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White paper:

Understanding the Upcoming NIST Post-Quantum Cryptographic Standards

PQShield

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The National Institute of Standards and Technology (or NIST) is a US Government agency responsible for standardizing several cryptographic primitives that are now ubiquitous: DES, AES, HMAC, SHA-{1,2,3}, (EC)DSA and much more. NIST is currently in the final phase of the process of standardizing post-quantum cryptography [NIS17b, NIS19a, NIS20b] and the winners are expected to be announced early 2022.

This document is a presentation of the 15 schemes (7 signature schemes and 8 key-establishment schemes) that are still under consideration by NIST to be the upcoming post-quantum cryptographic standards. As a follow-up to our overview of post-quantum cryptography [PQS20], it provides a more practical view of post-quantum schemes, some of which will become standards for decades to come. We present each of the 15 schemes in detail, and also provide extensive comparisons between them: bandwidth cost, computational cost, hardness assumptions and so on.

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1 The NIST Standardization Process

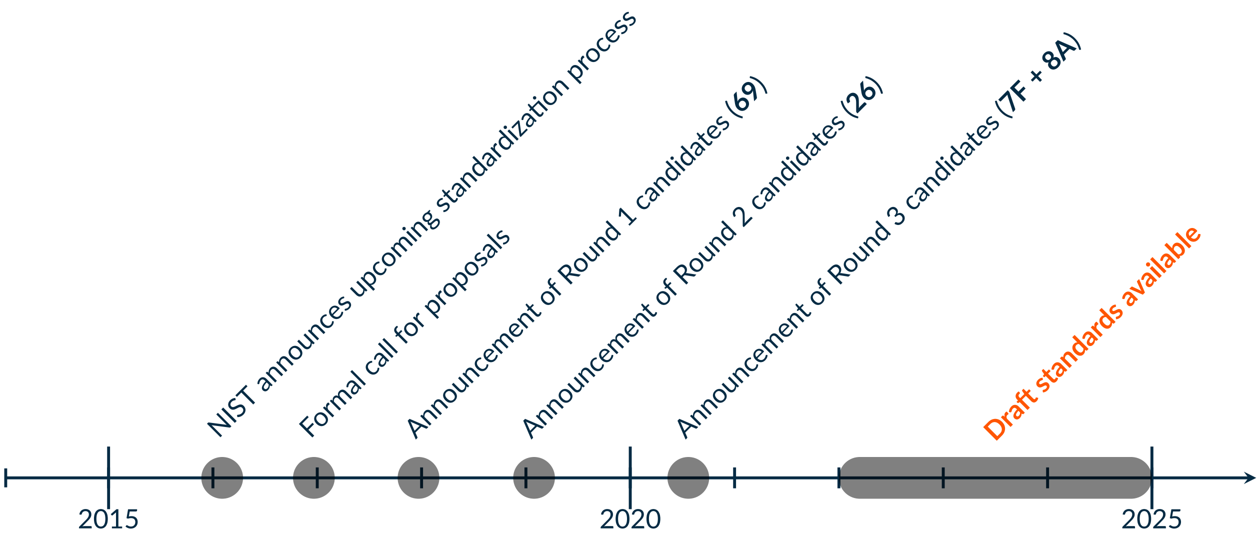
A question that may come to mind regarding standardization of post-quantum cryptography is:

Why standardize post-quantum cryptography now if quantum computers are not yet in practical use?

The reason is simple: standardizing and deploying new technology takes time. For example, the hash function SHA-2 has been standardized since 2001 to replace SHA-1; yet the latter can still be found in many places[1], despite several practical attacks against its collision resistance [SBK+17, LP19, LP20]. On the other hand, quantum computing is a fast-moving field, attracting hundreds of millions of dollars[2] in yearly funding. In this context, early standardization gives organizations more time and flexibility to carry out a smooth transition to quantum-safe cryptography.

There are a number of standardization efforts currently underway (by ETSI in Europe, CACR in China, etc.), but we focus on the one by NIST since it is by far the most documented and has attracted a significant amount of industrial and academic attention. NIST's post-quantum standardization process was announced in February 2016 [NIS16], with the goal to:

- Standardize post-quantum signature schemes;
- Standardize post-quantum key-establishment schemes.



By the initial deadline of November 2017, 82 submissions were made. Out of these, 69 were considered "complete and proper" as per NIST's submission requirements and minimal acceptance criteria and were selected as Round 1 candidates (49 for key-establishment, 20 for signatures). In January 2019, 26 schemes were selected as Round 2 candidates (17 for key-establishment, 9 for signatures). In July 2020, 15 schemes were selected as Round 3 candidates. These are separated as *finalists* and *alternates*. As per NIST [NIS20b], "*finalists will be considered for standardization at the end of the third round*", while alternates "*are still being considered for standardization, although this is unlikely to occur at the end of the third round*".

Finally, NIST intends to make draft standards available between 2022 and 2024. Whatever NIST's choice, the draft standard(s) will be among the schemes presented in this document.

[1] See for example The Github Blog: *Highlights from Git 2.29*.

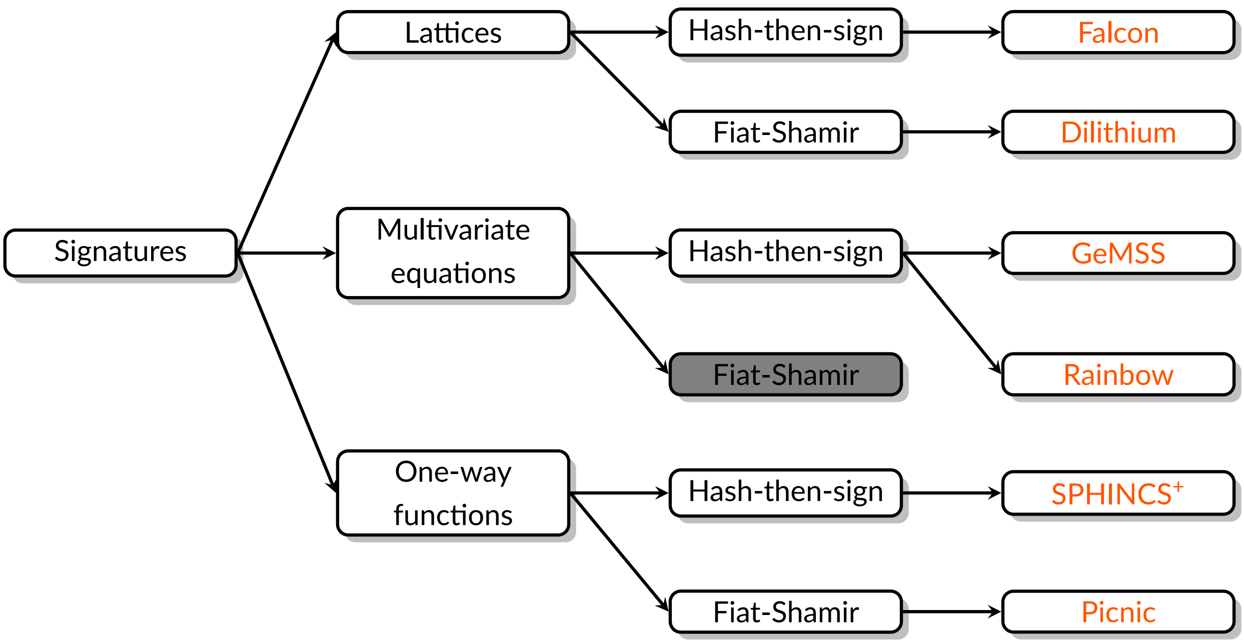
[2] Nature: *Quantum gold rush: the private funding pouring into quantum start-ups*

2 Signature Schemes

We first discuss signature schemes under consideration for standardization. In Round 1 of this process (December 2017), 20 digital signature schemes were accepted [NIS17b]. After a preliminary analysis by the cryptographic community, NIST selected 9 of these 20 schemes for Round 2 of the standardization process [NIS19a]. Out of these 9 schemes, 6 were selected for Round 3 of the standardization process [NIS20b]: 3 *finalists* and 3 *alternates*.

- Dilithium (finalist);
- Falcon (finalist);
- Rainbow (finalist);
- GeMSS (alternate);
- SPHINCS+ (alternate);
- Picnic (alternate).

These schemes are based on three families of hardness assumptions: lattices, multivariate equations, and one-way functions. Although there exist code-based or isogeny-based signature schemes, none are in this shortlist (because they were either eliminated at Round 1, or proposed after the submission deadline) so have not been included here. An overview of the signature schemes can be found in the figure below.



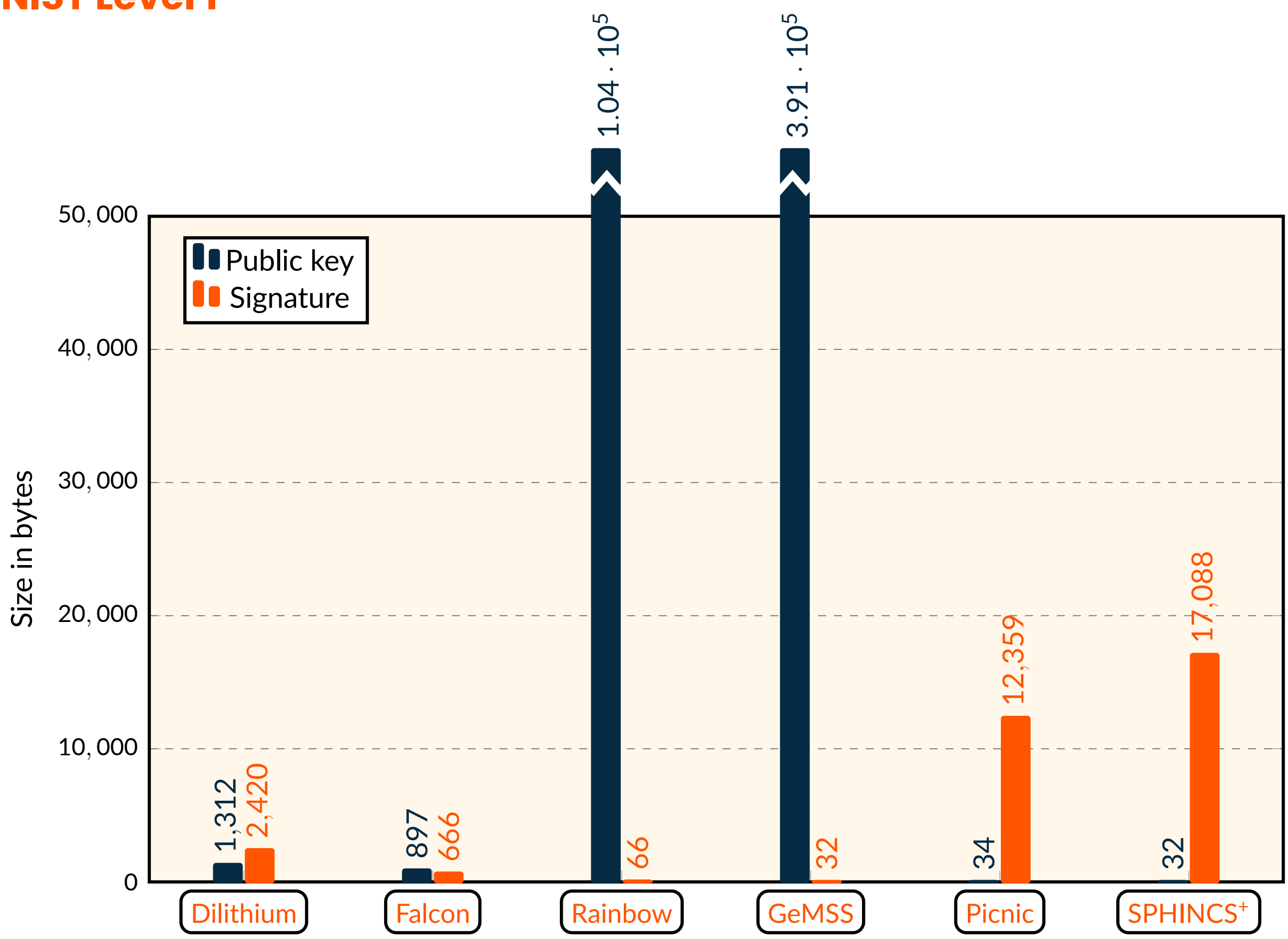
In addition, we provide the following data:

- A comparative performance study of the 6 signature schemes (pages 6 to 9);
- For each scheme, one page summarizing its main properties (pages 11 to 16).

