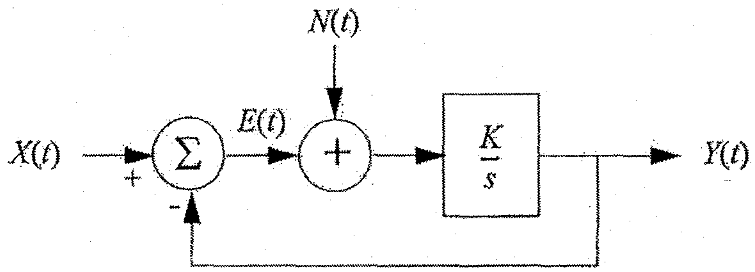
Stanford University
Department of Electrical Engineering
Qualifying Examination Winter 2005-06
Professor Joseph M. Kahn



This system attempts to make the output $Y(t)$ follow the input $X(t)$. The input is a Wiener process with power spectral density:

$$S_X(\omega) = \frac{X_0}{\omega^2}.$$

The signal $E(t)$ represents the tracking error. The error signal is corrupted by addition of the noise $N(t)$, which is statistically independent of $X(t)$. $N(t)$ is a zero-mean Gaussian random process with power spectral density:

$$S_N(\omega) = N_0.$$

You are free to choose the parameter K.

Question: what value of K minimizes σ_E^2, the variance of the tracking error?

Hint: you may need to use the integral:

$$\int_{-\infty}^{\infty} \frac{d\omega}{\omega^2 + K^2} = \frac{\pi}{K}.$$

Answer:

By linearity of system,

$$E(t) = E_1(t) + E_2(t)$$

where $X(t) \rightarrow E_1(t)$ and $N(t) \rightarrow E_2(t)$

By independence of $X(t)$ and $N(t)$, $E_1(t)$ and $E_2(t)$ are independent, and:

$$\sigma_E^2 = \sigma_{E_1}^2 + \sigma_{E_2}^2$$

Find $\sigma_{\epsilon_1}^2$:

If $X(t)$ and $E_1(t)$ were deterministic, we would have:

$$\frac{E_1(s)}{X(s)} = \frac{X(s) - Y(s)}{X(s)} = 1 - \frac{Y(s)}{X(s)} = 1 - \frac{K/s}{1+K/s} = \frac{s}{s+K}$$

Now consider stochastic $X(t)$:

$$\sigma_{E_1}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\left|\frac{s}{s+K}\Big|_{s=j\omega}\right|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{X_0}{\omega^2}\,\frac{\omega^2}{\omega^2+K^2}\, d\omega$$

$$= \frac{X_0}{2\pi}\int_{-\infty}^{\infty} \frac{d\omega}{\omega^2+K^2} = \frac{X_0}{2K}$$

Find $\sigma_{E_2}^2$:

If $N(t)$ and $E_2(t)$ were deterministic, we would have:

$$\frac{E_2(s)}{N(s)} = \frac{-K/s}{1+K/s} = \frac{-K}{s+K}$$

Now consider stochastic $N(t)$:

$$\sigma_{E_2}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_N(\omega)\left|\frac{-K}{s+K}\Big|_{s=j\omega}\right|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} N_0\,\frac{K^2}{\omega^2+K^2}\, d\omega$$

$$= \frac{N_0K^2}{2\pi}\int_{-\infty}^{\infty} \frac{d\omega}{\omega^2+K^2} = \frac{N_0K}{2}$$

Total:

$$\sigma^2 = \sigma_{E_1}^2 + \sigma_{E_2}^2 = \frac{X_0}{2K} + \frac{N_0K}{2}$$

$$\frac{d\sigma^2}{dK} = -\frac{X_0}{2K^2} + \frac{N_0}{2} = 0$$

$$K = \sqrt{\frac{X_0}{N_0}}$$

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Qualifying Examination Winter 2006-07
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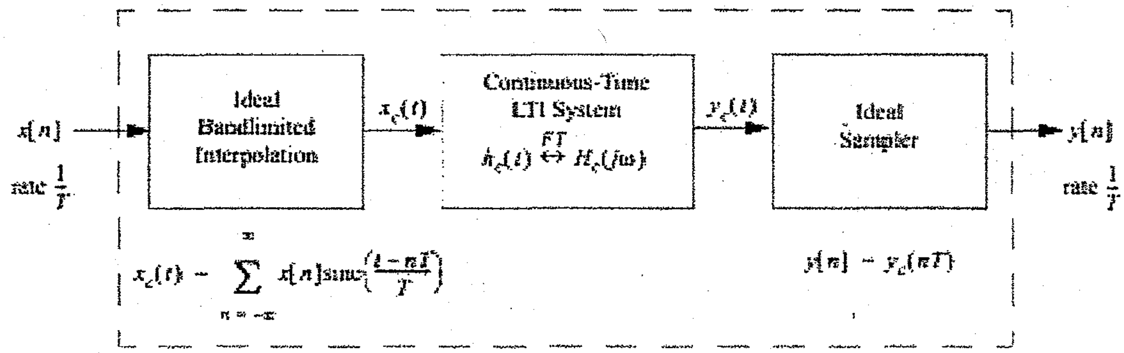


