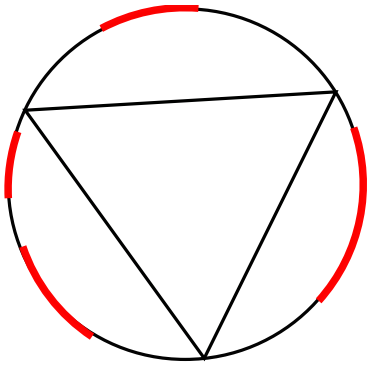
Ayfer Özgür

Ph.D. Qualifying Examination

Problem 1. The circumference of a circle is colored $a\%$ in red such that $a < 100/3$. Prove that it is always possible to inscribe an equilateral triangle into the circle such that all three vertices avoid the red, irrespective of how the coloring is done.



Hint: Consider an equilateral triangle that is randomly inscribed into the circle and evaluate the probability that the random triangle satisfies the desired property.