Prof. Andrea Montanari

Quals Question

Consider a checkerboard with $n \times n$ squares. Each square that is not on the boundary has four neighbors that share an edge with it: North, South, East and West. A subset S of the squares is infected at the beginning. Recursively, a square becomes infected if it has at least two neighbors that are infected.

The process stops when either all squares are infected, or when there is no longer a non-infected square that has two or more infected neighbors.

What is the minimum number of initially infected squares (i.e. the size of the smallest set S), so that the all the squares are infected at the end?