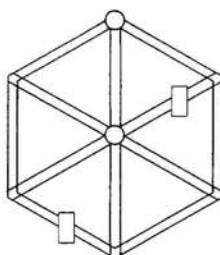


1999-2000 Qualifying Examination Questions

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The Hexagon has two sets of corridors as shown in the figure below. Each corridor segment is 1 km long. These corridors contain a very large number of light bulbs, which burn out at random independent times. Two battery-powered robots (represented by rectangles) are available to replace the light bulbs. There are two recharging stations (circles), one at the top of the Hexagon and one in the center.



One robot is dedicated to the outside (circumferential) corridors and the other robot serves the inside (radial) corridors.

Primary questions

1. Suppose that the outside robot returns to the charging station at the top of the Hexagon after replacing a light bulb. On average how far does it travel to change the light bulb and return to the charging station?
2. Suppose that the outside robot waits until two light bulbs have burned out. It then travels by the most efficient overall path to replace the two bulbs, finally returning to the charging station. On average how far does it travel?

Secondary and bonus questions

3. What is the average distance traveled by the outside robot to change a single light bulb if it remains at the location of the last replaced light bulb?
4. What is the average distance traveled by the inside robot to change a single light bulb and return to the charging station at the center?
5. What is the average distance traveled by the inside robot to change a single light bulb if it remains at the location of the last replaced light bulb?