Electrical Engineering Quals Questions 2013

http://ee.stanford.edu/academics/graduate-degree-progress/quals

January 2013

Solutions to R.M. Gray's 2013 qualifying exam problem.

The goal of the problem was to test understanding and familiarity with basic probability and expectation in an unfamiliar context.

The problem treats a notion of "distance" between two distributions. The quotes reflect the fact that this is not a distance or metric in the mathematical sense since it does not satisfy the triangle inequality. The square root of the quantity is a distance. This "distance" is very old and goes by many names, including Monge-Kantorovich, transportation, Gini, Wasserstein, and Mallow distance. Most recently it was rediscovered in 1998 in the CS literature and renamed the "earth mover's distance," but its primary origins were in work by Monge in 1781 and Kantorovich in 1942. Kantorovich shared the Nobel prize in economics for the development of linear programming, which is intimately connected with a general version of this distance. It is useful in signal processing and communications as a measure of the mismatch resulting when designing a system for one random variable, but then applying it to another. The distance extends naturally to random vectors and random processes. Here, however, only elementary probability is needed.

Let X be a (real-valued) random variable described by a cumulative distribution function (cdf) $F_X(x) = \Pr(X \leq x)$, which in turn is described either by a probability density function (pdf) $f_X(x) = dF_X(x)/dx$ if X is continuous, or a probability mass function (pmf) $p_X(x)$ if X is discrete. Let Y be another random variable with cdf F_Y etc. A joint cdf for both X and Y is denoted by $F_{XY}(x, y) = \Pr(X \leq x, Y \leq y)$.

Assume throughout that $E(X) = E(Y) = 0$,
$E(X^2) = \sigma_X^2$, $E(Y^2) = \sigma_Y^2$.
Both σ_X^2 and σ_Y^2 are assumed to be nonzero and finite.

A very old and very useful measure of "distance" between two given cdfs F_X and F_Y is defined by

$$\overline{d}(F_X, F_Y) = \min_{F_{XY}} E[(X - Y)^2],$$

where the expectation is with respect to the joint cdf F_{XY} and the minimum is over all joint cdfs F_{XY} having the given F_X and F_Y as marginals.

First Question: Given arbitrary cdfs F_X and F_Y describing random variables X and Y, give a *simple* example of a joint cdf F_{XY} with the prescribed marginals and use it to find an upper bound to $\overline{d}(F_X, F_Y)$ which depends only on σ_X^2 and σ_Y^2.

Solution: Assume that X and Y are independent random variables, in which case $F_{XY}(x, y) = F_X(x)F_Y(y)$ and $E(XY) = E(X)E(Y) = 0$ and hence

$$E[(X - Y)^2] = E(X^2) + E(Y^2) - 2E(XY) = \sigma_X^2 + \sigma_Y^2.$$

Second Question: Again assume you are given arbitrary cdfs F_X and F_Y describing random variables X and Y. Find a nontrivial *lower* bound to $\overline{d}(F_X, F_Y)$ which depends only on σ_X^2 and σ_Y^2.

Solution: The trivial lower bound is 0, since the expected value of the square of a real random variable is nonnegative. As before we know that

$$E[(X-Y)^2] = E(X^2) + E(Y^2) - 2E(XY) = \sigma_X^2 + \sigma_Y^2 - 2E(XY)$$

so to get a lower bound to $\overline{d}(F_X, F_Y)$ we want an upper bound to the correlation $E(XY)$. One of the most important bounds in probability has exactly this form. The Cauchy-Schwartz inequality states that

$$\mid E(XY) \mid \leq \sqrt{E(X^2)E(Y^2)} = \sigma_X \sigma_Y$$

so that for any joint cdf with the required marginals,

$$E[(X-Y)^2] \geq \sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y = (\sigma_X - \sigma_Y)^2$$

and hence

$$\overline{d}(F_X, F_Y) \geq (\sigma_X - \sigma_Y)^2$$

is the desired lower bound.
A few people used the equivalent fact that the correlation coefficient $E(XY)/\sigma_X\sigma_Y$ has magnitude less than 1.

Third Question: Suppose that both X and Y are Gaussian (0 means, variances σ_X^2 and σ_Y^2 respectively).

Find $\overline{d}(F_X, F_Y)$.

Solution: In a short oral exam there is no time for formal optimization. The idea here is to realize (possibly with a hint) that if you can think of a joint distribution with the given marginals which hits the previous lower bound, then that must be the minimum since no joint distribution can do any better. Here there is a natural guess — you can turn a 0 mean Gaussian X with variance σ_X^2 into a 0 mean Gaussian Y with variance σ_Y^2 by a simple scaling $Y = \sigma_Y X/\sigma_X$, which yields

$$\begin{aligned}
E[(X-Y)^2] &= \sigma_X^2 + \sigma_Y^2 - 2E(XY) \\
&= \sigma_X^2 + \sigma_Y^2 - 2\frac{\sigma_Y}{\sigma_X}E(X^2) \\
&= \sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y = (\sigma_X - \sigma_Y)^2.
\end{aligned}$$

Since this is an unbeatable lower bound from the previous part, it must solve the minimization.
Note that here one of the two random variables is defined as a deterministic random variable, which is how Monge defined his distance over two centuries ago. The definition in terms of fixed marginals and a best joint was Kantorovich's. A great deal of research has been done to determine when the two definitions are equivalent.

Fourth Question: Suppose that X is Gaussian with 0 mean and variance σ_X^2 as before, but now Y is discrete with pmf

$$p_Y(y) = \frac{1}{2}; y = \pm\sigma_Y.$$

Find an upper bound for $\overline{d}(F_X, F_Y)$ that is strictly better than the bound of the First Question.

Useful fact: X Gaussian, 0 mean, variance σ_X^2

$$E[|X|] = E[X \mid X \geq 0] = \sqrt{\frac{2}{\pi}}\sigma_X$$

Solution: To get an upper bound, a joint distribution is needed on X, Y with the correct marginals. The first question used a simple product distribution, making the random variables independent. If we want a close mean squared error match, however, we want the two random variables to be correlated, in fact we want them to be maximally correlated if we want to try to achieve the lower bound of Question 2. There are several ways to create a joint distribution with the desired properties. One way is to define Y as a deterministic function of X with

$$Y = \begin{cases} \sigma_Y & \text{if } X \geq 0 \\ -\sigma_Y & \text{otherwise} \end{cases}$$

the resulting joint distribution yields the desired marginals and using conditional expectation results in

$$\begin{aligned} E[(X-Y)^2] &= \sigma_X^2 + \sigma_Y^2 - 2E(XY) \\ &= \sigma_X^2 + \sigma_Y^2 - 2\left[E(XY \mid X \geq 0)\frac{1}{2} + E(XY \mid X < 0)\frac{1}{2}\right] \\ &= \sigma_X^2 + \sigma_Y^2 - 2\sqrt{\frac{2}{\pi}}\sigma_X\sigma_Y \end{aligned}$$

This is not as good as the lower bound of Question 2, but nonetheless it turns out to actually solve the minimization. Here the joint distribution results from one random variable being a deterministic function of the other, and the operation used here is simply a binary quantizer.
Another way to get the same joint distribution is to use a classic result often derived in elementary probability classes. If U is a uniform distribution on $[0, 1]$, then the random variable $F_X^{-1}(U)$ will have cdc F_X, where F_X^{-1} denotes the inverse cdf. Thus given a single uniform U, one can generate random variables with the correct marginals via $(X, Y) = F_X^{-1}(U), F_Y^{-1}(U)$. This results in the same joint distribution as using the quantizer, and hence yields the same bound. But everyone who made a guess chose the previous approach, which amounts to a binary quantizer.

Last Question: I did not expect anyone to get this far, but two people did.
Does the lower bound you found in the previous part actually solve the minimization? That is, does the lower bound equal the distance?

Solution: You might think the bound does not yield the maximum correlation and hence the minimum "distance" since it does not achieve the bound given by Cauchy-Schwartz in Question 2, but it turns out that it is the minimum and the lower bound of Question 2 is not achievable in this nonGaussian example. Intuitively, you can not make X and Y with the given distributions any more correlated then matching their signs (in this example).
To prove that our new bound actually yields the distance, we need to show that for *any* joint distribution *with the given marginals*, it must be true that

$$E(XY) \leq \sqrt{\frac{2}{\pi}}\sigma_X \sigma_Y,$$

the value we actually achieved by a specific joint distribution. While Cauchy-Schwartz is still true here, it is too optimistic, it is not achievable. We need a better bound to the maximum correlation. This question was intended to elicit thoughts on how such an inequality might be proved for this case. One way is to use the method of indicators.
Define the indicator function

$$1(X \geq 0) = \begin{cases} 1 & X \geq 0 \\ 0 & X < 0 \end{cases}$$

and define the other indicator $1(X < 0)$ similarly. Since $1 = 1(X \geq 0) + 1(X < 0)$, we have that

$$\begin{aligned} E(XY) &= E[XY(1(X \geq 0) + 1(X < 0))(1(Y \geq 0) + 1(Y < 0))] \\ &= E[XY1(X \geq 0)1(Y \geq 0)] + E[XY1(X \geq 0)1(Y < 0)] \\ &\quad + E[XY1(X < 0)1(Y \geq 0)] + E[XY1(X < 0)1(Y < 0)] \\ &\leq E[XY1(X \geq 0)1(Y \geq 0)] + E[XY1(X < 0)1(Y < 0)] \end{aligned}$$

since the removed terms are negative. Because Y is binary, the right hand side is

$$\begin{aligned} &\sigma_Y E[X1(X \geq 0)1(Y \geq 0)] - \sigma_Y E[X1(X < 0)1(Y < 0)] = \\ &\qquad \sigma_Y E[X1(X \geq 0)1(Y \geq 0)] + \sigma_Y E[-X1(X < 0)1(Y < 0)]. \end{aligned}$$

Again the terms in the brackets are nonnegative and indicator functions are bound above by 1, so we have

$$E(XY) \leq \sigma_Y E[X1(X \geq 0)] + \sigma_Y E[-X1(X < 0)] = \sigma_Y E(|X|) = \sqrt{\frac{2}{\pi}}\sigma_X \sigma_Y$$

as needed. This proves the bound is actually achieved and hence yields the transportation distance. I did not expect anyone to actually go through this (and there is probably a shorter proof), I was only looking for ideas on how to decompose the expectation using the structure of the given distributions.

2012-2013 PhD Qualifying Examination

Professor Yoshio Nishi

1. Draw band diagrams of 3 different semiconductor MOS diodes, where the semiconductor 1 has a band gap of 0.17eV, the semiconductor 2 has a band gap of 1.1eV and the semiconductor 3 has 3.2eV. All are doped with acceptor dopants of $10^{16}cm^{-3}$
2. Draw room temperature C-V characteristics of those 3 MOS diodes, and explain how they behave when you increase the temperatures to 500C and 900C, given that those semiconductors withstand at those temperatures.
3. Explain Id-Vg and Id-Vd characteristics of nMOS FETs made of those semiconductors, where metal-semiconductor work function difference is zero in all cases

2013 Qual Exam Questions

Prof. H.-S. Philip Wong

I would like to have a solid-state imaging device that takes a picture of the room in the visible wavelengths, in color.