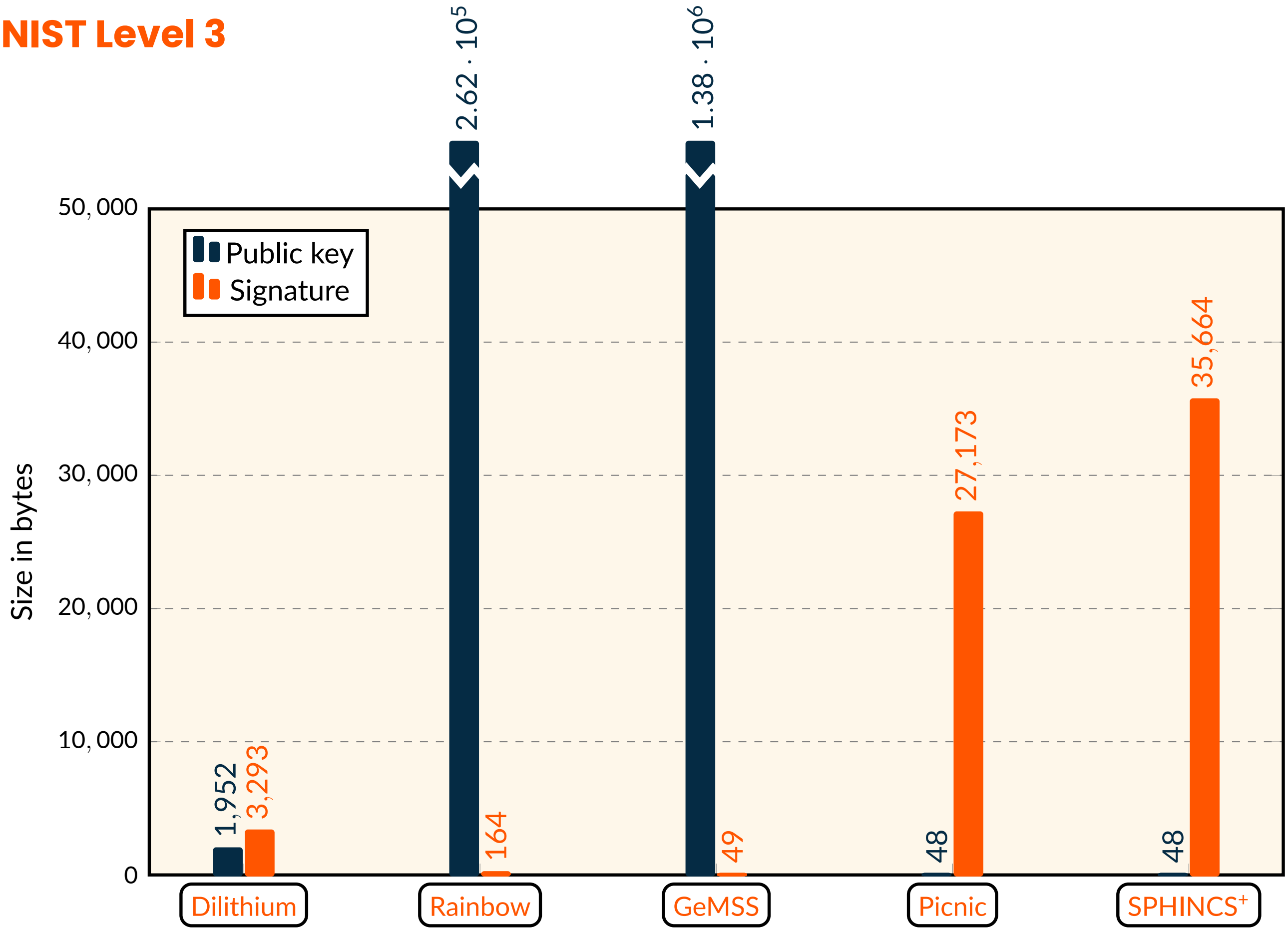
Communication Costs

We now provide a detailed comparison of the communication costs of the 6 signature schemes for three security levels: NIST Level 1, 3 and 5 (conjectured at least as secure as AES-128, AES-192 and AES-256, respectively). Interestingly, there can be huge differences between schemes: two schemes have small public keys and small signatures (lattice-based schemes Dilithium and Falcon), two provide extremely small public keys but large signatures (Picnic and SPHINCS^{+}) and two have enormous public keys but very small signatures (multivariate schemes GeMSS and Rainbow).

