Electrical Engineering

Quals Questions

2008

http://ee.stanford.edu/phd/quals/

Ask me about anything that isn't clear.

The *average* of a vector $x \in \mathbf{R}^n$ is defined as

$$\mathbf{avg}(x) = \frac{x_1 + \cdots + x_n}{n}.$$

Average-preserving linear transformation.
Under what conditions on $A \in \mathbf{R}^{m \times n}$ do we have

$$\mathbf{avg}(Ax) = \mathbf{avg}(x)$$

for all $x \in \mathbf{R}^n$?

Average-reducing linear transformation.
Under what conditions on $A \in \mathbf{R}^{m \times n}$ do we have

$$|\mathbf{avg}(Ax)| \leq |\mathbf{avg}(x)|$$

for all $x \in \mathbf{R}^n$?

Discussion/solution. We can write $\mathbf{avg}(x) = (1/n)\mathbf{1}^T x$, so

$$\mathbf{avg}(Ax) = \mathbf{avg}(x) \iff (1/m)\mathbf{1}^T Ax = (1/n)\mathbf{1}^T x. \iff \mathbf{1}^T Ax = (m/n)\mathbf{1}^T x.$$

This holds for all x if and only if $\mathbf{1}^T A = (m/n)\mathbf{1}^T$, which can be expressed as $A^T\mathbf{1} = (m/n)\mathbf{1}$. This means that all columns of A must sum to m/n.

Another way to say it is: If you add up the rows of A, you get a row vector all of whose entries as m/n.

If A is square (which it need not be), the condition also means that A has $\mathbf{1}$ as a left eigenvector, with associated eigenvalue m/n.

The second question is a bit trickier. The solution is: $|\mathbf{avg}(Ax)| \leq |\mathbf{avg}(x)|$ for all x if and only if $A^T\mathbf{1} = \alpha(m/n)\mathbf{1}$ for some α with $|\alpha| \leq 1$. In other words, all columns of A must sum to m/n, times a constant (which is the same for all columns) less than or equal to one in magnitude. In terms of eigenvectors, the condition can be expressed as: A has $\mathbf{1}$ as a left eigenvector, with associated eigenvalue λ, with $|\lambda| \leq m/n$.

The "if" direction is clear: If $A^T\mathbf{1} = \alpha(m/n)\mathbf{1}$, where $|\alpha| \leq 1$, then for any x we have

$$\mathbf{avg}(Ax) = (1/m)|(A^T\mathbf{1})^T x| = (|\alpha|/n)|\mathbf{1}^T x| \leq (1/n)|\mathbf{1}^T x| = \mathbf{avg}(x).$$

Now we'll show the opposite direction. Let $a = A^T\mathbf{1}$ and $b = (m/n)\mathbf{1}$. Then $|\mathbf{avg}(Ax)| \leq |\mathbf{avg}(x)|$ can be written as $|a^Tx| \leq |b^Tx|$. Suppose that $|a^Tx| \leq |b^Tx|$ for all x. We'll show that $a = \alpha b$, for some $\alpha \in [-1, 1]$. Note that this holds if $a = 0$, with $\alpha = 0$, so we will assume that $a \neq 0$.

Clearly if $b^Tx = 0$, then $a^Tx = 0$. Thus $\mathcal{N}(b^T) \subseteq \mathcal{N}(a^T)$. Taking orthogonal complements we get $\mathcal{R}(b) \supseteq \mathcal{R}(a)$. In particular $b \in \mathcal{R}(a)$, which means that $b = \alpha a$ for some $\alpha \in \mathbf{R}$. Taking $x = b$ in $|a^Tx| \leq |b^Tx|$ yields

$$|a^Tb| = |\alpha|a^Ta \leq |b^Tb| = \alpha^2 a^Ta,$$

so $|\alpha| \leq \alpha^2$. From this we conclude $|\alpha| \leq 1$.

Another way to come to the conclusion that the sums of the columns of A must be equal is to consider the particular values of x given by $x = e_i - e_j$, with $i \neq j$. Then $\mathbf{avg}(x) = 0$, so we have to have $\mathbf{avg}(Ax) = 0$. But $\mathbf{avg}(Ax)$ is exactly half the difference of the sum of column i and column j. We conclude that these column sums must be equal; since i and j were arbitrary, we see that all columns of A must have the same sum.

TOTAL - 10 pts

Random Variables, Processes and Linear Systems - 10 pts

A complex random variable x takes on the 4 values $\pm 1 \pm j$ with equal probability ($j = \sqrt{-1}$).

a). What is the mean value of x? (.5 pt)

b). What is the variance of x ? (1 pts)

Independent selections of this random variable at different discrete points in time, k, form the stationary random process x_k. Another random process is computed according to

$$y_k = \frac{x_k + x_{k-1} + x_{k-2} + x_{k-3}}{4} \quad .$$

c). What does the process y_k approximate? (.5 pt)

d). Find $E[y_k]$. (1 pt)

e). Find the variance of y_k (1 pt)

f). How many distinct values are there for the random process y_k ? (1 pt)

(hint you may want to consider each of real and imaginary components)

g). What is the autocorrelation function of x_k, $r_{x,k} = E[x_n \cdot x_{n-k}^*]$? (1 pt)

h). What is the autocorrelation function of y_k ? (2 pts)

i). What is the probability distribution of y_k ? (2 pts)

(hint, see hint in f).

2008 Quals Solution

a). 0

b). $Ex^2 = Ex_r^2 + Ex_i^2 = 1 + 1 = 2$

c). mean or ave. value

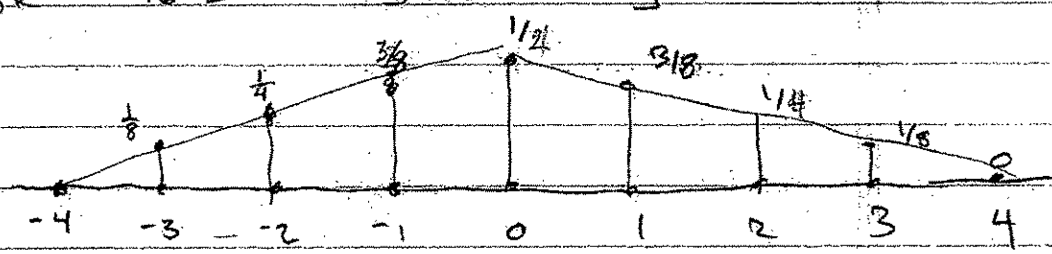
d). $Ey = 0$

e). $Ey^2 = \frac{1}{16}[2 + 2 + 2 + 2] = \frac{1}{2}$

F). $\pm 1, \pm \frac{1}{2}, 0$ for each component $5 \times 5 = \underline{25}$

g). $r_{x,k} = 2 \cdot \delta_k$

h). $r_{y,k} = \frac{1}{16}[1\ 1\ 1\ 1] * [1\ 1\ 1\ 1] * 2\,\delta_k$



i). ± 1 occurs 2 ways in either comp.

$\pm \frac{1}{2}$ " 8 " " " "

0 " 6 " " " "

$(0,0) = \frac{36}{256}$ $\quad (0, \pm\frac{1}{2}) = \frac{48}{256}$ $\quad (0, \pm 1) = \frac{12}{256}$

$(\pm\frac{1}{2}, 0) = 48/256$ $\quad (\pm\frac{1}{2}, \pm\frac{1}{2}) = \frac{64}{256}$ $\quad (\pm\frac{1}{2}, \pm 1) = \frac{16}{256}$

$(\pm 1, 0) = 12/256$ $\quad (\pm 1, \frac{1}{2}) = \frac{16}{256}$ $\quad (\pm 1, \pm 1) = \frac{4}{256}$

$\sum \rightarrow \frac{256}{256} = 1$ (checks)

Quals Question — Tom Cover
January 2008

Question 1: Gambling Scheme

Gamble \$1 each day on fair gambles. Stop when first ahead by \$1.

$$X_i = \begin{cases} 1, & \frac{1}{2} \\ \\ -1, & \frac{1}{2} \end{cases}, \quad X_i \text{ indep}$$

$$S_N = X_1 + X_2 \ldots + X_N = 1$$

$$N = \text{ stopping time}$$

1a. Are you eventually \$1 ahead (is $\Pr\{N < \infty\} = 1$)?

1b. Is $EN < \infty$, or $= \infty$? (How long does it take?)

Question 2

2a. Let X, Y be independent and identically distributed. What is $E\{X|X+Y\}$?

2b. Now let the joint distribution be arbitrary. Is

$$E[(X - E[X|X+Y])^2|X+Y] = E[(Y - E[Y|X+Y])^2|X+Y]?$$

Answers

1a. Yes, $\Pr\{N < \infty = 1\}$. You are sure to be \$1 ahead eventually.

1b. $EN = \infty$. The expected waiting time is infinite. (You are making money at the rate of (\$1)/(EN) trials and this rate can't be positive for fair gambles.)

2a. $E\{X|X+Y\} = ?$

Note: $E\{X|X+Y\} = E\{Y|X+Y\}$ since $f(x,y)$ is symmetric.

