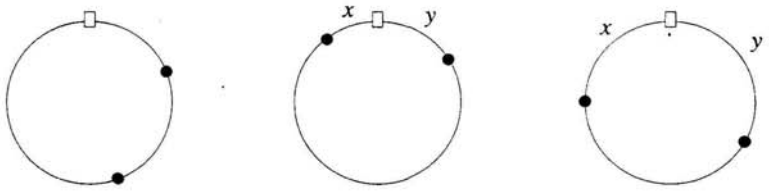
Answers to primary questions

1. The maximum round trip is 6 km, since the robot travels by the shortest path. The distance is uniform from 0 to 6, so the average round trip is 3 km.

2. With probability $\frac{2}{4} = \frac{1}{2}$, the two light bulbs are on the same side of the Hexagon In this case, the robot travels to the closer light bulb, then to the more distant one, then returns by the reverse path. The distance to the farther light bulb is a random variable that is the maximum of two independent random variables that are uniformly distributed from 0 to 3. The average value of the distance is $\frac{2}{3}$ of the maximum, so the average round trip in this case is 4 km.



When the light bulbs are on opposite sides of the Hexagon, the robot has two possible routes. If the bulbs are close together, the robot travels first to one, then returns to the charging station, then travels to the other and returns. The total distance is $2x + 2y$. If the light bulbs are far apart, it is more efficient to travel to one, then continue in the same direction to the second, then continue in the same direction to return to the charging station, for a total distance of 6. The robot chooses the shorter of these two paths, depending on whether $x + y < 3$. Since x and y are independent and uniformly distributed from 0 to 3, their sum $x+y$ ranges from 0 to 6 and $\Pr(x+y < 3) = \frac{1}{2}$. The conditional probability density for $z = x+y$ given $x+y < 3$ is the triangle $f(z) = 2z/9$, so the conditional expectation is $x + y$ is 2 and the average round trip in this case is 4. Obviously, when $x + y > 3$ the average distance is 6. The overall average round trip distance is

$$\tfrac{1}{2} \cdot 4 + \tfrac{1}{4} \cdot 4 + \tfrac{1}{4} \cdot 6 = 4\tfrac{1}{2},$$

which is obtained by combining the conditional expectations for the three cases.

Answers to secondary and bonus questions

3. The locations of successive light bulbs are independent, so distance from one to the next by the shortest path is a maximum of 3 km and an average of 1.5 km.

4. The maximum round trip for the inside robot is 2 km, so the average is 1 km.

5. With probability $\frac{5}{6}$ the next light bulb is in another radial corridor, in which case the average distance is $\frac{1}{2} \cdot 2 = 1$. With probability $\frac{1}{6}$, the next light bulb is in the same corridor, in which case the average distance is $\frac{1}{3}$. The overall average round trip for the inside robot is $\frac{5}{6} + \frac{1}{18} = \frac{8}{9}$.