Solution

(i) The electrostatic energy now stored on the capacitor is $\frac{1}{2}CV_{DD}^2$

Essentially all the students knew this answer, and they were not asked to prove or derive it.

(ii) A few students already knew or intelligently guessed that the answer to this part is also $(1/2)CV_{DD}^2$, though I would require them to justify this result. There are two main ways of answering this part. The first is to attempt some integration of the power or energy dissipated in the resistor. The most common approach students would take here was to integrate the known exponential behavior of the current or voltage in time, using either I^2R or V^2/R formulae for the power being dissipated in the resistor at a current I or voltage V. A more compact version is to integrate over voltage directly. The energy dissipated in flowing a charge δQ through a resistor at a voltage V is $\delta E = V\delta Q$. When flowing a charge δQ onto a capacitor C, the resulting change in voltage δV on the capacitor is such that $\delta Q = C\delta V$. In the circuit, the voltage V across the resistor R is $V = V_{DD} - V_{OUT}$. Hence the total energy dissipated in the resistor in charging the capacitor C from 0V to V_{DD} is

$$\Delta E_R = C\int_0^{V_{DD}} \left(V_{DD} - V_{OUT}\right) dV_{OUT} = \frac{1}{2}CV_{DD}^2$$

No student actually took this approach to start with. Anyway, once I could see that the student would be able to work out this energy by some integral approach, I generally stopped them, taking it for granted that eventually they would get to the right answer here.

The easy way to solve this problem is to solve part (iii) first, which avoids all this integration. Students were told to look at parts (ii) and (iii) together, though very few actually did that!

(iii) Some students would try to approach this problem (correctly, though not optimally) by figuring out an answer to part (ii) and adding it to the answer to part (i). The easy way to solve this is simply to calculate the energy that must be supplied by the power supply because it provides a charge $Q = CV_{DD}$ through the circuit to charge the capacitor. That energy is $E = QV_{DD} = CV_{DD}^2$. Few students got this without help. The temptation to integrate the power over time was generally too strong. Most students also had temporarily forgotten that Volts are Joules/Coulomb and that by definition in moving a 1 Coulomb charge through 1 Volt I must do 1 Joule of work. In general, students tended to be too frozen in to thinking in powers rather than energies in this whole problem, which made it harder for them than ideally it needed to be.

Armed with the result CV_{DD}^2 from this part, the answer to part (ii) is easy to deduce by conservation of energy.