From: "Ramesh Johari" <rjohari@stanford.edu>
To: "'Diane Shankle'" <shankle@ee.Stanford.EDU>
Subject: RE: Quals Meeting Time Change Please see below!
Date: Fri, 27 Jan 2006 16:31:55 -0800
Thread-Index: AcYjoSTBdzWaCh9JQ+GZBZdis/ufxwAALfYQ

Hi Diane,

My quals question is below. Am I required to attend the quals meeting?

Ramesh

There are two questions, you may do them in any order.

Q1. You are given three random variables, X_1, X_2, X_3.

X_1 is independent of X_2.
X_2 is independent of X_3.
X_3 is independent of X_1.

Is it true that (X_1, X_2, X_3) are jointly independent? Either prove or provide a counterexample.

Q2. Can you find three random variables X_1, X_2, X_3 such that:

P(X_1 > X_2) > 1/2
P(X_2 > X_3) > 1/2
P(X_3 > X_1 > 1/2 ?

If so, provide them.
If not, prove you cannot.

Does your answer to the question depend on whether or not (X_1, X_2, X_3) are assumed to be jointly independent?

> -----Original Message-----
> From: Diane Shankle [mailto:shankle@ee.Stanford.EDU]
> Sent: Friday, January 27, 2006 4:23 PM
> To: quals-examiners@ee.Stanford.EDU; EE-adminlist@ee.Stanford.EDU
> Subject: Quals Meeting Time Change Please see below!
>
>
>
>
> Quals Meeting
> Tuesday, January 31st.
> 4:30 P.M.
> CIX-X AUD
>
> Coffee,Tea and Cookies will be served before the meeting.
>
> Please send me a copy of your Quals Question either by email
> or a hard copy to the address listed below!
>
> Happy Friday,
> Diane
>
>
>
>
> --

2008 Quals

Examiner: Ramesh Johari

Problem 1. The Smiths have three children, of whom one is a boy. What is the probability he has two sisters as siblings? (Assume that a child is equally likely to be a boy or a girl, independent of other children.)

Problem 2. At each time period $t = 1, 2, 3, \ldots$, a red coin and a blue coin are flipped simultaneously. Assume that the red coin comes up heads with probability p_r, and the blue coin comes up heads with probability p_b.

(a) Calclulate the expected number of flips until the first head (either red or blue) is seen.

(b) Calculate the probability that at least 3 red coins come up heads before the first blue coin comes up heads.

Problem 3. Suppose that X and Y are two real-valued random variables. Show that:

$$\mathbf{E}[|X||Y|] \leq \sqrt{\mathbf{E}[X^2]\mathbf{E}[Y^2]},$$

where $|x|$ denotes the absolute value of x.

Partial credit will be given if the result is proven under the assumption that X and Y are both uniformly distributed on $\{0, 1, 2, \ldots, N\}$.

JOHARI

Problem 1. Suppose that X is a geometric random variable, with probability of success p. Let t, s be positive integers with $t > s$. Explain why $P(X > t | X > s) = P(X > t - s)$.

Problem 2. For what values of N does the following statement hold?

"If $X_1, \ldots, X_N$ are jointly Gaussian random variables such that $\operatorname{Var}(\sum_i X_i) = \sum_i \operatorname{Var}(X_i)$, then $X_1, \ldots, X_N$ are independent."

Problem 3. Suppose that $X_1, X_2, \ldots$ are a sequence of random variables on the nonnegative integers, such that $E[X_n] \to \infty$ as $n \to \infty$. Does it follow that $P(X_n = 0) \to 0$ as $n \to \infty$?

Now suppose that in addition, $\operatorname{Var}(X_n) = E[X_n]$. Does it follow that $P(X_n = 0) \to 0$ as $n \to \infty$?