Note:

$E\{X|X+Y\} + E\{Y|X+Y\}$

$= E\{(X+Y|X+Y\}$

$= X+Y$

Thus $E\{X|X+Y\} = \frac{X+Y}{2}$

2b. Yes, they are equal. We note that

$(X - E\{X|X+Y\}) + (Y - E\{Y|X+Y\})$

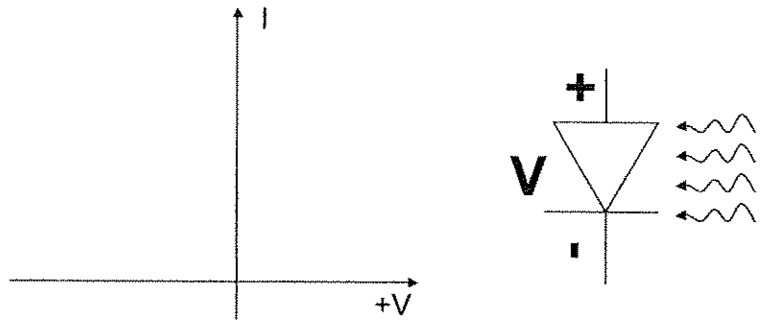
$= X + Y - E\{X+Y|X+Y\}$

$= X + Y - (X+Y) = 0$

Thus

$(X - E(X|X+Y))^2$

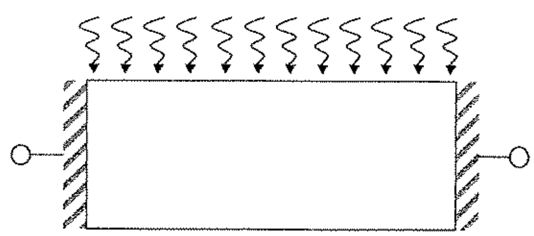
$= (Y - E\{Y|X+Y\})^2.$



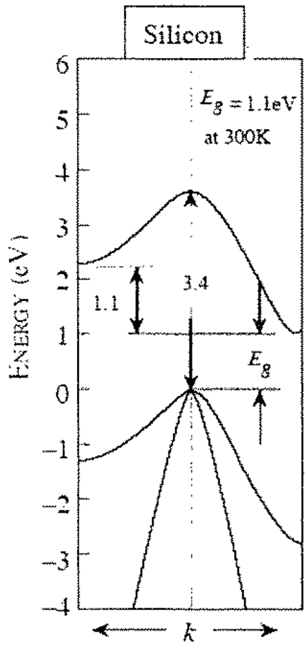
•Draw the I-V with and without light

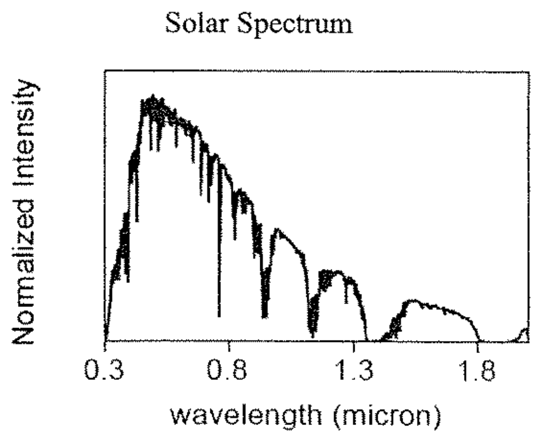
•Where do you operate the device for <u>optimum</u> power out (solar cell)

•What are the physical constraints between light in and electrons out (physical effects that must be considered)



•Sketch how the diode looks inside (doping etc.)
•For <u>negative bias</u> sketch the carrier profiles versus light everywhere through the device
•How do these distributions change with biasing (I.e. what do they look like at the "<u>optimum" bias</u> point)
•What determines the <u>maximum voltage</u> that can be measured
•How can you get more voltage





(a) For the sunlight, which part of the spectrum will be absorbed by silicon? (Note: a 1eV photon has a free space wavelength 1.24 micron.)

(b) Is Si a direct-bandgap semiconductor, or an indirect bandgap semiconductor?

(c) What is the difference in terms of optical properties, between these two classes of semiconductors? Why?

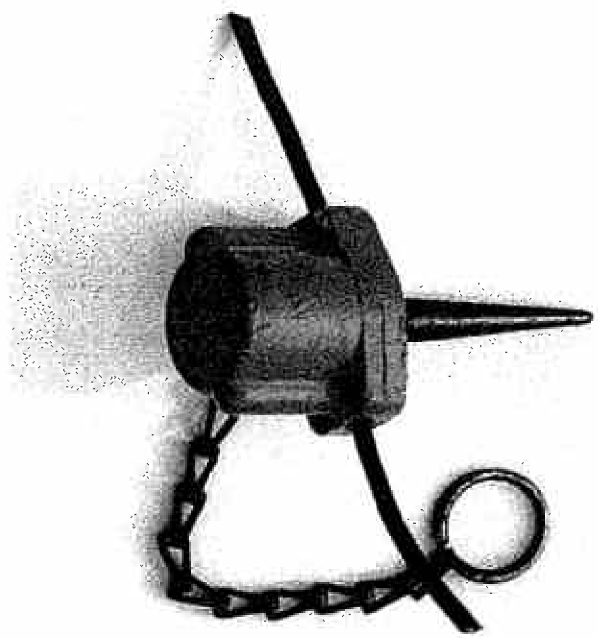
(d) What material systems are typically used to create a semiconductor laser?

(e) Any idea that you have that can make a silicon-based semiconductor laser?

Ph.D. Quals Question
January 2008
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

The Geophone

The picture below shows the device that was placed on the table in front of each student being examined. It is a geophone, a device that is used in large numbers by oil companies and other Earth resources companies to prospect for valuable minerals. What it does is measure Earth vibrations, i.e., it converts vibration of the ground into a voltage that can be measured by an appropriate instrument (e.g., a voltmeter) attached to the wires emerging from the device. The picture below doesn't show its scale; the ring on the chain is just large enough to hold a quarter. The student is shown the device, encouraged to shake it (there is clearly something loose inside its body) and asked how they think it works.



A Geophone

Scoring for this question consisted generally of 6 points for a scientifically-valid consideration of the way vibrations of the device are converted to a voltage, with 4 more points for a reasonable discussion of its actual frequency response as compared with what users might consider an ideal response.

A number of students thought the spike sticking out to the right in the picture above was an antenna. This is not an unreasonable assumption but the spike is really just that and it is meant to hold the device firmly in place on the ground (implications for frequency response?). There was no penalty for making the antenna assumption. The device contains a cylindrical magnet suspended by leaf springs and free to move along the device's axis. The magnet is surrounded by a coil and its motion produces a voltage in the coil through Faraday's Law. It is an analog, not digital, device, like the vast majority of sensors in EE and it only measures motion in one direction (dimension) – in this case, in the vertical direction. We would three such devices to measure a full 3D response.

Turning now to the frequency dependence, we have a weight suspended on a spring (actually springs in the geophone case). Most students quoted an analogy with an LC circuit, which was good. Now the problem is this: such a mechanical device, or its electrical analog, has a well-defined resonance frequency and possibly not much response apart from that resonance. Obviously users of the geophone would like to have a device that has some breadth to its frequency response. Some discussion of how the frequency response of the device might be broadened was therefore appropriate at this stage. Resistance added in an LC circuit; perhaps some damping in the actual spring/magnet setup. Returning to Faraday's Law, and the fundamental aspects of the device's response to ground motions, it is important to notice that the voltage induced in the coil surrounding the magnet varies as the rate of change of magnetic field and thus the device responds preferentially to higher-frequency ground vibrations. Finally, the spike! The device needs to be closely coupled to the ground to measure the ground vibrations properly. It is not hard to imagine a situation where the device is sitting loosely on the ground, responding to low-frequency motions but not responding at all to higher frequency motions.

$\boxed{\text{January 2008}}$

The questions are colored red.
Solutions to R.M. Gray's 2008 qualifying exam problem.

An ideal band-pass filter (BPF) with passband $\mathcal{B} = \{f : |f - f_c| \leq W\}$ is a linear system for which an input of the form $e^{j2\pi ft}$ produces an output of $Ae^{j2\pi f(t-t_0))}$ for $f \in \mathcal{B}$ and an output of 0 otherwise. f_c, W, A, and t_0 are system parameters.

Write a formula for the transfer function $H_{\text{BP}}(f) = \int_{-\infty}^{\infty} h_{\text{BP}}(t)e^{-j2\pi ft}$ where $h_{\text{BP}}(t)$ is the impulse response of the ideal BPF filter.

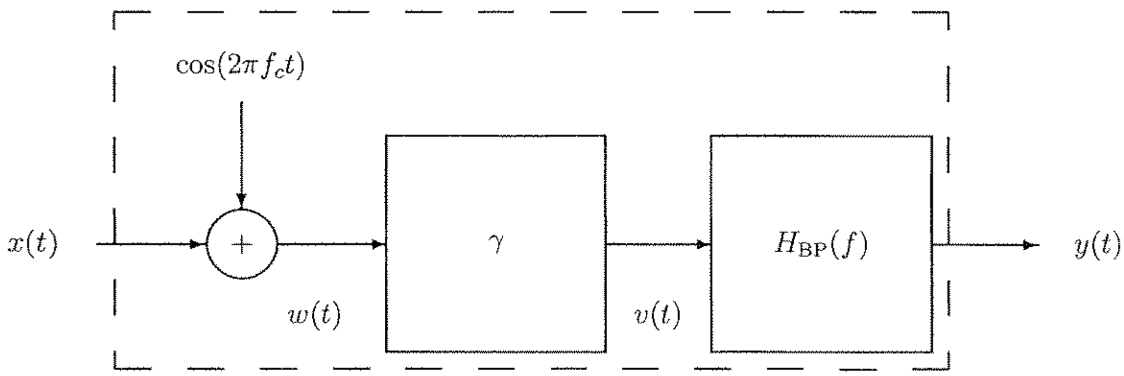
Is the filter causal?

