Near $w = -w_c$:

$$H(jw) \cong |H(jw_c)| e^{j[-\phi(w_c) + \phi'(w_c)(w+w_c)]}$$

$$y(t) \cong \frac{1}{4\pi} \left[\int_{-\infty}^{\infty} M(j(w-w_c)) |H(jw_c)| e^{j[\phi(w_c) + \phi'(w_c)(w-w_c)]} \right.$$
$$\cdot e^{jwt} dw$$

$$+ \int_{-\infty}^{\infty} M(j(w+w_c)) |H(jw_c)| e^{j[-\phi(w_c) + \phi'(w_c)(w+w_c)]} e^{jwt} dw$$

$$= \frac{1}{4\pi} |H(jw_c)| \left[e^{j\phi(w_c)} \int_{-\infty}^{\infty} M(j(w-w_c)) e^{j\phi'(w_c)(w-w_c)} e^{jwt} dw \right.$$
$$+ e^{-j\phi(w_c)} \int_{-\infty}^{\infty} M(j(w+w_c)) e^{j\phi'(w_c)(w+w_c)} e^{jwt} dw \left. \right]$$

Change variables:

First integral: $\Omega = w - w_c$

Second integral: $\Omega = w + w_c$

$$y(t) \cong \frac{1}{4\pi} |H(jw_c)| \left[e^{j\phi(w_c)} \int_{-\infty}^{\infty} M(j\Omega) e^{j\phi'(w_c)\Omega} e^{j(\Omega + w_c)t} d\Omega \right.$$
$$+ e^{-j\phi(w_c)} \int_{-\infty}^{\infty} M(j\Omega) e^{j\phi'(w_c)\Omega} e^{j(\Omega - w_c)t} d\Omega \left. \right]$$

$$= |H(jw_c)| \frac{1}{2} \left[e^{j(w_c t + \phi(w_c))} + e^{-j(w_c t + \phi(w_c))} \right]$$

$$\cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} M(j\Omega) e^{j\Omega(t + \phi'(w_c))} d\Omega$$