Electrical Engineering

Quals Questions

2009

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EE Ph.D. Qualifying Exam, January 2009 Question

David Miller

Computing without dissipation?

Notes:

There may not be a single "correct" answer to this question. The goal of this question is to see how you think about it.

If you finish the question on this sheet, subsequent questions will be asked.

[Examinees were also told verbally that the examiner was most interested in the way they thought rather than being concerned with right or wrong answers or statements, and that the exam would be conducted primarily by talking between the examinee and the examiner.]

Question:

Is it fundamentally possible to make a computer that dissipates no power while still performing a useful calculation?

If so, explain how to make the computer.

If not, explain why this cannot be done.

Answer

On this question as stated, essentially all the examinees stated that they believed (useful) computing without any dissipated power or energy was impossible (and, indeed, it is not possible to do useful computing, as defined later to the examinees, without dissipation of energy).

Most examinees were not initially clear exactly why a computer had to dissipate energy for a useful calculation, other than that they could not imagine computing parts that did not dissipate. Some students knew that the action of a logic gate, such as an AND gate, necessarily involves some dissipation because it is not reversible; knowing the output of an AND gate is not sufficient to tell you what the inputs had been.

There are, at least in principle, ways of performing computations without dissipating any energy. Some students knew that there have been proposals for reversible logic that does not throw away any information. (The Controlled-NOT gate of quantum computing is one such gate. Classical reversible gates were considered some years ago (e.g., by Fredkin and Toffoli) that also do not throw away information.) It is also true that ordinary analog physical systems, such as two balls bouncing elastically off one another, could be viewed as performing calculations of results without any particular dissipation; as an analog computer, such a system does obviously compute the result of two balls bouncing off of one another. But there is a catch in all such reversible systems, which relates to the idea of a useful calculation. At this point in the exam, after some discussion on points like these, the examinees were verbally asked the following question.

Question (second part)

A computation here will not be considered useful unless the result of the calculation is written down somewhere, such as in a memory register of some kind (e.g., in a USB memory stick). Given that requirement, and given a result that will be of some specific length (e.g., a 10 bit binary number), is there some lower bound to the amount of energy that must be dissipated for the computation to give a useful answer?

Answer (second part)

The key to the answer here is to consider entropy. The idea of entropy occurs both in discussing information and in thermodynamics, especially the Second Law of thermodynamics. Before the computation, I do not know the answer. Equivalently, the memory at the start contains an arbitrary 10 bit number. There are 2^{10} possible initial states of the memory. Hence it has an entropy

$$S = k_B \log\left(2^{10}\right) = 10 k_B \log 2$$

Here, k_B is Boltzmann's constant. This result follows from the more general statistical mechanics formula that the entropy $S = k_B \log \Omega$ where Ω is the "multiplicity" or the number of accessible states, each of which is presumed to be equally likely. (Many examinees knew this formula, either from a physics discussion of entropy or from the discussion of entropy in the context of information (in which case they would probably not have the Boltzmann's constant in the formula). If the examinee got the general point that the concept of entropy was the key to the answer and did not know this formula, some help was given to work towards an expression like this.)

After the calculation, the memory is in one specific state, so the multiplicity has been reduced to 1, leading to $S = k_B \log 1 = 0$. Hence, the change in entropy as a result of writing the answer into the memory is $\Delta S = 10 k_B \log 2$.

If the entropy of the memory has been reduced by an amount ΔS, then, by the Second Law of Thermodynamics, the entropy somewhere else or in some other aspect of the memory must have increased by an equal amount. If that is done by dissipation of energy, then we can use the thermodynamic formula

$$\Delta S = \frac{\Delta Q}{T}$$

where ΔQ is the dissipated energy (heat) and T is the temperature. (Not many examinees remembered a formula like this, though at this point any way of getting to an expression like this, including intelligent guesswork or even dimensional analysis based on the units of Boltzmann's constant being Joules/Kelvin, would have been sufficient.) Hence, finally, the dissipated energy is

$$\Delta Q = 10 k_B T \log 2$$

It is just possible that this increase in entropy does not appear as heat but instead as the disordering of some other previously ordered system, and if any examinee had given that answer it would certainly have been acceptable. With that minor caveat, we cannot make a computing system that gives us a useful result (i.e., one that is written down somewhere) without dissipating an energy ΔQ of this form.

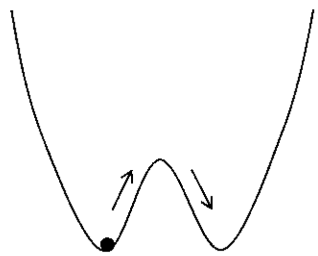
Question (third part)

If any examinee got this far in the question, then they were asked to consider mechanistically exactly how the dissipation of energy arises in setting the state of a memory in some toy example of a binary memory.

Answer (third part)

The key difficulty here is that, in writing the answer into a memory, we have to be able to write our answer (e.g., a logic one) into the memory *regardless of the current state of the memory*, and we have to make a mechanism that does that. (One might think that one could just measure the current state of the memory, and hence make a contingent mechanism that does different operations depending on the current state of the memory, but that merely postpones the problem. Our contingent mechanism would have to have a one bit memory in it to write down the answer to the measurement, and we would have to write down that answer independent of the current state of that memory, and so on.)

So, imagine that our memory element is in the form of a potential with two minima in it, and we have a ball that sits in one minimum or the other to represent either a logic zero (left minimum) or a logic one (right minimum).



Our proposed "set to logic one" mechanism is then one that simply pushes the ball to the right with sufficient pushing that it will push the ball over the top of the local maximum (the "hill") if it happens to be on the left. (Equivalently, we could temporarily lift the potential of the left well sufficiently that the ball will then roll gently into the right hand well, and then we reset the potential to its original form.) But this will not actually work yet, because the ball will simply then oscillate backwards and forwards between the wells because it now has enough energy to do so. To stop it oscillating, we need to introduce friction, such as immersing the entire apparatus in a viscous fluid, and it is in that addition that we introduce the dissipation to the system. To have the system settle into a specific state independent of its starting state, we need to introduce something irreversible, in this case the damping fluid.

Similarly, in a memory based on voltage stored on a capacitor, applying a voltage pulse through a wire will lead to oscillation from the combination of the capacitance together with the inductance of the wire unless there is resistance in the wire to damp out the oscillation.

A classic reference on the topics discussed in this question is

R. Landauer, "Dissipation and heat generation in the computing process," IBM J. Research and Development **5**, 183 – 191 (1961)

Subsequent discussions include

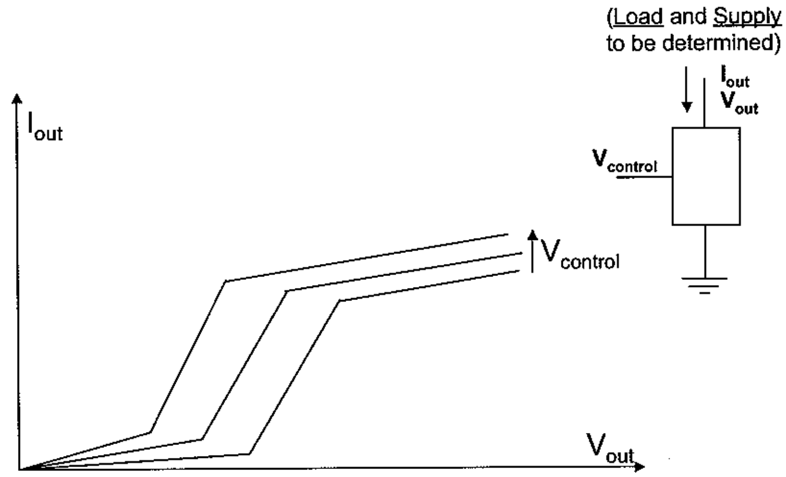
C. H. Bennett, "The thermodynamics of computation – a review," International Journal of Theoretical Physics **21**, 905 – 940 (1982)

C. H. Bennett, "Notes on Landauer's principle, reversible computation, and Maxwell's Demon," Studies in History and Philosophy of Modern Physics **34**, 501-510 (2003)

Do lower V_t devices have lower "off currents" (I.e. for V_{gs}=0V)?