Solution The solution I was looking for was this: You are told the system is linear and what the input/output relation is for complex exponentials. You can either assume it is also time-invariant or argue it is time invariant since you are told the system has an impulse response that depends on only a single time argument. (Or you could ask.) Since a complex exponential is an eigenvalue of an LTI system, an input time signal $e^{j2\pi ft}$ yields an output $H(f)e^{j2\pi ft}$, which you are told is $Ae^{j2\pi f(t-t_0)}$ for f in the pass band and 0 outside. Hence

$$H_{\text{BP}}(f) = \begin{cases} \frac{Ae^{j2\pi f(t-t_0))}}{e^{j2\pi ft}} = Ae^{-j2\pi ft_0} & f \in \mathcal{B} \\ 0 & \text{otherwise} \end{cases}$$

There were many more complicated ways to do this, but recognizing the input as a complex exponential and hence an eigenvalue of the system with $H(f)$ as the eigenvalue was the quickest. The inverse Fourier transform will give the impulse response, which is a modulated sinc function, which is not causal. (To be causal, the impulse response must be 0 for negative time.) I was not after detailed analysis here, rather I wanted to see what people could infer from the form of $H_{\text{BP}}(f)$ without grinding through the computation. An even more direct answer was to observe that since the spectrum is bandlimited and symmetric, its inverse Fourier transform could not be 0 for all $t \leq 0$.

Consider the following system:



where

$$w(t) = x(t) + \cos(2\pi f_c t)$$

$$v(t) = \gamma(w(t)) \quad , \quad \gamma(w) = a_0 + a_1 w + a_2 w^2$$

$H_{\mathrm{BP}}(f)$ as before
Require

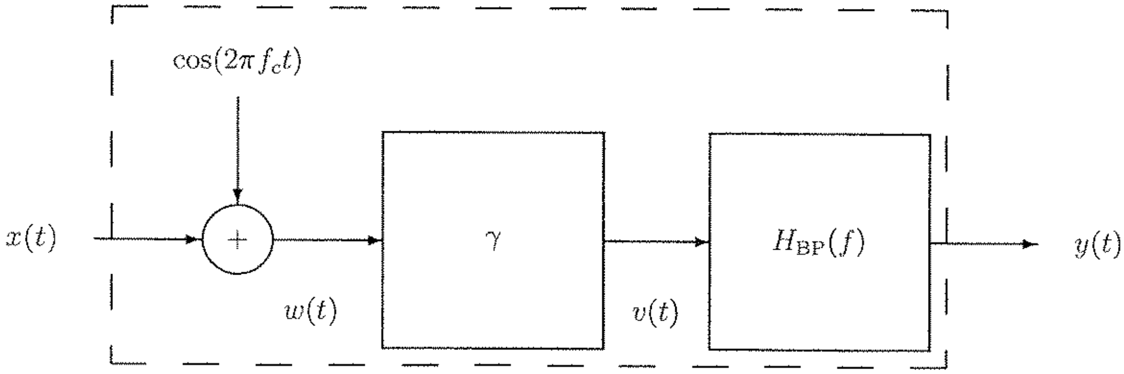
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\, dt = 0 \text{ for } \begin{cases} |f| \geq W \\ f = 0 \end{cases}$$

(bandlimited to $(-W, W)$ and no DC)

- Is this system time-invariant? linear?

Solution
Here I looked for one of two approaches. Either a student started trying to find the output $y(t)$ to see if the system was linear or time-invariant. When this approach was clear, I moved ahead to the next question to focus on $y(t)$ first. The other approach was to look at the system components and either argue for a property or say it looked like a property held or not. In this case the answers I sought were (1) that the system is probably not time-invariant because of the cosine, and probably not linear because it has nonlinear components (and also because of the cosine term that will get through the nonlinearity and bandpass filter). To answer this question definitively, you really need to find $y(t)$. This question usually served as a warmup for the next.

Continue with the system



$$w(t) = x(t) + \cos(2\pi f_c t)$$

$$v(t) = \gamma(w(t)) \quad , \quad \gamma(w) = a_0 + a_1 w + a_2 w^2$$

$H_{\mathrm{BP}}(f)$ as before

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft}\, dt = 0 \text{ for } \begin{cases} |f| \geq W \\ f = 0 \end{cases}$$

(bandlimited to $(-W, W)$ and no DC)

- Find a simple expression for $y(t)$.

Solution

$$\begin{aligned} v(t) &= a_0 + a_1 \left(x(t) + \cos(2\pi f_c t)\right) \\ &\quad + a_2 \left(x(t) + \cos(2\pi f_c t)\right)^2 \\ &= \underbrace{a_0 + a_1 x(t) + a_2 x(t)^2}_{baseband} \\ &\quad + \underbrace{a_1 \cos(2\pi f_c t) + 2a_2 x(t) \cos(2\pi f_c t)}_{passband} \\ &\quad + a_2 \cos(2\pi f_c t)^2 \end{aligned}$$

Since $\cos(2\pi f_c t)^2 = (1 + \cos(4\pi f_c t))/2$, this is

$$\begin{aligned} v(t) &= \underbrace{a_0 + a_2/2 + a_1 x(t) + a_2 x(t)^2}_{baseband} \\ &\quad + \underbrace{a_1 \cos(2\pi f_c t) + 2a_2 x(t) \cos(2\pi f_c t)}_{passband} \\ &\quad + \underbrace{(a_2/2) \cos(4\pi f_c t))}_{highband} \end{aligned}$$

This can also be expressed in the Fourier domain. Note that $x(t)^2$ occupies a frequency band of $(-2W, 2W)$ so we need at least $f_c \geq 3W$.
Passing $v(t)$ through the bandpass filter will produce a delayed and scaled version of the passband signal,

$$y(t) = Aa_1 \cos(2\pi f_c(t - t_0)) + 2Aa_2 x(t - t_0) \cos(2\pi f_c(t - t_0))$$

a classical AM modulated signal. This is clearly not time invariant, but it is a linear system if $a_1 = 0$.
I included the complete expansions for completeness, but most people only wrote down the terms that survive the BPF.

Suppose now that a signal $y(t) = (c_0 + c_1 x(t))\cos(2\pi f_c t + \theta)$ is received, but θ is unknown to the receiver.

- Can the signal $x(t)$ be recovered from $y(t)$ using only LTI filtering?

Solution

- No, to recover (or demodulate) $x(t)$ new frequencies must be introduced, which cannot occur with LTI systems. The system must be either nonlinear or time varying.

As before:
$y(t) = (c_0 + c_1 x(t)) \cos(2\pi f_c t + \theta)$ is received, but θ is unknown to the receiver.

- Can the signal $x(t)$ be recovered from $\gamma(y(t))$, $\gamma(w) = a_0 + a_1 w + a_2 w^2$, and LTI filtering?

Solution

-

$$
\begin{aligned}
&\gamma(y(t)) \\
&\quad = a_0 + a_1 \left[(c_0 + c_1 x(t)) \cos(2\pi f_c t + \theta)\right] \\
&\qquad + a_2 \left[(c_0 + c_1 x(t)) \cos(2\pi f_c t + \theta)\right]^2 \\
&\quad = \underbrace{a_0 + \frac{a_2}{2}(c_0^2 + 2c_1 x(t) + x(t)^2)}_{baseband} \\
&\qquad + \underbrace{a_1 \left[(c_0 + c_1 x(t)) \cos(2\pi f_c t + \theta)\right]}_{passband} \\
&\qquad + \underbrace{\frac{a_2}{2}(c_0 + c_1 x(t))^2 \cos(4\pi f_c t + \theta)}_{highpass}
\end{aligned}
$$

The bandpass and highpass information can be knocked out by a low pass filter, and they cannot be brought down to baseband by a LTI. So only the baseband terms can be used. There the $x(t)^2$ covers the $x(t)$, so there is no way in general to recover $x(t)$ alone from this signal using only LTI filtering.

2008 PhD Quals Questions
J. S. Harris

1. (I have a LED demo with Red, Green & Blue LEDs which I show to the student) Can you first draw a band diagram for light emitting diode (LED) and describe how it works?

2. What is different about the 3 LEDs I showed you, what is important? Does the semiconductor have to be a direct bandgap semiconductor?

3. I then show a white LED and ask, how can I get white light from a LED?

4. Can you draw the I vs V characteristic for the 3 diodes and label R, G, B? How would the LEDs differ and what insight can you gain about the material from the I-V characteristic?

5. Can you draw the emission spectrum for one of the diodes and compare it to that of an incandescent light bulb and explain the key features of the LED?

6. If you now compare the emission spectrum with the I-V characteristic, it seems that I'm getting a "free lunch" since I am able to get 2.5eV photons out with about 2V of applied bias. How is this possible? What is missing in this picture?

7. If we thermally isolate the diode, but have a window in which the photons can escape, what will happen to the LED and how does this effect the I-V and emission characteristics of the LED as a function of time (temperature)?

8. (I now show the student a Red semiconductor laser pointer) The semiconductor parts of this laser and the red LED are quite similar, but not identical. How does this semiconductor laser differ from the LED and in particular, why is the light emission so strongly directional in the laser compared to the LED?

9. On your graph of the LED spectral output, draw a similar curve for the laser? What are the notable differences?

10. If I were to put both the laser and LED in my thermally isolated environment so the temperature decreases, would you expect the optical output of the LED and laser to behave differently as a function of temperature? Explain.

