2015 Quals Questions – Amin Arbabian

Once they get to the oscillator part and plot the waveforms, I ask for calculation of the oscillation frequency and duty cycle (and ways to modify these).

EE Qualifying Exam 2015 – John Duchi

Question 1: Let $\delta \in (0,1)$. I have two unfair coins in my pocket. The first coin, when flipped, lands heads with probability 1. The second coin, when flipped, lands heads with probability $1-\delta$ and tails with probability δ. I give you one of the coins but do not tell you which one. How many flips do you need to guarantee that, no matter which coin I gave you, you can identify which coin it is with probability at least $3/4$?

Answer: Let P_1 be the distribution of heads under the first coin, P_2 the second. The probability of seeing no heads after n flips for coin 1 is always 1; the probability of seeing 0 heads after n flips for coin 2 is

$$P_2(\text{no heads after } n \text{ flips}) = (1-\delta)^n.$$

If our procedure is "guess coin 1 if there are no heads and guess coin 2 otherwise," then we never make a mistake with coin 1, while with coin 2 we have $P_2(\text{mistake}) = (1-\delta)^n$. To get $(1-\delta)^n \le 1/4$, it is sufficient that $n \log(1-\delta) \le \log \frac{1}{4}$, and using the approximation $\log(1-\delta) \approx -\delta$, we see that $n \ge \frac{\log 4}{\delta}$ coin flips are sufficient. $\square$

Question 2: Let $\delta \in (0,1)$. I have two unfair coins in my pocket. The first coin, when flipped, lands heads with probability $\frac{1+\delta}{2}$ and tails with probability $\frac{1-\delta}{2}$. The second coin, when flipped, lands heads with probability $\frac{1-\delta}{2}$ and tails with probability $\frac{1+\delta}{2}$. I give you one of the coins but do not tell you which one. How many flips do you need to guarantee that, no matter which coin I gave you, you can identify which coin it is with probability at least $3/4$?

Answer: Let P_1 and P_2 be the distributions of the coins as in Question 2. We flip the coin given n times, and if there are more heads report coin 1, more tails report coin 2. Now we must approximate the probability of success under this scheme.

There are several ways to answer this question. Let X_i be 1 if the ith flip is heads, 0 otherwise. The simplest is to recall the central limit theorem. We know that $\sigma^2 = \mathrm{Var}(X_i) = \frac{1-\delta^2}{4}$ for either distribution, and the CLT implies that

$$\frac{1}{\sqrt{n}} \sum_{i=1}^n (X_i - \mathbb{E}[X_i]) \xrightarrow{d} \mathsf{N}(0, \sigma^2).$$

Thus, if $\Phi(t) = \mathbb{P}(Z \le t)$, where Z is a standard normal random variable, we (approximately) have that

$$\mathbb{P}\left(\frac{1}{\sqrt{n \,\mathrm{Var}(X)}} \sum_{i=1}^n (X_i - \mathbb{E}[X_i]) \le t \right) \approx \Phi(t)$$

for $t \in \mathbb{R}$. Now, let us focus on coin 1 (the cases are symmetric). Then we have that

$$P_1(\text{wrong after } n \text{ flips}) = P_1\left(\sum_{i=1}^n X_i \le \frac{n}{2} \right) = P_1\left(\sum_{i=1}^n \left(X_i - \frac{1+\delta}{2} \right) \le \frac{-\delta n}{2} \right),$$

and by the CLT, we have (noting that $\mathrm{Var}(X) = \frac{1-\delta^2}{4}$)

$$P_1(\text{wrong after } n \text{ flips}) = P_1\left(\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left(X_i - \frac{1+\delta}{2}\right) \leq \frac{-\delta\sqrt{n}}{2}\right)$$
$$\approx \Phi\left(\frac{-\delta\sqrt{n}}{2\sqrt{\mathrm{Var}(X)}}\right) = \Phi\left(\frac{-\delta\sqrt{n}}{\sqrt{1-\delta^2}}\right).$$

For this probability to be small—i.e. for us to be unlikely to make a mistake—we see that we need $\sqrt{n}\delta$ to be large, or $n \approx \frac{1}{\delta^2}$.

A second way to see this result is to recall Hoeffding's bound (or the Chernoff bound), which says that if a sequence of independent random variables $X_i \in [0, 1]$, then

$$\mathbb{P}\left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \mathbb{E}[X_i]) \geq t\right) \leq \exp\left(-2nt^2\right) \quad \text{for all } t \geq 0.$$

(And an identical inequality holds for the event $\leq -t$.) Focusing on the first coin, we see that if $X_i = 1$ if the ith flip is heads, $X_i = 0$ if the ith flip is tails, then $\mathbb{E}[X_i] = (1+\delta)/2$, and

$$P_1(\text{wrong}) = P_1\left(\sum_{i=1}^{n} X_i \leq \frac{n}{2}\right) = P_1\left(\frac{1}{n}\sum_{i=1}^{n} X_i \leq \frac{1}{2}\right)$$
$$= P_1\left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \mathbb{E}[X_i]) \leq -\frac{\delta}{2}\right) \leq \exp\left(-\frac{n\delta^2}{2}\right),$$

by Hoeffding's bound. To guarantee that this bound is less than 1/4, we require

$$n \geq \frac{2\log 4}{\delta^2}.$$

This is much worse than $1/\delta$. □

Question 3: Can you design a procedure that does better than the one you identified in question 1? Can you design a procedure that does better than the one you identified in question 2? Can you give a lower bound on the number of flips needed to identify the coin in problems 1 and 2?

Answer: The procedures given above are both optimal, as they are likelihood ratio tests for the simple problem of deciding whether we have coin 1 or coin 2, so they are not improvable. (This is the Neyman-Pearson lemma.)

The bounds are also—in terms of δ—sharp. For the first question, this is an immediate consquence of the Neyman-Pearson lemma. For the second question, the normal approximation shows that asymptotically (as $n \to \infty$, or as $\delta \to 0$), the likelihood ratio test looks normal, so that we require $n \approx 1/\delta^2$ coin flips. It is possible to make this more rigorous.

I did not expect anyone to be able to answer for the second part, and you can look up a solution on your own. But for some intuition, note that there is a single bit of information in our system: whether we have coin 1 or coin 2. Each flip of the coin gives a number of bits roughly equal to $D_{\rm kl}\left(P_1 \| P_2\right)$, and n flips gives $D_{\rm kl}\left(P_1^n \| P_2^n\right) = n D_{\rm kl}\left(P_1 \| P_2\right)$ In this case, of course,

$$D_{\rm kl}\left(P_1 \| P_2\right) = \frac{1+\delta}{2} \log \frac{1+\delta}{1-\delta} + \frac{1-\delta}{2} \log \frac{1-\delta}{1+\delta} = \delta \log \frac{1+\delta}{1-\delta},$$

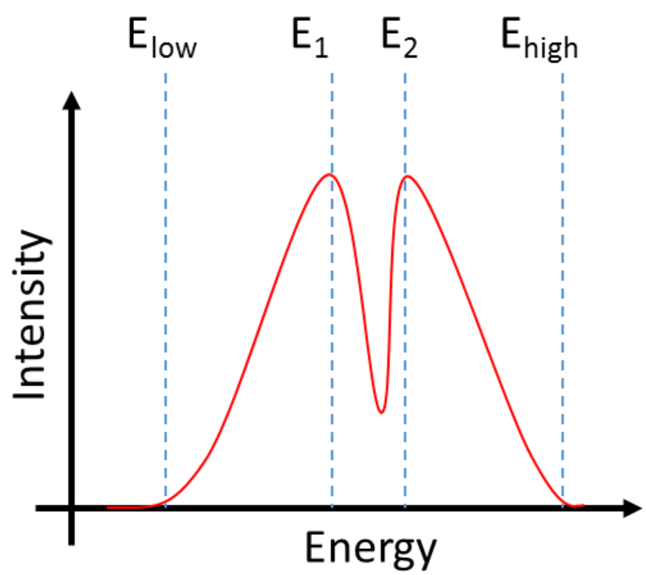
and for $\delta \in [0, \frac{1}{2}]$, we have $D_{\rm kl}\left(P_1 \| P_2\right) \le 3\delta^2$. Thus we observe at most $3n\delta^2$ bits of information after n flips, so that $n \gtrsim 1/\delta^2$ flips are necessary. $\square$

Quals 2015, Electromagnetism, Jonathan Fan

Question 1: What is a Michelson Interferometer? Draw it out. What response do you get when the source is monochromatic?