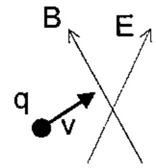
Series resistance (emitter) degrades bipolar transconductance more than (source resistance) for MOS (T/F). Explain

Does decreasing gate capacitance (C_{ox}) improve (I.e. decrease) intrinsic gate delay, independent of carrier mobility?

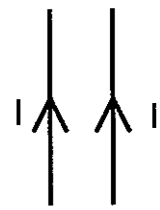


Is this device useful for building an amplifier; where (and how) would you bias it; what are the limitations?

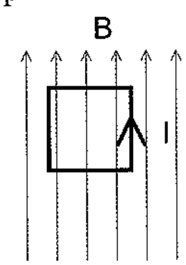
1. What is the Lorentz force?



2. Suppose we have two parallel wires, each carrying a current I flowing in the same direction. Could you use the Lorentz force formula above to predict whether the wires will attract or repel?



3. Instead, consider the following scenario, where a loop with a current I is subject to a constant magnetic field, what is the direction of the torque that is acting on the loop? What is the stable position of the loop?



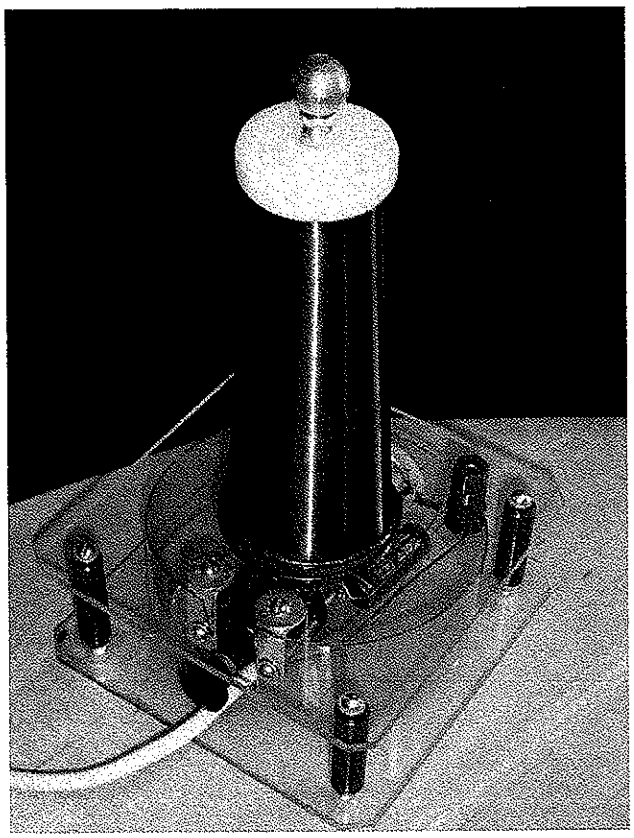
4. In the Maxwell's equation, a macroscopic material is usually characterized by ε and μ. What does μ stand for?

5. Could you think of a simple microscopic model that explains the existence of $\mu > \mu_0$ in some materials?

Ph.D. Quals Question
January 2009
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

Compact Tesla Coil

The figure below shows the compact Tesla Coil that was shown to each student. The white cord at the bottom connects the coil to a 110 V power outlet and when the black knurled knob next to it is screwed in it moves two electrical contacts closer together and sparking takes place between them. At this time, if a coin held firmly in the fingers is brought toward the round aluminum ball at the top of the coil, sparks up to 1–2 inches long can be drawn out of the sphere. Obviously there is a very high voltage being generated on the sphere; this observation leads to the first question asked of the students: (1) what is the electrical engineering basis for the generation of this high voltage, given that the source voltage is 110 V?



Two hints were given: First, the students were told, or guided, to remember Faraday's law of electromagnetic induction and then, second, an AM radio was turned on while the coil was sparking and it was shown the coil was generating radio interference across the entire AM band (i.e., covering many hundreds of kHz).

The answer to this question usually involved two steps: (i) identification of the key components of the device, and then (ii) a hypothesis for how these components generated the high voltage.

Points for (i) were awarded for identifying the many turns making up the red colored part of Tesla coil as the secondary of a **transformer,** with the two thick black coils at its base making up the primary. Obviously the much greater number of turns in the secondary will lead to higher voltages. During this inspection part of the test the students either noticed or had their attention drawn to the fact that one of the ends of the secondary coil was connected to the aluminum ball on the top and the other end, at the bottom, was connected to the green-colored socket on the right in the picture.; there was no direct electrical connection to any other part of the circuitry. At this time the students either noticed or had their attention drawn to the fact that the two ends of the primary disappeared into the circuitry containing the spark gap that was adjusted by means of the black knurled knob. Some students noticed that the wire comprising the secondary was much thinner than the wire for the primary, suggesting that the primary carried higher current.

Points for (ii) were awarded for sensible explanations for how the coil works based on Faraday's law. These explanations most succinctly made use of Faraday's law in the form:

