RSA®Conference2019

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BETTER.

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Automatic Search for A Variant of Division Property Using Three Subsets

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OUTLINE

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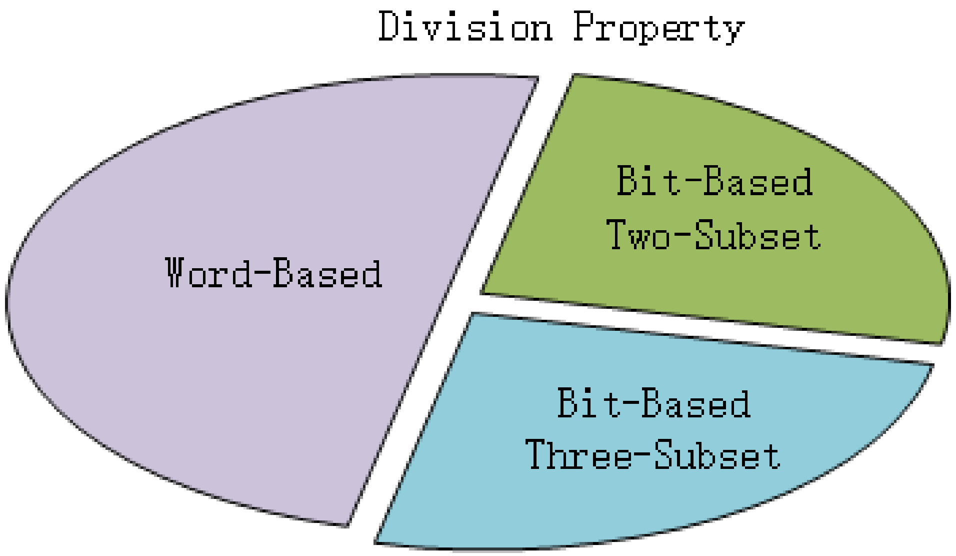
- ***1. Background of Division Property and Automatic Search***
- 2. Motivation and Contribution
- 3. A Variant of Three-Subset Division Property (VTDP)
- 4. Automatic Search for VTDP
- 5. Applications
- 6. Summary

What is Division Property (DP)?

- **A technique** to find **integral distinguishers** easily and efficiently
- Proposed by **Yosuke Todo** at **Eurocrypt'15**
- Divided into **Word-based DP** and **Bit-Based DP**
- **Bit-Based DP** is divided into **Two-Subset** and **Three-Subset**



What is Three-Subset Bit-Based Division Property ?

