Solution 2 In part 1 we found the formula for T as a function of R, C, and V. Because R, C, and V are independent,

$$E[T] = E\Big[-RC\ln\Big(1-\frac{V_1}{V}\Big)\Big] = E[R]E[C]E\Big[-\ln\Big(1-\frac{V_1}{V}\Big)\Big] = R_0C_0E\Big[-\ln\Big(1-\frac{V_1}{V}\Big)\Big].$$

The uniform pdf for V leads to the following integral.

$$E[T] = R_0C_0E\Big[-\ln\Big(1-\frac{V_1}{V}\Big)\Big] = \frac{R_0C_0}{2\delta V_0}\int_{V_0(1-\delta)}^{V_0(1+\delta)} -\ln\Big(1-\frac{V_1}{v}\Big)\,dv$$

The integrand is not defined when $V < V_1$, so $E[T]$ is undefined (or infinite) when $\delta \ge e^{-1}$.

For completeness, we perform the integration. This was *not* a requirement for the problem. From integration by parts, tables of integrals, or long term memory we obtain

$$\int \ln x \;=\; x\ln x - x\,.$$

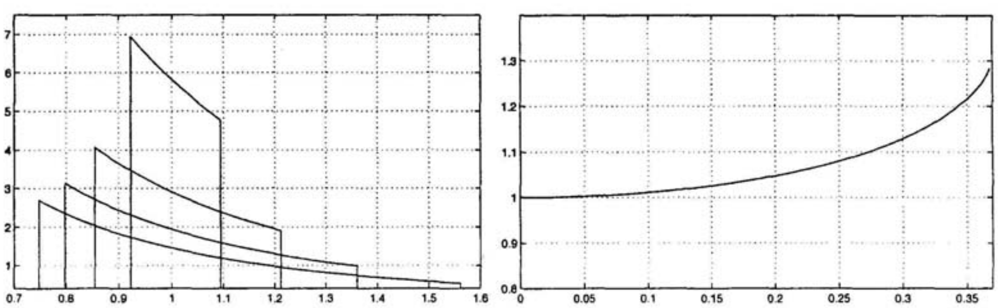
Next we find the *indefinite* integral needed for $E[T]$ (additive constants can be ignored).

$$\begin{aligned}\int \ln\Big(1-\frac{V_1}{v}\Big)\,dv \;&=\; \int \ln\frac{v-V_1}{v}\,dv \;=\; \int \Big(\ln(v-V_1) \,-\, \ln v\Big)\,dv\\ &=\; (v-V_1)\ln(v-V_1) - (v-V_1) - v\ln v + v \;=\; (v-V_1)\ln(v-V_1) - v\ln v\,.\end{aligned}$$

The final answer has a closed form but no obvious simplications.

$$\begin{aligned}E[T] \;&=\; \frac{R_0C_0}{2\delta V_0}\int_{V_0(1-\delta)}^{V_0(1+\delta)} -\ln\Big(1-\frac{V_1}{v}\Big)\,dv = \frac{R_0C_0}{2\delta V_0}\Big(v\ln v \,-\, (v-V_1)\ln(v-V_1)\Big)\Big[_{V_0(1-\delta)}^{V_0(1+\delta)}\\ &=\; \frac{R_0C_0}{2\delta V_0}\Big(V_0(1+\delta)\ln(V_0(1+\delta)) \,-\, V_0(1-\delta)\ln(V_0(1-\delta)) \,-\\ &\qquad (V_0(1+\delta) - V_1)\ln(V_0(1+\delta) - V_1) \,+\, (V_0(1-\delta) - V_1)\ln(V_0(1-\delta) - V_1)\Big)\end{aligned}$$

The left graph shows the conditional pdfs of T given $R=1$, $C=1$ for $\delta = 0.20, 0.15, 0.10, 0.05$ (left to right). The right graph plots the expected value of T as a function of δ for $0 < \delta < e^{-1}$.



The pdf and mean of T are not defined for $\delta \ge e^{-1}$. It is somewhat surprising that as $\delta \to e^{-1}$ the mean of T converges to a finite number $\frac{1}{2}(e+1)\ln(e+1) - \frac{1}{2}(e-1)\ln(e-1) - \ln 2 = 1.2833$