2008 Quals

Examiner: Ramesh Johari

Problem 1. The Smiths have three children, of whom one is a boy. What is the probability he has two sisters as siblings? (Assume that a child is equally likely to be a boy or a girl, independent of other children.)

Problem 2. At each time period $t = 1, 2, 3, \ldots$, a red coin and a blue coin are flipped simultaneously. Assume that the red coin comes up heads with probability p_r, and the blue coin comes up heads with probability p_b.

(a) Calclulate the expected number of flips until the first head (either red or blue) is seen.

(b) Calculate the probability that at least 3 red coins come up heads before the first blue coin comes up heads.

Problem 3. Suppose that X and Y are two real-valued random variables. Show that:
$$\mathbf{E}[|X||Y|] \leq \sqrt{\mathbf{E}[X^2]\mathbf{E}[Y^2]},$$
where $|x|$ denotes the absolute value of x.

Partial credit will be given if the result is proven under the assumption that X and Y are both uniformly distributed on $\{0, 1, 2, \ldots, N\}$.