Nishimura

Set-Up

This Wavetek function generator produces a signal that is sampled by the box behind it. The sampling box performs a simple sample-and-hold and its output is fed directly to a loudspeaker.

Question 1

Determine the sampling frequency of the sampling box by experimenting with the frequency and/or waveform of the function generator. Explain as you proceed.

Question 2

Set the function generator to a sinusoid at 10 Hz. What is causing the dominant tone that you hear and at what frequency is it?

Question 3

Sketch the magnitude of the frequency spectrum coming out of the loudspeaker.

Question 4

Set the function generator to about half the sampling frequency. Explain why you hear an oscillation in the loudness. What is the frequency of this oscillation? (Show me in the frequency domain what is going on).

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Satisfactory Responses

Question 1

To determine sampling frequency, should have looked for the input frequency where aliasing begins or where the sound goes away.

Question 2

Should have drawn a sketch in the time or frequency domain to determine the dominant frequency component. Dominant tone is at the sampling frequency.

Question 3

Should know how the zero-order hold affects the shape of the frequency spectrum. Spectrum is multiplied, not convolved, with a sinc.

Question 4

The wavering nature of the output is due to beating of two frequencies. Frequency-domain sketch clearly shows why the two frequency components exist. Time-domain explanation is possible too.

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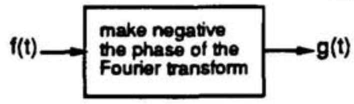
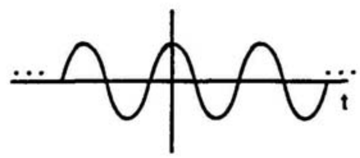
system

Problem 1

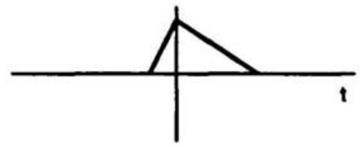


Input signal $f(t)$ and output signal $g(t)$ differ only in the phase of their Fourier transforms. The phases are the negative with respect to each other. Sketch the output for the following inputs:

a)



b)

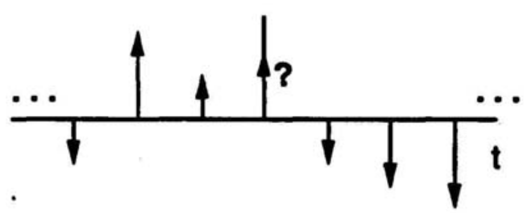


Is this system causal?

Is this system linear?

Is this system time-invariant?

Problem 2



A signal $f(t)$, bandlimited to ± 1 kHz, is sampled at the rates given below.

If the $t = 0$ sample is lost, is it possible to restore this sample point? If yes, describe how. If not possible to restore, why not?

a) Sampling rate = 5 kHz

b) Sampling rate = 2.5 kHz

c) Sampling rate = 1.25 kHz

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Solutions

Problem 1

Multiplying the phase of the Fourier transform by -1 amounts to conjugating the spectrum; hence,

$$G(s) = F^*(s) \ .$$

Therefore

$$g(t) = f^*(-t) \ .$$

If the input is real-valued, the output is a time-reversed version. The system is non-causal, non-linear, and time variant. If the input is constrained to be real valued, the system behaves linearly.

Problem 2

(a) Yes, one can throw away every other sample and still exceed the Nyquist frequency.

(b) Yes, model the "defective" sampling by $f(t)[1/T\,\text{comb}(t/T) - \delta(t)]$. The transform of this quantity is the original replicated spectrum shifted vertically based on the value $f(0)$. Therefore, $f(0)$ can be determined by evaluating the spectrum in the gaps between replication islands since the values there should be zero. A time domain approach is possible too.

(c) No, not possible because of aliasing.

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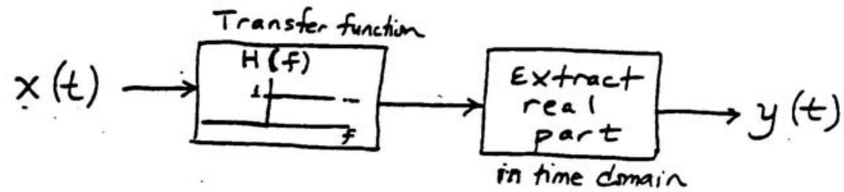
problem 1



1. Is this system linear?
2. Is this system time-invariant?
3. If $x(t)$ is real-valued, what is $y(t)$?

Answers

1) No. extracting real part is nonlinear (consider $i\,x(t)$)
2) Yes
3) $\frac{1}{2}x(t)$ time-domain or freq-domain approach ok

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Videotape Problem

A fan will start and eventually reach its fastest rotational speed. Consider what is happening when it reaches its fastest rotational speed. A marker (piece of tape) on 1 of the 3 fan blades will help you watch it.

Now Watch the Videotape

- At what speed is the fan rotating? Explain.

- Are there other rotational speeds that would give a video identical to what you have just seen (when the fan is rotating at its fastest speed)?

- Why does the fan appear to be nearly stationary in the video?

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Answers: Videotape Problem

- At what speed is the fan rotating? Explain.

Given a TV frame rate of 30 Hz and the fact that you see 3 markers appearing stationary, the fan is rotating at 10 Hz, one-third the sampling rate.

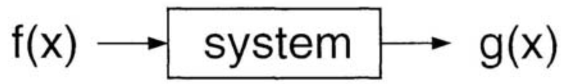
- Are there other rotational speeds that would give a video identical to what you have just seen (when the fan is rotating at its fastest speed)?

Yes, $(10 + 30n)$ Hz rotations would give the same video, frame by frame. Note that n could be a positive or negative integer. Note too that a 20 Hz rotation would appear to be the same ***perceptually***, but not on a frame-by-frame basis.

- Why does the fan appear to be nearly stationary in the video?

Because there are 3 fan blades and the rotation rate is one-third the sampling rate. If there were 2 fan blades and the rotation rate was one-half the sampling rate, it would also appear stationary.

Dwight Nishimura
system



Let $\mathcal{FT}\{f(x)\} = F(s)$ and $\mathcal{FT}\{g(x)\} = G(s)$,

Are the following systems linear? time-invariant?
What is the transfer function?

1.

$$G(s) = |F(s)|^2$$

2.

$$G(s) = F(-s)$$

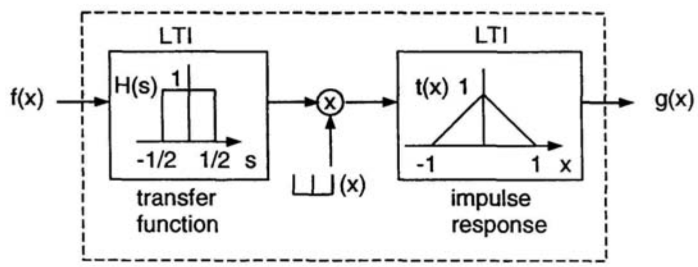
3.

$$g(x) = \frac{df(x-1)}{dx}$$

4.

$$g(x) = \sum_{n=-\infty}^{\infty} f(x-n)$$

5.



6.

$$g(x) = \int_{-\infty}^{x} f(u)\, du$$

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Answers

1. g(x) = autocorrelation of f(x). Nonlinear (consider 2 f(x)), Time-Variant (lose phase information).

2. g(x) = f(-x). Linear, Time-Variant (shift input one way, output shifts the other way).

3. Linear, Time-Invariant. H(s) = $j2\pi s\, e^{-i2\pi s}$

4. Linear, Time-Invariant. g(x) = f(x) * comb(x). H(s) = comb(s).

5. Linear (all elements linear), Time-Variant (If input shifted, linear phase in s-domain that should result does not because of replication islands that are not filtered out. Also, interpolation filter does linear interpolation. So consider a slightly shifted version of the input. The piecewise linear output cannot be a shifted version of the original piecewise linear output. Note that if the interpolation filter is a sinc interpolator of appropriate bandwidth, the system is LTI).

