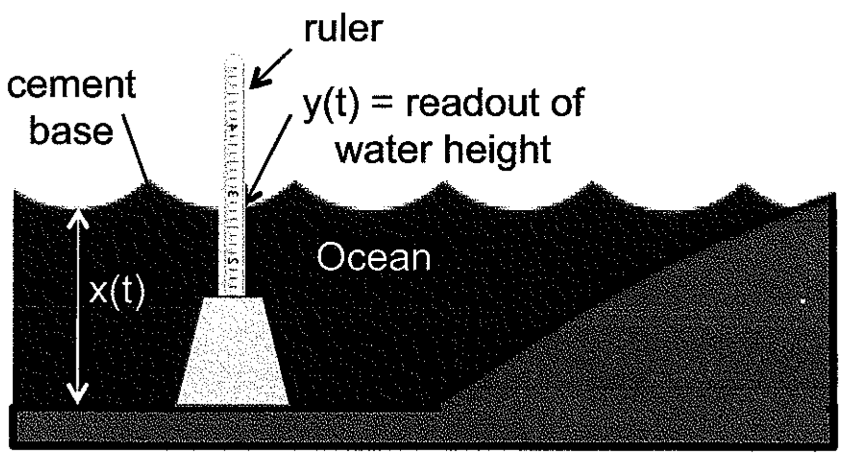
2010 EE PhD Quals

Prof. Daniel Spielman

Your goal is to build a system to measure the tidal fluctuations of the ocean.

Consider the following device:



1. Is this device linear and time-invariant? If so, sketch the frequency response.

2. What are the advantages and disadvantages of the design?

3. Can you design a better device?

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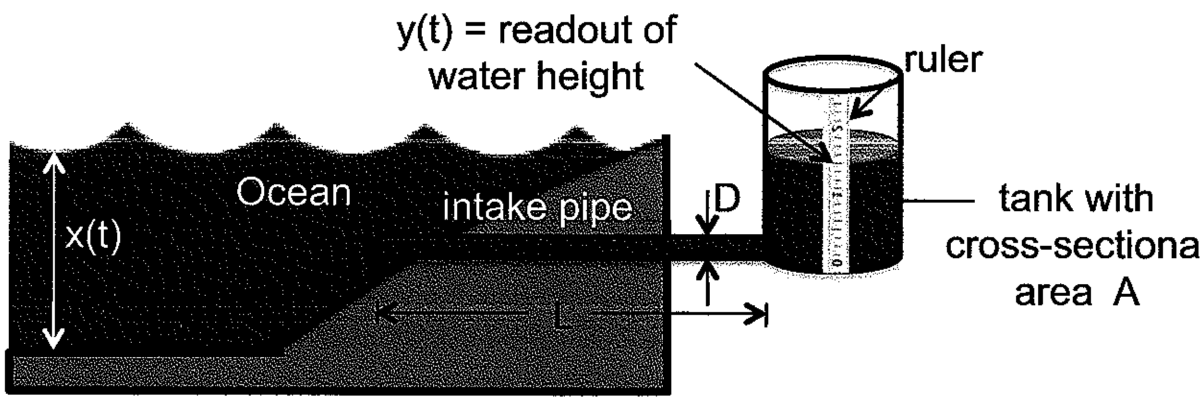
Consider the following alternative system:



4. Is this device linear and time-invariant? If so, sketch the frequency response.

5. Qualitatively describe the effects of:

   a) increasing A (cross-sectional area of the tank).

   b) increasing L (length of the intake pipe).

   c) increasing D (diameter of the intake pipe).

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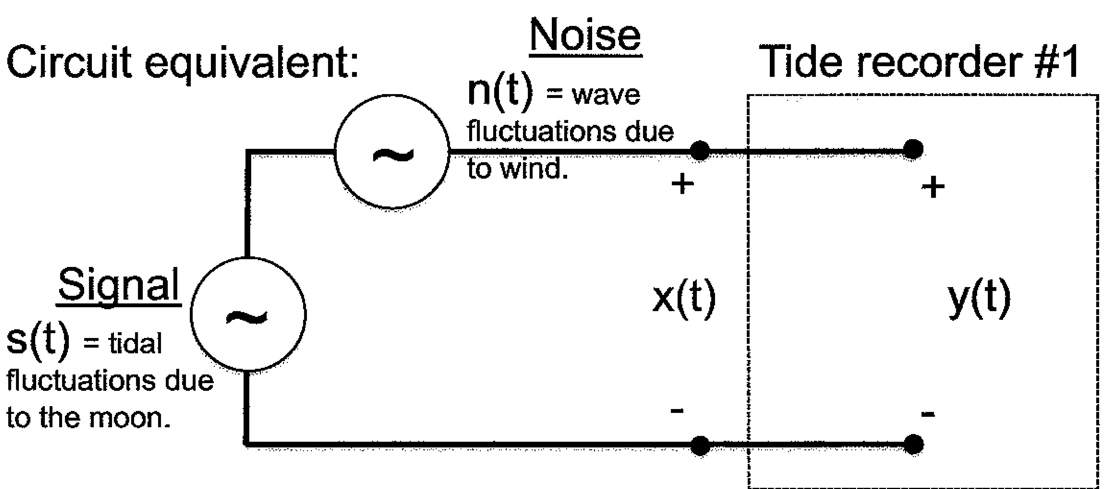
1. For this system, $y(t) = x(t)$, hence the device is linear and time-invariant. The frequency response is constant for all frequencies.

