(f) Consider the list, γ, of all the elements in S found by reading the elements in the tree, starting at the root y and traversing the tree from left to right along each row. Now pick two elements from γ, say $S_{(i)}$ and $S_{(j)}$. Let's figure out the probability that they are compared by the algorithm. i.e. the probability that one is in the sub-tree of the other. Let $S_{(k)}$ have rank such that $i \le k \le j$, and let $S_{(k)}$ appear earliest in γ of the elements in the range $S_{(i)}$ to $S_{(j)}$. If k is equal to neither i or j, will i and j appear in the same sub-tree as each other?

(g) If k is equal to either i or j, will they appear in the same sub-tree as each other?

(h) Given that there are j Ð $i + 1$ elements in the range $S_{(i)}$ to $S_{(j)}$, and we know that i and j will be compared iff either one of them is first in list γ, what is p_{ij}?

(i) Prove that the expected total number of comparisons equals $2n\sum_{k=1}^{n}\frac{1}{k}$.