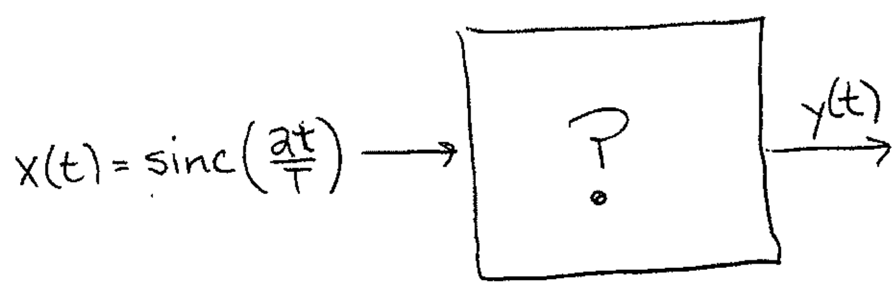
$$EMF = -\ \partial N/\partial t$$

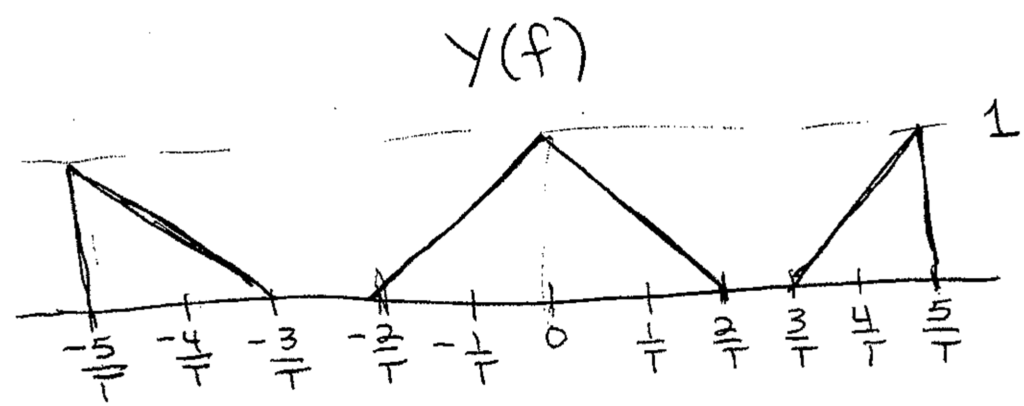
where EMF indicates the induced emf, N is the magnetic flux threading the circuit, and t is the time. The transformer action discussed above is one part of this explanation, and it involves N. Another part, however, involves the $\partial/\partial t$ term in the above equation. The noise produced by the Tesla coil in the AM radio indicates that high frequencies are involved, and high frequencies imply high $\partial/\partial t$, which in turn implies large emfs according to the Faraday equation. How are these high frequencies produced? This is where the students were expected to home in on the very noisy spark gap. The sparks were obviously very short lived and thus, eureka (for a Stanford EE student): the Fourier transform of an impulse is a function covering a wide range of frequencies in the frequency domain and the range of frequencies becomes larger as duration of the impulse gets smaller, thus the short-lived sparks give a big $\partial/\partial t$ which helps produce the high voltages in the Tesla coil.

To put this question into perspective, the earliest demonstrations of radio waves (e.g., by their discoverer Heinrich Hertz) made use of spark gaps to generate the waves, and the earliest commercial transmitters were all mostly based on spark gap technology.

Finally, a small fluorescent bulb was brought near the Tesla Coil while it was in operation, whereupon it began to glow. Thus the second, minor, question: (2) why does the bulb glow when it is not even connected to the Coil? A hint was given that the bulb contained gas at low pressure along with some mercury vapor. Here it was sufficient to point out that the strong em fields being produced by the Tesla Coil could ionize the gases in the bulb, generating em radiation in the UV and visible ranges. The UV radiation would make the phosphor coating on the inside of the bulb begin to glow.

What is in the box?





Let $n(t)$ be a WSS random process

Is $n(t) + n(t+T)$ WSS for some T?

Given a modulated signal $m(t)\cos(2\pi f_c t)$
build a demodulator using an ideal sampler

For received signal $m(t)\cos(2\pi f_c t) + n(t)$
for $n(t)$ WSS with mean zero and $S_n(f) = 3\,\mathrm{mW/Hz}$,
find the SNR at your demodulator output.

Given a signal plus interference $s(t) + i(t)$, how might you remove effects of $i(t)$?

$\boxed{\text{January 2009}}$

The questions are colored red.
Solutions to R.M. Gray's 2009 qualifying exam problem.

You are told that a discrete-time complex signal $x[n]$ for integer n has a Fourier series representation

$$x[n] = \sum_{k=0}^{K-1} a_k e^{j2\pi \frac{k}{K} n}$$

where $j = \sqrt{-1}$.
In particular, for this question an integer K and complex numbers $a_k, k = 0, 1, \ldots, K-1$ are given.

Evaluate the long-term time averages

$$\begin{aligned} < x[n] > &= \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} x[n] \\ < |x[n]|^2 > &= \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \end{aligned}$$

Solution
There are several ways to solve the problem.
First method
Those who remember their discrete time Fourier series will know that the $x[n]$ is periodic in n with period K and hence

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} x[n] = \frac{1}{K} \sum_{n=0}^{K-1} x[n]$$

and that the Fourier coefficients are given by

$$a_k = \frac{1}{K} \sum_{n=0}^{K-1} x[n] e^{-j2\pi \frac{k}{K} n} \tag{1}$$

and hence

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} x[n] = \frac{1}{K} \sum_{n=0}^{K-1} x[n] = a_0$$

If necessary, the formula for the coefficients could be derived e.g., as follows:

$$\begin{aligned} \sum_{n=0}^{K-1} x[n] e^{-j2\pi \frac{k}{K} n} &= \sum_{n=0}^{K-1} \left(\sum_{\ell=0}^{K-1} a_\ell e^{j2\pi \frac{\ell}{K} n} \right) e^{-j2\pi \frac{k}{K} n} \\ &= \sum_{\ell=0}^{K-1} a_\ell \sum_{n=0}^{K-1} e^{j2\pi \frac{\ell}{K} n} e^{-j2\pi \frac{k}{K} n} \\ &= \sum_{\ell=0}^{K-1} a_\ell \sum_{n=0}^{K-1} e^{j2\pi \frac{1}{K} n(\ell - k)} \end{aligned}$$

But

$$\sum_{n=0}^{K-1} e^{j2\pi \frac{n}{K} m} = \begin{cases} K & m = 0 \bmod K \\ \frac{1 - e^{j2\pi \frac{K}{K} m}}{1 - e^{j2\pi m/K}} = 0 & \text{otherwise} \end{cases} \tag{2}$$

and therefore

$$a_k = \frac{1}{K}\sum_{n=0}^{K-1} x[n]e^{-j2\pi\frac{k}{K}n}$$

Proving (1).

Second method Just plugging in and manipulating using the standard Fourier methods of interchanging summations with each other and limits works quite well here. This is what I looked for when students did not remember the basic properties of Fourier series.

$$\begin{aligned}\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} x[n] &= \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\left(\sum_{k=0}^{K-1} a_k e^{j2\pi\frac{k}{K}n}\right)\\ &= \sum_{k=0}^{K-1} a_k\left(\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{K}n}\right)\end{aligned}$$

Here two ways are possible: you can recognize that since complex exponentials of this form are periodic with period K,

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{K}n} = \frac{1}{K}\sum_{n=0}^{K-1} e^{j2\pi\frac{k}{K}n} = \begin{cases}1 & k = 0 \bmod K\\ 0 & \text{otherwise}\end{cases}$$

using (2), which implies that the result is a_0, or you can use the geometric progression to evaluate the limit (which saves time on the second question):

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{K}n} = \lim_{N\to\infty}\frac{1}{N}\left(\begin{cases}N & k = 0 \bmod K\\ \frac{1-e^{j2\pi\frac{k}{K}N}}{e^{j2\pi\frac{k}{K}N}} & \text{otherwise}\end{cases}\right) = \begin{cases}1 & k = 0 \bmod K\\ 0 & \text{otherwise}\end{cases}$$

which implies that the result is a_0.
Some students wrote down the geometric series formula to do the sum of the N exponential terms, but forgot that the formula requires a nonzero denominator (k cannot be 0). This led to the incorrect conclusion that the answer was 0 because the formula is bounded and the $1/N \to 0$. I gave a hint to the effect that the answer could not be correct and suggested the consider the $K = 1$ case, where $N^{-1}\sum_{n=0}^{N-1} x[n] = N^{-1}\sum_{n=0}^{N-1} a_0 = a_0$. Ideally people realized that by linearity this gave them the complete answer, their geometric progression formula forces the other terms to 0 and all that is left is a_0.
Some students tried to apply DTFTs here, which do not work since the signal does not have finite power. Others correctly recognized a connection with DFTs, but could not relate the DFTs to the Fourier coefficients correctly to get the answer.

