have constant speed, since $\dot{x} = (x_2(0), 0)$.