- **Sum all the ciphertexts together**

- Two-Subset DP **indicates** the sum of one bit of all the ciphertexts is

0 or Unknown

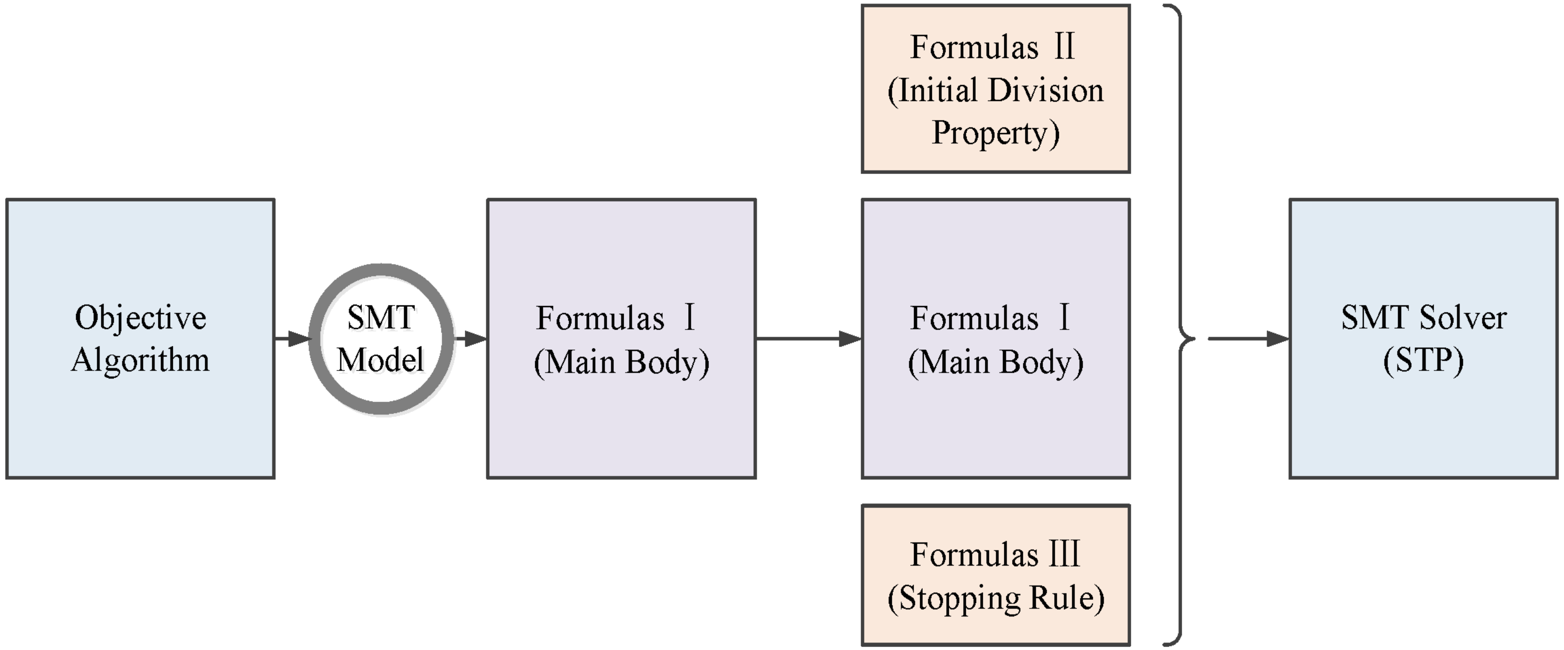
- Three-Subset DP **indicates** the sum of one bit of all the ciphertexts is

0 or 1 or Unknown

- Three-Subset DP is **more accurate** than any other division property

What is Automatic Search?

- **Tools** from **graph theory** can solve **constraint problems**
- Transform **cryptologic problems** into **constraint problems**
- **Solve** the constraint problems

