Stanford University
Department of Electrical Engineering
Qualifying Examination Winter 2008-09
Professor Joseph M. Kahn

Consider a carrier-modulated signal $s(t) = m(t)\cos\omega_c t$, having Fourier transform $S(j\omega)$. Assume that $s(t)$ is narrowband, i.e., $S(j\omega) = 0$ except for ω close to $\pm\omega_c$. The signal $s(t)$ is input to a LTI system having a real impulse response $h(t)$ and a frequency response $H(j\omega)$. Near $\omega = \pm\omega_c$, you can assume that $H(j\omega)$ varies slowly with ω.

- Show that the output $y(t)$ is given approximately by:

  $$y(t) \approx |H(j\omega_c)| m(t - \tau_g(\omega_c)) \cos(\omega_c(t - \tau_p(\omega_c))).$$

  Give expressions for the group delay $\tau_g(\omega_c)$ and the phase delay $\tau_p(\omega_c)$ in terms of $H(j\omega)$.

- Explain intuitively why the group delay $\tau_g(\omega_c)$ has the particular mathematical form it does.

Hint: Represent $y(t)$ as the inverse Fourier transform of $Y(j\omega) = S(j\omega)H(j\omega)$. Write the frequency response in polar form as

$$H(j\omega) = |H(j\omega)| e^{j\arg\{H(j\omega)\}} = |H(j\omega)| e^{j\phi(\omega)},$$

and expand $H(j\omega)$ in a Taylor series. Near $\omega = \omega_c$, write:

$$H(j\omega) \approx \{|H(j\omega_c)| + \ldots\} e^{j\left\{\phi(\omega_c) + \left.\frac{d\phi(\omega)}{d\omega}\right|_{\omega = \omega_c} \cdot (\omega - \omega_c) + \ldots\right\}}.$$

Near $\omega = -\omega_c$, write a similar Taylor series.