2. Advantages: inexpensive, easy to build, durable.

   Disadvantages: probably need a boat to actually make the measurements. More importantly, the data will be noisy. The measurements are subject to unwanted variations from surface wave fluctuations. An improved device would filter out this unwanted high frequency noise. Note: tidal fluctuations are on the order of 10^{-5} Hz, while wave action has a dominant component around 10^{-1} Hz.

Circuit equivalent:



3. There are lots of choices for better measurement systems which can incorporate the desired low-pass filtering. I like the device shown in Question 4.

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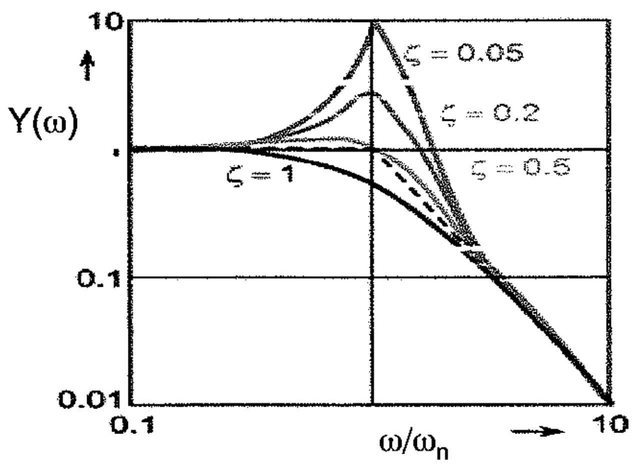
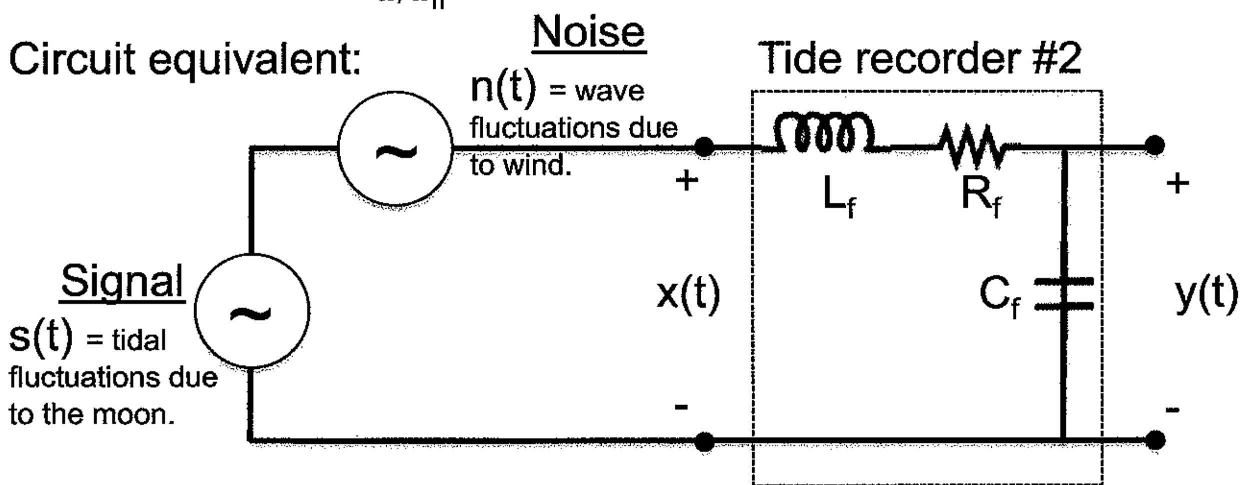
4. Device #2 is a low pass filter (equivalent to an LRC circuit). The frequency response would look something like (depending on the particular choices of A, L, and D):



In choosing A, L, and D, we want to avoid an underdamped system with a natural frequency in the range of ocean surface waves, typically on of the order of 0.1 Hz ($\omega_n = 1/\mathrm{sqrt}(L_fC_f)$ where L_f is the inertance [fluid analog of electrical inductance] and C_f is the fluid capacitance) . The damping ratio, ζ, equals $(R_f/2)\mathrm{sqrt}(C_f/L_f)$, where R_f is the fluid resistance. We want surface wave fluctuations >> ω_n and tidal variations << ω_n.

