# Efficient Leakage Resilient Circuit Compilers

Marcin Andrychowicz, Warsaw University, Poland Ivan Damgård, Aarhus University, Denmark Stefan Dziembowski, Warsaw University, Poland Sebastian Faust, EPFL, Switzerland

Antigoni Polychroniadou, Aarhus University, Denmark

#### Theory

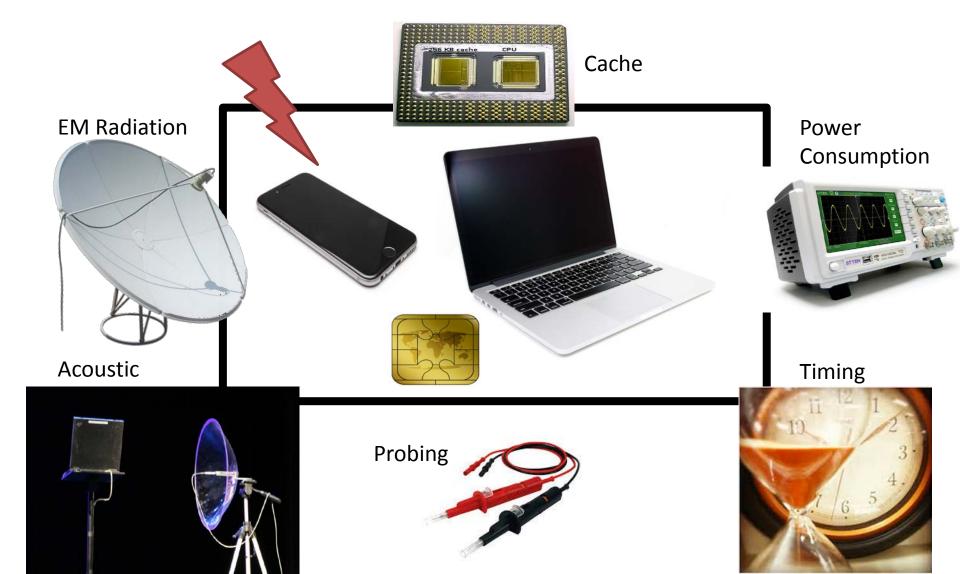
Cryptographic algorithms are often modeled as 'black boxes'



E.g. Internal computation is opaque to external adversaries.

Security is proven under various hardness assumptions.

### Reality Computation Internals Leak



#### Motivation

## Many provably secure cryptosystems can be broken by side-channel attacks



### Two Paradigms to Fight Leakage Attacks

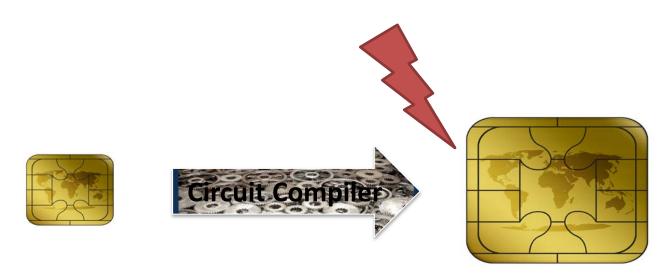
1. Consider Leakage at design level Only security of specific schemes.

How to securely implement any scheme?

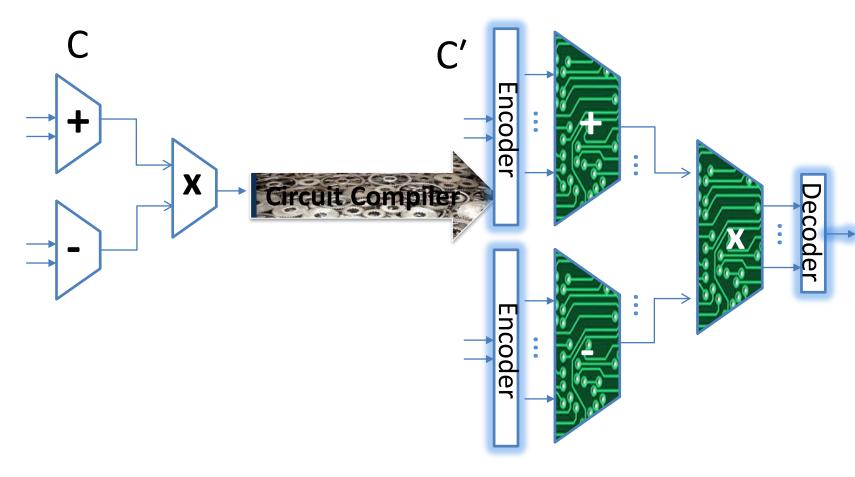
#### **Wanted:**

2. Leakage resilient Compiler

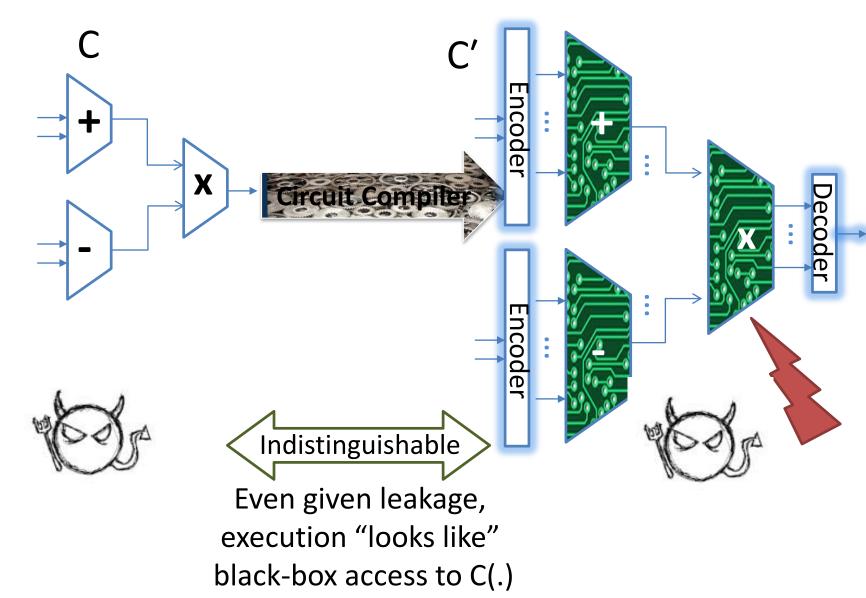
Transform any circuit to a leakage resilient circuit secure in a strong black-box sense.



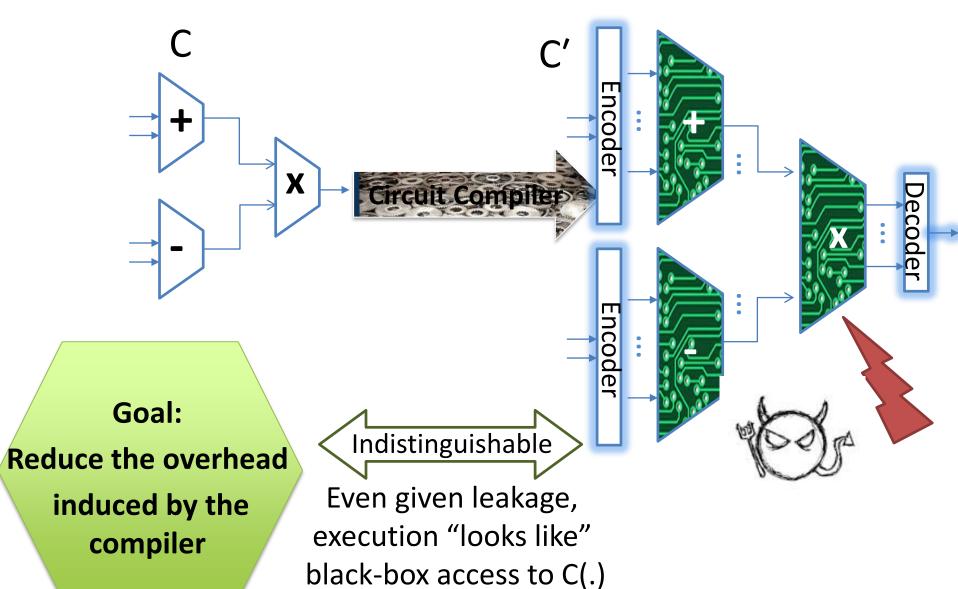




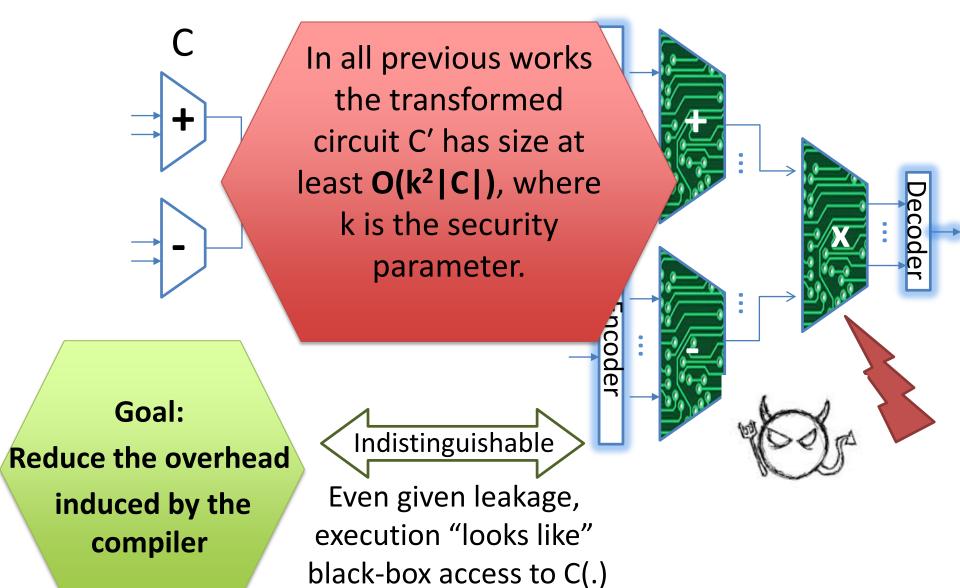












#### **Our Goal**

Build Efficient Leakage Resilient Compilers

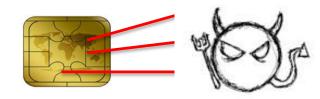
Is it possible to construct leakage resilient compilers with at most linear overhead?

• All previous works introduce at least quadratic overhead.

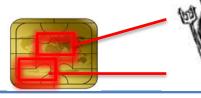
#### Prior Work on General Compilers

Three Leakage Models:

'Local' Bounded Wire-Probing: [ISW03,...]



'Local' Only Computation (OC) Leakage/ Split State Model: [MR04,...]





'Global' Computational Continuous Weak Leakage i.e. AC<sup>0</sup> leakage Functions [FRRTV10,...]



#### **Our Results**

**Efficient Compliers:** 

Using
Techniques
from secure
MPC

'Local' Wire-Probing: O(polylog(k) · | C | log | C |)

Previous Best Overhead: O(k² | C|) by [ISW03]

'Local' OC Leakage: O(k log k log log k | C | )

Previous Best Overhead:  $\Omega(k^4|C|)$  by [DF12] and  $\Omega(k^3|C|)$  by [GR12]

This talk

'Global' Computational Continuous Weak Leakage: O(k·|C|Iog|C|)

Previous Best Overhead: O(k<sup>2</sup>|C|) by [FRRTV10] and O(k<sup>3</sup>|C|) by [R13]



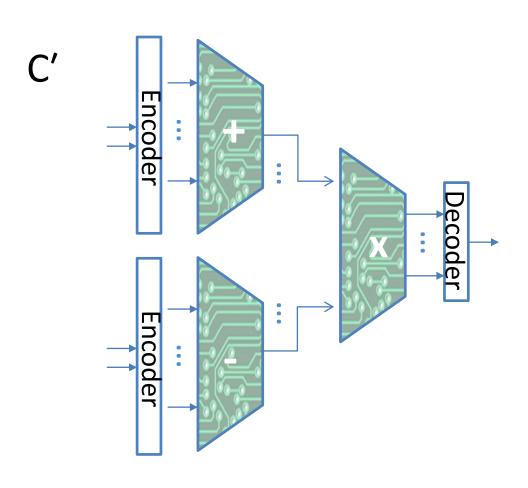
## Our Result on Global Computational Weak Leakage

- Informal Theorem: A compiler that makes any circuit resilient to computationally weak leakages. The compiler increases the circuit size by a factor of O(k).
- Global adaptive leakage
- Arbitrary total leakage

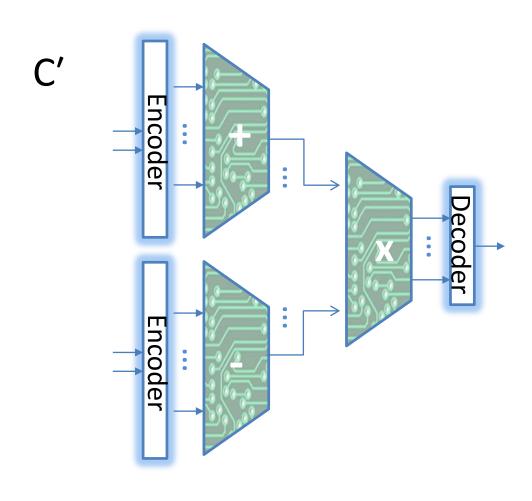
However we must assume something [MR04]:

- Leakage function is computationally weak.
- Simple opaque gates.

### The Compiler



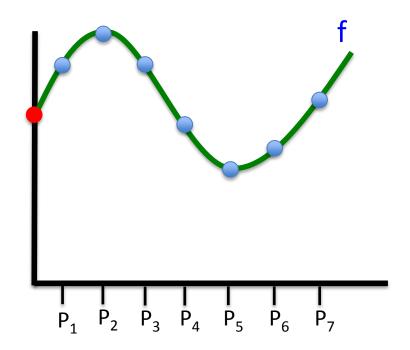
## The Compiler: From Wires to Wire Bundles



#### Packed Secret Sharing (PSS)

 PSS is a central tool in information theoretic secure MPC protocols.

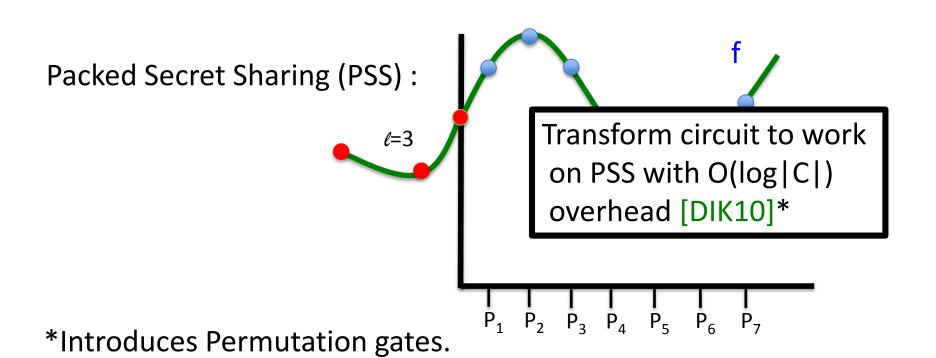
**Standard Secret Sharing:** 



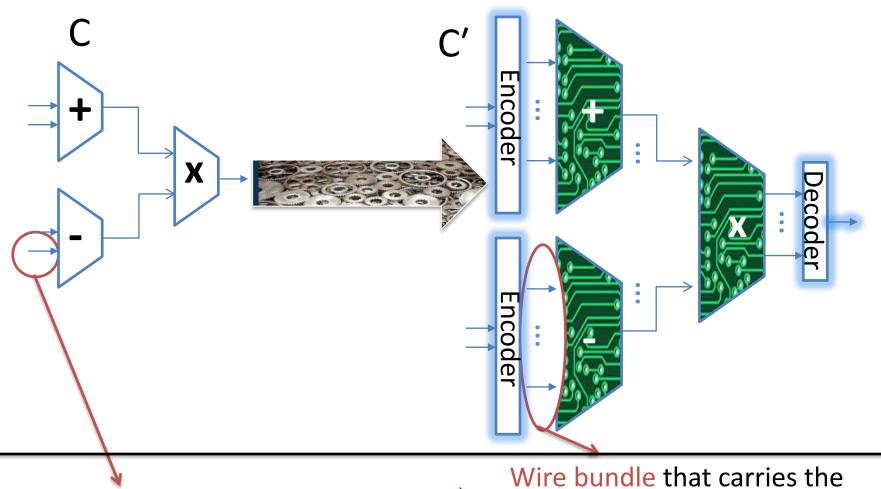
Degree of f denoted by d

#### Packed Secret Sharing (PSS)

 PSS is a central tool in information theoretic secure MPC protocols.



- Every wire is encoded with PSS.
- Inputs are encoded; outputs are decoded.



Each wire w



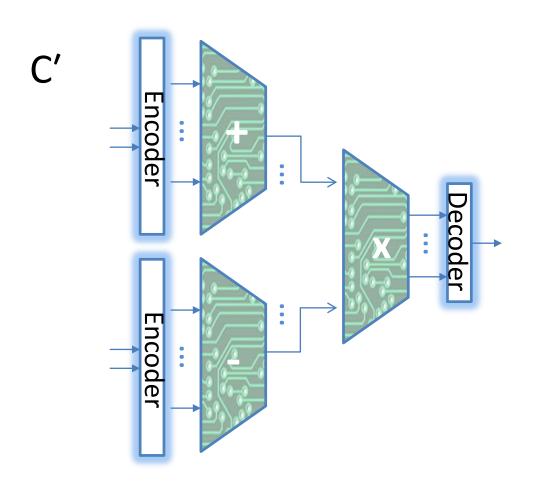
Wire bundle that carries the encoding of w, e.g. k shares of w. Notation:  $(w_1,...,w_k)=[w]_d$ 

#### PSS is Secure Against AC<sup>0</sup> Leakages

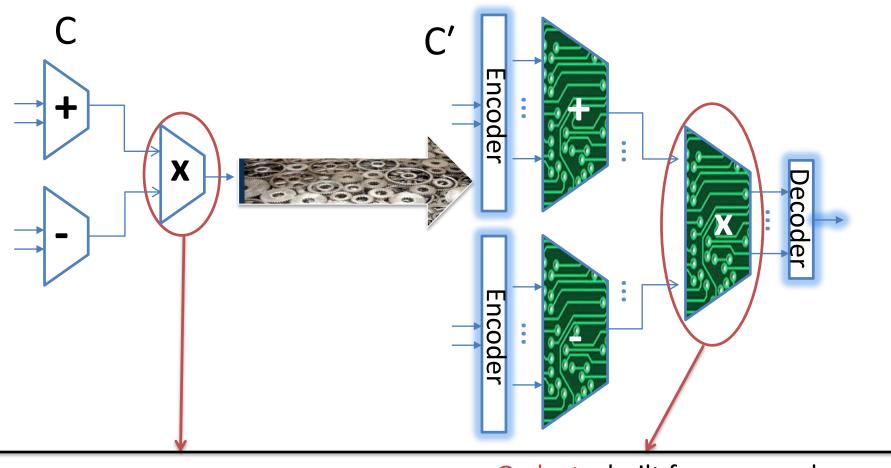
A function is in AC<sup>0</sup> if it can be computed by a poly-size O(1) depth Boolean circuit with unbounded fan-in AND, OR (and NOT) gates.

PSS Encoding is AC<sup>0</sup> indistinguishable, i.e.
 Inner product hard to compute in AC<sup>0</sup>.

# The Compiler: From Gates to Gadgets



 Every gate is replaced by a gadget operating on encoded PSS bundles.



Gates



Gadgets: built from normal gates and opaque gates and operate on encodings.

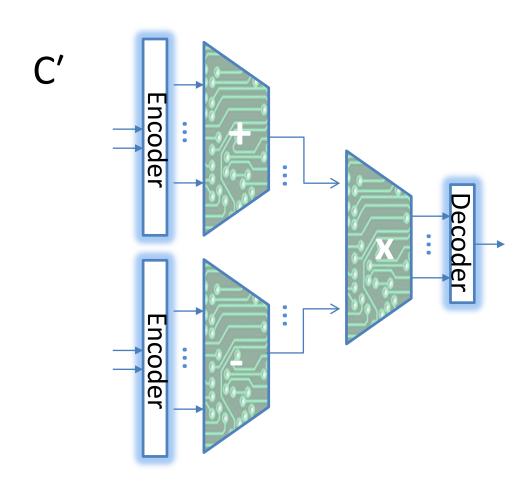
#### **Opaque Gates**

```
[G89,GoldOstr95]...Leak-free processor: oblivious RAM [MR04], [DP08], [GKR08], [DF12]...Leak-free memory: "only computation leaks", one-time programs [FRRTV10],... Opaque Gates [GR12],[R13]... Ciphertext banks
```

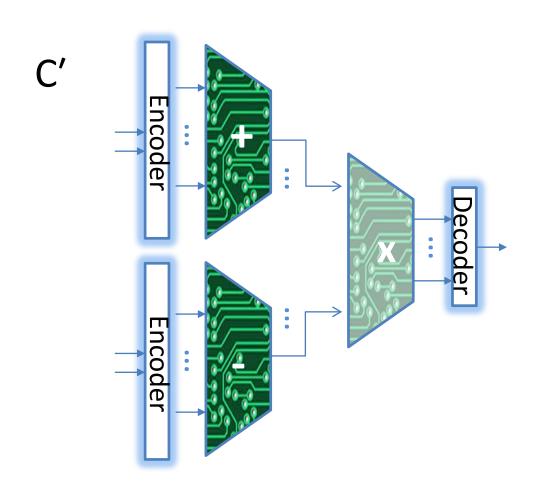
Opaque Gates: simple gates that sample from a fixed distribution e.g.: securely draw strings with inner product 0.

- ✓ Stateless: No secrets are stored
- ✓ Small and simple
- ✓ Computation independent: No inputs, so can be pre-computed

# The Compiler: Addition & Subtraction Gadgets



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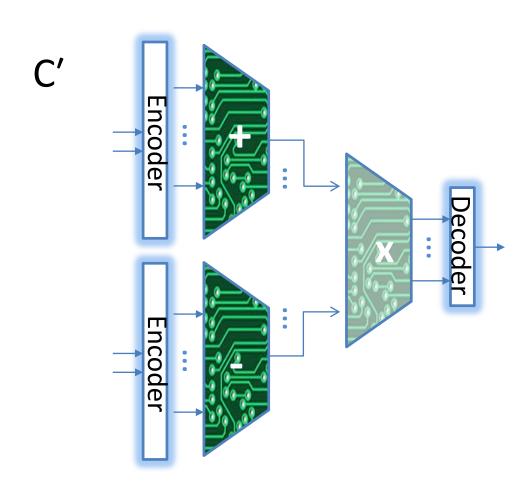


### The Compiler: Addition & Subtraction Gadgets

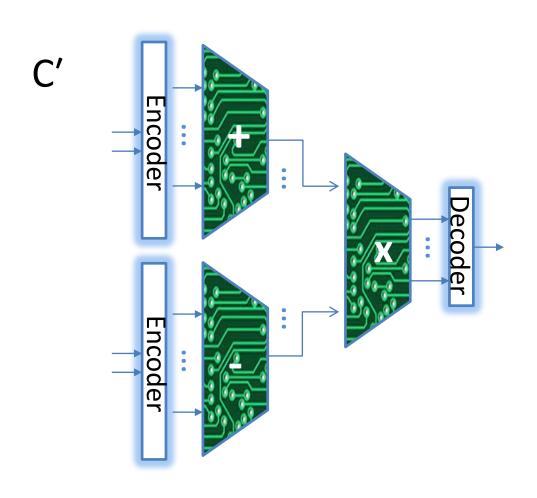
Goal: 
$$c=a+b \Rightarrow [a+b]_d \leftarrow [a]_d + [b]_d$$

1. 
$$[a+b]_d = [a]_d + [b]_d + [0]_d$$
 OR  $[a-b]_d = [a]_d - [b]_d + [0]_d$ 

# The Compiler: Multiplication Gadgets



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Goal: 
$$c = ab \Rightarrow [ab]_d \leftarrow [a]_d[b]_d$$

$$[r]_d$$
,  $[r]_{2d} \leftarrow Opaque gate$ 

- 1.  $[ab]_{2d} = [a]_{d}[b]_{d}$
- 2.  $[ab + r]_{2d} = [ab]_{2d} + [r]_{2d}$
- 3.  $(ab+r) \leftarrow Decode([ab+r]_{2d})$
- 4.  $[ab+r]_d \leftarrow Encode (ab+r)$
- 5.  $[ab]_d = [ab+r]_d [r]_d$

Permutation gadget follows in the same fashion.

#### Compiler: High-Level

- Circuit topology is preserved.
- Every wire is encoded yielding a wire bundle;
   Inputs are encoded; outputs are decoded.
- PSS Encoding is AC<sup>0</sup> indistinguishable
- Every gate is converted into a gadget operating on encodings.

#### Security of the Compiled Circuit

Prove security via 'shallow' Reconstructors per gadget (technique introduced in [FRRTV10])

 Reconstructor: on input the inputs and the outputs of a gadget is able to simulate its internals in a way that looks indistinguishable for leakages from AC<sup>0</sup>.



#### Conclusion

#### Three efficient circuit compilers ....

- ✓ compile any circuit
- √ 'Local' Wire-Probing
- √ 'Local' OC Leakage
- ✓ 'Global' Computational weak Leakage

#### Question

Connection to Obfuscation

#### Thank you!

#### Optimally Efficient Multi-Party Fair Exchange and Fair Secure Multi-Party Computation

#### Handan Kılınç <sup>1</sup> Alptekin Küpçü <sup>2</sup>

<sup>1</sup>EPFL, Koç University <sup>2</sup>Koç University

CT-RSA, 2015





#### **Outline**

- Introduction
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  - Definitions
- Our New Protocols
  - MFE Protocol
    - Resolve Protocols
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### MFE

#### **Exchange Protocol**

Two or more parties exchange their items with the other parties.

#### Fair Exchange Protocol

The exchange protocol is fair if in the end of

- All parties receive their desired items or,
- None of them receives any item.

### **MFE**

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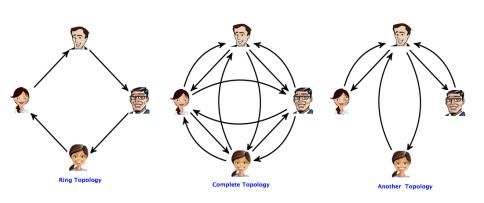








# MFE Topologies





- © Fairness is not possible without trusted third party (TTP).
- There is a lack of TTP. So the efficiency is important.



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#### Optimistic MFE ©

In an *optimistic* protocol, the TTP is involved in the protocol *only* when there is a malicious behavior.





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## Multi-Party Computation

#### **MPC**

A group of parties  $(P_1, P_2, ..., P_n)$  with their private inputs  $w_i$  desires to compute a function  $\phi$ .

- This computation is secure when the parties do not learn anything beyond what is revealed by the output of the computation.
- This computation is fair if either all of the parties learn their corresponding output in the end of computation, or none of them learns.

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### MFE is MPC

### Multi-party fair exchange is multi-party computation.

- Each party P<sub>i</sub> has item f<sub>i</sub>.
- ullet They need the compute the functionality  $\phi$  where

$$\phi(f_1, f_2, ..., f_n) = (\phi_1, \phi_2, ..., \phi_n)$$

#### MFE id MPC

For the complete topology:

$$\phi_i(f_1,...,f_n)=(f_1,...,f_{i-1},f_{i+1},...,f_n)$$

• For the ring topology:

if 
$$i = 1$$

$$\phi_i(f_1,...,f_n)=f_n$$

else

$$\phi_i(f_1,...,f_n) = f_{i-1}$$









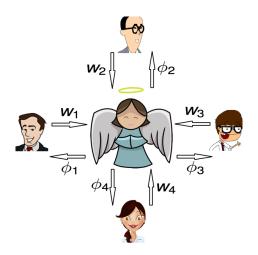






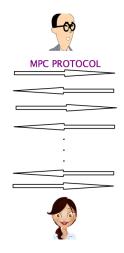






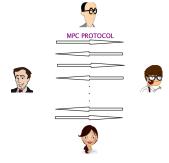
### Real World for MPC



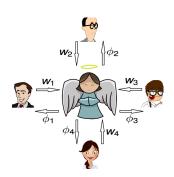




## Secure and Fair MPC







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## Overview of MFE protocol

The parties are  $P_1, P_2, ..., P_n$  and each party  $P_i$  has item  $f_i$ . They want the items of all parties (complete topology).

The TTP and his public key *pk* is known by all parties.

- Phase 1: Setup
- Phase 2: Encrypted Item Exchange
- Phase 3: Decryption Share Exchange





They agree on two timeouts  $t_1$  and  $t_2$  and know TTP's public key

























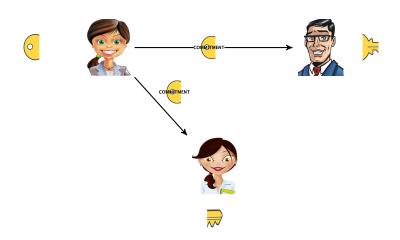


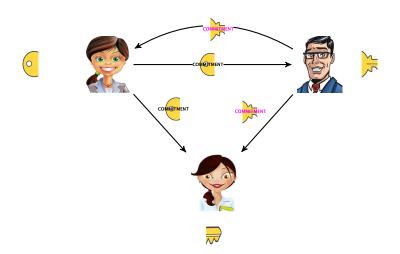


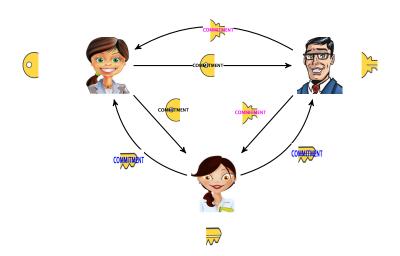


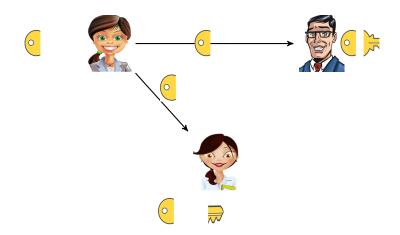


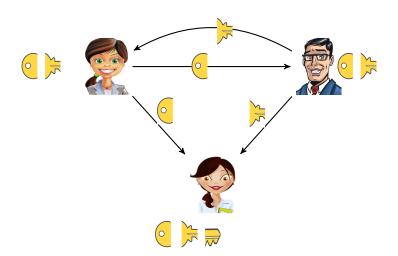


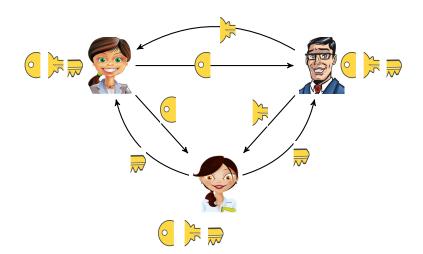












### Phase 1: Setup Phase



















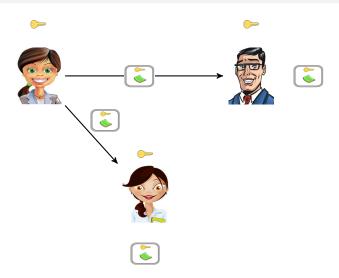


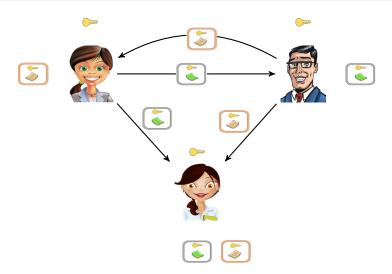




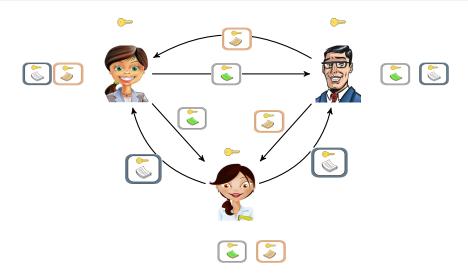






































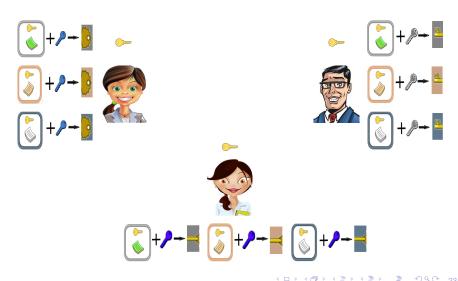
If any party does not receive verifiable encryption, (s)he aborts.





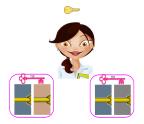


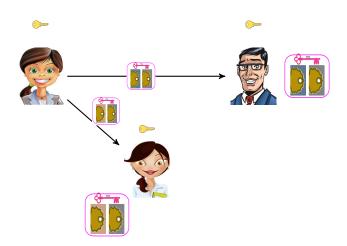


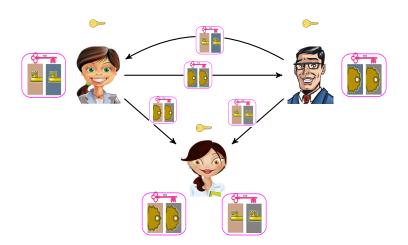


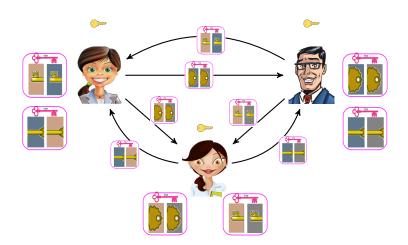
















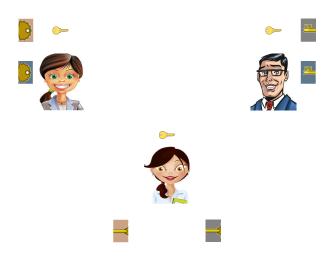
If any party does not receive verifiable escrow or receive wrong one(s) before  $t_1$ , (s)he does Resolve 1.

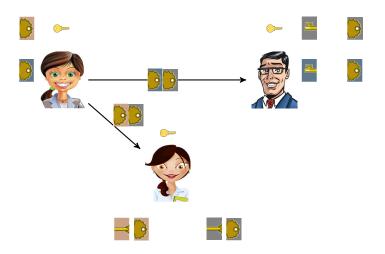




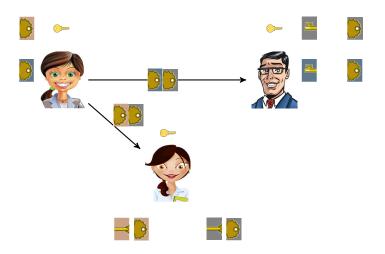




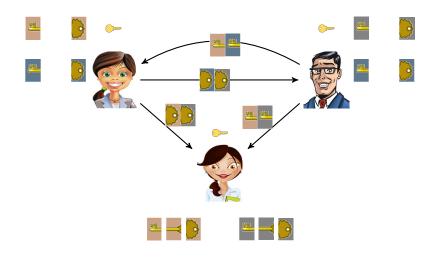


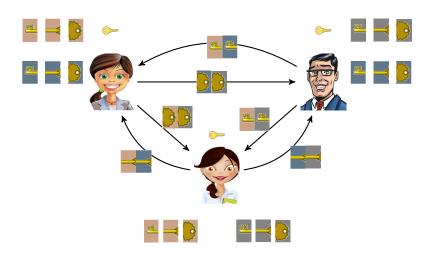




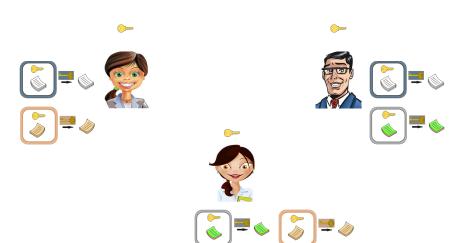














If any party does not receive decryption shares or receive wrong one(s) before  $t_2$ , (s)he does Resolve 2.











#### Resolve 1

- Parties do *not* learn any decryption shares here.
- They can just complain about other parties to the TTP.
- The TTP creates a fresh *complaintList* for the protocol with parameters id,  $t_1$ ,  $t_2$ .

### Resolve 2

- The party P<sub>i</sub>, who comes for Resolve 2 between t<sub>1</sub> and t<sub>2</sub>, gives all verifiable escrows that he has already received from the other parties and his own verifiable escrow to the TTP.
- The TTP uses these verifiable escrows to save the decryption shares required to solve the complaints in the complaintList.
  - If the complaintList is not empty in the end, P<sub>i</sub> comes after t<sub>2</sub> for Resolve 3.
  - Otherwise, TTP decrypts the verifiable escrow and gives decryption shares.

### Resolve 3

- If the complaintList still has parties, even after t<sub>2</sub>, the TTP answers each resolving party saying that the protocol is aborted, which means nobody is able to learn any item.
- If the *complaintList* is *empty*, the TTP decrypts any verifiable escrow that is given to him.

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#### **SMPC**

Parties are able to compute the following function in a secure way by using SMPC protocol.

$$\phi(\mathbf{w}_1,...,\mathbf{w}_n) = (\phi_1(\mathbf{w}_1,...,\mathbf{w}_n),...,\phi_n(\mathbf{w}_1,...,\mathbf{w}_n))$$

#### Fair SMPC

- Change input of the each  $P_i$  as  $z_i = (w_i, x_i)$ .
- Compute the following functionality with SMPC.

$$\psi_i(z_1, z_2, ..., z_n) = (E_i(\phi_i(w_1, ..., w_n)), \{g^{x_j}\}_{1 \le j \le n})$$

where

$$E_i(\phi_i(w_1,...,w_n)) = (g^{r_i},\phi_i h^{r_i})$$

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### Why MFE is fair?

- Parties do not learn anything without any missing decryption share.
  - $\Rightarrow$  All parties depend each other. So even though n-1 malicious party exist, they can not exclude an honest party.
- If an honest party does not receive verifiable escrow, (s)he does not continue.
  - ⇒ This obliges malicious party to send his verifiable escrow to the honest party, otherwise malicious one cannot learn anything.
- TTP does not decrypt verifiable escrow and send any decryption share until it is sure that he has all missing verifiable escrows.
  - ⇒ Resolve protocols preserve fairness.

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### Privacy in MFE and MPC

### Privacy

The **privacy against the TTP is preserved**. He just learns some decryption shares, but he cannot decrypt the encryption of exchanged items, since he never gets the encrypted items.

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    - Resolve Protocols
  - Fair and Secure MPC
- Conclusion
  - Security and Fairness
  - Comparison with Previous Works
  - Conclusion



# Previous works in Complete Topology

	Solution	Topology	Round Complexity	Number of Messages	Broad- cast
Garay & MacKenzie	MPCS	Complete	O(n <sup>2</sup> )	<i>O</i> ( <i>n</i> <sup>3</sup> )	Yes
Baum & Waidner	MPCS	Complete	O(tn)	O(tn²)	Yes
Mukhamedov & Ryan	MPCS	Complete	O(n)	<i>O</i> ( <i>n</i> <sup>3</sup> )	Yes
Mauw et al.	MPCS	Complete	O(n)	$O(n^2)$	Yes
Asokan et al.	MFE 🗸	Any 🗸	O(1) ✓	$O(n^3)$	Yes
Ours	MFE 🗸	Any 🗸	O(1) ✓	O(n²) ✓	No 🗸

# Previous works in Ring Topology

	Number Messages	All or None	TTP-Party Dependency	TTP Privacy
Bao et al.	O(n)	No	Yes	Not Private
González & Markowitch	O(n <sup>2</sup> )	No	Yes	Not Private
Liu & Hu	O(n)	No	Yes	Not Private
Ours	$O(n^2)$	Yes ✓	No 🗸	Private 🗸

### Previous works in fair SMPC

	Technique	TTP	Number of Rounds	Proof Technique
Garay et al.	Gradual Release	No	$O(\lambda)$	NFS
Bentov & Kumaresan	Bitcoin	Yes	Constant 🗸	NFS
Andrychowicz et al.	Bitcoin	Yes	Constant 🗸	NFS
Ours	MFE	Yes	Constant ✓	FS 🗸

### **Outline**

- Introduction
  - Multi-Party Fair Exchange
  - Definitions
- Our New Protocols
  - MFE Protocol
    - Resolve Protocols
  - Fair and Secure MPC
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- ✓ We design a MFE protocol requires only O(n²) messages and constant number of rounds for n parties.
- ✓ Our MFE **optimally** (in complete topology) guarantees fairness (for honest parties) even when n-1 out of n parties are malicious and colluding.
- We show how to employ our MFE protocol for any exchange topology, with the performance improving as the topology gets sparser.
- We formulate MFE as a secure multi-party computation protocol. We then prove security and fairness via ideal-real world simulation [9].



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### TTP Usage

- The TTP for fairness in our MFE is in the optimistic model The TTP has a very low workload.
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### Secure Multi-party Computation

- Our MFE can be employed on top of any SMPC protocol to obtain a fair SMPC protocol,
- ✓ We provide an ideal world definition for fair SMPC, and prove security and fairness of a SMPC protocol that use our MFE via simulation.

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### **Authors**



Handan Kılınç
PHD student at EPFL
handan.kilinc@epfl.ch



Asst. Prof. Alptekin Küpçü at Koç University akupcu@ku.edu.tr

# For Further Reading I



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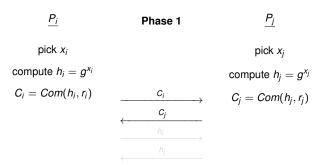
## Phase 1: Setup Phase

All participants agree on the prime p-order subgroup of  $\mathbb{Z}_q^*$ , where q is a large prime, and a generator g of this subgroup. Then each  $P_i$  does

<u> P</u> ;	Phase 1	$\underline{P_j}$
pick x <sub>i</sub>		pick x <sub>j</sub>
compute $h_i = g^{x_i}$		compute $h_j = g^{x_j}$
$C_i = Com(h_i, r_i)$	$\xrightarrow{C_i}$	$C_j = Com(h_j, r_j)$

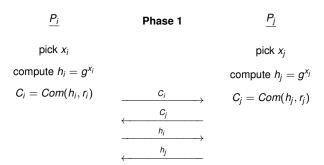
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If VE is not received from at least one of the parties

Abort

$$\begin{array}{ccc} \underline{P_i} & \textbf{Phase 2} & \underline{P_j} \\ \\ \text{compute } h = \prod_{k=0}^n h_k & \text{compute } h = \prod_{k=0}^n h_k \\ \\ \text{pick } r_i & \text{pick } r_j \\ \\ E_i = (a_i, b_i) = (g^{r_i}, f_i h^{r_i}) & \underbrace{{}^{VE_j = V(E_i, h;\emptyset)\{(v_i, f_i) \in B_{flem}\}}}_{VE_j = V(E_j, h;\emptyset)\{(v_j, f_j) \in B_{flem}\}} & E_j = (a_j, b_j) = (g^{r_j}, f_j h^{r_j}) \end{array}$$

If VE is not received from at least one of the parties

Abort

$$\frac{P_i}{\text{compute }h = \prod_{k=0}^n h_k} \qquad \qquad \text{compute }h = \prod_{k=0}^n h_k$$

$$\text{pick } r_i \qquad \qquad \text{pick } r_j$$

$$E_i = (a_i, b_i) = (g^{r_i}, f_i h^{r_i}) \xrightarrow{VE_i = V(E_i, h; \emptyset) \{(v_i, f_i) \in R_{ilem}\}} \qquad E_j = (a_j, b_j) = (g^{r_j}, f_j h^{r_j})$$

If VE is not received from at least one of the parties

Abort

If VE is not received from at least one of the parties

$$\underbrace{P_i}_{\text{compute }h = \prod_{k=0}^n h_k}_{\text{pick }r_i} \text{ pick }r_i \\ E_i = (a_i,b_i) = (g^{r_i},f_ih^{r_i}) \underbrace{\frac{VE_i = V(E_i,h;\emptyset)\{(v_i,f_i) \in R_{item}\}}{VE_j = V(E_j,h;\emptyset)\{(v_j,f_j) \in R_{item}\}}}_{\text{for }VE_i = V(E_j,h;\emptyset)\{(v_j,f_j) \in R_{item}\}} E_j = (a_j,b_j) = (g^{r_j},f_jh^{r_j}) \\ \underbrace{\frac{VE_i = V(E_j,h;\emptyset)\{(v_j,f_j) \in R_{item}\}}{VE_j = V(E_j,h;\emptyset)\{(v_j,f_j) \in R_{item}\}}}_{\text{for }I = Item} E_j = (a_j,b_j) = (g^{r_j},f_jh^{r_j})$$

**Abort** 

Note that  $\mathbf{a_k} = \mathbf{g^{r_k}}$  (First part of the  $k^{th}$  item's encryption). The relation  $R_s$  is  $\mathbf{log_gh_i} = \mathbf{log_{a_k}a_k^{x_i}}$  for each k.

$$\frac{P_{i}}{\text{compute }} \{d_{k}^{i} = a_{k}^{x_{i}}\}_{k=1}^{n} \xrightarrow{VS_{i} = V(E_{i}^{i}, pk; t_{1}, t_{2}, id, P_{i})\{(h_{i}, \{d_{k}^{i}\}) \in R_{S}\}} \xrightarrow{\text{compute }} \{d_{k}^{j} = a_{k}^{x_{j}}\}_{k=1}^{n}} \xrightarrow{VS_{i} = V(E_{i}^{i}, pk; t_{1}, t_{2}, id, P_{i})\{(h_{i}, \{d_{k}^{i}\}) \in R_{S}\}} \xrightarrow{\text{compute }} \{d_{k}^{j} = a_{k}^{x_{j}}\}_{k=1}^{n}} \xrightarrow{VS_{i} = V(E_{i}^{j}, pk; t_{1}, t_{2}, id, P_{i})\{(h_{i}, \{d_{k}^{i}\}) \in R_{S}\}} \xrightarrow{E_{i}^{t} = Enc_{pk}(\{d_{k}^{j}\}_{k=1}^{n})} \xrightarrow{\{d_{k}^{i}\}, PK(h_{i}, \{a_{k}\})\{(h_{i}, \{d_{k}^{i}\}) \in R_{S}\}} \xrightarrow{\{d_{k}^{i}\}, PK(h_{i}, \{a_{k}\})\{(h_{i}, \{d_{k}^{i}\}) \in R_{S}\}}$$

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Resolve

 $\{d_k^l\}_{j,PK(h_l,\{a_k\})}\{(h_l,\{d_k^l\})\in R_s\}$ 

 $\{d_k^J\}, PK(h_i, \{a_k\}) \{(h_i, \{d_k^J\}) \in R_s\}$ 

If  $d_k^i$  are not received before  $t_2$ 

Resolve 2

Note that  $\mathbf{a_k} = \mathbf{g^{r_k}}$  (First part of the  $k^{th}$  item's encryption). The relation  $R_s$  is  $\mathbf{log_gh_i} = \mathbf{log_{a_k}a_k^{x_i}}$  for each k.

Decelve

 $\{d_k^i\}_{j,PK(h_i,\{a_k\})}\{(h_i,\{d_k^i\})\in R_s\}$ 

 $\{d_k^j\}, PK(h_i, \{a_k\})\}\{(h_i, \{d_k^j\}) \in R_s\}$ 

If  $d_k^i$  are not received before  $t_2$ 

Resolve 2



Note that  $\mathbf{a_k} = \mathbf{g^{r_k}}$  (First part of the  $k^{th}$  item's encryption). The relation  $R_s$  is  $\mathbf{log_gh_i} = \mathbf{log_{a_k}a_k^{x_i}}$  for each k.

$$\begin{array}{c|c} \underline{P_i} & \textbf{Phase 3} & \underline{P_j} \\ \text{compute } \{d_k^j = a_k^{x_i}\}_{k=1}^n & \underbrace{v_{S_i = V(E_i^t, pk; t_1, t_2, id, P_i)\{(h_i, \{d_k^i\}) \in R_s\}}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & \text{compute } \{d_k^j = a_k^{x_j}\}_{k=1}^n \\ \hline E_i^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) & \underbrace{v_{S_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) \\ \hline E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) & \underbrace{v_{S_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) \\ \hline E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) & \underbrace{v_{S_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) \\ \hline E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) & \underbrace{v_{S_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) \\ \hline E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) \\ \hline E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) \\ \hline E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}_{k=1}^n)\}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\}} & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)} & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)} & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)} & \underbrace{E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n)}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)} & \underbrace{E_j^t = Enc$$

Resolve

 $\{d_k^l\}_{j,PK(h_i,\{a_k\})}\{(h_i,\{d_k^l\})\in R_s\}$ 

 $\left\langle \frac{(n_i, (a_k), (n_i, (a_k)) \in n_s)}{(n_i, (a_k)) \in n_s} \right\rangle$ 

If  $d_k^i$  are not received before  $t_2$ 

Note that  $\mathbf{a_k} = \mathbf{g^{r_k}}$  (First part of the  $k^{th}$  item's encryption). The relation  $R_s$  is  $\mathbf{log_gh_i} = \mathbf{log_{a_k}a_k^{x_i}}$  for each k.

$$\begin{array}{c|c} \underline{P_i} & \textbf{Phase 3} & \underline{P_j} \\ \text{compute } \{d_k^j = a_k^{x_i}\}_{k=1}^n & \underbrace{v_{S_i = V(E_i^t, pk; t_1, t_2, id, P_i)\{(h_i, \{d_k^i\}) \in R_s\}}_{VS_i = V(E_i^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & \text{compute } \{d_k^j = a_k^{x_j}\}_{k=1}^n \\ \hline E_i^t = Enc_{pk}(\{d_k^i\}_{k=1}^n) & \underbrace{v_{S_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}}_{VS_i = V(E_j^t, pk; t_1, t_2, id, P_j)\{(h_j, \{d_k^j\}) \in R_s\}} & E_j^t = Enc_{pk}(\{d_k^j\}_{k=1}^n) \\ \hline \\ If VS \text{ is not received from at least one of the parties before time } t_1 \\ \hline \end{array}$$

#### Resolve 1

 $\begin{cases} \{d_k^j\}, PK(h_i, \{a_k\})\{(h_i, \{d_k^j\}) \in R_s\} \\ \{d_k^j\}, PK(h_i, \{a_k\})\{(h_i, \{d_k^j\}) \in R_s\} \end{cases}$   $\begin{cases} \{d_k^j\}, PK(h_i, \{a_k\})\{(h_i, \{d_k^j\}) \in R_s\} \\ \{d_k^j\}, PK(h_i, \{a_k\})\{(h_i, \{d_k^j\}) \in R_s\} \end{cases}$ 

Note that  $\mathbf{a_k} = \mathbf{g^{r_k}}$  (First part of the  $k^{th}$  item's encryption). The relation  $R_s$  is  $\mathbf{log_gh_i} = \mathbf{log_{a_k}a_k^{x_i}}$  for each k.

#### Resolve 1

$$\underbrace{\{d_k^j\}_{j,PK}(h_i,\{a_k\})\{(h_i,\{d_k^j\})\in R_s\}}_{\{d_k^j\}_{j,PK}(h_i,\{a_k\})\{(h_i,\{d_k^j\})\in R_s\}}$$

If  $d_k^i$  are not received before  $t_2$ 

Note that  $\mathbf{a_k} = \mathbf{g^{r_k}}$  (First part of the  $k^{th}$  item's encryption). The relation  $R_s$  is  $\mathbf{log_gh_i} = \mathbf{log_{a_k}a_k^{x_i}}$  for each k.

#### Resolve 1

$$\underbrace{\{d_k^i\}_j, PK(h_i, \{a_k\})\{(h_i, \{d_k^i\}) \in R_s\}}_{\{d_k^i\}, PK(h_i, \{a_k\})\}\{(h_i, \{d_k^i\}) \in R_s\}}$$

If  $d_k^i$  are not received before  $t_2$ 

Resolve 2



Note that  $\mathbf{a_k} = \mathbf{g^{r_k}}$  (First part of the  $k^{th}$  item's encryption). The relation  $R_s$  is  $\mathbf{log_gh_i} = \mathbf{log_{a_k}a_k^{x_i}}$  for each k.

#### Resolve 1

$$\underbrace{\{g_k^i\}_j, PK(h_i, \{a_k\})\{(h_i, \{g_k^i\}) \in R_s\}}_{\neq (g_k^i), PK(h_i, \{a_k\})\{(h_i, \{g_k^i\}) \in R_s\}}$$

If  $d_k^i$  are not received before  $t_2$ 

Resolve 2