RSAConference2015

San Francisco | April 20-24 | Moscone Center

CHANGE

Challenge today's security thinking

SESSION ID: CRYP-F03

Communication Optimal Tardos-based Asymmetric Fingerprinting

Qiang Tang

University of Connecticut & University of Athens

joint work with Aggelos Kiayias, Nikos Leonardos, Helger Lipmaa, and Kateryna Pavlyk

A Motivational Example

A Movie Producer

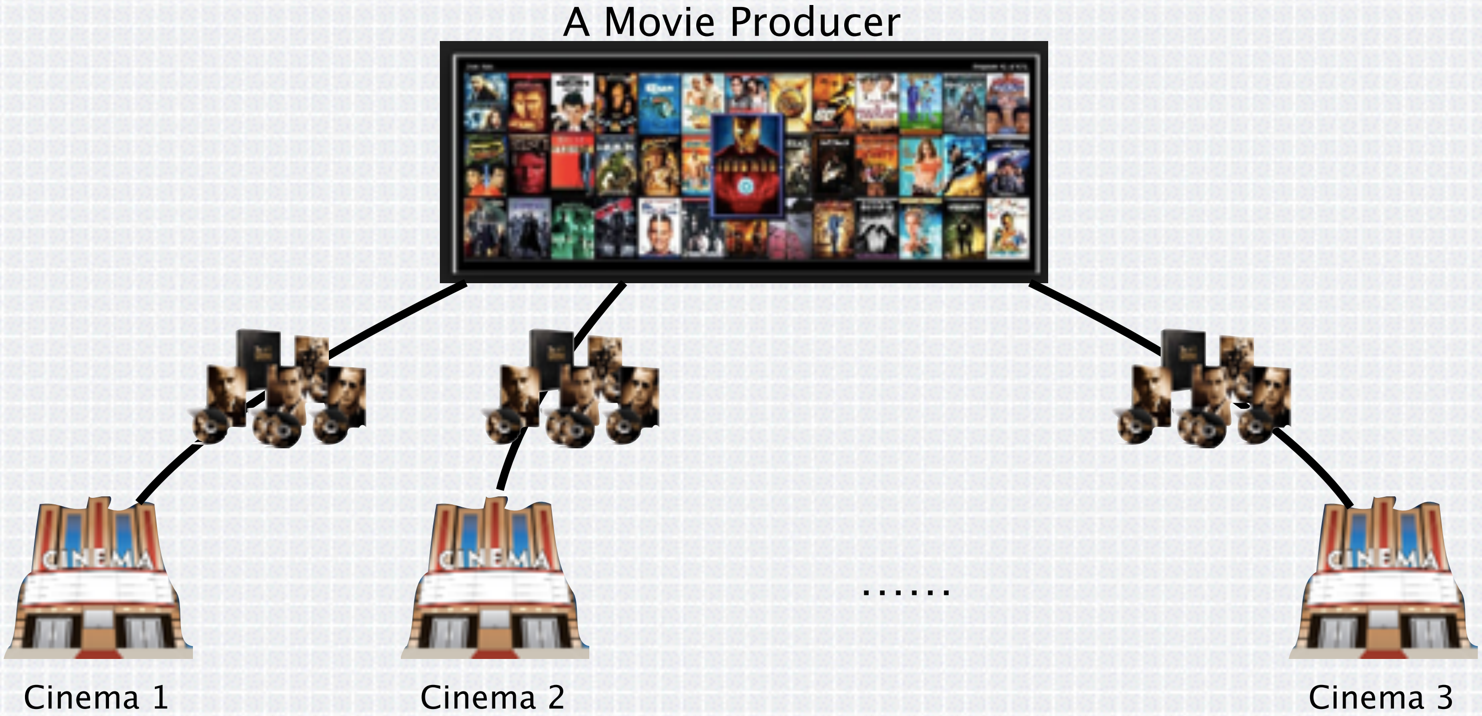
Cinema 1

Cinema 2

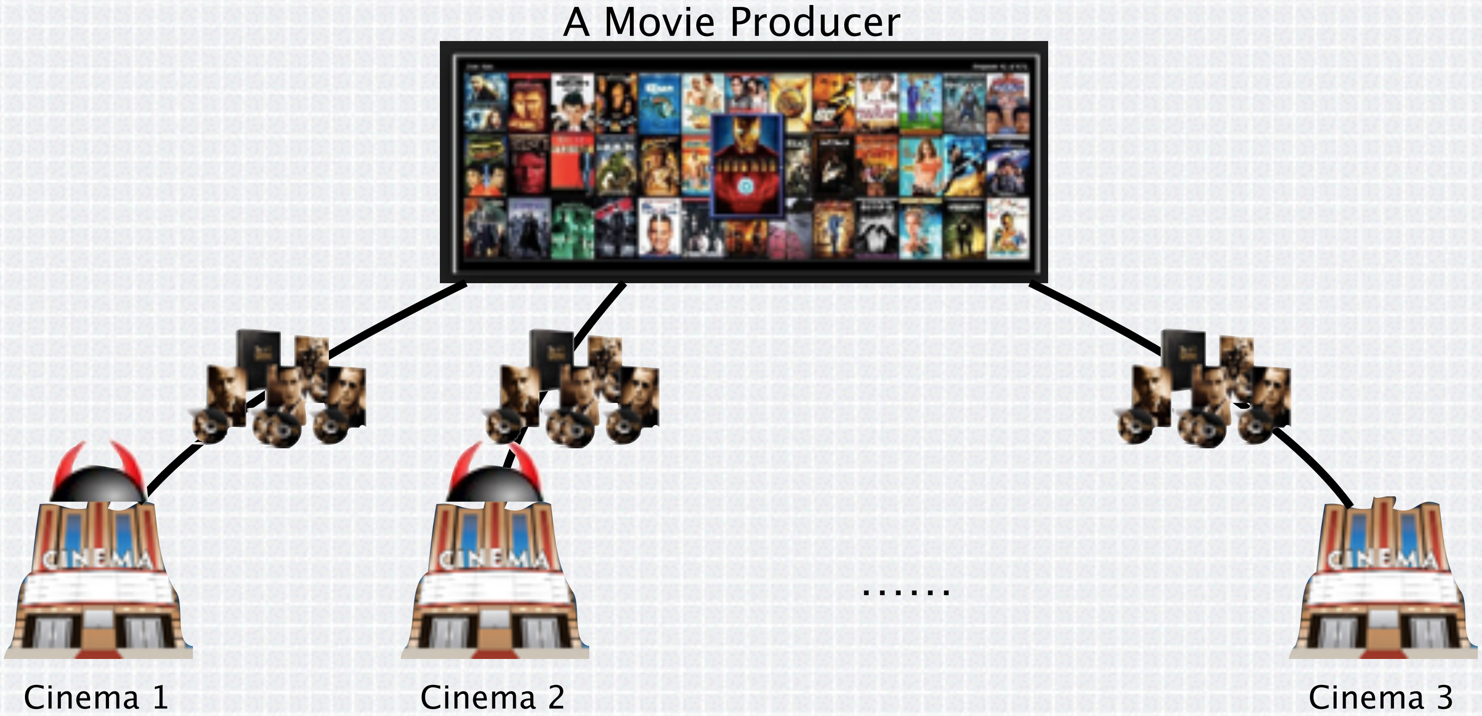
......

