Nicholas McKeown ?

Question 1.

A densely populated country decides to enforce a strict policy on the number of children per family. The problem is that there is a preference for sons, and every family would like to have at least one boy. The policy is as follows:

> *Each family may have as many daughters as they want, but they must stop having children after their first son is born.*

Assume the following:

1. Every family continues to have children until their first son.
2. All children survive.
3. Boys and girls are equally likely to be born.

Part a: Will the number of girls in the country exceed the number of boys?

Part b: Keeping assumptions (2) and (3), we try and devise a new policy such that the expected number of boys exceeds the expected number of girls in the population as a whole. Does such a policy exist?

Nicholas McKeown

Question 2.

Imagine a population in which the distribution of children per woman is constant over time, and is $a_i = 1/3$, where $i = 1,2,3$. We ask a number of women the following two questions:

1. How many children did you have? (Assume their childbearing days are over).
2. How many children did your mother have?

Part a: What is the average response to (1)?
Part b: What is the average response to (2)?
Part c: Will the population remain constant, increase or decrease?

Quals questions given by Professor Nick McKeown, January 2002.

Question #1:

I want to represent a counter with 8 different values using 3 incandescent light bulbs. The lifetime of the light bulbs is determined by how often they are switched on and off. The more often they are switched on or off, the more likely they are to fail.

a. If I want to maximize the time between replacing a light bulb, what code should I use to represent the 8 different values of the counter?

Question #2:

It's common for a room light to be switched on and off by two or more different light switches.

a. If two different switches control a light, how are the switches wired up?
b. What if there are three switches?

Question #3:

Network switches and routers process packets to decide where to send them, and to modify packet headers. Sometimes, a conventional CPU (such as a MIPs or Intel processor) is used to process the stream of packets.

a. What do you think are the pros and cons of using a general-purpose processor for this application?
b. If you were to design a "packet-processor", how would it differ from a conventional processor?

Qualifying Examinations

January 2003
Professor Nick McKeown

1. **Expectation.**

Explain intuitively why expectation is linear.

2. Randomized Quicksort.

Consider a set of n numbers, S, that we wish to sort using the following randomized algorithm.

Input: The set of numbers, S.
Output: Elements of S in increasing order.

Steps:

```
1. Choose $y$ uniformly and at random from $S$.
2. Compare every element with $y$ to find: $S_1$, the set of elements smaller than
   $y$, and $S_2$, the set of elements larger than $y$.
3. Recursively sort $S_1$ and $S_2$. Output $S_1$, $y$, $S_2$.
```

In this question, we're going to find out how long the algorithm takes to run. We'll define the expected running time to be the *expected number of comparisons* needed to sort the set S. Let $S_{(1)}$ be the smallest element of S, $S_{(n)}$ the largest, and $S_{(i)}$ the ith smallest.

(a) Consider two elements $S_{(i)}$ and $S_{(j)}$. How many times can they be compared to each other by the algorithm?

(b) Define X_{ij} to be equal to 1 if elements $S_{(i)}$ and $S_{(j)}$ are compared by the algorithm, and 0 otherwise. Write down an expression for the total number of comparisons in terms of X_{ij}.

(c) Find an expression for the expected number of comparisons as an expression of $E[X_{ij}] \equiv p_{ij}$. .

In what follows, we'll find an expression for p_{ij}.

(d) Draw a binary decision tree, rooted on y, that shows the sets evolving at each stage of the recursion. Show the set with smaller elements to the left. What order do we need to read out the elements in the tree to output them in sorted order?

(e) Pick two elements from S at random. Under what conditions will they be compared during the execution of the algorithm?

(f) Consider the list, γ, of all the elements in S found by reading the elements in the tree, starting at the root y and traversing the tree from left to right along each row. Now pick two elements from γ, say $S_{(i)}$ and $S_{(j)}$. Let's figure out the probability that they are compared by the algorithm. i.e. the probability that one is in the sub-tree of the other. Let $S_{(k)}$ have rank such that $i \le k \le j$, and let $S_{(k)}$ appear earliest in γ of the elements in the range $S_{(i)}$ to $S_{(j)}$. If k is equal to neither i or j, will i and j appear in the same sub-tree as each other?

(g) If k is equal to either i or j, will they appear in the same sub-tree as each other?