2008 Quals

Examiner: Ramesh Johari

Problem 1. The Smiths have three children, of whom one is a boy. What is the probability he has two sisters as siblings? (Assume that a child is equally likely to be a boy or a girl, independent of other children.)

Problem 2. At each time period $t = 1, 2, 3, \ldots$, a red coin and a blue coin are flipped simultaneously. Assume that the red coin comes up heads with probability p_r, and the blue coin comes up heads with probability p_b.

(a) Calclulate the expected number of flips until the first head (either red or blue) is seen.

(b) Calculate the probability that at least 3 red coins come up heads before the first blue coin comes up heads.

Problem 3. Suppose that X and Y are two real-valued random variables. Show that:

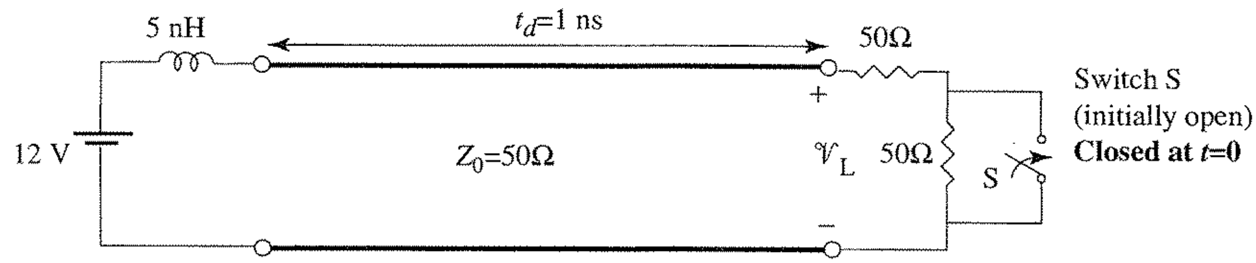
$$\mathbf{E}[|X||Y|] \leq \sqrt{\mathbf{E}[X^2]\mathbf{E}[Y^2]},$$

where $|x|$ denotes the absolute value of x.

Partial credit will be given if the result is proven under the assumption that X and Y are both uniformly distributed on $\{0, 1, 2, \ldots, N\}$.

Inan/Quals/2008
Question 1

Circuit connected for some long time before t=0



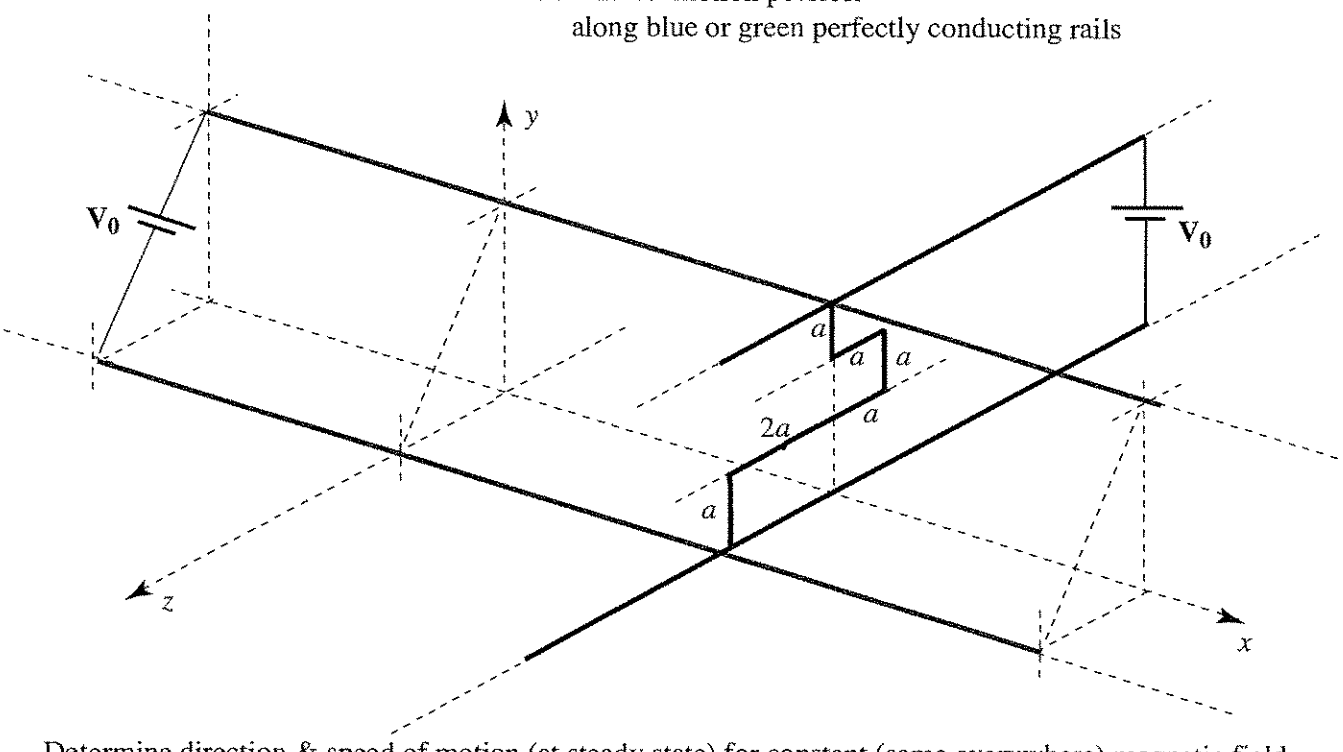
Determine and sketch the load voltage $\mathcal{V}_L(t)$

Answer: Initially, $\mathcal{V}_L(t)$ is 12 V. The switch closure launches a negative voltage wave towards the source, which is reflected at source end by the inductor (acting initially as open circuit). The voltage $\mathcal{V}_L(t)$ thus drops at t=0, and then drops again at t=2 ns, after which time it rises exponentially (as the inductor now charges) back to 12 V.

The time constant of the charging is L/Z_0=0.1 ns.

Inan/Quals/2008
Question 2

Rigid shaped bar, resistance R
Frictionless motion possible
along blue or green perfectly conducting rails



Determine direction & speed of motion (at steady state) for constant (same everywhere) magnetic field:

a) $\boldsymbol{B} = \mathbf{a}_x B_0$ **Answer:** +z-direction

b) $\boldsymbol{B} = \mathbf{a}_y B_0$ **Answer:** –x-direction

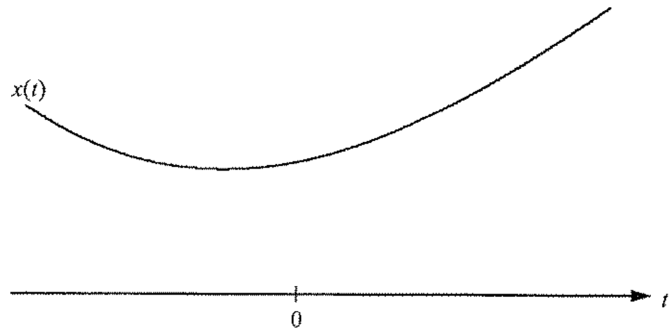
c) $\boldsymbol{B} = \mathbf{a}_z B_0$ **Answer:** –x-direction

Answer: The eventual speed of motion is determined by equating the induced emf to $\mathbf{V_0}$, e.g.,

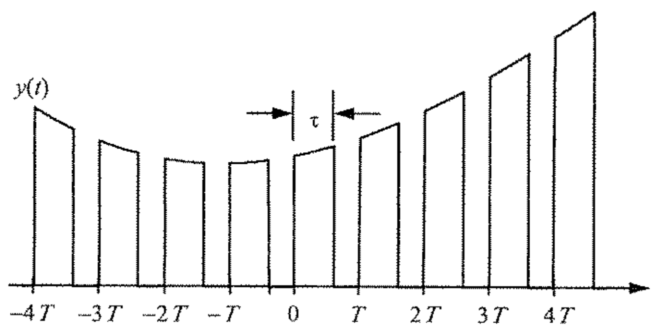
(a) emf= Integral [(v x B)·dl]= v B_0 (3a) = $\mathbf{V_0}$

Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2007-08
Professor Joseph M. Kahn

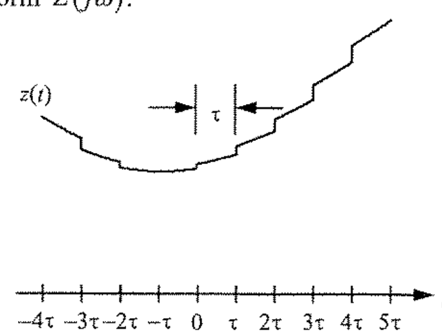
A continuous-time signal $x(t)$, $-\infty < t < \infty$, has a Fourier transform $X(j\omega)$.



We are only able to observe $x(t)$ for intervals of duration τ, which occur periodically with period T. So we observe the signal $y(t)$, $-\infty < t < \infty$, as shown. Find an expression for its Fourier transform $Y(j\omega)$.



Now we concatenate all of these observations as shown to form a new signal $z(t)$, $-\infty < t < \infty$. Find an expression for its Fourier transform $Z(j\omega)$.

