

## Solution

### Starter question:

If we shine the light beam onto the bottom of the beamsplitter, it will still come out of the top and right sides with equal powers.

(The reasons for asking this question are to make sure the examinee understands beamsplitters (so the exam is not a test of basic optics knowledge), and to set up the paradox in the examinee's mind that underlies the actual question – namely, if we are to run this beamsplitter backwards, what is it about the light beams that will make sure the light comes out of the left rather than out of the bottom of the beamsplitter.)

### Question:

The answer is that we can run the beamsplitter backwards to get the light to come out of only one port.

Often students starting to answer this question conclude we cannot run the beamsplitter backwards because they think in powers or intensities, which leads them to conclude that the power will always be split between two outputs when we attempt to run the beam splitter backwards. To see that it can be run backwards, one needs to think in terms of waves, and wave equations can typically be time reversed, rather like taking a movie of the wave splitting and then running the movie backwards. That idea of reversibility is sufficient to allow the student to conclude that, at least for some conditions, the beamsplitter can be run backwards.

The key concept is to understand that the property that determines whether the beams come out of the bottom or the left of the beamsplitter is the relative phase of the two beams. If they have one particular relative phase, they will both come out of the left face, and add perfectly. For that one particular phase, they add constructively out of the left face, and destructively out of the bottom face, and that is the relative phase required to run the beamsplitter backwards to get an output from the left. If the relative phase is changed by 180 degrees, the beam will come out of the bottom face.

It is also true that the two light beams have to be in the same spatial mode (beam shape and direction) on reflection off of, or transmission through, the mirror, and of the same polarization, though to keep the problem simple, I would not introduce these attributes unless the examinee brought them up. Without those two attributes being the same, the perfect cancellation of the waves in one output direction would not be possible.

### Supplementary question 1:

The answer is that we need to do something to change the relative phase of one path light beam path within the apparatus compared to the other. We could do this by very slightly displacing one of the mirrors, or we could add something into one of the paths, such as a thin piece of glass, to otherwise change the relative phase of the two beams going into the upper beamsplitter. In this way, we can actually arbitrarily change the relative power in the two output beams.

This structure, by the way, is known as a Mach-Zehnder interferometer.