6. Linear, Time-Invariant. g(x) = f(x) * step(x). H(s) = 1/2 $\delta(s)$ - i/2πs.

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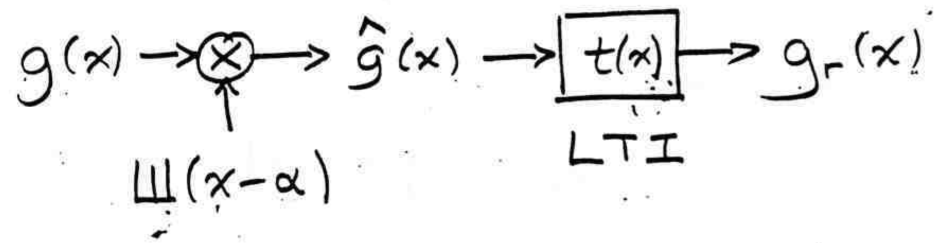
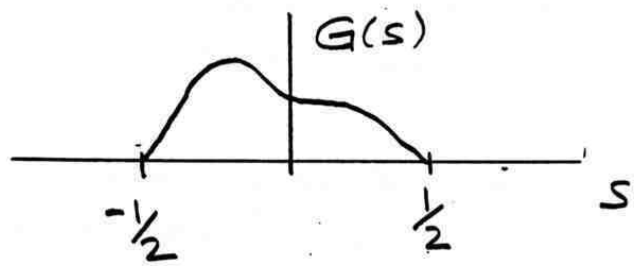
System

The Effect of Shifting the Sampling Function

- $g(x)$ is sampled to produce $\hat{g}(x)$.
- The LTI system has impulse response $t(x) = \operatorname{sinc}(x)$ (unless noted) and produces the recovered signal $g_r(x)$.
- The sampling function is shifted by α.



For problem 1, let $\mathcal{F}\{g(x)\} = G(s)$ as shown below.



1. Let the measure of error between $g_r(x)$ and $g(x)$ be

$$\epsilon = \int_{x=-\infty}^{\infty} |g_r(x) - g(x)|^2 \, dx .$$

   (a) Will this error depend on the shift α?

   (b) Repeat part (a) but let $t(x) = \wedge(x)$.

2. For the previous question, will $g_r(x)$ depend on α?

3. Now consider $g(x)$ that is not bandlimited.
   Will the error ϵ depend on α?

ANSWERS

1a) No, $g_r(x) = g(x)$, independent of α

1b) No, $g_r(x)$ depends on α but ϵ will not

2) No for $t(x) = \mathrm{sinc}\, x$
   Yes for $t(x) = \Lambda(x)$

3) Yes. Both frequency-domain and time-domain explanations can be given.

NISHIMURA JAN. 2000

Sample and Interpolate to Recover the Signal

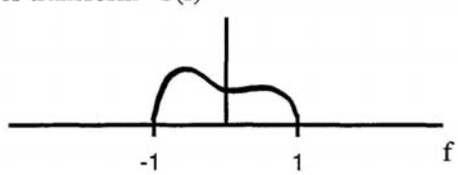
$$h(t) \longrightarrow \boxed{\text{sampler: frequency } f_S} \longrightarrow \hat{h}(t) \longrightarrow \boxed{\text{reconstruction operator}} \longrightarrow h_r(t)$$

For the following cases:

State the *lowest* sampling frequency f_S necessary to recover h(t) from $\hat{h}(t)$ (If possible)

Specify the required reconstruction operator.

1) h(t) = g(t), with Fourier transform G(f)



2) $h(t) = \dot{g}(t)$

3) $h(t) = g^3(t)$

4) $h(t) = g(t) \exp(-i2\pi t)$

5) $h(t) = g(t) \cos(6\pi t)$

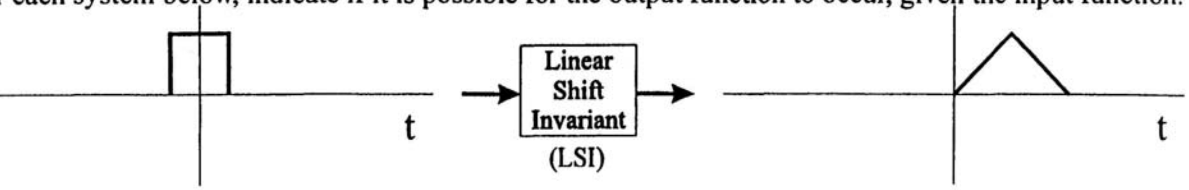
+ assorted questions throughout; e.g.,

2) how to reconstruct g(t)?

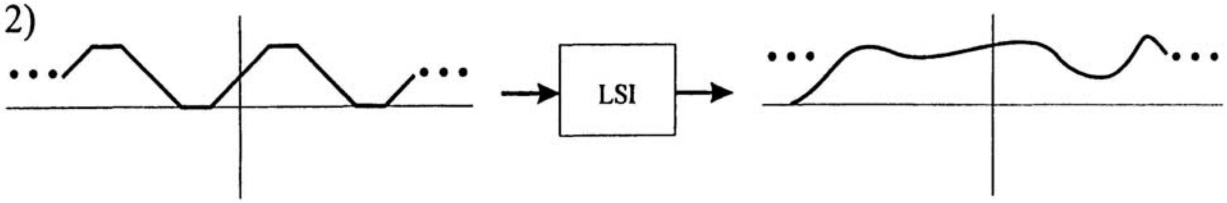
5) with your answer for fs, is h(t) recoverable if sampler delayed by some ε?

For each system below, indicate if it is possible for the output function to occur, given the input function.

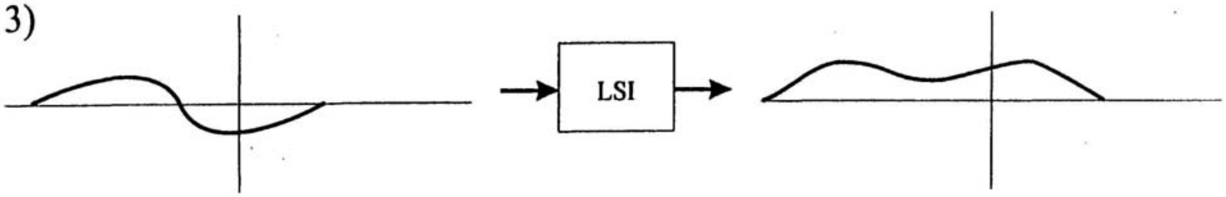
1)



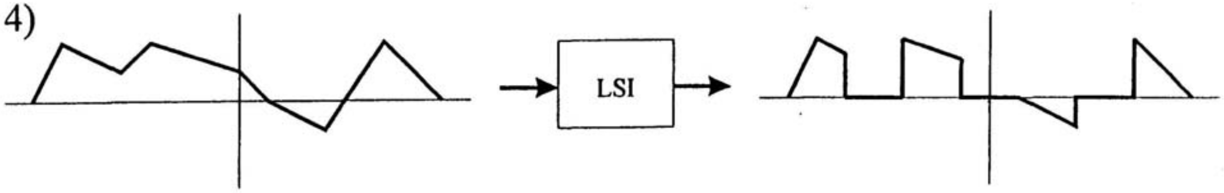
2)



3)



4)