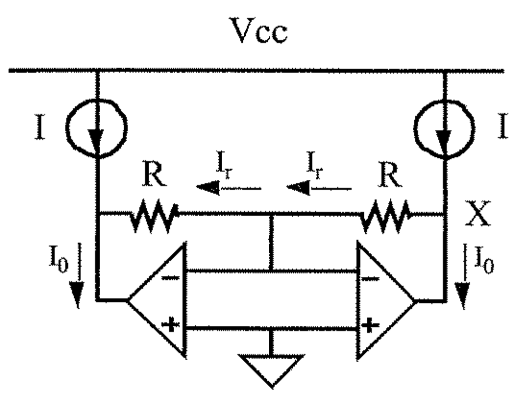


SOLUTIONS to Prof. Tom Lee's Ph.D. Qualifying Exam Question, 2008

Recall that an ideal operational transconductance amplifier (OTA) is simply a voltage-controlled current source (i.e., a transconductor, hence the name), in which the transconductance, bandwidth, output impedance and output voltage range are all infinite, and the input currents are zero.

Suppose we connect two identical, ideal OTAs in the following configuration:

FIGURE 1. OTA circuit



The two current sources shown are simply static (DC) sources.

What is the (small-signal) resistance seen between node "X" and ground? (As with most problems, there are many, many ways to solve this one. If you need a hint, consider exploiting the circuit's topological symmetry).

SOLUTION: We first make a couple of quick observations: The DC current sources can be removed right away for small-signal analysis. Next, the inverting terminals are at ground potential, given the assumptions of ideal OTA properties. Also, the matched OTAs are controlled by equal voltages, so their output currents match as well (labeled I_0). Finally, the current flowing in the two resistors must also be equal, because no current flows into the inverting terminals of the OTAs. These three observations allow a rapid deduction of the answer.

To discover resistances, the canonical method is to **drive the port in question with a source**, compute the response, and simply take the ratio of voltage to current. Here, suppose we arbitrarily drive node X with a test current source I_T. This current splits in an initially unknown way between the right-hand resistor and the OTA output. Call the current through the former I_r and that through the latter I_0. Now, the left-hand resistor carries the **same current** I_r. The left-hand OTA's output current is thus also I_r, because the left-hand resistor's (small-signal) current can only go through the OTA. So we now know that $I_r = I_0 = I_T/2$. That is, the test current applied at X splits *equally* between the resistor and the OTA. The voltage induced at node X is thus simply $(I_T/2)R$, because the other end of R is connected to a virtual ground. The equivalent load seen at X is just the ratio of voltage induced to current supplied, and is thus **R/2**.

So, what about the hint? Another way to solve this problem is to apply a differential excitation (say, I/2 into X, and I/2 out of its counterpart on the left-hand side), and then a common-mode excitation (I/2 into both nodes). The perfect symmetry of excitation and circuit in each case facilitates analysis. One need only sum the individual voltages induced at X. This method requires more steps, but depends much less on intuition, because the symmetry of the circuit AND excitations allows you to use the time-honored tricks of folding or bisecting, etc. to compute the induced voltages with a simplified circuit.

Common mistakes: The following is a partial list of widespread errors:

Almost everyone chose to retain the static DC current sources when computing small-signal resistance. Failure to remove these sources unnecessarily clutters derivations, and inhibits getting to the answer. When doing small-signal analysis, do yourself a favor and begin by setting all independent sources to their zero value.

Many students also didn't understand how to identify when something has symmetry. Yes, this *circuit by itself* is symmetrical, but adding the *excitation* required to investigate resistance introduces an asymmetry (the question asks for the single-ended resistance). So, you can't fold or bisect *first*, and *then* compute resistances. This error was disappointingly common. Study the alternate solution method above for an example of how to exploit symmetry.

Another common error was to hand-wave the OTA into irrelevance by saying "it's ideal, with infinite output impedance, so no current flows out of it." The OTA is a *dependent current source*, and is connected here in a feedback loop. Impedances can be modified in non-obvious ways. To avoid tripping up, just *measure* the impedance, as asked. There's no need to guess (and you shouldn't, if your circuit intuition is weak).

Another surprising error was the random grounding of the inverting inputs. Yes, it's a virtual ground, but the key word is *virtual*. It's not a real ground. The virtual ground's potential is indeed zero volts. However, arbitrarily adding a wire from this node to real ground then introduces a spurious branch through which currents can flow. It also deactivates the OTA (you've turned off the control voltage). Make sure you understand when and when not adding a wire to ground is correct.

To:
From: Diane Shankle <shankle@ee.stanford.edu>
Subject: 2008 Quals Questions
Cc:
Bcc:

Attachments:

From: Philip Levis <pal@cs.stanford.edu>

1) Threads vs. Processes
 - What is the difference between a thread and a process?
 - What is the relationship between the two?
 - When a program creates a new thread, what state does this allocate?
 - When a program creates a new process, what state does this allocate?

2) Consider two simplistic cases of 2-way parallel hardware. SMP, where two concurrent execution contexts have separate caches, and SMT, where the two share a cache. We have two threads, A and B, and can schedule them two ways. In option 1, we run *only* thread A for 10ms, then run *only* thread B for 10ms. In option 2, we run *both* thread A and thread B for 20ms.
 - In an SMT system, when might option 2 run *slower* than option 1?
 - Describe a memory access pattern for which option 2 would run *slowest* with respect to 1 in an SMT system.
 - In an SMP system, when might option 2 run slower than option 1?
 - Describe a memory access pattern for which option 2 would run *slowest* with respect to 1 in an SMP system.
 - Pretend you're an OS implementer and you get some bits from hardware that tell you whether you're an SMP or an SMT. Based on the above observations, what simple rules might the scheduler use to try to improve performance?

3) What is a file system extent? Why do file systems use extent-based allocation? What are its drawbacks?

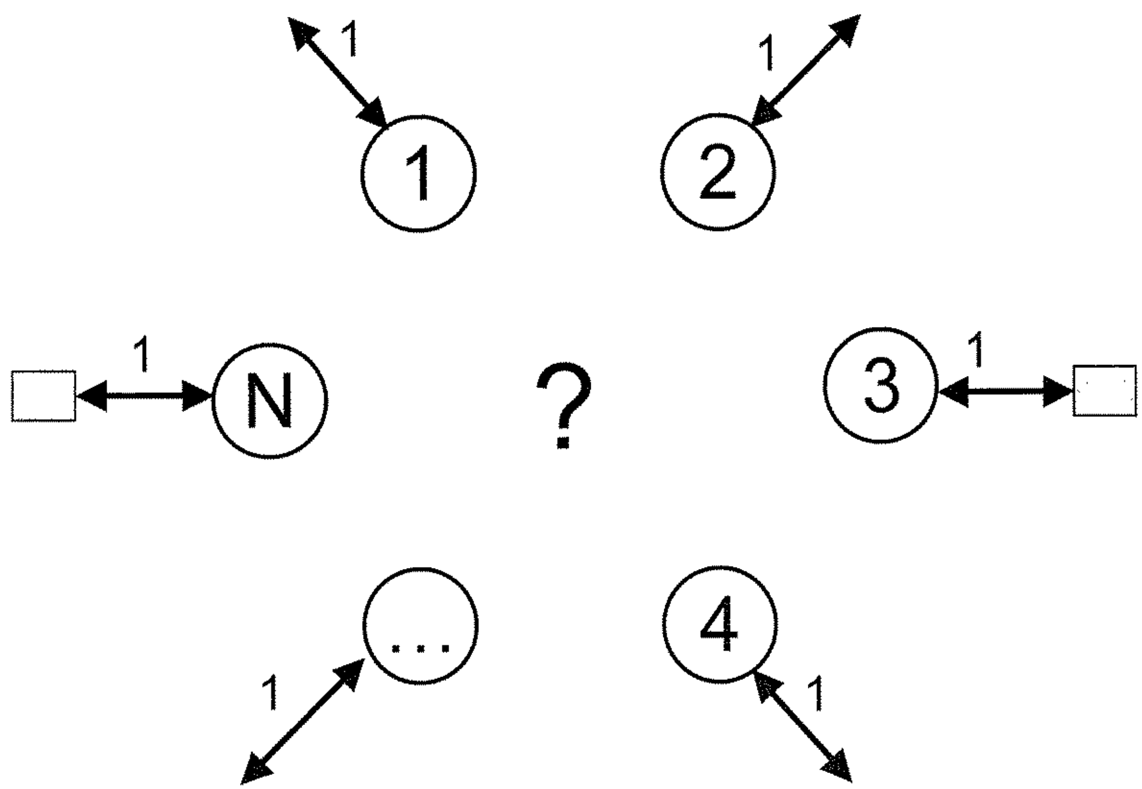
4) Describe how a kernel swap daemon frees memory pages. How does it know when a page can be freed?

Phil

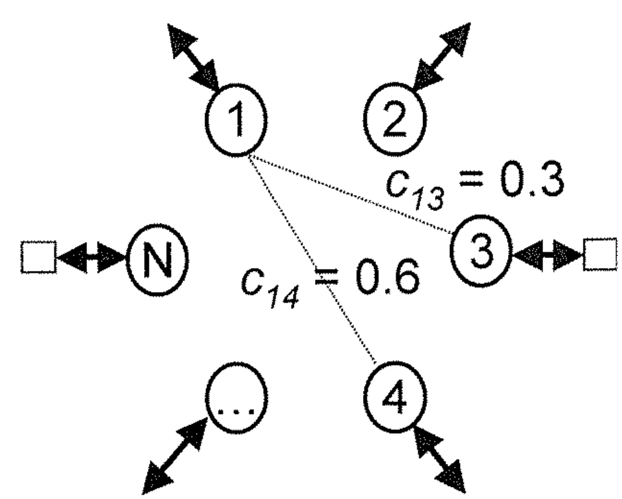
EE Quals 2008

Nick McKeown

Network



c_{ij} = data rate of link from node i to node j



Traffic Matrix

λ_{ij} = average rate of traffic from ingress *i* to egress *j*

$$\begin{pmatrix} \lambda_{11} & & \lambda_{1N} \\ & \ddots & \\ \lambda_{N1} & & \lambda_{NN} \end{pmatrix}$$

$$\sum_i \lambda_{ij} \le 1, \quad \sum_j \lambda_{ij} \le 1$$

Question 1

(a) If the traffic matrix is <u>known</u>, what is the most efficient network design that will support the traffic matrix? (*i.e.* how do you pick c_{ij} so as to minimize $\sum_{ij} c_{ij}$).