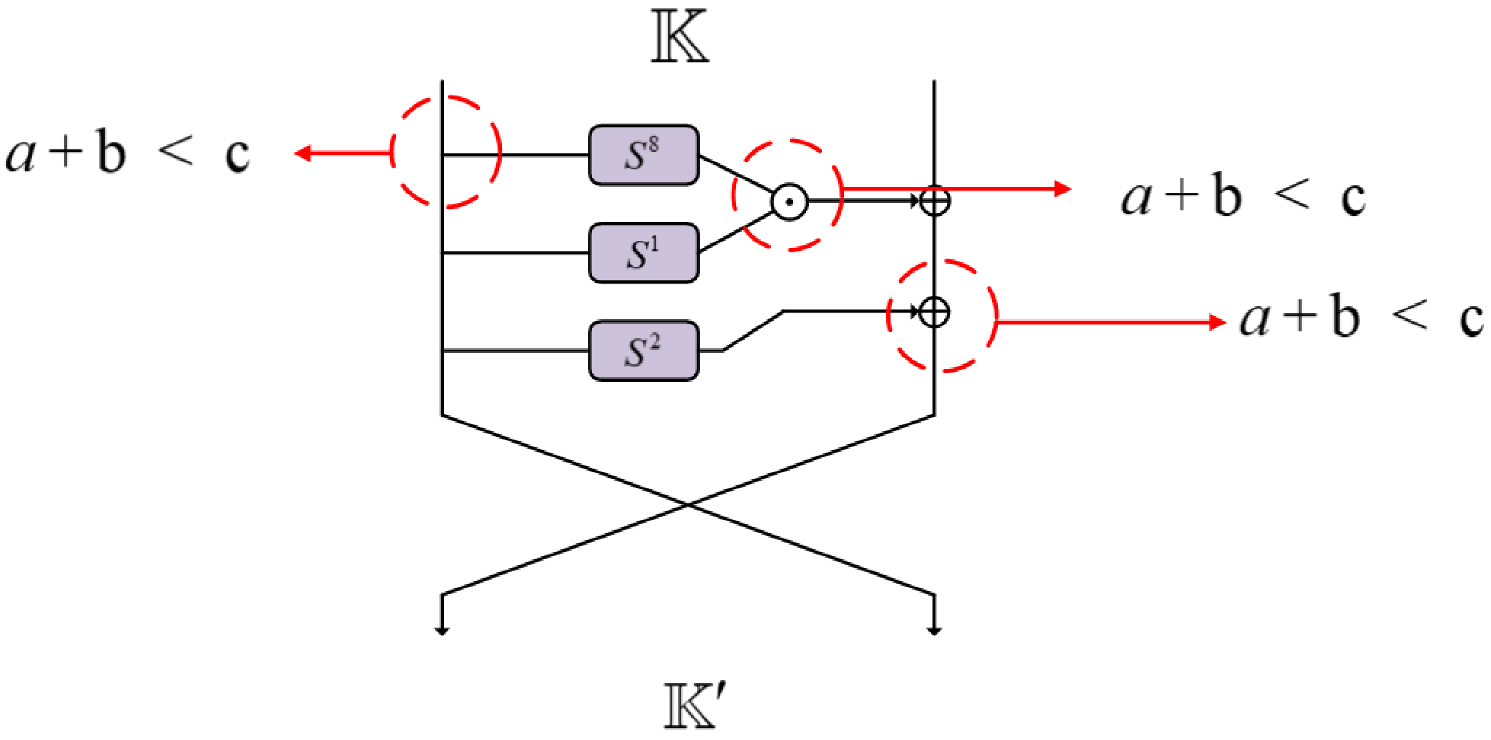


Why Automatic Search is Needed?

- C(python, etc.)-programming will cost **too much** time to write
- **Not easy** to optimize for the efficiency
- Concentration can be focused on the **problem itself**
- …

Automatic Search for Two-Subset Division Property

- Xiang et al. modeled **two-subset DP based MILP@Asiacrypt16**



Difficult to Model Three-Subset Division Property

- Propagation Rules of XOR for Two-Subset and Three-Subset DP are

ESSENTIALLY DIFFERENT!

- Two-Subset Division Property

$$\mathbb{K}' \leftarrow (\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4, \ldots, \boldsymbol{k}_m)$$

