Supplementary question 1 solution

Note that the energy calculated in part (ii) abo ve is quite independent of the value of the resistance R, and does not require any particular relation between current and voltage for that resistor; hence the resistor may be nonlinear and there will be no difference in the energy results here. This answer is particularly easy to see if the dissipation in the resistor is deduced by subtracting the energy stored in the capacitor from the total energy provided by the power supply; again, no particular relation between voltage and current is required in the resistor (at least as long as finite current flows at all voltages).

Supplementary question 2 solution

(a) The change in energy stored in the capacitor is the difference in energy between having a voltage V_B over the capitor and having a voltage $V_B + V_{DD}$ over it, which is

$$\Delta E = \frac{1}{2}C\left[\left(V_B + V_{DD}\right)^2 - V_B^2\right] = \frac{1}{2}CV_{DD}^2 + CV_{DD}V_B$$

(b) The answer to this is $(1/2)CV_{DD}^2$, exactly as in the main question, because the situation as seen by the resistor is exactly the same as it was in the first part, charging up a capacitor from 0V to V_{DD}, passing a charge $Q = CV_{DD}$ onto the capacitor just as before.

(c) The same charge $Q = CV_{DD}$ has to be passed through both power supplies (they are in series), so the energies they provide are, for the V_{DD} supply, CV_{DD}^2, and for the V_B supply $QV_B = CV_{DD}V_B$ (which we recognize as the second term on the far right on the above equation). Note that the energies do all add up correctly here, with the energy provided by the power supplies equaling ΔE plus the energy dissipated in the resistor.

$$CV_{DD}^2 + CV_B V_{DD} = \Delta E + \frac{1}{2}CV_{DD}^2$$

(d) When we discharge the system back to zero volts, an energy $(1/2)CV_{DD}^2$ is dissipated in the resistor, because as far as the resistor is concerned, we are discharging a capacitor from V_{DD} to 0V so all the currents and dissipations look the same as for such a problem. We are therefore left with an energy of magnitude $QV_B = CV_{DD}V_B$. This is energy that the circuit now tries to put back into the V_B power supply. Whether it can do that successfully depends on the nature of the power supply. If it is just an ordinary non-rechargeable battery, then the energy will end up dissipated as heat instead.

Supplementary question 3 solution

The answer is to make the V_B power supply effectively rechargeable, and the easiest way to do that is to put a large capacitor across it (sometimes called a bypass capacitor). A large capacitor effectively functions as a perfectly rechargeable power supply. In an actual circuit we might formally decouple the V_B power supply from the circuit with series resistance and/or inductance, leaving this bypass capacitor as the effective local, perfectly rechargeable “battery”.