

the finite sum

$$\begin{aligned}\frac{1}{K} \sum_{n=0}^{K-1} |x[n]|^2 &= \frac{1}{K} \sum_{n=0}^{K-1} x[n] \left(\sum_{k=0}^{K-1} a_k e^{j2\pi \frac{k}{K} n} \right)^* \\ &= \frac{1}{K} \sum_{k=0}^{K-1} a_k^* \sum_{n=0}^{K-1} x[n] e^{-j2\pi \frac{k}{K} n} \\ &= \frac{1}{K} \sum_{k=0}^{K-1} |a_k|^2\end{aligned}$$

This can also be done with more work by plugging in the Fourier series representation for $x[n]$, taking the magnitude squared, and manipulating the sums and limits.