EE Ph.D. Qualifying Exam, January 2015 Question

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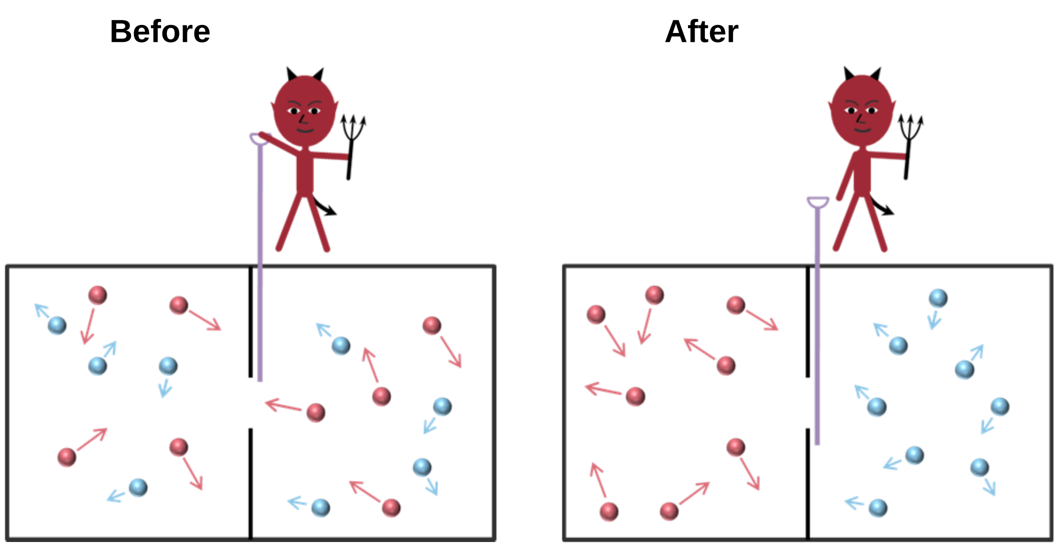
Notes: There may not be single "correct" answer to this question. The goal of this question is to see how you think about it. The answers are mostly qualitative, and little or no algebra should be required for them. If you finish the question on this sheet, subsequent questions will be asked.

Maxwell's demon

We imagine we have a box containing gas atoms in some thermal equilibrium distribution at a particular temperature. There is a divider or shutter that we can open and close in the middle of the box. Maxwell's demon is a small being or machine that opens or closes the shutter depending on what he observes.

In one version, the demon opens and closes the shutter appropriately so that he lets all the rapidly moving (and hence "hotter") atoms through into the left side of the box and all the slower moving (and hence "colder") atoms through into the right side of the box. As a result, he separates the gas into hotter and colder portions. (For simplicity in the pictures below, we show only "fast" atoms and "slow" atoms, but there will be a continuous range from slow to fast.)



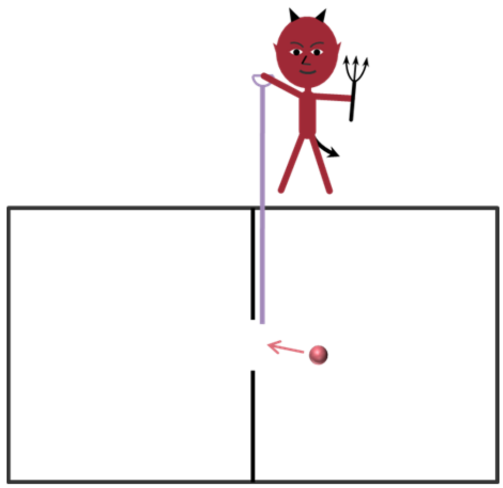
He can then run some sort of engine that extracts work from the gas, for example, by having the "hot" atoms push the divider to the right to lift some weight because there is now greater gas pressure on the divider from the left.

He has therefore apparently taken thermal energy at only one temperature and used it to perform work.

Can the demon actually do this? Does this violate any physical laws? Can I make a perpetual motion machine using this idea?

Supplementary question

Suppose we take a particularly simple and extreme version of the system where we imagine there is only one atom in the box overall.



(i) what is the reduction in entropy if we introduce the shutter when the atom is on the left of the shutter, thereby changing the situation from equal probabilities of the atom being on the left (L) and right (R) of the box to one in which, with probability one, it is on the left (L)?

(ii) what might this imply about any increase in entropy in the demon and, considering the demon as a simple machine, how might we view the mechanism of any increase of the demon's entropy?

Supplementary question 2

Is there a minimum energy cost to erase a one bit memory at finite temperature?

Answer

In using this as a question, I am much more interested to see how the examinee thinks. Whether or not the examinee got to supplementary questions did not itself directly influence the score, and depending on the kinds of responses the student gave, I might also ask them other things about the problem, such as how to make a "cyclic" engine out of this approach.

There is indeed a set of problems in the demon managing to separate out the atoms to the two sides. It is, of course, possible for the demon to do this separation, and it is possible to imagine him doing this with vanishing small expenditure of actual energy in moving the shutter itself. That movement of the shutter can presumably be performed with arbitrarily small energy by using a sufficiently light shutter and moving it sufficiently slowly. The issue is whether the demon can do this separation without otherwise increasing his own entropy or that of some other system. If there is no sufficient such increase in entropy elsewhere, then the demon's actions will violate the second law of thermodynamics by reducing entropy (by separating the atoms) with no apparent work performed on the system. And then the demon will manage to get the system to do some work for him, all while starting off with only a gas at one temperature, which would also violate the second law.

