EE Quals Problem
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Consider the following second-order system:

$$\begin{aligned}
x_1(n+1) &= x_1(n) - \epsilon\, x_2(n) + u(n) \\
x_2(n+1) &= \epsilon\, x_1(n+1) + x_2(n) \\
y(n) &= x_1(n) + x_2(n)
\end{aligned}$$

where $u(n)$ and $y(n)$ are the input and output signals at time n, respectively, and $x_1(n)$ and $x_2(n)$ denote the two *state variables* at time n. Assume all signals and states are zero for $n < 0$, and that typically $|\epsilon| \ll 1$.

1. Derive the *state-space description* for this system. That is, find matrices (A,B,C,D) such that $\underline{x}(n+1) = A\underline{x}(n) + Bu(n)$ and $y(n) = C\underline{x}(n)$, where $\underline{x}(n) = [x_1(n), x_2(n)]^T$ denotes the *state vector* at time n.
2. Write down or derive an expression for the system *transfer function* in terms of the state-space description.
3. Give a formula that can be solved to find the *poles* of the system.
4. Find the product of the poles of the system.
5. Give a formula for *diagonalizing* this state-space description, if possible.
6. Write an expression for the *maximum decay time-constant* in the impulse response, assuming the system is stable.
7. If time remains, work to find the impulse response in closed form. Otherwise, state how this should be carried out.