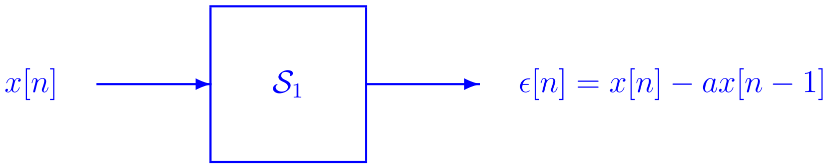
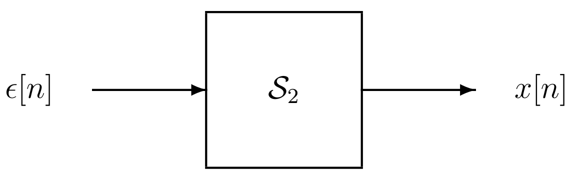
As before:



Notation: Kronecker delta function $\delta[n]$ for integer n defined by

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Next Question:
A system $\mathcal{S}_2$ is implied by the block diagram below.



$$\boxed{\text{Find the response } g[n] \text{ of } \mathcal{S}_2 \text{ to an input } \delta[n].}$$

Solution: The question effectively asks for the inverse filter. There are many ways to approach this, some of which can get quite complicated. The simplest way to solve the problem is to directly find linear difference equations that produce $x[n]$ from $\epsilon[n]$. To do this just rewrite the equation for the output to put $x[n]$ alone on the left and iterate back to time 0:

$$\begin{aligned} x[n] &= \epsilon[n] + a \underbrace{x[n-1]}_{\epsilon[n-1]+ax[n-2]} \\ &= \epsilon[n] + a\epsilon[n-1] + a^2 \underbrace{x[n-2]}_{\epsilon[n-2]+ax[n-3]} \\ &\vdots \\ &= \epsilon[n] + a\epsilon[n-1] + a^2\epsilon[n-2] + \cdots + a^n\epsilon[0] \\ &= \sum_{k=0}^{n} \epsilon[k]a^{n-k} \end{aligned}$$

If the input is $\epsilon[k] = \delta[k]$, then the output at time n is $g[n] = a^n$. The first line of the above equations can also be used to identify the filter as a first-order *autoregressive* or *all-pole* filter defined by the linear difference equation $x[n] = \epsilon[n] + ax[n-1]$