(b) What is the most efficient network design if the traffic matrix is <u>unknown</u>?

Question 2

Consider the same network, and assume the nodes are fully interconnected; i.e. $c_{ij} > 0 \quad , \forall ij$ but now packets are sent to a randomly picked intermediary, then routed directly to the correct output.

(a) Pick values of c_{ij} that will support a traffic matrix $\lambda_{ij} = \frac{1}{N}$.

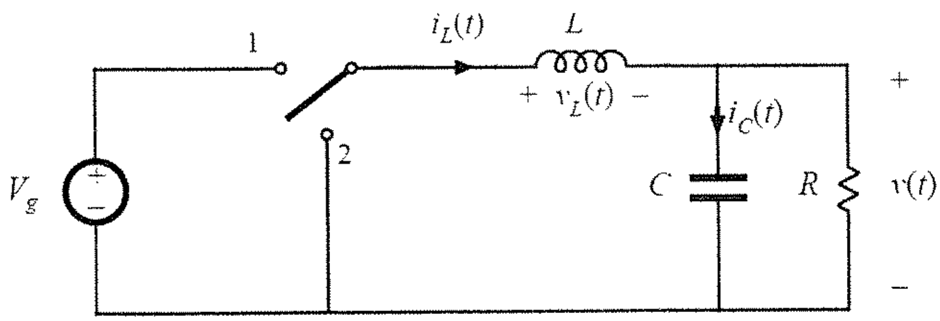
(b) What values of c_{ij} allow us to support <u>any</u> legal traffic matrix?

Question 3

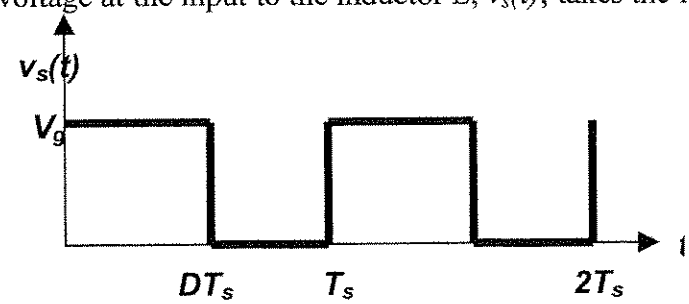
We want to design the network so that it will continue to support any legal traffic matrix if any k nodes fail.

(a) What values should we pick for c_{ij}?

(1) We have the following circuit:



- The switch has a duty cycle of D and a switching frequency of $f_s = 1/T_s$, so that the voltage at the input to the inductor L, $v_s(t)$, takes the following waveform:

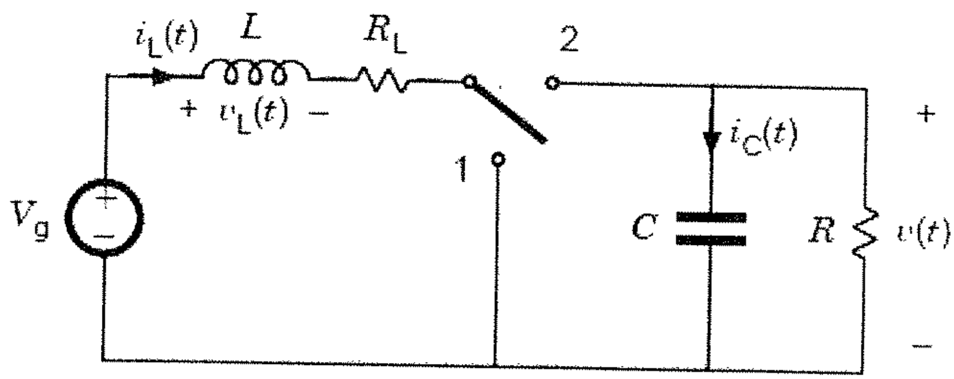


  Assume that the switching frequency f_s is higher than the bandwidth of the low-pass LC filter. Perform a steady-state DC analysis of the circuit and find out the relationship between the DC component of the output voltage v and input voltage V_g.

  Hint: Consider the principle of capacitor charge balance or the principle of inductor volt-second balance.

- Can you draw the approximate waveforms of the voltage across and the current through inductor L, $v_L(t)$ and $i_L(t)$, during the stead-state?

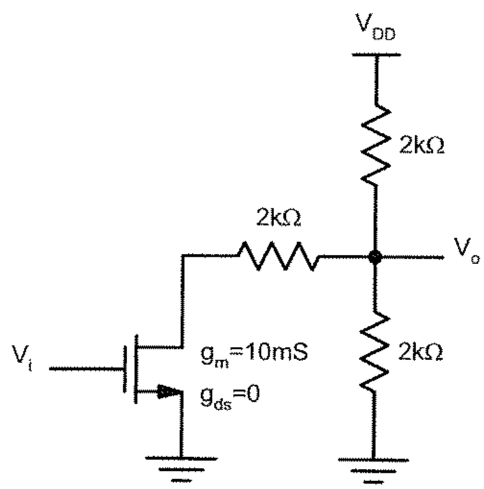
(2) Now consider the following more complicated circuit with the switch placed after the inductor L:



- Assume that that the switch is controlled similarly as in the previous case. Ignore R_L for now (which models the loss of the inductor L). Draw the waveform of $v(t)$ from time 0 to its steady-state value.
- Perform a steady-state DC analysis to find out the relationship between the DC component of output voltage $v(t)$ and input voltage V_g. How is the steady-state average output voltage influenced by R_L and D?

Name:	Stanford EE Quals 2008 Murmann

For the circuit below, calculate the low-frequency small-signal gain $A=v_o/v_i$. The MOS device is biased such that it operates in the active region.



Design a broadband circuit that interfaces a photo detector (as shown below) to a 50Ω load. The detector provides a short circuit current on the order of 100μA; the desired voltage swing across the load is on the order of 100mV.

Available components: transistors, resistors, capacitors and independent voltage and current sources for biasing (no ideal op-amps). Provide a first-order expression for the bandwidth of your circuit.

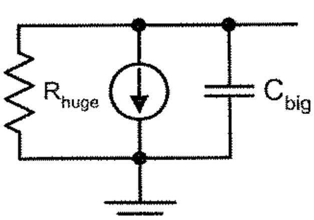
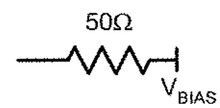


Photo detector
(small-signal model)



2007-2008 PhD Qualifying Examination

Professor Yoshio Nishi

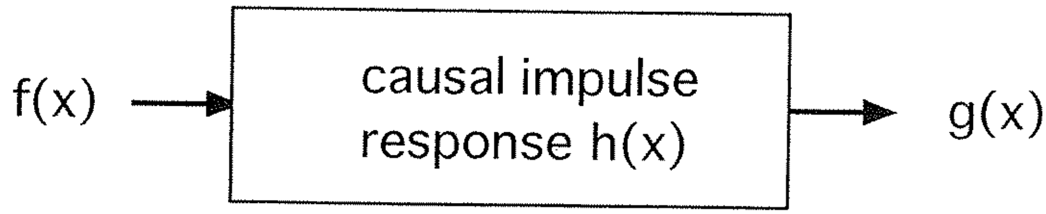
1. Please explain basic C-V characteristics of silicon MOS capacitor with p-type substrate measured at low enough frequency and at high enough frequency at room temperature. You can ignore any series resistances in gate electrode contact and substrate contact.

2. What would happen if you do the same measurements at 600K and at 50K for (a) highly doped substrate, (b) lightly doped substrate? Please assume the acceptor doping concentrations in the substrates are uniform for both cases.

3. If the silicon MOS capacitor has very high density of interface states (traps) at the middle of silicon forbidden gap, how would the C-V curve change, and why? You can assume it is acceptor-like interface states, and substrate silicon is p-Type.

1 Consider the following LTI system.

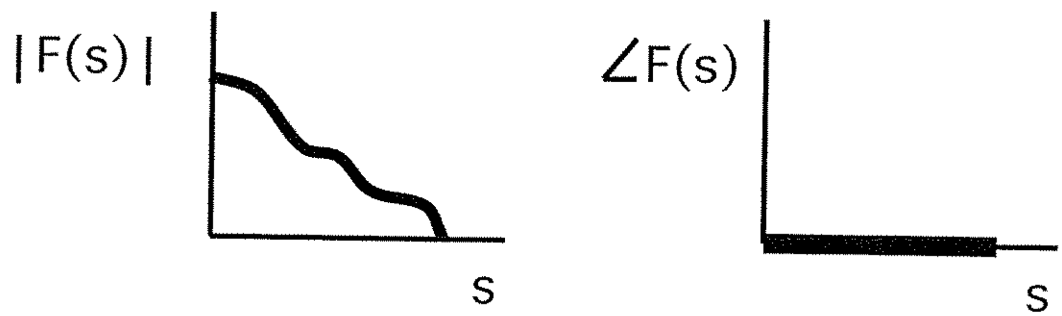
2008



$$\int h(x)\,dx = 0$$

f(x) and h(x) are real-valued

The input function f(x) has the following spectrum.



a) Will g(x) be real-valued?

b) Will g(x) be a causal function?

c) Sketch a possible output spectrum G(s) (magnitude and phase).

