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Quals Questions 2013

Let the 2-norm of a vector be defined by the square-root of the sum of the squares of its components. Let the max-norm of a vector be the maximum among absolute values of components.

- (1) Is there a vector  $x$  whose max-norm exceeds its 2-norm?
- (2) Suppose the 2-norm of a vector  $x$  exceeds the 2-norm of a vector  $y$ . Does this imply that the max-norm of  $x$  exceeds the max-norm of  $y$ ?
- (3) Suppose the 2-norm of a vector  $x$  exceeds the 2-norm of a vector  $y$  by a factor of  $b$ . If the vectors are 2-dimensional, are there values of  $b$  that would imply that the max-norm of  $x$  exceeds the max-norm of  $y$ ? What values? What about the general  $n$ -dimensional case?
- (4) Suppose the max-norm of a vector  $x$  exceeds the max-norm of a vector  $y$  by a factor of  $b$ . If the vectors are 2-dimensional, are there values of  $b$  that would imply that the 2-norm of  $x$  exceeds the 2-norm of  $y$ ? What values? What about the general  $n$ -dimensional case?
- (5) Why are the answers to (3) and (4) the same?