The dashed box encloses a discrete-time system having input $x[n] \overset{DTFT}{\leftrightarrow} X(e^{j\Omega})$ and output $y[n] \overset{DTFT}{\leftrightarrow} Y(e^{j\Omega})$. Give an explicit relationship between $X(e^{j\Omega})$ and $Y(e^{j\Omega})$. Is this system linear and time-invariant?

Solution: Let $\Omega = \omega T$. Define $\Pi(x) = \begin{cases} 1 & |x| \le 1/2 \\ 0 & |x| > 1/2 \end{cases}$.

Then $x_c(t) \overset{FT}{\leftrightarrow} X_c(j\omega) = T \cdot \Pi\left(\frac{\omega T}{2\pi}\right) \cdot X(e^{j\omega T}) = \begin{cases} T \cdot X(e^{j\omega T}) & |\omega| \le \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$.

$$Y_c(j\omega) = H_c(j\omega) \cdot X_c(j\omega) = T \cdot \Pi\left(\frac{\omega T}{2\pi}\right) \cdot H_c(j\omega) \cdot X(e^{j\omega T}).$$

$$Y(e^{j\omega T}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} Y_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) = X(e^{j\omega T}) \cdot \sum_{k=-\infty}^{\infty} H_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) \cdot \Pi\left(\left(\omega - k\frac{2\pi}{T}\right)\frac{T}{2\pi}\right).$$

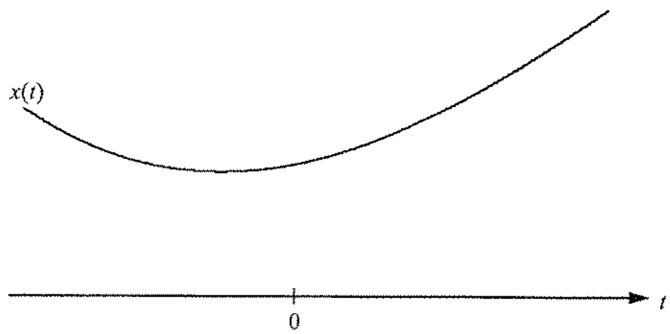
The system within the dashed box can be expressed in terms of the equivalent discrete-time frequency response:

$H(e^{j\Omega}) = H(e^{j\omega T}) = \frac{Y(e^{j\omega T})}{X(e^{j\omega T})} = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\omega - k\frac{2\pi}{T}\right)\right) \cdot \Pi\left(\left(\omega - k\frac{2\pi}{T}\right)\frac{T}{2\pi}\right)$, which is the periodic extension of $H_c(j\omega)$ bandlimited to $|\omega| \le \pi/T$. Since the system can be expressed in this way, it is linear and time-invariant.

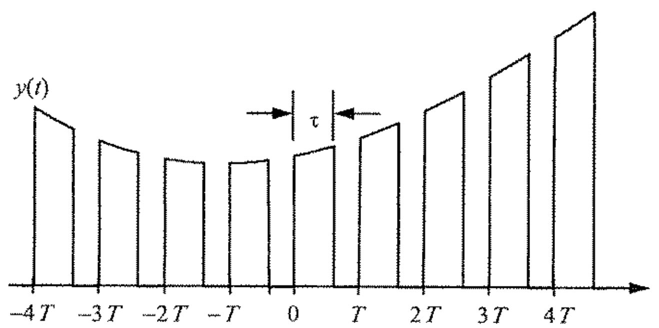
Kahn Signals 06-07.pdf

Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2007-08
Professor Joseph M. Kahn

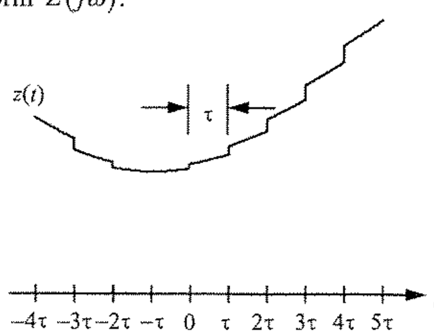
A continuous-time signal $x(t)$, $-\infty < t < \infty$, has a Fourier transform $X(j\omega)$.