Circuit equivalent:



5a. Increasing A increases the fluid capacitance, thereby decreasing the natural frequency ω_n.

5b. Increasing L increases the fluid inertance (thereby decreasing the natural frequency ω_n) and increases the fluid resistance (thereby increasing the damping ζ).

5c. Increasing D decreases the fluid inertance (thereby increasing the natural frequency ω_n) and decreases the fluid resistance (thereby decreasing the damping ζ).

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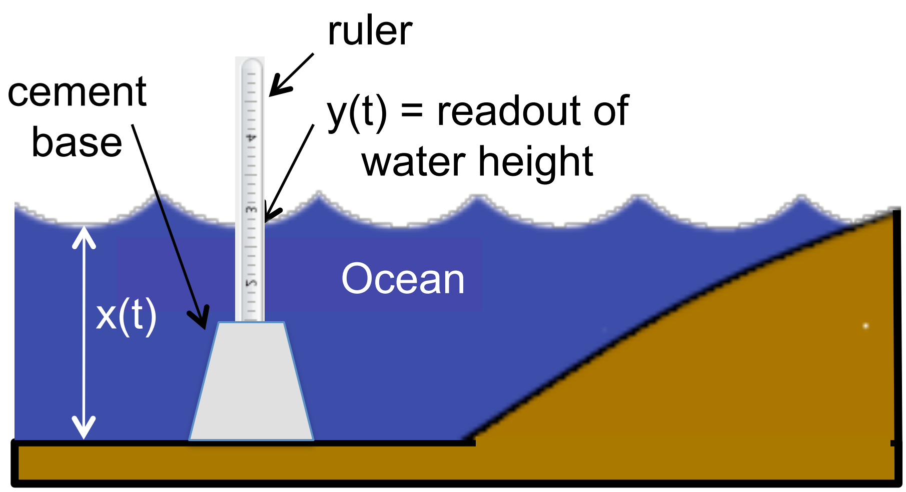

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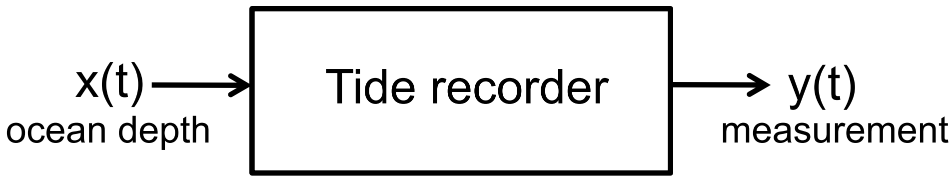

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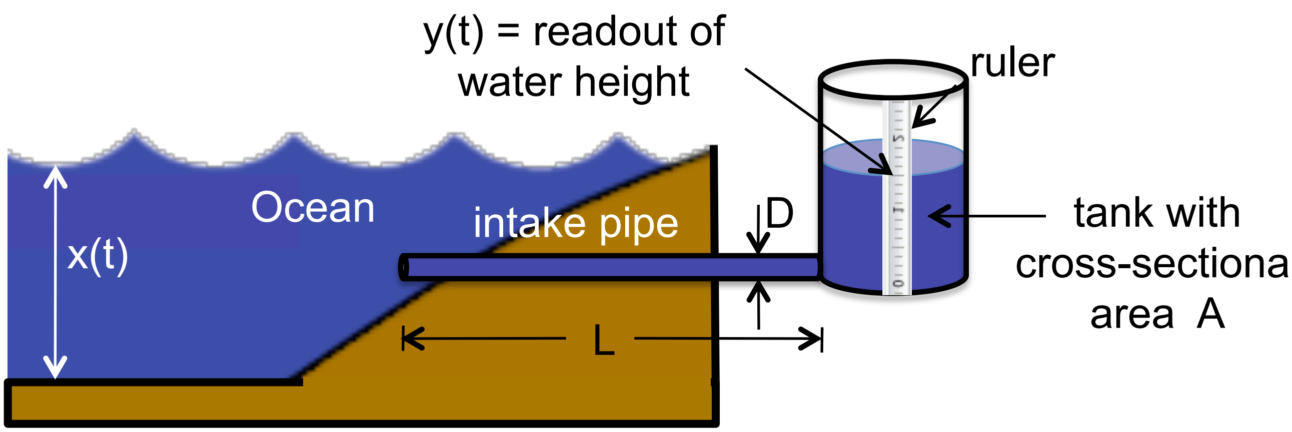