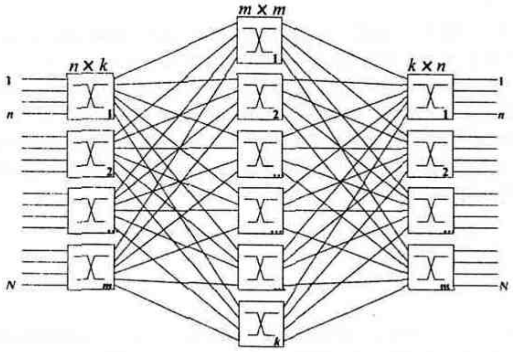
(h) Given that there are j Ð $i + 1$ elements in the range $S_{(i)}$ to $S_{(j)}$, and we know that i and j will be compared iff either one of them is first in list γ, what is p_{ij}?

(i) Prove that the expected total number of comparisons equals $2n\sum_{k=1}^{n}\frac{1}{k}$.

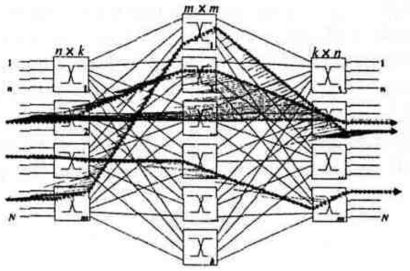
EE Quals 2004
Professor Nick McKeown

Q1: An Algorithm for a Switching Network

Consider the N-input and N-output switching network shown below. New connections are added one at a time between a single unoccupied input and a single unoccupied output.



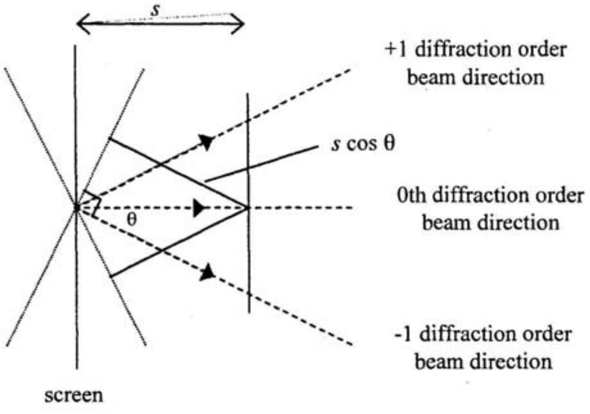
For example, here is the same switching network with three connections.



a) Write down N as a function of n and m.
b) In general, is k larger, smaller, or equal to n? Explain.
c) What is the minimum value of k (as a function of n) such that a new connection can always be added when its input and output are unoccupied?
d) Do you think the value of k depends on whether connections are all together, or one at a time (in an arbitrary order)?

Supplementary questions

The answer as to what happens at —— and multiples of that distance is called Talbot self-imaging. At the distance, all the diffraction orders have the same relative phase as they did at the screen, and the intensity pattern is therefore the same as it would be at the screen itself. Hence an image of the screen would appear if we put a piece of white card at this distance. To see how this happens at this specific distance, see the figure below.


s
+1 diffraction order beam direction
s cos θ
0th diffraction order beam direction
θ
-1 diffraction order beam direction
screen

In this figure, we see the first diffraction orders (both positive and negative) and the "straight through" beam (i.e., the zeroth diffraction order). The path length for the +1 (or -1) diffraction order phase front to hit the center of the observing screen at distance s is

Hence the path length difference between that and the zero order path is

Now, the diffraction angle θ is

and so we have

Hence, for the first order diffraction beams to be in phase with the zeroth order beam, we must have

Solution

Main questions

Huygens's principle states that each point on the surface of a phase front of a wave can be regarded as a source of spherical wavefronts, and the subsequent phase fronts can be constructed from the envelope formed by adding this together.

[If the examinee does not know Huygens's principle, it will be given to them so they can demonstrate their reasoning in the rest of the question.]

a. Only one slit.

The light from a narrow slit diffracts out in all directions, and so a screen at a very large distance would show essentially uniform illumination. [Note that, though the standard result for a slit of finite width is technically a "sinc" function, because the slit is stated as being much narrower than a wavelength, there really are no zeros in the actual pattern, because we never get to a condition of destructive interference between light from different parts of the slit.] Some students reasoned from their knowledge that there is a Fourier transform relationship between the field at the slits and the field in the far distance (or far field), reasoning that the slit is like an impulse, which transforms to a uniform distribution.

b. Two slits.