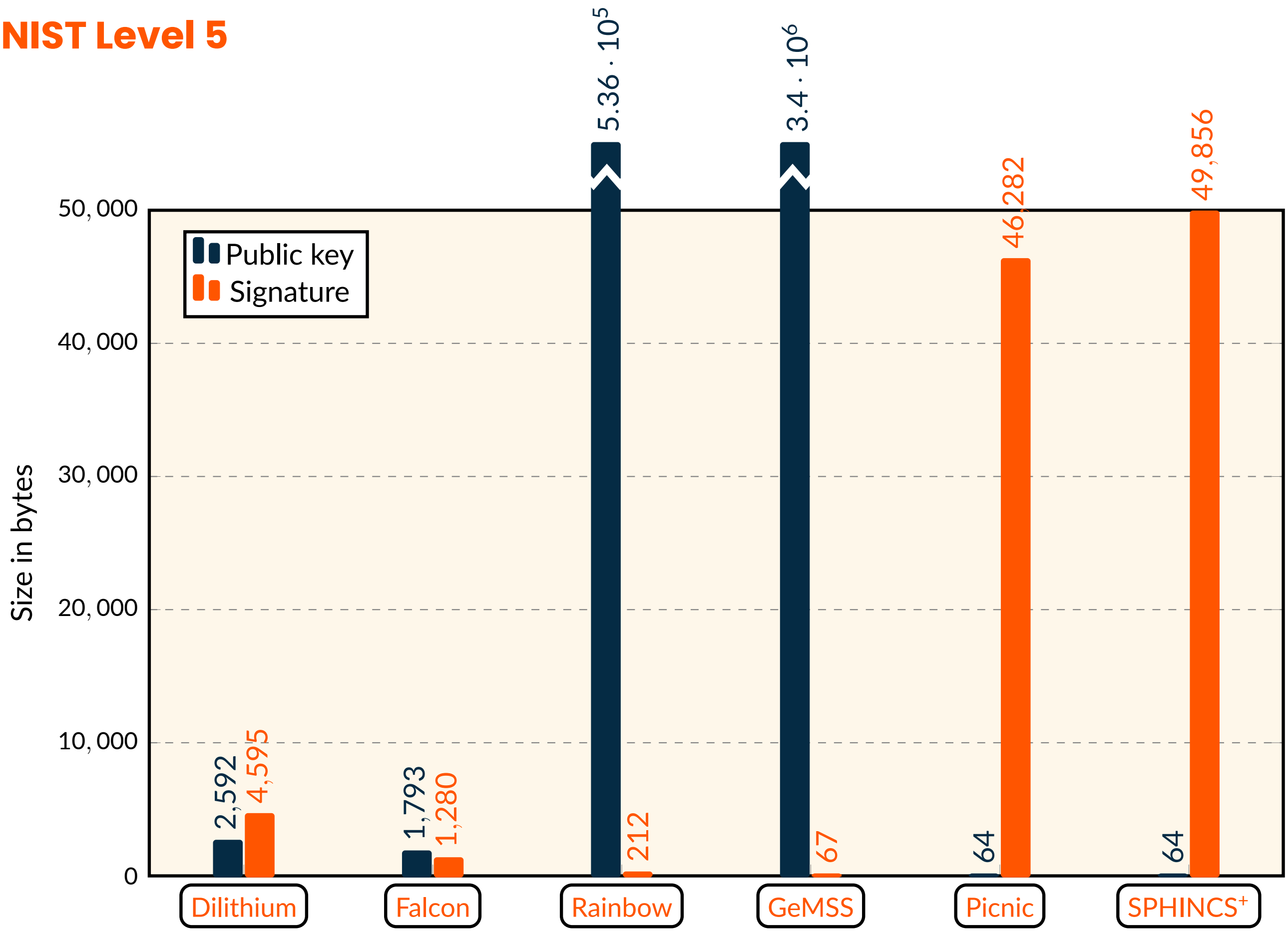
NIST Level 1



NIST Level 3



NIST Level 5

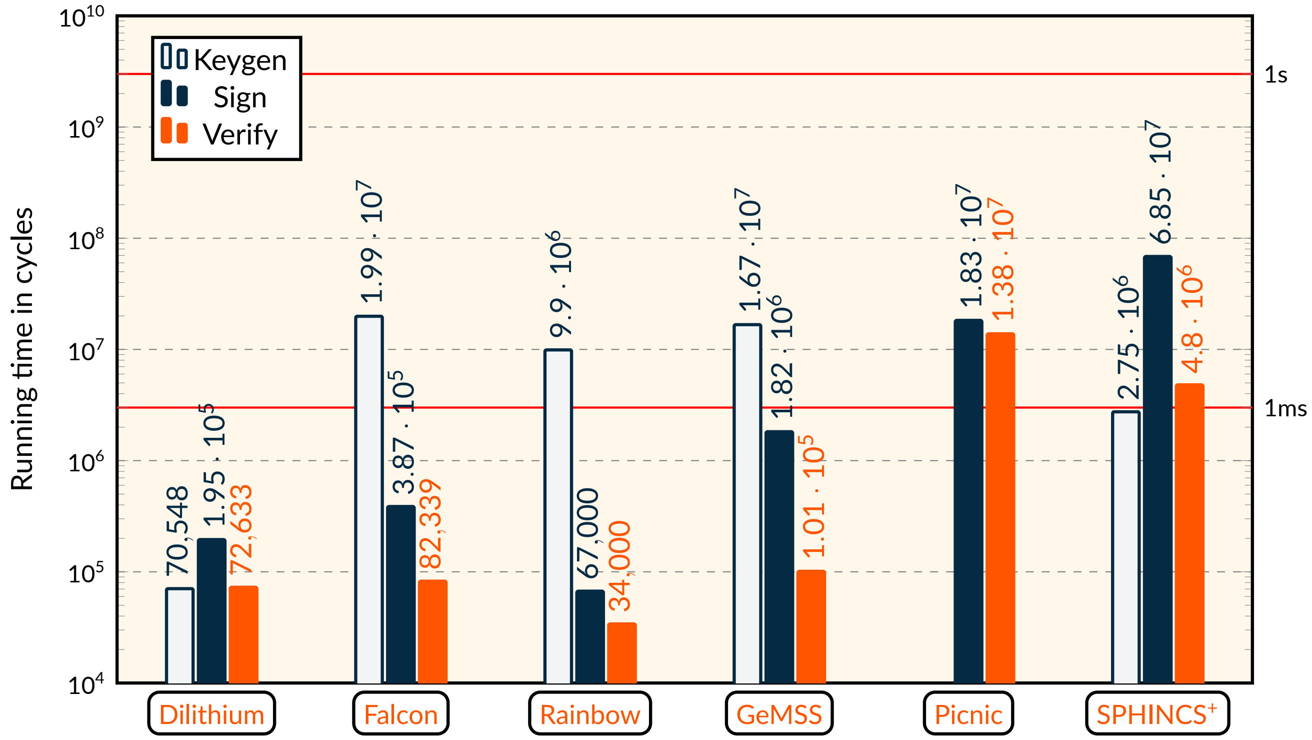


Computational Costs

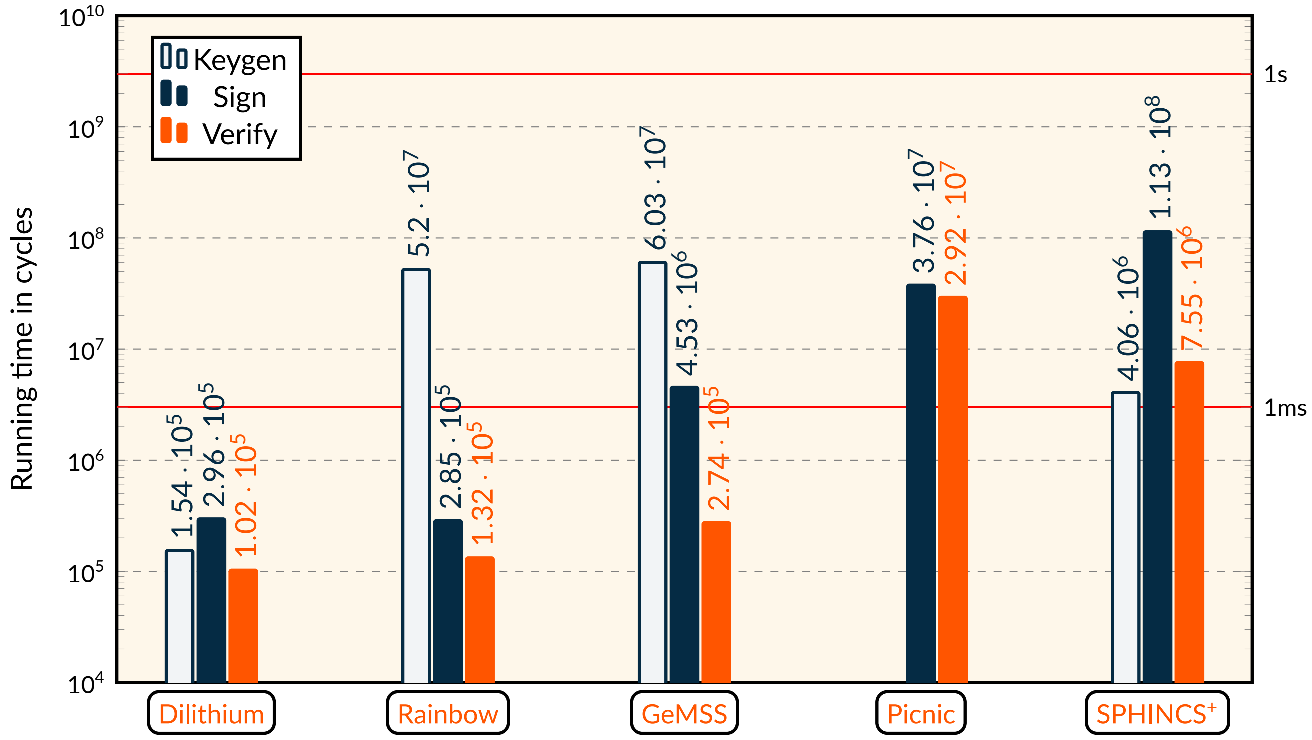
We now compare the running times in cycles of the 6 signature schemes, for optimized implementations targeting x64 platforms. All numbers are extracted from the specification documents of the schemes (which might be inaccurate) and were obtained on different platforms. Therefore, they may not enable a completely fair comparison. To make these numbers less abstract, each graph also contains two horizontal red lines that correspond respectively to 1 millisecond and 1 second on a microprocessor with a clock frequency of 3GHz, which is typical for microprocessors in personal computers.

We observe a high disparity between candidates. For example, the overall fastest signature scheme at the highest security level (Dilithium) has key generation, signing and verification procedures that are, respectively, about 140, 1200 and 100 times faster than the overall slowest one (SPHINCS$^+$). Note that raw performances do not tell the full story, since SPHINCS$^+$ relies on what appear to be more conservative assumptions than any other signature scheme presented here.

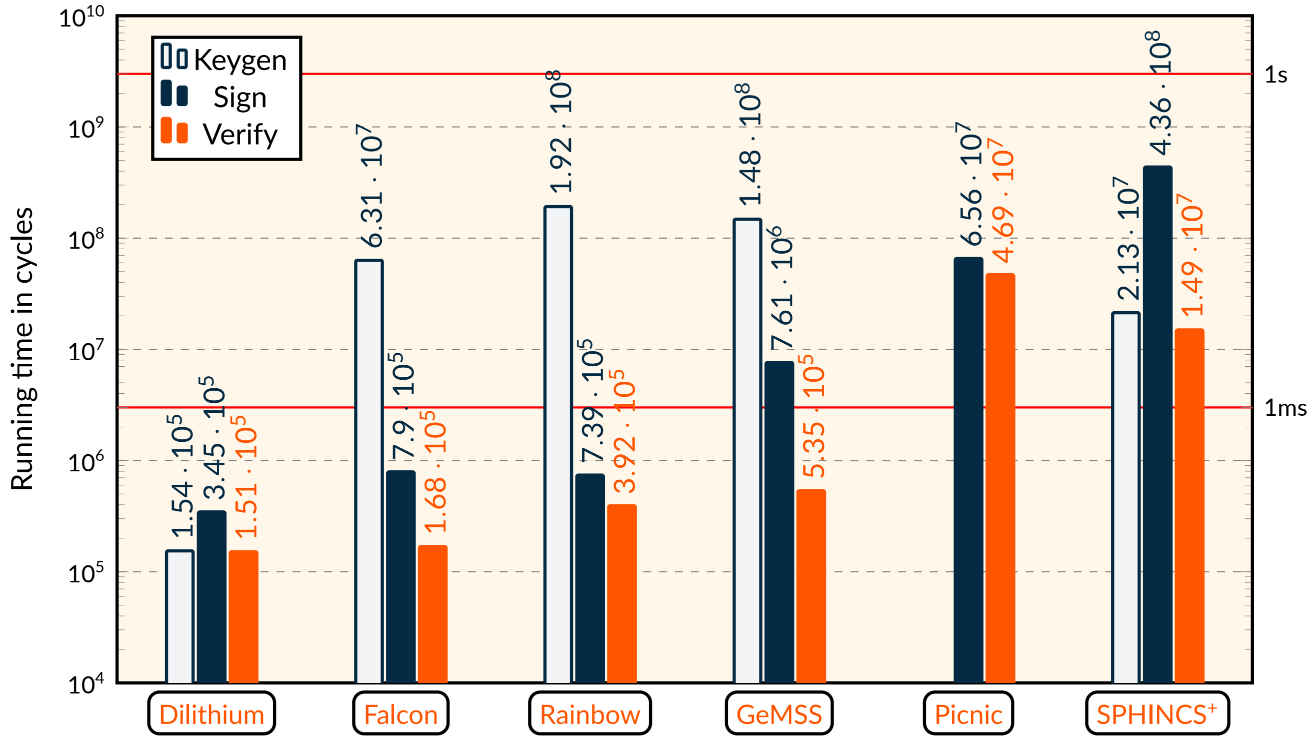
NIST Level 1



NIST Level 3



NIST Level 5



Breakdown of Each Scheme

For each signature scheme, we now provide the following information:

- **The paradigm** can be either Hash-then-sign or Fiat-Shamir. Even in the same family, two schemes based on different paradigms often end up with very different properties.
- **The family** can be either Lattices, Multivariate equations or One-way functions.
- **The underlying hard problem(s)** is specified.
- The **symmetric primitives** and the **type of randomness** used are specified. While these are not too important theoretically, they can have a huge impact on performance. For example, schemes such as Picnic and SPHINCS^{+} are very dependent on the symmetric primitive used, and Gaussian distributions (used in Falcon) can be hard to generate in a masked fashion.
- Links to **the specification**, **the website** (if any) and to **related works** are also provided.
- A **short summary** highlights the key facts about the scheme.
- Finally, a **performance table** is provided.

3 Dilithium (finalist)

Type:	Signature
Paradigm:	Fiat-Shamir
Family:	Lattices
Hard Problems:	Module-LWE (Learning With Errors), Module-SIS (Short Integer Solution)
Sym. primitives:	SHAKE, AES
Randomness:	Uniform, and uniform over the set $\mathcal{B}_\tau$ of ternary vectors with L_1 norm τ
Specification:	[LDK+20]
Website:	`https://pq-crystals.org/dilithium/`
Related Works:	[Lyu09, Lyu12, GLP12, DDLL13, BG14, KLS18, DKL+18, BP18b]

Paradigm

Dilithium is based on the Fiat-Shamir with Aborts paradigm, introduced in [Lyu09]. It implements two notable tricks: the first one, introduced in [GLP12], divides the size of the public key almost in half. A related trick by [BG14] reduces the size of the signature by half, by sending only one ring element instead of two. It also borrows elements of design from BLISS [DDLL13].

Hard Problems

Dilithium relies on the (decisional) Module-LWE and Module-SIS problems [LS15]. In addition, the security proof in the QROM relies on a new problem called SelfTargetMSIS [KLS18] (this problem might not be necessary after all, as discussed in the next paragraph).

Security Model

In the ROM, Dilithium is claimed to be SEU-CMA under the (decisional) Module-LWE and Module-SIS problems; SEU-CMA stands for the classical notion of *Strong Existential Unforgeability under Chosen-Message Attack*. In the QROM, it is claimed to be SEU-CMA under Module-LWE, Module-SIS and SelfTargetMSIS. New results [DFMS19, LZ19] seem to imply that the security proof of Dilithium can be made stronger, and that the hypothesis SelfTargetMSIS may not be necessary after all.

Design Rationale and Physical Attacks

The design of Dilithium has been heavily influenced by the numerous side-channel attacks to which its precedessor, BLISS, has been subjected [BHLY16, PBY17, EFGT17, BDE+18]. To guard against these attacks, Dilithium discards BLISS's use of Gaussian distributions, and relies on uniform distributions instead. A masked implementation of Dilithium has been proposed in [MGTF19].

NIST level	|SK| (bytes)	|PK| (bytes)	|sig| (bytes)	KG (cycles)	Sign (cycles)	Verify (cycles)
2	-	1312	2420	70548	194892	72633
3	-	1952	3293	153856	296201	102396
5	-	2592	4595	153936	344578	151066

4 Falcon (finalist)

Type: Signature
Paradigm: Hash-then-sign
Family: Lattices
Hard Problems: NTRU
Sym. primitives: SHAKE-256
Randomness: Noncentered discrete Gaussians
Specification: [PFH+20]
Website: `https://falcon-sign.info/`
Related Works: [HHP+03, GPV08, SS13, DLP14, DP16, OSHG19]

Design

Falcon is based on the GPV framework [GPV08] for obtaining hash-then-sign schemes over lattices. As first suggested by [SS13, DLP14], the design is instantiated over the very compact class of NTRU lattices [HHP+03] in order to minimize the bandwidth cost. Falcon is the Round 3 signature with the smallest communication cost (public key + signature).

Algorithmic Optimisations

Falcon exploits the algebraic structure of cyclotomic rings in order to optimize its efficiency, notably via the use of a *Fast Fourier Sampling* algorithm [DP16] in the signing procedure, and of a *tower-of-rings* algorithm [PP19] during key generation. Both algorithms yields a $\tilde{O}(n)$-factor improvement compared to previous algorithms, n being the degree of the base ring $\mathbb{Z}[x]/(x^n + 1)$.

Modes of Operation

The specification of Falcon highlights a few possible modes of operation; in addition to the classical mode, the key-recovery mode doubles the size of the signature but shrinks the key size to 64 bytes. The message-recovery mode recovers part of the message (similarly to RSA's key-recovery mode). Finally, Falcon can be converted to an identity-based encryption scheme *à la* [DLP14].

Implementation

Falcon uses floating-point arithmetic (FPA), which can make its implementation delicate on platforms that don't support FPA natively. In this case, FPA needs to be emulated. [OSHG19, Por19] have proposed implementations of Falcon on ARM Cortex-M4; both use memory-laziness tricks in order to reduce its memory footprint.