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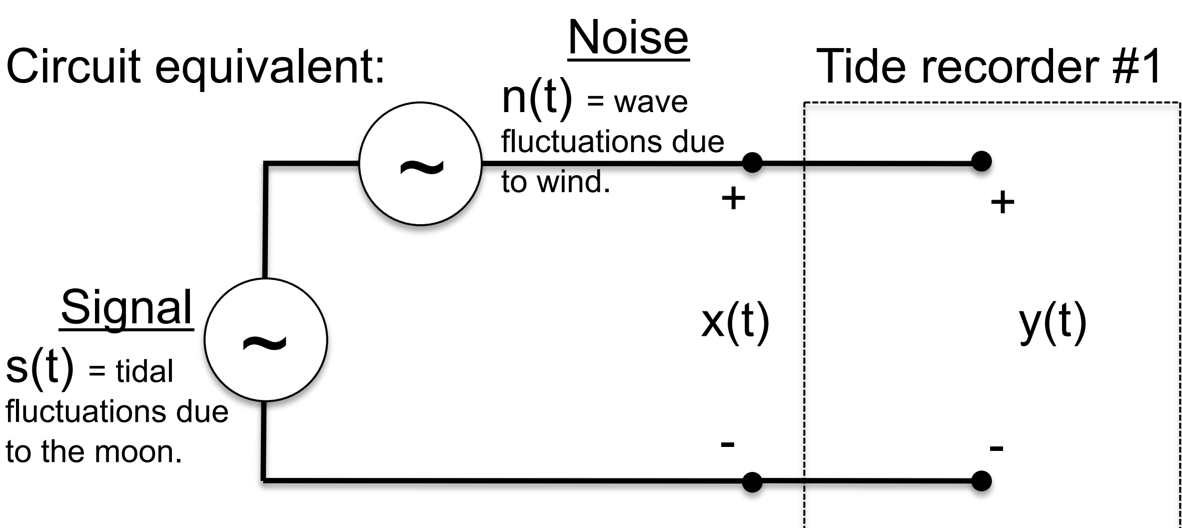

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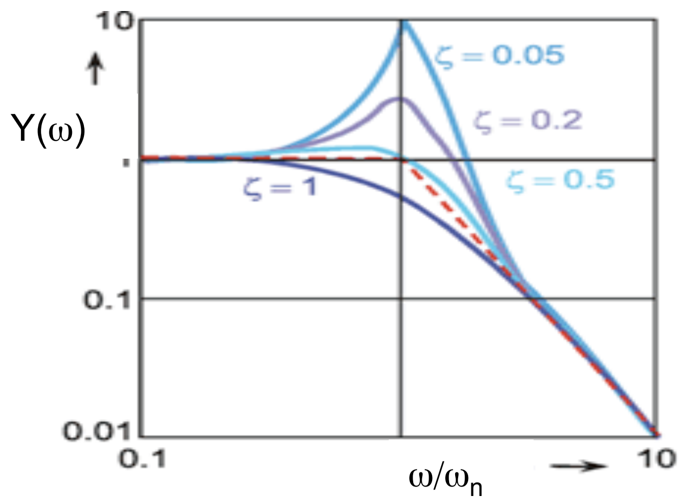

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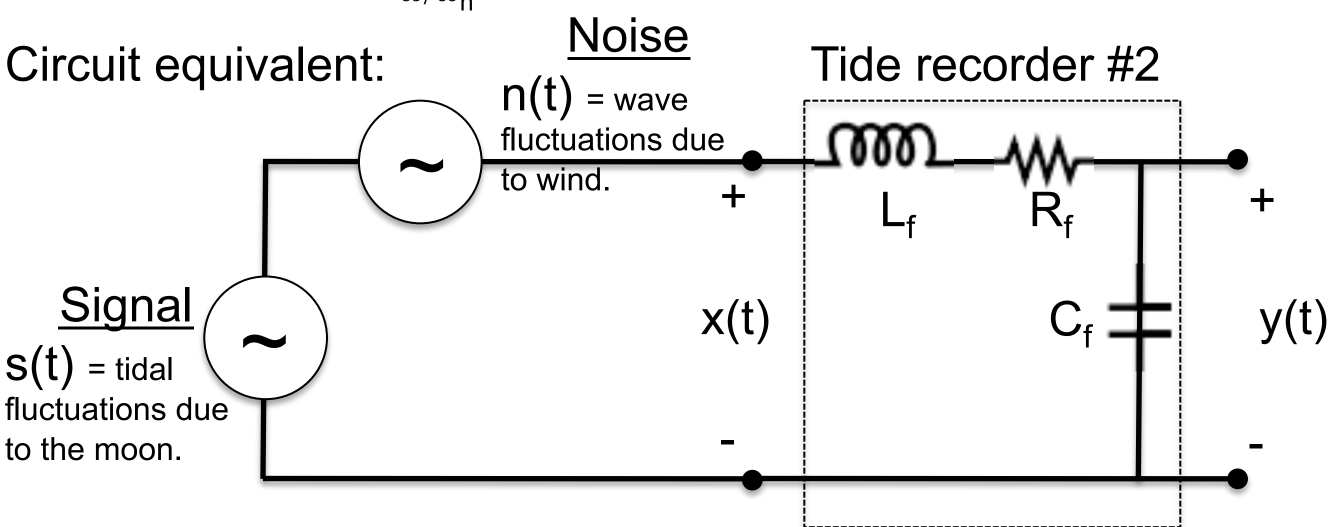
Circuit equivalent:



5a. Increasing A increases the fluid capacitance, thereby decreasing the natural frequency ω_n.

