

Quals Question

Suppose I give you a random variable $U \sim \text{Unif}[0,1]$.

Can you use it (and only it) to generate

1. An unbiased bit $B \sim \text{Bernoulli}(1/2)$?

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2. Two independent unbiased bits B_1, B_2 ?

3. An infinite stream of independent unbiased bits B_1, B_2, B_3, \dots ?

4. Two independent $\text{Unif}[0, 1]$ random variables U_1, U_2 ?

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5. An infinite sequence of i.i.d. $\sim \text{Unif}[0, 1]$ random variables U_1, U_2, U_3, \dots ?

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Keywords: Random variables; Uniform distribution

ABSTRACT

Consider a sequence of independent random variables U_1, U_2, U_3, \dots

(1) A random variable U_i is said to be

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(4) A random variable U_i is said to be

(5) A random variable U_i is said to be

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(19) A random variable U_i is said to be

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Let U_1, U_2, U_3, \dots be a sequence of independent random variables

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Quals Question

Let the random variable X have CDF F . Suppose that X_i iid $\sim X$.

(a) What is the CDF of $\max_{1 \leq i \leq n} X_i$?

(b) What is the CDF of $\min_{1 \leq i \leq n} X_i$?

Suppose now that $X_{i,j}$ iid $\sim X$

(c) What is the CDF of $f(n, m) = \max_{1 \leq i \leq n} \min_{1 \leq j \leq m} X_{i,j}$?

(d) Suppose $X \sim \text{exponential}(\lambda)$. Let $Y_m = f(e^{\beta m}, m)$, for some parameter $\beta > 0$. What does Y_m converge to as $m \rightarrow \infty$? In what sense?

(e) Repeat the previous part for the general case $X \sim F$ (can assume that F is continuous and strictly increasing).

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- (e) Repeat the previous part for the general case $X \sim F$ (can assume that F is continuous and strictly increasing).

Suppose you are required to guess the value of a discrete random variable X , of a known PMF, via a sequence of questions of the form “is $X = x$?”.

- What questioning strategy is optimal in the sense of minimizing the expected number of trials until guessing correctly?
- What is the value of this minimum expected number of guesses until correctly guessing (inclusive) for $X \sim \text{Poisson}(1)$?
- Repeat the question for $X \sim \text{Poisson}(2)$.

Quals Question

You are drawing variables U_0, U_1, \dots iid $\sim U[0, 1]$.

- (a) T_1 is the number of attempts until you surpass your first draw (i.e., break your first record):

$$T_1 = \min\{n \geq 1 : U_n > U_0\}$$

What is the PMF of T_1 ?

- (b) What is the PDF of U_{T_1} ?

- (c) T_2 is the number of attempts it takes you to set your next record:

$$T_2 = \min\{n \geq 1 : U_{T_1+n} > U_{T_1}\}$$

What is the PMF of T_2 (bring to as explicit a form as you can)?

- (d) Repeat parts (a) and (c) for U_0, U_1, \dots iid $\sim N(0, 1)$.