Quals Question — Tom Cover
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Question 1: Gambling Scheme

Gamble \$1 each day on fair gambles. Stop when first ahead by \$1.

$$X_i = \begin{cases} 1, & \frac{1}{2} \\ -1, & \frac{1}{2} \end{cases}, \; X_i \text{ indep}$$

$$S_N = X_1 + X_2 \ldots + X_N = 1$$

$$N = \text{ stopping time}$$

1a. Are you eventually \$1 ahead (is $\Pr\{N < \infty\} = 1$)?

1b. Is $EN < \infty$, or $= \infty$? (How long does it take?)

Question 2

2a. Let X, Y be independent and identically distributed. What is $E\{X|X+Y\}$?

2b. Now let the joint distribution be arbitrary. Is

$$E[(X - E[X|X+Y])^2|X+Y] = E[(Y - E[Y|X+Y])^2|X+Y]?$$