Cinema 3

A Motivational Example



A Motivational Example

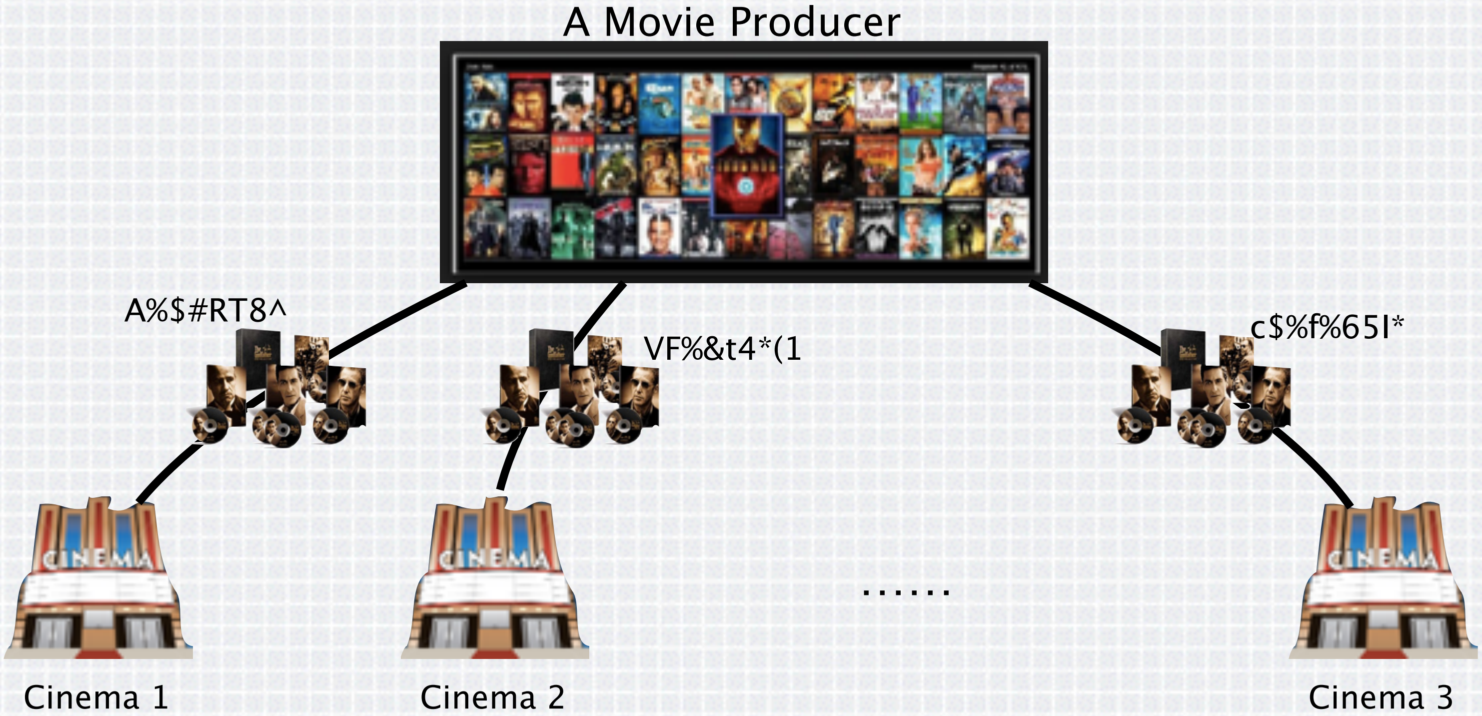


A Motivational Example

A Motivational Example

How to identify the source of the pirate?

Fingerprinting



A Motivational Example

Fingerprinting

Fingerprinting

VF%&#RT8^

Fingerprinting

pirate codeword

VF%&#RT8^

Fingerprinting

pirate codeword

VF%&#RT8^ → VF%&t4*(1

A%$#RT8^

Fingerprinting

pirate codeword

VF%&#RT8^

VF%&t4*(1

A%$#RT8^

tracing algorithm

Fingerprinting

pirate codeword

VF%&#RT8^