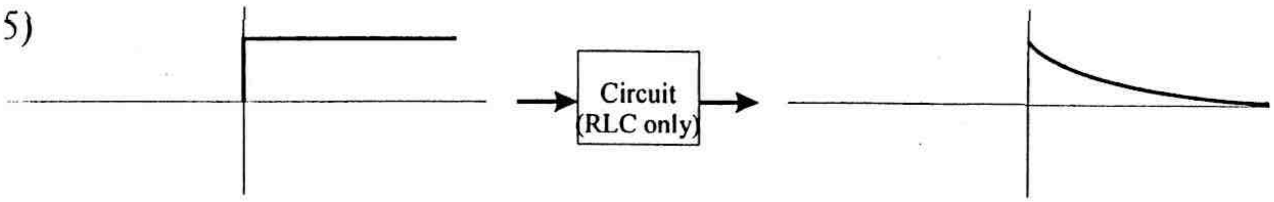


Quals 2001

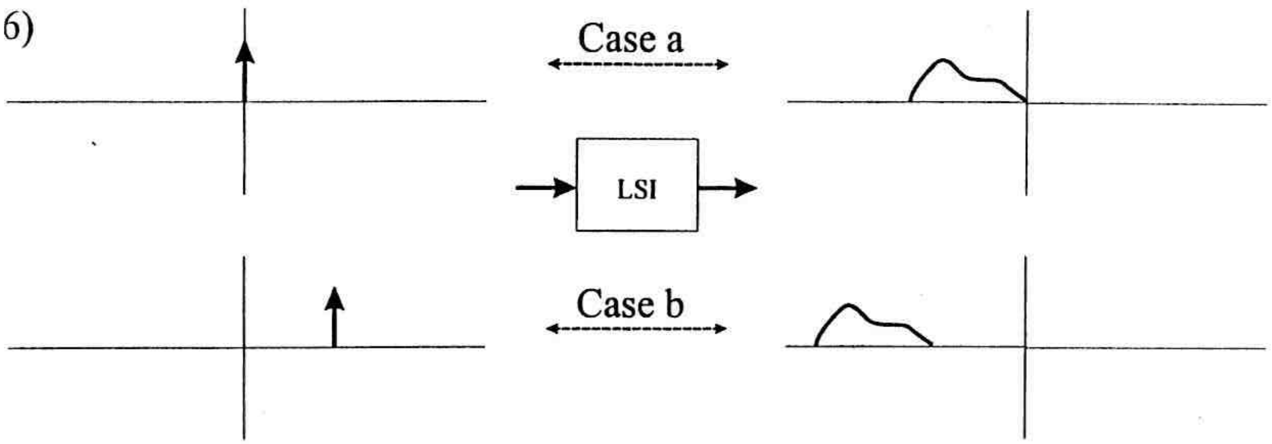
Nishimura

page 1

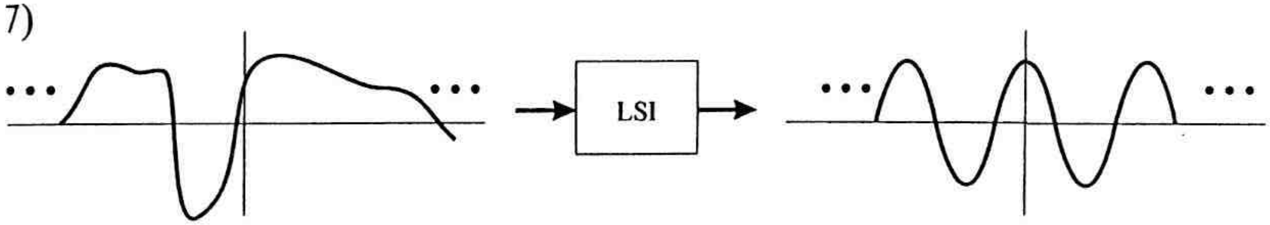
5)



6)



7)



Nishimura

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Quals Solutions

1. Yes, impulse response is a shifted rectangle.

2. No

3. No

4. No

5. Yes, impulse response is the derivative of the step response.

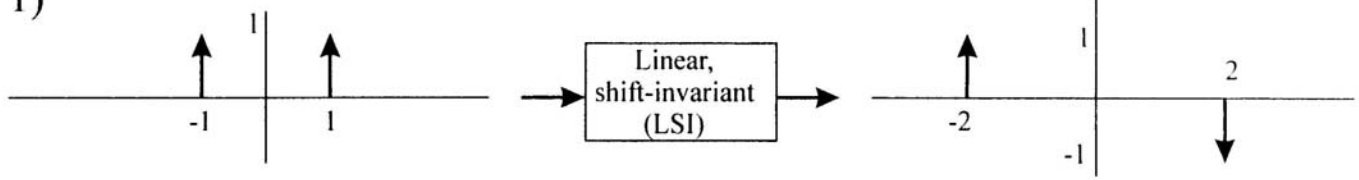
6. No

7. Yes, narrow bandpass filter would do it.

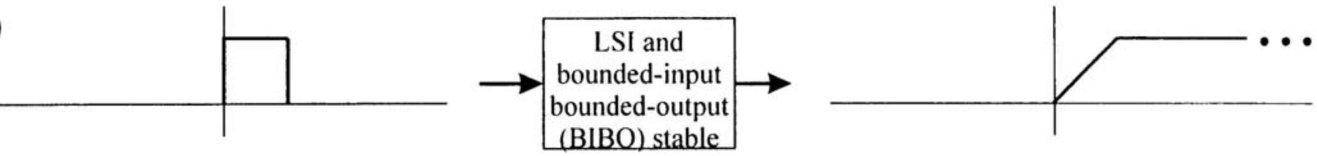
Quals Question : Nishimura ' 2002

For each system below, indicate if it is possible for the output function to occur, given the input function. (You may do them in any order you wish.)

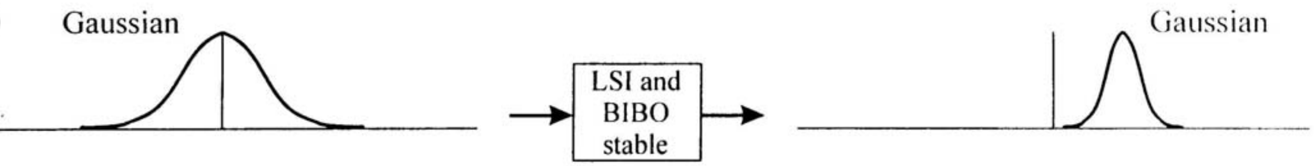
1)



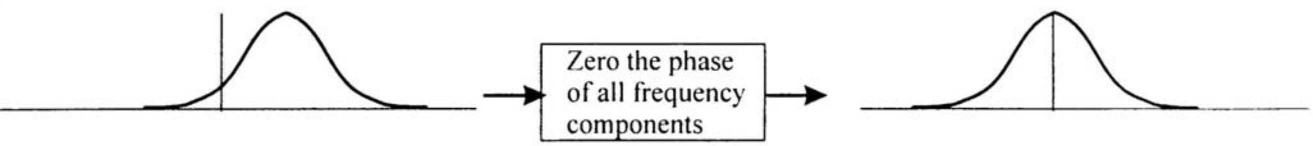
2)



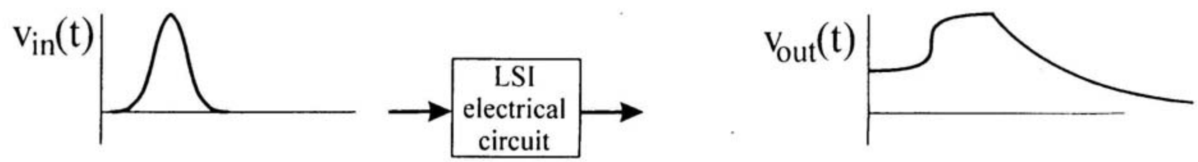
3)



4)

