

January 2013

Solutions to R.M. Gray's 2013 qualifying exam problem.

The goal of the problem was to test understanding and familiarity with basic probability and expectation in an unfamiliar context.

The problem treats a notion of “distance” between two distributions. The quotes reflect the fact that this is not a distance or metric in the mathematical sense since it does not satisfy the triangle inequality. The square root of the quantity is a distance. This “distance” is very old and goes by many names, including Monge-Kantorovich, transportation, Gini, Wasserstein, and Mallow distance. Most recently it was rediscovered in 1998 in the CS literature and renamed the “earth mover’s distance,” but its primary origins were in work by Monge in 1781 and Kantorovich in 1942. Kantorovich shared the Nobel prize in economics for the development of linear programming, which is intimately connected with a general version of this distance. It is useful in signal processing and communications as a measure of the mismatch resulting when designing a system for one random variable, but then applying it to another. The distance extends naturally to random vectors and random processes. Here, however, only elementary probability is needed.