OFFICE MEMORANDUM ◇ STAR LABORATORY

February 6, 1997

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1997

Electromagnetics

Question: Explain how you might send an electromagnetic signal down to the center of the earth from the surface. Ignore the possibility that the inner core of the earth consists of molten iron and assume the earth is wholly a conducting material of conductivity $\sigma = 0.01$ S/m. Other possibly useful information: The radius of the earth is 6370 km, and at 1 Hz the skin depth (δ) for an electromagnetic wave propagating in a medium with a conductivity of 0.01 S/m is 5 km.

Answer: There are many different ways to answer the above question. An ideal answer would include most or all of the following: (1) A brief discussion of good conductors and poor conductors, leading to the conclusion that the above problem must treat the earth as a good conductor ($\sigma/\omega\epsilon \gg 1$, where ω is the angular frequency and ϵ is the permittivity). (2) Some technical discussion in which the student must either remember or derive an expression for the attenuation of electromagnetic fields in a good conductor in terms of the attenuation constant (α) or the skin depth (δ), where $\delta = 1/\alpha = [2/(\omega\mu\sigma)]^{1/2}$, and where μ is the permeability. (3) The student should demonstrate some knowledge of how the wave is exponentially attenuated (with attenuation constant $\alpha = 1/\delta$).

Good conductor is prerequisite to use skin depth

Using the above information, the ideal answer would then include (4) a scaling of the skin depth information given at 1 Hz to derive the frequency corresponding to a skin depth of around 6370 km. The answer obtained is typically around 10^{-6} Hz, which is essentially dc as far as most electrical engineering students are concerned. The students are then asked if there are any electric or magnetic fields EMERGING from great depths in the earth, which would be subject to the same attenuation. This should lead into a discussion of (5) the earth's magnetic field, which is very nearly steady, but not quite, and (6) how there can be some very long term variations, which are consistent with the above frequency estimate. Finally, the students are told that there are some shorter term variations with periods around 1 Hz, and asked to explain them in the context of our discussion. The final conclusion (7) is that the variations either originate in the earth very close to the surface or they reach the earth's surface from above and thus are not subject to the same attenuation as that which takes place in the earth.

$\frac{\sigma}{\omega\epsilon} = \frac{0.01}{1.8 \times 10^{-12}}$ $\quad \epsilon_0 = 8.8 \times 10^{-12}$ F/m