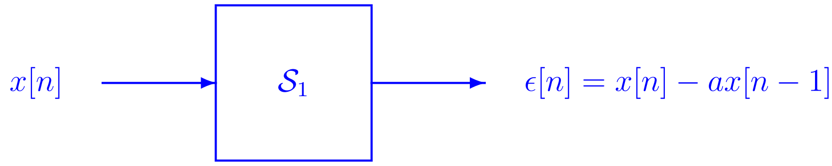
As before:



Autocorrelation $r_x(k) = \sum_{n=k}^{\infty} x[n]x[n-k]$

Error energy $\mathcal{E}_\epsilon = \sum_{n=0}^{\infty} \epsilon[n]^2 = \sum_{n=0}^{\infty} (x[n] - ax[n-1])^2$

Next Question:
Suppose $ax[n-1]$ is interpreted as a linear prediction of $x[n]$ based on a single past sample so that $\epsilon[n]$ is the linear prediction error sequence.

$$\boxed{\text{What value of } a \text{ minimizes } \mathcal{E}_\epsilon?}$$

Solution: Use calculus: Solve for

$$0 = \frac{d}{da}\mathcal{E}_\epsilon = \frac{d}{da}\big((1+a^2)r_x(0) - 2ar_x(1)\big) = 2ar_x(0) - 2r_x(1)$$

or $a = r_x(1)/r_x(0)$. Note that since $r_x(0) = \mathcal{E}_x \geq 0$, the second derivative satisfies

$$\frac{d^2}{da^2}\mathcal{E}_\epsilon \geq 0$$

so that the $a = r_x(1)/r_x(0)$ indeed minimizes $\mathcal{E}_\epsilon$. The resulting minimum is

$$\begin{aligned}
\mathcal{E}_\epsilon &= (1+a^2)r_x(0) - 2ar_x(1) = \left[1 + \left(\frac{r_x(1)}{r_x(0)}\right)^2\right] r_x(0) - 2\left(\frac{r_x(1)}{r_x(0)}\right) r_x(1) \\
&= r_x(0) + \frac{r_x(1)^2}{r_x(0)} - 2\frac{r_x(1)^2}{r_x(0)} = r_x(0) - \frac{r_x(1)^2}{r_x(0)}
\end{aligned}$$