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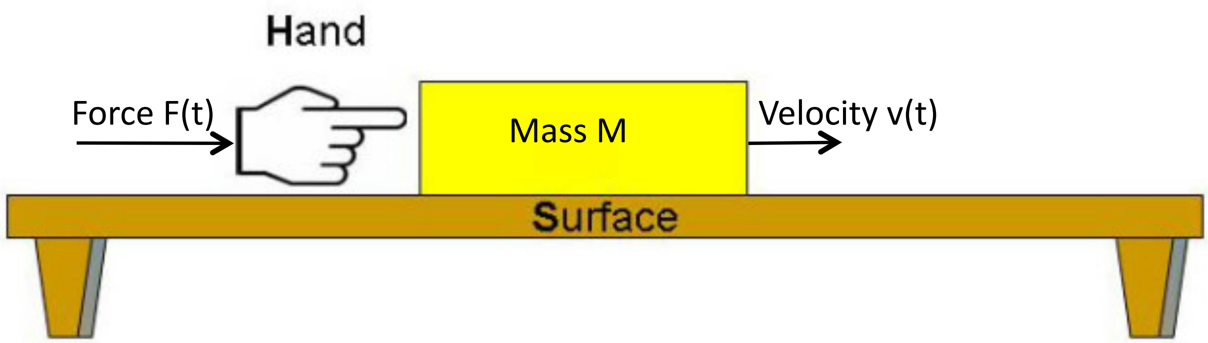
2013 EE PhD Quals

Prof. Daniel Spielman

Consider a stone block on a table:



1. Can you model this as a linear time-invariant system with force, $F(t)$, as the input and velocity, $v(t)$, the output?

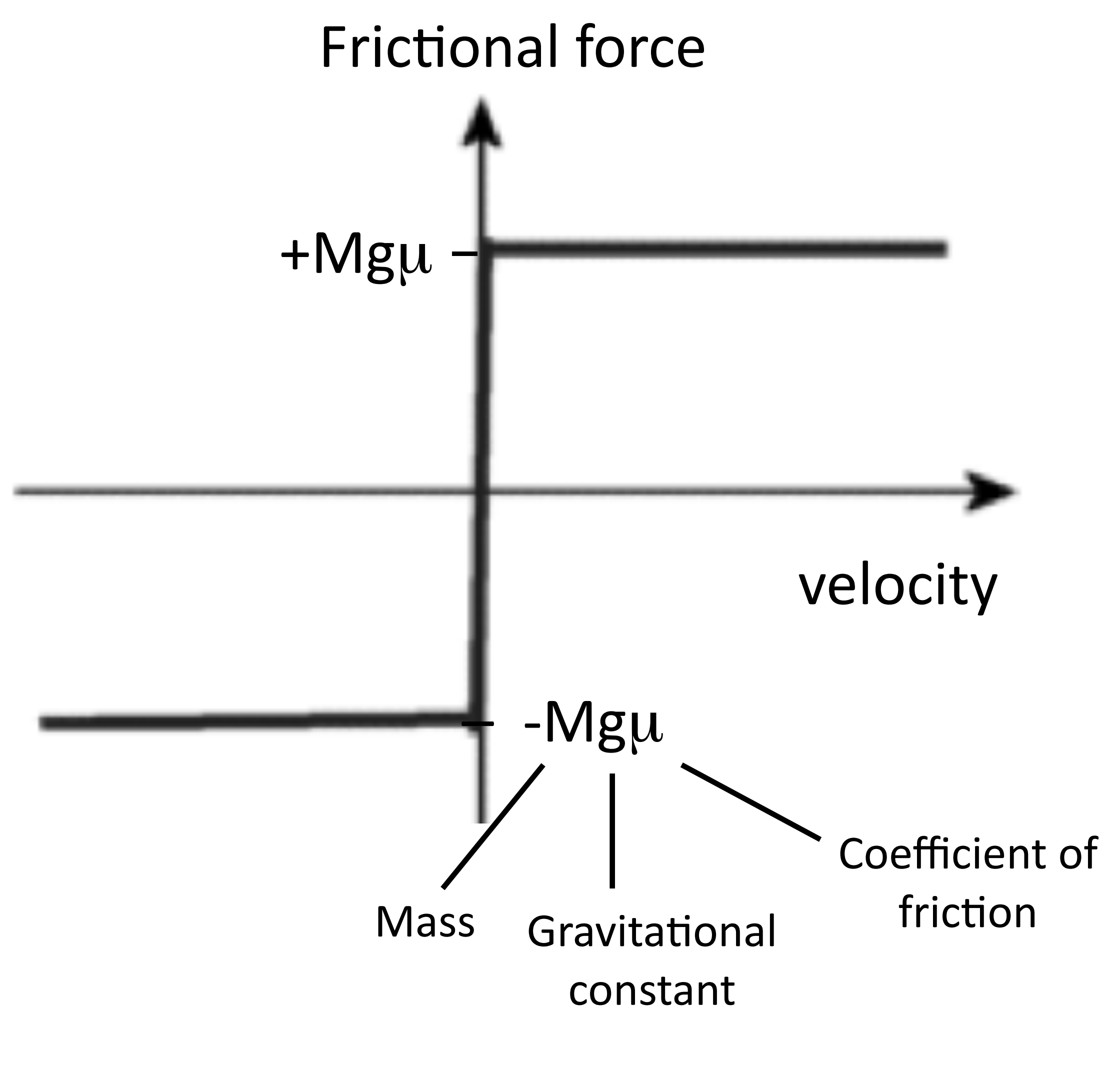
2. Is there an equivalent electrical circuit?