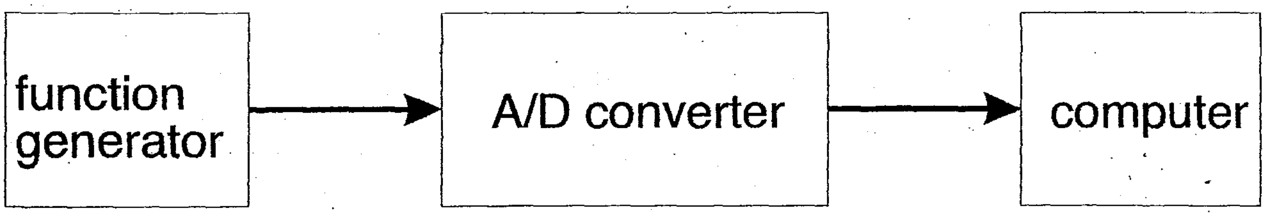


5)



Class Demo Set-up

NISHIMURA 2006



knob set for sinusoidal output

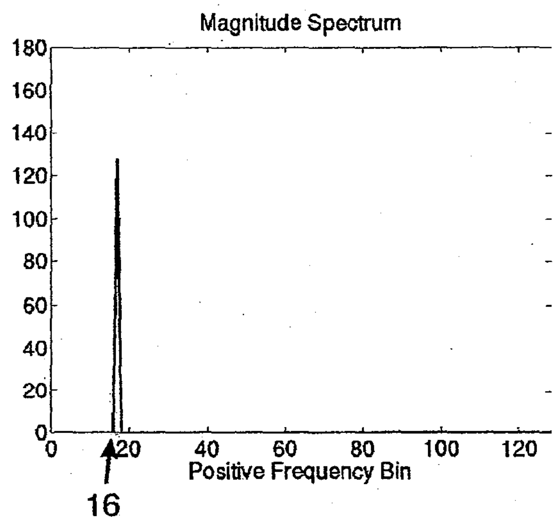
adjustable knob changes frequency.

adjustable knob changes sampling rate.

creates file of 256 data pts

A program was written to read in file and take 256-pt FFT of data.

We hope to show the following magnitude spectrum of the collected data.



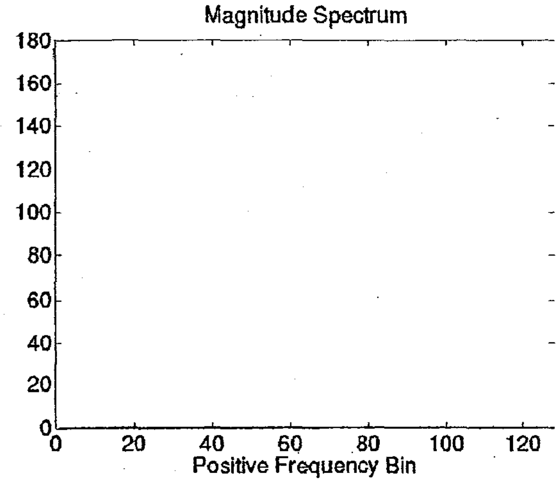
What sinusoidal frequency and A/D sampling rate could you use?

Debug the Situation

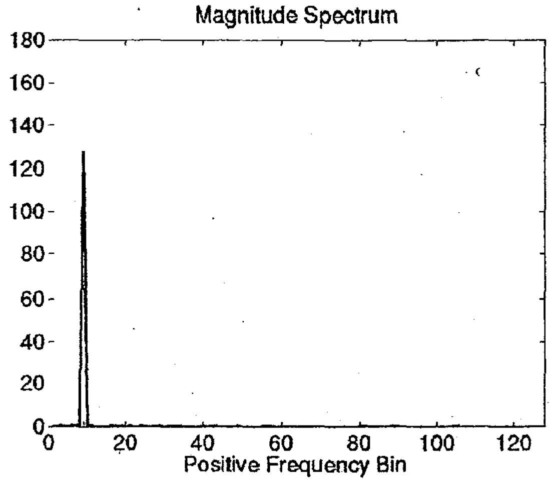
It works perfectly in the office a week before class. On class day however, the set-up creates a new data file during lecture that results in a different magnitude spectrum. Something has unexpectedly changed!

For each case below, assess the nature and possible cause(s) of the problem.

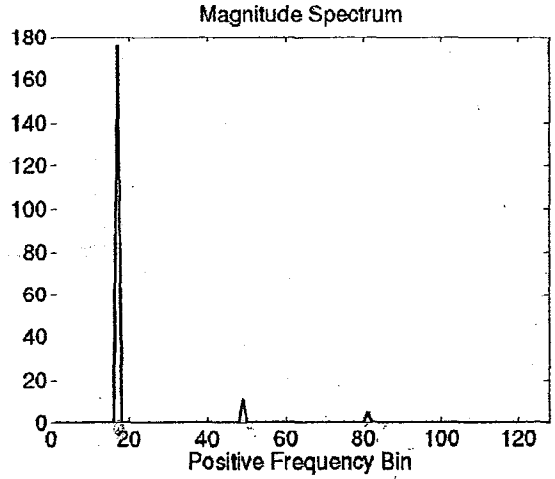
(1)



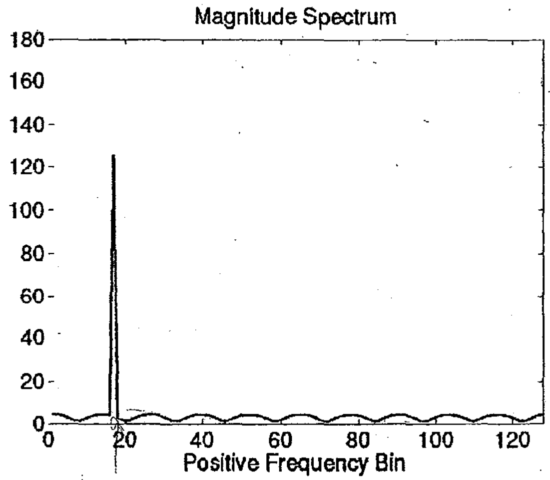
(2)



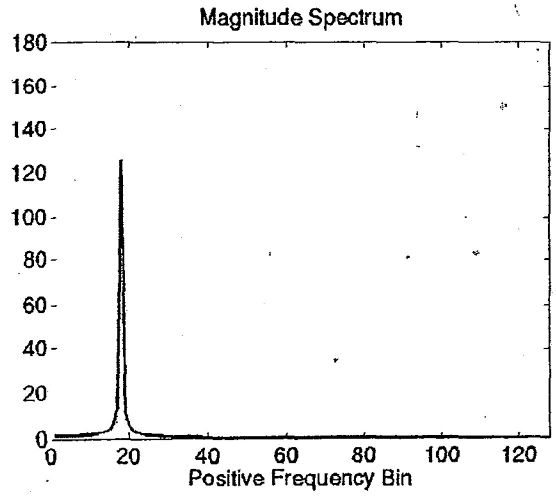
(3)



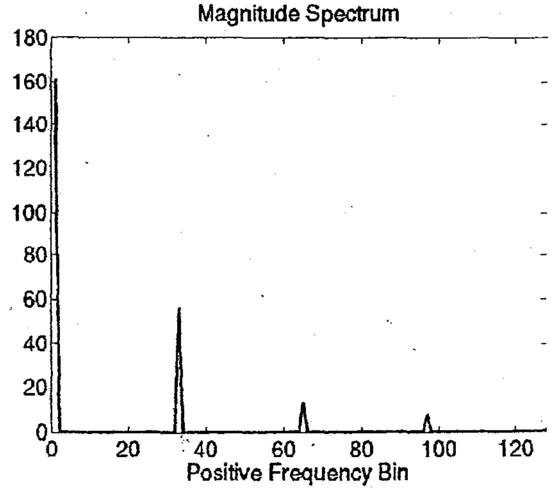
(4)



(5)



(6)



ANSWERS

Page 1

$$f_{in} = \frac{16}{256} f_{samp}$$

Page 2

1) check power, connections

2) f_{IN} was reduced and/or f_{samp} was increased

3) signal being clipped

4) spike noise in time signal

5) leakage

6) time signal rectified

> General debugging approach was judged.