tracing algorithm

VF%&t4*(1 → Cinema 1

A%$#RT8^ → Cinema 5

Fingerprinting

pirate codeword

compare to the DB

VF%&#RT8^

VF%&t4*(1 → Cinema 1

A%$#RT8^ → Cinema 5

tracing algorithm

The Goals of Fingerprinting

- Individualize contents

The Goals of Fingerprinting

- Individualize contents
- Trace back to the sources

A Catch

Does fingerprinting really de-incentivize illegal content re-distribution?

A Catch

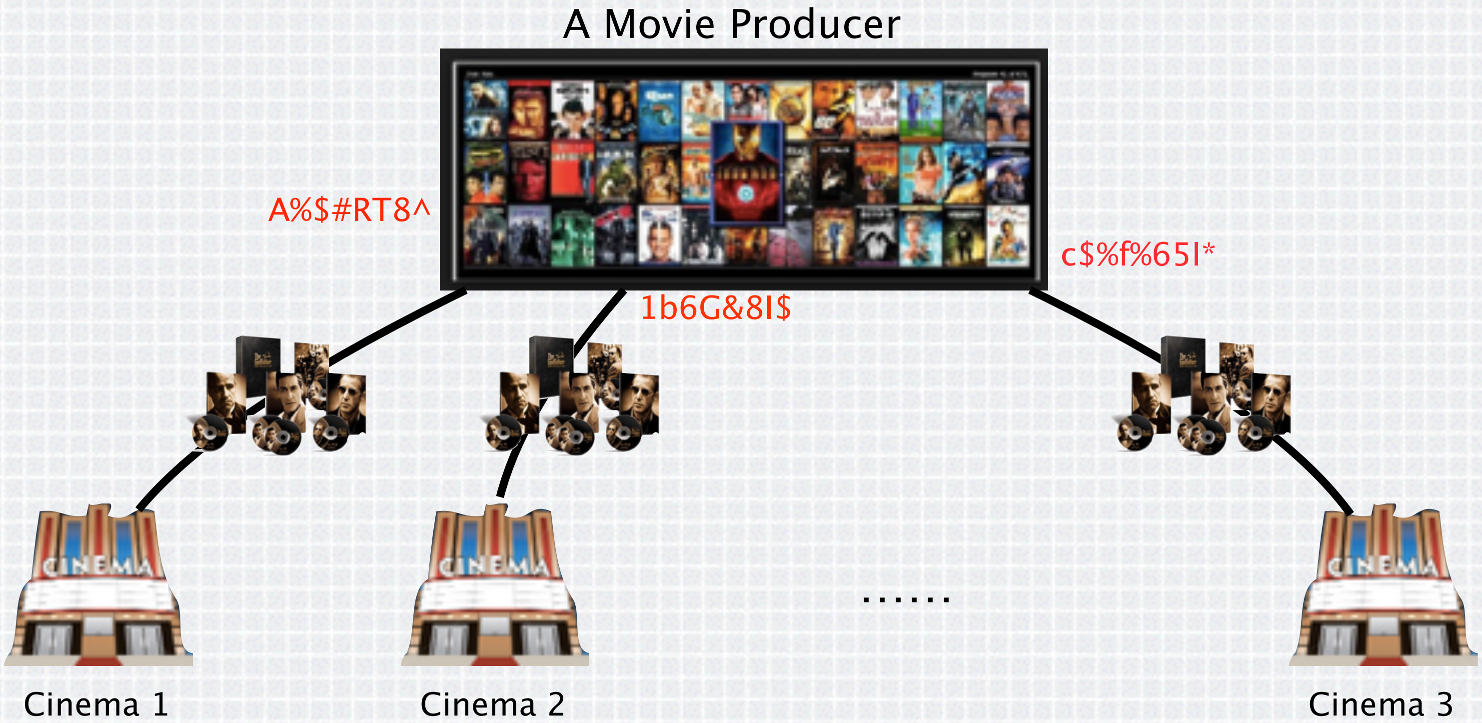
- Both the content provider and the content receiver can leak a copy

