Electrical Engineering

Quals Questions

2012

http://ee.stanford.edu/phd/quals

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1. Computer Architecture and Logic Design

EE Quals Questions 2012 - Mark Horowitz

The goal of this question was to get to a point where the student did not know the answer off the top of their head, and needed to figure it out. It did not matter much where the point occurred, the score really depended on how the student solved a new problem.

This question is going to look at the hardware needed to compress a video stream.

1. Video consists of a stream of picture. Often motion estimation is used to compress frames. Why is estimating motion useful in compression?

2. Show a picture of an image block (8x8 pixels) being compared against many different 8x8blocks of pixels. Ask about the trade-off in choosing the size of the comparison region.

3. The actually computation is a funny one. It is not a squared error metric $(a-b)^{\hat{}2}$, but rather the sum of absolute differences $|a-b|$. What practical reasons might have favored this metric

4. If you create an implementation on a simple processor, it runs to slowly, and takes too mch energy. What can you change, to either the hardware or the algorithm, to make this computation more efficient.

Mark

Computation on Meshes

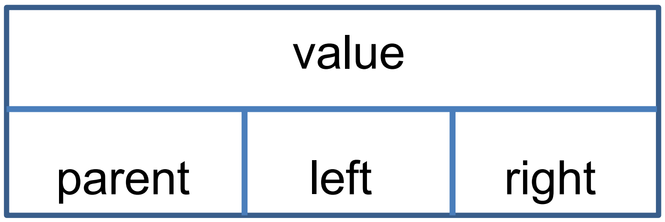

```
for(i = 0; i < 100M; i++) {
     edge = edges_of_mesh[i]
   flux = flux_calc(edge)  /* millions of instructions */
   v0 = head(edge)
   v1 = tail(edge)
   Flux[v0] += flux
   Flux[v1] -= flux
}
```

1. What types of data locality exist in this loop?
2. How would you exploit this data locality? (two ways)
3. Instruction stream parallelism vs. data stream parallelism. What is benefit of each and which works here?
4. What are issues with data parallelism here?
5. Name three ways of dealing with them?

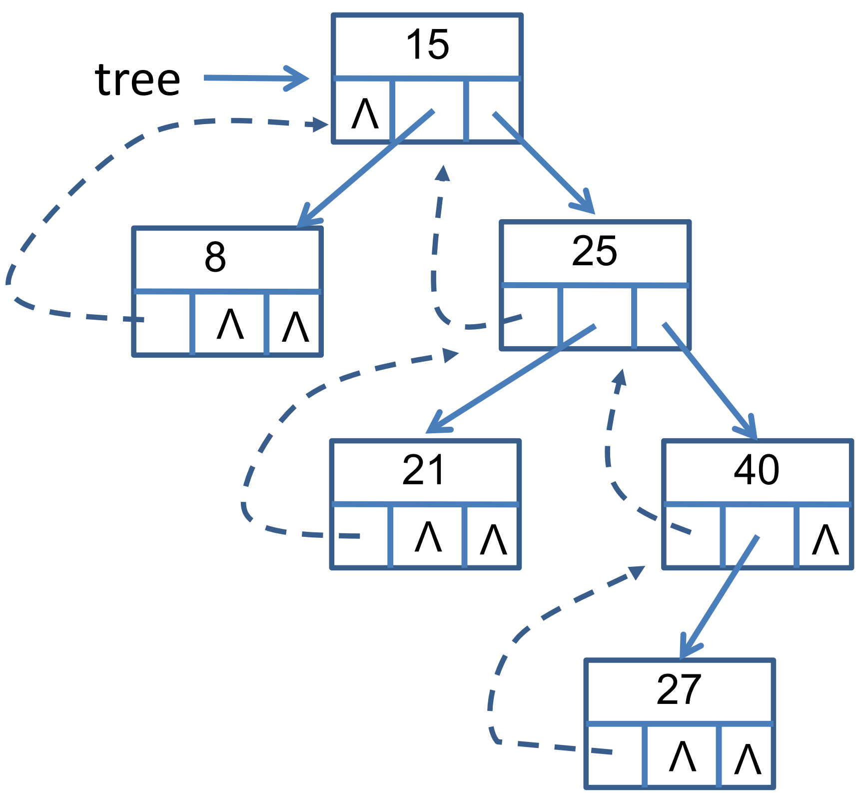
2. Computer Systems Software

Binary Tree

- Node x in tree:



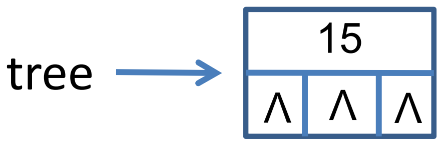
- x.value = search value
  x.parent = pointer to parent node
  x.left = pointer to left node
  x.right = pointer to right node
- Example:



- Empty tree:

tree → null

- Tree with a single value:



- Question 1
  Write <u>recursive</u> pseudo code for lookup procedure
  lookup(tree, value): returns pointer to node
  null if value not found

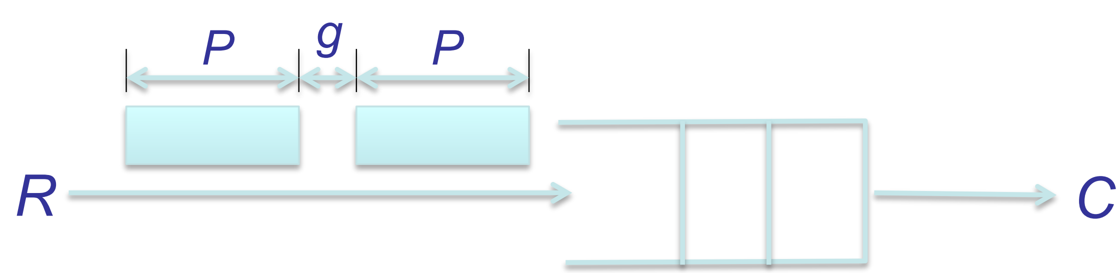
- Question 2
  Write pseudo code for delete procedure
  delete(tree, value): removes node with value;
  does nothing if value not in

EE Quals 2012 (Software)

Nick McKeown

Question 1

- The figure below shows a simple FIFO packet queue. Packets of length P arrive on the input link at rate R bits/s, and are stored in the buffer.
- If there is a complete packet at the head of the queue, it is immediately transmitted onto the output link at rate C bits/s. $R > C$.



Question 1

a) Write down an expression for the minimum interpacket gap, g, so we can be sure that the buffer will never overflow.

b) Write down an expression for the size of the buffer, B, in bits so we can be sure that the buffer will never overflow.

c) If the gap is a random variable, G, where $E[G] = 2g$ from part (a), can you find B so that the buffer will never overflow?

