Several systems are described below by their input/output relations.
For each system:

1. is the system linear?
2. is the system time-invariant?
3. Is the system stable?
4. is the system invertible?

Systems:

- *Input:* $x(t)$; all real t

  Output: $y(t) = \int_{-\infty}^{t} x(\tau)e^{-\alpha(t-\tau)}d\tau$; all real t

- *Input:* $x(t)$; all real t

  Output: $y(t) = [a + mx(t)]\cos(2\pi f_0 t + \theta)$; all real t

- *Input:* $x(t)$; all real t

  Output: $y[n] = x(n)$; all integer n

- *Input:* $x[n]$; all integer n

  Output: satisfies difference equation $y[n] = ay[n-1] + x[n]$; all integer n. The system is assumed to be causal.

Solution

- The system is defined by a convolution, so it is linear and time invariant. The impulse response can be recognized as $h(t) = e^{-\alpha t}u(t)$, where $u(t)$ is the unit step function. The system is stable provided $\alpha > 0$, otherwise it is an integrator (and has a Fourier transform only in the generalized, limiting, sense). Assuming that $\alpha > 0$, The Fourier transform of the impulse response is

  $$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt = \frac{1}{\alpha + j2\pi f}.$$

  The inverse filter will have transform $\alpha + j2\pi f$, which has inverse Fourier transform $\alpha\delta(t) + \delta'(t)$, a scaled identity plus differentiation.

- The system is ordinary amplitude modulation. It is linear if $a = 0$, but only affine otherwise. It is time varying because of the cosine. It is invertible if θ is known provided the positive bandwidth of $x(t)$, say W Hz, satisfies $W < f_0$. E.g., multiply by $\cos(2\pi f_0 t + \theta)$ and low pass filter and DC block. If θ is not known, then demodulation is still possible but takes more work.

- This is a sampling system. It is linear. The system is not time invariant because an input shift of an arbitrary amount does not correspond to an output shift by the same amount (unless the shift is by an integer). The system will be invertible if the sampling theorem holds, which means that the sampling frequency f_s (here 1) satisfies $f_s > 2W$, where W is