NIST level	|SK| (bytes)	|PK| (bytes)	|sig| (bytes)	KG (cycles)	Sign (cycles)	Verify (cycles)
1	-	897	666	19872000	386678	82339
3	-	-	-	-	-	-
5	-	1793	1280	63135000	789564	168498

5 Rainbow (finalist)

Type: Signature
Paradigm: Hash-then-sign
Family: Multivariate equations
Hard Problems: MQ (*Multivariate Quadratic*), UOV (*Oil and Vinegar*), MinRank
Sym. primitives: SHA (2), AES
Randomness: Uniform
Specification: [DCP+20]
Website: `https://www.pqcrainbow.org/`
Related Works: [DS05, PBB10, KPG99, SSH11, Pet20, Beu20]

Paradigm

Rainbow is based on the Hash-then-sign paradigm. It implements an upgraded version of UOV [KPG99] in order to propose more efficient parameters. For instance, for a specific choice of parameters one directly obtains the original UOV scheme.

Hard Problems

Rainbow relies on the MQ (*Multivariate Quadratic*) and UOV (*Oil and Vinegar*) problems. Note that this is heuristic, and there is no known security reduction of Rainbow to precisely defined instances of these problems.

Design Rationale

Rainbow is based on a quadratic central map and its inversion function. This inversion function is based on a parameter that can be seen as the depth or layer of inversion. Depending on the layer value, one can achieve different efficiency results for the underlying signature schemes: the more layers, the larger the key but the more efficient the implementation. It is worth noting that for a layer equal to one, the underlying signature scheme is the original UOV signature.

Variants

There are a few variants of Rainbow: standard Rainbow, CZ-Rainbow and compressed Rainbow. CZ-Rainbow uses a trick by [Pet20] to reduce the public key size, whereas the latter allows better memory usage with a compressed private key form but a slower signature generation process. Overall, Rainbow's main advantage is the shortness of the signatures. The performance numbers provided here are those of standard Rainbow.

Recent Attacks

Two papers [NIW+20, PS20] recently revisited the complexity of the *Rainbow Band Separation* attack. A new work by Beullens [Beu20] has shown that Rainbow failed to take into account another attack, and has put a significant dent in the security of currently proposed parameters.

NIST level	|SK| (bytes)	|PK| (bytes)	|sig| (bytes)	KG (cycles)	Sign (cycles)	Verify (cycles)
1	-	103628	66	9900000	67000	34000
3	-	261602	164	52000000	285000	132000
5	-	536166	212	192000000	739000	392000

6 GeMSS (alternate)

Type: Signature
Paradigm: Hash-then-sign
Family: Multivariate equations
Hard Problems: MinRank, HFEv-
Sym. primitives: SHA, AES
Randomness: Uniform, random invertible matrices
Specification: [CFM+20]
Website: `https://www-polsys.lip6.fr/Links/NIST/GeMSS.html`
Related Works: [PCG01, DY13, Pat96, KPG99]

Paradigm

GeMSS is based on the Hash-then-sign paradigm. It implements a direct lineage from QUARTZ [PCG01] and takes some design rationale from the Gui multivariate signature scheme [DY13]. Both schemes descend from the Hidden Field Equations cryptosystem.

Hard Problem

GeMSS relies on a variant of the Hidden Field Equations problem (HFE, [Pat96]). This variant, called HFEv-, was introduced in [PCG01] and adds two new parameters to HFE (*vinegar* and *minus* to HFE, hence the "v-").

Design and Variants

The design of GeMSS is heavily influenced by QUARTZ [PCG01], proposed in the early 2000's. It allows fast verification and short signature sizes at the cost of large public keys. The specification of GeMSS proposes 6 variants (GeMSS and Blue/Red/Cyan/White/Magenta-GeMSS), and each comes in 3 security levels, hence there are 18 parameter sets for GeMSS. The performance numbers we give are for MagentaGeMSS.

New Attack

A very recent work [TPD20] has shown that the *vinegar* and *minus* modifications only marginally increase the security of the HFE problem. As a consequence, they show that GeMSS, RedGeMSS and BlueGeMSS fall short of their claimed security levels.

NIST level	|SK| (bytes)	|PK| (bytes)	|sig| (bytes)	KG (cycles)	Sign (cycles)	Verify (cycles)
1	16	390615	32	16700000	1820000	101000
3	24	1380383	49	60300000	4530000	274000
5	32	3401441	67	148000000	7610000	535000

7 Picnic (alternate)

Type: Signature
Paradigm: Fiat-Shamir
Family: One-way functions
Hard Problems: Invertibility of a one-way function
Sym. primitives: LowMC (underlying one-way function), SHAKE
Randomness: Uniform
Specification: [ZCD+20]
Website: `https://microsoft.github.io/Picnic/`
Related Works: [ARS+15, CDG+17, KKW18, DN19, DKP+19]

High-Level Design

Picnic stands out among signature schemes because of its unique design, as it relies on multiparty computation (or MPC), a paradigm in which N parties collaborate to compute the output of a function F. The public key is $pk = F(sk)$, where F is a publicly-known one-way function. The signing procedure simulates a MPC protocol (this technique is called *MPC-in-the-head*, and is similar to what Fiat-Shamir does for sigma protocols), and the signature is a transcript of this simulation.

Picnic

Present in all three rounds, this version simulates a MPC protocol with 3 parties. It relies on the proof system ZKB++ [CDG+17], and its communication cost as well as its running times are comparable to those obtained by hash-based signatures.

Picnic2

Only present in Round 2 [ZCD+19], Picnic2 simulates a MPC protocol with 64 parties. It uses new techniques introduced by [KKW18], which divides the signature size by 3 but increases signing and verification times by an order of magnitude compared to Picnic.

Picnic3

Compared to Picnic2, this variant reduces the number of parties from 64 to 16, and makes some tweaks to the MPC protocol and to LowMC. As a consequence, it is an order of magnitude faster than Picnic2. See [KZ20] for more details. The performance numbers we provide are for Picnic3.

LowMC

Both Picnic and Picnic2 require a one-way function at their core: this function must of course be hard to invert, but should have as few multiplications as possible, since these are costly to evaluate in MPC. Thus they use LowMC [ARS+15], a block cipher tailored for MPC and designed to have a very small multiplicative complexity. Although recent, LowMC has been studied in a fair amount of works [DKP+19, JNRV20, LIM20].

Multi-Target Attack on Round 1 Picnic

A recent attack [DN19] has shown that in the Round 1 version of Picnic, an adversary knowing Q signatures could recover the private key about $128 \cdot Q$ times faster than predicted by the theory. This attack has been mitigated in the Round 2 submission.

NIST level	|SK| (bytes)	|PK| (bytes)	|sig| (bytes)	KG (cycles)	Sign (cycles)	Verify (cycles)
1	17	34	12359	4151	18252055	13811201
3	24	48	27173	6567	37595772	29243365
5	32	64	46282	9504	65555710	46887830

8 SPHINCS+ (alternate)

Type: Signature
Paradigm: Hash-then-sign
Family: One-way functions
Hard Problems: Multi-target second-preimage resistance of a hash function family
Sym. primitives: SHAKE-256, SHA-256 or Haraka (underlying hash function)
Randomness: Uniform
Specification: [HBD+20]
Website: `https://sphincs.org/`
Related Works: [BDH11, Hül13, BHH+15, HRS16, AE17, AE18]

Design Rationale and Optimizations

SPHINCS+ is a stateless hash-based signature scheme. It follows the framework introduced in [BHH+15], which combines Merkle trees, Goldreich trees and hash-based few-times signatures (or FTS). SPHINCS+ introduces a few optimizations such as the use of tweakable hash functions [HRS16] to protect against multi-target attacks. HORST, an FTS used in [BHH+15], has been replaced by FORS, a more secure FTS which also provides smaller signatures. See also [BHK+19] for an up-to-date presentation of the framework.

Variants

SPHINCS+ admits several variants: in addition to the 3 security levels (128, 192 or 256), there are 3 choices for the underlying building block (SHAKE-256, SHA-256 or Haraka). Additionally, one could choose between a "small" and a "fast" variant: these provide a trade-off between the size of the signature and the running time of the signing procedure. Finally, one could choose between a "simple" variant which is simpler and faster, and a "robust" variant which has a more conservative security argument. Hence there are $3 \times 3 \times 2 \times 2 = 36$ variants. The performance numbers provided here are those of SPHINCS+-SHA-256-fast-robust.

Security Proof?

While some simple hash-based signatures have security reductions to standard assumptions over generic hash functions, SPHINCS+ is one of the more complex schemes in this family, and no security proof is known for it (yet). See also [BH19].

Physical Attacks

While no black-box attack has been proposed against SPHINCS+, a side-channel attack [KGB+18] has shown how an unprotected implementation can leak part of the private key. Similarly, [CMP18] showed theoretically that one single, mildly-controlled fault injection can lead to the recovery of the private key, and [GKPM18] carried this attack on an ARM Cortex M3.

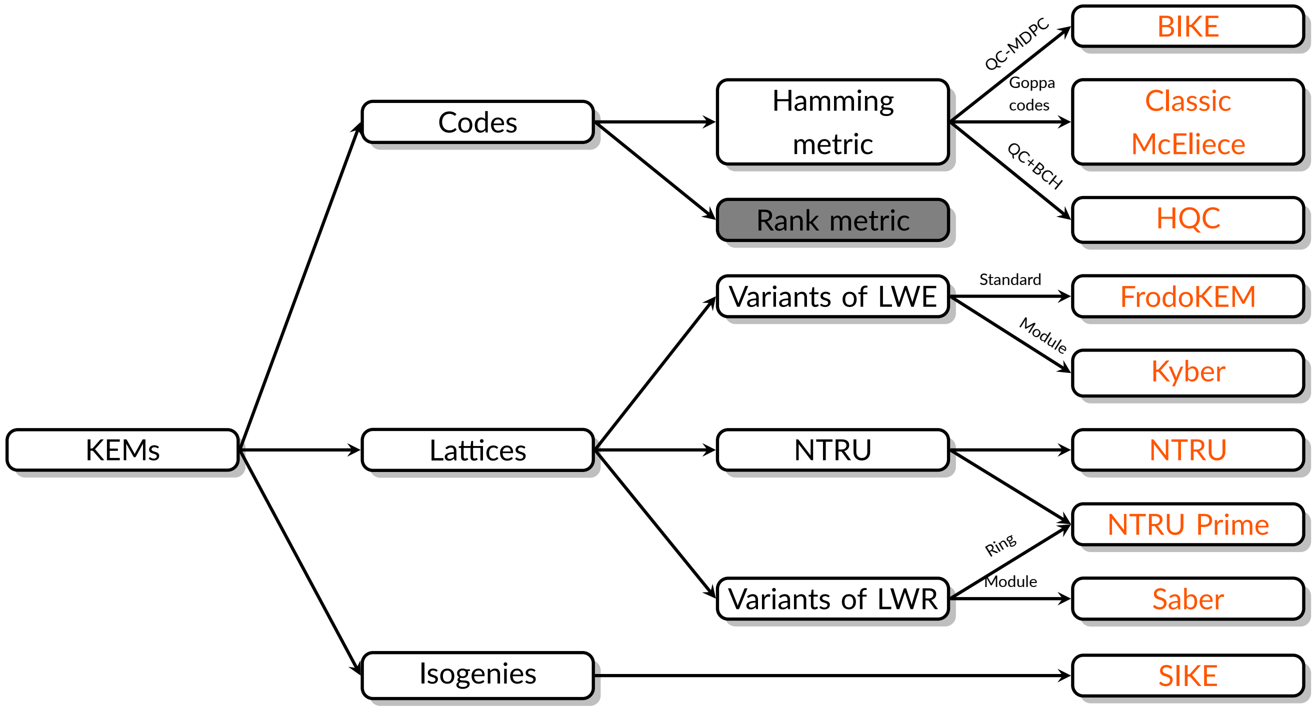
NIST level	|SK| (bytes)	|PK| (bytes)	|sig| (bytes)	KG (cycles)	Sign (cycles)	Verify (cycles)
1	64	32	17088	2748026	68541826	4801338
3	96	48	35664	4063066	113484456	7552358
5	128	64	49856	21327470	435984168	14938510

9 Key-Encapsulation Mechanisms

We now study the key-establishment schemes under consideration for standardization. In Round 1 of NIST's standardization process (December 2017), 49 submissions for key-establishment were accepted [NIS17b]. After a preliminary analysis by the cryptographic community, NIST selected 17 of these 49 submissions for Round 2 of the standardization process [NIS19a]. Out of these 17 submissions, 9 were selected to Round 3 of the standardization process [NIS20b]: 4 *finalists* and 5 *alternates*.

