1996 Qualifying Exam Answers

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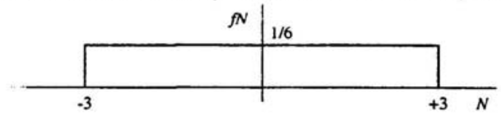
The first three questions are warmup questions. The last two questions are bonus questions; about 15% of the examinees reached the last two questions.

1. The pdf of N has the constant value $1/6$ between -3 and $+3$, zero elsewhere.



2. The variance of a random variable uniformly distributed on the interval $[a, b]$ is $(b-a)^2/12$. So the variance of the noise N is $6^2/12 = 3$. The variance can also be calculated from the definition:

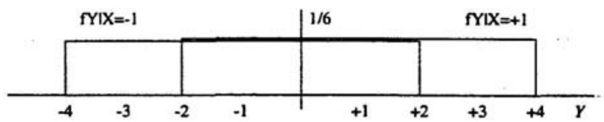
$$\sigma_N^2 = E[N^2] - E[N]^2 = \int_{-3}^{+3} \frac{1}{6} x^2 \, dx = \frac{x^3}{18} \Big[_{-3}^{+3} = \frac{(+3)^3 - (-3)^3}{18} = 3 \, .$$

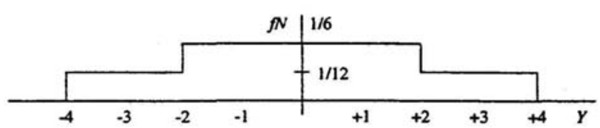
3. The power of the signal X is 1 since the magnitude of X is always 1. The variance of X is also 1 because

$$\sigma_X^2 = E[X^2] - E[X]^2 = \frac{1}{2}(+1)^2 + \frac{1}{2}(-1)^2 = 1 \, .$$

   Therefore the signal-to-noise ratio is $1/3$.

4. For each value of X, the conditional probability density of Y is uniformly distributed about that value of X. The conditional densities $f_Y(y \mid X = \pm 1)$ and the unconditional density $f_Y(y)$ are shown below.





   Combining the two conditional densities is the same as convolving the pdf of X, which is two impulses of height $1/2$ located at ± 1 with the pdf of N.

5. The obvious decision rule is to estimate that $X = +1$ when $Y > 0$ and that $X = -1$ when $Y < 0$. This decision rule is optimal, as shown below.