Question 2

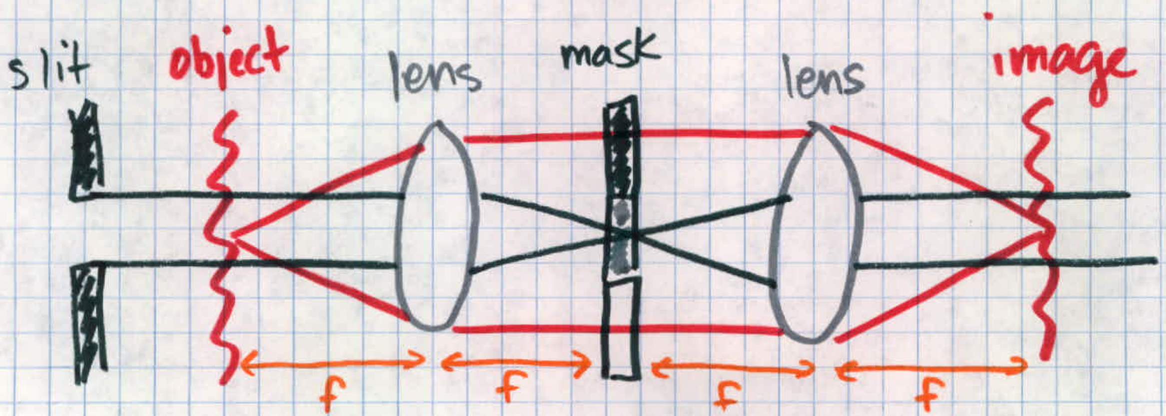
a) Explain how the command *traceroute* finds the path between a source and a destination, and the round-trip-time from the source to any router along the path.

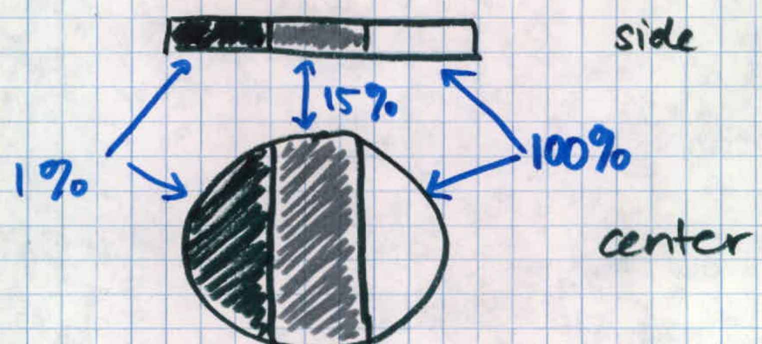
b) Describe a method in which a source and destination can determine the rate of the bottleneck link between them, by sending just two packets.

c) How could you extend the method in (b) to find the rate of any link up to and including the bottleneck?

3. Electromagnetics

A special microscope is designed as follows:





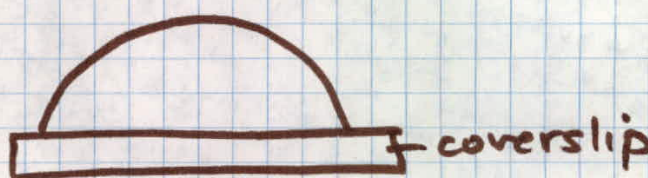
Assume the object is a pure phase object (ie - it does not change the amplitude of the transmitted beam). Ignore diffraction.

Q1: What does it do?

Q2: Draw a front-view (center) image of a cell with the following front and side profiles:

FRONT | SIDE

a)



b)

Assume the cell has a constant index of refraction.

Ph.D. Quals Question
January 23-27, 2012
A.C. Fraser-Smith
Department of Electrical Engineering
Stanford University

Super Capacitors

Quals students were told that the advent of electric cars has led to a strong interest in the development of "super" or "ultra" capacitors, which might just conceivably replace batteries or, more likely, given current progress, be used in conjunction with batteries. The students were warned that my questioning would be directed toward the development of these high-capacity capacitors and that it would largely involve basic electromagnetism, or more particularly, basic electricity. To start with they were asked to describe Gauss's theorem relating the surface integral of the electric field **E** over a closed surface to the enclosed charge Q:

$$\int \mathbf{E}.\mathbf{ds} = Q/\varepsilon_0$$

Next, when we apply this to the surface of a plane conducting plate of infinite extent carrying surface charge σ we obtain the electric field perpendicular to the surface:

$$E = \sigma/\varepsilon_0$$

This same expression also applies to the electric field close to the surface of any conductor, with σ the surface charge density in the immediate vicinity.

For two parallel conducting planes of area A separated by a distance d carrying equal but opposite charges Q, -Q, the electric field has the same form as above (we will assume it is everywhere perpendicular to the surfaces of the plates, with no fringing) and the capacitance of the plates is

$$C = Q/V = \sigma A/(\sigma d/\varepsilon_0) = \varepsilon_0 A/d$$

Although all isolated conducting bodies have some capacitance, these capacitances are generally very small and measured in microfarads or micromicrofarads. [To illustrate, the capacitance of a sphere of radius R is given by $C = 4\pi\,\varepsilon_0 R$ and for a sphere as large as the Earth, with R = 6370 km, we only have C = 708 μF]. Once again in general, large capacitances can only be achieved by using two close conductors bearing opposite charges (this arrangement is called a **capacitor**) and here the above equation for the capacitance between two plates is representative. Not only that, most attempts to produce very large capacitances are based on the conducting plate model. We will now consider how "super" or "ultra" capacitors might be produced using this model.

Obviously, we can make C large by increasing A and decreasing d, and this is what is done in conventional capacitors. However, these conventional capacitors are not "super" nor "ultra." **Background material**: The best of these conventional capacitors is the electrolytic capacitor, invented around 1890, which can have a capacitance in the mF range. In these capacitors a very thin (small d) layer of non-conducting aluminum oxide is formed between two aluminum plates containing an electrolyte or an electrolytesoaked paper spacer (the electrolyte can have many different compositions, but borax has been successfully used in the past).

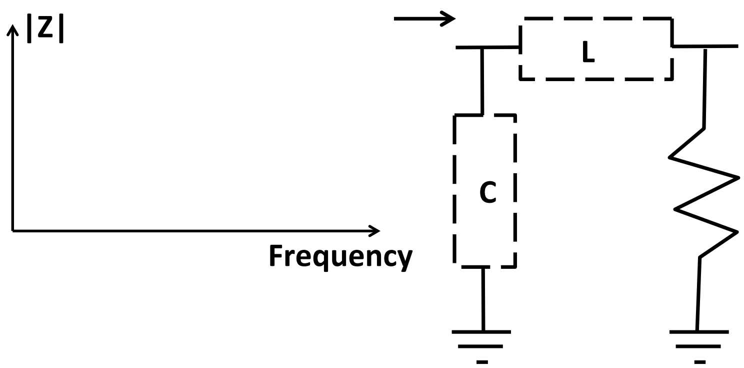
In a “nano” world, increasing A is difficult without increasing the size of the capacitor whereas decreasing d can lead to small capacitors and this is in tune with our present “nano” world. However, we have to remember that $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (value given to students) and so even d = 1 nm and A = 1 m^2 (is this a reasonable value?) will leave C less than 1 F. At this stage the students were told that there were now commercial electrolytic supercapacitors (or ultracapacitors) available with capacitances on the order of a few farads or more. How did they think this was achieved (given what we have already discussed)? What was looked for here in the wrap-up was a suggestion that the area A needed to be increased. The existing supercapacitors make use of carbon materials with large effective areas due to their porosity; areas of as much as 250 m^2 are quoted. Small d’s (yes, on the order of 1 nm) are achieved by producing electrical double layers. There will be much research done on these capacitors over the next few decades because of the large commercial interest in their applications.