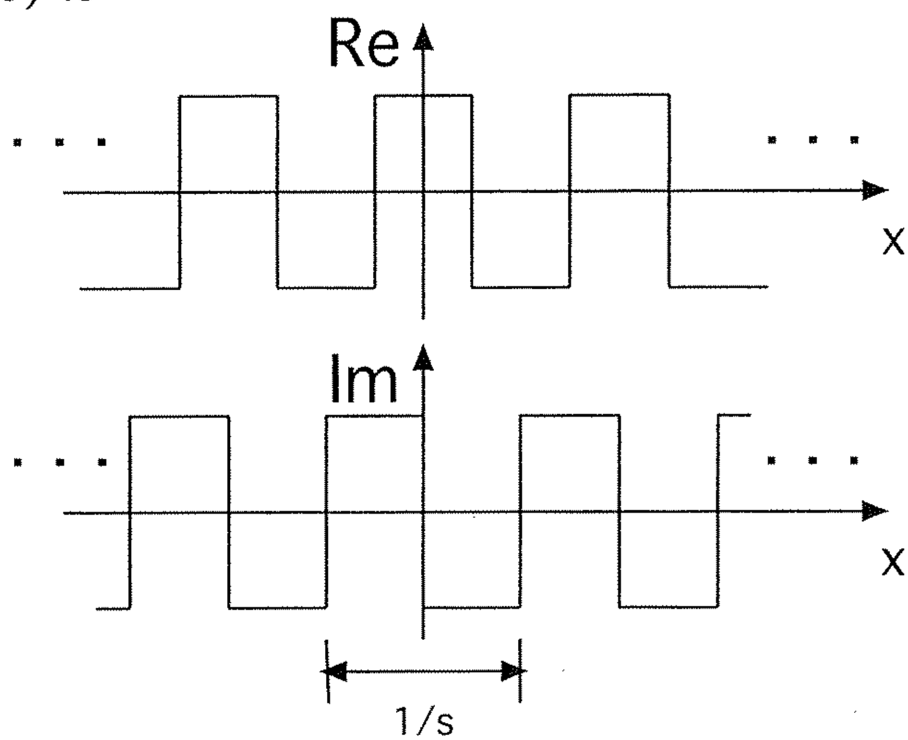
d) You are told that $\angle G(s) \propto s$.
Sketch a possible impulse response h(x).

2008

(2) Consider the following transform: $g(x) \xrightarrow{\text{transforms to}} \tilde{G}(s)$

$$\tilde{G}(s) = \int g(x)\, w(x,s)\, dx$$

where w(x,s) is



Indicate if the following transform properties still apply.

a) $a\, f(x) + b\, g(x) \longrightarrow a\,\tilde{F}(s) + b\,\tilde{G}(s)$

b) $g(ax) \longrightarrow \frac{1}{|a|}\tilde{G}(s/a)$

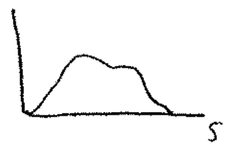
c) $g(x) = f(x) * h(x) \longrightarrow \tilde{G}(s) = \tilde{F}(s)\,\tilde{H}(s)$

Quals 2008 Answers

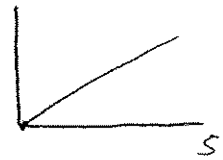
① a) yes

b) no

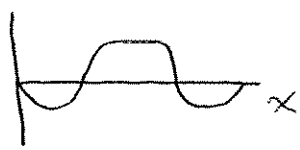
c) $|G(s)|$



$\angle G(s)$



d) $h(x)$



② a) yes

b) yes

c) no

Quals Question – 2008
Prof. A. Paulraj

Consider y(t) a sum of three (A, B and C) continuous sine (or CW) waves

$$y(t) = a1 \sin(2\pi f1\, t + p1) + a2 \sin(2\pi f2\, t + p2) + a3 \sin(2\pi f3\, t + p3); \qquad t = [\,0, 1\,] \text{ sec}$$

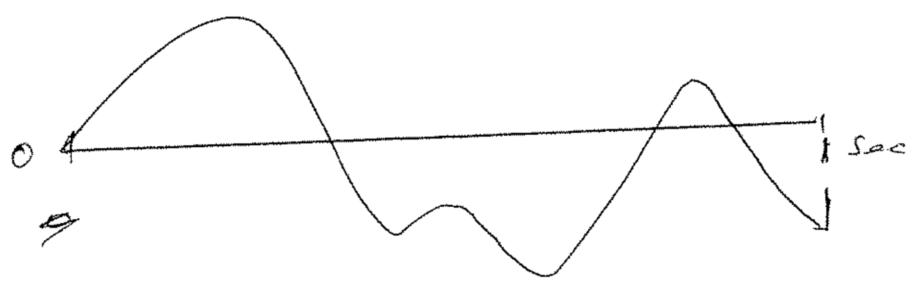
Let f1 = 2 , f2 = 4 and f3 = 3.3 Hz/ Sec

Questions:

1. Sketch y(t) if a1= a2= 1 v and a3=0. p1=p2=p3=0 degrees
2. Sketch y(t) if a1= 1 v, a2= 0.5 v and a3= 0.5 v. p1= 0 , p2= 90 and p3=180 degrees
3. If a1 = 1 v and a2=a3= 0 v, how can you picture this signal in the frequency domain, can you device a some kind of Fourier transform (say DFT ?) that will reveal the frequency information properly

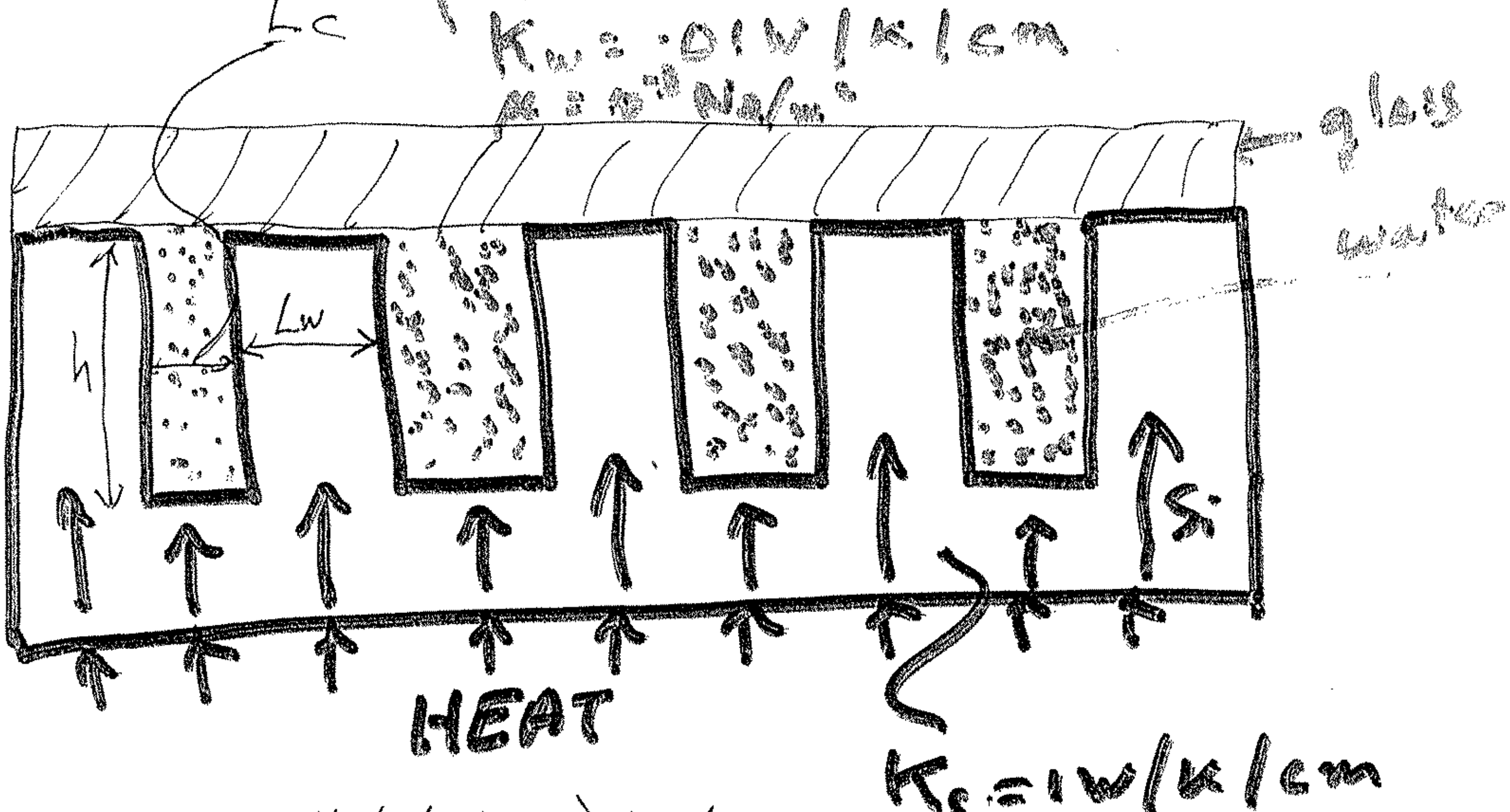
5. Now if all three sine waves are present (say a1= 1 v, a2= a3= 0.5 v. p1= 0 , p2= 90 and p3=180 degrees) what will your Fourier domain picture look like.