The second sum is the time average power and it follows from Parseval's relation or by direct computation, again using (1): First, since the signal is periodic, the limit as $N \to \infty$ is given by the finite sum

$$\begin{aligned}\frac{1}{K}\sum_{n=0}^{K-1}|x[n]|^2 &= \frac{1}{K}\sum_{n=0}^{K-1}x[n]\left(\sum_{k=0}^{K-1}a_k e^{j2\pi\frac{k}{K}n}\right)^* \\ &= \frac{1}{K}\sum_{k=0}^{K-1}a_k^*\sum_{n=0}^{K-1}x[n]e^{-j2\pi\frac{k}{K}n} \\ &= \frac{1}{K}\sum_{k=0}^{K-1}|a_k|^2\end{aligned}$$

This can also be done with more work by plugging in the Fourier series representation for $x[n]$, taking the magnitude squared, and manipulating the sums and limits.

Before: $x[n] = \sum_{k=0}^{K-1} a_k e^{j2\pi \frac{k}{K} n}$

Next, you are told that $x[n]$ instead has the following form for integer n:

$$x[n] = \sum_{k=0}^{K-1} a_k e^{j2\pi k f_0 n}$$

where f_0 is a fixed real number.

Evaluate the time averages

$$\begin{aligned} < x[n] > &= \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} x[n] \\ < |x[n]|^2 > &= \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \end{aligned}$$

Solution

If f_0 is rational, the answer is the same as before. So the question is what happens when f_0 is irrational. This case occurs in modeling real-world systems, such as A/D converter error with sinusoidal inputs.

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{k=0}^{K} a_k e^{j2\pi k f_0 n} \right) = \sum_{k=0}^{K} a_k \lim_{N\to\infty} \left(\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi k f_0 n} \right)$$

As in (2)

$$\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi k f_0 n} = \begin{cases} 1 & k f_0 = 0 (\mathrm{mod} N) \\ \frac{1}{N} \frac{1-e^{j2\pi k f_0 N}}{1-e^{j2\pi k f_0}} & \text{otherwise} \end{cases}$$

If f_0 is irrational, then the first case only occurs for $k = 0$. The second case converges to 0 as $N \to \infty$. Therefore

$$< x[n] >= a_0$$

as in the periodic case.

Altenatively, by linearity you can separately consider what happens for each k in the sum. As in the periodic case, the $k = 0$ term immediately gives a_0 and the limit is trivial. The remaining terms are summing rotations around the circle and then dividing by N. The sum is bounded and the N blows up, so hey all go to zero, leaving only a_0.

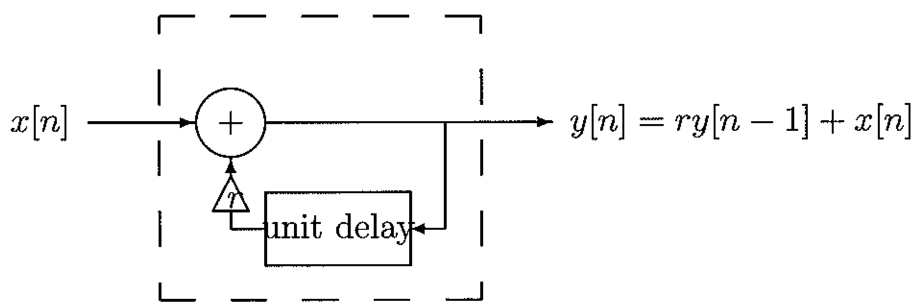
The average power follows similarly:

$$\begin{aligned}
\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 &= \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{\ell=0}^{K} a_\ell e^{j2\pi k\ell f_0 n} \right) \left(\sum_{k=0}^{K} a_k e^{j2\pi k f_0 n} \right)^* \\
&= \lim_{N\to\infty} \sum_{\ell=0}^{K} \sum_{k=0}^{K} a_\ell a_k^* \left(\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi k\ell f_0 n} e^{-j2\pi k f_0 n} \right) \\
&= \lim_{N\to\infty} \sum_{\ell=0}^{K} \sum_{k=0}^{K} a_\ell a_k^* \left(\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi k(\ell-k) f_0 n} \right) \\
&= \lim_{N\to\infty} \sum_{k=0}^{K} |a_k|^2
\end{aligned}$$

A signal representation of the type just considered,

$$x[n] = \sum_{k=0}^{K-1} a_k e^{j2\pi k f_0 n},$$

where f_0 is not a rational number, is an example of what is called a *generalized Fourier series* or an *almost periodic function.*
Suppose that such a signal is put into a discrete-time system as depicted below with $|r| < 1$:



Find a generalized Fourier series for $y[n]$.

Solution: The system is a linear system described by a linear difference equation with constant coefficients. There are many ways to find the output. The standard method from linear systems would be to recognize that (as in the case of Fourier series) the input signal is a linear combination of complex exponentials, and complex exponentials are eigenfunctions of linear systems. Together these facts lead to a solution for $y[n]$. Before going into details, the output must be in the form

$$y[n] = \sum_{k=0}^{K} a_k b_k e^{j2\pi k f_0 n}$$

and the problem is solved by finding b_k.
Observe that $y[n]$ can be found directly:

$$\begin{aligned}
y[n] &= ry[n-1] + x[n] \\
&= r(ry[n-2] + x[n-1]) + x[n] = r^2 y[n-2] + rx[n-1] + x[n] \\
&= r^3 y[n-3] + r^2 x[n-2] + rx[n-1] \\
&\vdots \\
&= \sum_{m=0}^{\infty} r^m x[n-m]
\end{aligned}$$

Therefore

$$\begin{aligned}
y[n] &= \sum_{m=0}^{\infty} r^m \left(\sum_{k=0}^{K} a_k e^{j2\pi k f_0 (n-m)} \right) \\
&= \sum_{k=0}^{K} a_k e^{j2\pi k f_0 n} \sum_{m=0}^{\infty} r^m e^{-j2\pi k f_0 m} \\
&= \sum_{k=0}^{K} a_k e^{j2\pi k f_0 n} \frac{1}{1 - re^{-j2\pi f_0}}
\end{aligned}$$

Alternatively, if a signal $e^{j2\pi k f_0 n}; n = 0, 1, 2, \ldots$ is put into the system, the output must be $H(kf_0)e^{j2\pi k f_0 n}$, where $H(kf_0)$ is the discrete-time Fourier transform of the Kronecker delta (discrete-time impulse) response $h_k = r^k$, $k = 0, 1, 2, \ldots$:

$$H(kf) = \sum_{n=0}^{\infty} r^k e^{-j2\pi k f_0} = \frac{1}{1 - re^{-j2\pi f_0}},$$

which yields the same answer.

2009 PhD Quals Questions
J. S. Harris

1. What is the Depletion Approximation for a p/n junction?

2. Can you draw the charge distribution, electric field and potential for an abrupt p/n junction under the depletion approximation?

3. Can you draw an energy band diagram for the p/n junction, including the vacuum level?

4. What happens to the above sketches if I now insert a plane of positive charge right at the p/n junction interface which is exactly $1/2(N_d x_n)$ of the original depletion region. Go back to your original drawings for the idealized p/n junction and using a different color pen, draw in the charge distribution, electric field and potential for the new situation.