You are given the following materials and the use of a semiconductor fabrication facility: Si, Ge, SiO_2, Si_3N_4, TiO_2, HfO_2, Al_2O_3, Al, Cu, W.

Would you please make a proposal for such a COLOR solid-state imaging device using only those materials listed above? Trace elements such as dopants of semiconductors can be assumed to be available.

Please explain the physics and the operation principles of the proposed device.

2013 Qualifying Exam
Simon Wong

An n-channel MOSFET with $V_{TH} = 0.5V$, $V_G = 1V$, $V_S = 0V$, sketch I_D versus V_D.

Why does I_D saturate ?
Channel pinches off near the drain.

Will I_D stay saturated at very high V_D ?
No, at high V_D, I_D will increase rapidly.

What is the mechanism responsible for the rapid increase in I_D ?
There are at least 4 possible mechanisms:

1. Darin-induced-barrier-lowering, also known as punch through
2. Avalanche (in the channel) induced breakdown
3. Drain-substrate junction breakdown
4. Drain-gate oxide breakdown

The dominant mechanism depends on the channel length, gate oxide thickness, and doping levels in the junction, channel and substrate.

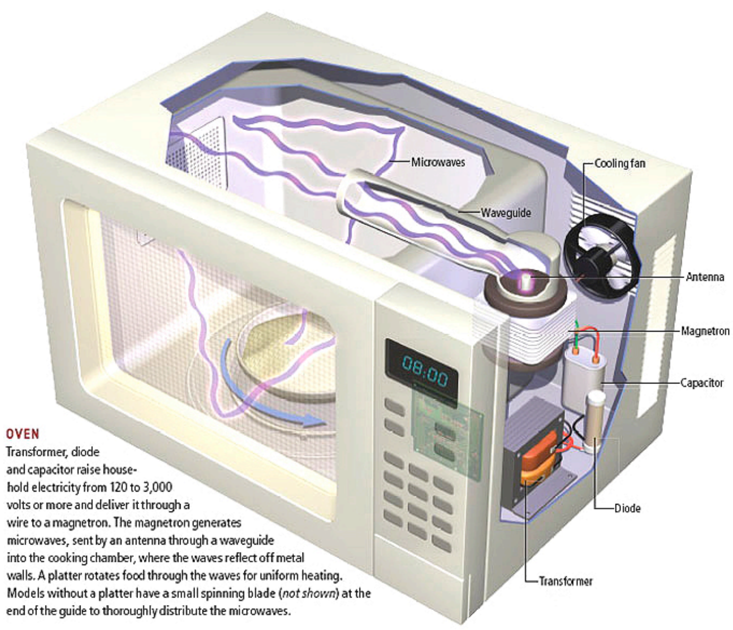
How can you distinguish which mechanism is responsible for the rapid increase in I_D ?
Measure all currents, I_D, I_S, I_G, and I_{SUB}. Assume the device has been destroyed yet.

1. Drain-induced-barrier-lowering
   $I_D \cong I_S, I_G \cong 0, I_{SUB} \cong 0$
2. Avalanche induced breakdown
   $I_D \cong I_S, I_G \cong 0, I_{SUB} \neq 0$ but $<< I_D$
3. Drain-substrate junction breakdown
   $I_D \cong I_{SUB}, I_S < I_D, I_G \cong 0$
4. Drain-gate oxide breakdown
   $I_D \cong I_G, I_S < I_D, I_{SUB} \cong 0$

Microwave Ovens: EE Quals Question 2013
A. C. Fraser-Smith

(1) I start by stating that we are going to talk about how microwave ovens work and I then ask what is actually meant by ‘microwaves’ here? Are we dealing with ultrasound or electromagnetic waves? Micro implies something small – how small?”

It is not all that simple distinguishing between the use of ultrasound or em waves in a microwave oven simply by looking at one. The prohibition against putting metal objects inside was considered a clue, as was the grill (assumed to be metal) in the window. Having been told that em waves were used is not a particularly bad answer. Turning to the “micro,” many students state that it relates to the wavelength of the waves, i.e., they are around 10^(-6) of a meter. At this stage they are told that the frequency of operation of typical commercial microwave ovens is 2.45 GHz – some students know this! So, what is the wavelength (in free space), assuming, say that the frequency is ~3 GHz? A quick computation of $\lambda = c/f$ gives $\lambda = 0.1$m or 10 cm. Obviously the “micro” is a misnomer. A few students knew that when it first became possible to generate microwaves in the late 1930’s the wavelengths were “micro” in comparison with the wavelengths of the radio waves in common use at the time. (2 points)



(2) Given that there is some device within a microwave oven that generates microwaves and radiates them into the chamber of the oven, what, in your opinion, are the important features of the oven from the electromagnetic point of view?

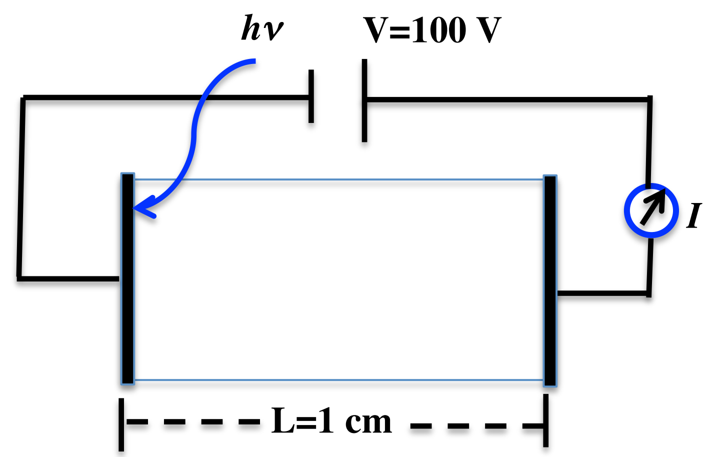
The device generating the microwaves is a magnetron with an attached antenna. Many students knew this but it was treated as being knowledge and not considered part of the exam. However, the following were important. (i) The microwaves enter the oven's "cavity" and it obviously must have metal walls to contain the radiation. Plastic, i.e., non-conducting, walls would allow it to leak out, reducing the oven's efficiency and making the radiation a safety hazard. (ii) The window in the oven's door contains a metal mesh that can be seen though but which prevents the microwaves from passing through. (iii) the holes in the mesh are roughly 1 mm in size, which is much less that the ~10 cm wavelength of the radiation. The mesh is seen as a solid metal sheet. Light waves can easily penetrate, though, since their wavelengths are much less than ~ 1mm. (iv) With its radiation confined to a metal cavity, without much loss, it must be undesirable to operate a microwave oven empty. Often the instructions for the ovens include a warning about this use. (v) Pointed, or sharp-edged, metal objects will have voltages induced in them that might lead to sparking at the points and edges, causing them to become a fire-hazard. (vi) Although the oven's cavity is not necessarily a resonant cavity its em fields will have some form of standing wave pattern. Thus the oven may not heat uniformly. We can guess that the spacing between the hot spots will be about half a wavelength (two peaks of electric field per wavelength), or around 5 cm. This is why the ovens often have a rotating plate on the bottom (and in some cases there is a metal "paddle" rotating where the microwaves come in, to help disperse them). (5 points)

(3) How do you think the microwaves heat food?

It is common knowledge that the microwave heating involves the water molecules in the food but from this point the process becomes murky. Students thought the microwaves might resonate with the molecules and/or cause them to rotate vigorously. Some thought the microwaves would heat the food from the inside! We investigated these possibilities after the water molecule was described as being dipolar with a dipole moment (negative on the oxygen end and positive on the end where the two hydrogen atoms are bound). The students were also told that although it is possible to set up internal resonances in individual water molecules the frequencies involved are outside the microwave range. With this information we decided: (i) that the water molecules would first rotate in one direction to line up with the electric field of the wave and then rotate in the opposite direction as the electric field oscillated. There would be no net rotation and the molecule would feed energy into the water and its surroundings by oscillating backwards and forwards and interacting with the surrounding molecules as it did so, i.e., heating by collisions. (ii) we dismissed the concept of the microwaves heating food from the inside out, since they would have to penetrate into the food from the outside, heating it and losing intensity as they penetrated. Nevertheless, (iii) because food is unlikely to be a good conductor, the skin depth at microwave frequencies could well be on the order of a wavelength and the food should be heated relatively uniformly. (3 points)

2013 PhD Qualifying Exam Questions—J. Harris

1. I have the structure illustrated here with two metal electrodes separated by 1cm and I put it into vacuum. I then illuminate the cathode with a pulse of light.



   a. Is there any requirement on the threshold energy of the photons to measure a current at the anode?
   b. When I measure a current, what is the current density as a function of time? Plot it.
   c. Since the external circuit requires current continuity, how do I explain the current through vacuum?
   d. What is the maximum current density?
   e. What is the length of time (known as the transit time) that there will be a measurable current?

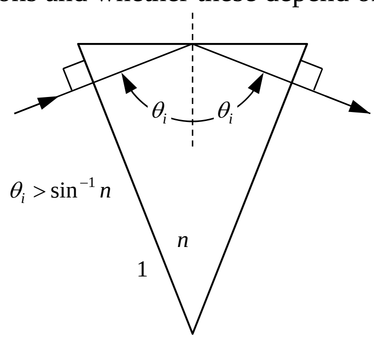
2. I now fill the void between the electrodes with a semiconductor and the electrodes form Ohmic contacts to the semiconductor and I illuminate it with a pulse of light.
   a. Is there any difference in the photon energy from the prior case? Why?
   b. What is the current density in this case? Plot it as a function of time. Why is it different than the prior case?
   c. What is the transit time in this case?

3. The Einstein Relationship is used extensively in describing the transport of carriers in semiconductors.
   a. Do you know what physical foundation or assumptions are to derive this relationship?
   b. Can you derive of describe how to simply derive the Einstein Relationship?
   c. Would this apply in the description of transport at very high electric fields?

