Since T is a monotonically decreasing function of V, the range of T is

$$-RC\ln\left(1-\frac{V_1}{V_0(1+\delta)}\right) \le T \le -RC\ln\left(1-\frac{V_1}{V_0(1-\delta)}\right).$$

If the random voltage V is too small—namely, $V < V_1$—then the capacitor voltage $V_C(t)$ never reaches V_1. In this case $T = +\infty$. (The upper bound in the above equation is meaningless.) We assume from now on that $\delta < e^{-1}$.

Given the formula for T, there are two standard ways to find the pdf of T: find the cdf $F_T(t)$ and differentiate, or express the pdf $f_T(t)$ in terms of the pdf $f_V(v)$ of V.

We can find the cdf $F_T(t)$ by using its definition.

$$\begin{aligned}
\mathbf{P}\{T \le t\} &= \mathbf{P}\left\{-RC\ln\left(1-\tfrac{V_1}{V}\right) \le t\right\} = \mathbf{P}\left\{\ln\left(1-\tfrac{V_1}{V}\right) \ge -\frac{t}{RC}\right\} \\
&= \mathbf{P}\left\{1-\tfrac{V_1}{V} \ge e^{-t/RC}\right\} = \mathbf{P}\left\{\tfrac{V_1}{V} \le 1-e^{-t/RC}\right\} = \mathbf{P}\left\{V \ge \frac{V_1}{1-e^{-t/RC}}\right\} \\
&= 1-\mathbf{P}\left\{V \le \frac{V_1}{1-e^{-t/RC}}\right\} = F_V\left(\frac{V_1}{1-e^{-t/RC}}\right) = \frac{1}{2\delta V_0}\left(\frac{V_1}{1-e^{-t/RC}} - V_0(1-\delta)\right)
\end{aligned}$$

Finding the pdf is now an exercise in using the Chain Rule to differentiate the cdf.

$$\begin{aligned}
f_T(t) &= \frac{d}{dt}\frac{1}{2\delta V_0}\left(\frac{V_1}{1-e^{-t/RC}} - V_0(1-\delta)\right) = \frac{V_1}{2\delta V_0}\frac{d}{dt}\frac{1}{1-e^{-t/RC}} \\
&= \frac{V_1}{2\delta V_0}\left(-\frac{1}{(1-e^{-t/RC})^2}\right)\left(-\frac{e^{-t/RC}}{RC}\right) = \frac{V_1 e^{-t/RC}}{2RC\delta V_0(1-e^{-t/RC})^2}
\end{aligned}$$

The pdf of $T = g(V)$ can be obtained directly from the pdf of V using a formula familiar to EE 278 students. Let $v_1, v_2, \ldots$ be the solutions of the equation $t = g(v)$ and let $g'(v_i)$ be the derivative of g evaluated at v_i. Then

$$f_T(t) = \sum_i \frac{f_V(v_i)}{|g'(v_i)|},$$

Since $g(v) = -RC\ln(1-V_1/V)$ is monotonically decreasing, there is at most one solution to the equation. By definition of T, the value of v corresponding to t satisfies $v(1-e^{-t/RC}) = V_1$, hence $v = V_1/(1-e^{-t/RC})$. Therefore

$$\begin{aligned}
\frac{d}{dv}\left(\ln\left(1-\frac{V_1}{v}\right)\right) &= \frac{d}{dv}\left(\ln\left(\frac{V_1-v}{v}\right)\right) = \frac{d}{dv}\Big(\ln(v-V_1) - \ln v\Big) = \frac{1}{v-V_1} - \frac{1}{v} \\
&= \frac{1}{v-v(1-e^{-t/RC})} - \frac{1-e^{-t/RC}}{V_1} = \frac{1}{ve^{-t/RC}} - \frac{1-e^{-t/RC}}{V_1} \\
&= \frac{1-e^{-t/RC}}{V_1e^{-t/RC}} - \frac{1-e^{-t/RC}}{V_1} = \frac{1-e^{-t/RC}-e^{-t/RC}(1-e^{-t/RC})}{V_1e^{-t/RC}} = \frac{(1-e^{-t/RC})^2}{V_1e^{-t/RC}}
\end{aligned}$$

Putting it all together, we find the pdf of T:

$$f_T(t) = \frac{f_V(v)}{|g'(v)|} = \frac{1}{2\delta V_0}\left(RC\frac{(1-e^{-t/RC})^2}{V_1e^{-t/RC}}\right)^{-1} = \frac{V_1e^{-t/RC}}{2RC\delta V_0(1-e^{-t/RC})^2}$$

We note with satisfaction and relief that both methods yield the same answer.