- Classic McEliece (finalist);
- Kyber (finalist);
- NTRU (finalist);
- Saber (finalist);
- BIKE (alternate);
- FrodoKEM (alternate);
- HQC (alternate);
- NTRU Prime (alternate);
- SIKE (alternate).

These submissions are based on three families of hardness assumptions: codes, lattices or isogenies. Candidates based on multivariate equations were eliminated at Round 1. Some submissions also propose an encryption scheme or a key-exchange protocol, but all submissions propose a key encapsulation mechanism (henceforth KEM). This KEM is typically obtained by applying a CCA transform to a base key-exchange/encryption scheme, and therefore usually claims the best security guarantees (security against active attackers). Thus, for simplicity we will only consider KEMs. An overview of the KEMs can be found in the figure below.



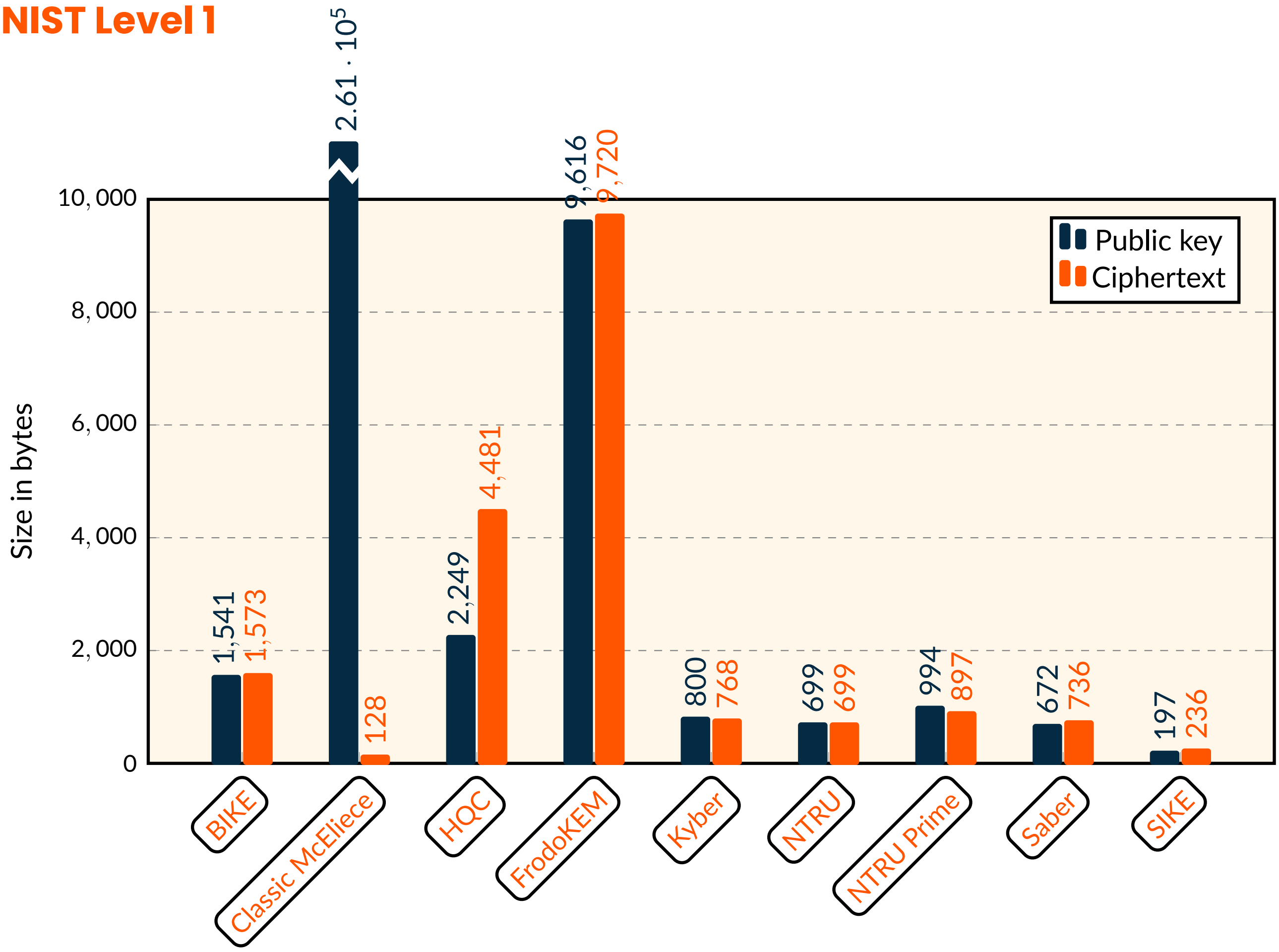
In addition, we provide the following data:

- A comparative performance study of the 9 KEMs (pages 18 to 21);
- For each scheme, one page summarizing its main facts (pages 23 to 31).

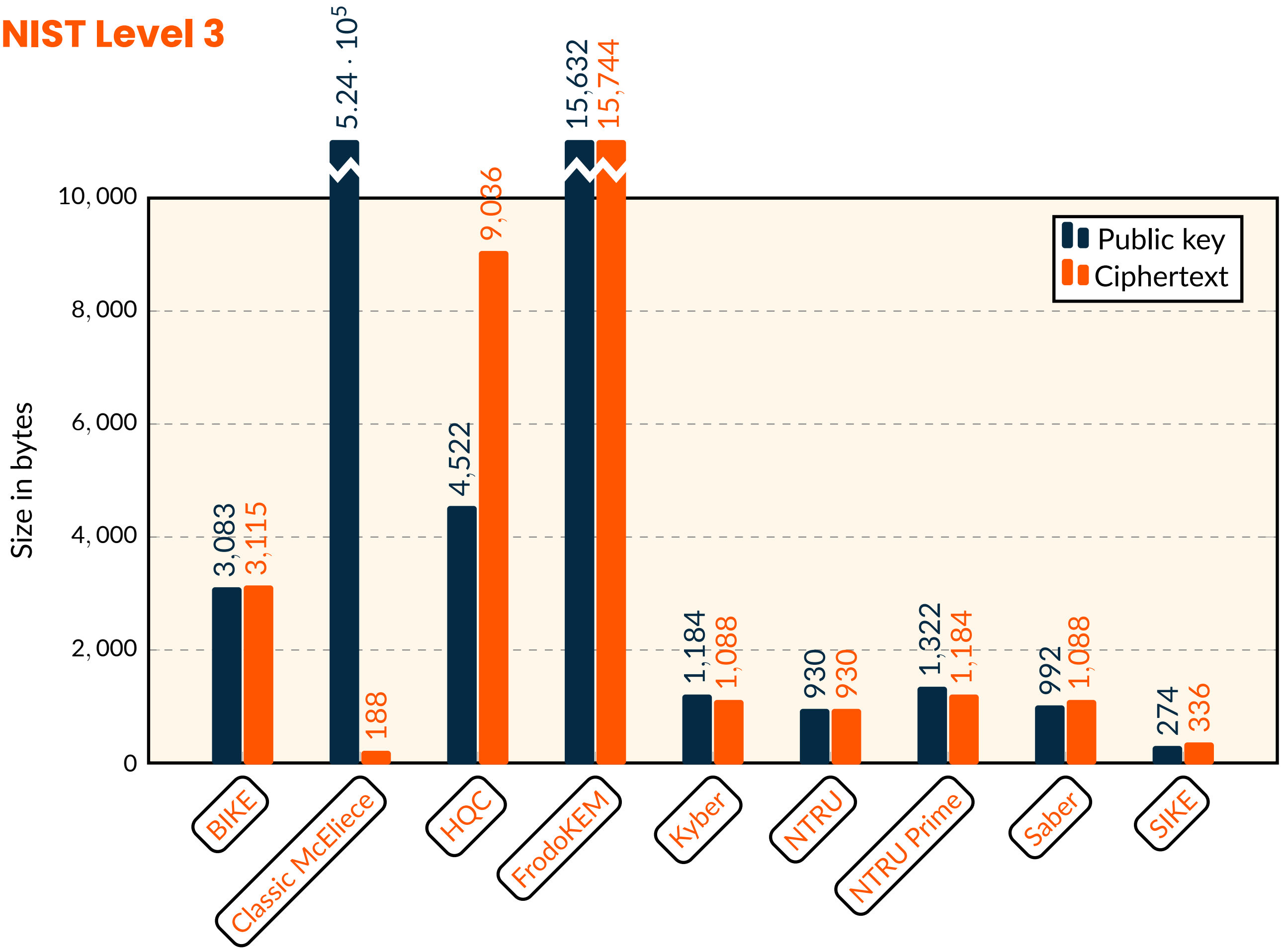
Communication Costs

The following section provides a detailed comparison of the communication costs of the 9 Round 3 KEMs for three security levels: NIST Level 1 (conjectured at least as secure as AES-128), NIST Level 3 (conjectured at least as secure as AES-192) and NIST Level 5 (conjectured at least as secure as AES-256). At the lowest security level (NIST Level 1), most schemes manage to keep their total communication cost below 2000 bytes. In that regard, the most efficient scheme is SIKE and the least efficient is Classic McEliece, which has very large public keys, although it manages to have the smallest ciphertexts across all schemes.