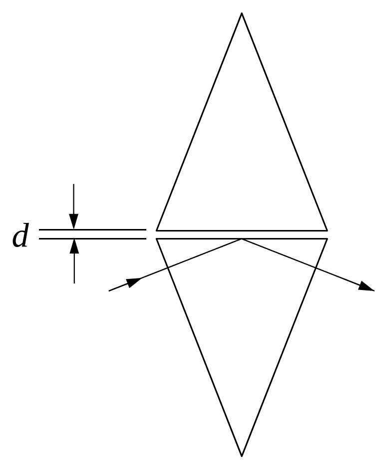
2013 EE Qualifying Examination
Electromagnetics
Professor Joseph M. Kahn

Questions

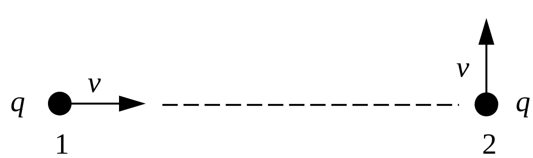
1. a. Total internal reflection occurs at the upper surface of the prism. Describe any fields in the region above that surface. Sketch surfaces of constant phase. Sketch surfaces of constant amplitude. Describe any propagation of the fields and any associated energy flow. Describe their relation to boundary conditions and whether these depend on the vector nature of the fields.



   b. A second identical prism is brought close to the first prism. Describe what happens to the propagating beam as a function of the gap spacing *d*.

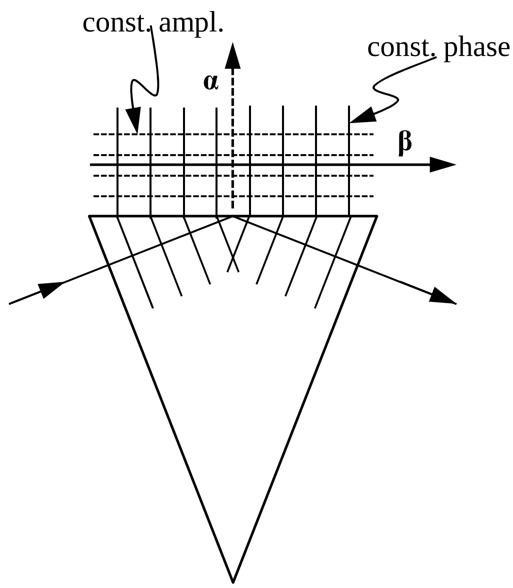


2. At a particular instant of time, two particles of equal charge q are positioned as shown and are moving at speed v as shown. Describe the forces on the two particles. Are these equal and opposite?

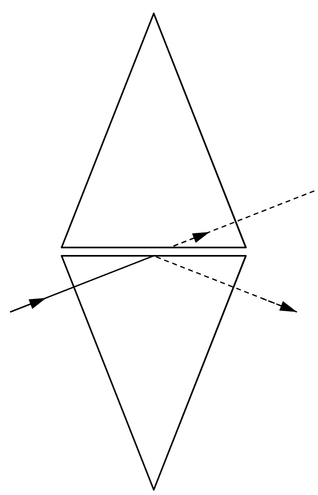


Answers

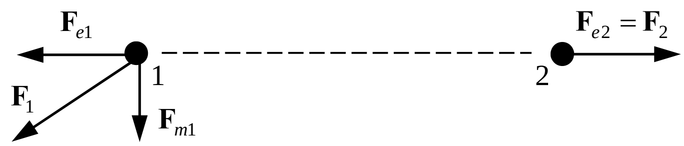
1. a. The fields in the upper region are an evanescent wave. This is a nonuniform plane wave with complex propagation vector $\boldsymbol{\gamma} = \boldsymbol{\alpha} + j\boldsymbol{\beta}$. In a lossless medium, a nonuiform plane wave must have $\boldsymbol{\alpha} \perp \boldsymbol{\beta}$. The evanescent wave propagates parallel to the boundary but is attenuated exponentially away from the boundary. Surfaces of constant phase and amplitude are perpendicular to $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$, respectively, as shown. There is a flow of energy along the surface, but since the plane wave is implicitly assumed to be infinite in cross section, no energy enters or leaves the system. The surfaces of constant phase are continuous across the boundary. This continuity is a consequence of the wave nature of the fields, and has nothing to do with their vector nature (as do the the continuity of normal **B**, **D** and tangential **H**, **E**).



b. When the gap width *d* is zero, all of the light propagates forward into the second prism. As *d* increases, the transmitted amplitude decreases exponentially.



2. The two particles are subject to repulsive electric forces $\mathbf{F}_{e1}$ and $\mathbf{F}_{e2}$, which are equal and opposite. The motion of particle 1 creates a magnetic field but this is zero at particle 2, so it exerts no force on particle 2. The motion of particle 2 creates a magnetic field that is nonzero at particle 1, and this exerts a magnetic force $\mathbf{F}_{m1} = q\mathbf{v} \times \mathbf{B}$ on particle 1. The total force on particle 1 is $\mathbf{F}_1 = \mathbf{F}_{e1} + \mathbf{F}_{m1}$, while that on particle 2 is $\mathbf{F}_2 = \mathbf{F}_{e2}$. These total forces are not equal and opposite. An explanation lies beyond the scope of the exam, but lies in the fact that the fields contain linear momentum that is changing as the particles move. It has nothing to do with relativistic effects, nor with radiation.



Prof. Andrea Montanari

Quals Question

Consider a checkerboard with $n \times n$ squares. Each square that is not on the boundary has four neighbors that share an edge with it: North, South, East and West. A subset S of the squares is infected at the beginning. Recursively, a square becomes infected if it has at least two neighbors that are infected.

The process stops when either all squares are infected, or when there is no longer a non-infected square that has two or more infected neighbors.

What is the minimum number of initially infected squares (i.e. the size of the smallest set S), so that the all the squares are infected at the end?

2013 Quals Questions

Hi --

Here they are.

Thanks!

-- John Pauly

Question 1:
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f(t) is a real, causal signal.

Given Re{F(s)}, for s>=0, can you find f(t)?

Question 2:
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f(t) is a signal bandlimited to ± B.

How many times can it cross zero over an interval from 0 to A?

Ben Van Roy
Quals Questions 2013

Let the 2-norm of a vector be defined by the square-root of the sum of the squares of its components. Let the max-norm of a vector be the maximum among absolute values of components.

(1) Is there a vector x whose max-norm exceeds its 2-norm?

(2) Suppose the 2-norm of a vector x exceeds the 2-norm of a vector y. Does this imply that the max-norm of x exceeds the max-norm of y?

(3) Suppose the 2-norm of a vector x exceeds the 2-norm of a vector y by a factor of b. If the vectors are 2-dimensional, are there values of b that would imply that the max-norm of x exceeds the max-norm of y? What values? What about the general n-dimensional case?

(4) Suppose the max-norm of a vector x exceeds the max-norm of a vector y by a factor of b. If the vectors are 2-dimensional, are there values of b that would imply that the 2-norm of x exceeds the 2-norm of y? What values? What about the general n-dimensional case?

(5) Why are the answers to (3) and (4) the same?

2013 Quals Questions
Fabian Pease

What do you mean by the term 'active device'; give examples of active and passive devices.
What do you think was the earliest active device?
Here is a vacuum tube. Hot filament down the center emits electrons that are attracted to the anode. In between is a grid electrode that modulates the electron current between anode and cathode.
Now using the board show me how a MOSFET works. Why is the substrate silicon (or other single crystal semiconductor)? Why not graphite (or other cheap amorphous or polycrystalline material)? i.e. how is it that the gate voltage affects the conductance of a silicon layer but not of a graphite one?

EE Ph.D. Qualifying Exam, January 2013 Question

David Miller

Waves and transmission lines

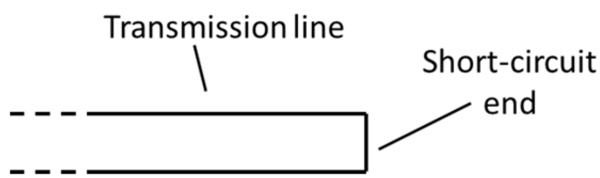
Notes: There may not be single "correct" answer to parts of this question. The goal of this question is to see how you think about it. The answers are mostly qualitative, and little or no algebra should be required for them. If you finish the question on this sheet, subsequent questions will be asked.

Question:

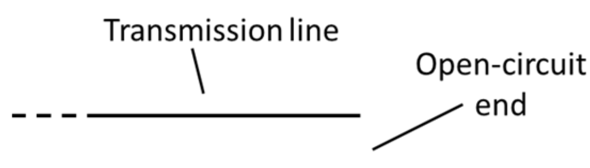
Consider a transmission line – that is, a two-conductor electrical line with some well-defined impedance (for example, 50 Ω). Presume for simplicity that the line has no loss (e.g., perfectly conducting wires) and that the wave propagation velocity on the line is *c*, the velocity of light in free space. A monochromatic (i.e., single-frequency) wave of frequency *f* has been launched onto the line from some source on the far left.

a) Sketch the form of the voltage on the line near the right end at some specific time for the two cases below, in both cases indicating any characteristic length involved and giving its magnitude.

(i) a line that is short-circuited at its right end, as shown in the diagram below

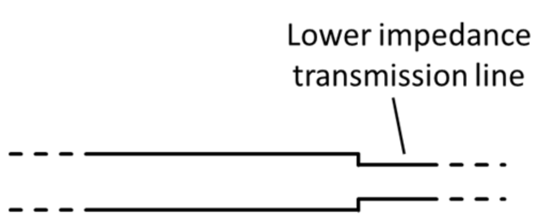


(ii) a line that is open-circuited at its right end, as shown in the diagram below



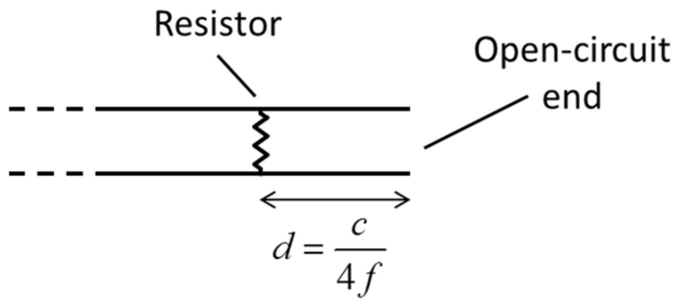
b) Sketch the form of the time-averaged electrostatic energy density in the line (which you can take to be proportional to the square of the voltage from your answers above) for both cases above.

c) Suppose that, instead of an open circuit at the right, we connect to another, lower impedance (for example, 25 Ω) line. Sketch the form of the energy density (i.e., the square of the voltage) in the line to the left of this connection (you need not calculate any numbers, but you should show the qualitative behavior).



Supplementary question 1

a) What happens if we add a resistor between the two conductors on the open-circuited line, at a distance $d = c/(4f)$ from the right (open-circuit) end?



b) Where else could you position the resistor to have the same effect?

Supplementary question 2

I want to have some signal that varies with frequency so that I can monitor possible frequency changes in a single-frequency electrical voltage, but I want to avoid constructing any resonant filter or frequency counter. I don't need an accurate measurement of the frequency – just some signal that varies predictably as the frequency changes. I do, however, want to be able to distinguish between the frequency increasing or decreasing relative to some central frequency.

