Andrea Montanari

Here is the question I asked
(it is a bit difficult to type it in plain text. I could do it in Tex but don't know if this is ok with you).

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Let H be a matrix with n columns, m rows and entries in {0,1}. Assume that H has 3 ones per row. In the following x denotes a binary vector of length n.

1)
Consider the linear system Hx=0 mod 2.
Let Z(H) be the number of solutions of such a system.
What is the maximum and minimum of Z(H) over all the matrices H.

2)
Assume H to be a uniformly random matrix as above.
(The tree non-vanishing positions in each row are chosen uniformly at random among the n choose 3 possible ones). Let b be a uniformly random binary vector of length m, and Z(H,b) be the number of solutions of the linear system Hx=b mod 2. What is the expectation of Z(H,b)?

3) How does the calculation change if yu want to compute the expectation of Z(H)?

Prof. Andrea Montanari

Quals Question

Consider a checkerboard with $n \times n$ squares. Each square that is not on the boundary has four neighbors that share an edge with it: North, South, East and West. A subset S of the squares is infected at the beginning. Recursively, a square becomes infected if it has at least two neighbors that are infected.

The process stops when either all squares are infected, or when there is no longer a non-infected square that has two or more infected neighbors.

What is the minimum number of initially infected squares (i.e. the size of the smallest set S), so that the all the squares are infected at the end?