2. The goal of the second problem was to find out what the student knew about stationarity. The problem counted for about 1/2 point, mainly for seeing this process is not stationary and why. The process cannot be stationary because the mean at time 0 is different from the mean at all other times. If the process were begun at time $n = 1$, however, this problem goes away. In that case, the fact that the autocorrelation depends only on the time difference would mean the process is weakly or wide-sense stationary.

3. Most students spent most of their time on this problem. The intent was that most students would spend the remaining time on this problem. The first sum counted for about 2.5 points and the remainder for about 1.5 points each or about 7 total.

   - $\frac{1}{N}\sum_{n=0}^{N-1} X_n$ if the law of large numbers holds, this should converge to the mean of the process, which is 0. Except for $n = 0$, the process is weakly stationary and it is an uncorrelated process, this is enough to ensure that the weak las of large numbers and hence the sum from $n = 1$ will converge in both mean-square and in probability to the common mean, which is 0. The $n = 0$ term does not effect the limit, so the answer is 0. Some students remembered this fact and a few derived the result as follows:

$$\begin{aligned} E\left[\left(\frac{1}{N}\sum_{n=0}^{N-1} X_n\right)^2\right] &= (\frac{1}{N})^2 \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} R_X(n,k) \\ &= (\frac{1}{N})^2 \sum_{n=0}^{N-1} R_X(n,n) = \frac{1}{N} \to 0. \end{aligned}$$

     Convergence in probability follows from this result and the Tchebychev inequality. Alternately and equally good was to use the method described in the next item and simply directly prove that the sum goes to zero by evaluating it using the geometric progression.

   - The $\sqrt{N}$ term instead of the N term in the denominator was intended to make the student think of the central limit theorem, but that does not work here because this process does not meet any of the conditions required for the central limit theorem to hold. So here something different is needed, and the trick is to try to find the sum exactly. Here the geometric progression can be used to write

$$\sum_{n=0}^{N-1} X_n = \sum_{n=0}^{N-1} e^{jn\Theta} = \frac{1 - e^{j\Theta(N+1)}}{1 - e^{j\Theta}}$$

     With probability 1 Θ will take on a sample value that is not 0, so the denominator has some fixed value independent of N and the numerator is a well behaved function of N that can never have magnitude greater than 2. Thus dividing by $\sqrt{N}$ in the denominator will drive $(1/\sqrt{N})\sum_{n=0}^{N-1} X_n$ to zero.
     If this method were used for the first sum, then this part was an obvious variation.

   - $\frac{1}{N}\sum_{k=0}^{N-1} Y_k$ The fast way to do this one is to realize that the Y_k constitute the DFT of the X_n, so this is just the inverse DFT for $n = 0$, which is 1.
     If the inverse DFT was not remembered, it could be derived as

$$\begin{aligned} \frac{1}{N}\sum_{k=0}^{N-1} Y_k e^{\frac{j2\pi kn}{N}} &= \frac{1}{N}\sum_{k=0}^{N-1}\left(\sum_{m=0}^{N-1} X_m e^{\frac{-j2\pi km}{N}}\right) e^{\frac{j2\pi kn}{N}} \\ &= \sum_{m=0}^{N-1} X_m \frac{1}{N}\sum_{k=0}^{N-1} e^{\frac{j2\pi k(n-m)}{N}} = X_n \end{aligned}$$