For two slits, this experiment is known as Young's two slit experiment. The result is that we see an interference pattern of alternating bright and dark stripes on a screen at a very large distance. This can be deduced from the notion of expanding circular phase fronts from the two narrow slit sources and the resulting interference at a screen at a very large distance. Again, some students reasoned from Fourier transforms to deduce that the transform of two impulse functions leads to a cosinusoidal tranform, which also explains the alternating bright and dark stripes.

c. A very large number of slits.

The result at a screen at a very large distance will be a set of relatively narrow beams at equally spaced angles. There are several ways this can be deduced. The standard way an optics person would likely go at this would be to consider the screen as a diffraction grating, and look at the directions that give constructive interference between the beams. Those knowing the Fourier transform relationship between the fields could reason that the The Fourier transform of a set of equally spaced impulses (or delta functions) is a set of equally spaced impulses (or delta functions). Another way of deducing the result is to consider the superposition of the interference patterns of slits of separation d and those of slits of separation $3d$, and those of separation $5d$, etc., thereby gradually building up the pattern for all the slits. The only angles for which these all add up in phase are the angles of strongest interference in the two beam case. This leads to a sharpening of the interference pattern from the two beam case, but no change in the position of the interference peaks. Again, this is equivalent to at set of beams at specific, equally spaced angles.

Supplementary questions

In the region moderately close to the screen, for the case where we have a very large number of equally spaced slits, something special happens at a distance

—

behind the screen. What is it, and can you describe why it is seen at this distance? (The same thing can also happen at integer multiples of this distance.)

EE PhD Quals Question
January 2006
Professor Nick McKeown

1. What is *congestion control* and why do we use it in the Internet?

2. Flows in the Internet commonly use TCP (Transmission Control Protocol). TCP uses window-based flow control, in which a maximum number of packets, W, are allowed to be outstanding (i.e. transmitted but not yet acknowledged) at any one time. How many packets should a transmitter hold onto, just in case they need to be retransmitted?

3. To control congestion, TCP does not use a fixed value for W. Instead, W varies over time, depending on the current congestion in the network. Specifically, TCP follows two rules to control congestion:
   a. When a packet is successfully acknowledged: $W \rightarrow W + 1$.
   b. When a packet is dropped: $W \rightarrow \frac{W}{2}$.

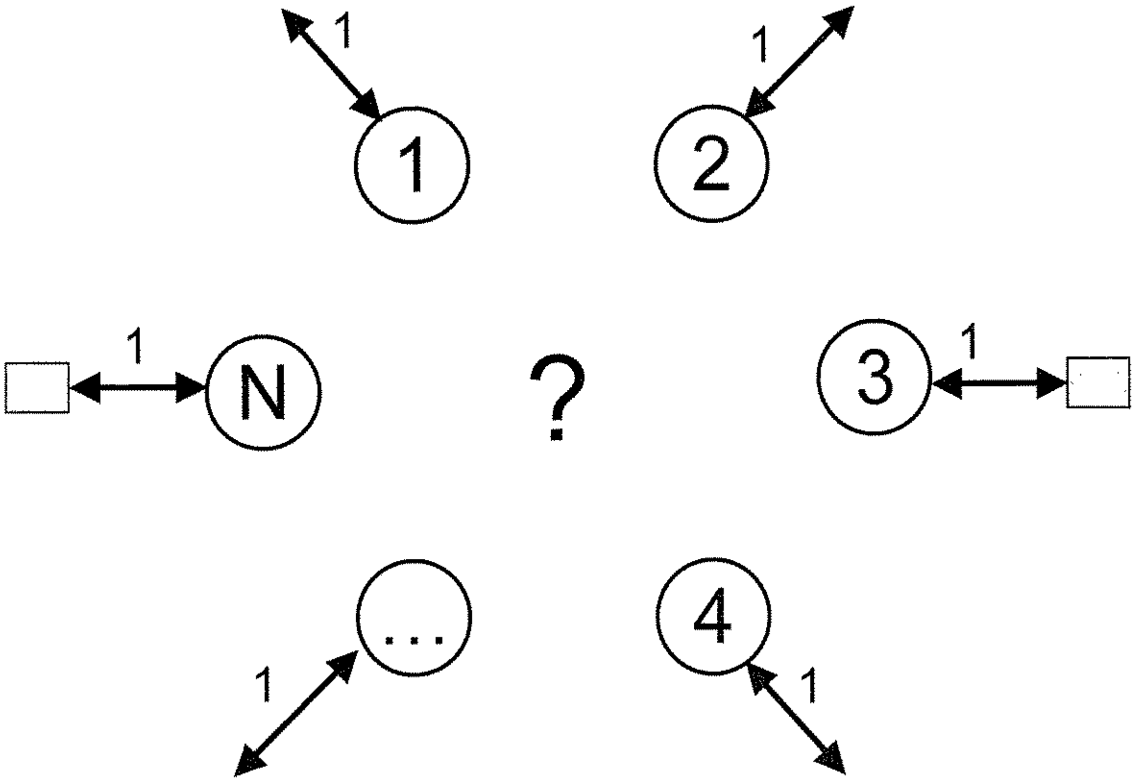
   Sketch the evolution of W as a function of time, assuming that exactly one packet is dropped every time W reaches $\hat{W}$.

