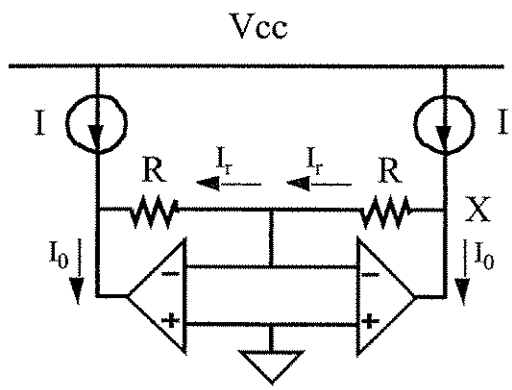
SOLUTIONS to Prof. Tom Lee's Ph.D. Qualifying Exam Question, 2008

Recall that an ideal operational transconductance amplifier (OTA) is simply a voltage-controlled current source (i.e., a transconductor, hence the name), in which the transconductance, bandwidth, output impedance and output voltage range are all infinite, and the input currents are zero.

Suppose we connect two identical, ideal OTAs in the following configuration:

FIGURE 1. OTA circuit



The two current sources shown are simply static (DC) sources.

What is the (small-signal) resistance seen between node "X" and ground? (As with most problems, there are many, many ways to solve this one. If you need a hint, consider exploiting the circuit's topological symmetry).

SOLUTION: We first make a couple of quick observations: The DC current sources can be removed right away for small-signal analysis. Next, the inverting terminals are at ground potential, given the assumptions of ideal OTA properties. Also, the matched OTAs are controlled by equal voltages, so their output currents match as well (labeled I_0). Finally, the current flowing in the two resistors must also be equal, because no current flows into the inverting terminals of the OTAs. These three observations allow a rapid deduction of the answer.

To discover resistances, the canonical method is to **drive the port in question with a source**, compute the response, and simply take the ratio of voltage to current. Here, suppose we arbitrarily drive node X with a test current source I_T. This current splits in an initially unknown way between the right-hand resistor and the OTA output. Call the current through the former I_r and that through the latter I_0. Now, the left-hand resistor carries the **same current** I_r. The left-hand OTA's output current is thus also I_r, because the left-hand resistor's (small-signal) current can only go through the OTA. So we now know that $I_r = I_0 = I_T/2$. That is, the test current applied at X splits *equally* between the resistor and the OTA. The voltage induced at node X is thus simply $(I_T/2)R$, because the other end of R is connected to a virtual ground. The equivalent load seen at X is just the ratio of voltage induced to current supplied, and is thus **R/2**.