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Problem 1. Suppose that X is a geometric random variable, with probability of success p. Let t, s be positive integers with $t > s$. Explain why $P(X > t|X > s) = P(X > t - s)$.

Problem 2. For what values of N does the following statement hold?

"If $X_1, \ldots, X_N$ are jointly Gaussian random variables such that $\mathrm{Var}(\sum_i X_i) = \sum_i \mathrm{Var}(X_i)$, then $X_1, \ldots, X_N$ are independent."

Problem 3. Suppose that $X_1, X_2, \ldots$ are a sequence of random variables on the nonnegative integers, such that $E[X_n] \to \infty$ as $n \to \infty$. Does it follow that $P(X_n = 0) \to 0$ as $n \to \infty$?

Now suppose that in addition, $\mathrm{Var}(X_n) = E[X_n]$. Does it follow that $P(X_n = 0) \to 0$ as $n \to \infty$?