Question 2: Thermal sources, such as a light bulb, are "incoherent," as oppose to a laser, which is "coherent." What is the difference between the two? And can the spectrum of a thermal source be resolved in the interferometer? Why or why not?

Question 3: Suppose the source has a spectrum with the following form:



The movable arm of the interferometer can move in increments of ΔL over an entire length L. Discuss how ΔL and L should be specified in order to resolve the spectrum above.

Ph.D. Quals Question
January 26-30, 2015
A. C. Fraser-Smith
Department of Electrical Engineering
Stanford University

The "Shake" Flashlight

The students enter the examiner's office find the device pictured below sitting on the table in front of them. They are asked what they think it is and what is does. How does it work? They are reminded that the subject of the quals exam is electromagnetism.

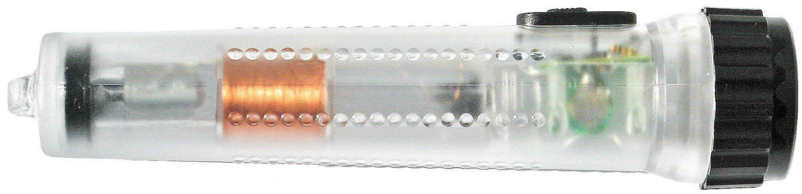


Figure 1. Picture of the "Shake" Flashlight used in this quals exam. It was chosen because it was transparent and its internal parts could be seen reasonably clearly.

On first inspection the item is clearly a flashlight, and when (most) students press the switch they find an LED light comes on – with the beam emerging from the right side of the flashlight as pictured above. Also, most students find that the silver-colored cylindrical item toward the rear of the flashlight's tube (in the picture) has a disconcerting habit of sliding backwards and forwards through the hollow orange-colored unit, which is quite obviously a coil of fine copper wire. There are black rubber bumpers at both ends of the tube containing the sliding silver cylinder and the coil, which suggests the the backwards and forwards motion of the cylinder is a design feature. There are no obvious batteries and when this lack of batteries is noticed, or if queried, the examiner tells them that the flashlight is advertised as needing no batteries. Needless to say, a developing electrical engineer would be expected to notice that there is no provision for batteries to be inserted!

At this stage the students are expected to deduce that the silver unit is a magnet (a small iron nail will stick to the flashlight next to the sliding unit) and that its function is to produce a time-varying magnetic field through the coil, which produces an EMF in the coil, that (1) powers a current that (2) is fed into the circuitry that can be seen at the front of the flashlight (right under the switch). A careful inspection reveals two thin wires leaving the coil and entering the circuitry.

When these conclusions are passed on to the examiner he asks what is the electrical engineering behind the induced current and this should lead to a discussion of Faraday's Law:

$$\xi = -\frac{d\Phi}{dt}$$

where we are considering the EMF ξ induced in a closed circuit threaded by a time-varying magnetic flux Φ. It was expected that this discussion would include a brief explanation of the negative sign (the examiner asks "why is it not a positive sign?"), a mention of Lenz's Law, and how conservation of energy is involved. In addition, because the coil in the flashlight pictured above obviously contains a large number of turns, it was expected that mention would also be made of how the number of these turns, N, would appear in Faraday's Law:

$$\xi = -N\frac{d\Phi}{dt}$$

where the magnetic flux Φ is the flux passing through just one of the coil's turns. Obviously increasing N is a good way to increase the EMF being generated in the coil and thus increasing the efficiency of the generation process.

Some students started the above discussion with the Maxwell equation derived from Faraday's Law : $\nabla \times \mathbf{E} = -\partial B/\partial t$. However, although there is nothing wrong with the equation, it is not particularly useful in discussing how the flashlight works.

The question now arises: The sliding magnet does not generate a steady EMF as it slides back and forth. Fairly obviously it must be an oscillating EMF producing an oscillating current into the circuit part of the flashlight. How do we account for the steady flashlight beam produced?

At this stage most students look really hard at the small electronics section of the flashlight. Apart from the switch there is a small resistor in series with the LED, four small black items into which the wires from the coil disappear, and what looks like a moderately large capacitor. That's all. The correct response here, which a surprisingly large number of the students gave, was to identify the four black items as diodes and to identify their role as components of a full wave rectifier. After some discussion the electrical diagram for the flashlight was derived as shown here:

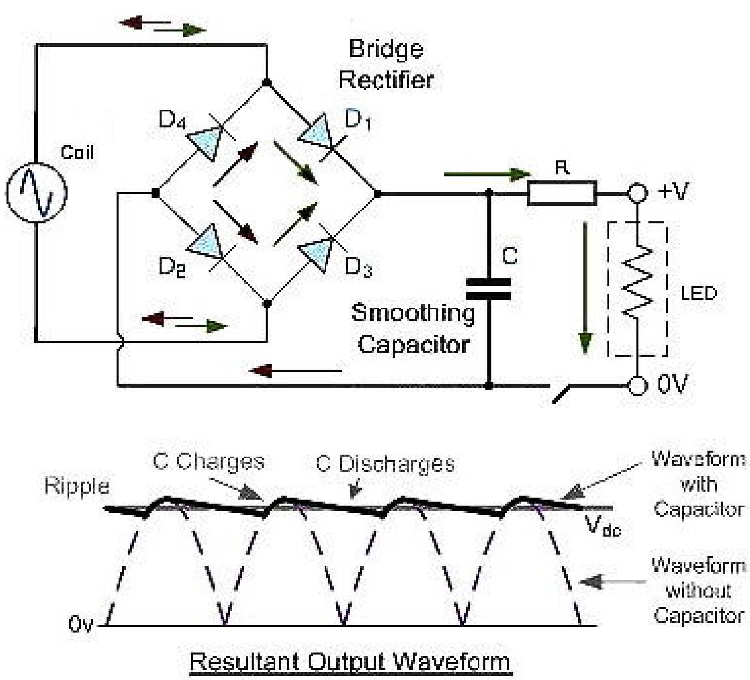


Figure 2. Circuit diagram for the "Shake" Flashlight used in this quals exam (modified from an internet version). The four diodes are denoted by D_1, D_2, D_3, and D_4. The switch is shown open here in the circuit but is closed when the flashlight is being operated.

Note 1: In the flashlight used in the quals, what appeared to be a capacitor was in fact a small nickel metal hydride battery (located in exactly the same place in the circuit as the capacitor shown above). Its function in the circuit differed little from a capacitor, with the charging current simply recharging the battery instead of charging a capacitor. However, your examiner has another “shake” flashlight with capacitor instead of a battery and he greatly prefers the version with the small battery. The capacitor version must be shaken for around 30 seconds to generate 3–4 minutes of light. The shaking required to keep its light going soon becomes VERY tedious.

Note 2: In his best selling 2013 novel *Inferno*, author Dan Brown introduces a “Faraday pointer” as an important plot element. It is nothing but a small “shake” flashlight with a transparent image on the lens that gets projected onto a viewing surface.