Nishimura 2007

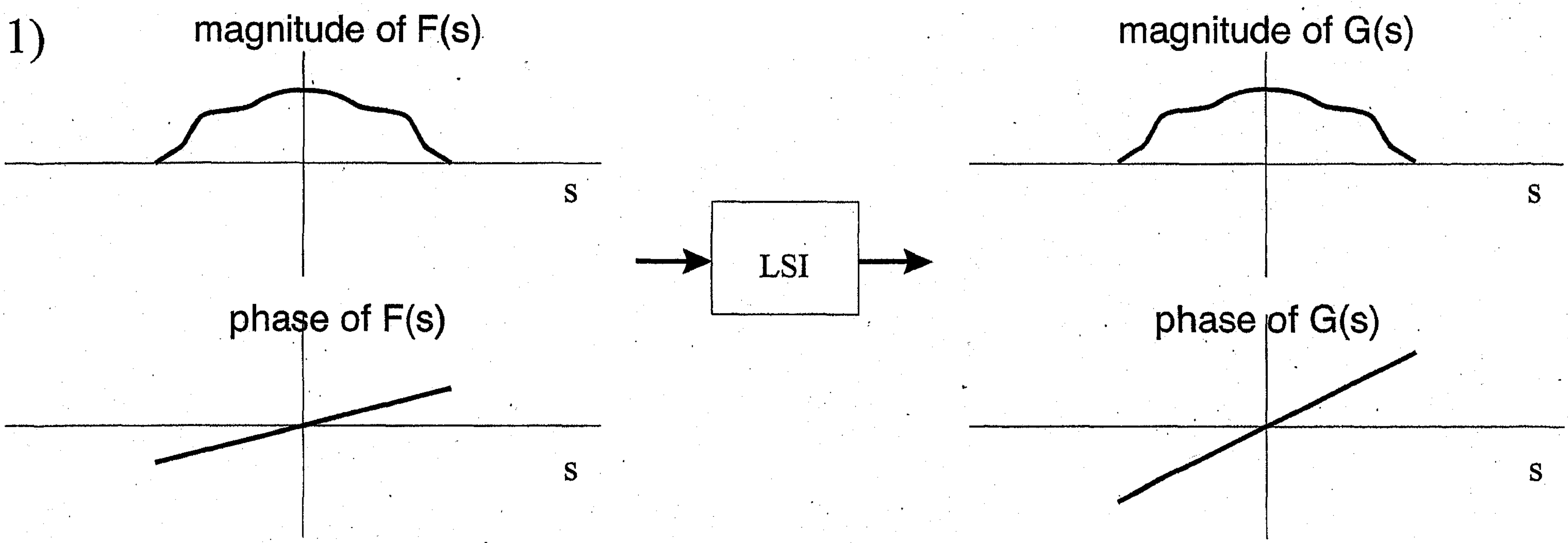
Consider

complex-valued input $f(x)$ → system → output $g(x)$

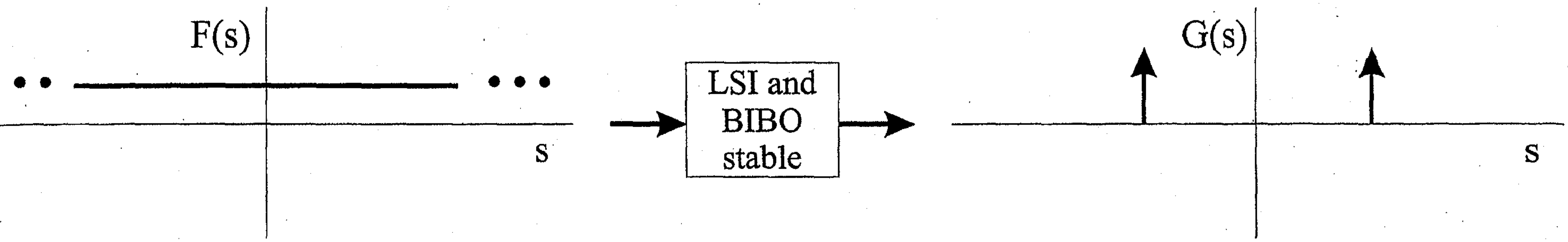
Fourier transform$\{f(x)\} = F(s)$
Fourier transform$\{g(x)\} = G(s)$

For each case below, is it possible to achieve that output? Explain.

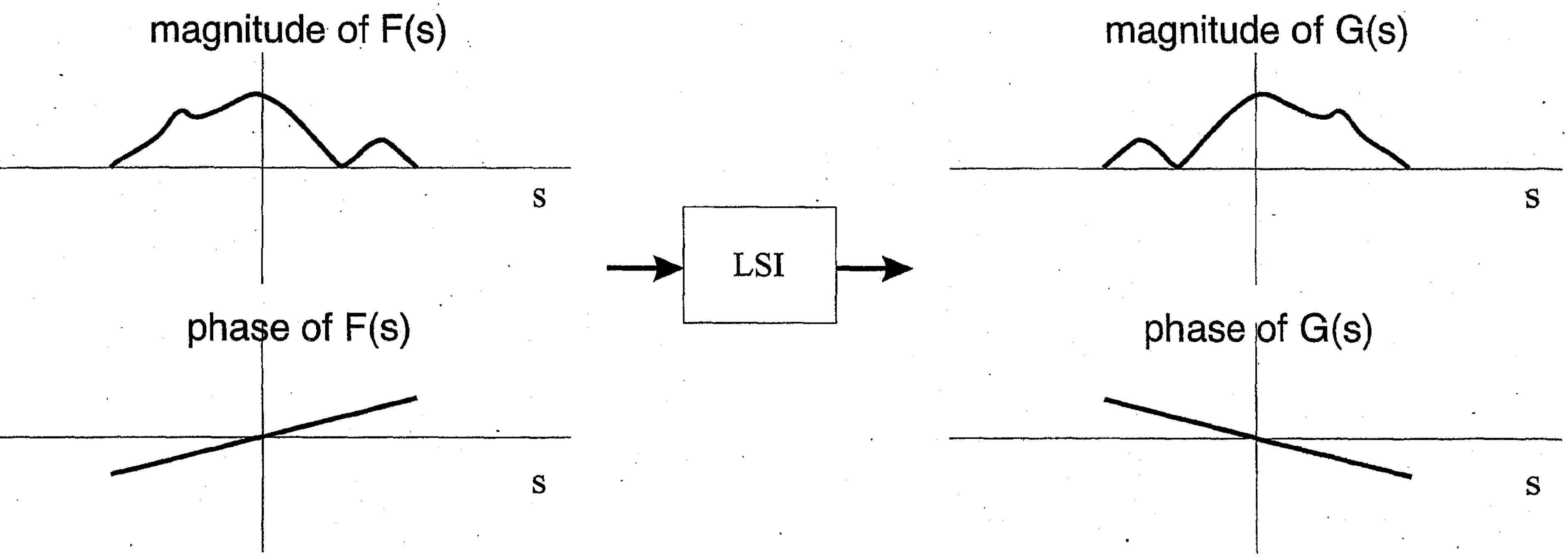
1)