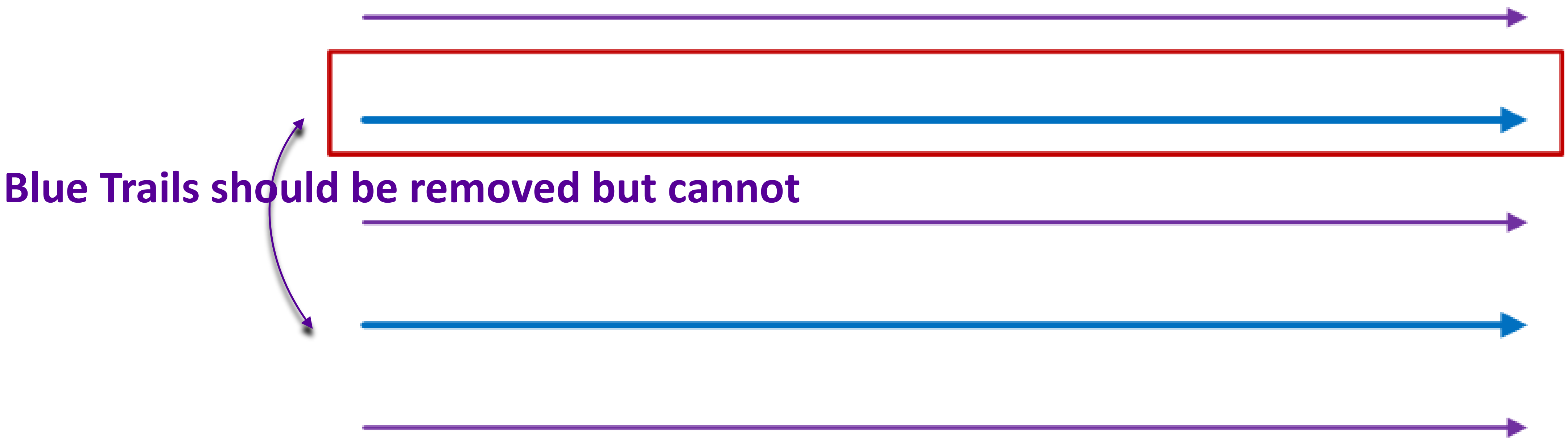
- Three-Subset Division Property

$$\mathbb{L}' \xleftarrow{\boldsymbol{x}} (\boldsymbol{l}_1 + \boldsymbol{l}_2, \boldsymbol{l}_3, \boldsymbol{l}_4, \ldots, \boldsymbol{l}_m)$$

Removed if exits

Why Is It So Difficult?

At any time, the automatic search tool can process only one trial



Division Trails

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Motivations

- Three-Subset Division Property can **find more distinguishers**
- It still **cannot be modeled** by automatic search methods

Contributions

- A **new division property** is proposed
- **More** integral distinguishers than two-subset division property
- **Improvement** of the results of SIMON, SPECK and KATAN

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Variant Three-Subset Division Property

Rule (Variant XOR)

Let F be a function compressed by an XOR, where the input $(x_1, x_2, \ldots, x_m)$ takes values of $(\mathbb{F}_2)^m$, and the output is calculated as $(x_1 \oplus x_2, x_3, \ldots, x_m)$. Let $\mathbb{X}$ and $\mathbb{Y}$ be the input and output multiset, respectively. Assuming that $\mathbb{X}$ has $\mathcal{D}^{1^m}_{\mathbb{K},\mathbb{L}}$, $\mathbb{Y}$ has $\mathcal{D}^{1^{m-1}}_{\mathbb{K}',\mathbb{L}'}$, where $\mathbb{K}'$ is computed from $\boldsymbol{k} \in \mathbb{K}$ s.t. $(k_1, k_2) = (0,0), (1,0),$ or $(0,1)$ as

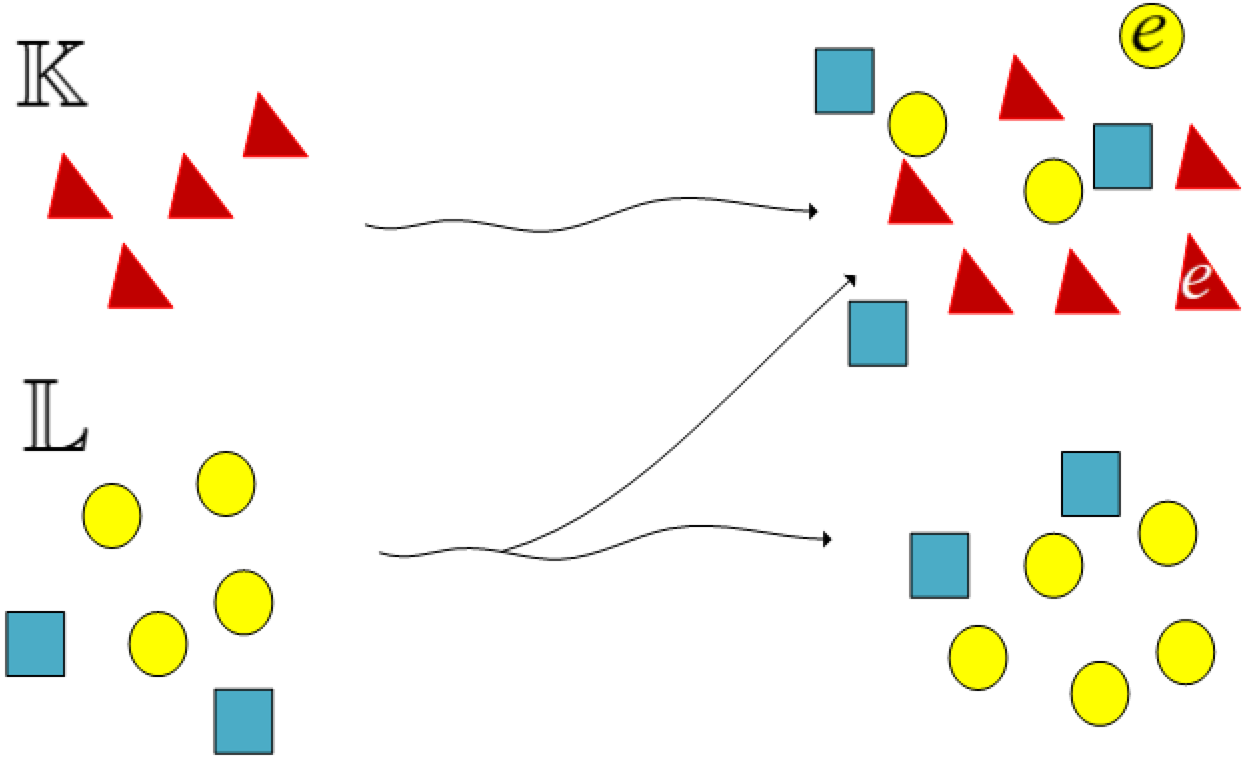
$$\mathbb{K}' \leftarrow (k_1 + k_2, k_3, k_4, \ldots, k_m).$$

Moreover, $\mathbb{L}'$ is computed from $\boldsymbol{l} \in \mathbb{L}$ s.t. $(l_1, l_2) = (0,0), (1,0),$ or $(0,1)$ as

$$\mathbb{L}' \leftarrow (l_1 + l_2, l_3, l_4, \ldots, l_m),$$

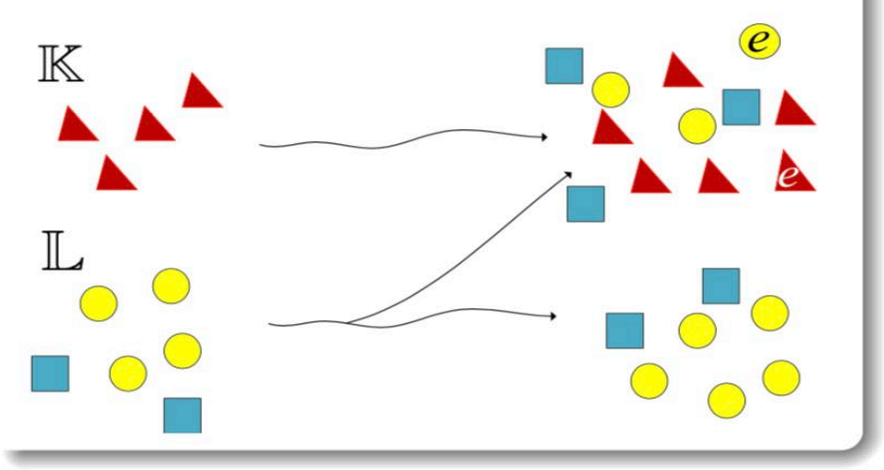
Variant XOR Propagation Rules

- Duplicated vectors will not be removed
- $\mathbb{L}' \overset{x}{\leftarrow} (l_1 + l_2, l_3, l_4, \dots, l_m) \longrightarrow \mathbb{L}' \leftarrow (l_1 + l_2, l_3, l_4, \dots, l_m)$

