

9. The optimum decision rule is the obvious decision rule:

$$\hat{X} = \begin{cases} +1 & \text{if } Y > 0 \\ -1 & \text{if } Y < 0 \end{cases}$$

Since the values of  $X$  are equiprobable, minimum error decoding is the same as maximum-likelihood decoding. Because the pdfs of  $Y$  given  $X$  are triangular,  $\Pr(Y = y | X = -1) > \Pr(Y = y | X = +1)$  when  $y < 0$ , whereas  $\Pr(Y = y | X = -1) < \Pr(Y = y | X = +1)$  when  $y > 0$ .

The overall error probability is the conditional error probability given either value of  $X$ . For  $X = -1$  this conditional probability is the area under the left triangle to the right of the vertical axis. Thus  $\Pr(\hat{X} \neq X) = \Pr(N_1 + N_2 > 2) = 2/9$ . As expected, this error probability is smaller than  $1/3$ , the error probability for a single transmission of  $X$ .