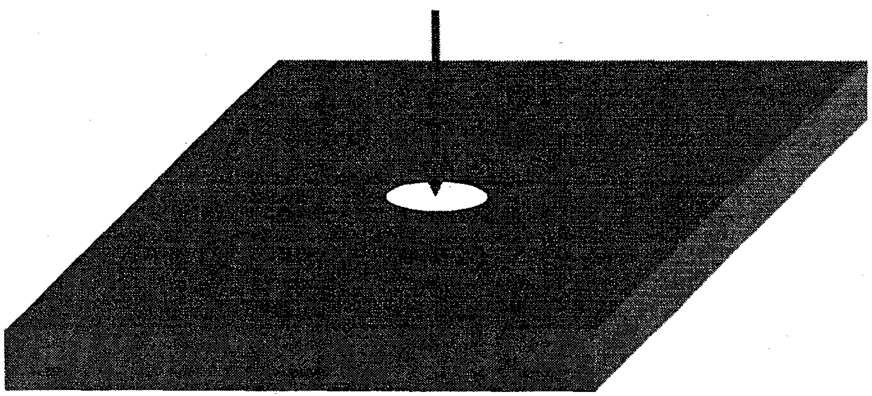
Shanhui Fan

In this exam, the metal is all assumed to be perfect metal. (i.e. perfect electrical conductor).

(a) What are the boundary conditions for the electric fields at the metal-air interfaces?
(b) What are the boundary conditions for the magnetic fields at the metal-air interfaces? Why?
(c) Consider an infinitely long cylindrical air hole inside a metal, sketch qualitatively the $\omega \sim \beta$ diagram for the lowest order propagating modes inside the hole, where $\omega$ is the angular frequency for the waves and $\beta$ is the propagation constant. (i.e. the field varies along the $z$-direction, defined as the axis of the cylinder, as $e^{-i\beta z}$.)
(d) Sketch the electric field vector distribution of the lowest order propagating inside the hole.
(e) Consider the following experiment, where a plane wave is incident upon a metal film with a hole introduced in it. Could you sketch the amount of transmitted power as the wavelength is varied from $\lambda >> a$ to $\lambda \sim a$, where $a$ is the radius of the hole?



Scoring: (a) 1; (b) 2; (c) 2; (d) 2; (e) 2; (f) 3.
In (c), a rough estimate of the cut-off 1 point
In (f), plot of the cut off 2 point, Fabry Perot oscillation 1 point.

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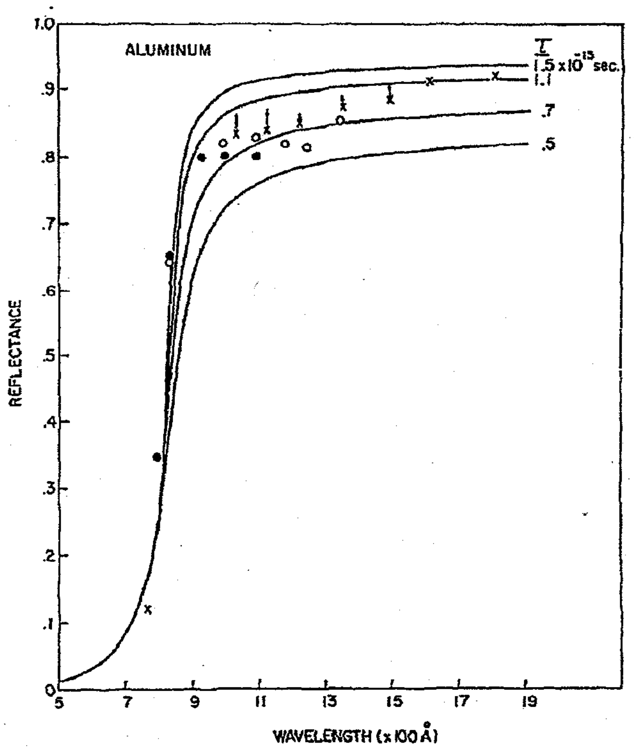
1. Why is metal highly reflecting for incident electromagnetic wave?
2. How do you explain the following measured reflectivity spectrum of aluminum, which shows a drastic reduction of reflectivity for aluminum at ultraviolet wavelength range? (Taken from R. C. Vehse, E. T. Arakawa, and J. L. Stanford, Journal of Optical Society of America, 57, 551, 1967)
3. Could you provide a simple microscopic model for this?