3. What is the impulse response?

4. How could you linearize either your model or the system?

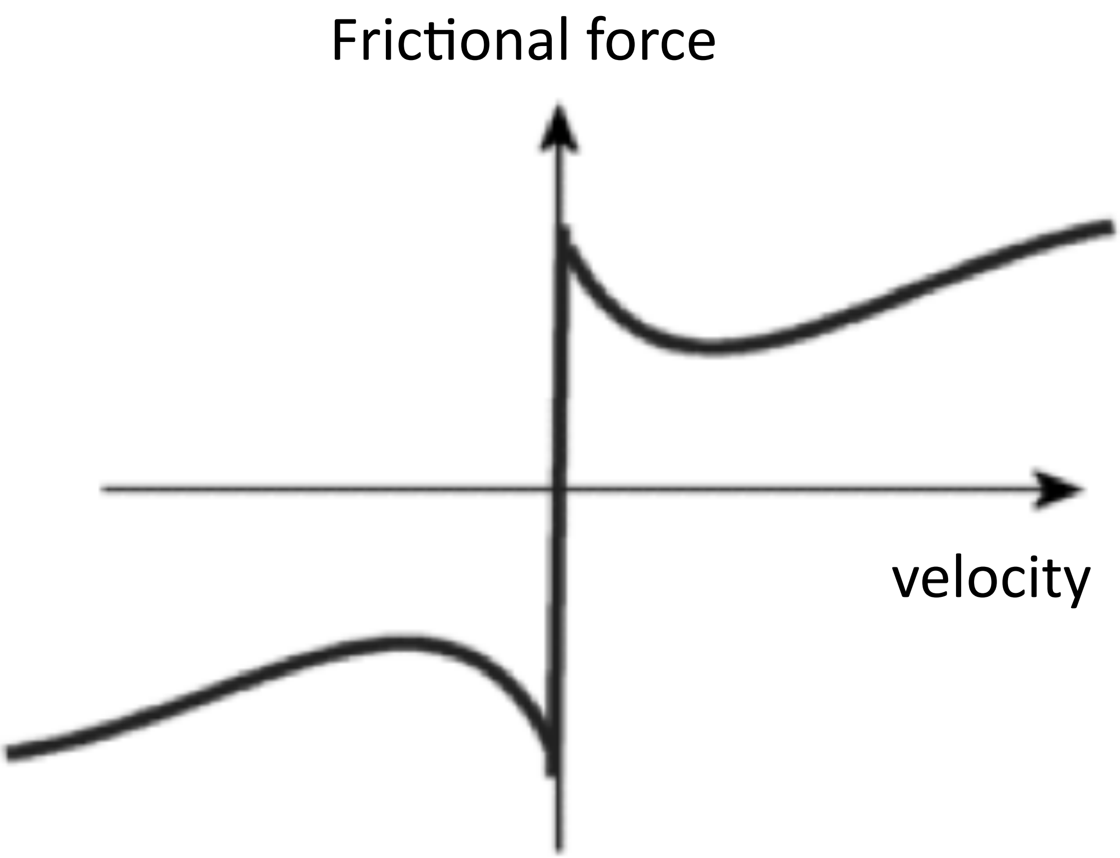
Hint:

A simple model of friction.



Hint #2:

Frictional model for a greased block.