Can you think of a way of doing this, exploiting some of the concepts explored here with transmission lines?

(Note: there is certainly no single correct answer to this question!)

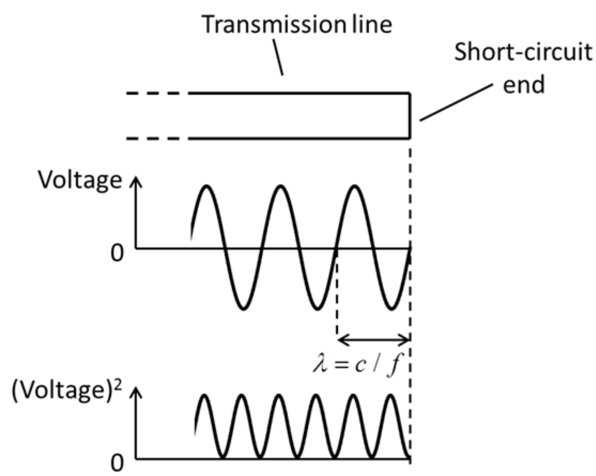
Answers

Main question

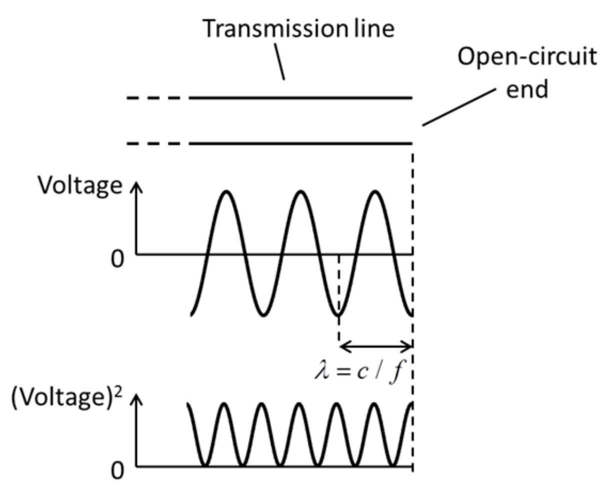
In both cases, of shorted or open ends on the line, we will form standing waves on the line.

a)

For case (i), the short circuit, there can be no voltage at the right end of the line, so that point will be a node (i.e., a zero) in the standing wave pattern. At any specific time, the voltage on the line will be a sine wave of wavelength $\lambda = c/f$ with a zero at the right end. So, the voltage will look as sketched below, or possibly minus this.



For case (ii), the open circuit, the voltage is maximum at the right end of the line, so that point will be an antinode (a maximum) in the standing wave pattern. The result is similar to the above, but shifted to have the maximum at the end.

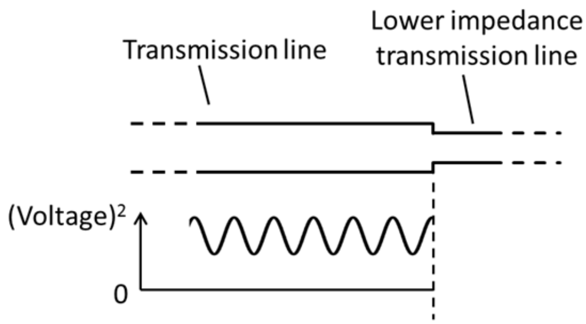


b)

The squares of the voltages are also sketched on the figures above for part (a).

c)

This situation is intermediate between an open-circuit and a short-circuit end. The phase of the reflection is the same as that for a short-circuit end, but the amplitude is smaller (the reflection cannot be total because some power is transmitted on into the lower-impedance line). Consequently, we have a partial standing wave, not going to zero at the minima, but with the same phase as that of the short-circuited line.



Notes on how students answered this

Most students had some notions of transmission lines. For those that did not, it was possible to recast this question in terms of waves on a string, waves in other kinds of waveguides, or waves propagating up to dielectric interfaces.

The most common minor error was that, instead of drawing a $\sin^2$ function for the standing wave, students drew a rectified sine wave. (In fact, nearly all students did this!) I took little or nothing off for that error in itself because it is not such big problem for parts (a) and (b), but it made it conceptually more difficult to answer part (c).

In part (c), some students correctly reasoned that the answer should be between that of parts (a) and (b) in some sense. This could cause them to guess (incorrectly) that the phase on reflection was somewhere between 0 and 180 degrees. I took no marks off for that, though, because it is not easy to get the phase here based on purely logical (rather than algebraic) reasoning. The main difficulty most students had was in realizing that the partial reflection would lead to a standing wave pattern that does not reach zeros. If so, I tried to help them reason towards the right answer by asking them what the pattern would look like if both lines had the same impedance (the answer is a completely flat line, of course).

Supplementary question 1

Since this position (one quarter wavelength from the open end) is a node, this resistor has no effect; there is no voltage across it at any time.

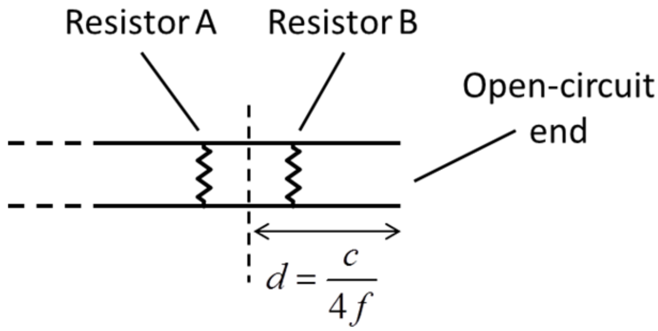
The resistor could be positioned at any node in the line for the same effect. The nodes are a one-quarter wavelength, 3/4 wavelength, 5/4 wavelength, … and so on, from the right end of the line.

Notes

The students who got to this point generally got this answer correct.

Supplementary question 2

There are many possible ways of doing this. One possible solution here is shown below.



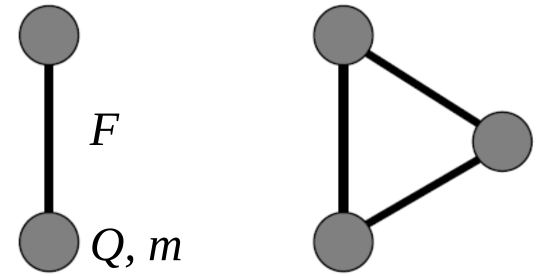
Here, two resistors are placed on either side of the position of the node in the open-circuit line case. We presume we make these resistors rather large values compared to the line impedance so that, though they are not sitting at the node position, there is relatively little current flowing through them and so they do not greatly perturb the standing wave on the line. If the frequency rises higher, the node moves to the right, so there is less power dissipated in Resistor B and more in Resistor A. Hence, if we measured the temperature difference between these resistors, higher temperature in Resistor B compared to Resistor A would indicate somewhat higher frequency in the line. Conversely, moving to lower frequency would move the node towards Resistor A giving the opposite temperature difference. Hence measuring the temperature difference between these resistors could gives a measure, with an appropriate sign, of changes in the signal frequency in the line.

Notes

It is not hard to come up with an approach that tells you that the frequency is off – just looking for a finite voltage across the resistor in the circuit of Supplementary Question 1 will tell you that. The tricky part is to come up with something that also tells you the sign – i.e., whether your frequency is above or below. That requires that you compare to something. The solution shown above does that, in this case in a symmetric way that makes it easy to get the sign without any calculations.

EE Qual 13, Engineering Phys.
Shan Wang

You are given a large collection of identical balls and strings. All the balls have the same electric charge, Q, and mass, m. The lengths of the strings are initially fixed at l. When two balls are placed at the ends of one string, the tensile force in the string is F. Next, three balls and three strings are placed at the vertexes and edges of an equilateral triangle.



a) What is the tensile force in each string in the latter case? [2 pt]

b) If the strings are ideally elastic with a spring constant of k and an original length of l. What is the new size of the triangle at equilibrium? [3 pt]

c) There can be many resonance modes for the triangle in (b). Find the frequency of one of the any resonance modes. [5 pt]

Binary Search

Prof. Hector Garcia-Molina

- We are given an array A of sorted integers
  Example:

A

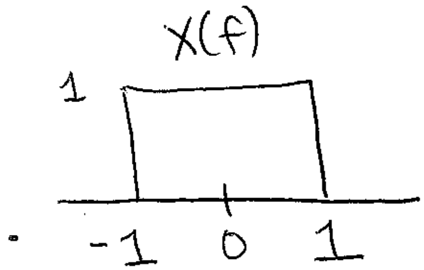
3	7	12	13	21	43	44	45	62	69	75

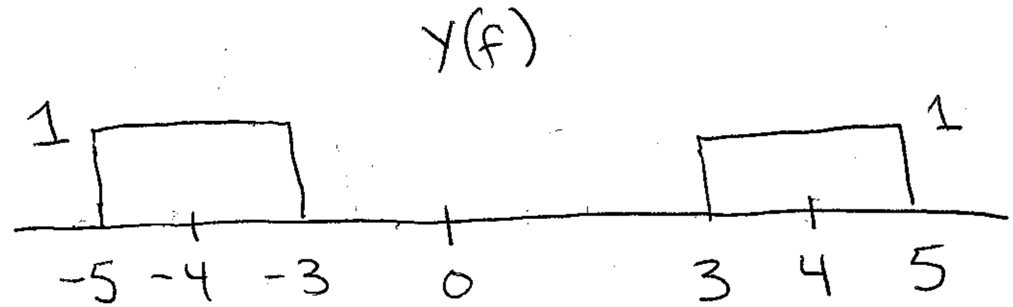
N=11

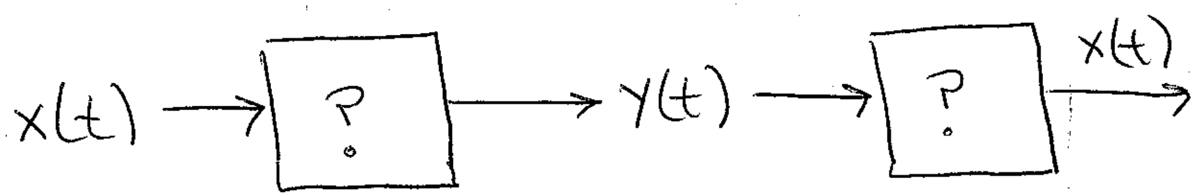
  - N is number of elements (N > 0)
  - A[1] is first element, A[N] is last
  - Variables A, N are global
  - There are no duplicates in array
- Your task: code following function
  Procedure FIND(V): returns FOUND, INDEX
  - V is integer we are looking for
  - If V is in array, FOUND is set to TRUE, else FALSE
  - If FOUND=TRUE, INDEX is location of V
  - Example:
    FIND(13) returns FOUND=TRUE, INDEX=4
    FIND(14) returns FOUND=FALSE
    FIND(87) returns FOUND=FALSE
- Use <u>binary search</u> for your procedure
- Extra credit: handling duplicates in array

Prof. Andrea Goldsmith

Goldsmith







- Describe 2 "black boxes" that generate y(t) as output for x(t) as input

- How can you recreate x(t) from y(t)?

