Ph.D. Quals Question
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Super Capacitors

Quals students were told that the advent of electric cars has led to a strong interest in the development of "super" or "ultra" capacitors, which might just conceivably replace batteries or, more likely, given current progress, be used in conjunction with batteries. The students were warned that my questioning would be directed toward the development of these high-capacity capacitors and that it would largely involve basic electromagnetism, or more particularly, basic electricity. To start with they were asked to describe Gauss's theorem relating the surface integral of the electric field **E** over a closed surface to the enclosed charge Q:

$$\int \mathbf{E}.\mathbf{ds} = Q/\varepsilon_0$$

Next, when we apply this to the surface of a plane conducting plate of infinite extent carrying surface charge σ we obtain the electric field perpendicular to the surface:

$$E = \sigma/\varepsilon_0$$

This same expression also applies to the electric field close to the surface of any conductor, with σ the surface charge density in the immediate vicinity.

For two parallel conducting planes of area A separated by a distance d carrying equal but opposite charges Q, -Q, the electric field has the same form as above (we will assume it is everywhere perpendicular to the surfaces of the plates, with no fringing) and the capacitance of the plates is

$$C = Q/V = \sigma A/(\sigma d/\varepsilon_0) = \varepsilon_0 A/d$$

Although all isolated conducting bodies have some capacitance, these capacitances are generally very small and measured in microfarads or micromicrofarads. [To illustrate, the capacitance of a sphere of radius R is given by $C = 4\pi\,\varepsilon_0 R$ and for a sphere as large as the Earth, with R = 6370 km, we only have C = 708 μF]. Once again in general, large capacitances can only be achieved by using two close conductors bearing opposite charges (this arrangement is called a **capacitor**) and here the above equation for the capacitance between two plates is representative. Not only that, most attempts to produce very large capacitances are based on the conducting plate model. We will now consider how "super" or "ultra" capacitors might be produced using this model.

Obviously, we can make C large by increasing A and decreasing d, and this is what is done in conventional capacitors. However, these conventional capacitors are not "super" nor "ultra." **Background material**: The best of these conventional capacitors is the electrolytic capacitor, invented around 1890, which can have a capacitance in the mF range. In these capacitors a very thin (small d) layer of non-conducting aluminum oxide is formed between two aluminum plates containing an electrolyte or an electrolyte-