and therefore

$$a_k = \frac{1}{K}\sum_{n=0}^{K-1} x[n]e^{-j2\pi\frac{k}{K}n}$$

Proving (1).

**Second method** Just plugging in and manipulating using the standard Fourier methods of interchanging summations with each other and limits works quite well here. This is what I looked for when students did not remember the basic properties of Fourier series.

$$\begin{aligned}
\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} x[n] &= \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\left(\sum_{k=0}^{K-1} a_k e^{j2\pi\frac{k}{K}n}\right)\\
&= \sum_{k=0}^{K-1} a_k\left(\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{K}n}\right)
\end{aligned}$$

Here two ways are possible: you can recognize that since complex exponentials of this form are periodic with period $K$,

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{K}n} = \frac{1}{K}\sum_{n=0}^{K-1} e^{j2\pi\frac{k}{K}n} = \begin{cases}1 & k = 0 \bmod K\\ 0 & \text{otherwise}\end{cases}$$

using (2), which implies that the result is $a_0$, or you can use the geometric progression to evaluate the limit (which saves time on the second question):

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{K}n} = \lim_{N\to\infty}\frac{1}{N}\left(\begin{cases}N & k = 0 \bmod K\\ \frac{1-e^{j2\pi\frac{k}{K}N}}{e^{j2\pi\frac{k}{K}N}} & \text{otherwise}\end{cases}\right) = \begin{cases}1 & k = 0 \bmod K\\ 0 & \text{otherwise}\end{cases}$$

which implies that the result is $a_0$.
Some students wrote down the geometric series formula to do the sum of the $N$ exponential terms, but forgot that the formula requires a nonzero denominator ($k$ cannot be 0). This led to the incorrect conclusion that the answer was 0 because the formula is bounded and the $1/N \to 0$. I gave a hint to the effect that the answer could not be correct and suggested the consider the $K = 1$ case, where $N^{-1}\sum_{n=0}^{N-1} x[n] = N^{-1}\sum_{n=0}^{N-1} a_0 = a_0$. Ideally people realized that by linearity this gave them the complete answer, their geometric progression formula forces the other terms to 0 and all that is left is $a_0$.
Some students tried to apply DTFTs here, which do not work since the signal does not have finite power. Others correctly recognized a connection with DFTs, but could not relate the DFTs to the Fourier coefficients correctly to get the answer.

The second sum is the time average power and it follows from Parseval's relation or by direct computation, again using (1): First, since the signal is periodic, the limit as $N \to \infty$ is given by