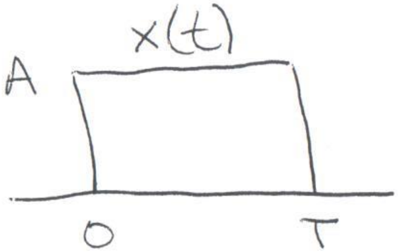
Consider a causal LTI system with frequency response $H(j\omega)$.

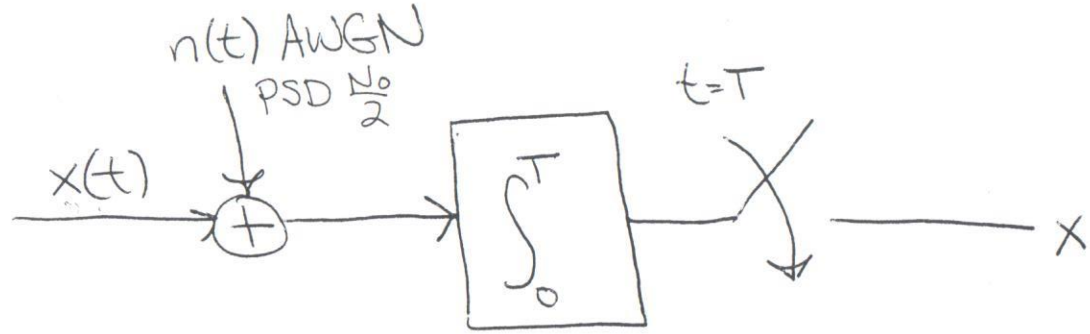
Only the real part of the frequency response is known:

$$\mathrm{Re}\left\{H(j\omega)\right\} = 1 + \cos\omega$$

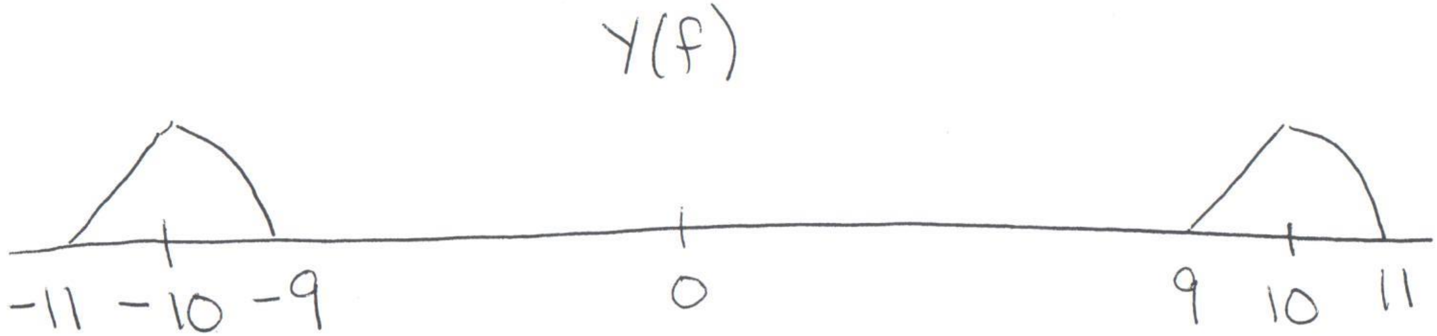
Show that the system possesses a linear-phase frequency response.

Andrea Goldsmith 2015 quals





- What is x?
- What is $p(x<0)$?
- If $A=\pm 1$, how would you determine its value from x?



- What is the minimum sampling rate at which you can recreate $y(t)$ from its samples

- How would you reconstruct $y(t)$ from these samples?

Let X and Y be independent random variables

$$X \sim U[0,3] \quad , \quad Y \sim U[-4,4]$$

- What is $p(X+Y<0)$?

- How does your answer change if $p(x,y) \neq p(x)p(y)$

2015 PhD Quals Questions
J. S. Harris

1. Can you first tell me what a solar cell does and describe how it works?

2. Can you sketch the Current vs Voltage (I-V) characteristic for a solar cell under illumination?

3. On your sketch, what are the different parameters by which we characterize solar cells and what does each represent?

4. If I say that is the I-V characteristic for a Si solar cell with a 1.1eV bandgap, what would the characteristic look like for a GaAs solar cell with a 1.5eV bandgap?

5. What do you think is a typical range of solar cell efficiency? There is a now famous paper by Shockley and Queisser defining a limit efficiency for a solar cell. Why is there a "limit efficiency" and why is it so low? What happens to the energy that isn't converted? Upon what parameter of the solar cell does the limit depend?

6. What strategy might you employ to overcome this limit and significantly increase solar cell efficiency?

7. Can you draw the energy band diagram for your solar cell at thermal equilibrium and then show me how you would modify that diagram to show what it looks like to describe the non-equilibrium situation when the cell is illuminated and first, in the short circuit current condition and explain? And second, what does the band diagram look like in the open circuit voltage condition and explain?

8. One of the proposed means to increase cell efficiency has been to create an intermediate band midgap level with quantum dots in a wider bandgap material such that we could use 2 IR photons with half of the bandgap that are otherwise not absorbed so do not contribute to the power generation to excite an electron first to the midgap level and then to the conduction band to produce an electron-hole pair in the wide bandgap material a higher V_{oc}. From 2 photons that are otherwise wasted. This looks good on paper, but has never worked. Can you guess what is the fundamental problem with this approach? (Hint—the lowest threshold, most efficient LEDs and semiconductor lasers are all based upon quantum dots)

2015 Quals Questions – Mark Horowitz

Here is a keyboard with 50+ key on it. Assume some mechanical designer created the physical keys giving you 50+ switches. How do you wire up the switches to a chip to generate the information that needs to be sent to a computer to tell what key was pressed?

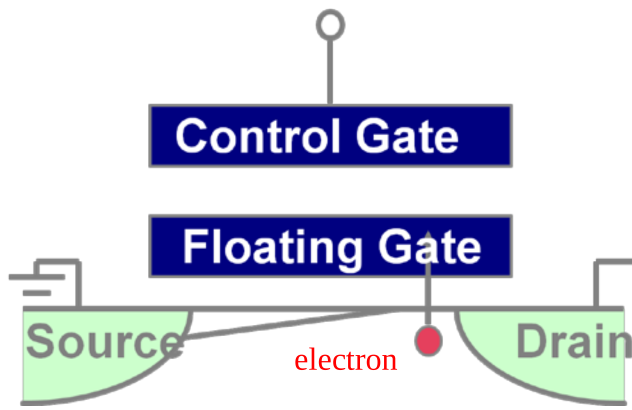
2015 Qualifying Exam Questions

Prof. H.-S. Philip Wong

You got some extra money from working as a course grader. A new phone is on the market with 128 GB of storage. You got yourself one of these new phones and started loading it up with songs and videos. Say you transferred all the data from your old 16 GB phone (which was practically loaded to full capacity) and bought some new songs to fill the storage up to 64BG. And you did this all over wi-fi.

Assume you can measure the weight of the phone to infinitesimally small values. If you measure the weight of the phone before and after you load in the data and the new songs.
1. Would there be any difference in the weight? Please explain your reasoning.

2. Now down to the chip level. If you can somehow weigh the data storage chip separately from the phone, would you observe any difference in the weight of the data storage chip when the phone was brand new vs when the phone had 64GB data loaded on it?



3. Now, on to the device. Data in the phone are stored in Flash memories. Flash memory has a floating gate (consider it a metal) and a control gate (consider it like the gate of a MOSFET) with oxides in between them. Draw the Id vs Vgs curve of the transistor (show above) for the case before and after electrons are stored in the floating gate.
4. If the threshold voltage difference between the case of having electrons and having no electron in the floating gate is 1 V, what is the total amount of charge stored in the floating gate?
5. I have not given you all the information you need to calculate this total amount of charge. You need to ask me for the necessary device information. You need to be judicious and not just ask for everything.
6. Show how you arrive at your answer. Accurate numerical answer is not required. Just the method to get to the solution.

