Recall that

$$
\begin{aligned}
\int_{-\infty}^{\infty} p(t)p(t-n)dt &= \delta_n \qquad (1)\\
P(f) &= \int_{-\infty}^{\infty} p(t)e^{-j2\pi ft}dt
\end{aligned}
$$

Third Question:
Find a waveform $p(t)$ which has the desired property (1) and has the additional property that $P(t)$ *also* satisfies property (1); that is,

$$\int_{-\infty}^{\infty} P(t)P(t-n)dt = \delta_n \qquad (2)$$

Solution Here are two possible approaches:

1. Guess and show it works. There are not many signals p which satisfy (1), so it is easy to see if they also satisfy (2) if you either know or can find the CTFT.

2. Look at the formulas the signals must satisfy and find a solution.

First Approach: A simple signal which satisfies (1) is the box or rect function or square pulse

$$p_0(t) = \begin{cases} 1 & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Its Fourier transform is easily found (or recalled from memory) as

$$
\begin{aligned}
P_0(f) &= \int_{-1/2}^{1/2} e^{-j2\pi ft}dt = \left.\frac{e^{-j2\pi ft}}{-j2\pi ft}\right|_{-1/2}^{1/2}\\
&= \frac{e^{-j\pi f}}{-j\pi f} - \frac{e^{j\pi f}}{-j\pi f} = \frac{\sin(\pi f)}{\pi f}
\end{aligned}
$$

which is the sinc function, $P_0(f) = \text{sinc}(f)$. But these are also orthogonal as in (1) from Parseval's theorem and the modulation theorem:

$$\int_{-\infty}^{\infty} \text{sinc}(f-n)\,\text{sinc}(f)df = \int_{-\infty}^{\infty} p_0(t)e^{-j2\pi tn}p_0(t)dt = \int_{-1/2}^{1/2} e^{-j2\pi tn}dt = \delta_n.$$

Second Approach: Combining the Second Question with (2) yields

$$\int_{-\infty}^{\infty} |P(f)|^2 e^{j2\pi fn}df = \int_{-\infty}^{\infty} P(f)P(f-n)df = \delta_n$$

so $P(f)$ must satisfy

$$\int_{-\infty}^{\infty} P(f)\left[P(f)e^{j2\pi fn} - P(f-n)\right]df = 0.$$