Stanford University, Department of Electrical Engineering
Qualifying Examination, Winter 2011-12
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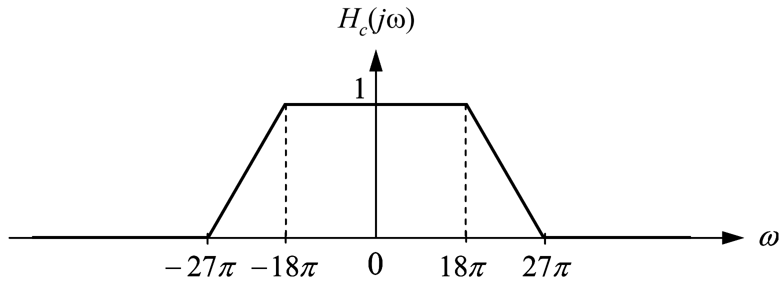
Consider a continuous-time filter $h_c(t) \overset{FT}{\leftrightarrow} H_c(j\omega)$ having the frequency response shown below.



A sampling frequency $\omega_s = 2\pi/T = 48\pi$ rad/s is assumed. Using three different approaches, a discrete-time filter $h[n] \overset{DTFT}{\leftrightarrow} H(e^{j\Omega})$ is derived from the continuous-time filter. In each case, you are asked to sketch the magnitude response $|H(e^{j\Omega})|$ and answer a few questions. The discrete-time and continuous-time frequencies are related by $\Omega = \omega T$.

(a) An infinite impulse response filter $h_1[n] \overset{Z}{\leftrightarrow} H_1(z)$ is designed using the impulse invariance criterion:

$$h_1[n] = T \cdot h_c(t)\big|_{t=nT} .$$

Sketch the magnitude response $|H_1(e^{j\Omega})|$. Does aliasing occur?

(b) An infinite impulse response filter $h_2[n] \overset{Z}{\leftrightarrow} H_2(z)$ is designed using the bilinear transformation:

$$H_2(z) = H_c(s)\big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} .$$

Sketch the magnitude response $|H_2(e^{j\Omega})|$. Does aliasing occur? The continuous-time frequencies $\omega_c = 18\pi$ and $\omega_s = 27\pi$ map to discrete-time frequencies Ω_c and Ω_s. Can you obtain expressions for Ω_c and Ω_s?

(c) A finite impulse response filter $h_3[n] \overset{Z}{\leftrightarrow} H_3(z)$ is designed by performing a Fourier series expansion of:

$$H_c\left(j\tfrac{\Omega}{T}\right)$$

over the frequency range $-\pi < \Omega < \pi$. (This is equivalent to performing a Fourier series expansion of $H_c(j\omega)$ over the range $-24\pi < \omega < 24\pi$.) Sketch the magnitude response $|H_3(e^{j\Omega})|$. Does aliasing occur? Will the Gibbs phenomenon be observed if $h_3[n]$ is not multiplied by a window function?