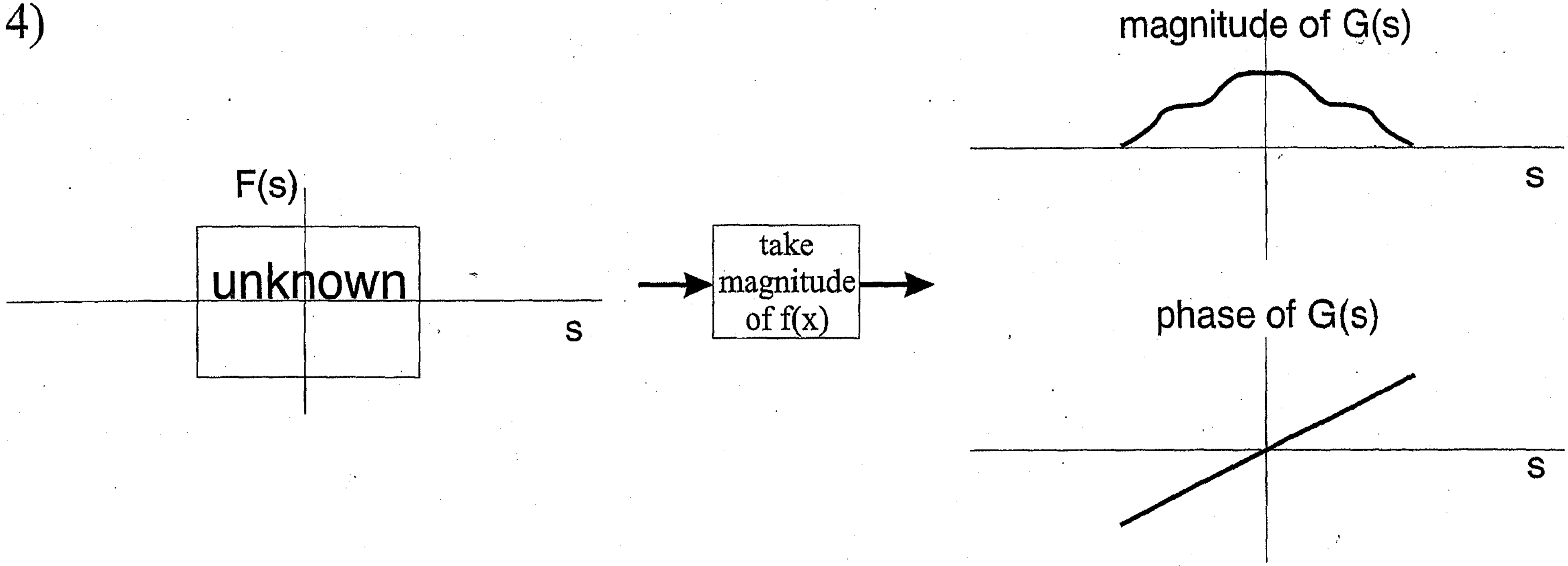
2)


F(s)
s
LSI and
BIBO
stable
G(s)
s

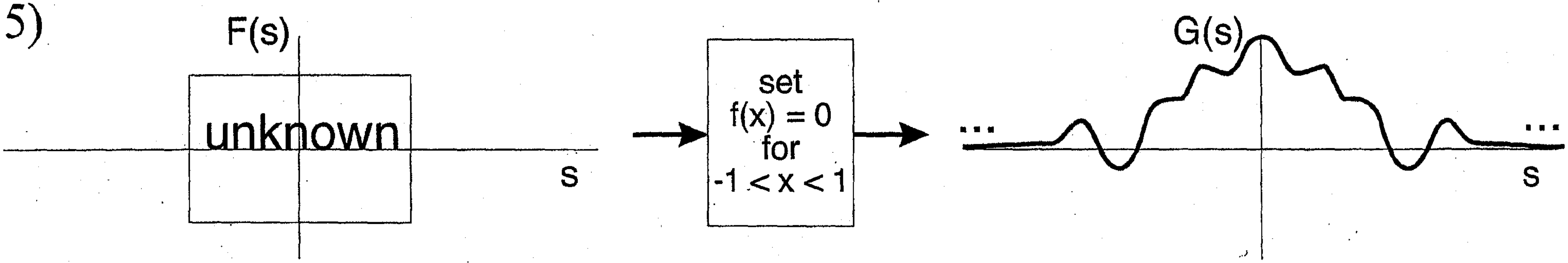
3)


magnitude of F(s)
s
phase of F(s)
s
LSI
magnitude of G(s)
s
phase of G(s)
s

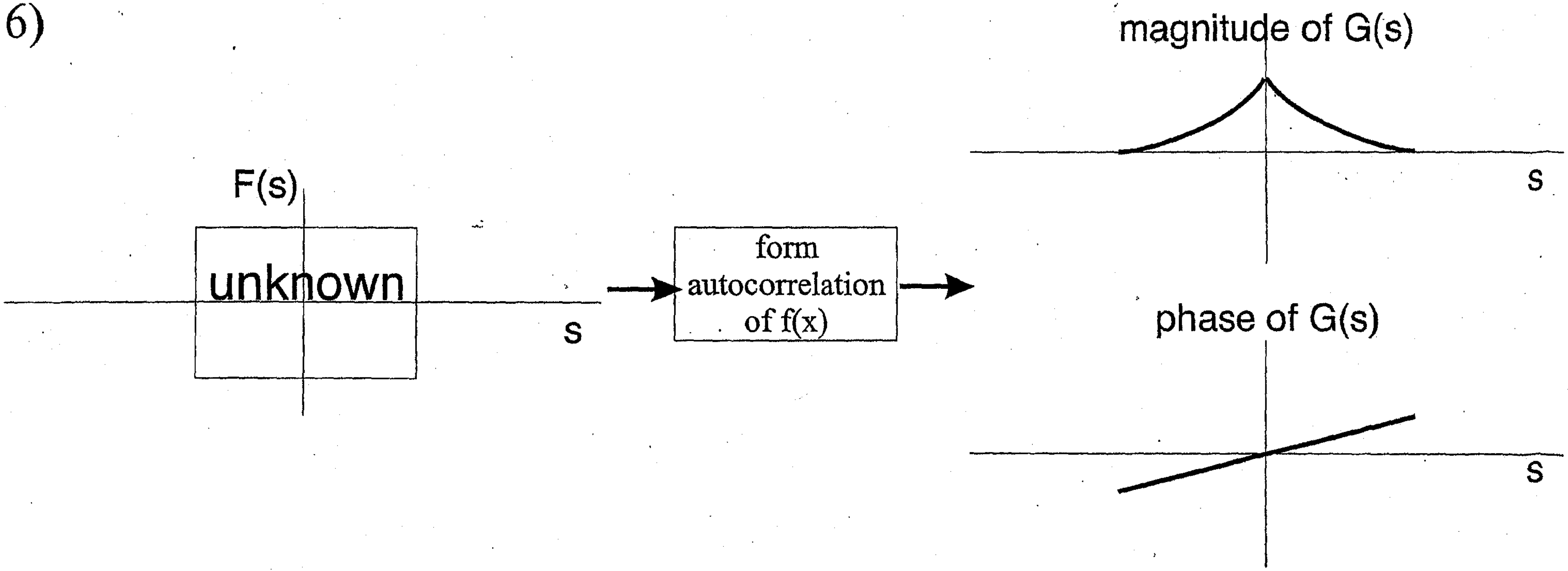
4)


F(s)
unknown
s
take magnitude of f(x)
magnitude of G(s)
s
phase of G(s)
s

5)


F(s)
unknown
s
set f(x) = 0 for -1 < x < 1
G(s)
...
...
s

6)


F(s)
unknown
s
form autocorrelation of f(x)
magnitude of G(s)
s
phase of G(s)
s

Answers: Nishimura 2007

1) Yes

2) LSI: Yes BIBO: No

3) No (almost Yes in degenerate case)

