A sufficient condition is that the bracketed term is itself 0 for all integer n. It is 0 for $n = 0$ and it will be 0 for all nozero integers if we pick

$$P(f) = P_1(f) = \begin{cases} 1 & |f| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}.$$

Similar to the first approach, taking the (inverse) Fourier transform yields $p_1(t) = \text{sinc}(t)$.
Thus both the box function $p_0(t)$ and the sinc function $p_1(t) = \text{sinc}(t)$ have the desired properties. The signal $p(t)$ is time limited, but occupies an infinite bandwidth in the time domain (although most of the energy is between the first zeros of the sinc function), while the second has infinite extent in time, but is bandlimited to $[-1/2, 1/2]$.