Quals Question — Tom Cover
January 2008

Question 1: Gambling Scheme

Gamble \$1 each day on fair gambles. Stop when first ahead by \$1.

$$X_i = \begin{cases} 1, & \frac{1}{2} \\ -1, & \frac{1}{2} \end{cases}, \; X_i \text{ indep}$$

$$S_N = X_1 + X_2 \ldots + X_N = 1$$

$$N = \text{ stopping time}$$

1a. Are you eventually \$1 ahead (is $\Pr\{N < \infty\} = 1$)?

1b. Is $EN < \infty$, or $= \infty$? (How long does it take?)

Question 2

2a. Let X, Y be independent and identically distributed. What is $E\{X|X+Y\}$?

2b. Now let the joint distribution be arbitrary. Is

$$E[(X - E[X|X+Y])^2|X+Y] = E[(Y - E[Y|X+Y])^2|X+Y]?$$

Answers

1a. Yes, $\Pr\{N < \infty = 1\}$. You are sure to be \$1 ahead eventually.

1b. $EN = \infty$. The expected waiting time is infinite. (You are making money at the rate of (\$1)/(EN) trials and this rate can't be positive for fair gambles.)

2a. $E\{X|X+Y\} = ?$

Note: $E\{X|X+Y\} = E\{Y|X+Y\}$ since $f(x,y)$ is symmetric.

Note:

$E\{X|X+Y\} + E\{Y|X+Y\}$

$= E\{(X+Y|X+Y\}$

$= X+Y$

Thus $E\{X|X+Y\} = \frac{X+Y}{2}$

2b. Yes, they are equal. We note that

$(X - E\{X|X+Y\}) + (Y - E\{Y|X+Y\})$

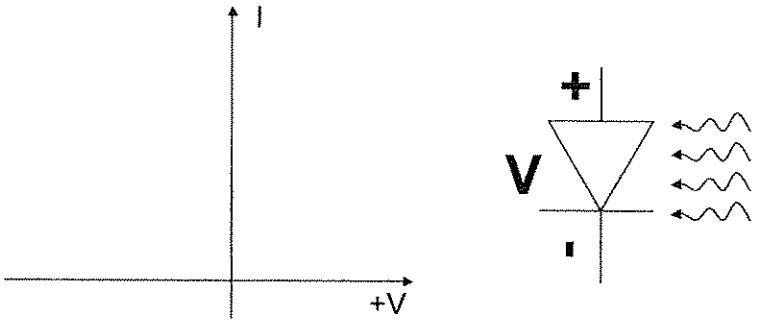
$= X+Y - E\{X+Y|X+Y\}$

$= X+Y-(X+Y) = 0$

Thus

$(X - E(X|X+Y))^2$

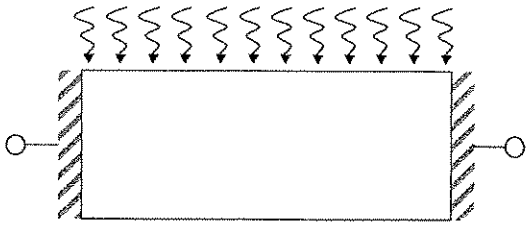
$= (Y - E\{Y|X+Y\})^2.$



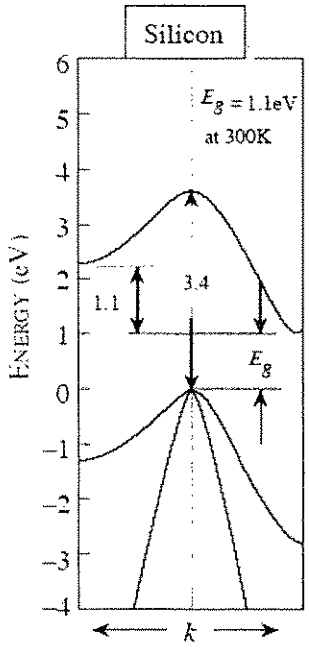
•Draw the I-V with and without light

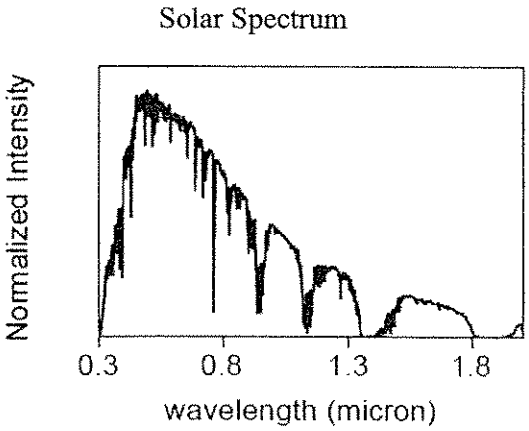
•Where do you operate the device for optimum power out (solar cell)

•What are the physical constraints between light in and electrons out (physical effects that must be considered)



•Sketch how the diode looks inside (doping etc.)
•For negative bias sketch the carrier profiles versus light everywhere through the device
•How do these distributions change with biasing (I.e. what do they look like at the "optimum" bias point)
•What determines the maximum voltage that can be measured
•How can you get more voltage





(a) For the sunlight, which part of the spectrum will be absorbed by silicon? (Note: a 1eV photon has a free space wavelength 1.24 micron.)

(b) Is Si a direct-bandgap semiconductor, or an indirect bandgap semiconductor?

(c) What is the difference in terms of optical properties, between these two classes of semiconductors? Why?

(d) What material systems are typically used to create a semiconductor laser?

(e) Any idea that you have that can make a silicon-based semiconductor laser?