We are only able to observe $x(t)$ for intervals of duration τ, which occur periodically with period T. So we observe the signal $y(t)$, $-\infty < t < \infty$, as shown. Find an expression for its Fourier transform $Y(j\omega)$.



Now we concatenate all of these observations as shown to form a new signal $z(t)$, $-\infty < t < \infty$. Find an expression for its Fourier transform $Z(j\omega)$.



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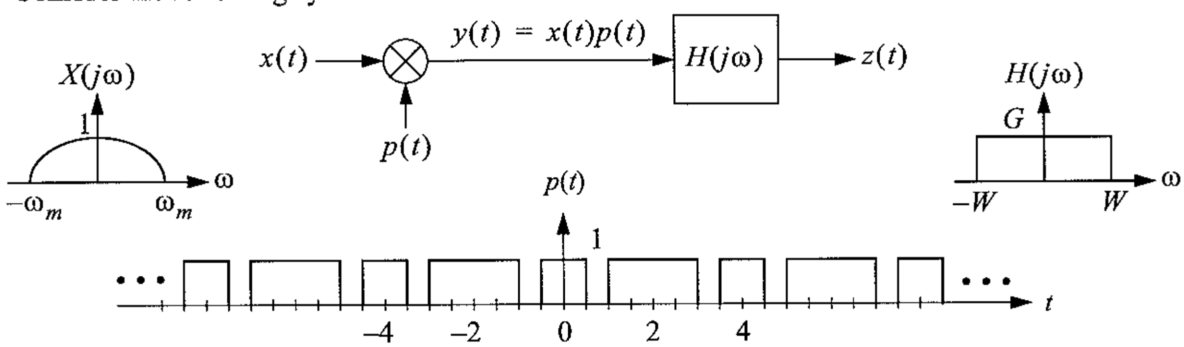
Qualifying Examination Winter 2008-09
Professor Joseph M. Kahn

Consider the following system.



The signal $x(t)$ is bandlimited to $|\omega| \leq \omega_m$. It is multiplied by $p(t)$, the periodic signal shown. The product, $y(t)$, is passed through the ideal lowpass filter $H(j\omega)$, which has cutoff frequency W and gain G.

Since $p(t)$ is periodic with period T_0 and fundamental frequency $\omega_0 = 2\pi/T_0$, it can be represented as a Fourier series:

$$p(t) = \sum_{n = \infty}^{\infty} P_n e^{jn\omega_0 t}.$$

- Without explicitly calculating the P_n, find an expression for $Y(j\omega)$, the Fourier transform of $y(t)$, in terms of $X(j\omega)$ and the P_n.

- State the conditions on ω_m, W and G such that $z(t) = x(t)$. Be as specific as possible, replacing variables by specific numbers when possible.

$$y(t) = x(t) \cdot p(t)$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

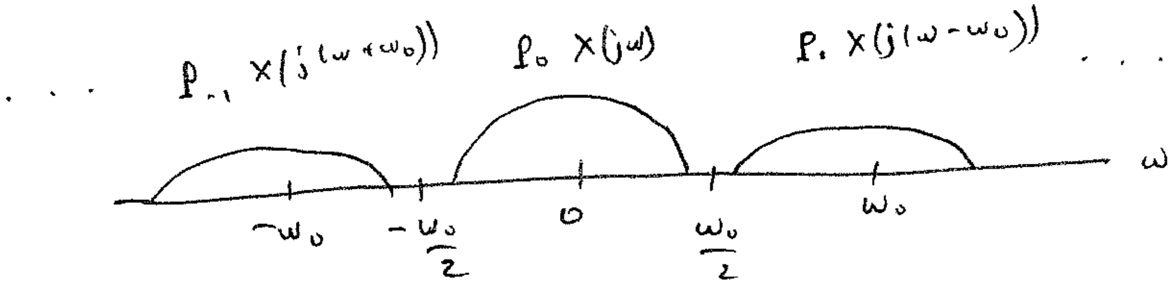
$$p(t) = \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_0 t}$$

$$P(j\omega) = 2\pi \sum_{n=-\infty}^{\infty} P_n \delta(\omega - n\omega_0)$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * 2\pi \sum_{n=-\infty}^{\infty} P_n \delta(\omega - n\omega_0)$$

$$= \sum_{n=-\infty}^{\infty} P_n X(j(\omega - n\omega_0))$$

If we can have $z(t) = x(t)$, we must have the following picture



Need $\quad \omega_m \leq \frac{\omega_0}{2}$

$$\omega_m \leq W \leq \omega_0 - \omega_m$$

$$G = \frac{1}{P_0}$$

$$P_0 = \frac{1}{T_0} \int_{T_0} p(t)\,dt = \frac{6}{8} = \frac{3}{4}$$

So $\quad G = \frac{4}{3}$

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Consider a carrier-modulated signal $s(t) = m(t)\cos\omega_c t$, having Fourier transform $S(j\omega)$. Assume that $s(t)$ is narrowband, i.e., $S(j\omega) = 0$ except for ω close to $\pm\omega_c$. The signal $s(t)$ is input to a LTI system having a real impulse response $h(t)$ and a frequency response $H(j\omega)$. Near $\omega = \pm\omega_c$, you can assume that $H(j\omega)$ varies slowly with ω.