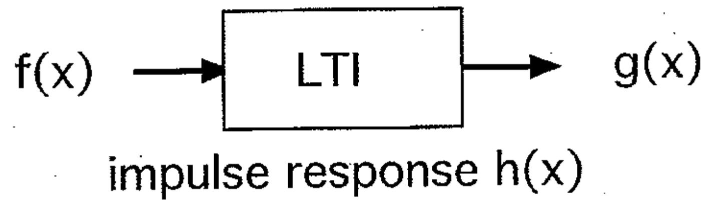
4) Yes

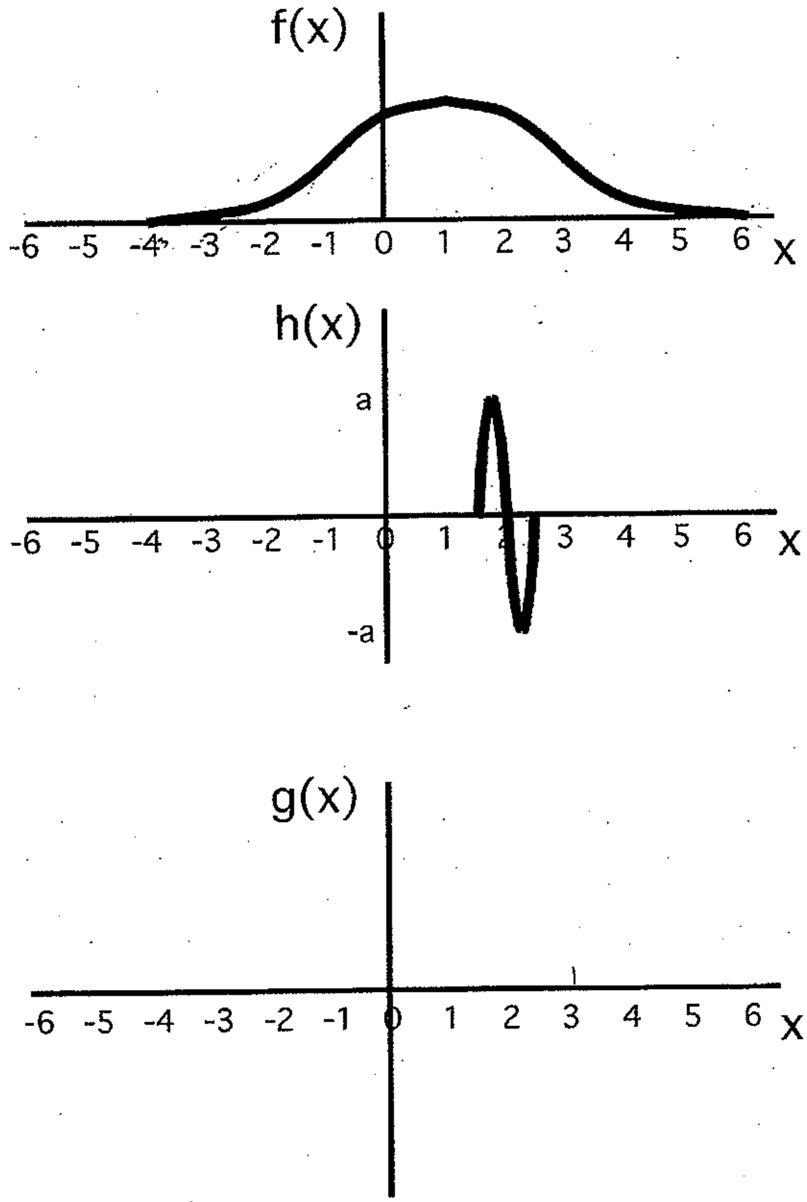
5) No

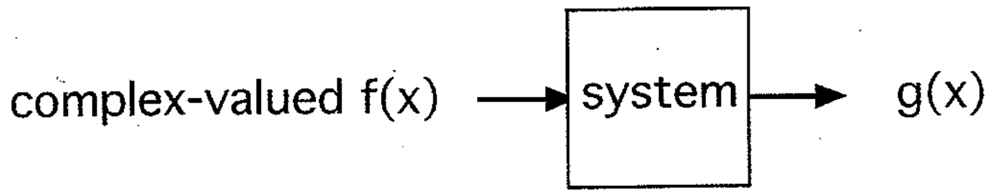
6) No

Nishimura 2009

Sketch the output g(x).







The output g(x) has the same magnitude as f(x). However, the system passes the phase of f(x) through a linear, time-invariant system.

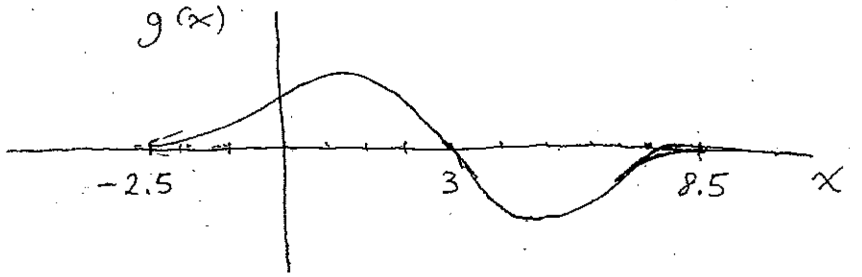
a) Is the overall system linear? Explain.

b) Is the overall system time invariant? Explain

c) Is the overall system causal? Explain.

Solutions 2009 Nishimura

(1)



