

EE Qualifying Exam
January 2016

A population R of robins interacts with a population W of worms. An interaction between the two populations is good for the robins and bad for the worms. Find a system of differential equations of the form

$$\begin{aligned}\frac{dW}{dt} &= f(R, W) \\ \frac{dR}{dt} &= g(R, W)\end{aligned}$$

that describes how the populations evolve. Hint: First ask what happens to the worm population if there are no robins, and what happens to the robin population if there are no worms, and then modify the model to take account of the interactions.

A system that models the interaction is

$$\begin{aligned}\frac{dW}{dt} &= aW - bRW \\ \frac{dR}{dt} &= -cR + dRW\end{aligned}$$

where a, b, c, d are positive constants. Then

$$\frac{dW}{dR} = \frac{aW - bRW}{-cR + dRW}.$$

Here is a sketch of the slope field associated with the system in the (W, R) -plane for one choice of a, b, c, d . Sketch a few trajectories and interpret what you see in terms of how W and R are varying together, and be sure to indicate the direction of the trajectory. For what values of W and R do the populations remain constant?