Quals questions, L. Hesselink, 2012

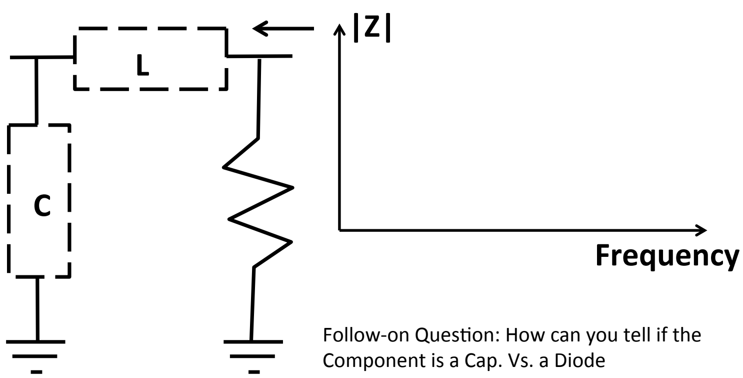
1. Please consider a two-slit diffraction problem. (a picture of the set up is provided).
    a. Sketch the diffraction pattern that results.
        i. Describe the solution in mathematical terms
2. Now let the slit size become significantly less (say 10x) than the wavelength of the illumination light
    a. Describe the changes in the diffraction problem between case 1 and 2
3. In case 2, what is the effect of the polarization of the light source?
4. Now consider shortening the wavelength, i.e. use electrons
5. What is the wavelength of an electron for a given energy?
6. What happens when a lot of electrons flow through a looped wire?
    a. Calculate the magnetic field some distance R away from the loop
7. How can we create a bundle of closely spaced parallel B-field lines?
8. Describe the diffraction pattern that results when we use electrons of certain energy in set up of question 2.
9. We now place a small bundle of magnetic field lines close to the center between the slits.
    a. What will be the effect on the diffraction pattern? Why?

4. Electronic Circuits

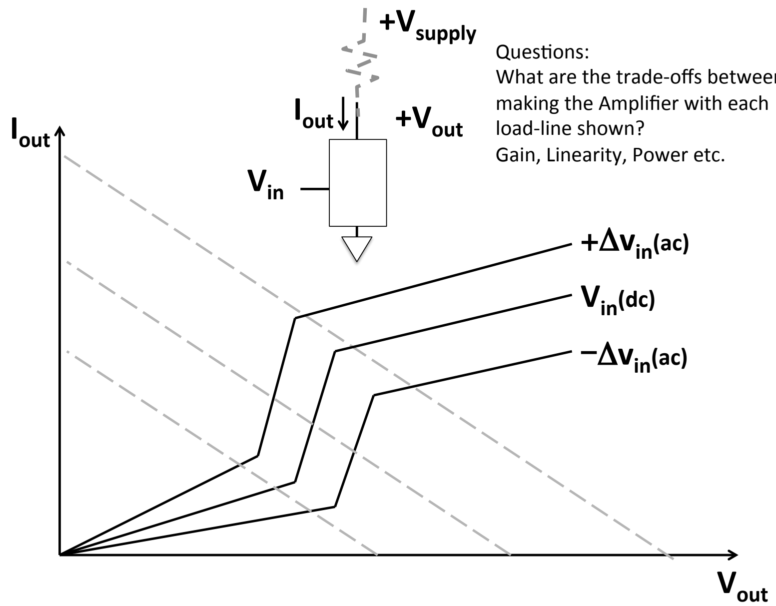
<u>Choices For Components</u>:
R, L, C, Diode



<u>Choices For Components</u>:
R, L, C, Diode



Follow-on Question: How can you tell if the Component is a Cap. Vs. a Diode



Questions:
What are the trade-offs between making the Amplifier with each load-line shown?
Gain, Linearity, Power etc.

2012 Quals question:

Students were asked to examine a simple electronic circuit consisting of six commercially available components. The circuit produced rhythmic flashes of light and students were asked to explain the operation of the circuit.

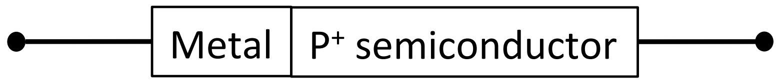
Thanks,
Greg Kovacs

5. Electronic Devices

2011-2012 PhD Qualifying Examination
Professor Yoshio Nishi

1. Draw typical drain current-drain voltage characteristics of n channel MOSFET(long channel) made of Si if measured at room temperature for Vgs=0, 0.5, 2.0v. Let's assume the channel doping concentration is 10E17cm^{-3}, V_T=0.5V and V_{BD} (drain breakdown voltage at Vg=0) =5V at room temperature.
2. What would happen if you measure it at 50K?
3. What would happen if you measure it at 700K?
4. If there is no scattering of electrons, how would the Id-Vd characteristics look like at room temperature, and why?

Can light emission take place from a metal/p$^+$ semiconductor junction?



2012 Qual Exam Questions

Prof. H.-S. Philip Wong

1. Consider an n-channel MOSFET designed for 1V operation with a metal to semiconductor source/drain contact (sometimes, this is called a Schottky barrier FET). Draw the band diagram in the direction normal to the Si/SiO2 interface.
2. Next, draw the band diagram from the source to the drain assuming the nMOSFET is above the (small, positive, say 0.4V) threshold voltage and the drain voltage is small compared to the applied gate bias.
3. Now, if there is a gap between the edge of the gate and the source and drain metal to semiconductor junction, draw the band diagram again. Assume the gate dielectric extends across the ungated gap region.
4. If I now insert a dielectric layer on top of the gate dielectric. This gate dielectric has a dielectric constant of 20 and there are trapped positive charges between this 2^{nd} gate dielectric and the original gate dielectric. Draw the band diagram again.
5. What does the band diagram look like if instead of isolated positive charges, there are electric dipoles (+ve on the 2^{nd} gate dielectric side and –ve on the 1^{st} gate dielectric side).
6. Sketch the Id vs Vgs and Id vs Vds curves for case #4 and case 5 above.

The more questions you can answer, the more points you will get in the exam.