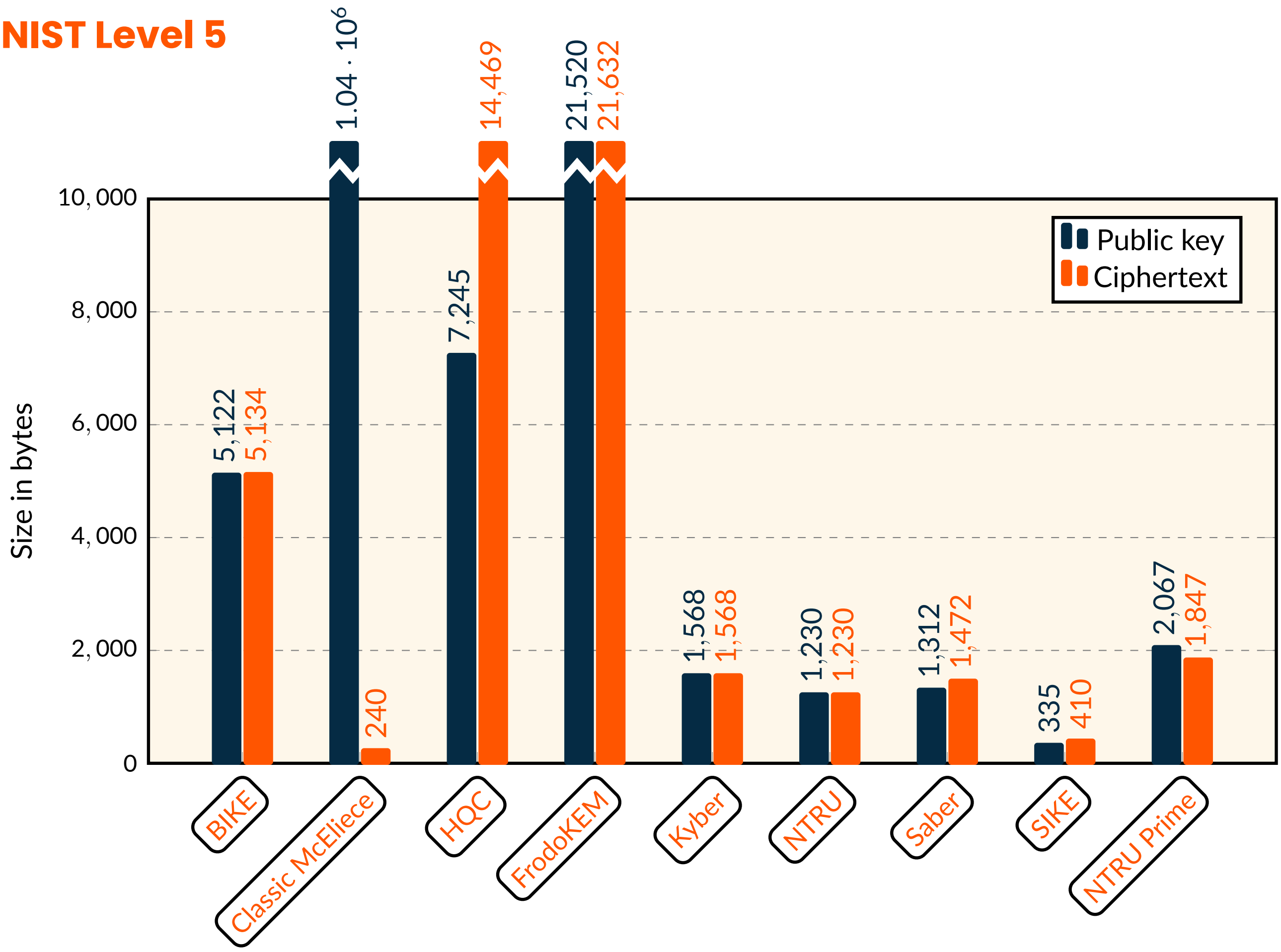
NIST Level 1



NIST Level 3



NIST Level 5

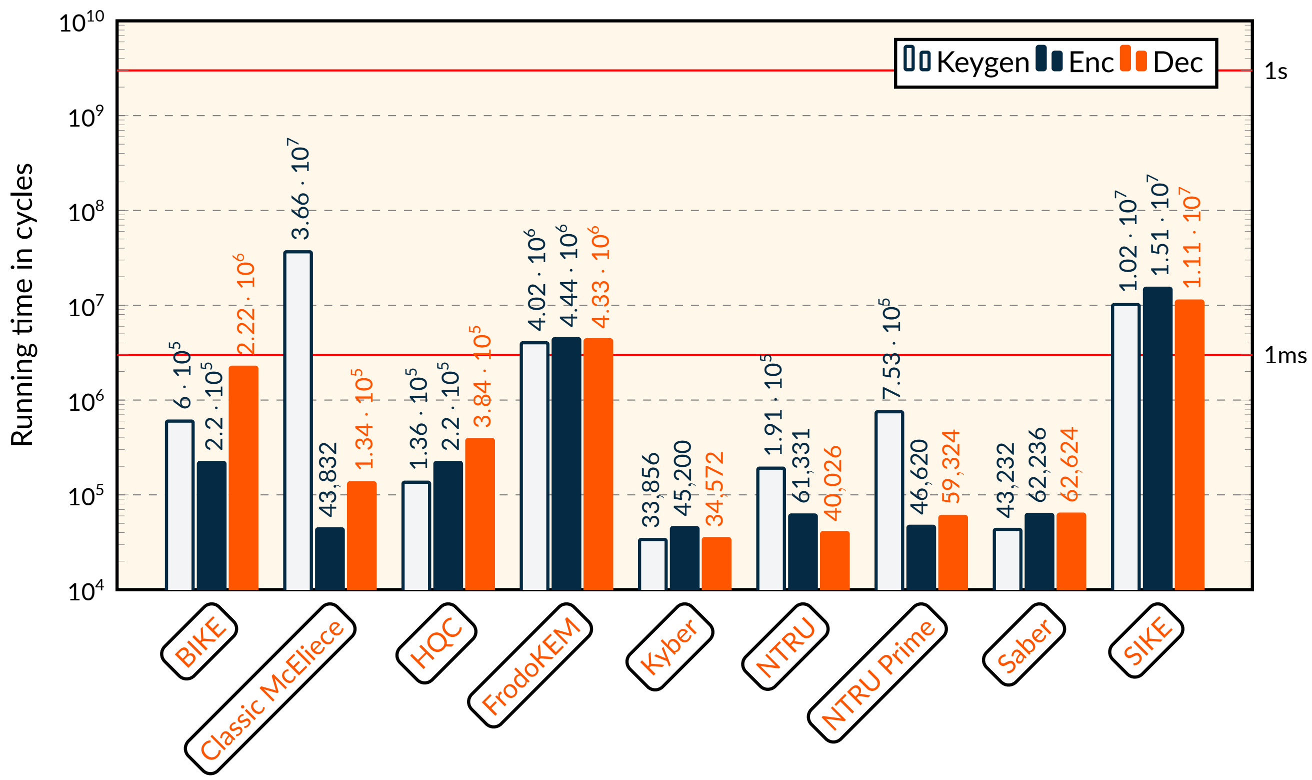


Computational Costs

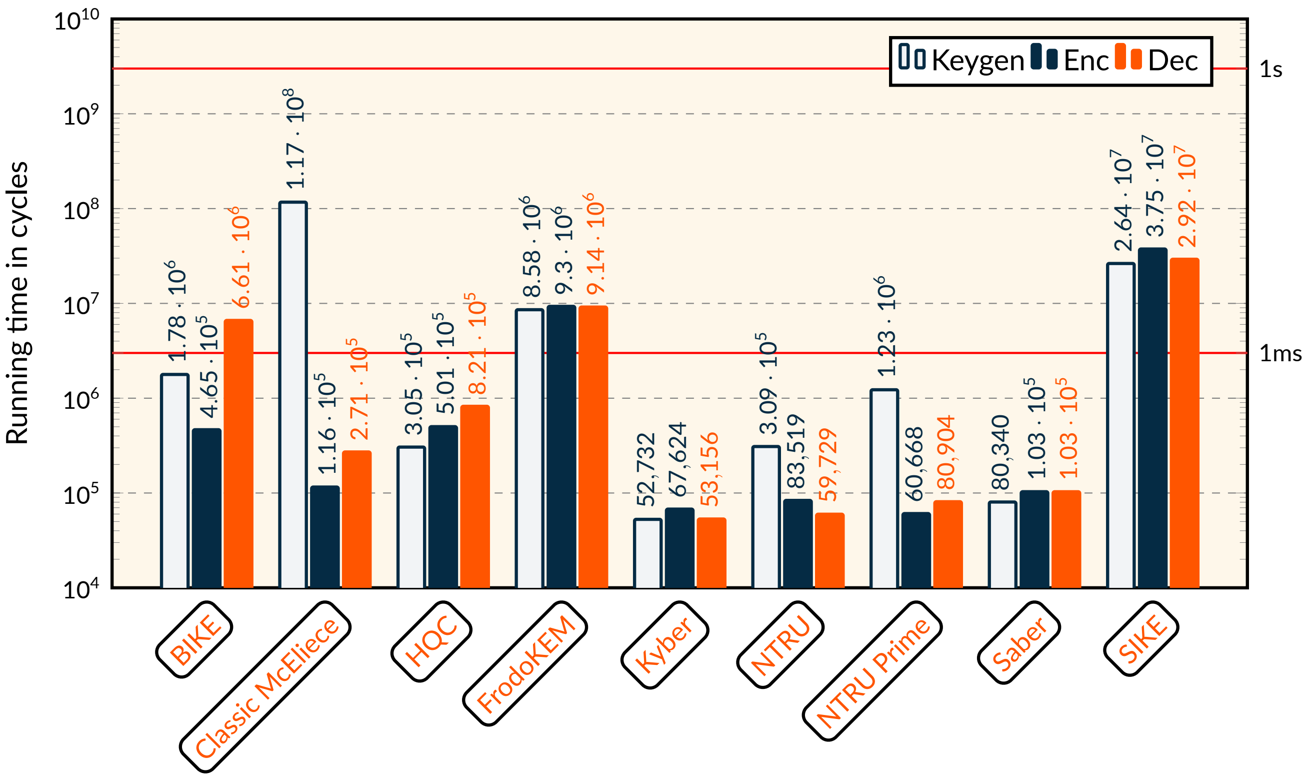
We now compare the running times in cycles of the 9 KEMs, for optimized implementations targeting x64 platforms. All numbers are extracted from the specification documents of the schemes (which might be inaccurate) and were obtained on different platforms. Therefore, they may not enable a completely fair comparison. To make these numbers less abstract, each graph also contains two horizontal red lines that correspond respectively to 1 millisecond and 1 second on a microprocessor with a clock frequency of 3GHz, which is typical for microprocessors in personal computers.

As with signatures, there can be a large disparity between candidates. At the highest security level, the fastest scheme overall (Kyber) is about 200 times faster than FrodoKEM, and about 600 times faster than SIKE. Again, these numbers do not tell the full story, as FrodoKEM relies on hardness assumptions that are far less structured than Kyber's, and SIKE is much cheaper in terms of bandwidth.

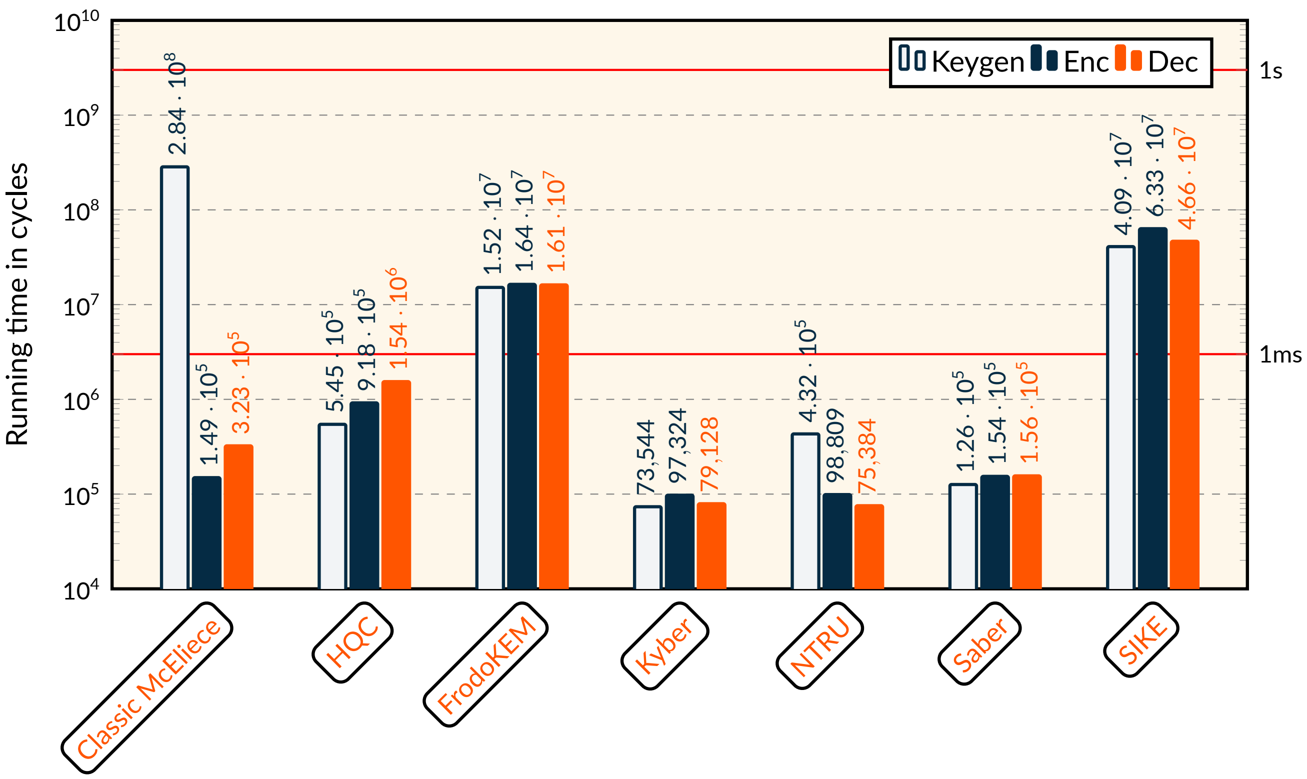
NIST Level 1



NIST Level 3



NIST Level 5



Breakdown of Each Scheme

For each KEM, we now provide the following information:

- **The transform** is the generic conversion used to turn an IND-CPA scheme into an IND-CCA scheme. We recall that IND-CPA stands for *Indistinguishability under Chosen Plaintext Attack*, and IND-CCA stands for *Indistinguishability under Chosen Ciphertext Attack*. The former is simpler to achieve, but does not guarantee resistance against an attacker that can tamper with ciphertexts (for example in a man-in-the-middle attack). Therefore, IND-CPA schemes are usually converted to IND-CCA schemes using a *CCA transform*.
- **The family** can be either Error-correcting codes, Lattices or Isogenies.
- **The underlying hard problem** is specified.
- The **symmetric primitives** and the **type of randomness** used are specified. These can impact performance: in some schemes, the call to a symmetric primitive actually takes most of the running time. The type of randomness impacts how easy it is to protect a scheme against side-channel attacks, for example via the *masking* countermeasure.
- Links to **the specification**, **the website** (if any) and to **related works** are also provided.
- A **short summary** highlights the key facts about the scheme.
- Finally, a **performance table** is provided.