A Catch

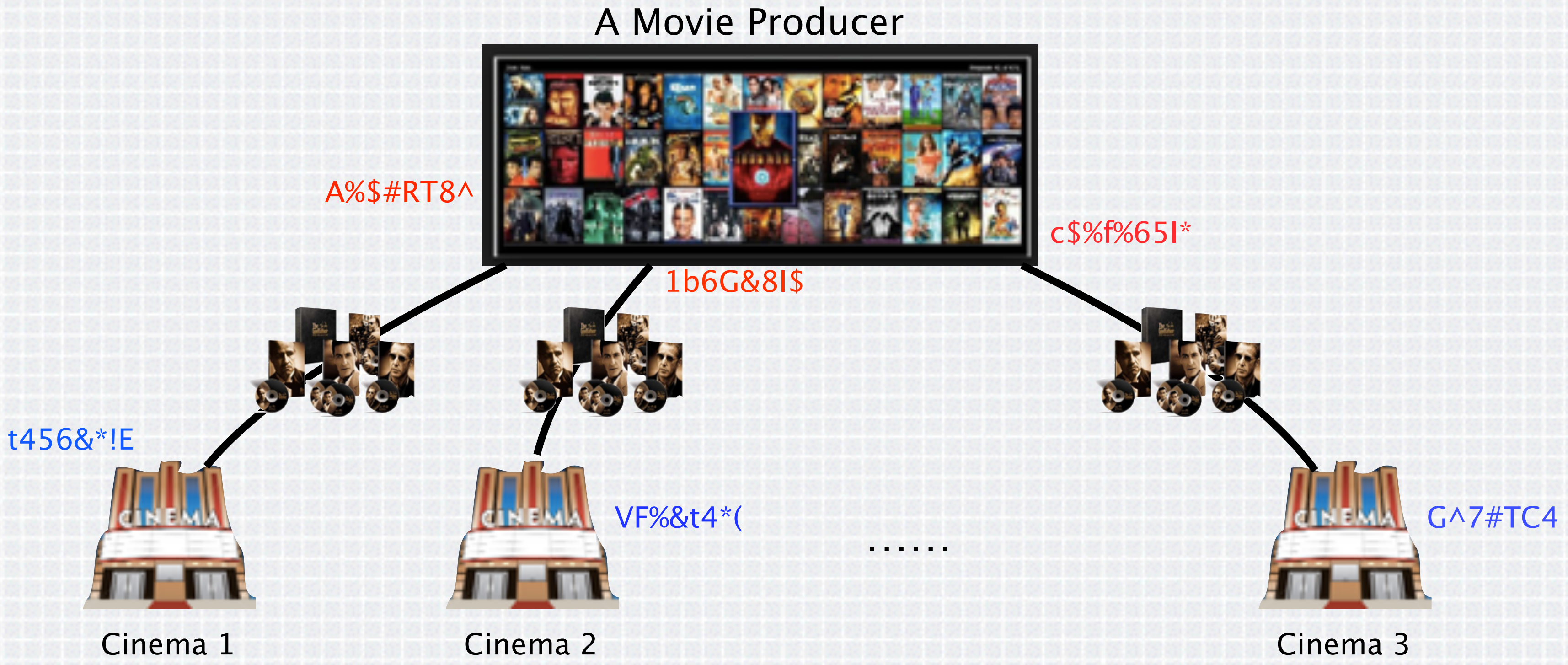
- Both the content provider and the content receiver can leak a copy
- The copy found in the public can not serve as a undeniable proof