6. Engineering Physics

EE Ph.D. Qualifying Exam, January 2012 Question

David Miller

Energy in charging and discharging capacitors

Notes:

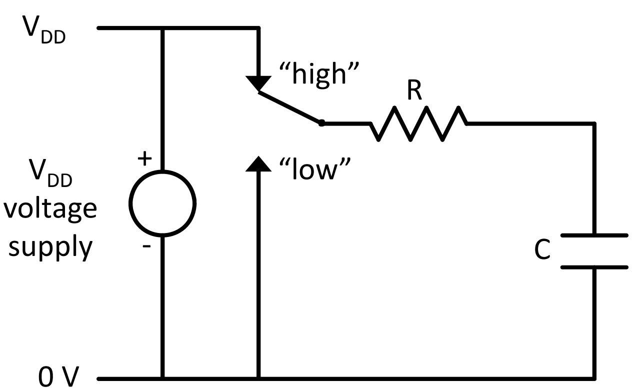
There may not be single "correct" answer to parts of this question. The goal of this question is to see how you think about it.

If you finish the question on this sheet, subsequent questions will be asked.

[In the exam, most students got through the main question, possibly with some help. Most of those then got through the first supplementary question. A fair fraction got to the second supplementary question, though only a few had time to finish that. Those that did finish mostly also got through the third supplementary question.]

Question:

In the circuit below, the switch has initially been connected in the "low" position for a long time; at the start of our experiment, therefore, the capacitor C is completely discharged. Then we move the switch to the "high" position and leave it there for a long time, charging the capacitor up to a voltage V_{DD}.



(i) What electrostatic energy is now stored in the capacitor C?

(ii) What energy has been dissipated in the resistor R during the charging process?

(iii) What total energy has been provided by the V_{DD} power supply?

Solution

(i) The electrostatic energy now stored on the capacitor is $\frac{1}{2}CV_{DD}^2$

Essentially all the students knew this answer, and they were not asked to prove or derive it.

(ii) A few students already knew or intelligently guessed that the answer to this part is also $(1/2)CV_{DD}^2$, though I would require them to justify this result. There are two main ways of answering this part. The first is to attempt some integration of the power or energy dissipated in the resistor. The most common approach students would take here was to integrate the known exponential behavior of the current or voltage in time, using either I^2R or V^2/R formulae for the power being dissipated in the resistor at a current I or voltage V. A more compact version is to integrate over voltage directly. The energy dissipated in flowing a charge δQ through a resistor at a voltage V is $\delta E = V\delta Q$. When flowing a charge δQ onto a capacitor C, the resulting change in voltage δV on the capacitor is such that $\delta Q = C\delta V$. In the circuit, the voltage V across the resistor R is $V = V_{DD} - V_{OUT}$. Hence the total energy dissipated in the resistor in charging the capacitor C from 0V to V_{DD} is

$$\Delta E_R = C\int_0^{V_{DD}} \left(V_{DD} - V_{OUT}\right) dV_{OUT} = \frac{1}{2}CV_{DD}^2$$

No student actually took this approach to start with. Anyway, once I could see that the student would be able to work out this energy by some integral approach, I generally stopped them, taking it for granted that eventually they would get to the right answer here.

The easy way to solve this problem is to solve part (iii) first, which avoids all this integration. Students were told to look at parts (ii) and (iii) together, though very few actually did that!

(iii) Some students would try to approach this problem (correctly, though not optimally) by figuring out an answer to part (ii) and adding it to the answer to part (i). The easy way to solve this is simply to calculate the energy that must be supplied by the power supply because it provides a charge $Q = CV_{DD}$ through the circuit to charge the capacitor. That energy is $E = QV_{DD} = CV_{DD}^2$. Few students got this without help. The temptation to integrate the power over time was generally too strong. Most students also had temporarily forgotten that Volts are Joules/Coulomb and that by definition in moving a 1 Coulomb charge through 1 Volt I must do 1 Joule of work. In general, students tended to be too frozen in to thinking in powers rather than energies in this whole problem, which made it harder for them than ideally it needed to be.

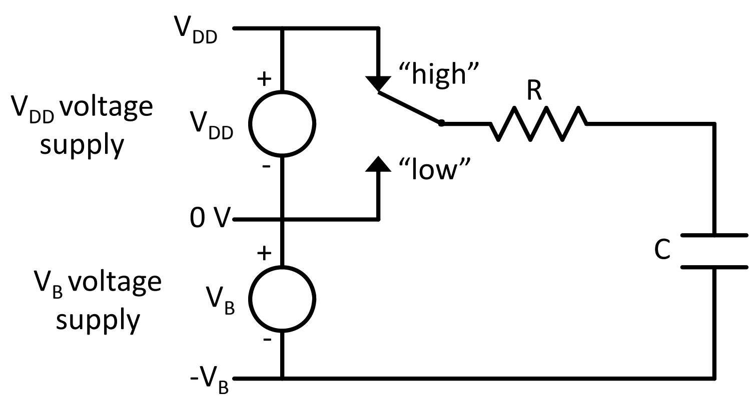
Armed with the result CV_{DD}^2 from this part, the answer to part (ii) is easy to deduce by conservation of energy.

Supplementary question 1

How would the above answers change if the resistor was nonlinear – i.e., the current through it was not proportional to voltage (as could be the case in a transistor or a diode, for example)?

Supplementary question 2

Now consider the circuit below. Here, initially, the switch has been connected in the "low" position for a long time; at the start of our experiment, therefore, the capacitor C has a voltage V_B across it. Then we move the switch to the "high" position and leave it there for a long time, charging the capacitor up to a voltage $V_{TOT} = V_{DD} + V_B$.



(a) What has been the change in the electrostatic energy stored in the capacitor C as a result of moving the switch to the "high" position?

(b) What energy has been dissipated in the resistor R during the charging process?

(c) What energies have been provided by each power supply (i.e., by the V_{DD} power supply and by the V_B power supply)?

(d) What happens to the energy provided by the capacitor C to the circuit when we now connect the switch back to the "low" position?

Supplementary question 3

Can you think of a way in which we can avoid dissipating the energy that the capacitor tries to put back into the V_B power supply?

Supplementary question 1 solution

Note that the energy calculated in part (ii) abo ve is quite independent of the value of the resistance R, and does not require any particular relation between current and voltage for that resistor; hence the resistor may be nonlinear and there will be no difference in the energy results here. This answer is particularly easy to see if the dissipation in the resistor is deduced by subtracting the energy stored in the capacitor from the total energy provided by the power supply; again, no particular relation between voltage and current is required in the resistor (at least as long as finite current flows at all voltages).

