Answers

1a. Yes, $\Pr\{N < \infty = 1\}$. You are sure to be \$1 ahead eventually.

1b. $EN = \infty$. The expected waiting time is infinite. (You are making money at the rate of (\$1)/(EN) trials and this rate can't be positive for fair gambles.)

2a. $E\{X|X+Y\} = ?$

Note: $E\{X|X+Y\} = E\{Y|X+Y\}$ since $f(x,y)$ is symmetric.

Note:

$E\{X|X+Y\} + E\{Y|X+Y\}$

$= E\{(X+Y|X+Y\}$

$= X+Y$

Thus $E\{X|X+Y\} = \frac{X+Y}{2}$

2b. Yes, they are equal. We note that

$(X - E\{X|X+Y\}) + (Y - E\{Y|X+Y\})$

$= X+Y - E\{X+Y|X+Y\}$

$= X+Y-(X+Y) = 0$

Thus

$(X - E(X|X+Y))^2$

$= (Y - E\{Y|X+Y\})^2.$