Prof. Ramesh Johari

Name:

ID:

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1. Roger's father challenges him to a peculiar series of tennis games against him and the tennis teacher. The rules are: Roger can choose to play a three game series, either in the order teacher-father-teacher, or in the order father-teacher-father. To win the series, Roger must win two *consecutive* games. The teacher is a better player than the father.

   Which ordering should Roger select?

2. M males and F females are seated at random in a single row of a theater with $M + F$ seats.

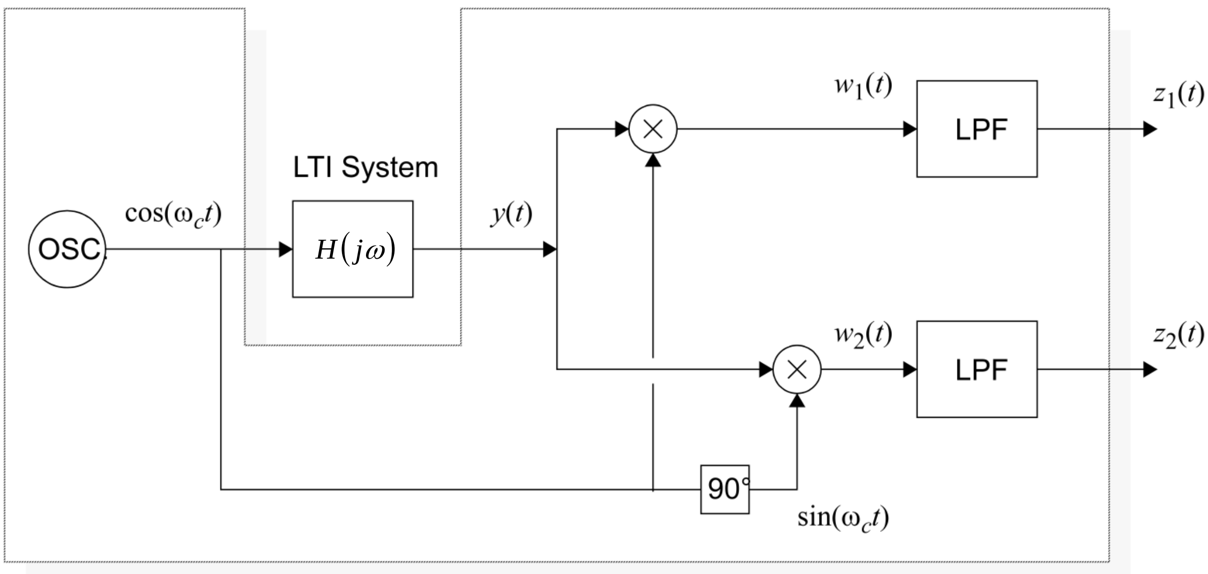
   What is the expected number of adjacent pairs that consist of one male and one female?

3. Ballots are collected after an election between two candidates A and B, and placed into a ballot box. Ballots are then removed one at a time (randomly) from the box and counted. Suppose that A received a votes and B received b votes, with $a > b$.

   What is the chance that, at some point after the counting begins, A and B are tied in the vote tally?

Stanford University, Department of Electrical Engineering
Qualifying Examination, Systems Area, Winter 2014-15
Professor Joseph M. Kahn

The instrument shown below (within the dashed box) is said to be useful for measuring the frequency response of a linear time-invariant (LTI) system, but someone misplaced the instruction manual. Evidently, the user connects an LTI system as shown. The oscillator frequency ω_c is swept over a frequency range of interest, and the two outputs $z_1(t)$ and $z_2(t)$ yield information about the frequency response $H(j\omega)$. Explain how the instrument works by deriving expressions for $y(t)$, $w_1(t)$ and $w_2(t)$ and $z_1(t)$ and $z_2(t)$. Assume the LTI system has a real impulse response $h(t)$. Also assume the lowpass filters (LPFs) are ideal and have cutoff frequencies much less than ω_c.



Solution

Since the LTI system's impulse response $h(t)$ is real, the frequency response $H(j\omega)$ has conjugate symmetry, $H(-j\omega) = H^*(j\omega)$. Equivalently, $|H(-j\omega_c)| = |H(j\omega_c)|$ and $\angle H(-j\omega_c) = -\angle H(j\omega_c)$.

We write the system input signal as

$$\cos\omega_c t = \frac{1}{2}e^{j\omega_c t} + \frac{1}{2}e^{-j\omega_c t}.$$

The system output signal is

$$\begin{aligned} y(t) &= \frac{1}{2}H(j\omega_c)e^{j\omega_c t} + \frac{1}{2}H(-j\omega_c)e^{-j\omega_c t} \\ &= \frac{1}{2}|H(j\omega_c)|\left[e^{j\angle H(j\omega_c)}e^{j\omega_c t} + e^{-j\angle H(j\omega_c)}e^{-j\omega_c t}\right]. \\ &= |H(j\omega_c)|\cos(\omega_c t + \angle H(j\omega_c)) \end{aligned}$$

Recall the identities

$$\cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)].$$

Using these identities, the multiplier outputs are

$$\begin{aligned} w_1(t) &= |H(j\omega_c)|\cos\omega_c t\cos(\omega_c t + \angle H(j\omega_c)) \\ &= \frac{1}{2}|H(j\omega_c)|[\cos(2\omega_c t + \angle H(j\omega_c)) + \cos\angle H(j\omega_c)] \end{aligned}$$

$$\begin{aligned} w_2(t) &= |H(j\omega_c)|\sin\omega_c t\cos(\omega_c t + \angle H(j\omega_c)) \\ &= \frac{1}{2}|H(j\omega_c)|[\sin(2\omega_c t + \angle H(j\omega_c)) - \sin\angle H(j\omega_c)] \end{aligned}$$

Each of the multiplier outputs contains a term at $\omega = 2\omega_c$ and a term at $\omega = 0$. The lowpass filters pass only the terms at $\omega = 0$, yielding

$$z_1(t) = \frac{1}{2}|H(j\omega_c)|\cos\angle H(j\omega_c)$$

$$z_2(t) = -\frac{1}{2}\left|H(j\omega_c)\right| \sin \angle H(j\omega_c).$$

Using $z_1(t)$ and $z_2(t)$, the magnitude and phase of $H(j\omega_c)$ can be computed as

$$\left|H(j\omega_c)\right|^2 = 4\left[z_1^2(t) + z_2^2(t)\right]$$

$$\angle H(j\omega_c) = -\tan^{-1}\left(\frac{z_2(t)}{z_1(t)}\right)$$

2015 Quals Questions – Greg Kovacs

Students were given (with important equations) a single-transistor amplifier and asked to derive its gain via simple circuit analysis. If they finished, they were asked a more nuanced question about a slightly modified version of the circuit. The question was undergrad-level only and hints were given if needed.

I asked one question: please tell me about your research.

In the case of a student who hadn't started research yet but who had a background in software engineering, I asked:

Show what a call stack looks like and how a program modifies it as it enters and leaves a function.

Phil Levis

EE Quals 2015

Computer architecture and digital design

Nick McKeown

Question 1

Draw a 1-bit full adder from 2-input logic gates.

Question 2

Show how you can create a 32bit adder from your 1bit full adder building block.

Question 3

Draw/describe other ways to create a 32-bit adder from a 1bit full adder.

Describe the gate-count/delay tradeoff for each design.

EE Ph.D. Qualifying Exam, January 2015 Question

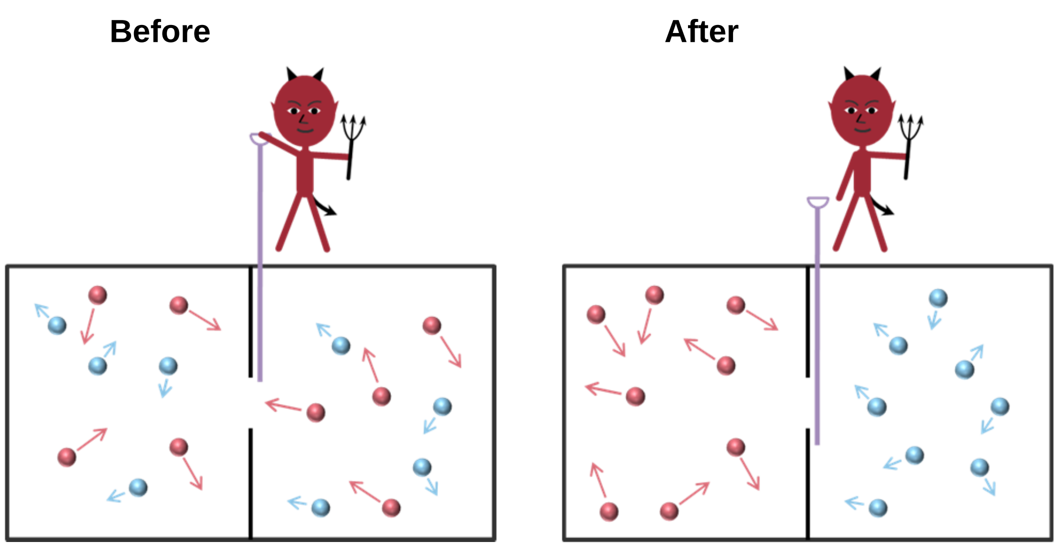
David Miller

Notes: There may not be single "correct" answer to this question. The goal of this question is to see how you think about it. The answers are mostly qualitative, and little or no algebra should be required for them. If you finish the question on this sheet, subsequent questions will be asked.

Maxwell's demon

We imagine we have a box containing gas atoms in some thermal equilibrium distribution at a particular temperature. There is a divider or shutter that we can open and close in the middle of the box. Maxwell's demon is a small being or machine that opens or closes the shutter depending on what he observes.

In one version, the demon opens and closes the shutter appropriately so that he lets all the rapidly moving (and hence "hotter") atoms through into the left side of the box and all the slower moving (and hence "colder") atoms through into the right side of the box. As a result, he separates the gas into hotter and colder portions. (For simplicity in the pictures below, we show only "fast" atoms and "slow" atoms, but there will be a continuous range from slow to fast.)



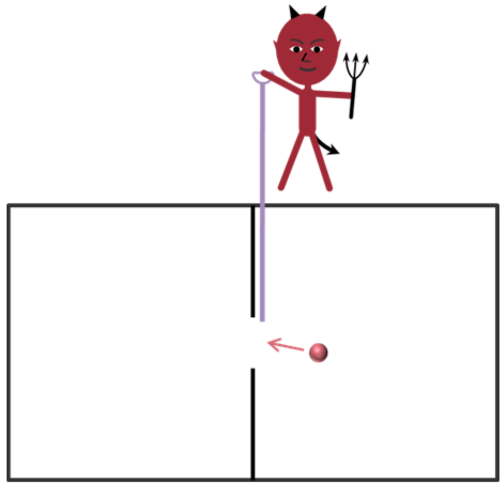
He can then run some sort of engine that extracts work from the gas, for example, by having the "hot" atoms push the divider to the right to lift some weight because there is now greater gas pressure on the divider from the left.

He has therefore apparently taken thermal energy at only one temperature and used it to perform work.

Can the demon actually do this? Does this violate any physical laws? Can I make a perpetual motion machine using this idea?

Supplementary question

Suppose we take a particularly simple and extreme version of the system where we imagine there is only one atom in the box overall.



(i) what is the reduction in entropy if we introduce the shutter when the atom is on the left of the shutter, thereby changing the situation from equal probabilities of the atom being on the left (L) and right (R) of the box to one in which, with probability one, it is on the left (L)?

(ii) what might this imply about any increase in entropy in the demon and, considering the demon as a simple machine, how might we view the mechanism of any increase of the demon's entropy?

Supplementary question 2

Is there a minimum energy cost to erase a one bit memory at finite temperature?

Answer

In using this as a question, I am much more interested to see how the examinee thinks. Whether or not the examinee got to supplementary questions did not itself directly influence the score, and depending on the kinds of responses the student gave, I might also ask them other things about the problem, such as how to make a "cyclic" engine out of this approach.

There is indeed a set of problems in the demon managing to separate out the atoms to the two sides. It is, of course, possible for the demon to do this separation, and it is possible to imagine him doing this with vanishing small expenditure of actual energy in moving the shutter itself. That movement of the shutter can presumably be performed with arbitrarily small energy by using a sufficiently light shutter and moving it sufficiently slowly. The issue is whether the demon can do this separation without otherwise increasing his own entropy or that of some other system. If there is no sufficient such increase in entropy elsewhere, then the demon's actions will violate the second law of thermodynamics by reducing entropy (by separating the atoms) with no apparent work performed on the system. And then the demon will manage to get the system to do some work for him, all while starting off with only a gas at one temperature, which would also violate the second law.

A more subtle point is that this system can be simply extended to continue to extract work cyclically. If we let the divider be pushed to the right after separating atoms, we can then remove it from its right-most point and reinsert it in the middle, or open the shutter and push it back over, and start all over again, repeating the work extraction. Such a cyclic engine causes us particular problems with the second law of thermodynamics, giving us a "perpetual motion machine of the second kind"[1]. (In such a case, we would likely connect the system to a heat reservoir at our one starting temperature to heat the gas up again in between cycles, but this still leaves us with a cyclic machine that can continually perform work starting with only one temperature.)

Part of the difficulty here is in understanding the demon as a physical system. Certainly in the original proposals of this thought experiment by James Clerk Maxwell (yes, that Maxwell!) in his book *Theory of Heat* in 1871, there was no clear understanding of just how we should view the demon[2] as a physical system other than thinking of him as having "free will", whatever that is, and the abilities to perform his task.

As far as the "answer" I am looking for in this part of the question, it is the understanding that there is at least potentially a substantial issue here with violating the second law of thermodynamics through the apparent reduction of entropy here, and/or the ability

[1] A perpetual motion machine of the second kind would be one that did not necessarily violate overall conservation of energy (which would be a "perpetual motion machine of the first kind"), but that violated the second law. An example of such a machine would be a ship that could power its motors by extracting energy for work from the heat energy of the ocean, ejecting sufficient ice cubes out the back so as not to violate conservation of energy.

[2] Incidentally, Maxwell himself did not introduce the term "demon"; that term was apparently introduced by William Thomson (also known as Lord Kelvin) in 1874. And "demon" here is likely not meant to indicate any malevolence, just referring to "an intelligent being endowed with free will, and fine enough tactile and perceptive organization to give him the faculty of observing and influencing individual molecules of matter" (from W. Thomson, "The kinetic theory or the dissipation of energy," Nature **IX**, 441-444 (1874)). It is, of course, more fun to draw the demon as a little devil-like character!

apparently to perform work starting only from the thermal energy in a gas at a uniform temperature in a system that can be cycled. I would also look for examinees to be able to come up with various possible resolutions, and, ideally, for them to criticize these resolutions themselves. Reasonable suggestions include (i) that there must be some energy associated with the actual raising and lowering of the shutter, and (ii) the process of measurement must itself involve some energy. Actually, depending on how you interpret these answers, they could be viewed as correct, though I would thank the examinees for either of these suggestions (and give them credit), but ask them to come up with something else. See the answers to the supplementary questions for another possible resolution of this problem.

Incidentally, the many issues and arguments and the history of Maxwell's Demon are gathered in the book "Maxwell's Demon 2 – Entropy, Classical and Quantum Information, Computing," H. S. Leff and A. F. Rex (eds.) (IoP Publishing, Bristol and Philadelphia, 2003).