Asymmetric Fingerprinting

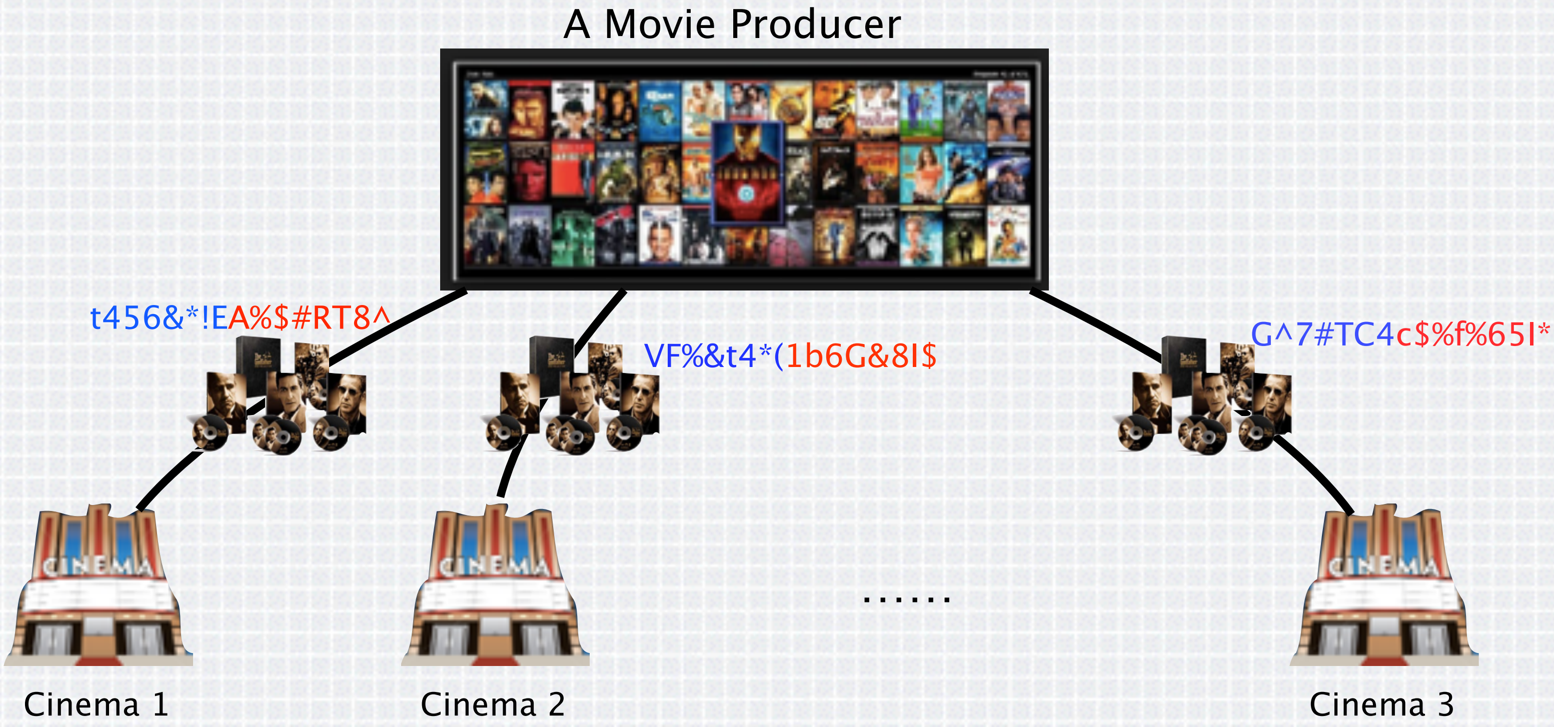
Asymmetric Fingerprinting



Asymmetric Fingerprinting



Asymmetric Fingerprinting



Asymmetric Fingerprinting

Asymmetric Fingerprinting

Asymmetric Fingerprinting

half code

VF%&t4*(11b6G&8I$

G^7#TC4A%$#RT8^

Asymmetric Fingerprinting

half code

half DB

VF%&t4*(11b6G&8I$

G^7#TC4A%$#RT8^

Asymmetric Fingerprinting

half code

half DB

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

t456&*!EA%$#RT8^

Cinema 1

VF%&t4*(1b6G&8I$

Cinema 2

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

t456&*!EA%$#RT8^

Cinema 1

t456&*!E

VF%&t4*(

VF%&t4*(1b6G&8I$

Cinema 2

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

t456&*!EA%$#RT8^

Cinema 1

t456&*!E

VF%&t4*(

VF%&t4*(1b6G&8I$

Cinema 2

Cinema 1 ✗

Cinema 2 ✓

A Subtle Security Consideration

t456&*!EA%$#RT8^

VF%&t4*(1b6G&8I$

VF%&t4*(A%$#RT8^

A Subtle Security Consideration

t456&*!EA%$#RT8^

VF%&t4*(1b6G&8I$

→

VF%&t4*(A%$#RT8^

Accusation Withdraw

A Subtle Security Consideration

1. User should not know how the two halves are mixed

A Subtle Security Consideration

1. User should not know how the two halves are mixed

2. Lower down the tracing parameter at the judge side

Important Efficiency Consideration

Even transmitting 2* movie size kills the bandwidth

Important Efficiency Consideration

Even transmitting 2* movie size kills the bandwidth

And will hinder the adoption of this technique

Important Efficiency Consideration

Size of 

Rate= ―――――――――――――――

Size of actual transmission

Important Efficiency Consideration

Size of 

Rate= ――――――――――――――――――――― ⟶ 1

Size of actual transmission

Important Efficiency Consideration

Fight Piracy without Extra Bandwidth Cost !