Goldsmith

Let $x(t)$ be a Gaussian WSS random process with mean 2 and PSD 100.

Let $h(t) = \text{rect}\left(\frac{t}{20}\right)$

What is $Z = .5\int_{0}^{30} h(t)\,x(t)\,dt$

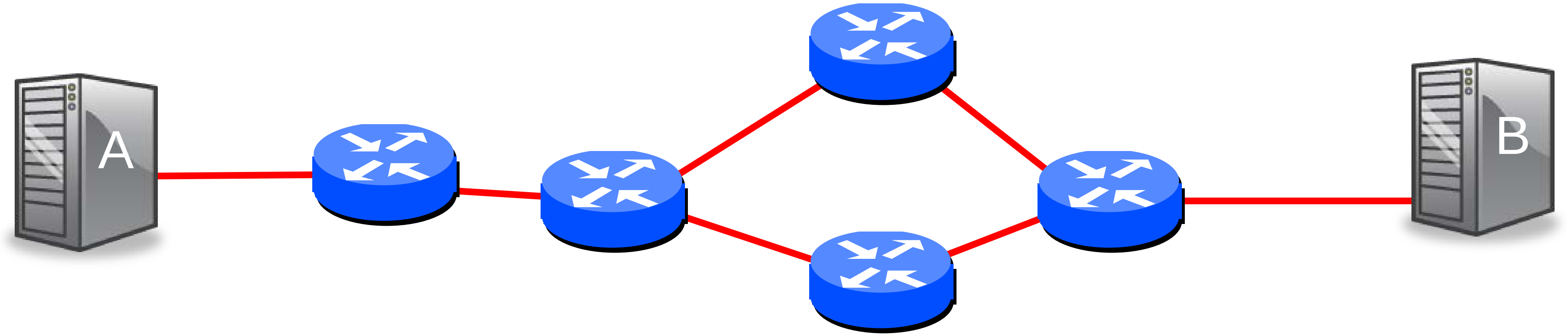
EE Quals 2013 (Software)

Nick McKeown

Question 1

Why does the Internet use <u>packet switching</u>?

Question 2



(a) What is the simplest way we can find the data rate of the slowest link from A to B, observing only at the end points?

(a) How can we do it by sending only two packets?

(b) What difficulties or noise are we likely to encounter?

Question 3

How might we find the rate of the 2nd slowest link?

Qualifying Exam 2013
EM, Shanhui Fan

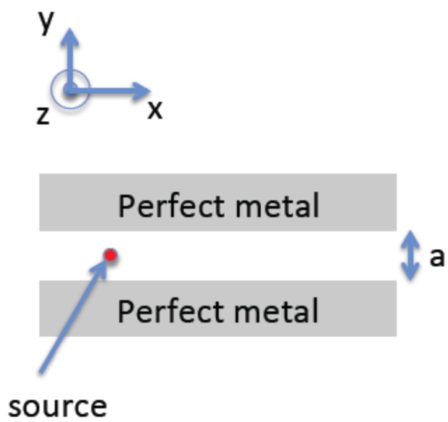


Figure 1

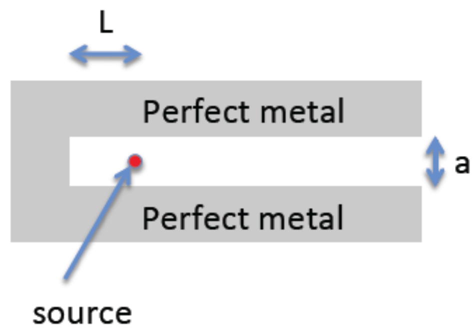


Figure 2

Consider a parallel-plate waveguide, with PEC (perfect electric conductor) sidewalls and air in between. The width of the waveguide is a. We consider two-dimensional system. (i.e. the fields and the structures are uniform in the z-direction).

(1) Sketch the ω-k relation for the lowest-order TE mode (with E-field polarized along the z-direction), and for the lowest-order TM mode (with E-field polarized along the y-direction).

(2) Suppose we put in an oscillating line source as indicated in Figure 1. The source oscillates at a frequency ω_0. How does the power radiated into the waveguide changes as a function of ω_0? Consider both cases, the TE case with the line source polarized along the z-direction, and the TM case with the line source polarized along the y-direction.

(3) Suppose we truncate the waveguide with a perfect electric conductor at the end as shown in Figure 2. For the TM case, suppose we choose $\omega_0 = 0.1\frac{2\pi c}{a}$, how does the power radiated into the waveguide vary as a function of the distance L between the source and the truncation?

Prof. Brad Osgood

EE Qualifying Exam
January 2013

From Wikipedia:

> Pink noise or 1/f noise (sometimes also called flicker noise) is a signal or process with a frequency spectrum such that the power spectral density is inversely proportional to the frequency. In pink noise, each octave carries an equal amount of noise power. (The name arises from the pink appearance of visible light with this power spectrum.)

- Please explain the terms in the description above. How would you go about measuring a signal to see if it's pink noise? How would you generate a signal that has pink noise? How does pink noise differ from white noise?

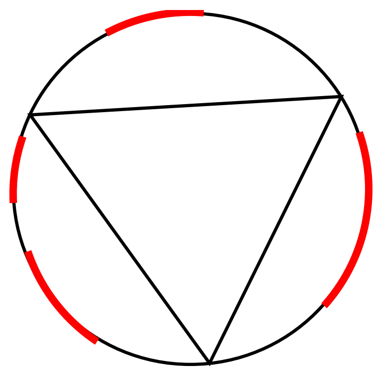
1/f noise occurs in many physical, biological and economic systems. Some researchers describe it as being ubiquitous. In physical systems, it is present in some meteorological data series, the electromagnetic radiation output of some astronomical bodies, and in almost all electronic devices (referred to as flicker noise). In biological systems, it is present in, for example, heart beat rhythms, neural activity, and the statistics of DNA sequences. ... Also, it describes the statistical structure of many natural images (images from the natural environment).

Richard F. Voss and J. Clarke claim that almost all musical melodies, when each successive note is plotted on a scale of pitches, will tend towards a pink noise spectrum. There are many theories of the origin of 1/f noise. Some theories attempt to be universal, while others are applicable to only a certain type of material, such as semiconductors. Universal theories of 1/f noise remain a matter of current research interest.

Ayfer Özgür

Ph.D. Qualifying Examination

Problem 1. The circumference of a circle is colored $a\%$ in red such that $a < 100/3$. Prove that it is always possible to inscribe an equilateral triangle into the circle such that all three vertices avoid the red, irrespective of how the coloring is done.



Hint: Consider an equilateral triangle that is randomly inscribed into the circle and evaluate the probability that the random triangle satisfies the desired property.

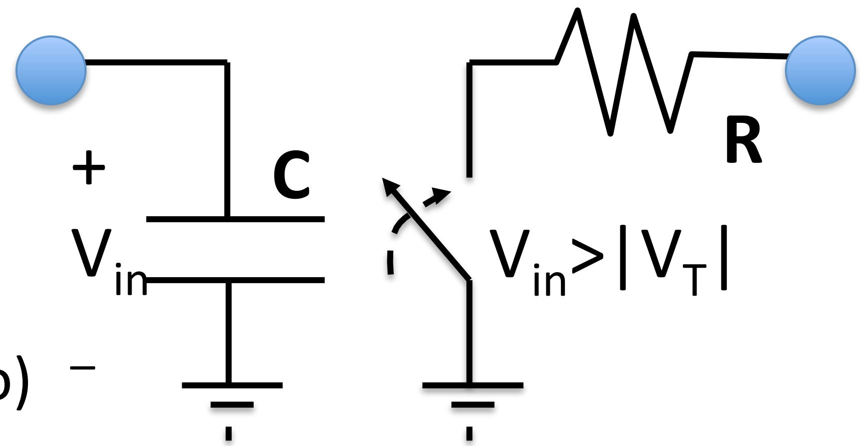
Quals Question 2013
Greg Kovacs

You have been asked by your boss to evaluate a circuit board. She needs you to figure out as much as you can about it in ten minutes and leave her with a one-page written report on the form provided. You may not modify, power-down, break, or pull components from the circuit.

The digital world deals with "1" and "0" so…why do we need to consider the amplification of signals? (What does amplification mean and why do we need it?)

What does an amplifier do and how does it work?

Suppose the world only had ideal switches with series resistance as shown. How would that work from a digital time-domain perspective? (Explain what your amplifier would do)



What would you do if we want to amplify current? (How would you do it?)

1. For each of the following systems, determine whether the system is stable, causal, linear, and time invariant:

(a) $T(x(n)) = (\cos(\pi n))x(n)$

(b) $T(x(n)) = x(n^2)$

(c) $T(x(n)) = x(n)\sum_{k=0}^{\infty}\delta(n-k)$

(d) $T(x(n)) = \sum_{k=n-1}^{\infty}x(k)$

(e) $T(x(n)) = n^3 x(n)$

a	Stable	Causal	Linear	Time-Invariant
b				
c				
d				
d				

2. If the Nyquist rate for $x_a(t)$ is W_s, what is the Nyquist rate for each of the following signals that are derived from $x_a(t)$?

(a) $\dfrac{dx_a(t)}{dt}$

(b) $x_a(2t)$

(c) $x_a^2(t)$

(d) $x_a(t)\cos(W_0 t)$

3. Consider the system shown in Figure 1, where

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi / L, \\ 0, & \pi / L < |\omega| \le \pi. \end{cases}$$

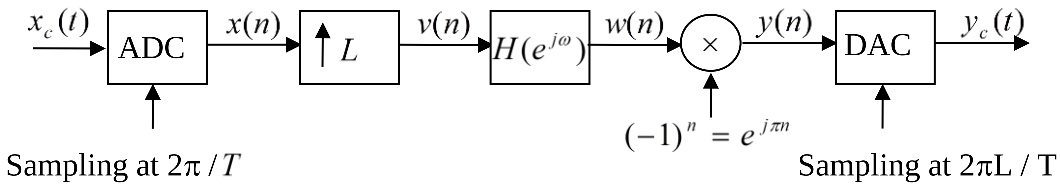


Figure 1

Sketch $Y_c(j\Omega)$ if $X_c(j\Omega)$ is as shown in Figure 2.

