

skin depth $\delta = \left(\frac{2}{\omega \mu \sigma} \right)^{1/2}$
Good conductor approximation $\frac{\sigma}{\omega \epsilon} \gg 0$

Electromagnetics

OFFICE MEMORANDUM ♦ STAR LABORATORY

January 23, 1999

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1999

Penetration of Low-Frequency Electromagnetic Fields

The student is presented with a thin sheet of plastic insulation and asked about its electric and magnetic properties. He/she comes up with, or is led to, an electrical conductivity $\sigma = 0$, electrical permittivity $\epsilon = \epsilon_0$ (free space), and magnetic permeability $\mu = \mu_0$ (free space).

$\omega \rightarrow 0$
 $\sigma \rightarrow \epsilon$

A strong horseshoe magnet is produced; the steel keeper is removed, and placed on the desk. The piece of plastic is placed over the keeper and the student asked if the magnet will attract the keeper through the plastic sheet. In the subsequent discussion the student is asked about the skin depth (δ) and inevitably he/she can write it down as $\delta = \sqrt{2/(\omega \mu \sigma)}$, where ω is the angular frequency, μ is the permeability, and where σ is the electrical conductivity. We discuss the significance of each of the factors in the expression for δ and its applicability. At this stage we will probably discuss the "good conductor" approximation $(\sigma/\omega \epsilon) \gg 0$ and how it applies at low frequencies (particularly very low frequencies). Needless to say, applicability of the good conductor approximation is a prerequisite for a material to have a skin depth.

The magnet is now brought up to the plastic sheet and it is seen that the keeper is strongly attracted through the plastic. This experimental result is discussed.

Next, a sheet of aluminum is produced and its electrical properties discussed (it is a good conductor, so σ is relatively large; $\mu \approx \mu_0$; $\epsilon \approx \epsilon_0$). Is the keeper attracted through the aluminum sheet? After the student arrives at an answer, a test is carried out and it is found that the keeper is attracted. A similar test is carried out with a sheet of copper, one of the best metallic conductors. Once again the keeper is attracted strongly through the metal. These results are then discussed in the context of the skin depth equation above. At this stage it is concluded that the low value of the frequency (in fact, $\omega \approx 0$) must be a crucial factor. For low frequency, skin depth is high

there is ←
no
magnetization
kind of
effect?

A thin sheet of steel is produced. The student is asked about its properties in the context of the skin depth equation. He/she is expected to come up with, or is led to, an electrical conductivity $\sigma \approx \sigma_{\text{copper}}, \sigma_{\text{aluminum}}$, electrical permittivity $\epsilon \approx \epsilon_0$ (free space), and magnetic permeability $\mu \gg \mu_0$ (free space). Will the magnet attract the steel keeper through the steel sheet? At this time the student is expected to fret over the fact that the high conductivity and permeability values will reduce the skin depth but the frequencies involved are still extremely small, thus keeping the skin depth large. Given the previous results, it is hoped that the conclusion will be that the keeper is attracted. Experiment shows that the keeper is strongly attracted.

Finally, the student is asked if there was any means for preventing a low-frequency or DC magnetic field from penetrating through a material. High conductivity/high permeability materials might be briefly discussed, but it is hoped that reference will be made to the use of a superconducting material.