A more subtle point is that this system can be simply extended to continue to extract work cyclically. If we let the divider be pushed to the right after separating atoms, we can then remove it from its right-most point and reinsert it in the middle, or open the shutter and push it back over, and start all over again, repeating the work extraction. Such a cyclic engine causes us particular problems with the second law of thermodynamics, giving us a "perpetual motion machine of the second kind"[1]. (In such a case, we would likely connect the system to a heat reservoir at our one starting temperature to heat the gas up again in between cycles, but this still leaves us with a cyclic machine that can continually perform work starting with only one temperature.)

Part of the difficulty here is in understanding the demon as a physical system. Certainly in the original proposals of this thought experiment by James Clerk Maxwell (yes, that Maxwell!) in his book *Theory of Heat* in 1871, there was no clear understanding of just how we should view the demon[2] as a physical system other than thinking of him as having "free will", whatever that is, and the abilities to perform his task.

As far as the "answer" I am looking for in this part of the question, it is the understanding that there is at least potentially a substantial issue here with violating the second law of thermodynamics through the apparent reduction of entropy here, and/or the ability

[1] A perpetual motion machine of the second kind would be one that did not necessarily violate overall conservation of energy (which would be a "perpetual motion machine of the first kind"), but that violated the second law. An example of such a machine would be a ship that could power its motors by extracting energy for work from the heat energy of the ocean, ejecting sufficient ice cubes out the back so as not to violate conservation of energy.

[2] Incidentally, Maxwell himself did not introduce the term "demon"; that term was apparently introduced by William Thomson (also known as Lord Kelvin) in 1874. And "demon" here is likely not meant to indicate any malevolence, just referring to "an intelligent being endowed with free will, and fine enough tactile and perceptive organization to give him the faculty of observing and influencing individual molecules of matter" (from W. Thomson, "The kinetic theory or the dissipation of energy," Nature **IX**, 441-444 (1874)). It is, of course, more fun to draw the demon as a little devil-like character!

apparently to perform work starting only from the thermal energy in a gas at a uniform temperature in a system that can be cycled. I would also look for examinees to be able to come up with various possible resolutions, and, ideally, for them to criticize these resolutions themselves. Reasonable suggestions include (i) that there must be some energy associated with the actual raising and lowering of the shutter, and (ii) the process of measurement must itself involve some energy. Actually, depending on how you interpret these answers, they could be viewed as correct, though I would thank the examinees for either of these suggestions (and give them credit), but ask them to come up with something else. See the answers to the supplementary questions for another possible resolution of this problem.

Incidentally, the many issues and arguments and the history of Maxwell's Demon are gathered in the book "Maxwell's Demon 2 – Entropy, Classical and Quantum Information, Computing," H. S. Leff and A. F. Rex (eds.) (IoP Publishing, Bristol and Philadelphia, 2003).

Answer to supplementary question

We remember that the entropy of some system can be described statistically by

$$S = k \log g$$

Here g is the number of different microstates corresponding to the one macrostate we are considering. In physics the logarithm is taken to the base e and k is Boltzmann's constant. In information theory we more typically use logs to the base 2 with no prefactor constant in considering the related quantity. In physics, all the different microstates are considered to be equally likely.

For a situation where there is one atom in the box, we could consider the contribution to the entropy from the atom being either on the left, L, or the right, R, of the middle of the box, which is also the position of the divider or shutter. Initially, then, the atom can be on either side of the box, in which case that uncertainty of those two "microstates" of being on the left or the right gives a contribution to the entropy of

$$S_{LR} = k \log 2$$

If we wait until we observe the atom to be in the left side of the box and then insert the shutter to keep it there, there is now only one microstate of the two left, and so no entropy contribution, i.e., the entropy contribution corresponding to this state is

$$S_L = k \log 1 = 0$$

So the action of inserting the shutter when we know for sure the atom is in the left half of the box has decreased the entropy of the system by an amount of magnitude $k \log 2$. The argument then is that, because the second law of thermodynamics tells us that we cannot decrease entropy overall in a closed system without doing work, and we seem not to have done any work here, something associated with the demon must have increased its entropy by $k \log 2$.

One view of this is to assert that there is a one-bit memory associated with the demon, a memory that is where the result of the measurement is stored. The act of measuring the atom to be on the left and writing that into the memory sets the memory to its "1" state, representing the "L" condition. This memory is to be set to "1" in such a case regardless of its previous state. Hence, the entropy of this memory has been reduced by $k \log 2$ on the average, which, according to the second law of thermodynamics much have been accomplished by an increase of entropy of at least $k \log 2$ in the environment surrounding the memory (i.e., in the demon or in his environment). Thus there is no net decrease in entropy of the entire system if we presume this process in the demon. And the demon has to remember to eat his breakfast each day so as to get the necessary fuel to allow him to perform these actions!

Answer to supplementary question 2

In an environment of temperature *T*, the entropy change ΔS associated with a flow of heat *Q* into a system is

$$\Delta S = Q / T$$

So the energy involved in erasing a one bit memory is at least

$$E = T \Delta S = kT \log 2$$

This idea that the erasure of a memory cell requires this increase in entropy in the environment was introduced by Landauer in 1961[3]. It is this resolution and proposal that finally offers a way to link physical entropy and information entropy, since all information must be written down somewhere. Though this idea has continued to be debated, this idea of Landauer's principle is now viewed as being quite a serious idea. For a modern attempt at an experiment to test this idea, see J. V. Koski, V. F. Maisi, J. P. Pekola, and D. V. Averin, "Experimental realization of a Szilard engine with a single electron," PNAS **111**, 13786–13789 (2014) doi: 10.1073/pnas.140696611 .

[3] R. Landauer, "Irreversibility and heat generation in the computing process," IBM J. Research and Development 5, 183 – 191 (1961)