- Show that the output $y(t)$ is given approximately by:

$$y(t) \approx |H(j\omega_c)| m(t - \tau_g(\omega_c)) \cos(\omega_c(t - \tau_p(\omega_c))).$$

  Give expressions for the group delay $\tau_g(\omega_c)$ and the phase delay $\tau_p(\omega_c)$ in terms of $H(j\omega)$.

- Explain intuitively why the group delay $\tau_g(\omega_c)$ has the particular mathematical form it does.

Hint: Represent $y(t)$ as the inverse Fourier transform of $Y(j\omega) = S(j\omega)H(j\omega)$. Write the frequency response in polar form as

$$H(j\omega) = |H(j\omega)| e^{j\arg\{H(j\omega)\}} = |H(j\omega)| e^{j\phi(\omega)},$$

and expand $H(j\omega)$ in a Taylor series. Near $\omega = \omega_c$, write:

$$H(j\omega) \approx \{ |H(j\omega_c)| + \ldots \} e^{j\left\{\phi(\omega_c) + \left.\frac{d\phi(\omega)}{d\omega}\right|_{\omega = \omega_c} \cdot (\omega - \omega_c) + \ldots\right\}}.$$

Near $\omega = -\omega_c$, write a similar Taylor series.

4.
$$s(t) = m(t)\cos w_c t$$

$$S(jw) = \frac{1}{2}\Big[\underbrace{M(j(w-w_c))}_{\substack{\text{nonzero} \\ |w-w_c| \le w_m}} + \underbrace{M(j(w+w_c))}_{\substack{\text{nonzero} \\ |w+w_c| \le w_m}}\Big]$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(jw)e^{jwt}\,dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S(jw)H(jw)e^{jwt}\,dw$$

$$= \frac{1}{4\pi}\Big[\int_{-\infty}^{\infty} M(j(w-w_c))H(jw)e^{jwt}\,dw + \int_{-\infty}^{\infty} M(j(w+w_c))H(jw)e^{jwt}\,dw\Big]$$

Near $w = w_c$:

$$H(jw) \cong |H(jw_c)|e^{j[\phi(w_c)+\phi'(w_c)(w-w_c)]}$$

Near $w = -w_c$:

$$H(jw) \cong |H(-jw_c)|e^{j[\phi(-w_c)+\phi'(-w_c)(w+w_c)]}$$

$$|H(-jw_c)| = |H(jw_c)|$$

$$\phi(-w) = -\phi(w) \Rightarrow \phi(-w_c) = -\phi(w_c)$$

$$\phi'(-w_c) = \phi'(w_c)$$

Near $w = -w_c$:

$$H(jw) \simeq |H(jw_c)|\, e^{j[-\phi(w_c) + \phi'(w_c)(w+w_c)]}$$

$$y(t) \simeq \frac{1}{4\pi}\Big[\int_{-\infty}^{\infty} M(j(w-w_c))\, |H(jw_c)| e^{j(\phi(w_c)+\phi'(w_c)(w-w_c))} \cdot e^{jwt}\, dw$$

$$+ \int_{-\infty}^{\infty} M(j(w+w_c))|H(jw_c)|\, e^{j[-\phi(w_c)+\phi'(w_c)(w+w_c)]} e^{jwt}\, dw$$

$$= \frac{1}{4\pi}|H(jw_c)|\Big(e^{j\phi(w_c)} \int_{-\infty}^{\infty} M(j(w-w_c)) e^{j\phi'(w_c)(w-w_c)} e^{jwt}\, dw$$

$$+ e^{-j\phi(w_c)} \int_{-\infty}^{\infty} M(j(w+w_c))\, e^{j\phi'(w_c)(w+w_c)} e^{jwt}\, dw \Big]$$

Change variables:

First integral: $\Omega = w - w_c$

Second integral: $\Omega = w + w_c$

$$y(t) \simeq \frac{1}{4\pi}|H(jw_c)| \Big[e^{j\phi(w_c)} \int_{-\infty}^{\infty} M(j\Omega) e^{j\phi'(w_c)\Omega} e^{j(\Omega+w_c)t}\, d\Omega$$

$$+ e^{-j\phi(w_c)} \int_{-\infty}^{\infty} M(j\Omega)\, e^{j\phi'(w_c)\Omega} e^{j(\Omega-w_c)t}\, d\Omega \Big]$$

$$= |H(jw_c)|\, \frac{1}{2}\Big[e^{j(w_c t+\phi(w_c))} + e^{-j(w_c t+\phi(w_c))} \Big]$$

$$\cdot \frac{1}{2\pi}\int_{-\infty}^{\infty} M(j\Omega)\, e^{j\Omega(t+\phi'(w_c))}\, d\Omega$$

$$= |H(j\omega_c)| \cos(\omega_c t + \phi(\omega_c))\, m(t + \phi'(\omega_c))$$

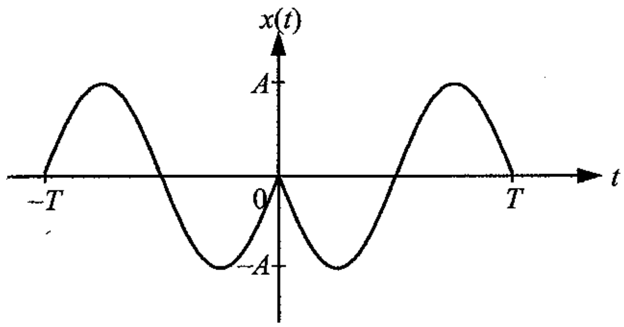
Define $\tau_g(\omega_c) = -\phi'(\omega)\big|_{\omega=\omega_c}$

$$\tau_p(\omega_c) = -\frac{\phi(\omega_c)}{\omega_c}$$

$$\boxed{y(t) = |H(j\omega_c)| \cos(\omega_c(t - \tau_p(\omega_c)))\, m(t - \tau_g(\omega_c))}$$

Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2009-10
Professor Joseph M. Kahn

A continuous-time signal $x(t)$, $-\infty < t < \infty$, has a Fourier transform $X(j\omega)$, $-\infty < \omega < \infty$.



Without computing $X(j\omega)$, answer the following:

a. What is $X(j0)$?

b. What is $\int_{-\infty}^{\infty} X(j\omega) e^{j\frac{\omega T}{4}} d\omega$?

c. What is $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$?

d. By what power of $|\omega|$ does $|X(j\omega)|$ decrease as $|\omega| \to \infty$?

Answer

a. $X(j0) = \int_{-\infty}^{\infty} x(t) e^{-j0t} dt = 0$.

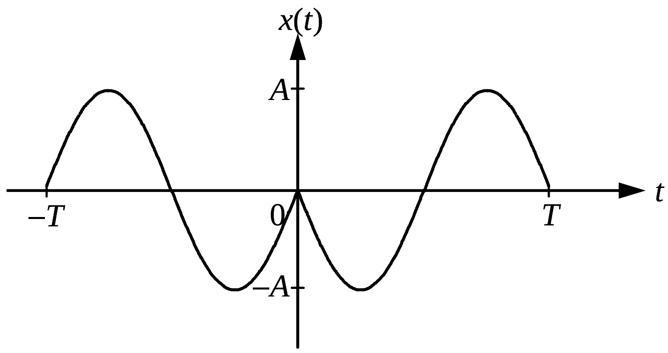
b. $\int_{-\infty}^{\infty} X(j\omega) e^{j\frac{\omega T}{4}} d\omega = 2\pi x\left(\frac{T}{4}\right) = -2\pi A$.

c. $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\pi \cdot 2 \cdot \frac{A^2 T}{2} = 2\pi A^2 T$.

d. Since $x(t)$ and $\frac{dx}{dt}$ do not have any impulses (delta functions), but $\frac{d^2x}{dt^2}$ has impulses, $|X(j\omega)| \propto |\omega|^{-2}$ as $|\omega| \to \infty$.

Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2009-10
Professor Joseph M. Kahn

A continuous-time signal $x(t)$, $-\infty < t < \infty$, has a Fourier transform $X(j\omega)$, $-\infty < \omega < \infty$.



Without computing $X(j\omega)$, answer the following:

a. What is $X(j0)$?

b. What is $\int_{-\infty}^{\infty} X(j\omega) e^{j\frac{\omega T}{4}} d\omega$?

c. What is $\int_{-\infty}^{\infty} \left| X(j\omega) \right|^2 d\omega$?

d. By what power of $|\omega|$ does $|X(j\omega)|$ decrease as $|\omega| \to \infty$?

Answer

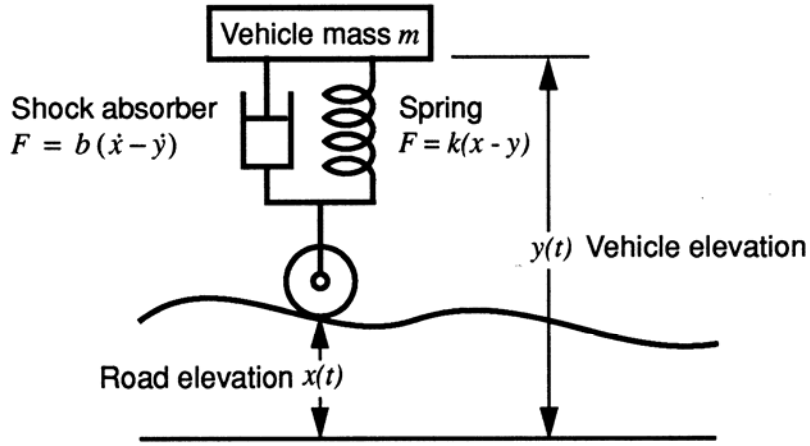
a. $X(j0) = \int_{-\infty}^{\infty} x(t) e^{-j0t} dt = 0$.

b. $\int_{-\infty}^{\infty} X(j\omega) e^{j\frac{\omega T}{4}} d\omega = 2\pi x\left(\frac{T}{4}\right) = -2\pi A$.

c. $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\pi \cdot 2 \cdot \frac{A^2 T}{2} = 2\pi A^2 T$.

d. Since $x(t)$ and $\frac{dx}{dt}$ do not have any impulses (delta functions), but $\frac{d^2x}{dt^2}$ has impulses, $|X(j\omega)| \propto |\omega|^{-2}$ as $|\omega| \to \infty$.

Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2010-11
Professor Joseph M. Kahn



A one-wheeled vehicle rolls along a road, supported by a suspension that includes a spring and a shock absorber. At time t, the road elevation is $x(t)$ and the vehicle elevation is $y(t)$. The suspension can be considered as a linear time-invariant system H with input $x(t)$ and output $y(t)$, $H\{x(t)\} = y(t)$, which is governed by the differential equation:

$$m\ddot{y} + b\dot{y} + ky = b\dot{x} + kx \, ,$$

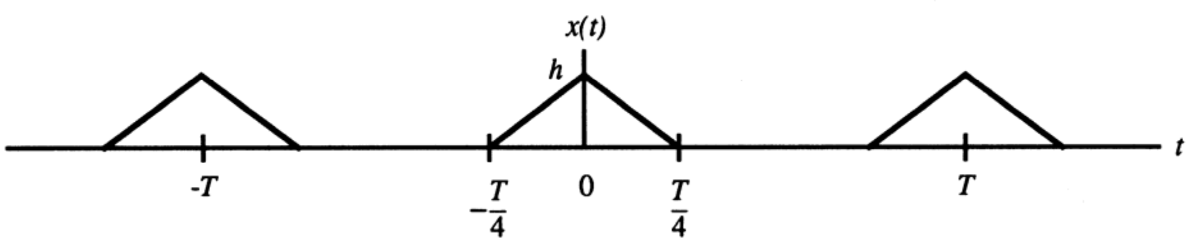
where m, b and k are positive real constants.

1. Find the frequency response $H(j\omega)$ of the system, which satisfies:

$$H\{e^{j\omega t}\} = H(j\omega) \cdot e^{j\omega t} \, .$$

2. Assume $m = 1$, $b = 1$ and $k = 1$. Make a sketch of the magnitude response of the system, $|H(j\omega)|$, and describe the system qualitatively.

3. The vehicle rolls along an infinitely long road with regularly spaced triangular speed bumps, so the road elevation $x(t)$ is the periodic signal indicated below. Give an exponential Fourier series representation of this $x(t)$.



4. Assuming general values of m, b and k, find an expression for the vehicle elevation $y(t)$ when the vehicle rolls over the road pictured above.

Answers

1. Taking the Fourier transform of the differential equation and using the property

$$\dot{x}(t) \leftrightarrow (j\omega)X(j\omega),$$

we obtain:

$$\left[m(j\omega)^2 + b(j\omega) + k\right]X(j\omega) = \left[b(j\omega) + k\right]Y(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{b(j\omega) + k}{m(j\omega)^2 + b(j\omega) + k}.$$

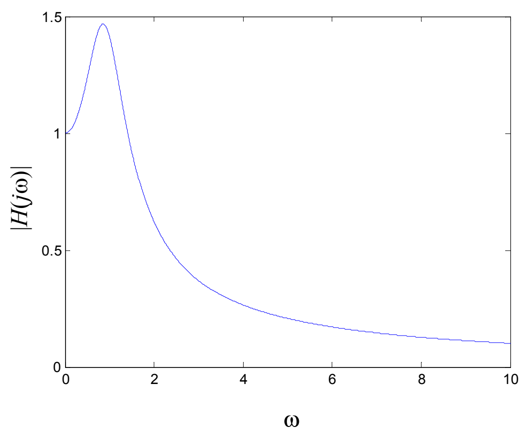
2. Setting $m = b = k = 1$:

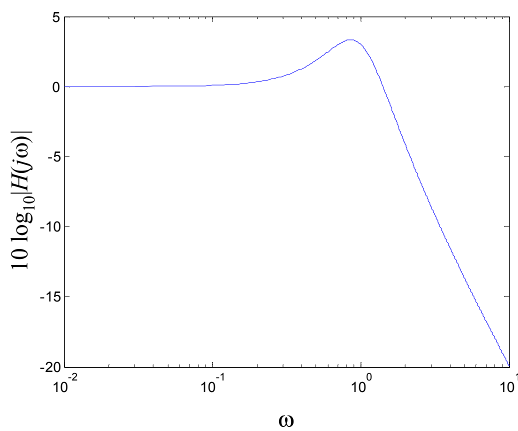
$$H(j\omega) = \frac{j\omega + 1}{j\omega + 1 - \omega^2}$$

$$\left|H(j\omega)\right| = \sqrt{\frac{\omega^2 + 1}{\omega^2 + \left(1 - \omega^2\right)^2}}$$

We evaluate $\left|H(j\omega)\right|$ for several values of ω:

ω	$\left\|H(j\omega)\right\|$
0	1
1	$\sqrt{2}$
∞	0





3. Let $q(t)$ denote one period of $x(t)$. Note that $q(t) = h\Lambda\left(\frac{t}{T/4}\right)$, and use the Fourier transform pair

$$\Lambda\left(\frac{t}{\tau}\right) \leftrightarrow \tau \mathrm{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$$

to obtain

$$q(t) \leftrightarrow Q(j\omega) = \frac{Th}{4}\mathrm{sinc}^2\left(\frac{\omega T}{8\pi}\right).$$

Representing $x(t)$ by a Fourier series:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} ,$$

where the fundamental frequency is $\omega_0 = \frac{2\pi}{T}$, the Fourier series coefficients are given by:

$$a_n = \frac{1}{T}Q(j\omega)\Big|_{\omega=n\omega_0} = \frac{h}{4}\mathrm{sinc}^2\left(\frac{n\omega_0 T}{8\pi}\right) = \frac{h}{4}\mathrm{sinc}^2\left(\frac{n}{4}\right).$$

4. Given an input $x(t)$, the output is:

$$y(t) = H\{x(t)\}.$$

Representing $x(t)$ as a Fourier series, and using the fact that complex exponentials are eigenfunctions of any LTI system, so that $H\{e^{jn\omega_0 t}\} = H(jn\omega_0)e^{jn\omega_0 t}$, the output is:

$$\begin{aligned}
y(t) &= H\left\{\frac{h}{4}\sum_{n=-\infty}^{\infty}\mathrm{sinc}^2\left(\frac{n}{4}\right)e^{jn\omega_0 t}\right\} \\
&= \frac{h}{4}\sum_{n=-\infty}^{\infty}\mathrm{sinc}^2\left(\frac{n}{4}\right)H\{e^{jn\omega_0 t}\} \\
&= \frac{h}{4}\sum_{n=-\infty}^{\infty}\mathrm{sinc}^2\left(\frac{n}{4}\right)H(jn\omega_0)e^{jn\omega_0 t} \\
&= \frac{h}{4}\sum_{n=-\infty}^{\infty}\mathrm{sinc}^2\left(\frac{n}{4}\right)\frac{b(jn\omega_0)+k}{m(jn\omega_0)^2 + b(jn\omega_0)+k}e^{jn\omega_0 t}
\end{aligned}$$

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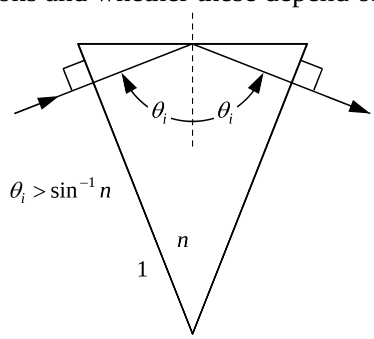
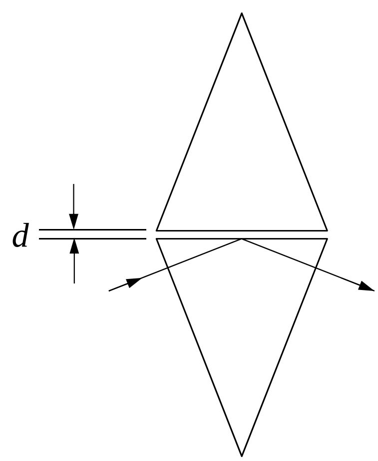
2013 EE Qualifying Examination
Electromagnetics
Professor Joseph M. Kahn

