6. The conditional probability of error given any particular received value y is

$$\Pr(X \neq \hat{X} \mid Y = y) = \frac{\Pr(X \neq \hat{X} \text{ and } Y = y)}{\Pr(Y = y)}.$$

When $Y = +1$ the estimate of X is $\hat{X} = +1$, so the conditional error probability is

$$\frac{\Pr(X \neq -1 \text{ and } Y = +1)}{\Pr(Y = +1)} = \frac{\Pr(X = -1)\Pr(Y = +1 \mid X = -1)}{\Pr(Y = +1)} = \frac{(1/2)\cdot(1/6)}{(1/6)} = \frac{1}{2}.$$

Both $\Pr(Y = +1)$ and $\Pr(Y = +1 \mid X = -1)$ are values of probability density functions, so their quotient is meaningful.

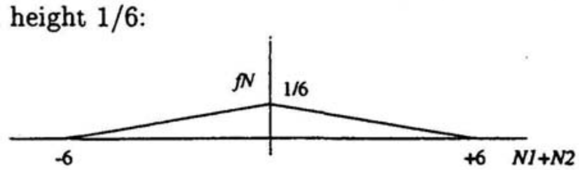
The same conditional error probability of $1/2$ is obtained for all values of Y between -2 and $+2$. For these values of Y, the receiver knows no more about X after receiving Y than before X was transmitted. Therefore $\hat{X} = +1$ and $\hat{X} = -1$ are equally good estimates, so the simple decision rule based on the sign of Y is optimal. (Or $\hat{X}$ could be decided by tossing a coin when $-2 < Y < +2$.)

7. When $Y < -2$ it is certain that $X = -1$, and when $Y > +2$ it is certain that $X = +1$. For Y in these two intervals, the conditional error probability is 0. When $-2 < Y < +2$, the conditional error probability is $1/2$. The overall error probability is

$$\Pr(|Y| > 2)\cdot 0 + \Pr(|Y| < 2)\cdot \tfrac{1}{2} = \tfrac{2}{3}\cdot\tfrac{1}{2} = \tfrac{1}{3}.$$

Another way to evaluate the error probability is to condition on X. When $X = -1$, an error occurs when $Y = X + N > 0$, that is, when $N > 1$. From the pdf for N, we see that $\Pr(N > 1) = 1/3$, so $\Pr(\hat{X} \neq X \mid X = -1) = 1/3$. By a similar calculation, $\Pr(\hat{X} \neq X \mid X = +1) = 1/3$. The overall error probability, which is the average of these two conditional probabilities, is $1/3$.

8. The combined noise is the sum of two independent uniformly distributed random variables. The pdf of the sum is the convolution of two rectangle functions, which is a triangle with range $[-6, +6]$ and height $1/6$:



The pdf for $Y = 2X + N_1 + N_2$ is obtained by averaging the conditional pdfs: