Stanford University
Department of Electrical Engineering
Qualifying Examination Winter 2006-07
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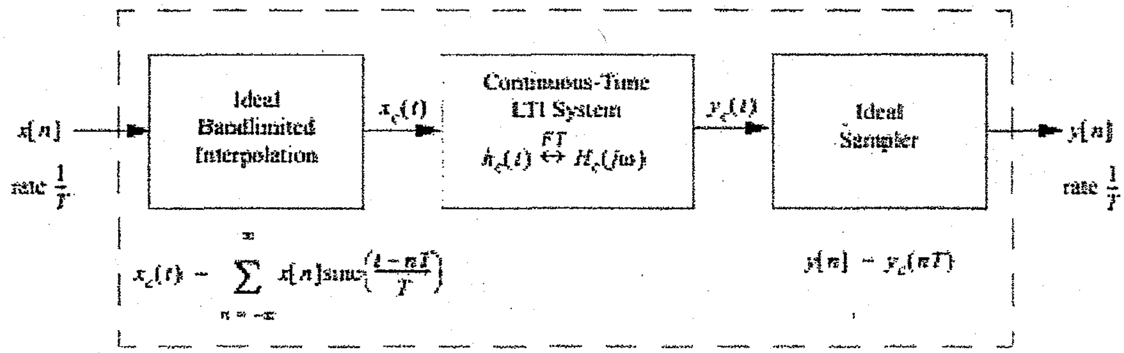

The dashed box encloses a discrete-time system having input $x[n] \overset{DTFT}{\leftrightarrow} X(e^{j\Omega})$ and output $y[n] \overset{DTFT}{\leftrightarrow} Y(e^{j\Omega})$. Give an explicit relationship between $X(e^{j\Omega})$ and $Y(e^{j\Omega})$. Is this system linear and time-invariant?

Solution: Let $\Omega = \omega T$. Define $\Pi(x) = \begin{cases} 1 & |x| \le 1/2 \\ 0 & |x| > 1/2 \end{cases}$.

Then $x_c(t) \overset{FT}{\leftrightarrow} X_c(j\omega) = T \cdot \Pi\left(\frac{\omega T}{2\pi}\right) \cdot X(e^{j\omega T}) = \begin{cases} T \cdot X(e^{j\omega T}) & |\omega| \le \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$.

$$Y_c(j\omega) = H_c(j\omega) \cdot X_c(j\omega) = T \cdot \Pi\left(\frac{\omega T}{2\pi}\right) \cdot H_c(j\omega) \cdot X(e^{j\omega T}).$$

$$Y(e^{j\omega T}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} Y_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) = X(e^{j\omega T}) \cdot \sum_{k=-\infty}^{\infty} H_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) \cdot \Pi\left(\left(\omega - k\frac{2\pi}{T}\right)\frac{T}{2\pi}\right).$$

The system within the dashed box can be expressed in terms of the equivalent discrete-time frequency response:

$H(e^{j\Omega}) = H(e^{j\omega T}) = \dfrac{Y(e^{j\omega T})}{X(e^{j\omega T})} = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) \cdot \Pi\left(\left(\omega - k\frac{2\pi}{T}\right)\frac{T}{2\pi}\right)$, which is the periodic extension of $H_c(j\omega)$ bandlimited to $|\omega| \le \pi/T$. Since the system can be expressed in this way, it is linear and time-invariant.

Kahn Signals 06-07.pdf