Recall that $p(t)$ is real and

$$\begin{aligned}\int_{-\infty}^{\infty} p(t)p(t-n)dt &= \delta_n \qquad (1)\\ P(f) &= \int_{-\infty}^{\infty} p(t)e^{-j2\pi ft}dt\end{aligned}$$

Second Question:
Find a *simple* expression for

$$\int_{-\infty}^{\infty} |P(f)|^2 e^{j2\pi fn} df$$

Solution There are *many* ways to do this problem.
The most straightforward approach is the standard Fourier proof method of substitution and interchanging order of integration.

$$\begin{aligned}\int_{-\infty}^{\infty} |P(f)|^2 e^{j2\pi fn} df &= \int_{-\infty}^{\infty} P(f)\left[\int_{-\infty}^{\infty} p(t)e^{-j2\pi ft}dt\right]^* e^{j2\pi fn}df\\ &= \int_{-\infty}^{\infty} p(t)\left[\int_{-\infty}^{\infty} P(f)e^{j2\pi f(t+n)}df\right]dt\\ &= \int_{-\infty}^{\infty} p(t)p(t+n)dt\end{aligned}$$

where we have used the Fourier inversion formula to recover p from P. From (1) the answer is

$$\int_{-\infty}^{\infty} |P(f)|^2 e^{j2\pi fn} df = \delta_n$$

Some people got bogged down by substituting the time domain intergral for both occurrences of $P(f)$, which is messier because of the triple integration. I tried to warn people who took a path that was likely to get tangled in details.
A shortcut to the answer is to recognize the integral as the continuous-time inverse Fourier transform of $|P(f)|^2$ evaluated at time n and that $|P(f)|^2$ is the transform of the CT autocorrelation of p,

$$r_p(\tau) = \int_{-\infty}^{\infty} p(t)p(t-\tau)dt$$

(from the correlation theorem for continuous time Fourier transforms), which when evaluated at an integer time yields the Kronecker delta δ_n from (1).
Equivalently, the integral asked for is the integral of the product of $P(f)$ and $P^*(f)e^{j2\pi fn} = \left(P(f)e^{-j2\pi fn}\right)^*$. From the generalized Parseval's theorem this is the integral in the time domain of the product of the inverse Fourier transforms of these signals, which are $p(t)$ and $p^*(t-n) = p(t-n)$, which from (1) is the Kronecker delta δ_n.
Several people tried another short cut that does not work. They correctly recognized (1) as an autocorrelation and reasoned that therefore if they transformed both sides the left hand side should be $|P(f)|^2$ (the transform of a correlation) and the right hand side should be 1 (the transform of a delta function), thus $|P(f)|^2 = 1$ for "all" f. But the correlation is a continuous time correlation, while the equation is a discrete time relation – the delta function is a Kronecker