Supplementary question 2 solution

(a) The change in energy stored in the capacitor is the difference in energy between having a voltage V_B over the capitor and having a voltage $V_B + V_{DD}$ over it, which is

$$\Delta E = \frac{1}{2}C\left[\left(V_B + V_{DD}\right)^2 - V_B^2\right] = \frac{1}{2}CV_{DD}^2 + CV_{DD}V_B$$

(b) The answer to this is $(1/2)CV_{DD}^2$, exactly as in the main question, because the situation as seen by the resistor is exactly the same as it was in the first part, charging up a capacitor from 0V to V_{DD}, passing a charge $Q = CV_{DD}$ onto the capacitor just as before.

(c) The same charge $Q = CV_{DD}$ has to be passed through both power supplies (they are in series), so the energies they provide are, for the V_{DD} supply, CV_{DD}^2, and for the V_B supply $QV_B = CV_{DD}V_B$ (which we recognize as the second term on the far right on the above equation). Note that the energies do all add up correctly here, with the energy provided by the power supplies equaling ΔE plus the energy dissipated in the resistor.

$$CV_{DD}^2 + CV_B V_{DD} = \Delta E + \frac{1}{2}CV_{DD}^2$$

(d) When we discharge the system back to zero volts, an energy $(1/2)CV_{DD}^2$ is dissipated in the resistor, because as far as the resistor is concerned, we are discharging a capacitor from V_{DD} to 0V so all the currents and dissipations look the same as for such a problem. We are therefore left with an energy of magnitude $QV_B = CV_{DD}V_B$. This is energy that the circuit now tries to put back into the V_B power supply. Whether it can do that successfully depends on the nature of the power supply. If it is just an ordinary non-rechargeable battery, then the energy will end up dissipated as heat instead.

Supplementary question 3 solution

The answer is to make the V_B power supply effectively rechargeable, and the easiest way to do that is to put a large capacitor across it (sometimes called a bypass capacitor). A large capacitor effectively functions as a perfectly rechargeable power supply. In an actual circuit we might formally decouple the V_B power supply from the circuit with series resistance and/or inductance, leaving this bypass capacitor as the effective local, perfectly rechargeable “battery”.

Pease's Quals question 2012

When troubleshooting electrical equipment containing voltages 100 to 300 we are advised to stand on an insulating sheet. Why?

BUT when the equipment has voltages about 50,000V we are advised to be sure we are grounded. Why?

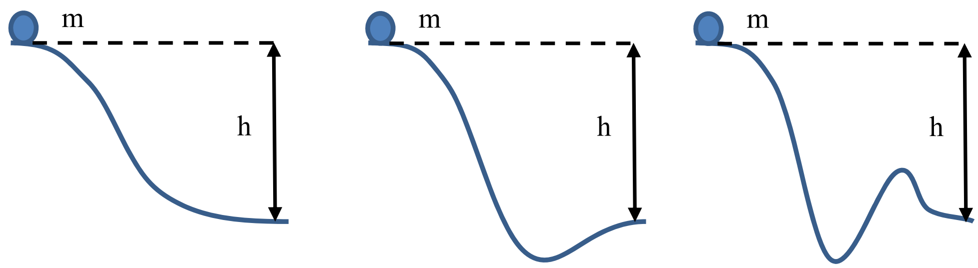
How much energy do we store when we are charged to 50,000V?

Estimate your capacitance. Either isolated or standing on a 1cm thick insulator.

How would you measure your capacitance?

EE Quals Question 2012, Olav Solgaard

1. What is the velocity of the ball after it has rolled down the ramp? Assume negligible friction and that the initial velocity is zero in each example.



$$mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh}$$

2. How can you use this to find an expression for the resonance frequency of a mechanical harmonic oscillator (spring-mass system)?

Set the maximum stored kinetic energy equal to the maximum stored potential energy, and assume harmonic motion of the mass to find: $\omega = \sqrt{k}/m$

3. How does the resonance frequency change (up, down, or unchanged) if the spring sets the mass into rotation as it is elongated? Assume that the rotation speed is proportional to the linear speed.

The resonance frequency goes down.

4. How can you use the same thinking to find the shape of a beamed curved by positioning and twisting its ends (positioning the ends and fixing their angles)?

The curve with the minimum stored elastic energy is the correct solution.

EE QUALIFYING EXAMINATION

January 23 – 27, 2012

Yoshihisa Yamamoto

1. What is the difference between amplifiers and oscillators? Discuss the difference in terms of external pumping levels with respect to oscillation threshold and applications.

2. What is the difference between negative conductance oscillators/amplifiers and nonlinear susceptance oscillators/amplifiers?

3. What is the difference between pre-amplifiers and on-line amplifiers?

Clearly state any assumptions you make while solving the problems. Good luck!

1. Multiple slit interference

Consider a setup consisting of two parallel screens, separated by distance d.
The first screen has very narrow slits on it, while the second screen is used for imaging. The separation between the slits (s) is much smaller than the distance between the screens (d).
Figures 1a and 1b show the first screen with 2 and 3 slits, respectively. A monochromatic light source is used to illuminate the first screen from the back, as shown in the figures.

a) How would the projected image (intensity distribution) on the second screen look for 2 slits on the first screen?

b) What about the intensity distribution on the second screen in case of 3 slits on the first screen? How is it different from the result in (a)?

c) How would the image on the second screen look for N slits on the first screen, where N is an arbitrary positive integer? (Same as in parts (a) and (b), assume that the separation between neighboring slits is s.)

d) Do you expect the central point (O) of the second screen to be bright or dark for an arbitrary N? Why?

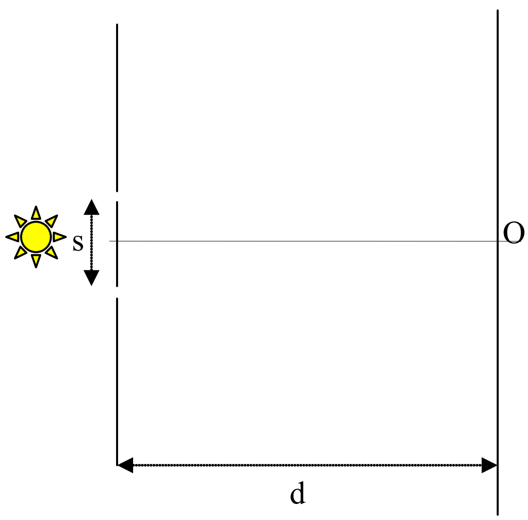


Figure 1a

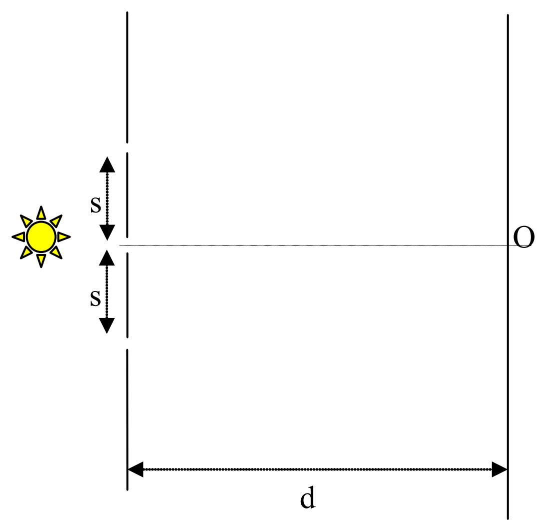


Figure 1b

2. Quantum box

Suppose there is a tiny, quantum box with dimension L, and a particle with mass m confined inside of it. What can you tell about the momentum of this particle?