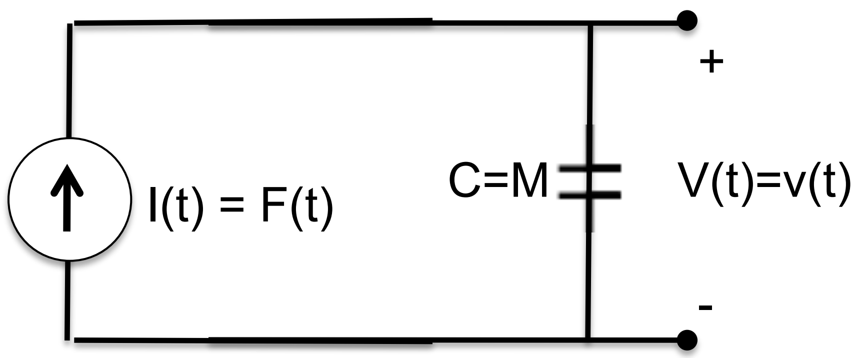


Answers

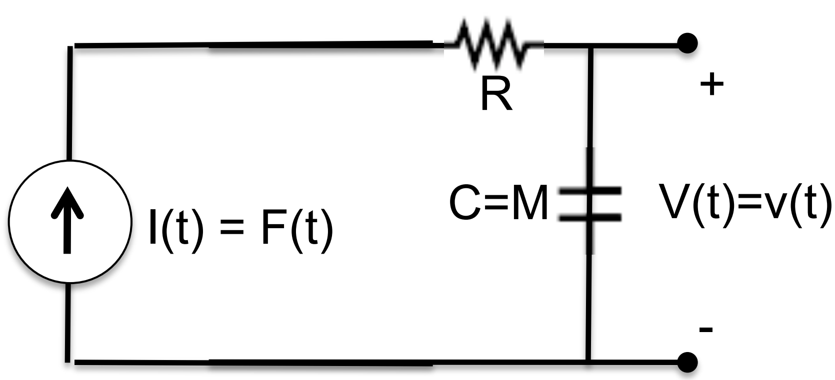
Without friction, the system is LTI an can easily be modeled as a simple integrator.

$$v(t) = \frac{1}{M}\int_{-\infty}^{t}\left(F(t')\right)dt'$$

For the equivalent circuit, let current, I(t), equal F(t) and the voltage, V(t), equal v(t), then the equivalent circuit is…



However, for a stone block on a table, there is friction, in which case the system is clearly nonlinear. Try pushing the block very lightly and observing the output. One might be tempted to try the following RC model.



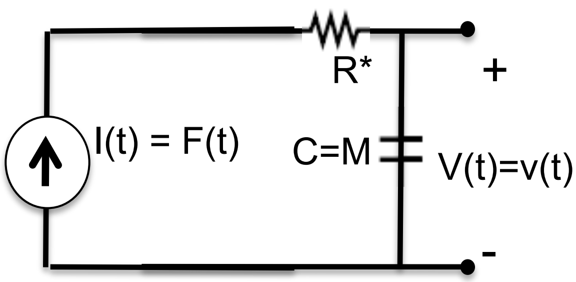
Unfortunately, a resistor is a device such that voltage and current are linearly proportional (V=IR), clearly not the case with respect to kinetic friction. Hence the RC model is wrong.

Answers (cont.)

Mathematically, the input/output relationship is given by:

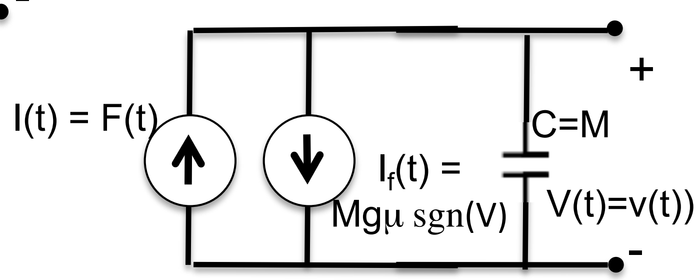
$$v(t) = \frac{1}{M}\int_{-\infty}^{t}\left(F(t') - Mg\mu\,\mathrm{sgn}(v(t'))\right)dt'$$

Using the simple model of friction given in Hint 1, the equivalent "circuit" would be:



where R* is a peculiar "resistor" in which I=sgn(V)/R instead of the usual I=V/R for which R=Mgμ.

Alternatively…

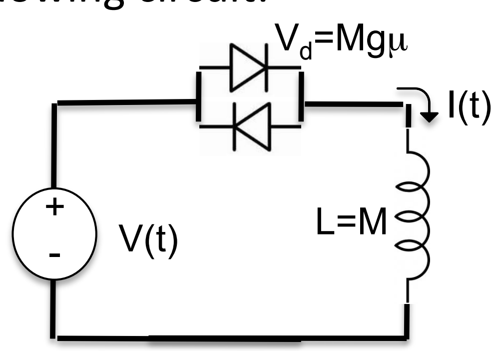


One clever student came up the following circuit:

Using the analogy,

$$\text{velocity } v(t) \leftrightarrow \text{ current } I(t)$$
$$\text{Force } F(t) \leftrightarrow \text{ voltage } V(t),$$