Answer to supplementary question

We remember that the entropy of some system can be described statistically by

$$S = k \log g$$

Here g is the number of different microstates corresponding to the one macrostate we are considering. In physics the logarithm is taken to the base e and k is Boltzmann's constant. In information theory we more typically use logs to the base 2 with no prefactor constant in considering the related quantity. In physics, all the different microstates are considered to be equally likely.

For a situation where there is one atom in the box, we could consider the contribution to the entropy from the atom being either on the left, L, or the right, R, of the middle of the box, which is also the position of the divider or shutter. Initially, then, the atom can be on either side of the box, in which case that uncertainty of those two "microstates" of being on the left or the right gives a contribution to the entropy of

$$S_{LR} = k \log 2$$

If we wait until we observe the atom to be in the left side of the box and then insert the shutter to keep it there, there is now only one microstate of the two left, and so no entropy contribution, i.e., the entropy contribution corresponding to this state is

$$S_L = k \log 1 = 0$$

So the action of inserting the shutter when we know for sure the atom is in the left half of the box has decreased the entropy of the system by an amount of magnitude $k \log 2$. The argument then is that, because the second law of thermodynamics tells us that we cannot decrease entropy overall in a closed system without doing work, and we seem not to have done any work here, something associated with the demon must have increased its entropy by $k \log 2$.

One view of this is to assert that there is a one-bit memory associated with the demon, a memory that is where the result of the measurement is stored. The act of measuring the atom to be on the left and writing that into the memory sets the memory to its "1" state, representing the "L" condition. This memory is to be set to "1" in such a case regardless of its previous state. Hence, the entropy of this memory has been reduced by $k\log 2$ on the average, which, according to the second law of thermodynamics much have been accomplished by an increase of entropy of at least $k\log 2$ in the environment surrounding the memory (i.e., in the demon or in his environment). Thus there is no net decrease in entropy of the entire system if we presume this process in the demon. And the demon has to remember to eat his breakfast each day so as to get the necessary fuel to allow him to perform these actions!

Answer to supplementary question 2

In an environment of temperature *T*, the entropy change ΔS associated with a flow of heat *Q* into a system is

$$\Delta S = Q/T$$

So the energy involved in erasing a one bit memory is at least

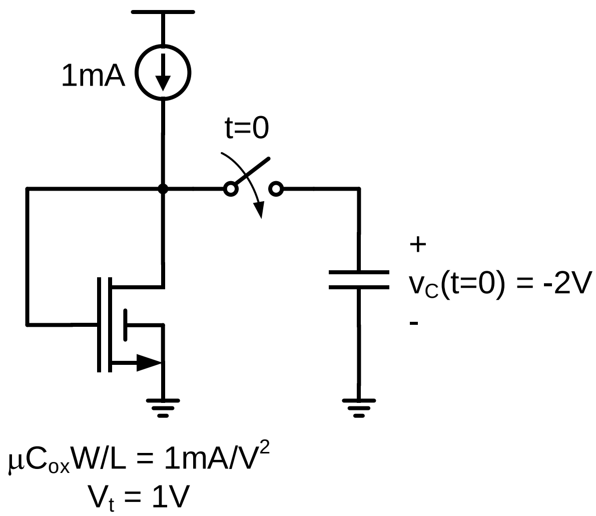
$$E = T\Delta S = kT\log 2$$

This idea that the erasure of a memory cell requires this increase in entropy in the environment was introduced by Landauer in 1961[3]. It is this resolution and proposal that finally offers a way to link physical entropy and information entropy, since all information must be written down somewhere. Though this idea has continued to be debated, this idea of Landauer's principle is now viewed as being quite a serious idea. For a modern attempt at an experiment to test this idea, see J. V. Koski, V. F. Maisi, J. P. Pekola, and D. V. Averin, "Experimental realization of a Szilard engine with a single electron," PNAS **111**, 13786–13789 (2014) doi: 10.1073/pnas.140696611 .

[3] R. Landauer, "Irreversibility and heat generation in the computing process," IBM J. Research and Development 5, 183 – 191 (1961)

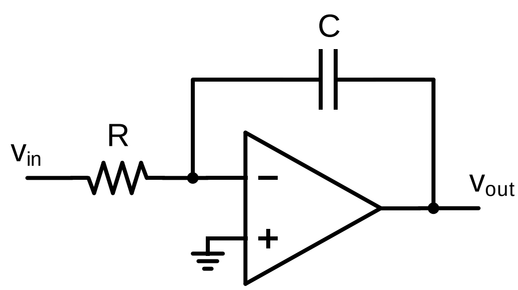
Name:	Stanford EE Quals 2015 Murmann

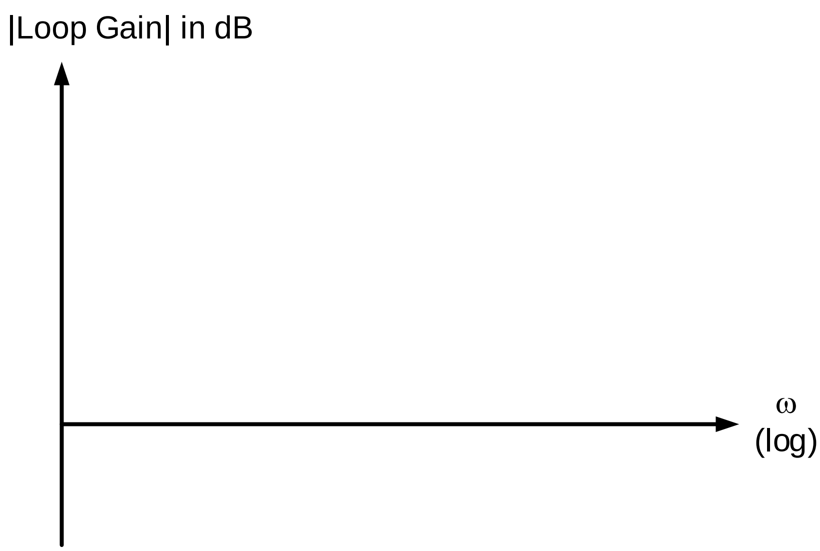
Sketch $v_C(t)$ versus time. Ignore the capacitance of the MOSFET and assume that it obeys the ideal square law model.



The op-amp in the circuit below has a DC gain of 80dB and two left half plane poles at 1 rad/s.

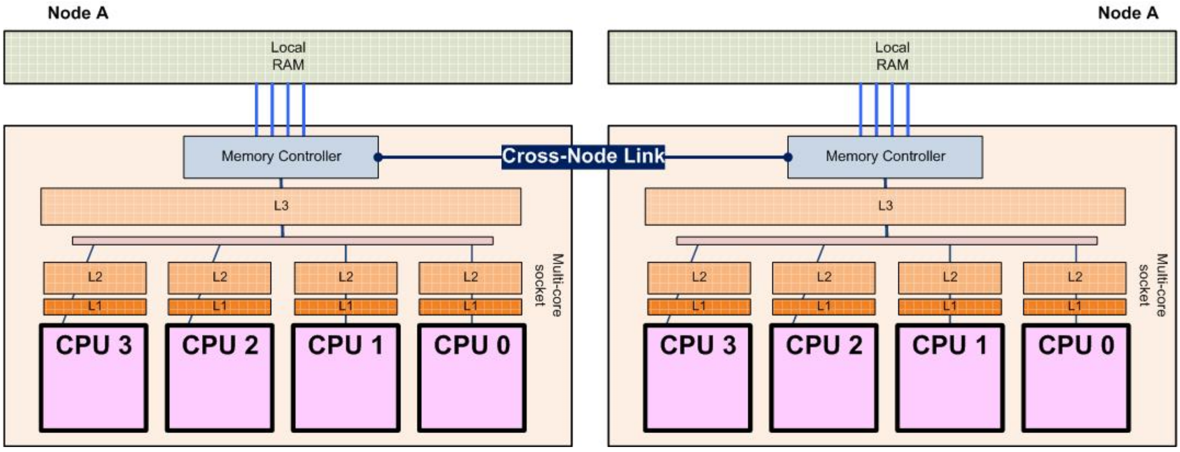
a) Sketch the magnitude of the circuit's loop gain.
b) Is this circuit stable? If yes, what is its phase margin assuming RC = 10ms?