7. Signals

January 2012

The questions are boxed for emphasis.

Reminders of useful facts from previous slides are in blue.

Solutions to R.M. Gray's 2012 qualifying exam problem.
Solutions include much more information than was expected from the student during the exam, specific exams could emphasize one thread or another depending on the student's performance on the earlier material.

Consider the following discrete-time system with input $x[n]$ and output $\epsilon[n]$ defined by the input/output relation

$$x[n] \longrightarrow \boxed{\mathcal{S}_1} \longrightarrow \epsilon[n] = x[n] - ax[n-1]$$

Only causal signals are allowed, i.e., $x[n] = 0$ **for all** $n < 0$**.**

First Question: Given the following definitions and assumptions:

$$\begin{aligned}
< x > &\triangleq \sum_{n=0}^{\infty} x[n] = 0 \quad \text{mean} \\
< x^2 > &\triangleq \sum_{n=0}^{\infty} x[n]^2 = \mathcal{E}_x < \infty \quad \text{energy} \\
r_x(k) &\triangleq \sum_{n=k}^{\infty} x[n]x[n-k] \quad \text{autocorrelation } (\mathcal{E}_x = r_x(0)) \\
X(f) &\triangleq \sum_{n=0}^{\infty} x[n]e^{-j2\pi fn}; -\frac{1}{2} \le f \le \frac{1}{2} \qquad \text{DTFT,}
\end{aligned}$$

Find simple expressions for $< \epsilon >, \mathcal{E}_\epsilon$, and the DTFT $E(f)$ of $\epsilon[n]$.

Solution: Using the fact that $x[-1] = 0$ since inputs must be causal,

$$\begin{aligned}
< \epsilon > &= \sum_{n=0}^{\infty} \epsilon[n] = \sum_{n=0}^{\infty} (x[n] - ax[n-1]) = \sum_{n=0}^{\infty} x[n] - a \sum_{n=0}^{\infty} x[\underbrace{n-1}_{n'}] \\
&= < x > -a \sum_{n'=0}^{\infty} x[n'] =< x > -a < x >= 0 \\
\mathcal{E}_\epsilon &= \sum_{n=0}^{\infty} \epsilon[n]^2 = \sum_{n=0}^{\infty} (x[n] - ax[n-1])^2 \\
&= \sum_{n=0}^{\infty} \left(x[n]^2 - 2ax[n]x[n-1] + a^2x[n-1]^2\right) \\
&= r_x(0) - 2ar_x(1) + a^2r_x(0) = (1 + a^2)r_x(0) - 2ar_x(1) \\
E(f) &= \sum_{n=0}^{\infty} \epsilon[n]e^{-j2\pi fn} = \sum_{n=0}^{\infty} (x[n] - ax[n-1])\, e^{-j2\pi fn} \\
&= \sum_{n=0}^{\infty} x[n]e^{-j2\pi fn} - a \sum_{n=0}^{\infty} x[\underbrace{n-1}_{n'}]e^{-j2\pi fn} = X(f) - a \sum_{n'=0}^{\infty} x[n']e^{-j2\pi f(n+1)} = X(f)(1 - ae^{-j2\pi f})
\end{aligned}$$

or just quote linearity and the shift property of DTFTs

As before:

$$x[n] \longrightarrow \boxed{\mathcal{S}_1} \longrightarrow \epsilon[n] = x[n] - ax[n-1]$$

Autocorrelation $r_x(k) = \sum_{n=k}^{\infty} x[n]x[n-k]$

Error energy $\mathcal{E}_\epsilon = \sum_{n=0}^{\infty} \epsilon[n]^2 = \sum_{n=0}^{\infty} (x[n] - ax[n-1])^2$

Next Question:
Suppose $ax[n-1]$ is interpreted as a linear prediction of $x[n]$ based on a single past sample so that $\epsilon[n]$ is the linear prediction error sequence.

$$\boxed{\text{What value of } a \text{ minimizes } \mathcal{E}_\epsilon\text{?}}$$

Solution: Use calculus: Solve for

$$0 = \frac{d}{da}\mathcal{E}_\epsilon = \frac{d}{da}\big((1+a^2)r_x(0) - 2ar_x(1)\big) = 2ar_x(0) - 2r_x(1)$$

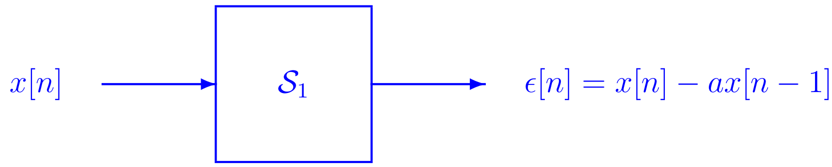
or $a = r_x(1)/r_x(0)$. Note that since $r_x(0) = \mathcal{E}_x \geq 0$, the second derivative satisfies

$$\frac{d^2}{da^2}\mathcal{E}_\epsilon \geq 0$$

so that the $a = r_x(1)/r_x(0)$ indeed minimizes $\mathcal{E}_\epsilon$. The resulting minimum is

$$\begin{aligned}
\mathcal{E}_\epsilon &= (1+a^2)r_x(0) - 2ar_x(1) = \left[1 + \left(\frac{r_x(1)}{r_x(0)}\right)^2\right] r_x(0) - 2\left(\frac{r_x(1)}{r_x(0)}\right) r_x(1) \\
&= r_x(0) + \frac{r_x(1)^2}{r_x(0)} - 2\frac{r_x(1)^2}{r_x(0)} = r_x(0) - \frac{r_x(1)^2}{r_x(0)}
\end{aligned}$$

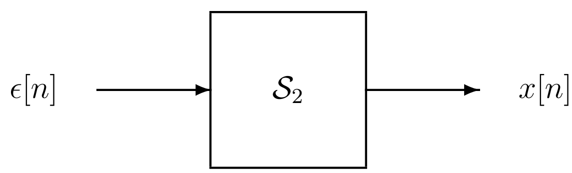
As before:



Notation: Kronecker delta function $\delta[n]$ for integer n defined by

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Next Question:
A system $\mathcal{S}_2$ is implied by the block diagram below.