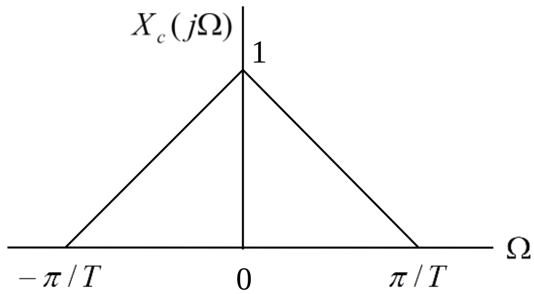
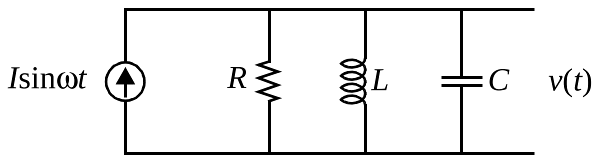


Figure 2

Consider the following circuit:

FIGURE 1. Parallel *RLC* circuit



Assume that the current source has an amplitude of 1A and a frequency of 1rad/s. Assume also that the inductance is 1H and that the resistance is 1kΩ.

a) Suppose that the capacitance is 1F. Sketch the steady-state voltage $v(t)$. Identify the amplitude and frequency (or period). What is the average power dissipated in R?

The component values are chosen to produce a resonance at 1rad/s, at which the inductive and capacitive impedances cancel to contribute a net infinite impedance. The 1A current thus flows through the 1kΩ resistor to produce a 1kV-amplitude sinusoidal $v(t)$*. The average dissipation in the resistor is just* $I^2R/2 = 500W$*.*

Common mistake**:** Many students immediately and mechanically started writing node equations and Laplace transformed them (or wrote differential equations and tried solving them directly), instead of thinking about the problem a bit first.

b) Now suppose that, starting from $t = 0$, the capacitance instantaneously decreases by 1% at the voltage extrema. Sketch the voltage $v(t)$, assuming that the capacitance returns to its nominal value at each zero crossing. Make reasonable assumptions/approximations.

Capacitive charge cannot change instantaneously if currents are finite (as they are here). Since Q=CV and Q is continuous, a discontinuous change in C must be balanced by a discontinuous change in V. Here, a drop in C implies a jump in V; the capacitor's voltage therefore increases by an amount Q/ΔC (or ~1%) at each extremum, growing without bound. This behavior is exploited in the parametric amplifier*, of which a child's swing is an example.*

At the zero-crossings of v(t), the capacitive charge is zero, so there is no change in capacitor voltage there.

Common mistake**:** Many students attempted to deduce the behavior using concepts of shifts in resonant frequency. Now, resonant phenomena are evident only over the order of Q cycles (where Q here is 1000), so deducing circuit responses to perturbations occurring on a per-cycle basis (and here, more often than that) is somewhat of an unnatural act for a resonance-based approach.

Quals Question

You are drawing variables $U_0, U_1, \cdots$ iid$\sim U[0,1]$.

(a) T_1 is the number of attempts until you surpass your first draw (i.e., break your first record):

$$T_1 = \min\{n \geq 1 : U_n > U_0\}$$

What is the PMF of T_1?

(b) What is the PDF of U_{T_1}?

(c) T_2 is the number of attempts it takes you to set your next record:

$$T_2 = \min\{n \geq 1 : U_{T_1+n} > U_{T_1}\}$$

What is the PMF of T_2 (bring to as explicit a form as you can)?

(d) Repeat parts (a) and (c) for $U_0, U_1, \cdots$ iid$\sim N(0,1)$.

The entries of an invertible $n \times n$ matrix A are integers.

When are all the entries of A^{-1} integers?

(Always? Never? Sometimes?)

Discussion/solution.

As always, the point is not the solution; the point is the clarity of the arguments used.

The identity matrix is an example showing it's possible for all entries of A and A^{-1} to be integers. Another more interesting example is an upper or lower triangular matrix, with its diagonal entries all 1 or -1.

Let's start with 1×1 matrices, $i.e.$, scalars. Here the inverse is an integer only if $A = 1$ or $A = -1$.

Now let's look at 2×2 matrices. We have

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

so if $\det A = 1$ or -1, then all entries of A^{-1} are integers. The converse is also true: if all entries of the inverse are integers, then $\det A = 1$ or -1. To see this, we note that

$$1 = \det I = \det(AA^{-1}) = (\det A)(\det A^{-1}).$$

If A and A^{-1} have all integer entries, then $\det A$ and $\det A^{-1}$ are both integers (since they are sums of products of entries). These two integers have a product equal to 1, so they can only be both 1, or both -1.

Now we can guess the general case: A^{-1} has integer entries if and only if $\det A$ is 1 or -1. To show one way, assume that $\det A = 1$ or -1. Cramer's formula for the inverse is

$$(A^{-1})_{ij} = \frac{(-1)^{i+j} \det \tilde{A}}{\det A},$$

where $\tilde{A}$ is formed from A by removing a column and a row. The numerator is an integer, and the denominator is 1 or -1, so $(A^{-1})_{ij}$ is an integer. To prove the converse, the argument above works: if A and A^{-1} both have integer entries, then $(\det A)(\det A^{-1})1$, and we conclude that $\det A = 1$ or -1.

Name

Start time

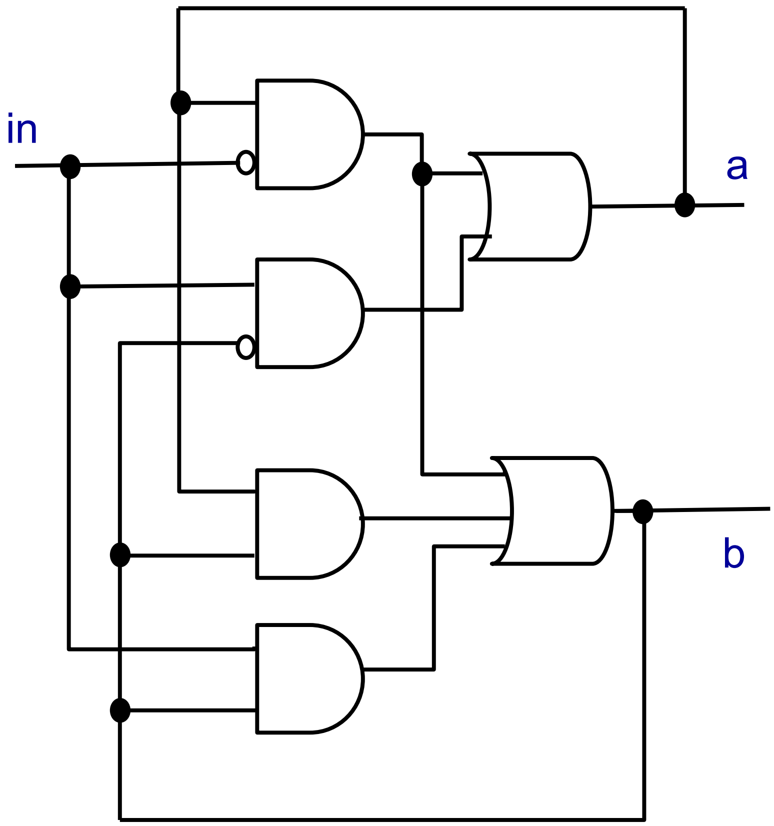
1. What are hazards in a logic circuit?

2. Is there a hazard problem for output “a” of the circuit below?

3. If yes, please fix it. If not, why not?

4. Is there any other hazard problem in this circuit? Which ones? What’s the fix? Is it possible to fix those problems without inserting delays?

Name



State	Code	Next	
	b a	in = 0	in = 1
A	0 0	A	B
B	0 1	C	B
C	1 1	C	D
D	1 0	A	D

Truth table for a

b a \ in	0	1
0 0		1
0 1	1	1
1 1	1	
1 0		

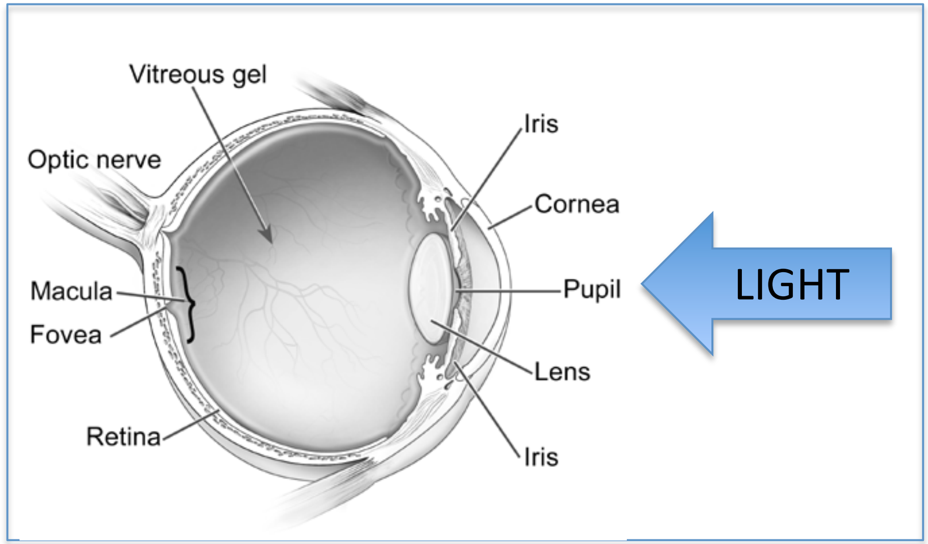
Truth table for b

b a \ in	0	1
0 0		
0 1	1	
1 1	1	1
1 0		1

The eye as an optical system

Your eye is an optical system with a built-in lens (lens) and detector (retina). Today you will design and carry out a series of experiments to measure its various properties. Assume the distance from your lens to your retina is roughly 22.4 mm.

If you need me to help me hold something for you, please ask. If there is an experiment you believe you do not have the right materials to perform, draw and write out an experimental protocol that would be appropriate, including materials needed.



- Size of aperture stop (pupil)
- Range of focal lengths
- Resolution
- Field of view
- Depth of field
- Sensitivity
- Location of your blind spot
- Number of photoreceptors

NOTE: It is not expected that you complete all of them, but rather that you demonstrate rational thinking skills in a timely manner.

You are giving N = 2 identical fuses. Their rating is the same; it is one of the following values:

0.5 mA, 1 mA, 2 mA, 5 mA, 10 mA, 20 mA, 50 mA, 100 mA, 200 mA, 500 mA, 1 A

Since the fuses are not marked, you test them repeatedly with different currents to determine the unknown rating. When the current exceeds the rating, the fuse is irreversibly destroyed.

Devise a strategy that minimizes the maximum number of tests to reliably determine the fuses' current rating.

How many tests T are required to distinguish R different ratings, if you have N fuses?