10 Classic McEliece (finalist)

Type: KEM
CCA Transform: Dent [Den03], SXY [SXY18], see also [BP18a]
Family: Error-correcting codes (Goppa codes)
Hard Problems: Syndrome Decoding, Indistinguishability of Goppa codes from random codes
Sym. primitives: SHAKE
Randomness: Uniform, fixed weight
Specification: [ABC+20]
Website: `https://classic.mceliece.org`
Related Works: [McE78, Den03, NIE86, SXY18]

Design

Despite its name, Classic McEliece is not exactly based on McEliece's scheme [McE78], but rather on a dual variant by Niederreiter [NIE86], which is equivalent security-wise. One of the selling points of Classic McEliece is its very conservative design: the original designs by [McE78, NIE86] have been extensively studied, and Classic McEliece makes no fundamental change to them.

Chosen-Ciphertext Security

Design-wise, the most notable novelty of Classic McEliece is perhaps the CCA transform that is used to obtain an IND-CCA KEM from a OW-CPA public-key encryption scheme. This transform is inspired by Dent [Den03] and Saito-Xagawa-Yamakawa [SXY18]. See also [BP18a] for discussions on the QROM security of this transform.

Size Constraints

Classic McEliece has very large public keys but very small ciphertexts. Although this may make it unsuitable in some contexts, applications for which ciphertext size is more important than key size may benefit from it. This is demonstrated in [HNS+20], which uses Classic McEliece in a post-quantum version of the WireGuard protocol. See also [BL20] for a protocol built around these constraints.

Hardware Implementation and Attacks

Hardware implementations of the core mathematical elements of Classic McEliece have been provided in [WSN18], and the specification provides performance numbers on Artix-7 and Virted-7 FPGAs. Note that this is not a full implementation *per se* (it does not include, e.g., hashing).

The implementation of [WSN18] implements the Berlekamp-Massey decoder in constant-time to prevent timing attacks. However [LNPS20] showed that it is still vulnerable to an electromagnetic side-channel attack, and shows it is possible to recover a plaintext in a few hundred power traces.

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	6492	261120	128	36627388	43832	134184
3	13608	524160	188	116914656	115540	270856
5	13932	1044992	240	284468140	149080	322988

11 Kyber (finalist)

Type: KEM
CCA Transform: Tweaked Fujisaki-Okamoto [FO99] with implicit rejection [HHK17]
Family: Lattices
Hard Problems: Module-LWE
Sym. primitives: SHA 3-256/512 and SHAKE-128/256
Randomness: Binomial
Specification: [SAB+20]
Website: `https://pq-crystals.org/kyber/`
Related Works: [LPR10, LP11, LS15, ADPS16b, ADPS16a, BDK+18]

Design

Kyber follows the Lindner-Peikert framework [LPR10, LP11], also used by Saber, FrodoKEM and NTRU Prime (NTRU LPRime). We give a simplified (CPA-secure) description below.

Key generation goes as follows:

1. Sample a pseudo-random matrix A.
2. Sample short matrices S, E.
3. Compute $B = AS + E$.
4. The public key is $pk = (A, B)$, and the private key is $sk = S$.

Encryption goes as follows:

1. Sample short matrices R, E', E''.
2. Compute $U = RA + E'$ and $V = RB + E'' + \mathit{Encode}(\mathit{msg})$.
3. The ciphertext is $\mathit{ctxt} = (U, V)$.

Decryption goes as follows:

1. $\mathit{msg} = \mathit{Decode}(V - US)$.

Module Lattices

Kyber uses *module lattices*: it manipulates matrices and vectors with entries in $\mathcal{R} = \mathbb{Z}_q[x]/(x^{256} + 1)$; the security is adjusted by changing the dimensions of these matrices and vectors. Rationales behind this choice is to provide a trade-off between efficiency and conservatism, to make implementation simpler and to easily change security levels.

Hashing the Public Key

Kyber achieves CCA security by performing a variant of Fujisaki-Okamoto's transform. One interesting fact is that Kyber also hashes the public key as part of that process; it has been argued [BDK+18, SAB+20] that this provides protection against multi-target attacks and other useful properties.

Round 2 Changes

Between the Round 1 and Round 2, Kyber has reduced the modulus q by a factor of about two, due to improvements in NTT techniques. The Round 1 version of Kyber [SAB+17] also included a technique for compressing public keys by dropping least significant bits. D'Anvers pointed out in [NIS17a] that this technique could invalidate Kyber's security proof (though in practice, it does not seem to introduce a concrete weakness). Consequently, the compression technique was removed in the Round 2 specification.

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	1632	800	768	33856	45200	34572
3	2400	1184	1088	52732	67624	53156
5	3168	1568	1568	73544	97324	79128

12 NTRU (finalist)

Type: KEM (and Encryption)
CCA Transform: $U_m^{\not\perp}$ [HHK17], Saito-Xagawa-Yamada [SXY18], SimpleKEM [BP18a]
Family: Lattices
Hard Problems: One-Wayness under Chosen Plaintext Attacks (OW-CPA) of the underlying DPKE
Sym. primitives: SHAKE-256, SHA 3-256
Randomness: Ternary polynomials (sometimes with bounded weight)
Specification: [CDH+20]
Website: `https://ntru.org/`
Related Works: [HPS98, Den03, HPS+17, HRSS17, Sch18]

History

NTRU has a long story as it was first proposed 20 years ago [HPS98]. Since then, the scheme has known a few evolutions. It was the first scheme for which decryption failure attacks (a common caveat of many lattice-based KEMs) were highlighted [HNP+03], and a fix was proposed via the NAEP transform [HSSW03]. Over the years, updated parameters were proposed [HHHW09, HPS+17] to account for cryptanalytic advances.

Design

NTRU is based on a variant of the eponymous assumption. By tweaking the parameters of the original NTRU scheme [HPS98], it makes it easy to implement in constant time and eliminates decryption failures [HNP+03], "evaluate-at-1" attacks and invertibility checks. There are several ways to interpret and prove the CCA transform used by NTRU: either as the $U_m^{\not\perp}$ transform of [HHK17], the one of [SXY18] or the SimpleKEM transform from [BP18a].

A Merge of Two Schemes

NTRU is the merge of two Round 1 schemes: NTRU-HRSS-KEM [SHRS17] and NTRUEncrypt [ZCHW17]. NTRU-HRSS-KEM aimed at perfect correctness and used a CCA transform inspired by Dent [Den03].
On the other hand, NTRUEncrypt relied on the NAEP transform [HSSW03], and proposed parameters with decryption failures, parameters inspired by a construction by Stehlé and Steinfeld [SS13] and (optionally) the use of Gaussian distributions.

Experimental TLS deployments

Google [Lan18] and Cloudflare [Kwi19] have experimentally deployed NTRU-HRSS-KEM (as well as SIKE) on TLS as an effort to assess the feasibility of a post-quantum TLS. Conclusions can be found at [KV19].
Similar deployment efforts were conducted by Amazon [Hop19, Wei20], this time on BIKE and SIKE.

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	935	699	699	191279	61331	40026
3	1234	930	930	309216	83519	59729
5	1590	1230	1230	431667	98809	75384

13 Saber (finalist)

Type: KEM
CCA Transform: $FO^{\not\perp}$ transform [HHK17, JZC+18]
Family: Lattices
Hard Problems: Module-LWR (Learning With Rounding)
Sym. primitives: SHA-3, SHAKE-128
Randomness: Uniform
Specification: [DKR+20]
Website: `https://www.esat.kuleuven.be/cosic/pqcrypto/saber/`
Related Works: [DKRV18, JZC+18]

High-Level Design

Just like Kyber, Saber is based on the Lindner-Peikert framework. The main difference is that it uses Module-LWR instead of Module-LWE: the "random noise" is replaced with "deterministic rounding", making the implementation simpler but changing the underlying hardness assumption. The conversion into a IND-CCA scheme is done via the $FO^{\not\perp}$ transform [HHK17, JZC+18]. An early version [DKRV18] of Saber relied on *Noisy Diffie-Hellman* key-exchange, but its design has switched to LPR-style [LPR10] encryption since then.

A Simple Design

Saber makes several design choices oriented at simplicity. Like Kyber, it only works with elements over $\mathbb{Z}_q[x]/(x^{256}+1)$. Moreover, the use of LWR simplifies its description. Finally, the integer modulus q is taken to be a power of two. As a consequence of this choice of q, polynomial multiplications are done via the Toom-Cook and Karatsuba algorithms.

Implementations

Saber has been implemented on several platforms. The specification document reports implementations on ARM Cortex-M4, HW/SW codesigns and/or complete hardware implementations on Artix-7 and Virtex Ultrascale+, a masked implementation on ARM Cortex-M4 and even an implementation on the RSA coprocessor ESP32 (inspired from a similar result [AHH+18] for Kyber).

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	1568	672	736	43232	62236	62624
3	2304	992	1088	80340	103204	103092
5	3040	1312	1472	126220	153832	155700

14 BIKE (alternate)

Type: KEM
CCA Transform: $FO^{\not\perp}$ [HHK17]
Family: Error-correcting codes (QC-MDPC codes)
Hard Problems: Quasi-Cyclic Syndrome Decoding and Codeword Finding problems
Sym. primitives: AES, SHA
Randomness: Uniform, fixed weight, odd weight
Specification: [ABB+20]
Website: `https://bikesuite.org/`
Related Works: [MTSB12, BGG+17, HHK17]

	BIKE-1	BIKE-2	BIKE-3
SK	(h_0, h_1) with $\|h_0\| = \|h_1\| = w/2$		
PK	$(f_0, f_1) \leftarrow (gh_1, gh_0)$	$(f_0, f_1) \leftarrow (1, h_1 h_0^{-1})$	$(f_0, f_1) \leftarrow (h_1 + gh_0, g)$
Enc	$(c_0, c_1) \leftarrow (mf_0 + e_0, mf_1 + e_1)$	$c \leftarrow e_0 + e_1 f_1$	$(c_0, c_1) \leftarrow (e + e_1 f_0, e_0 + e_1 f_1)$
	$K \leftarrow \mathbf{K}(e_0, e_1)$		
Dec	$s \leftarrow c_0 h_0 + c_1 h_1$; $u \leftarrow 0$	$s \leftarrow ch_0$; $u \leftarrow 0$	$s \leftarrow c_0 + c_1 h_0$; $u \leftarrow t/2$
	$(e_0', e_1') \leftarrow \texttt{Decode}(s, h_0, h_1, u)$		
	$K \leftarrow \mathbf{K}(e_0', e_1')$		

Design and Variants

BIKE is based on QC-MDPC codes – this acronym stands for *Quasi-Cyclic Moderate Density Parity Check*. The quasi-cyclicity allows dramatic gains in compactness and speed. BIKE originally came in three variants, BIKE-{1,2,3}, presented in the above table extracted from the Round 1 presentation of BIKE. In a simplification effort, only BIKE-2 was kept in the last iteration. Note that BIKE-2 was the most compact of the three variants; it also used to have the slowest key generation procedure, but this was recently mitigated in [DGK20a].

