the maximum frequency in the signal. Thus provided the Fourier transform of the signal is nonzero only for $f \in (-1/2, 1/2)$, the signal can be reconstructed from its samples using the sampling expansion or by low pass filtering the a signal with the samples imbedded on impulses.

- A difference equation defines a linear system, and the system is time invariant since the coefficients are. The system can be characterized as convolving the input with the response to a Kronecker delta $x[n] = \delta(n) = 1$ for $n = 0$, and 0 otherwise. Since the system is assumed causal, $y[n] = 0$ for $n < 0$ and hence $y[0] = 1$, $y[1] = a$, $y[2] = a^2$, etc. so that the response to a Kronecker delta is $h[n] = a^n$ for $n = 0, 1, 2, \ldots$ and 0 otherwise. The inverse filter has Kronecker delta response $g[n] = \delta[n] - a\delta[n-1]$, the discrete time convolution of h and g is δ. Alternatively, from the geometric series (assuming $|a| < 1$)

$$H(f) = \sum_{n=0}^{\infty} a^n e^{-j2\pi fn} = \frac{1}{1 - ae^{-j2\pi f}}$$

and the inverse filter is $1/H(f)$.

Alternatively, taking the transform of the difference equation and changing variables (or using the delay theorem)

$$\begin{aligned} Y(f) &= \sum_{n=-\infty}^{\infty} y[n]e^{-j2\pi fn} \\ &= \sum_{n=-\infty}^{\infty} ay[n-1]e^{-j2\pi fn} + \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi fn} \\ &= \sum_{n'=-\infty}^{\infty} ay[n']e^{-j2\pi f(n'+1)} + X(f) \\ &= e^{-j2\pi f}Y(f) + X(f) \end{aligned}$$

so that

$$Y(f) = \frac{X(f)}{1 - e^{-j2\pi f}}$$

If $a = 1$, then the first argument still works and the system is still invertible, but the transform arguments get more complicated.