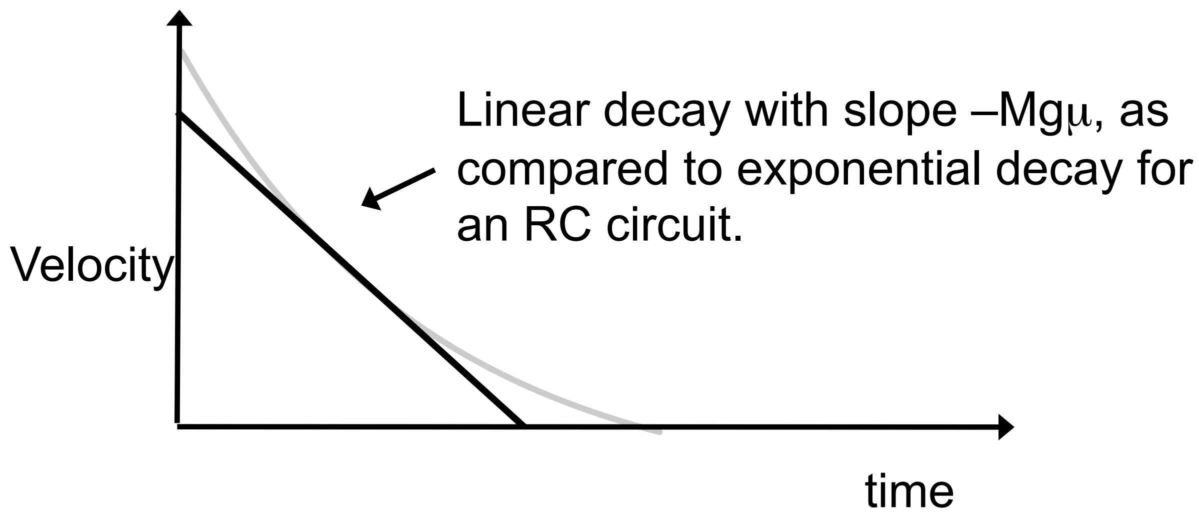
find I(t) as a function of V(t).



Note, even restricting our model to velocities > 0, this system is still not linear, but rather affine.

Answers (cont.)

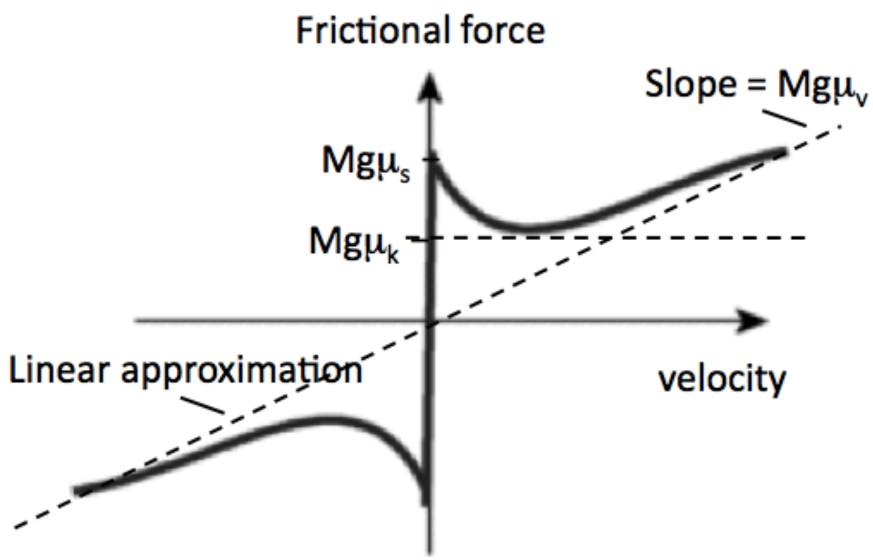
3. Plotting the impulse response (and yes the system does have an impulse response even though it is not LTI) is quite straight forward (just use F=Ma). For the case where the applied impulse is strong enough to cause the block to move…



For smaller impulses, the impulse response is simply zero.

4. One approach to linearize a system is to pick an operating point and try to linearize about that point. As mentioned before, picking, say, some non-zero velocity, v_0,technically gives you a affine (non-linear system). However, if, for example, we also restrict ourselves to $v(t) > 0$ and only large input forces, i.e. and $F(t) >> Mg\mu$, (or use $F'(t)=F(t)- Mg\mu$) then the system looks like a simple capacitor.

My favorite approach is to grease the block! Although no one actually came up with this solution, greasing changes the system from having kinetic/static friction to one having viscous friction for which force is indeed linearly proportional to velocity. Adding some labels to the frictional model for a greased block...



μ_s = cofficient of static friction
μ_k = cofficient of kinetic friction
μ_v = cofficient of viscous friction

Now, one reasonable model would be just to use the linear approximation drawn on the plot, i.e. model using a capacitor and a resistor. This model would be accurate for intermediate velocities, breaking down for velocities near zero and at very high velocities where fiscous friction, or drag, becomes proportional to v^2 not v.