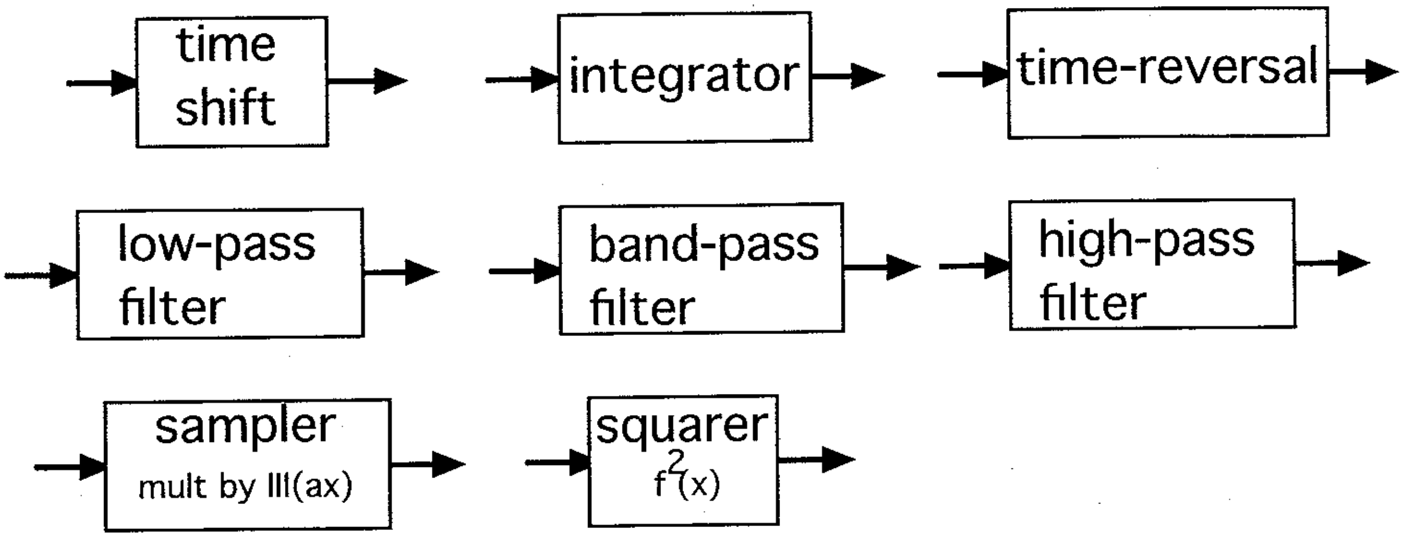
(2) a) probably nonlinear

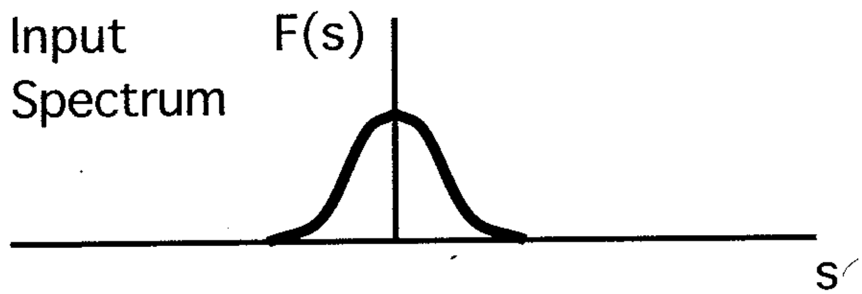
b) time invariant

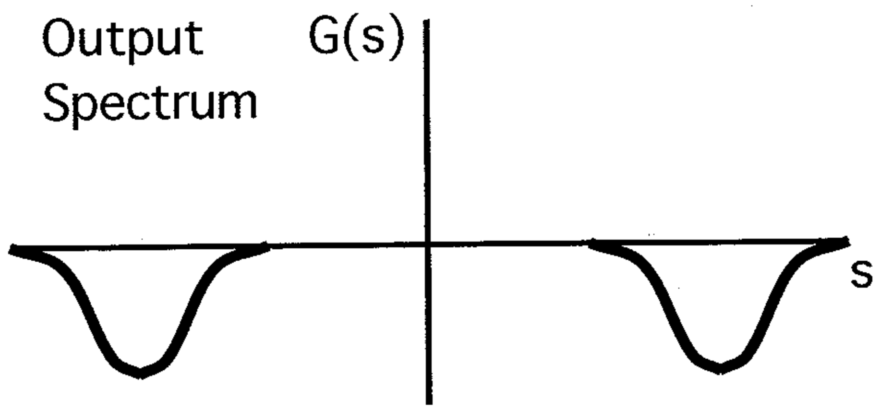
c) It depends on causality or noncausality of LTI system.

Nishimura 2010

(1) Selecting the appropriate building block(s) from those below, design a system to achieve the output spectrum.







2

Let f(x) represent an unknown distribution of mass for an object along x. You are given its Fourier transform F(s).

Is it possible to determine the object's center of mass from F(s) without taking the inverse Fourier transform? Explain.

(3)

Dr. T says the center of mass can be determined from just a single value of $F(s)$, call it $F(s_o)$, if $f(x)$ is symmetric about its center of mass.

Do you agree with Dr. T? Explain.

Answers

Nishimura 2010

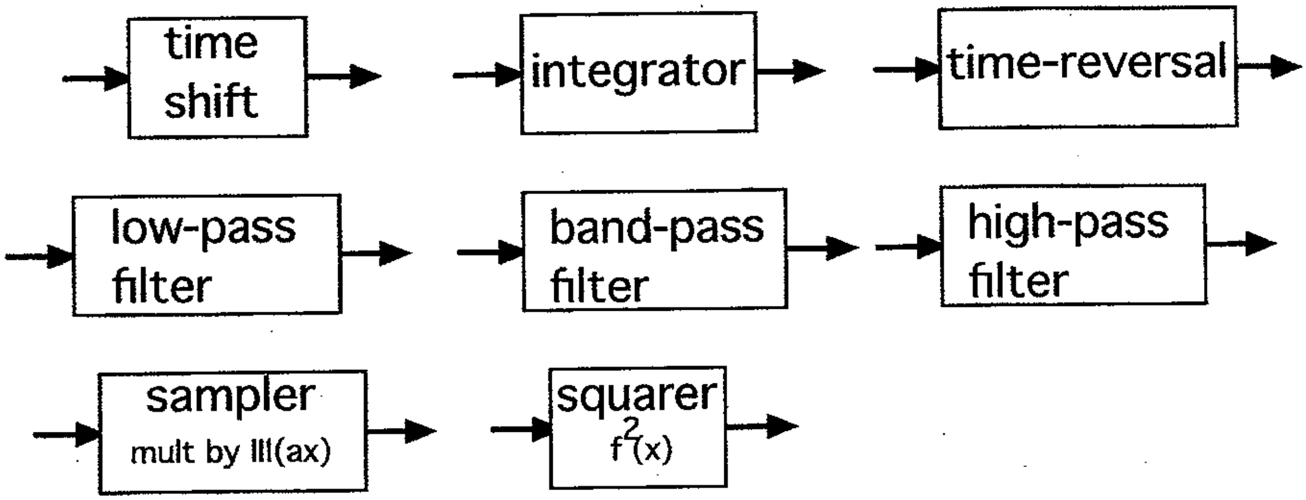
1) Sampler + bandpass filter gets you close.
The time shift plays a role in achieving the inverted spectrum but it requires some finesse.

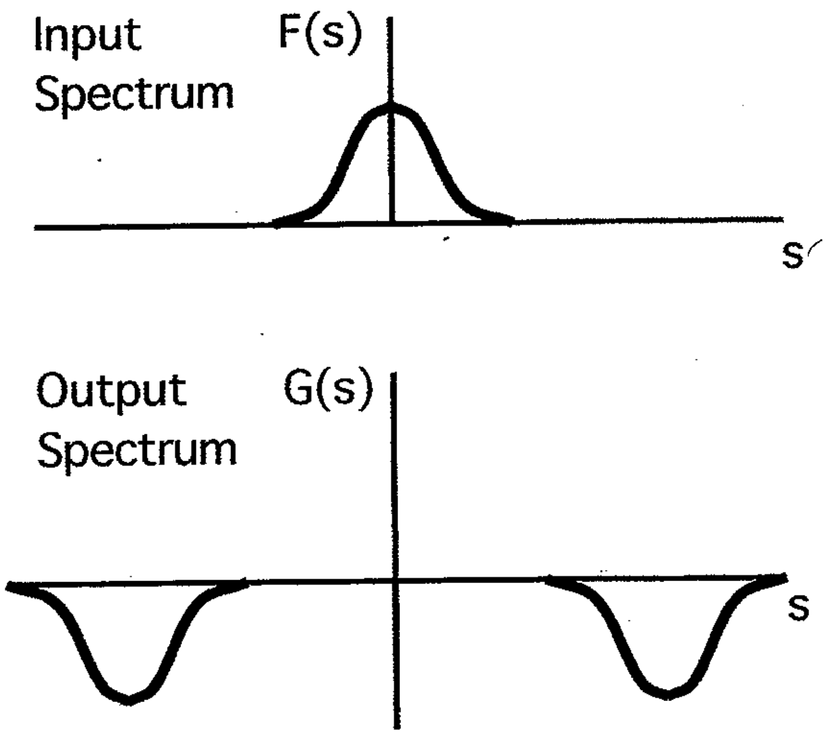
2) Yes. The center of mass is $\int x f(x)\,dx$ which relates nicely to $F(s)$.

3) It would be best to agree and disagree with Dr. T.
Dr T. is partially correct but additional constraints on s_0 and $F(s_0)$ are needed to avoid potential ambiguities in determining the center of mass.

Nishimura 2010

(1) Selecting the appropriate building block(s) from those below, design a system to achieve the output spectrum.





(2)

Let f(x) represent an unknown distribution of mass for an object along x. You are given its Fourier transform F(s).

Is it possible to determine the object's center of mass from F(s) without taking the inverse Fourier transform? Explain.

(3)

Dr. T says the center of mass can be determined from just a single value of $F(s)$, call it $F(s_o)$, if $f(x)$ is symmetric about its center of mass.

Do you agree with Dr. T? Explain.

Answers

Nishimura 2010

1) Sampler + bandpass filter gets you close.
The time shift plays a role in achieving the inverted spectrum but it requires some finesse.

2) Yes. The center of mass is $\int x\, f(x)\, dx$ which relates nicely to $F(s)$.

3) It would be best to agree and disagree with Dr. T.
Dr T. is partially correct but additional constraints on s_0 and $F(s_0)$ are needed to avoid potential ambiguities in determining the center of mass.