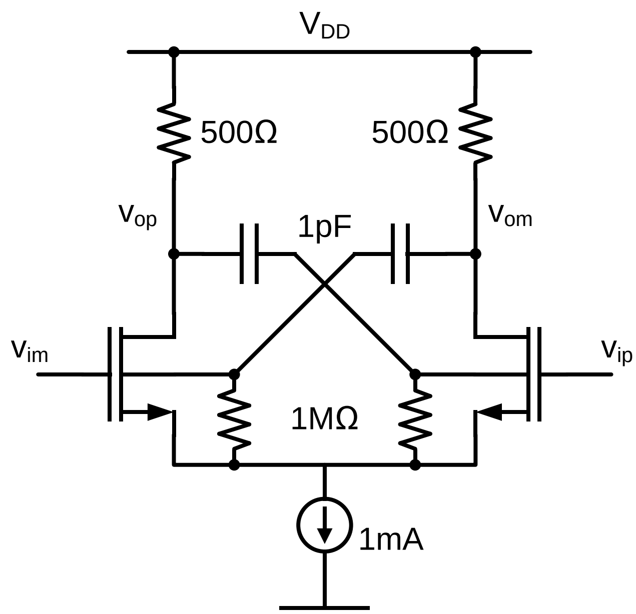
Note: All N fuses may be destroyed in the process.

QUALS

Knock once, then wait

Name:	Stanford EE Quals 2013 Murmann

In the circuit below, all MOSFETs obey the ideal square law equations. The transistors are sized such that $|V_{GS}\text{-}V_t|$ = 200 mV. The backgate transconductance is $g_{mb} = 0.2\ g_m$. Ignore all device capacitances.



1. Sketch the frequency response (magnitude only) of the differential small-signal voltage gain $(v_{op}\text{-}v_{om})/(v_{ip}\text{-}v_{im})$.
2. Is this circuit stable? Discuss in terms of gain and phase margin, as applicable.
3. Can this circuit work with the 1-pF decoupling capacitors shorted? Discuss potential issues.

Clearly state any assumptions you make while solving the problems. Good luck!

1. Electron source

Suppose you need to generate an electron beam for your experiment.

You have access to the following equipment:

- a source of ultraviolet radiation (e.g., a UV lamp with radiation spectrum covering 200nm-400nm wavelength range);
- a high voltage source (DC);
- a strong magnet (generating DC magnetic fields of up to 4T);
- a metal evaporator, that you can use to coat substrates with gold, platinum, or aluminium;
- a variety of substrates (e.g., quartz and glass microscope slides and silicon wafers);
- a sensitive screen detecting electrons, which you can use to characterize the profile of the electron beam

(a) Explain how you would generate an electron beam using the available equipment.

(b) What would you do to focus the electron beam, so that the spot on the detector screen is as small as possible?

Hint: The following parameters can be useful for your analysis:

- the workfunctions of gold, platinum, and aluminium are 5.1eV, 6.35eV, and 4.08eV, respectively;
- electron mass $m=9.1 \cdot 10^{-31}$kg; electron charge $e=-1.6 \cdot 10^{-19}$ C;
- Planck's constant $h=6.626 \cdot 10^{-34}$Js

2. Coffee cooling

Explain the process by which a hot cup of coffee cools.

From sunrise to sunset is there a change in the energy produced by a solar cell?

- Number of photons reduces at sunrise and sunset.
- Spectrum changes, more red light at sunrise and sunset.
- Temperature increases during mid day.

How will you improve the energy produced during early and late hours of the day?

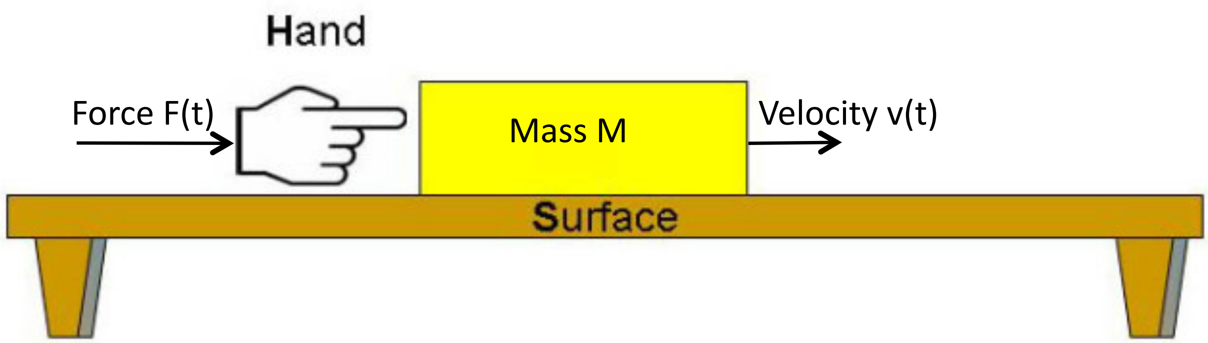
- Multi-junction cell

If an LED is heated at 800C what will be the emission spectrum?

2013 EE PhD Quals

Prof. Daniel Spielman

Consider a stone block on a table:



1. Can you model this as a linear time-invariant system with force, F(t), as the input and velocity, v(t), the output?

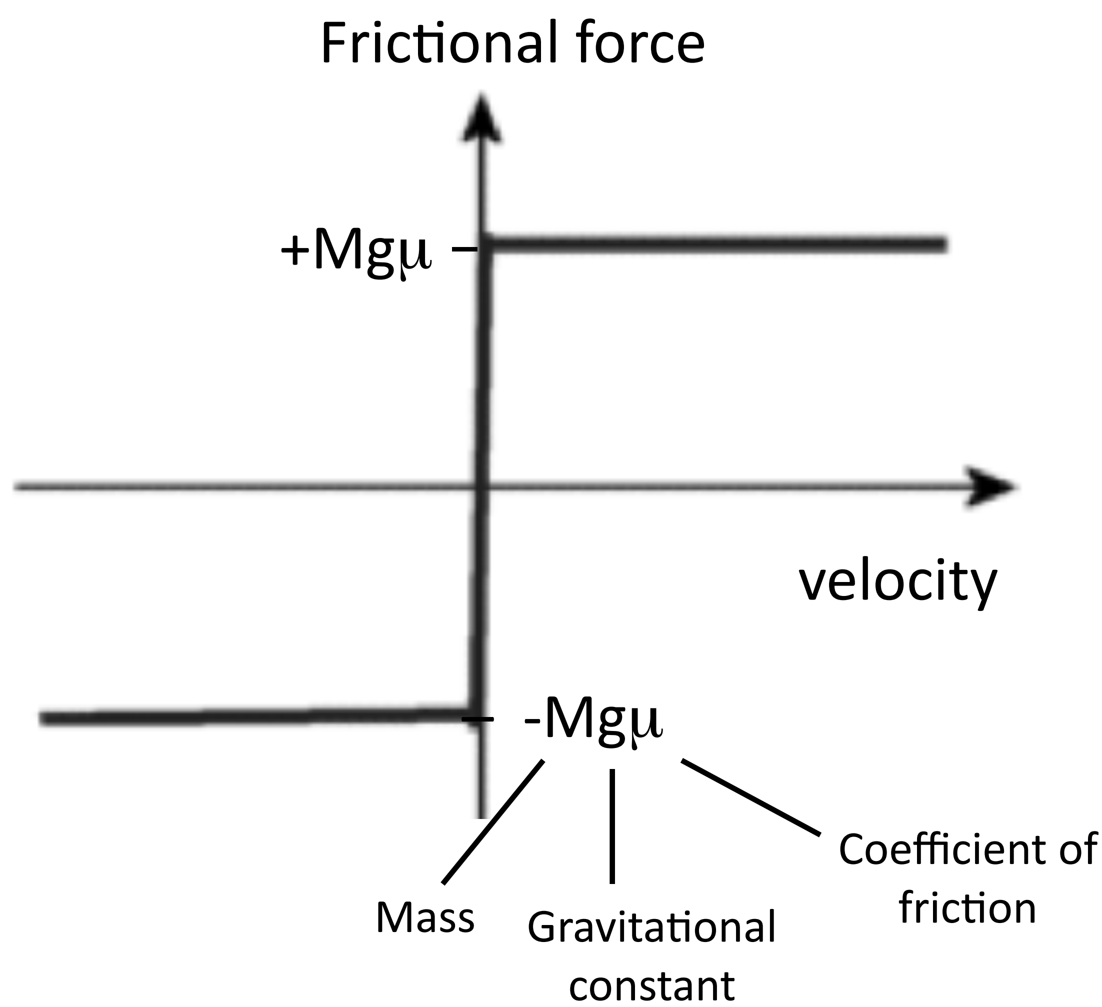
2. Is there an equivalent electrical circuit?

3. What is the impulse response?

4. How could you linearize either your model or the system?

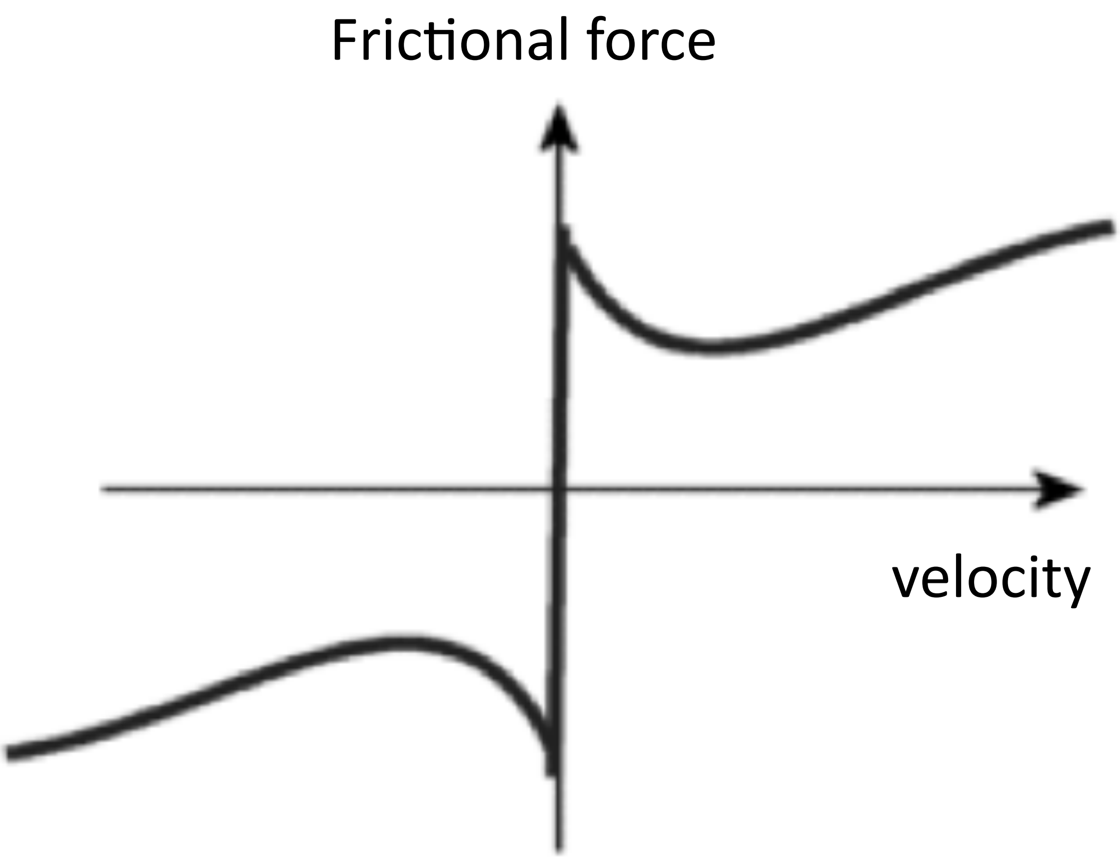
Hint:

A simple model of friction.