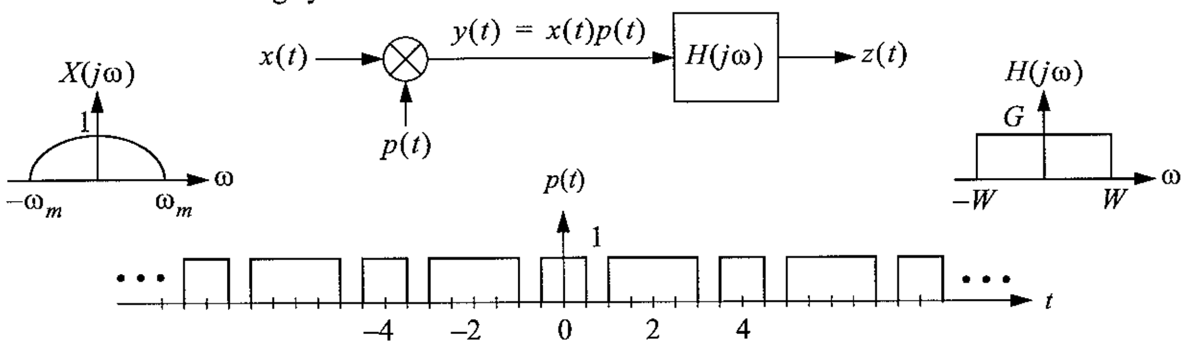
5. Please draw the I-V characteristic for the first "ideal" junction at room temperature. What would the I-V characteristic look like at -100°C and explain the differences based upon the physical processes for current in the diode.

6. Would there be any significant differences between the I-V characteristic for the "ideal" diode and the one where we introduced the sheet of charge? Why or why not?

7. If I have a p/N heterojunction in which the bandgap of the n-region is 1.5 eV and that of the p-region is 1.0 eV and both materials have exactly the same electron affinity. Draw the energy band diagram for this p/N heterojunction, including the vacuum level. Why is there a discontinuity in the valence band and not the conduction band?

Stanford University
Department of Electrical Engineering
Qualifying Examination Winter 2008-09
Professor Joseph M. Kahn

Consider the following system.



The signal $x(t)$ is bandlimited to $|\omega| \le \omega_m$. It is multiplied by $p(t)$, the periodic signal shown. The product, $y(t)$, is passed through the ideal lowpass filter $H(j\omega)$, which has cutoff frequency W and gain G.

Since $p(t)$ is periodic with period T_0 and fundamental frequency $\omega_0 = 2\pi/T_0$, it can be represented as a Fourier series:

$$p(t) = \sum_{n=\infty}^{\infty} P_n e^{jn\omega_0 t}.$$

- Without explicitly calculating the P_n, find an expression for $Y(j\omega)$, the Fourier transform of $y(t)$, in terms of $X(j\omega)$ and the P_n.
- State the conditions on ω_m, W and G such that $z(t) = x(t)$. Be as specific as possible, replacing variables by specific numbers when possible.

$$y(t) = x(t) \cdot p(t)$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

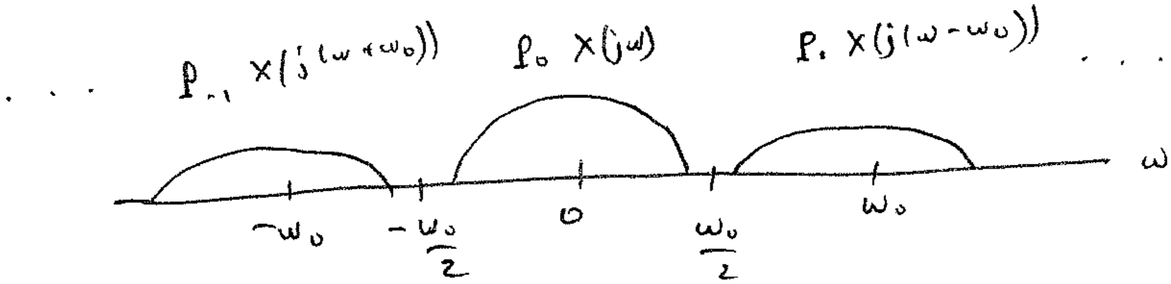
$$p(t) = \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_0 t}$$

$$P(j\omega) = 2\pi \sum_{n=-\infty}^{\infty} P_n \delta(\omega - n\omega_0)$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * 2\pi \sum_{n=-\infty}^{\infty} P_n \delta(\omega - n\omega_0)$$

$$= \sum_{n=-\infty}^{\infty} P_n X(j(\omega - n\omega_0))$$

If we can have $z(t) = x(t)$, we must have the following picture



Need $\omega_m \le \frac{\omega_0}{2}$

$$\omega_m \le W \le \omega_0 - \omega_m$$

$$G = \frac{1}{P_0}$$

$$P_0 = \frac{1}{T_0} \int_{T_0} p(t)\,dt = \frac{6}{8} = \frac{3}{4}$$

So $G = \frac{4}{3}$

Stanford University
Department of Electrical Engineering
Qualifying Examination Winter 2008-09
Professor Joseph M. Kahn

Consider a carrier-modulated signal $s(t) = m(t)\cos\omega_c t$, having Fourier transform $S(j\omega)$. Assume that $s(t)$ is narrowband, i.e., $S(j\omega) = 0$ except for ω close to $\pm\omega_c$. The signal $s(t)$ is input to a LTI system having a real impulse response $h(t)$ and a frequency response $H(j\omega)$. Near $\omega = \pm\omega_c$, you can assume that $H(j\omega)$ varies slowly with ω.

- Show that the output $y(t)$ is given approximately by:

$$y(t) \approx |H(j\omega_c)| m(t - \tau_g(\omega_c)) \cos(\omega_c(t - \tau_p(\omega_c))).$$

Give expressions for the group delay $\tau_g(\omega_c)$ and the phase delay $\tau_p(\omega_c)$ in terms of $H(j\omega)$.

- Explain intuitively why the group delay $\tau_g(\omega_c)$ has the particular mathematical form it does.

Hint: Represent $y(t)$ as the inverse Fourier transform of $Y(j\omega) = S(j\omega)H(j\omega)$. Write the frequency response in polar form as

$$H(j\omega) = |H(j\omega)| e^{j\arg\{H(j\omega)\}} = |H(j\omega)| e^{j\phi(\omega)},$$

and expand $H(j\omega)$ in a Taylor series. Near $\omega = \omega_c$, write:

$$H(j\omega) \approx \{|H(j\omega_c)| + \ldots\} e^{j\left\{\phi(\omega_c) + \left.\frac{d\phi(\omega)}{d\omega}\right|_{\omega=\omega_c} \cdot (\omega - \omega_c) + \ldots\right\}}.$$

Near $\omega = -\omega_c$, write a similar Taylor series.