4. Based on your sketch in (3), derive an approximate expression for the throughput of a TCP flow as function of p (the loss probability) and RTT (the round-trip-time, which we will assume is constant).

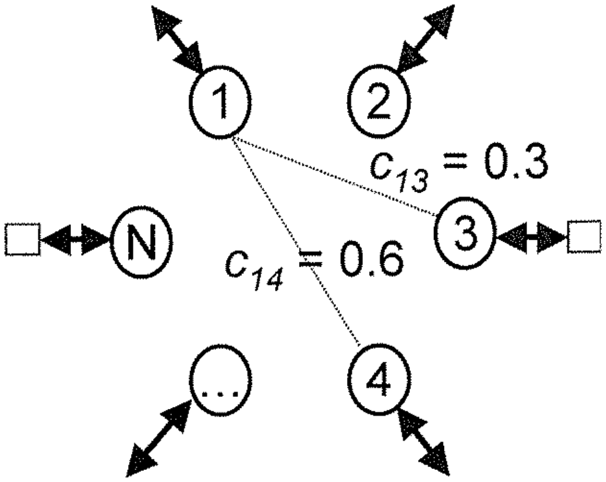
EE Quals 2008

Nick McKeown

Network



c_{ij} = data rate of link from node *i* to node *j*



Traffic Matrix

λ_{ij} = average rate of traffic from ingress *i* to egress *j*

$$\begin{pmatrix} \lambda_{11} & & \lambda_{1N} \\ & \ddots \quad \iddots & \\ \lambda_{N1} & & \lambda_{NN} \end{pmatrix}$$

$$\sum_i \lambda_{ij} \le 1, \quad \sum_j \lambda_{ij} \le 1$$

Question 1

(a) If the traffic matrix is known, what is the most efficient network design that will support the traffic matrix? (*i.e.* how do you pick c_{ij} so as to minimize $\sum_{ij} c_{ij}$).

(b) What is the most efficient network design if the traffic matrix is unknown?

Question 2

Consider the same network, and assume the nodes are fully interconnected; i.e. $c_{ij} > 0 \quad , \forall ij$ but now packets are sent to a randomly picked intermediary, then routed directly to the correct output.

(a) Pick values of c_{ij} that will support a traffic matrix $\lambda_{ij} = \frac{1}{N}$.

(b) What values of c_{ij} allow us to support <u>any</u> legal traffic matrix?

Question 3

We want to design the network so that it will continue to support any legal traffic matrix if any *k* nodes fail.

(a) What values should we pick for c_{ij}?

EE Quals 2010 (Hardware)