Hint #2:

Frictional model for a greased block.

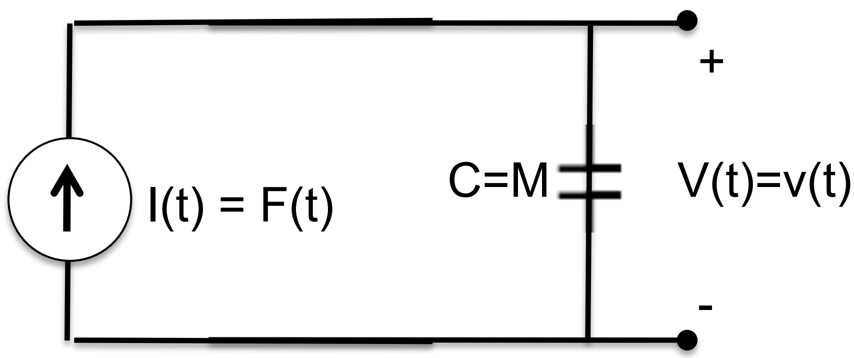


Answers

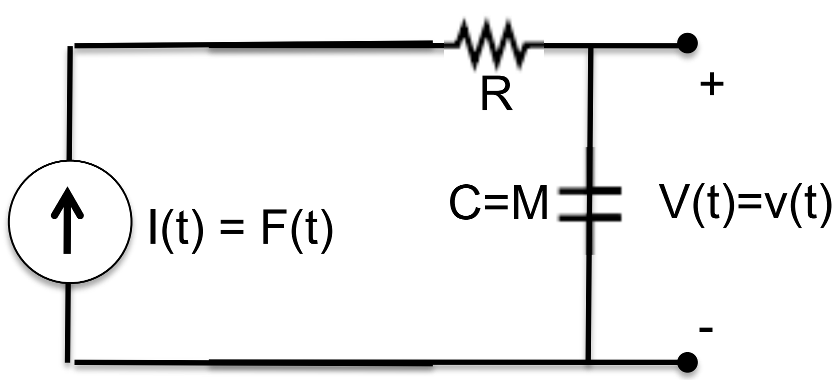
Without friction, the system is LTI an can easily be modeled as a simple integrator.

$$v(t) = \frac{1}{M}\int_{-\infty}^{t}\left(F(t')\right)dt'$$

For the equivalent circuit, let current, I(t), equal F(t) and the voltage, V(t), equal v(t), then the equivalent circuit is…



However, for a stone block on a table, there is friction, in which case the system is clearly nonlinear. Try pushing the block very lightly and observing the output. One might be tempted to try the following RC model.



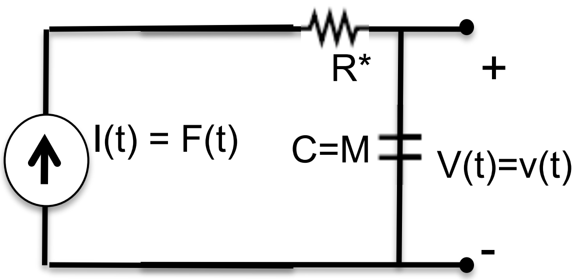
Unfortunately, a resistor is a device such that voltage and current are linearly proportional (V=IR), clearly not the case with respect to kinetic friction. Hence the RC model is wrong.

Answers (cont.)

Mathematically, the input/output relationship is given by:

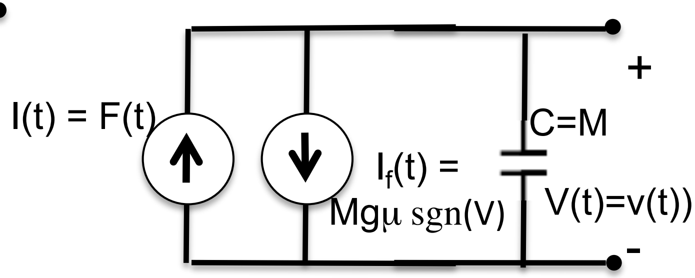
$$v(t) = \frac{1}{M}\int_{-\infty}^{t}\left(F(t') - Mg\mu \operatorname{sgn}(v(t'))\right)dt'$$

Using the simple model of friction given in Hint 1, the equivalent "circuit" would be:



where R* is a peculiar "resistor" in which I=sgn(V)/R instead of the usual I=V/R for which R=Mgμ.

Alternatively…

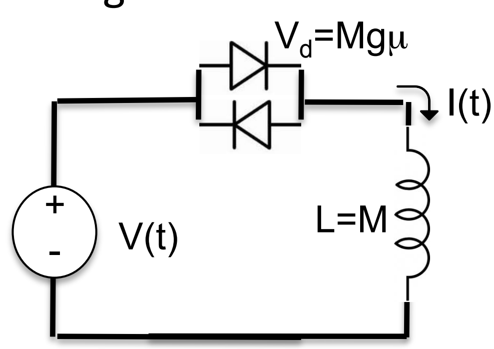


One clever student came up the following circuit:

Using the analogy,

$$\text{velocity } v(t) \leftrightarrow \text{ current } I(t)$$
$$\text{Force } F(t) \leftrightarrow \text{ voltage } V(t),$$

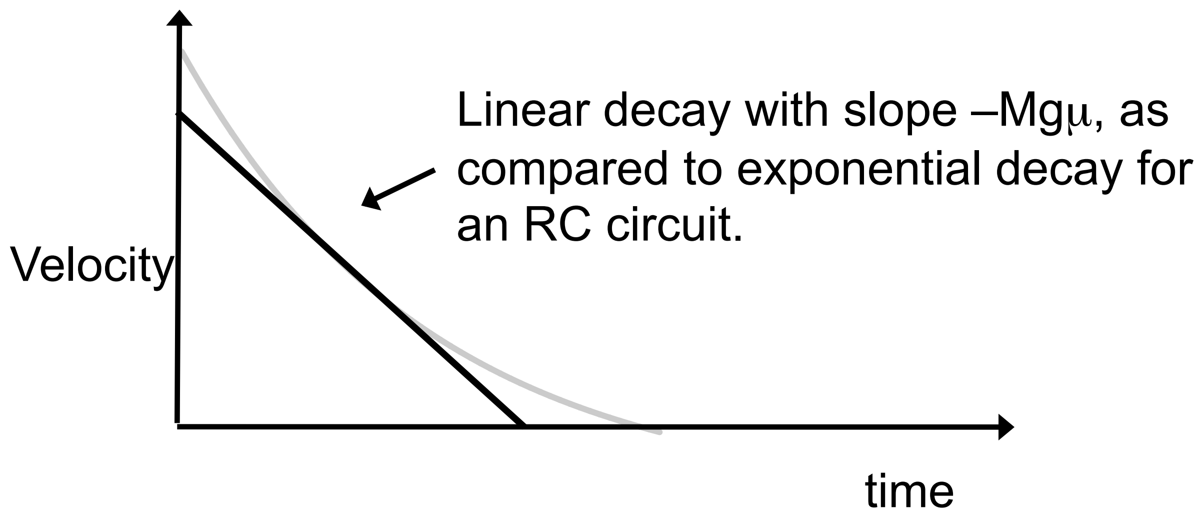
find I(t) as a function of V(t).



Note, even restricting our model to velocities > 0, this system is still not linear, but rather affine.

Answers (cont.)

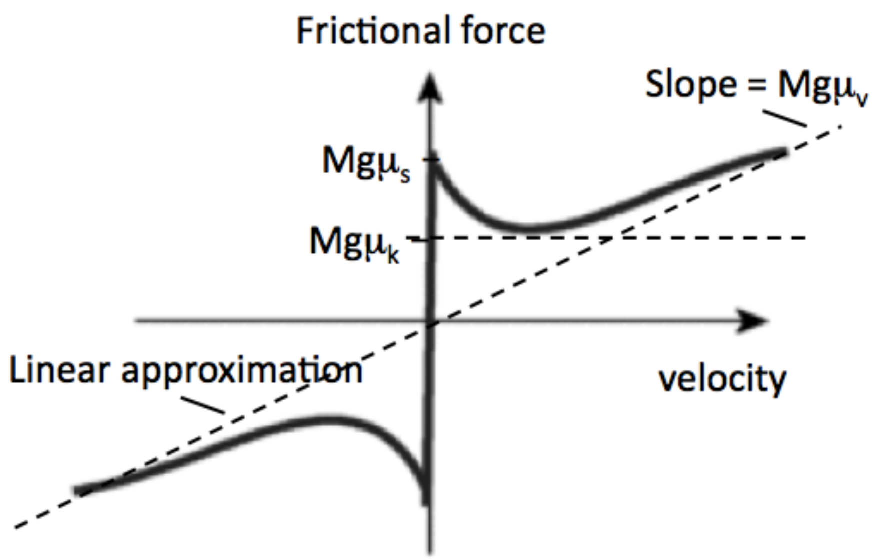
3. Plotting the impulse response (and yes the system does have an impulse response even though it is not LTI) is quite straight forward (just use F=Ma). For the case where the applied impulse is strong enough to cause the block to move…



For smaller impulses, the impulse response is simply zero.

4. One approach to linearize a system is to pick an operating point and try to linearize about that point. As mentioned before, picking, say, some non-zero velocity, v_0,technically gives you a affine (non-linear system). However, if, for example, we also restrict ourselves to $v(t) > 0$ and only large input forces, i.e. and $F(t) >> Mg\mu$, (or use $F'(t)=F(t)- Mg\mu$) then the system looks like a simple capacitor.

My favorite approach is to grease the block! Although no one actually came up with this solution, greasing changes the system from having kinetic/static friction to one having viscous friction for which force is indeed linearly proportional to velocity. Adding some labels to the frictional model for a greased block...



μ_s = cofficient of static friction
μ_k = cofficient of kinetic friction
μ_v = cofficient of viscous friction

Now, one reasonable model would be just to use the linear approximation drawn on the plot, i.e. model using a capacitor and a resistor. This model would be accurate for intermediate velocities, breaking down for velocities near zero and at very high velocities where fiscous friction, or drag, becomes proportional to v^2 not v.