

To: shankle@ee.Stanford.EDU
 Subject: EE quals question/solution
 Date: Fri, 25 Jan 2002 17:54:18 -0800
 From: Jennifer Widom <widom@DB.Stanford.EDU>

Diane,

I suddenly realized I forgot to turn in my EE Quals question/solution. I can't remember if you're the one to take it, but if not please forward accordingly.

Thanks,
 Jennifer Widom

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2002 EE Quals, Prof. Jennifer Widom

Problem
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Consider directed graphs with a single source (root) node R from which there is at least one path to every other node N in the graph.

We can represent such graphs in a couple of ways:

- (1) As a data structure of nodes and edges. Each node N has $i \geq 0$ out-neighbors (children) accessed as N.1, N.2, ..., N.i
 [show example]
- (2) As a $K \times K$ square matrix M for a graph with K nodes. $M[x,y] = 1$ if there is an edge from node x to node y; $M[x,y] = 0$ otherwise
 [show same example]

Write a program that determines whether such a graph contains a cycle. The program should return YES if there is one or more cycles, NO otherwise.

- * You may use whichever of the two graph representations you prefer.
- * You may use any pseudocode notation you like, including function definitions and calls if it helps you.
- * Your solution will be graded on simplicity as well as correctness.

Solution
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Here are two possible solutions but there are many correct variants.

Using representation (1):

```
main program: cycle(R, {R})

function cycle(N, seen-set)
  if N has no children return(NO)
  else if any of N.1, N.2, ..., N.i are in seen-set return(YES)
  else if cycle(N.i, seen-set U {N.i})=YES for any i
    then return(YES) else return(NO)
```

Using representation (2):

```
repeat until M is unchanged:
  M <- M + M*M
  if M[x,x] > 0 for any x then return(YES) else return(NO)
```