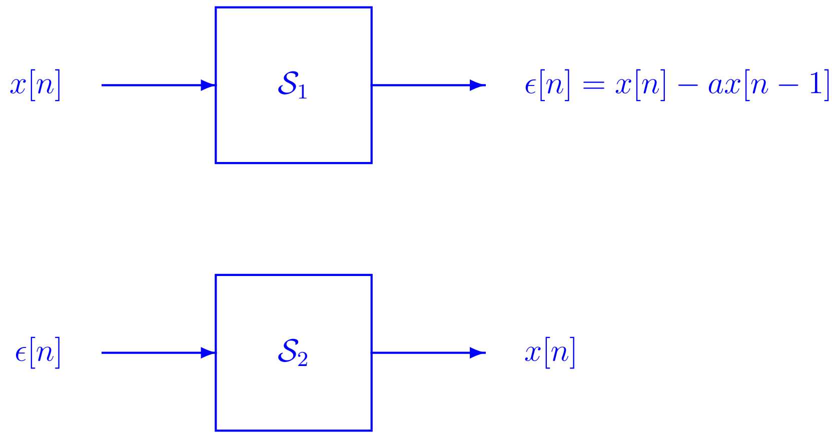
$$\boxed{\text{Find the response } g[n] \text{ of } \mathcal{S}_2 \text{ to an input } \delta[n].}$$

Solution: The question effectively asks for the inverse filter. There are many ways to approach this, some of which can get quite complicated. The simplest way to solve the problem is to directly find linear difference equations that produce $x[n]$ from $\epsilon[n]$. To do this just rewrite the equation for the output to put $x[n]$ alone on the left and iterate back to time 0:

$$\begin{aligned} x[n] &= \epsilon[n] + a \underbrace{x[n-1]}_{\epsilon[n-1]+ax[n-2]} \\ &= \epsilon[n] + a\epsilon[n-1] + a^2 \underbrace{x[n-2]}_{\epsilon[n-2]+ax[n-3]} \\ &\vdots \\ &= \epsilon[n] + a\epsilon[n-1] + a^2\epsilon[n-2] + \cdots + a^n\epsilon[0] \\ &= \sum_{k=0}^{n} \epsilon[k]a^{n-k} \end{aligned}$$

If the input is $\epsilon[k] = \delta[k]$, then the output at time n is $g[n] = a^n$. The first line of the above equations can also be used to identify the filter as a first-order *autoregressive* or *all-pole* filter defined by the linear difference equation $x[n] = \epsilon[n] + ax[n-1]$

As before:



Next Question:

$$\boxed{\text{What are the eigenvalues and eigenfunctions of the system } \mathcal{S}_1\text{?}}$$

$$\boxed{\text{of the system } \mathcal{S}_2\text{?}}$$

Solution: An eigenfunction $u[n]$ of a discrete-time system is a nonzero signal with the property that an input of $u[n]$ yields an output of $\lambda u[n]$, where λ is the associated eigenvalue. From linear systems theory, discrete-time time-invariant systems have complex exponentials as eigenvalues, that is, signals of the form

$$u[n] = e^{j2\pi fn}$$

with a corresponding eigenvalue of $H(j2\pi f)$, where H is the DTFT of the Kronecker delta response (the *system function*) and where f is any real number in $[-1/2, 1/2]$. (Note that the eigenfunctions are not in the family of inputs considered up until now since they do not have finite energy.) Both $\mathcal{S}_1$ and $\mathcal{S}_2$ are LTI systems and hence have such eigenfunctions.

For $\mathcal{S}_1$, using the previously found expression for $E(f)$ yields $H(f) = E(f)/X(f) = 1 - e^{-j2\pi f}$. Alternatively, take the DTFT of the Kronecker delta response $h[n] = \delta[n] - a\delta[n-1]$ to get the same result.

For $\mathcal{S}_2$ you can either observe from the properties of linear systems that

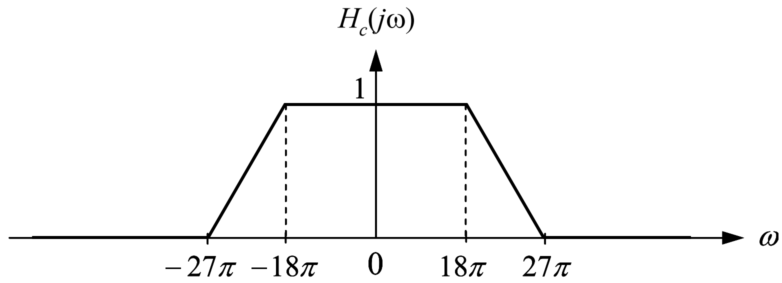
$$G(f) = \frac{1}{H(f)} = \frac{1}{1 - e^{-j2\pi f}}$$

or you can compute $G(f)$ directly using the geometric progression and the geometric Kronecker delta response found earlier.

It is straightforward to verify directly that these eigenfunctions and eigenvalues work by plugging them into the specific linear difference equations describing the two systems.

Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2011-12
Professor Joseph M. Kahn

Consider a continuous-time filter $h_c(t) \overset{FT}{\leftrightarrow} H_c(j\omega)$ having the frequency response shown below.



A sampling frequency $\omega_s = 2\pi/T = 48\pi$ rad/s is assumed. Using three different approaches, a discrete-time filter $h[n] \overset{DTFT}{\leftrightarrow} H(e^{j\Omega})$ is derived from the continuous-time filter. In each case, you are asked to sketch the magnitude response $\left|H(e^{j\Omega})\right|$ and answer a few questions. The discrete-time and continuous-time frequencies are related by $\Omega = \omega T$.

(a) An infinite impulse response filter $h_1[n] \overset{Z}{\leftrightarrow} H_1(z)$ is designed using the impulse invariance criterion:

$$h_1[n] = T \cdot h_c(t)\big|_{t=nT} \ .$$

Sketch the magnitude response $\left|H_1(e^{j\Omega})\right|$. Does aliasing occur?

(b) An infinite impulse response filter $h_2[n] \overset{Z}{\leftrightarrow} H_2(z)$ is designed using the bilinear transformation:

$$H_2(z) = H_c(s)\big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} \ .$$

Sketch the magnitude response $\left|H_2(e^{j\Omega})\right|$. Does aliasing occur? The continuous-time frequencies $\omega_c = 18\pi$ and $\omega_s = 27\pi$ map to discrete-time frequencies Ω_c and Ω_s. Can you obtain expressions for Ω_c and Ω_s?

(c) A finite impulse response filter $h_3[n] \overset{Z}{\leftrightarrow} H_3(z)$ is designed by performing a Fourier series expansion of:

$$H_c\left(j\tfrac{\Omega}{T}\right)$$

over the frequency range $-\pi < \Omega < \pi$. (This is equivalent to performing a Fourier series expansion of $H_c(j\omega)$ over the range $-24\pi < \omega < 24\pi$.) Sketch the magnitude response $\left|H_3(e^{j\Omega})\right|$. Does aliasing occur? Will the Gibbs phenomenon be observed if $h_3[n]$ is not multiplied by a window function?