4.

$$s(t) = m(t)\cos w_c t$$

$$S(jw) = \frac{1}{2}\left[\underbrace{M(j(w-w_c))}_{\substack{\text{nonzero} \\ |w-w_c| \le w_m}} + \underbrace{M(j(w+w_c))}_{\substack{\text{nonzero} \\ |w+w_c| \le w_m}}\right]$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(jw)e^{jwt}\,dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S(jw)H(jw)e^{jwt}\,dw$$

$$= \frac{1}{4\pi}\left[\int_{-\infty}^{\infty} M(j(w-w_c))H(jw)e^{jwt}\,dw\right.$$

$$\left.+\int_{-\infty}^{\infty} M(j(w+w_c))H(jw)e^{jwt}\,dw\right]$$

Near $w = w_c$:

$$H(jw) \simeq |H(jw_c)|e^{j[\phi(w_c)+\phi'(w_c)(w-w_c)]}$$

Near $w = -w_c$:

$$H(jw) \simeq |H(-jw_c)|e^{j[\phi(-w_c)+\phi'(-w_c)(w+w_c)]}$$

$$|H(-jw_c)| = |H(jw_c)|$$

$$\phi(-w) = -\phi(w) \Rightarrow \phi(-w_c) = -\phi(w_c)$$

$$\phi'(-w_c) = \phi'(w_c)$$

Near $w = -w_c$:

$$H(jw) \simeq |H(jw_c)|\, e^{j[-\phi(w_c) + \phi'(w_c)(w+w_c)]}$$

$$y(t) \simeq \frac{1}{4\pi}\Big[\int_{-\infty}^{\infty} M(j(w-w_c))\, |H(jw_c)| e^{j(\phi(w_c)+\phi'(w_c)(w-w_c))} \cdot e^{jwt}\, dw$$

$$+ \int_{-\infty}^{\infty} M(j(w+w_c)) |H(jw_c)|\, e^{j[-\phi(w_c)+\phi'(w_c)(w+w_c)]} e^{jwt}\, dw$$

$$= \frac{1}{4\pi} |H(jw_c)| \Big(e^{j\phi(w_c)} \int_{-\infty}^{\infty} M(j(w-w_c)) e^{j\phi'(w_c)(w-w_c)} e^{jwt}\, dw$$

$$+ e^{-j\phi(w_c)} \int_{-\infty}^{\infty} M(j(w+w_c)) e^{j\phi'(w_c)(w+w_c)} e^{jwt}\, dw \Big]$$

Change variables:

First integral: $\Omega = w - w_c$

Second integral: $\Omega = w + w_c$

$$y(t) \simeq \frac{1}{4\pi} |H(jw_c)| \Big[e^{j\phi(w_c)} \int_{-\infty}^{\infty} M(j\Omega) e^{j\phi'(w_c)\Omega} e^{j(\Omega + w_c)t}\, d\Omega$$

$$+ e^{-j\phi(w_c)} \int_{-\infty}^{\infty} M(j\Omega) e^{j\phi'(w_c)\Omega} e^{j(\Omega - w_c)t}\, d\Omega \Big]$$

$$= |H(jw_c)| \frac{1}{2} \Big[e^{j(w_c t + \phi(w_c))} + e^{-j(w_c t + \phi(w_c))} \Big]$$

$$\cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} M(j\Omega) e^{j\Omega(t + \phi'(w_c))}\, d\Omega$$

$$= |H(jw_c)| \cos(w_c t + \phi(w_c))\, m(t + \phi'(w_c))$$

Define $\tau_g(w_c) = -\phi'(w)\big|_{w=w_c}$

$$\tau_p(w_c) = -\frac{\phi(w_c)}{w_c}$$

$$\boxed{y(t) = |H(jw_c)| \cos(w_c(t - \tau_p(w_c)))\, m(t - \tau_g(w_c))}$$

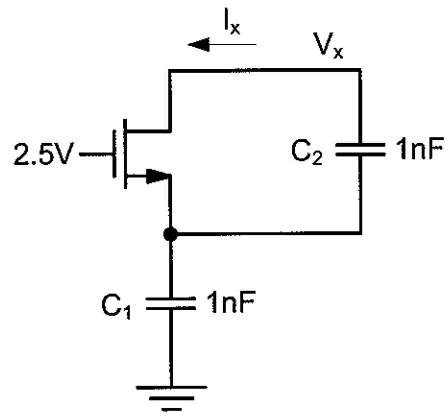
2009 Qualifying Exam Questions
S. Mitra

1.What is the difference between a combinational and a sequential circuit?
2. Consider a ripple carry adder with an End Around Carry (i.e., last carry out fed back into first carry in). Is this a combinational or a sequential circuit? Under what circumstances?
3. Given an arbitrary circuit with feedback loops, how will you analyze whether the circuit is combinational or not?
4. How will you perform static timing analysis for the ripple carry adder with End Around Carry feedback?
5. Can you extend such a static timing analysis methodology for general designs?

Name:	Stanford EE Quals 2009 Murmann

For the circuit below, sketch V_x and I_x versus time. Annotate pertinent break points and slopes.

At t=0, the initial voltages across C_1 and C_2 are 1V and 3V, respectively. Device parameters: V_t=0.5V, $\mu C_{ox}W/L$ = 1mA/V^2.



Consider a CMOS circuit in which the features and voltages are scaled by a factor S such that all electric fields remain constant. For this case, it can be shown that all performance parameters of the circuit improve by a factor of S^k.

Determine the values of k that apply to integration density, speed and energy per operation.

2008-2009 PhD Qualifying Examination

Professor Yoshio Nishi

1. Please describe mobility of electrons in semiconductor in which the size of semiconductor is enough large as compared to the mean free path of electrons.

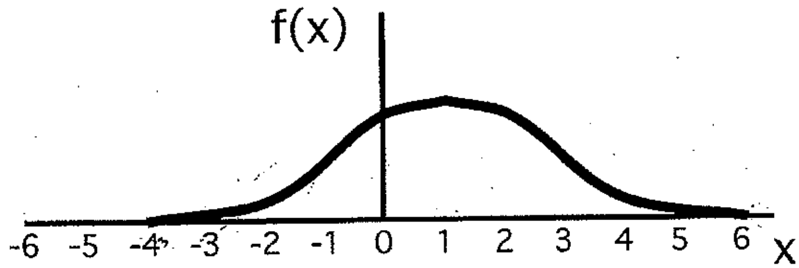
2. Given that electrons can be scattered by many different scatterers such as ionized impurity atoms, lattice vibrations, etc, how can you describe overall mobility of electrons? Please derive the formula.

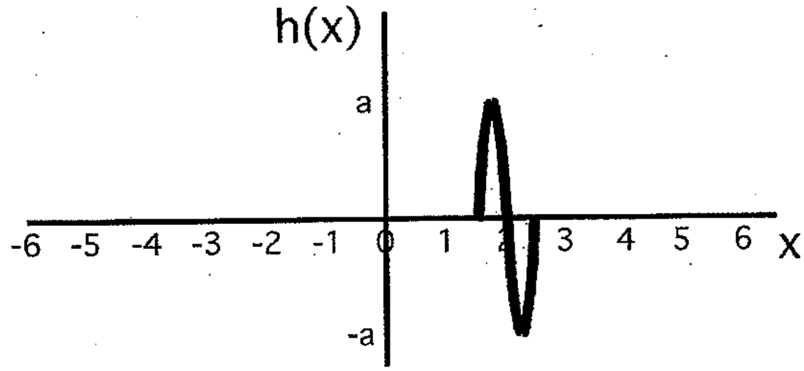
3. What conditions would make the formula inaccurate?