**Qualifying Exam 2011**
**Engineering Physics, Shanhui Fan**

As a reminder, the time-dependent Schodinger equation of electrons:

$$i\hbar \frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \phi}{\partial x^2} + V(x)\phi$$

and the time-independent Schodinger equation of electrons:

$$E\phi = -\frac{\hbar^2}{2m}\frac{\partial^2 \phi}{\partial x^2} + V(x)\phi$$

(a) Suppose an electron is confined in an infinite potential well

$$V(x) = \begin{cases} 0, \ 0 < x < a \\ \infty, \text{ everywhere else} \end{cases}$$

sketch the ground state $\phi_0(x)$ and the first excited state $\phi_1(x)$ for the electron in the potential well. Provide the eigen-energy of these two states.

(b) Suppose at $t = 0$, the electron has a wavefunction $\phi_0(x)$, what is the electron wavefunction at a time $t$ later?

(c) Suppose at $t = 0$, the electron has a (un-normalized) wavefunction of $\phi_0(x) + \phi_1(x)$, could you sketch the shape of the electron probability density distribution as a function of time?

(d) Instead, consider the potential well:

$$V(x)=\begin{cases} \infty, x<0 \\ 0, 0<x<a \\ U>0, a<x<a+b \\ 0, x>b \end{cases}$$

suppose at $t=0$ the electron has a wavefunction $\phi_0(x)$, how does the probability of finding the electron in the potential well vary as a function of time?

**Qualifying Exam 2013**
**EM, Shanhui Fan**

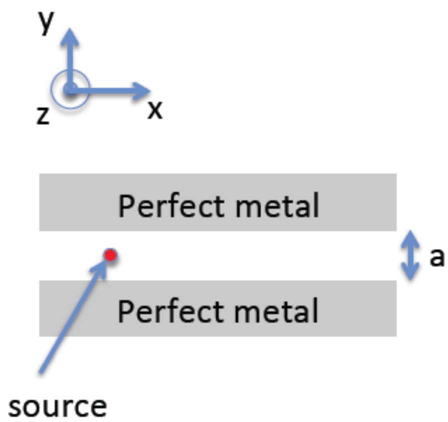



Figure 1

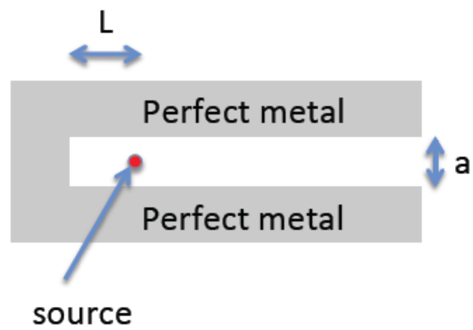



Figure 2

Consider a parallel-plate waveguide, with PEC (perfect electric conductor) sidewalls and air in between. The width of the waveguide is $a$. We consider two-dimensional system. (i.e. the fields and the structures are uniform in the $z$-direction).

(1) Sketch the ω-k relation for the lowest-order TE mode (with E-field polarized along the z-direction), and for the lowest-order TM mode (with E-field polarized along the y-direction).

(2) Suppose we put in an oscillating line source as indicated in Figure 1. The source oscillates at a frequency $\omega_0$. How does the power radiated into the waveguide changes as a function of $\omega_0$? Consider both cases, the TE case with the line source polarized along the z-direction, and the TM case with the line source polarized along the y-direction.

(3) Suppose we truncate the waveguide with a perfect electric conductor at the end as shown in Figure 2. For the TM case, suppose we choose $\omega_0 = 0.1\frac{2\pi c}{a}$, how does the power radiated into the waveguide vary as a function of the distance $L$ between the source and the truncation?