Solution

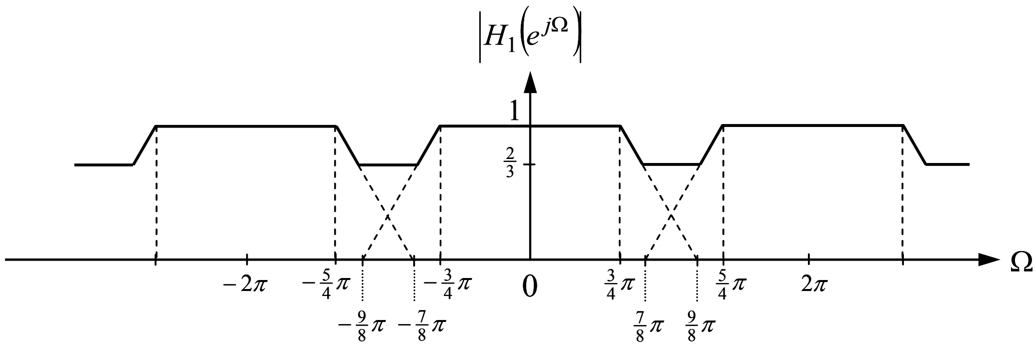
(a) Since $h_1[n] \overset{Z}{\leftrightarrow} H_1(z)$ is designed using the impulse invariance criterion, we have:

$$H_1\left(e^{j\omega T}\right) = \sum_{l=-\infty}^{\infty} H_c\left(j\left(\omega - \frac{l2\pi}{T}\right)\right),$$

In terms of discrete-time frequency Ω, we have:

$$H_1\left(e^{j\Omega}\right) = \sum_{l=-\infty}^{\infty} H_c\left(j\left(\frac{\Omega - l2\pi}{T}\right)\right).$$

A plot of $\left|H_1\left(e^{j\Omega}\right)\right|$ is shown below. Aliasing occurs because the sampling rate is less than twice the highest frequency in $H_c(j\omega)$.



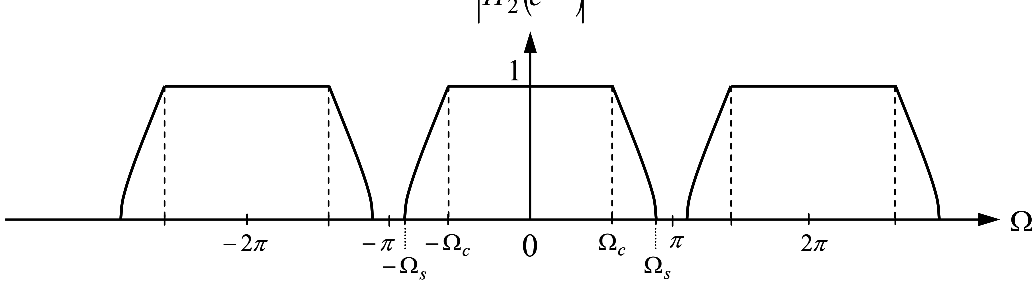
(b) Since $h_2[n] \overset{Z}{\leftrightarrow} H_2(z)$ is designed using bilinear transformation:

$$H_2\left(e^{j\Omega}\right) = H_c(j\omega),$$

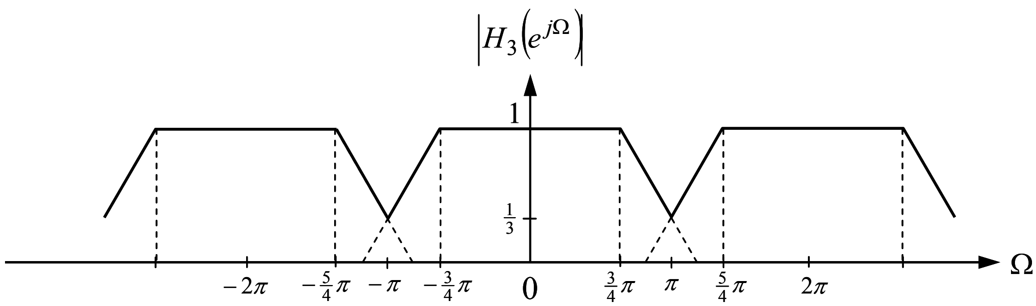
where the discrete-time frequency ω and continuous-time frequency Ω are related by:

$$\Omega = 2\tan^{-1}(\omega T/2).$$

Thus, the passband edge frequency $\omega_c = 18\pi$ maps to $\Omega_c = 2\tan^{-1}(3\pi/8)$, while the stopband edge frequency $\omega_s = 27\pi$ maps to $\Omega_s = 2\tan^{-1}(9\pi/16)$. A plot of $\left|H_2\left(e^{j\Omega}\right)\right|$ is shown below.



(c) Since $h_3[n] \overset{Z}{\leftrightarrow} H_3(z)$ is designed by frequency matching over the Nyquist bandwidth $H_3(e^{j\Omega}) = H_c\left(j\frac{\Omega}{T}\right)$, the magnitude response $\left|H_3(e^{j\Omega})\right|$ is shown below. As in part (b), there is no aliasing. Since $\left|H_3(e^{j\Omega})\right|$ has no discontinuities, there is no Gibbs phenomenon.



1) What is the relationship between the Laplace transform and the Fourier transform of a continuous-time signal?

2) What is the relationship between the Z transform and the discrete-time Fourier transform (DTFT) of a discrete-time signal?

3) What is the relationship between the discrete Fourier transform (DFT) and DTFT of a finite-length discrete-time signal?

4) If an analog sinusoidal signal is input to a digital spectrum analyzer, which samples the analog signal and performs a DFT of the sampled sequence to display its frequency content, what do you expect to see at the display of the spectrum analyzer?

5) If an analog signal $x(t)$ has been sampled according to the Nyquist criterion to generate $x(n)$ where the samples are taken at $t = nT$, can you use a digital filter to generate $y(n)$ from $x(n)$, y(n) being the sequence as if it had been generated by sampling $x(t)$ at $t = nT + 0.67T$?

Suppose you are required to guess the value of a discrete random variable X, of a known PMF, via a sequence of questions of the form "is $X = x$?".

- What questioning strategy is optimal in the sense of minimizing the expected number of trials until guessing correctly?
- What is the value of this minimum expected number of guesses until correctly guessing (inclusive) for $X \sim \text{Poisson}(1)$?
- Repeat the question for $X \sim \text{Poisson}(2)$.

8. Systems

EE Qualifying Exam
January 2012

1. Define
$$\mathrm{III}_p(t) = \sum_{k=-\infty}^{\infty} \delta(t - pk), \quad p > 0.$$
What is $\mathcal{F}\mathrm{III}_p$, the Fourier transform of III_p? Deduce from your answer that III_p is even.

2. Suppose we input III_p into a linear time-invariant system L and measure the output, $w = L\mathrm{III}_p$. Is it possible to recover the impulse response h of the system from this information?