Questions

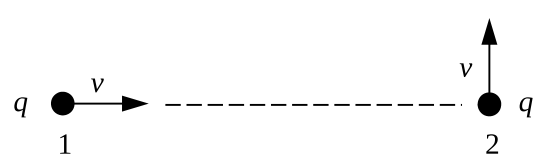
1. a. Total internal reflection occurs at the upper surface of the prism. Describe any fields in the region above that surface. Sketch surfaces of constant phase. Sketch surfaces of constant amplitude. Describe any propagation of the fields and any associated energy flow. Describe their relation to boundary conditions and whether these depend on the vector nature of the fields.



b. A second identical prism is brought close to the first prism. Describe what happens to the propagating beam as a function of the gap spacing *d*.

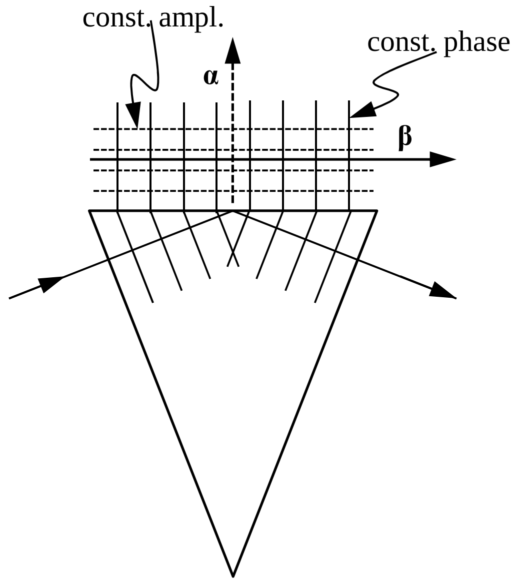


2. At a particular instant of time, two particles of equal charge q are positioned as shown and are moving at speed v as shown. Describe the forces on the two particles. Are these equal and opposite?

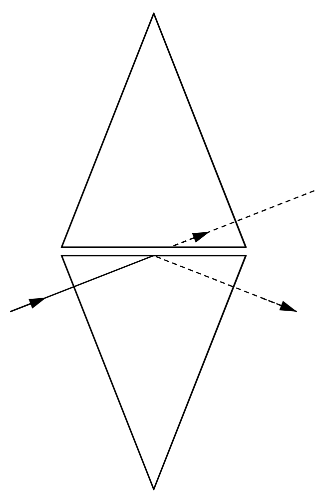


Answers

1. a. The fields in the upper region are an evanescent wave. This is a nonuniform plane wave with complex propagation vector $\boldsymbol{\gamma} = \boldsymbol{\alpha} + j\boldsymbol{\beta}$. In a lossless medium, a nonuiform plane wave must have $\boldsymbol{\alpha} \perp \boldsymbol{\beta}$. The evanescent wave propagates parallel to the boundary but is attenuated exponentially away from the boundary. Surfaces of constant phase and amplitude are perpendicular to $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$, respectively, as shown. There is a flow of energy along the surface, but since the plane wave is implicitly assumed to be infinite in cross section, no energy enters or leaves the system. The surfaces of constant phase are continuous across the boundary. This continuity is a consequence of the wave nature of the fields, and has nothing to do with their vector nature (as do the the continuity of normal **B**, **D** and tangential **H**, **E**).



b. When the gap width *d* is zero, all of the light propagates forward into the second prism. As *d* increases, the transmitted amplitude decreases exponentially.



2. The two particles are subject to repulsive electric forces $\mathbf{F}_{e1}$ and $\mathbf{F}_{e2}$, which are equal and opposite. The motion of particle 1 creates a magnetic field but this is zero at particle 2, so it exerts no force on particle 2. The motion of particle 2 creates a magnetic field that is nonzero at particle 1, and this exerts a magnetic force $\mathbf{F}_{m1} = q\mathbf{v} \times \mathbf{B}$ on particle 1. The total force on particle 1 is $\mathbf{F}_1 = \mathbf{F}_{e1} + \mathbf{F}_{m1}$, while that on particle 2 is $\mathbf{F}_2 = \mathbf{F}_{e2}$. These total forces are not equal and opposite. An explanation lies beyond the scope of the exam, but lies in the fact that the fields contain linear momentum that is changing as the particles move. It has nothing to do with relativistic effects, nor with radiation.