2006 Quals J. Cioffi

TOTAL - 10 pts

Noise- 10 pts

A continuous uniform random variable is used to model the quantization error (or noise) of a digital-to-analog converter (DAC). This error is between $-\frac{\Delta}{2}$ and $+\frac{\Delta}{2}$.

a). What is the mean of this error. (0.5 pt)

b). What is the variance of this error (or energy of the noise)? (1 pt)

c). Two independent signals both occupy the same electrical medium and both were generated by identical DACs of the form in this exam. Provide the distribution, mean, and variance of the sum of the errors (or new noise). (2 pts)

d). Extend part c to 20 signals (say an unlicensed wireless band) to determine the distribution, mean, and variance. (3 pts)

e). Suppose all the DACs have $\Delta = .001$ and the signals themselves have unit energy. What is the signal to noise ratio for a single signal with no others occupying the medium. (1 pt)

f). Now suppose that all signals but one have very small transmit energy but their DACs continue to have the same noise even when zero signal is transmitted (the designer saved some money and did not care about quantization noise if not transmitting). What is the new SNR for that one signal? (1 pt)

g). Qualitatively describe what you expect would happen to the distribution of the noise if the situation in part f applied, but each of the DACs had a different Δ. (1.5 pts).

TOTAL - 10 pts

Random Variables, Processes and Linear Systems - 10 pts

A complex random variable x takes on the 4 values $\pm 1 \pm j$ with equal probability ($j = \sqrt{-1}$).

a). What is the mean value of x? (.5 pt)

b). What is the variance of x ? (1 pts)

Independent selections of this random variable at different discrete points in time, k, form the stationary random process x_k. Another random process is computed according to

$$y_k = \frac{x_k + x_{k-1} + x_{k-2} + x_{k-3}}{4} .$$

c). What does the process y_k approximate? (.5 pt)

d). Find $E[y_k]$. (1 pt)

e). Find the variance of y_k (1 pt)

f). How many distinct values are there for the random process y_k ? (1 pt)

(hint you may want to consider each of real and imaginary components)

g). What is the autocorrelation function of x_k, $r_{x,k} = E[x_n \cdot x_{n-k}^*]$? (1 pt)

h). What is the autocorrelation function of y_k ? (2 pts)

i). What is the probability distribution of y_k ? (2 pts)

(hint, see hint in f).

2008 Quals Solution

a). 0

b). $Ex^2 = Ex_r^2 + Ex_i^2 = 1 + 1 = 2$

c). mean or ave. value

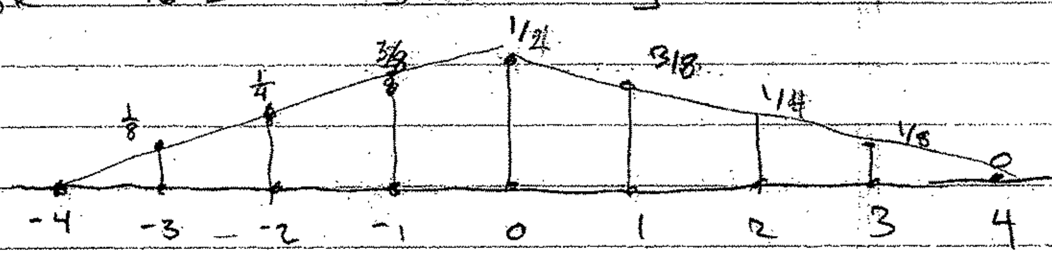
d). $Ey = 0$

e). $Ey^2 = \frac{1}{16}[2 + 2 + 2 + 2] = \frac{1}{2}$

f). $\pm 1, \pm \frac{1}{2}, 0$ for each component $5 \times 5 = \underline{25}$

g). $r_{x,k} = 2 \cdot \delta_k$

h). $r_{y,k} = \frac{1}{16}[1\ 1\ 1\ 1] * [1\ 1\ 1\ 1] * 2\delta_k$



i). ± 1 occurs 2 ways in either comp.

$\pm \frac{1}{2}$ " 8 " " " "

0 " 6 " " " "

$(0,0) = \frac{36}{256}$ $\quad (0, \pm\frac{1}{2}) = \frac{48}{256}$ $\quad (0, \pm 1) = \frac{12}{256}$

$(\pm\frac{1}{2}, 0) = 48/256$ $\quad (\pm\frac{1}{2}, \pm\frac{1}{2}) = \frac{64}{256}$ $\quad (\pm\frac{1}{2}, \pm 1) = \frac{16}{256}$

$(\pm 1, 0) = 12/256$ $\quad (\pm 1, \frac{1}{2}) = \frac{16}{256}$ $\quad (\pm 1, \pm 1) = \frac{4}{256}$

$\sum \rightarrow \frac{256}{256} = 1$ (checks)