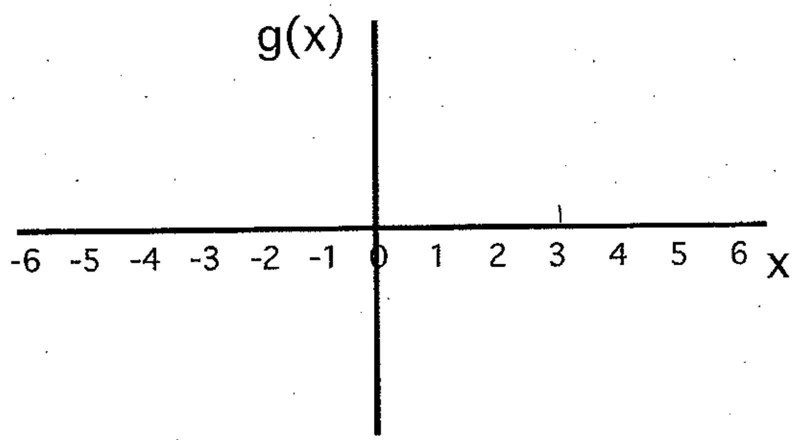
Nishimura 2009

Sketch the output g(x).

impulse response h(x)







complex-valued $f(x) \longrightarrow \boxed{\text{system}} \longrightarrow g(x)$

The output g(x) has the same magnitude as f(x). However, the system passes the phase of f(x) through a linear, time-invariant system.

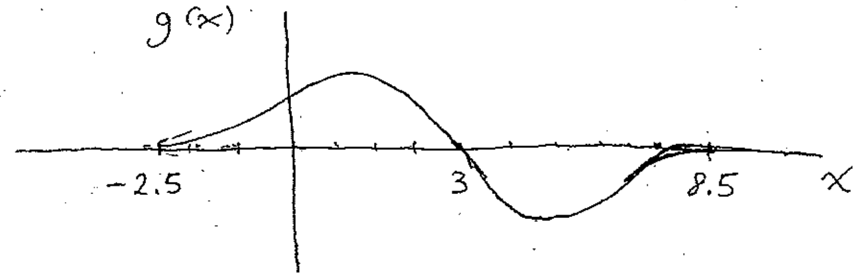
a) Is the overall system linear? Explain.

b) Is the overall system time invariant? Explain

c) Is the overall system causal? Explain.

Solutions 2009 Nishimura

(1)



(2) a) probably nonlinear

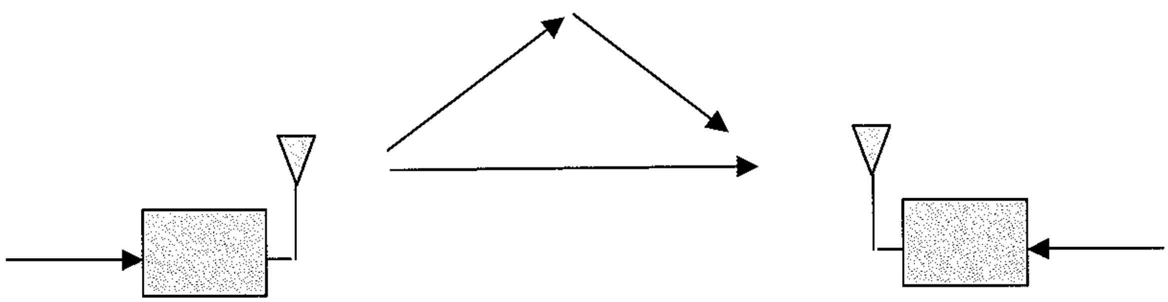
b) time invariant

c) it depends on causality or noncausality of LTI system.

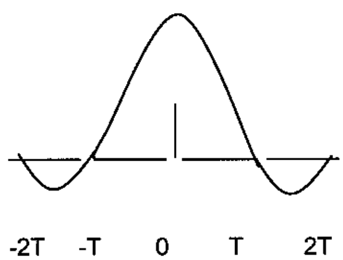
A. Paulraj

PhD Oral Exam 2009

1. Given a digital transmission link and a multi-path environment shown. Let $s(t)$ be the transmitted signal. $s(t)$ is a sequence of BPSK modulated pulses, each pulse is $b(*)$ is sent at intervals T and is sent either as $+b(*)$ or $-b(*)$ for +1 or -1 resp.



b()



Questions

1. Sketch a random sample Tx sequence $s(t)$

2. Let Rx signal be

$$y(t) = \sum a_i s(t - \partial_i) + n(t)$$

Sketch $y(t) = \sum a_i s(t - \partial_i)$

If there is only path $a_1 = 1, a_2 = 0; \partial_1 = 0, \partial_2 = T/2$,

If there are two paths $a_1 = 1, a_2 = 0.5; \partial_1 = 0, \partial_2 = T/2$

3. How can we detect the Tx data bits +1 or -1 given $y(t) = \sum a_i s(t - \partial_i) + n(t)$

4. What is equalization?

5. If $a_1 = 1, a_2 = 0.5; \partial_1 = 0, \partial_2 = T/2$ design a equalization filter

6. If $a_1 = 0.5, a_2 = 1; \partial_1 = 0, \partial_2 = T/2$, design a equalization filter

J. Pauly 2009

A bandlimited signal x(t) has been sampled at the Nyquist rate to produce a discrete time signal x[n].

x[n] is then applied to a zero-order hold to produce x_r(t), a reconstruction of the original continuous time signal x(t).

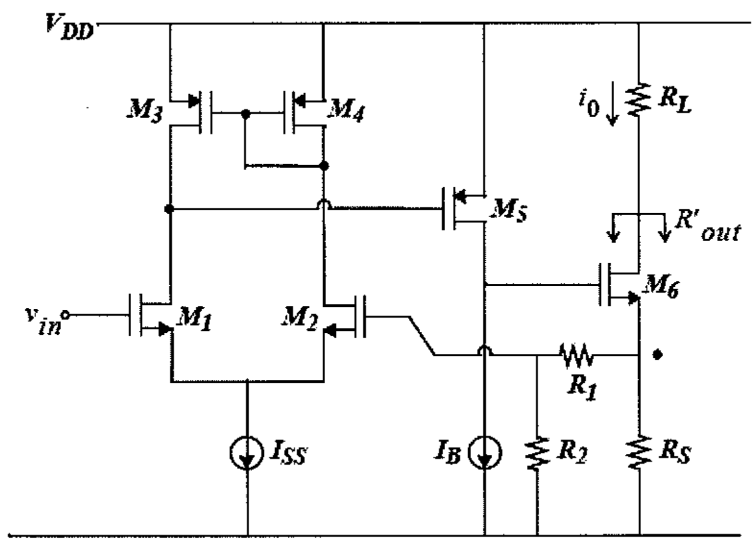
1. Sketch the spectrum of the output X_r(f), assuming you know the input spectrum X(f).

2. Assuming we still want to use a zero-order hold, how can we improve the fidelity of the reconstructed signal?

Qualifying Exam 2009

Ada S. Y. Poon

January 11, 2009



This amplifier supplies current i_0 to the load R_L. Assume all transistors operate in saturation and body effect is negligible.

1. What type of feedback is used in this circuit?
2. What is the feedback factor?
3. Determine the amplifier open-loop gain with loading. Take $R_L = 0$ for this calculation.
4. Determine the closed-loop output resistance R'_{out} (impedance at the drain of M_6).

EE QUALS, 2009

Prof. Balaji Prabhakar

1. (20 Points)

A survey of Bay Area reports: Due to a recent baby boom, the average age of the Bay Area population has come down to 30 years. Moreover, since there is a trend for people to have children later in life, the fraction of people older than 50 years of age is 65% ...

Can this be accurate?

2. Let X_i, i = 1,2,...,1000 be i.i.d Bernoulli random variables with

$$P(X_1 = 1) = 1/1000.$$

Let $S = \sum_{i=1}^{1000} X_i$. Establish one of the following bounds.

Note: The stronger the bound, the higher the points you receive.

