Quals Question

Let the random variable X have CDF F. Suppose that $X_i \overset{\text{iid}}{\sim} X$.

(a) What is the CDF of $\max_{1 \le i \le n} X_i$?

(b) What is the CDF of $\min_{1 \le i \le n} X_i$?

Suppose now that $X_{i,j} \overset{\text{iid}}{\sim} X$

(c) What is the CDF of $f(n, m) = \max_{1 \le i \le n} \min_{1 \le j \le m} X_{i,j}$?

(d) Suppose $X \sim$ exponential(λ). Let $Y_m = f\left(e^{\beta m}, m\right)$, for some parameter $\beta > 0$. What does Y_m converge to as $m \to \infty$? In what sense?

(e) Repeat the previous part for the general case $X \sim F$ (can assume that F is continuous and strictly increasing).