#RSAC

High-level Construction Idea

Fingerprint Phase

- The content provider and a movie theater jointly samples a codeword (during the transmission of the movies), which is oblivious to the CP (using a conditional OT)

Fingerprint Phase

- The content provider and a movie theater jointly samples a codeword (during the transmission of the movies), which is oblivious to the CP (using a conditional OT)
- The CP and the movie theater runs OT protocols to see half of the codeword

Fingerprint Phase

- The content provider and a movie theater jointly samples a codeword (during the transmission of the movies), which is oblivious to the CP (using a conditional OT)
- The CP and the movie theater runs OT protocols to see half of the codeword

CP only knows half of the codeword

Fingerprint Phase

- The content provider and a movie theater jointly samples a codeword (during the transmission of the movies), which is oblivious to the CP (using a conditional OT)
- The CP and the movie theater runs OT protocols to see half of the codeword

CP only knows half of the codeword

Theaters don't know which part is known to the CP

Fingerprint Phase

- The content provider and a movie theater jointly samples a codeword (during the transmission of the movies), which is oblivious to the CP (using a conditional OT)
- The CP and the movie theater runs OT protocols to see half of the codeword

CP only knows half of the codeword

Theaters don’t know which part is known to the CP

Rate optimal OT and COT are needed

Identify Phase

- Run the tracing algorithm of the underlying fingerprinting code on the half known to the CP

Dispute Phase

- The accused movie theaters submit the other halves of the codewords (with the proof)

Dispute Phase

- The accused movie theaters submit the other halves of the codewords (with the proof)
- The judge also runs the tracing algorithm with a less restrict parameter on these halves

Dispute Phase

- The accused movie theaters submit the other halves of the codewords (with the proof)
- The judge also runs the tracing algorithm with a less restrict parameter on these halves

Weaker judge side parameter is to avoid accusation withdraw

Communication Optimal Tardos-Based Asymmetric Fingerprinting

Linearly Homomorphic Encryption from DDH

Guilhem Castagnos[1] Fabien Laguillaumie[2]

[1] Université de Bordeaux
INRIA Bordeaux - Sud-Ouest - LFANT
Institut de Mathématiques de Bordeaux UMR 5251,

[2] Université Claude Bernard Lyon 1
CNRS/ENSL/INRIA/UCBL LIP
Laboratoire de l'Informatique du Parallélisme

CT-RSA 2015

Outline

Outline

Linearly Homomorphic Encryption ?

- Public key encryption scheme with the following properties:
- Suppose that the set of plaintexts $\mathscr{M}$ is a ring
- $c \leftarrow \mathsf{Encrypt}(pk, m)$, $c' \leftarrow \mathsf{Encrypt}(pk, m')$
- $c_1 \leftarrow \mathsf{EvalSum}(pk, c, c')$ s.t.

$$\mathsf{Decrypt}(sk, c_1) = m + m'$$

- For $\alpha \in \mathscr{M}$, $c_2 \leftarrow \mathsf{EvalScal}(pk, c, \alpha)$ s.t.

$$\mathsf{Decrypt}(sk, c_2) = \alpha m$$

- Applications: Electronic Voting, Private Information Retrieval, Mix-Net, Oblivious Transfer, Fingerprinting...

Examples from Factoring

- Goldwasser Micali (84)
  - Plaintext space $\mathscr{M} = \mathbf{Z}/2\mathbf{Z}$
  - Ciphertext space : $\mathbf{Z}/N\mathbf{Z}$ where $N = pq$ is an RSA integer
- Paillier (99)
  - Plaintext space $\mathscr{M} = \mathbf{Z}/N\mathbf{Z}$
  - Ciphertext space : $\mathbf{Z}/N^2\mathbf{Z}$ where $N = pq$ is an RSA integer
  - Plaintext encoding :

$$m \in \mathbf{Z}/N\mathbf{Z} \mapsto (1+N)^m \equiv 1 + mN \pmod{N^2}$$

From DDH: ElGamal "in the exponent"

- Folklore message encoding: $m \in \mathbf{N} \mapsto g^m$
- $(c_1, c_2) = (g^r, h^r g^m) \leftarrow \mathsf{Encrypt}(pk, m)$
- $\mathsf{Decrypt}(pk, c) : c_2/c_1^x = g^m \rightsquigarrow m$
- m must be small. Can only do a bounded number of homomorphic operations:
  - $(c_1, c_2) = (g^r, h^r g^m) \leftarrow \mathsf{Encrypt}(pk, m)$,
  - $(c_1', c_2') = (g^{r'}, h^{r'} g^{m'}) \leftarrow \mathsf{Encrypt}(pk, m')$,

$$(c_1 c_1', c_2 c_2') = (g^{r+r'}, h^{r+r'} g^{m+m'})$$

$$(c_1^\alpha, c_2^\alpha) = (g^{r\alpha}, h^{r\alpha} g^{m\alpha})$$

DDH group with an easy DL subgroup

- $(G, \times) = \langle g \rangle$ a cyclic group of order n
- $n = ps$, $\gcd(p, s) = 1$
- $\langle f \rangle = F \subset G$ subgroup of G of order p
- The DL problem is easy in F: There exists, Solve, a deterministic polynomial time algorithm s.t.

$$\mathsf{Solve}(p, f, f^x) \rightsquigarrow x$$

- The DDH problem is hard in G even with access to the Solve algorithm

A Generic Linearly Homomorphic Encryption Scheme

- $\mathcal{M} = \mathbf{Z}/p\mathbf{Z}$
- $pk : h = g^x$, $sk : x$, where g has order $n = ps$ for an unknown s
- $\mathsf{Encrypt} : c = (c_1, c_2) = (g^r, f^m h^r)$, where $f \in \langle g \rangle$ has order p
- $\mathsf{Decrypt} : \mathrm{A} \leftarrow c_2/c_1^x$, $\mathsf{Solve}(p, f, \mathrm{A}) \rightsquigarrow m$
- $\mathsf{EvalSum}$:

$$(c_1c_1', c_2c_2') = (g^{r+r'}, h^{r+r'} f^{m+m'})$$

- $\mathsf{EvalScal}$:

$$(c_1^{\alpha}, c_2^{\alpha}) = (g^{r\alpha}, h^{r\alpha} f^{m\alpha})$$

An Unsecure Instantiation

- p a prime and $\mathrm{G} = \langle g \rangle = (\mathbf{Z}/p^2\mathbf{Z})^\times$ of order $n = p(p-1)$
- $f = 1 + p \in \mathrm{G}$, $\mathrm{F} = \langle f \rangle = \{1 + kp, k \in \mathrm{Z}/p\mathrm{Z}\}$
- $f^m = 1 + mp$.
- There exist a unique $(\alpha, r) \in (\mathbf{Z}/p\mathbf{Z}, (\mathbf{Z}/p\mathbf{Z})^\times)$ such that $g = f^\alpha r^p$

$$g^{p-1} = f^{\alpha(p-1)} = f^{-\alpha}$$

- Public key : $h = g^x$,

$$h^{p-1} = f^{-\alpha x} \rightsquigarrow x \mod p$$

- $(c_1, c_2) = (g^r, h^r f^m)$

$$c_1^{p-1} = f^{-\alpha r} \rightsquigarrow r \mod p$$

$$c_2^{p-1} = f^{-\alpha x r - m} \rightsquigarrow m \mod p$$

Partial Discrete Logarithm Problem

- $(G, \times) = \langle g \rangle$ a cyclic group of order n
- $n = ps$, $\gcd(p, s) = 1$
- $\langle f \rangle = F \subset G$ subgroup of G of order p
- Partial Discrete Logarithm (PDL) Problem:

$$\text{Given } X = g^x \text{ compute } x \mod p.$$

- The knowledge of s makes the PDL problem easy.

s must be hidden or unknown !

A Secure Instantiation

- Bresson, Catalano, Pointcheval (03)
- Let N be an RSA integer, $G = \langle g \rangle \subset (\mathbf{Z}/N^2\mathbf{Z})^\times$
- $n = \mathrm{Card}(G) = Ns$ with $s \mid \varphi(N)$,
- $f = 1 + N \in G$, $F = \langle f \rangle = \{1 + kN, k \in Z/NZ\}$, of order N
- Public key : $h = g^x$, x secret key
- $(c_1, c_2) = (g^r, h^r f^m)$
- Based on DDH in $(\mathbf{Z}/N^2\mathbf{Z})^\times$ and the Factorisation problem.
- The factorisation of N acts as a second trapdoor.

Outline

Definitions

Imaginary Quadratic Fields

- $K = \mathbf{Q}(\sqrt{\Delta_K})$, $\Delta_K < 0$
- Fundamental Discriminant:
  - $\Delta_K \equiv 1 \pmod 4$ square-free
  - $\Delta_K \equiv 0 \pmod 4$ and $\Delta_K/4 \equiv 2, 3 \pmod 4$ square-free
- Non Fundamental Discriminant:
  - $\Delta_\ell = \ell^2 \Delta_K$
  - ℓ is the conductor

Class Group of Discriminant Δ

- Finite Group denoted $C(\Delta)$
- Elements: Equivalence classes of Ideals
- Class Number: $h(\Delta) \approx \sqrt{|\Delta|}$

ElGamal in Class Group

- Buchmann and Williams (88): Diffie-Hellman key exchange and ElGamal
- Düllmann, Hamdy, Möller, Pohst, Schielzeth, Vollmer (90-07): Implementation
- Size of Δ_K? Index calculus algorithm to compute $h(\Delta_K)$ and Discrete Logarithm in $C(\Delta_K)$
- Security Estimates from Biasse, Jacobson and Silvester (10):
  - Complexity conjectured $L_{|\Delta_K|}(1/2, o(1))$
  - Δ_k : 1348 bits as hard as factoring a 2048 bits RSA integer
  - Δ_k : 1828 bits as hard as factoring a 3072 bits RSA integer

Map between two Class Groups

- Let Δ_K be a fondamental negative discriminant, $\Delta_K \neq -3, -4$, ℓ a conductor, and $\Delta_\ell = \ell^2 \Delta_K$
- There exists a surjective morphism, denoted $\bar{\varphi}_\ell$, between $C(\Delta_\ell)$ and $C(\Delta_K)$
- $\bar{\varphi}_\ell$ is effective, can be computed if ℓ is known
- Used by the NICE cryptosystem by Paulus and Takagi (00), $\Delta_K = -q$, $\Delta_p = -qp^2$, p, q primes, p is the trapdoor
- C., Laguillaumie (09) :

  In each non trivial class of $\ker \bar{\varphi}_p$, there exists an ideal of the form $\left[p^2\mathbf{Z} + \frac{bp+\sqrt{\Delta_p}}{2}\mathbf{Z}\right]$

Outline

A Subgroup with an Easy DL Problem

- $\Delta_K = -pq$, $\Delta_p = -qp^3$, p, q primes and $pq \equiv 3 \pmod 4$

$$h(\Delta_p) = p \times h(\Delta_K)$$

- Let $f = \left[p^2\mathbf{Z} + \frac{p+\sqrt{\Delta_p}}{2}\mathbf{Z}\right] \in C(\Delta_p)$

- $F = \ker \bar{\varphi}_p = \langle f \rangle$ is of order p, and

$$f^m = \left[p^2\mathbf{Z} + \frac{[m^{-1} \mod p]p + \sqrt{\Delta_p}}{2}\mathbf{Z}\right]$$

A New Linearly Homomorphic Encryption Scheme

- $\Delta_K = -pq$, $\Delta_p = -qp^3$, p, q primes and $pq \equiv 3 \pmod 4$ and $(p/q) = -1$, $q > 4p$
- Let g be an element of $C(\Delta_p)$, $h = g^x$ where x secret key
- g has order ps for an unknown $s | h(\Delta_K)$
- $(c_1, c_2) = (g^r, h^r f^m)$ where f has order p
- Based on DDH in $C(\Delta_p)$ (and the Class number problem).
- Linearly homomorphic over $\mathbf{Z}/p\mathbf{Z}$ where p can be chosen (almost) independently from the security parameter

Some Variants

- Faster Variant: most of the work in $C(\Delta_K)$ (based on a non standard problem)

- More general message spaces:
  - $\mathbf{Z}/N\mathbf{Z}$ with $N = \prod_{i=1}^{n} p_i$, with a discriminant of the form $\Delta_K = -Nq$
  - $\mathbf{Z}/p^t\mathbf{Z}$ for $t > 1$, with discriminants of the form $\Delta_{p^t} = p^{2t}\Delta_K$, and $\Delta_K = -pq$

Performance comparison

Cryptosystem	Parameter	Message Space	Encryption (ms)	Decryption (ms)
Paillier	2048 bits modulus	2048 bits	**28**	**28**
BCP03	2048 bits modulus	2048 bits	107	54
New Proposal	1348 bits Δ_K	80 bits	93	49
Fast Variant	1348 bits Δ_K	80 bits	82	45
Fast Variant	1348 bits Δ_K	256 bits	105	68
Paillier	3072 bits modulus	3072 bits	**109**	109
BCP03	3072 bits modulus	3072 bits	427	214
New Proposal	1828 bits Δ_K	80 bits	179	91
Fast Variant	1828 bits Δ_K	80 bits	145	**78**
Fast Variant	1828 bits Δ_K	512 bits	226	159
Fast Variant	1828 bits Δ_K	912 bits	340	271

Timings performed with Sage and PARI/GP.

Linearly Homomorphic Encryption from DDH

Guilhem Castagnos[1] Fabien Laguillaumie[2]

[1] Université de Bordeaux
INRIA Bordeaux - Sud-Ouest - LFANT
Institut de Mathématiques de Bordeaux UMR 5251,

[2] Université Claude Bernard Lyon 1
CNRS/ENSL/INRIA/UCBL LIP
Laboratoire de l'Informatique du Parallélisme

CT-RSA 2015