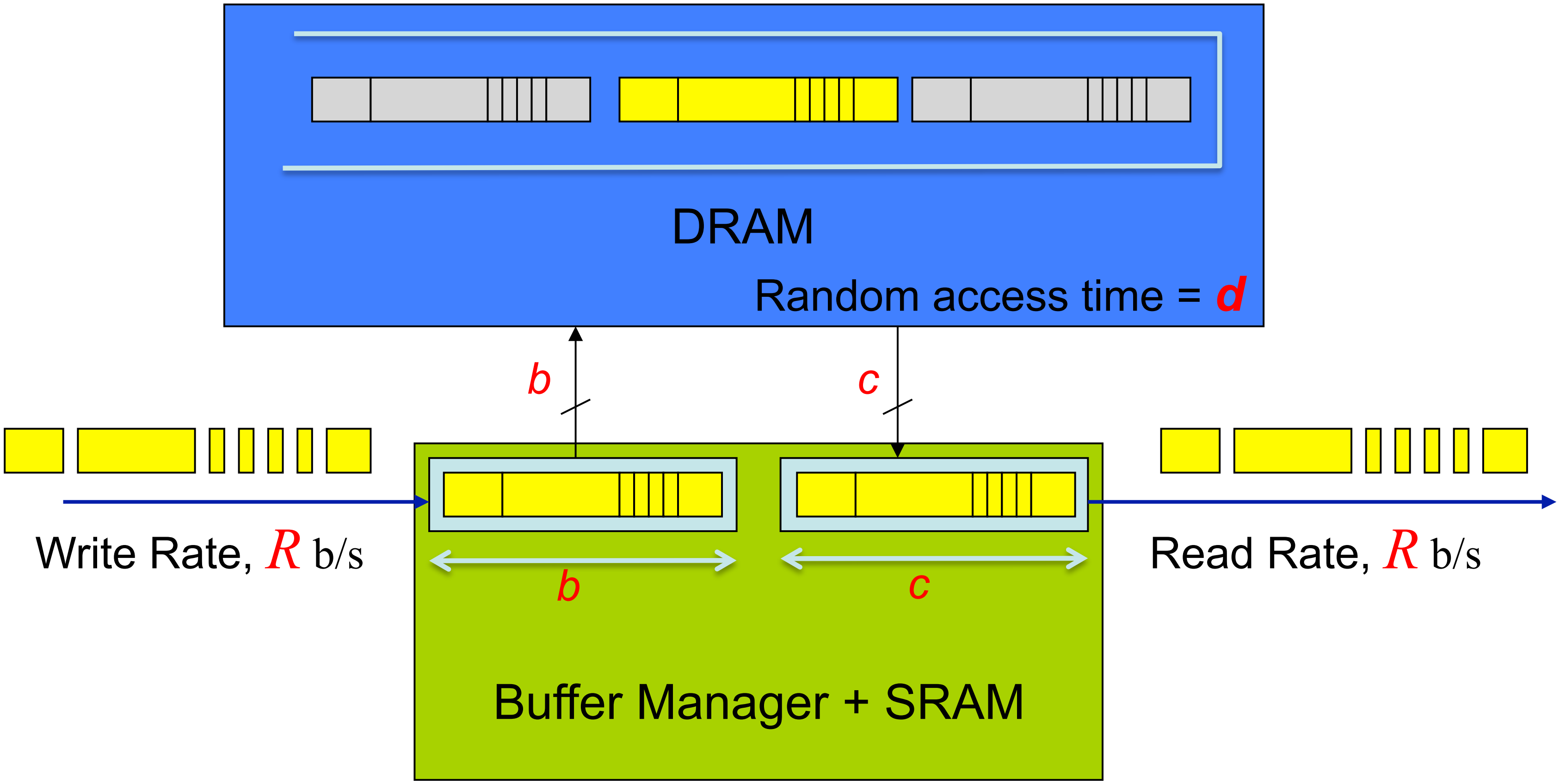
Nick McKeown

Question 1

(a) Why do we have a cache in a computer?

(b) I have an SRAM with random access time s, and a DRAM with random access time d. The probability of finding an entry in the SRAM is p. If I assume $d = 100s$, what value of p do I need so that the expected lookup time is twice as long as the random access time of the SRAM?

Single FIFO Queue



Question 2

Consider a cache for a FIFO queue in a network switch, router or network interface card, built from the same SRAM and DRAM.

