

**Answer (second part)**

The key to the answer here is to consider entropy. The idea of entropy occurs both in discussing information and in thermodynamics, especially the Second Law of thermodynamics. Before the computation, I do not know the answer. Equivalently, the memory at the start contains an arbitrary 10 bit number. There are  $2^{10}$  possible initial states of the memory. Hence it has an entropy

$$S = k_B \log(2^{10}) = 10k_B \log 2$$

Here,  $k_B$  is Boltzmann's constant. This result follows from the more general statistical mechanics formula that the entropy  $S = k_B \log \Omega$  where  $\Omega$  is the "multiplicity" or the number of accessible states, each of which is presumed to be equally likely. (Many examinees knew this formula, either from a physics discussion of entropy or from the discussion of entropy in the context of information (in which case they would probably not have the Boltzmann's constant in the formula). If the examinee got the general point that the concept of entropy was the key to the answer and did not know this formula, some help was given to work towards an expression like this.)

After the calculation, the memory is in one specific state, so the multiplicity has been reduced to 1, leading to  $S = k_B \log 1 = 0$ . Hence, the change in entropy as a result of writing the answer into the memory is  $\Delta S = 10k_B \log 2$ .

If the entropy of the memory has been reduced by an amount  $\Delta S$ , then, by the Second Law of Thermodynamics, the entropy somewhere else or in some other aspect of the memory must have increased by an equal amount. If that is done by dissipation of energy, then we can use the thermodynamic formula

$$\Delta S = \frac{\Delta Q}{T}$$

where  $\Delta Q$  is the dissipated energy (heat) and  $T$  is the temperature. (Not many examinees remembered a formula like this, though at this point any way of getting to an expression like this, including intelligent guesswork or even dimensional analysis based on the units of Boltzmann's constant being Joules/Kelvin, would have been sufficient.) Hence, finally, the dissipated energy is

$$\Delta Q = 10k_B T \log 2$$

It is just possible that this increase in entropy does not appear as heat but instead as the disordering of some other previously ordered system, and if any examinee had given that answer it would certainly have been acceptable. With that minor caveat, we cannot make a computing system that gives us a useful result (i.e., one that is written down somewhere) without dissipating an energy  $\Delta Q$  of this form.

**Question (third part)**

If any examinee got this far in the question, then they were asked to consider mechanistically exactly how the dissipation of energy arises in setting the state of a memory in some toy example of a binary memory.