The Decoding Algorithm

Decoding algorithms for code-based KEMs has been the topic of intensive research. Decryption failures have been shown [GJS16] to lead to practical attacks, hence the decryption failure rate (DFR) must be kept negligible. However, constant-time decoding algorithms with negligible DFR have been difficult to obtain. BIKE currently uses the Black-Gray-Flip decoder [DGK20b].

Hardware Implementation

BIKE is one of the few Round 3 candidates to have proposed a hardware implementation (on Artix-7 FPGA), see: `https://github.com/Chair-for-Security-Engineering/BIKE`.

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	281	1541	1573	600000	220000	2220000
3	419	3083	3115	1780000	465000	6610000
5	580	5122	5134	-	-	-

15 HQC (alternate)

Type: KEM
CCA Transform: Variant [HHK17] of FO
Family: Error-correcting codes
Hard Problems: Quasi-Cyclic Syndrome Decoding
Sym. primitives: AES, SHA
Randomness: Uniform, fixed weight
Specification: [MAB+20]
Website: `http://pqc-hqc.org`
Related Works: [Ale03, Gab05, ABD+16a, DGZ17]

- $\mathsf{Setup}(1^\lambda)$: generates and outputs the global parameters $\mathsf{param} = (n, k, \delta, w, w_\mathbf{r}, w_\mathbf{e})$.
- $\mathsf{KeyGen}(\mathsf{param})$: samples $\mathbf{h} \xleftarrow{\$} \mathcal{R}$, the generator matrix $\mathbf{G} \in \mathbb{F}_2^{k \times n}$ of $\mathcal{C}$, $\mathsf{sk} = (\mathbf{x}, \mathbf{y}) \xleftarrow{\$} \mathcal{R}^2$ such that $\omega(\mathbf{x}) = \omega(\mathbf{y}) = w$, sets $\mathsf{pk} = (\mathbf{h}, \mathbf{s} = \mathbf{x} + \mathbf{h} \cdot \mathbf{y})$, and returns $(\mathsf{pk}, \mathsf{sk})$.
- $\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m})$: generates $\mathbf{e} \xleftarrow{\$} \mathcal{R}$, $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2) \xleftarrow{\$} \mathcal{R}^2$ such that $\omega(\mathbf{e}) = w_\mathbf{e}$ and $\omega(\mathbf{r}_1) = \omega(\mathbf{r}_2) = w_\mathbf{r}$, sets $\mathbf{u} = \mathbf{r}_1 + \mathbf{h} \cdot \mathbf{r}_2$ and $\mathbf{v} = \mathbf{m}\mathbf{G} + \mathbf{s} \cdot \mathbf{r}_2 + \mathbf{e}$, returns $\mathbf{c} = (\mathbf{u}, \mathbf{v})$.
- $\mathsf{Decrypt}(\mathsf{sk}, \mathbf{c})$: returns $\mathcal{C}.\mathsf{Decode}(\mathbf{v} - \mathbf{u} \cdot \mathbf{y})$.

Design

HQC stands for Hamming Quasi-Cyclic. Just like BIKE, HQC relies on quasi-cyclic codes. Its high-level design is presented above. Lattice practitioners will recognize a design similar to lattice-based schemes such as Kyber, Saber, FrodoKEM and NTRU Prime (in its LPRime variant). While this analogy can be useful at a very high level, the mathematical objets used are different (codes vs lattices) and therefore HQC relies on completely different problems and algorithms.

Hardware Implementation

The specification document of HQC gives performance numbers for an implementation on the Artix-7 FPGA.

Attacks against the BCH Decoder

Implementation attacks were proposed against the BCH decoder used in earlier versions of HQC. In [WTBB+19], co-authors of HQC displayed a timing attack exploiting the BCH decoder running time, and proposed a constant-time variant as a countermeasure. [SRSWZ20] mounted a power side-channel against the BCH decoder. The last iteration of HQC has replaced the BCH decoder with a Reed-Muller Reed-Solomon decoder.

A Decryption Failure Attack

A decryption failure attack against a Round 2 parameter set of HQC has been proposed in [GJ20]. This parameter set is not present in the last iteration of HQC.

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	40	2249	4481	136000	220000	384000
3	40	4522	9036	305000	501000	821000
5	40	7245	14469	545000	918000	1538000

16 FrodoKEM (alternate)

Type: KEM (and Encryption)
CCA Transform: Variant of $\mathsf{FO}_m^{\not\perp}$ [HHK17, JZC+18]
Family: Lattices
Hard Problems: LWE
Sym. primitives: SHAKE-128/256 (and optionally AES-128/256)
Randomness: Centered rounded Gaussians (using a CDT)
Specification: [NAB+20]
Website: `https://frodokem.org/`
Related Works: [LP11, BCD+16, ATT+18, BFM+18, HOKG18, BFM+19]

Standard Lattices

FrodoKEM is an instantiation of the Lindner-Peikert framework [LP11] (see Kyber). It is the only remaining scheme that works over unstructured lattices. These lattices entail working with matrices having entries in $\mathbb{Z}$, whereas most other lattice-based candidates take entries in $\mathbb{Z}_q[x]/(f)$ for some polynomial f. As a result, FrodoKEM has larger communication costs, but relies on more conservative (or at least less structured) hardness assumptions.

FrodoCCS

FrodoKEM is an evolution of FrodoCCS [BCD+16]. FrodoCCS was based on *Noisy Diffie-Hellman*, but FrodoKEM switched in favor of public key encryption. In addition, they use different error distributions and symmetric primitives.

Implementations

Despite its somewhat large memory footprint, many works studied the implementation of FrodoKEM on embedded devices. For example, [HOKG18] proposes designs for FPGA and microprocessors, with the goal to minimize area consumption on FPGAs (resp. peak stack usage on microprocessors). This is achieved mainly by using memory-laziness tricks. [BFM+18] informally argue that the generation of a very large public matrix (A) doesn't require a cryptographically secure PRNG, and uses a PRNG known as xoshiro** for this step.

Side-Channel Attacks

[ATT+18] studied horizontal attacks against FrodoCCS, and [BFM+19] studied single-trace attacks against FrodoKEM. A timing attack [GJN20] was recently displayed against the reference implementation of Round 2 FrodoKEM, which performed comparison (during decapsulation) in variable time. This has been fixed in Round 3. The flaw was also present in LAC, BIKE, HQC, ROLLO , RQC .

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	19888	9616	9720	4022000	4440000	4325000
3	31296	15632	15744	8579000	9302000	9143000
5	43088	21520	21632	15191000	16357000	16148000

17 NTRU Prime (alternate)

Type: KEM
CCA Transform: Variant of Dent [Den03] with confirmation hash
Family: Lattices
Hard Problems: NTRU, Ring-LWR
Sym. primitives: SHA-512, AES-256-CTR
Randomness: Uniform in $\{-1, 1\}$ or $\{-1, 0, 1\}$, sometimes with bounded weight
Specification: [BBC+20]
Website: `https://ntruprime.cr.yp.to/`
Related Works: [Den03, BCLv17]

Design

The high-level design of NTRU Prime was introduced in [BCLv17]. Just like NTRU, it tweaks the original NTRU encryption scheme [HPS98] in order to make it easily implementable in constant time, and to eliminate decryption failures [HNP+03], "evaluate-at-1" attacks and invertibility checks.

Reducing the Attack Surface

A point emphasized by the specification of NTRU Prime is the choice of the base field, which is $\mathbb{Z}_q[x]/(x^p - x - 1)$, where p, q are two primes such that $(x^p - x - 1)$ is prime in $\mathbb{Z}_q[x]$. The explicit goal behind this choice is to hedge against future cryptanalytic attacks similar to those exploiting the presence of subfields [ABD16b, BBdV+17] or computable homomorphisms [CGS14, CDPR16], while keeping the efficiency gains provided by a ring structure.

Polynomial Multiplication

The rings $\mathbb{Z}_q[x]/(x^p - x - 1)$ chosen by NTRU Prime do not natively support the number theoretic transform (NTT). Until recently, NTRU Prime used Karatsuba and Toom-Cook for multiplying polynomials, but recent progress [ACC+21] has made the NTT competitive for NTRU Prime rings.

Two Variants

The NTRU Prime submission proposes two variants: Streamlined NTRU Prime, and NTRU LPRime. At a high level, the first variant is close to NTRU, but low-level details differ significantly, see [SHRS17, Section 6.1] and [Sch18]. The second variant instantiates the Lindner-Peikert framework (see Kyber) using a variant of Ring-LWR over the NTRU Prime ring. The performance numbers we provide are for Streamlined NTRU Prime.

Embedded Implementations

The specification and official website of NTRU Prime report optimized implementations on ARM Cortex-M4 and AVR ATmega1284 microcontrollers, and Xilinx Zynq Ultrascale+ FPGA.

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	1518	994	897	752904	46620	59324
3	1999	1322	1184	1227380	60668	80904
5	3059	2067	1847	-	-	-

18 SIKE (alternate)

Type: Key exchange and KEM
CCA Transform: Variant of [HHK17]
Family: Isogenies
Hard Problems: SIDH problem
Sym. primitives: SHAKE-256
Randomness: Uniform
Specification: [JAC+20]
Website: `https://sike.org/`
Related Works: [JD11, CJL+17, CLN16, JS19, CLN+20, MLRB20]

History and design

SIKE is the only isogeny-based candidate scheme. At a high level, it implements the SIDH key-exchange [JD11]. Unlike the classical Diffie-Hellman, it is not fully interactive. An attack in [GPST16] can be adapted to break the CCA security of the basic SIDH design. Hence SIKE uses a conversion inspired from [HHK17] to ensure IND-CCA security. It is to be noted that while SIKE is the KEM with the lowest communication cost, it is one of those with the higher computational costs.

Compressed Variant

SIKE comes in two variants, a basic one, and a second one that uses point compression [CJL+17], which reduces the public key size by about 41%, but multiplies the overall running time by about a factor of two. Our performance figures are for the variant with point compression.

Implementations

SIKE has attracted several implementations for embedded devices, including over ARM processors [SLLH18, sJA19], Xilinx Artix-7, Virtex-7, and Kintex UltraScale+ FPGAs [KAK18, KAK+19, MLRB20] or even for the RISC architecture [KPHS18]. Although SIKE is slower than other candidates, recent works consistently report running times of a few dozens milliseconds over these platforms.

Cryptanalysis

The current best attack against SIKE is via claw-finding. The best classical algorithm is due to van Oorschot and Wiener [vW99], and the best quantum one to Tani. Jaques and Schanck [JS19] recently showed that in reasonable computation models, the classical attack is better than the quantum one. See also [CLN+20] for a state-of-the-art analysis.

Implementation Flaws

Two flaws in the reference implementation of SIKE has been recently unearthed [NIS19b]. The comparison step in the re-encryption part of the decapsulation procedure was improperly computed, seemingly voiding out the CCA security claim. This has been subsequently corrected.

NIST level	|SK| (bytes)	|PK| (bytes)	|ctxt| (bytes)	KG (cycles)	Enc (cycles)	Dec (cycles)
1	350	197	236	10158000	15120000	11077000
3	491	274	336	26360000	37470000	29216000
5	602	335	410	40935000	63254000	46606000

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