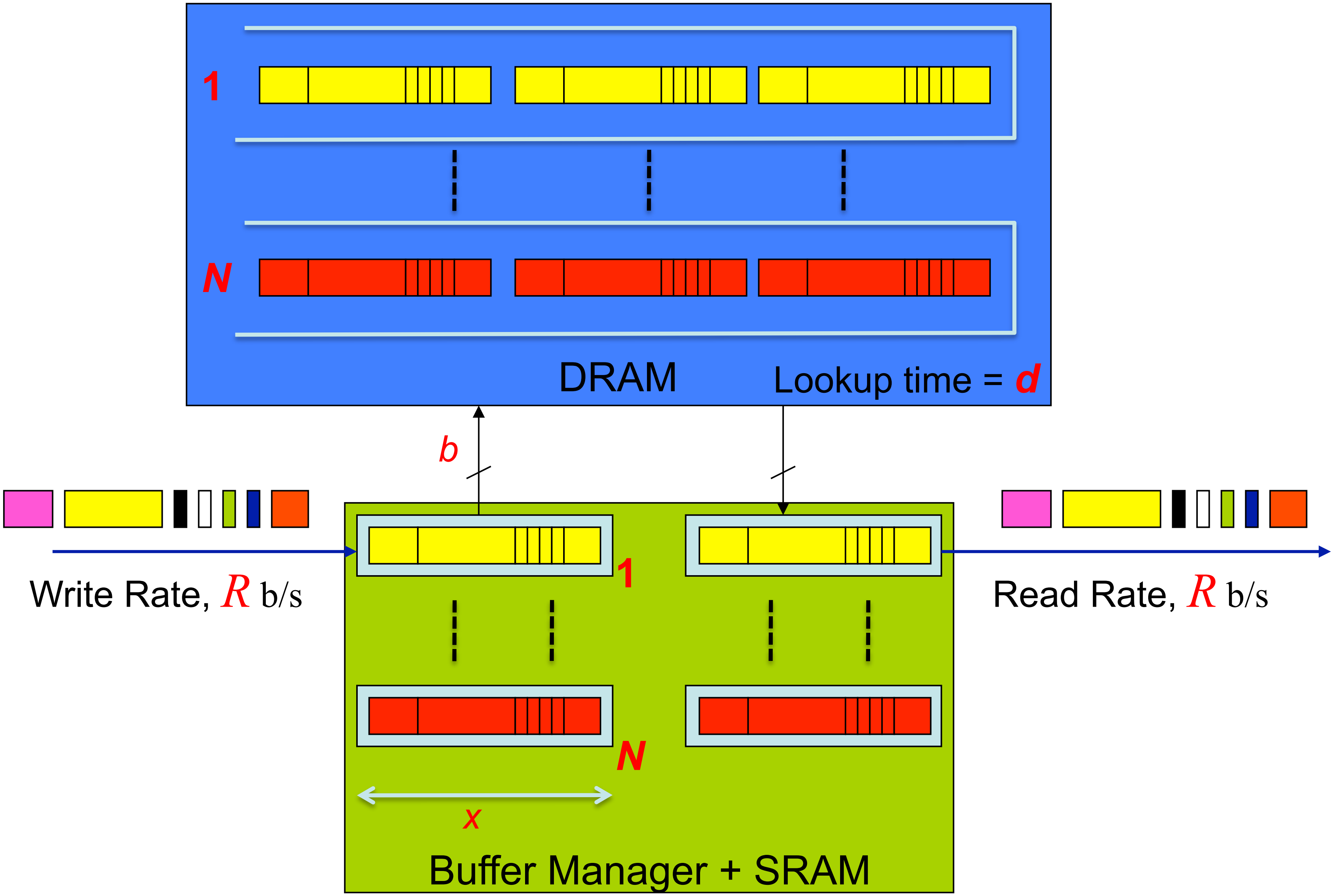
(a) Explain how it works.

(b) How large does the block size, b, need to be so that it won't overflow?

(c) How about c, so that the "head cache" won't underflow?

(d) What problems do we run into if we want to build a cache for N FIFO queues, instead of just 1?

Multiple FIFO Queues



Question 3

We want to figure out how large x needs to be, so the SRAM will never overflow.

(a) Is $x = b$ big enough? Explain.

(b) How can we figure out how large the SRAM needs to be?

CS Quals 2011 (Networking)

Nick McKeown

Question 1

Why does the Internet use packet switching?

Question 2

How do we define statistical multiplexing gain?

How could we define it for a network?

Question 3

What do you think the statistical multiplexing gain is at different points in the Internet? E.g. WAN, enterprise, home.

How could we measure it?

Question 4

In the 1980s and 90s, the telecommunication companies proposed an alternative to TCP/IP called “ATM” or “B-ISDN”. It had the following characteristics:

1. Virtual circuits were established end-to-end.
2. A separate control plane for establishing circuits
3. Virtual circuits carried small, fixed length packets (called “cells”)
4. Virtual circuits could have different qualities of service.