Prof. Kunle Olukotun

Modern 2-socket System



Prof. Kunle Olukotun

- Tell me about key characteristics of this system
  - What are two purposes for the cross-node link?
- How will you program to reach peak performance on this system?
- How will you know you have reached peak performance?

John Pauly Quals Questions 2015

1) How does the broadcast AM signal work? Why was broadcast AM done this way?

2) Do you need both sidebands of the AM signal? If not, why would you do this?

3) How does FM work? What are the advantages and disadvantages?

4) If you have a fixed transmit power on a specified frequency, which one of these could be detected at the greatest range?

Students could do well without knowing the specifics of these systems. I would provide prompts for the aspects they were unsure of. I was mostly looking for how comfortable students were with thinking in the frequency domain, and intuition for concepts such as modulation, bandwidth, and power.

Fabian Pease Quals Questions 2015

1) Sketch the output (Id/Vd) characteristics of a typical MOSFET.
2) TO maximize power in a load resistor how should its value compare with that of the internal resistance of a power source?
3) Your MOSFET feeds a long microstrip electrode 1 micron wide on 250nm SiO2 on a Si substrate. Estimate its characteristic impedance. How well does this value match the output resistance of your MOSFET? Any suggestions on how to overcome this mismatch?

Piero Pianetta Quals Questions 2015

Consider two semiconductors with different band gaps. What happens to their respective band diagrams when you put them together? What properties are you considering in your analysis?

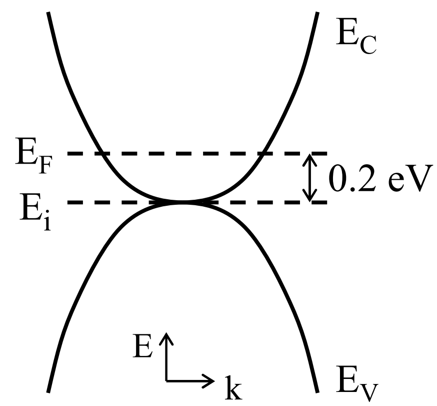
Now put a metal contact on the side with the larger band gap. What happens? Now apply a voltage, what happens?

What is the difference between the case of an undoped large band gap semiconductor and the case where the larger bandgap semiconductor is replaced by an insulator?

Prof. Eric Pop

1. Estimate how many modern transistors can fit within the period at the end of this sentence.

2. Consider a <u>semimetal</u> with the band diagram shown in the figure. The band gap is zero ($E_G = 0$) and the conduction and valence bands "touch" at a point with zero density of states. The semimetal has electron affinity $\chi = 4.5$ eV and it is n-doped, resulting in the Fermi level E_F.



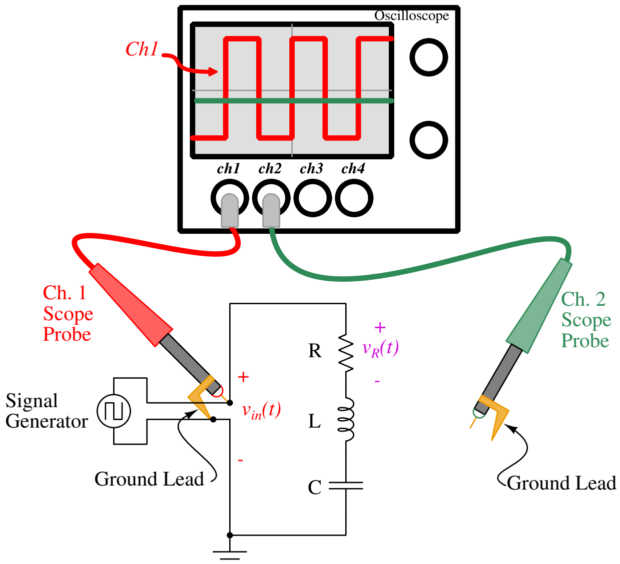
a) The semimetal is brought in contact with a metal of workfunction $\Phi_M = 5$ eV. Draw and numerically label (where possible) the energy vs. position (E vs. x) band diagram of this junction.

b) We shine infrared light of 4 μm wavelength on the junction. Please explain where absorption happens (or does not happen) and why.

Prof. Juan Rivas

Name:

Put on the safety glasses. You'll be measuring a live circuit with non differential probes. Please consider the circuit in Fig. 1 which is also implemented on the desk and connected to a signal generator. You can not use the autoset button of the oscilloscope. You can not change the setting on the signal generator, or move/disconnect the probe on channel 1 of the oscilloscope. The circuit is driven by a 10 $V_{pk,pk}$, 500 kHz with 0 dc offset square wave, which is measured by Channel 1 and displayed on the oscilloscope. At each edge of the square wave, the RLC circuit will be exited and exhibit a transient response. In this circuit $C = 120$ pF.



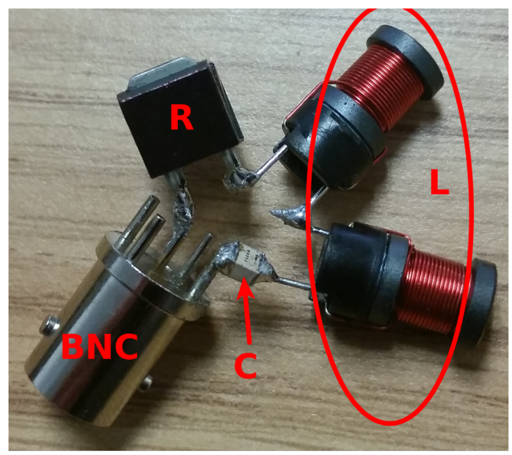


Figure 1: RLC circuit driven by a square wave

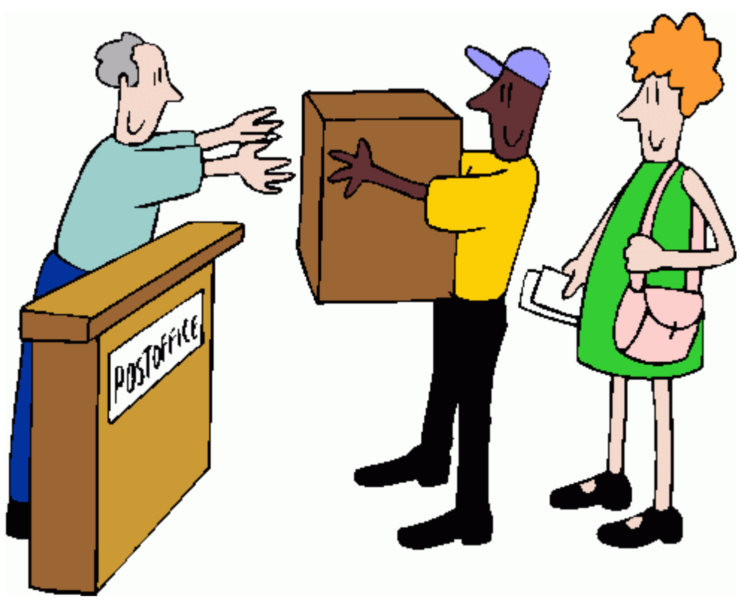
- Using the probe attached to channel 2 of the scope, display (and capture) on the screen the voltage across the resistor $v_R(t)$.
- From the measured waveform, find the values of R and L in the circuit
- What value of R would you need to choose to get a critically damped transient response?