(a) $P(S > 600) < 1/600$ (40 points)

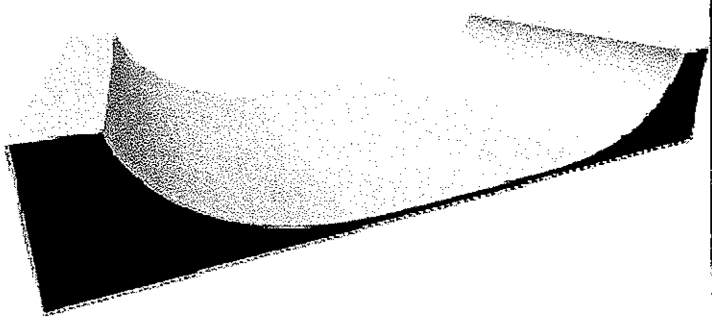
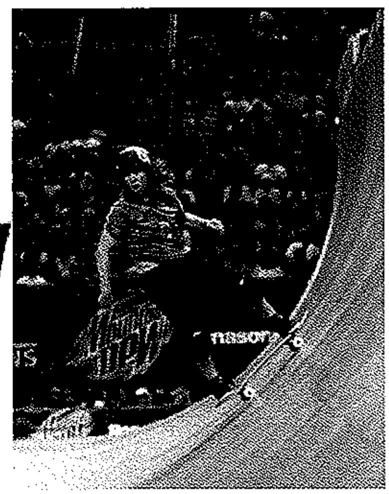
(b) $P(S > 600) < 1/599^2$ (60 points)

(c) $P(S > 600) < e^e/e^{600}$ (80 points)

2008-2009 EE Ph.D. Qualifying Exam
Question area: *Engineering Physics*
Examiner: *Jelena Vuckovic*

Please give order of magnitude answers to all of the questions below, and make any approximations you believe are reasonable.

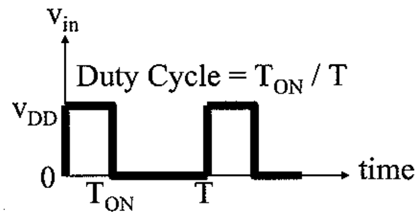
1. [7 pts]
   Skateboarding or snowboarding on a half-pipe (ramp), as shown in the figures below is a very popular sport (half-pipe snowboarding is now even a Winter Olympics sport).
   Suppose the half-pipe is 10m high and 20m wide, and the skateboarder's weight is 70kg (the skateboarder releases him/herself from the top of the half-pipe).
   a) What is the maximum velocity that the skateboarder has at any point of the half-pipe?
   b) What is the approximate skateboarder's oscillation period?
   c) Under your assumptions, how would your answers to (a) and (b) change if the skateboarder is lighter or heavier?

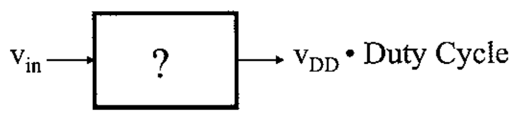



(Figures from Wikipedia)

2. [3pts]
   What area of Nevada would you have to cover with solar cells in order to provide all energy needed for the US?
   The solar flux is 340 Wm^{-2} at the surface of the Earth, and the current energy consumption in the US is 3.35TW. State all your assumptions.

2009 Qualifying Exam
Simon Wong





1. Design a circuit such that with the periodic input waveform shown, the output is approximately a DC voltage of V_{DD} • Duty Cycle.

   A low pass filter with bandwidth << 1/T;

   Possible Answers :

   R-C low pass filter with $2\Pi RC >> T$

   R-L low pass filter with $2\Pi L/R >> T$

   L-C filter with $2\Pi(LC)^{1/2} >> T$
   (Except for the peaking at resonant frequency, this filter has a low-pass behavior.)

2. If the output has to drive a heavy load (e.g., 1A), how will you modify the circuit ?

   Possible Answers :

   R-C low pass filter will not be appropriate as the small load resistance will increase the effective bandwidth.

   R-L low-pass filter will be fine, but the small load resistance will decrease the effective bandwidth.

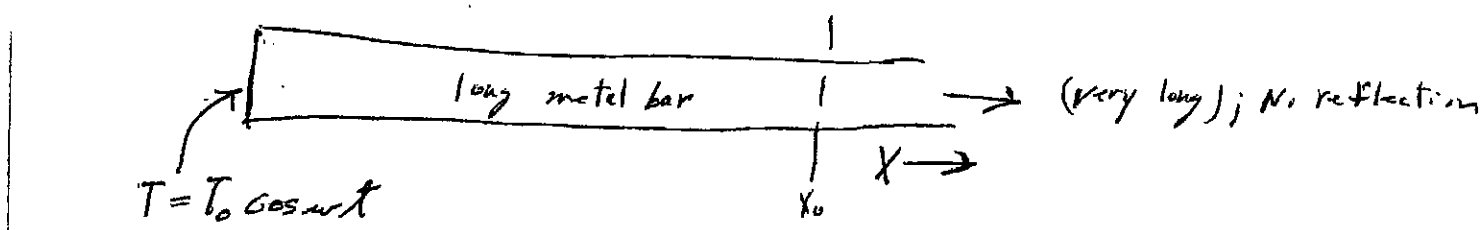
   L-C filter will not be significantly affected by the load resistance. The peaking will be reduced.

   Add a unity gain voltage buffer that is capable of driving the heavy load.

EE Quals (2009)
S.E. Harris

The first portion of the exam was qualitative and took somewhat different directions with different students. The overall discusion was, in all cases, on properties of the one-dimensional wave equation as compared to the one dimensional heat flow (diffusion) equation.

We Then considered the following problem:



One end of a metal bar is held at a sinusoidally (steady state) temperature $T = T_0 \cos \omega t$.

Making use of the heat equation

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2}$$

Find The temperature as a function of time at The dashed line.

→ First Recognize That any linear system driven by a sinusoid Responds at The drive frequency. Therfore The waveform at $X = X_0$ will differ, at most, in amplitude and phase from That at the boundary.

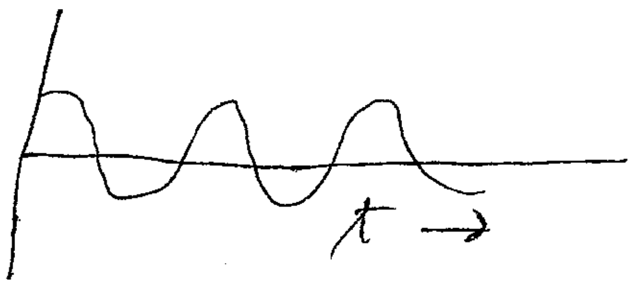
assume $T \sim \exp j\omega t \exp -\gamma X$

$$j\omega T = K\gamma^2 T$$

$$\gamma^2 = j\omega/K$$

$$\gamma = \pm \frac{(1+j)}{\sqrt{2}} \left(\frac{\omega}{K}\right)^{1/2} = \alpha + j\beta$$

$$T = \mathrm{Re}\left[T_0 \exp j\omega t \exp -\alpha X \exp -j\beta X \right]$$



at $X = X_0$, The amplitude is reduced and there is a phase change.

Now assume That a rectangular temperature pulse is applied at $X = 0$;

how does it look at $X = X_0$

why ~~does it~~ does the pulse broaden

$$\alpha \sim \sqrt{\omega}$$

higher frequencies decay more rapidly with distance.