6. Given a waveform below of the class of y(t) (ie different frequencies, amplitudes and phases), how can we estimate the frequencies of these component sine waves.



Pease's Question
2008

$\rho C = 4 J/K/cc$
$K_w = .01 W/K/cm$
$\mu = 10^{-3} Ns/m^2$



Length (into board) 1cm
Water Pressure 1atm

- Why do heat sinks have fins?
- Express the area advantage of the above finned structure over an un-finned structure in terms of h, Lc, Lw.
- Choose h, Lc, Lw.

Piero Pianetta

How would you use a silicon device to efficiently detect photons with an energy as high as 10,000 eV? Note that the 1/e penetration depth of such a photon is about 150 microns versus a few microns for visible light photons

Acceptable answers would be a PIN diode but recognizing the fact that the intrinsic region of the PIN diode must be thick enough to accommodate the long penetration depth of the high energy photon. A Schottky diode would also be acceptable with the same considerations.

Could you use such a device to measure the energy of an individual photon?

Yes, using the fact that the electron created by the high energy photon has a very high kinetic energy which will be converted into a cascade of secondary electrons and can be swept out of the device as a current pulse.

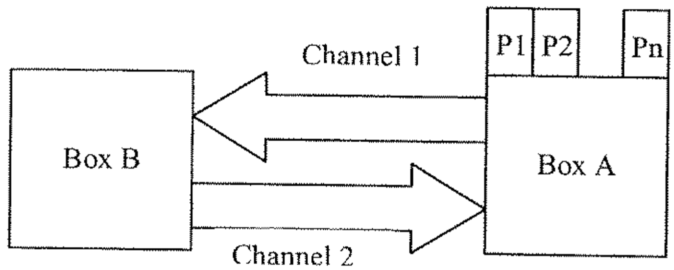
What determines the response time of the detector?

Concepts that should be understood include the time required to sweep the electrons out of the depletion region and the RC time constant of the device.

How could you make the detector faster?

Builds on answer to previous question including reducing sweep out time with higher voltage, reducing R and C. Ideas for reducing C include shaping the electrodes to have a small back electrode that can still collect the charge.

Professor Rosenblum EE quals questions – Jan 2008



Observation Points:
1) CPU-A Utilization
2) CPU-B Utilization
3) Channel 1 Utilization
4) Channel 2 Utilization

I assume you have given a system with to boxes (Box A and Box B) with communication channels between them. Channel 1 allows Box A to talk to Box B while Channel 2 allows Box B to talk to Box A. Processes (P1 … Pn) run on Box A send small request to Box B over Channel 1 and get replies back over Channel 2. A process blocks while waiting for the request to return. The system has 4 observation points where the utilizations of CPUs in Box A and Box B and the utilizations of the communication channels can be seen.

Questions
1) Someone studying the system claims that there is a bandwidth problem on Channel 2 that is causing the system to run slow. What would you expect to observe at each of the Observation points?
2) What if the claim was a latency problem on Channel 1? What would be seen at the observation points.
3) Suggest how you could fix a bandwidth problem by adding software to either box A or B. State what assumptions you fix requires.
4) Could increasing the value of n (the number of processes on Box A) help with a bandwidth problem? How about a latency problem?

Integrated Circuits Lab
Department of Electrical Engineering
Stanford University

Qualifying exam
January 2008
Prof. K.V. Shenoy

1. Does the transconductance of a MOSFET depend linearly on I_{DS}? Why (please derive)?

2. To carry the same amount of current under the same bias conditions, should a PMOS transistor be made wider, narrower, or the same as a nearby NMOS? Why (in physical terms)?

3. Please sketch an NMOS differential amplifier with tail current of 1 mA (i.e., a current source of 1 mA between source node and ground). What happens to V_{SOURCE} when the common-mode input voltage is increased? Why?

4. Before the days of digital computers, or even calculators, if you wanted to electronically calculate the natural logarithm (ln) of a number how could you do it? Please sketch and/or write equations to explain.

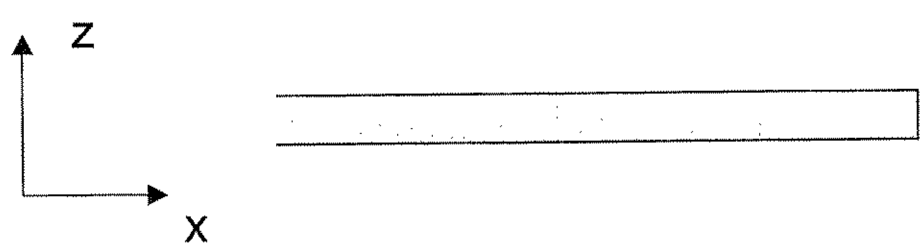
5. If you wanted to amplify the signal from a photodiode (which acts like a weak current source) with a common source amplifier, should you connect a resistor so as to form a series-x or a shunt-x feedback amplifier? Or is feedback not needed? Please explain.

6. Please describe the major sub-circuits of an op-amp, and the critical design considerations of each.

EE Qual 08, Engineering Phys.

Shan Wang

1. Write down the Maxwell equations. [2 pt]
2. Electric field is in general equivalently described by electric potential, why is magnetic field often described by both vector potential and scalar potential? [2 pt]
3. Consider a semi-infinitely long (x direction) and infinitely wide (y direction) but very thin (z direction) magnetic bar with a uniform magnetization ***M*** along the z direction. Derive the magnetic field outside the magnetic bar. [4 pt]

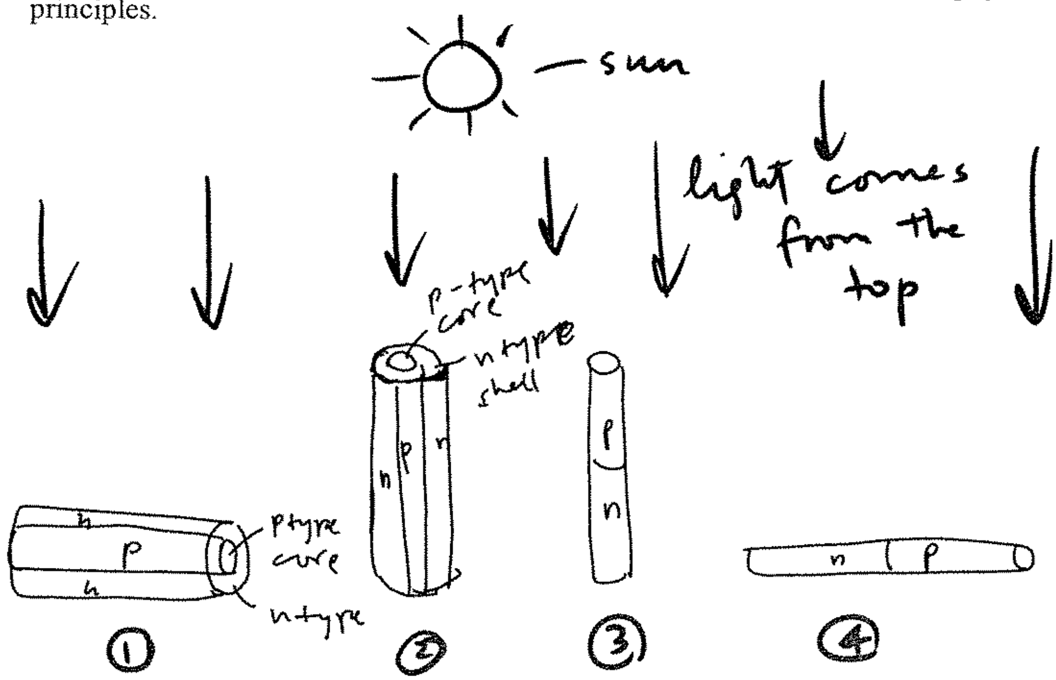


4. Sketch the scalar potential of the magnetic field above. [2 pt]

2008 Qual Exam Questions

Prof. H.-S. Philip Wong

1. Tell me how a semiconductor solar cell works.
2. Show the I-V characteristics.
3. Show the band diagram of a silicon pn-junction solar cell with and without light illumination. Show where the Fermi levels are and where the conduction band and valence band is. Explain how you would determine the Fermi level.
4. Consider a semiconductor pn-junction nanowire of diameter 30 nm and about 1 um long. Which one of the following four device configurations will give you the best solar cell? Light is coming from the top. Explain your answers using device physics principles.



5. If you are allowed to change the device configuration, how would you change it to improve this solar cell?

EE Qualifying Examination
January 2008

1. The average particle occupation number per state in the Bose-Einstein distribution is given by

$$n_{\mathrm{BE}}(E) = \frac{1}{\exp[(E-\mu)/k_B T]-1}$$

where E, μ and T are the energy of the particle state, chemical potential and temperature of the gas. If $n_{\mathrm{BE}}(E)$ becomes greater than one in the ground state, such a situation is called quantum degeneracy.

(1) Derive the condition of quantum degeneracy.

(2) If the system has a fixed number of particles, which (higher or lower) temperature is required to reach quantum degeneracy?

2. The average photon number per state in the photon statistics is given by

$$n_{\mathrm{PH}}(\hbar\omega) = \frac{1}{\exp(\hbar\omega/k_{\mathrm{B}}T)-1}$$

where $\hbar\omega$ is the energy of photon.

(1) What is a fundamental difference from the BE distribution?

(2) If the photon energy $\hbar\omega$ is fixed, which (higher or lower) temperature is required to reach quantum degeneracy?