2015 EE PhD Quals

Prof. Daniel Spielman

1. A post office has two clerks who are currently serving customers A and B. Customer C walks into the office and is told she will be served as soon as one of the clerks is free. Assuming the time it takes a clerk to serve a customer is given by an exponential random variable with mean λ, what is the probability that customer C is NOT the last to leave the office?

2. Is this a good model for a post office?



2015 EE PhD Quals

Prof. Daniel Spielman

Answers

1. The probability that customer C is NOT the last to leave = 0.5. The reasoning is as follows…

An exponential random variable has the pdf

$$\Pr\left\{X = x\right\} = f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

from which it is easily shown that mean(X)=λ, Var(X)=$\lambda^2/2$, and X is memoryless, i.e.

$$\Pr\left\{X > s + t \middle| X > t\right\} = \Pr\left\{X > s\right\}$$

Based on the memoryless property, once customer A or B leaves, customer C has equal probability of finishing after the other remaining customer.

2. Whether or not this is a good model for service at a post office is the heart of this quals question. Re-assessing the pdf is a good place to start. Perhaps considering sums of exponential variables? I was looking for a discussion of how one develops a mathematical model for a physical process. Taking data is almost always a good thing to do. What are the assumptions and are they reasonable? What are the sources of error? How is a model tested and refined?

Have you used Netflix's streaming service?
What impairments have you experienced?
What causes these impairments?
What control schemes can be used to mitigate these impairments.

Fouad Tobagi

2015 Quals Question – David Tse

The carpet on the floor of my office is a grid of 1 by 1 squares. I throw a stick of length 1 randomly onto the floor. What is the probability the stick will cross exactly two lines?

Ben Van Roy Quals Questions 2015

1. What is a band-limited signal?

2. How frequently do I need to sample such a signal if I want enough data to reconstruct it?

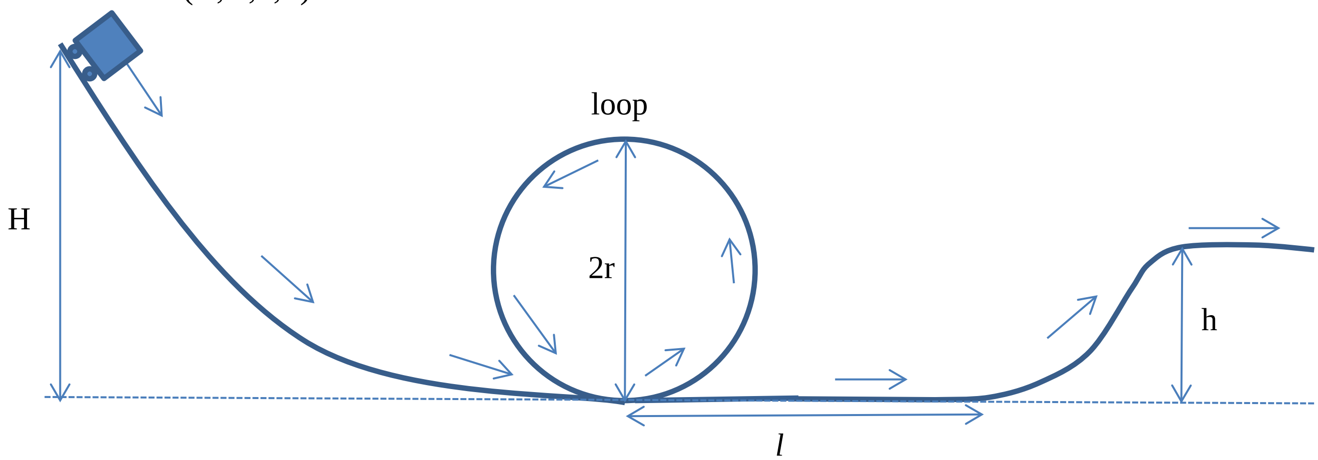
3. If I modulate the signal by multiplying it by a sine wave. How frequently do I need to sample the modulated signal if I want enough data to reconstruct it?

4. If the sample times are generated by a Poisson process with an average frequency equal to the frequency you specified, will the samples still be sufficient to reconstruct the signal?

Clearly state any assumptions you make while solving the problems. Good luck!

1. Rollercoaster design

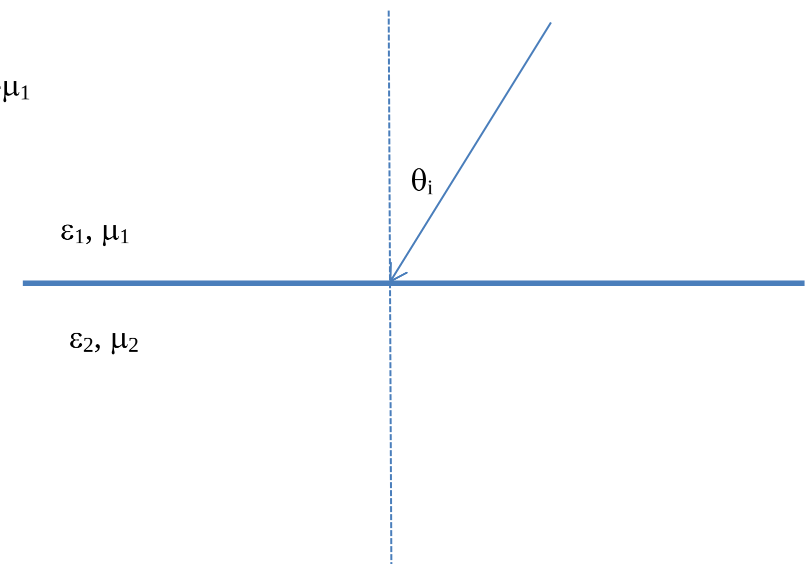
You are asked to design a rollercoaster shaped as in the figure below. How would you choose the dimensions (H, h, *l*, r) relative to each other?



2. Reflection and refraction

Plane wave is incident from a medium with permittivity and permeability ε_1, μ_1 ($\varepsilon_1>0$, $\mu_1>0$) onto a boundary with a medium described with ε_2, μ_2. The angle of incidence is θ_i, as shown in the figure. What happens with a reflected and a refracted wave in following situations?

- $\varepsilon_2>0$, $\mu_2>0$
- $\varepsilon_2<0$, $\mu_2<0$
- $\varepsilon_2=-\varepsilon_1$, $\mu_2=-\mu_1$
- $\varepsilon_2<0$, $\mu_2>0$



EE Quals 15, Engineering Physics
Shan Wang

1. A point source of light radiates 1 W radially at 1 mm underneath a very large metallic plane.

a) How would you estimate the range of lifting force on the plane due to the light?

b) Derive an expression of the force.

c) Give the numerical value of the force if you can.

quals question 2015

From where he stands, one step toward the cliff would send the drunken man over the edge. He takes random steps, either toward or away from the cliff. At any step his probability of taking a step away is p and of a step toward the cliff is $1-p$. What are his chances of escaping the cliff for:

1. $p = 1/3$

2. $p = 2/3$

2015 Quals Questions – Gordon Wetzstein

I usually started with general questions and then adapted the questions a little bit based on how far we got in the 10 minutes and what the areas of the students were.

General Questions for Everyone:

- what are your research interests, what courses are you taking, are you working with any faculty yet?

- signal processing basics:

    - explain the Nyquist-Shannon sampling theorem

    - how to downsample an image (filtering followed by sampling)

    - explain the convolution theorem (convolution in primal domain is multiplication in frequency domain)

    - what is aliasing?

Medical imaging specifics (for people working in that area):

    - explain the fourier slice theorem

    - what are some of the challenges with metal in MRI/CT

Other questions (for a few students who were working in related areas, but not all students):

    - hows does lidar work?

    - how does it compare with interferrometry?