What are the pros and cons of such a design?

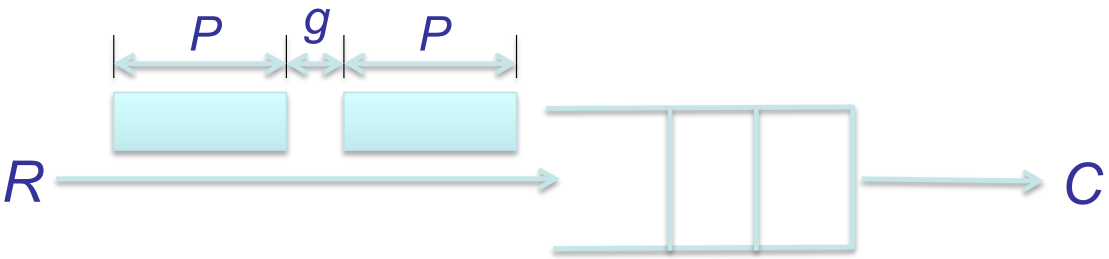
Why do you think it didn’t succeed?

EE Quals 2012 (Software)

Nick McKeown

Question 1

- The figure below shows a simple FIFO packet queue. Packets of length *P* arrive on the input link at rate *R* bits/s, and are stored in the buffer.
- If there is a complete packet at the head of the queue, it is immediately transmitted onto the output link at rate *C* bits/s. *R* > *C*.



Question 1

a) Write down an expression for the minimum interpacket gap, g, so we can be sure that the buffer will never overflow.

b) Write down an expression for the size of the buffer, B, in bits so we can be sure that the buffer will never overflow.

c) If the gap is a random variable, G, where $E[G] = 2g$ from part (a), can you find B so that the buffer will never overflow?

Question 2

a) Explain how the command *traceroute* finds the path between a source and a destination, and the round-trip-time from the source to any router along the path.

b) Describe a method in which a source and destination can determine the rate of the bottleneck link between them, by sending just two packets.

c) How could you extend the method in (b) to find the rate of any link up to and including the bottleneck?

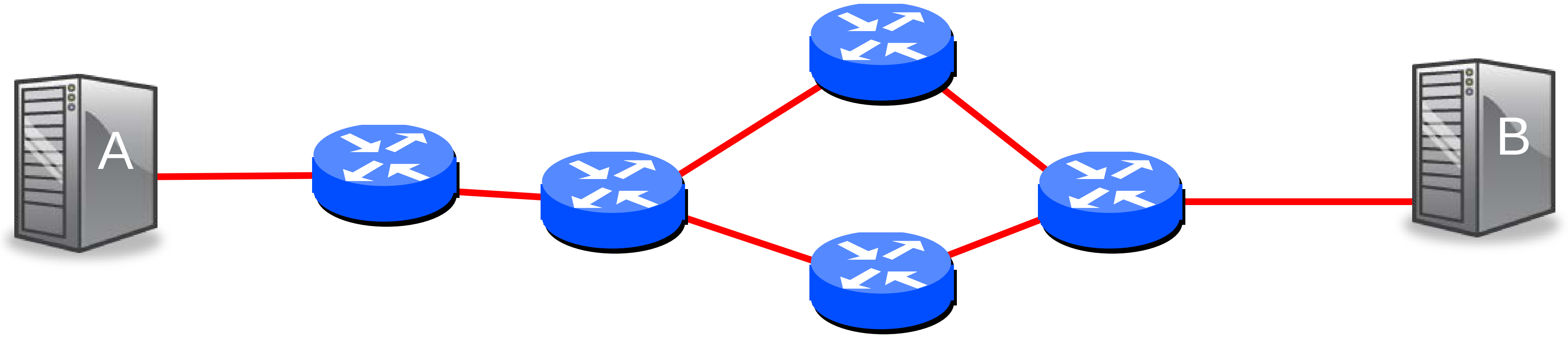
EE Quals 2013 (Software)

Nick McKeown

Question 1

Why does the Internet use packet switching?

Question 2



(a) What is the simplest way we can find the data rate of the slowest link from A to B, observing only at the end points?

(a) How can we do it by sending only two packets?

(b) What difficulties or noise are we likely to encounter?

Question 3

How might we find the rate of the 2nd slowest link?