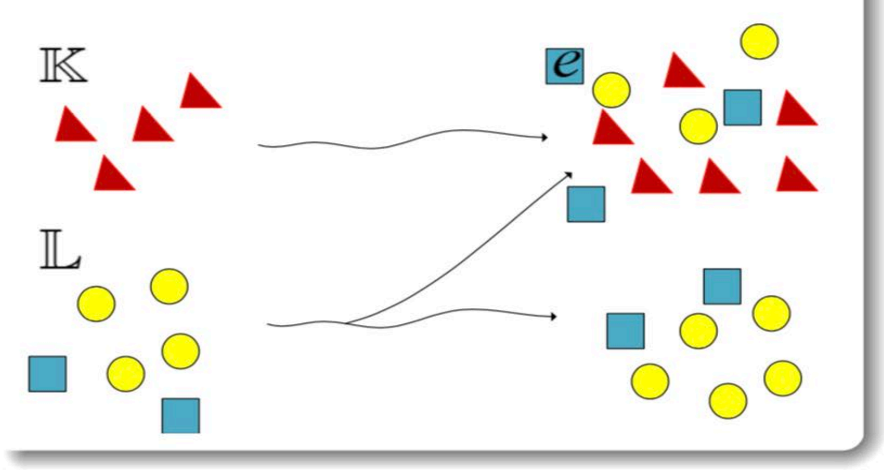


Relationship of OTDP and VTDP

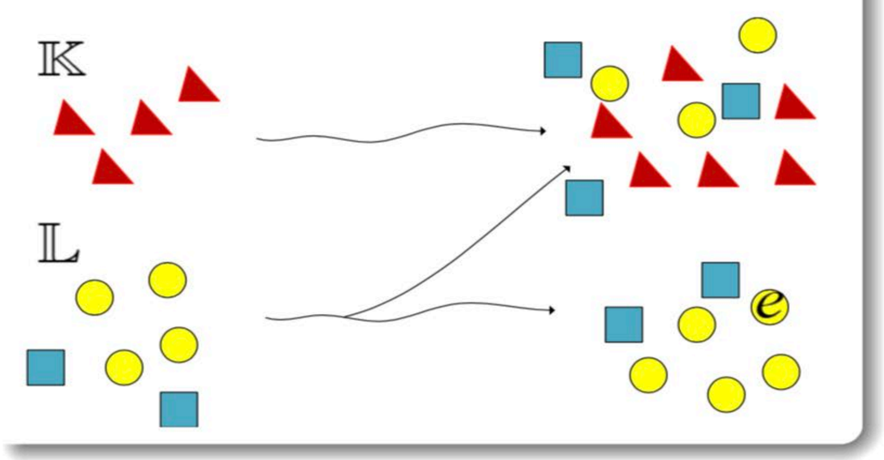
1 OTDP: unknown VTDP: unknown



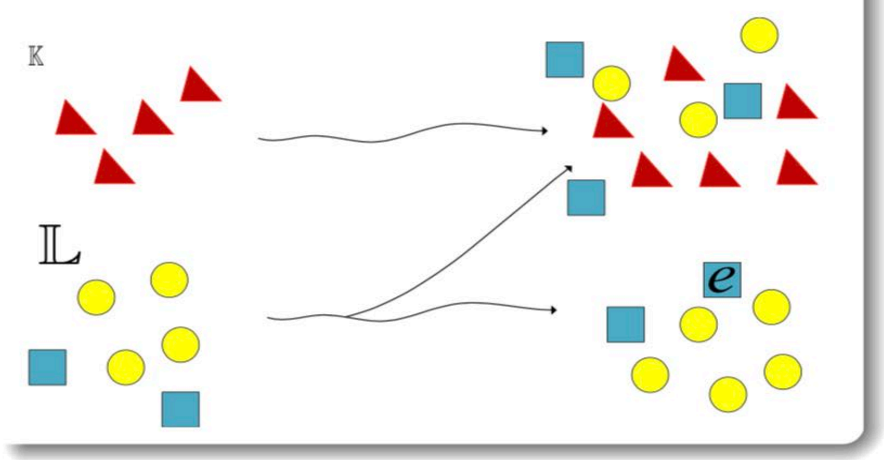
2 OTDP: constant VTDP: unknown



3 OTDP: odd VTDP: odd

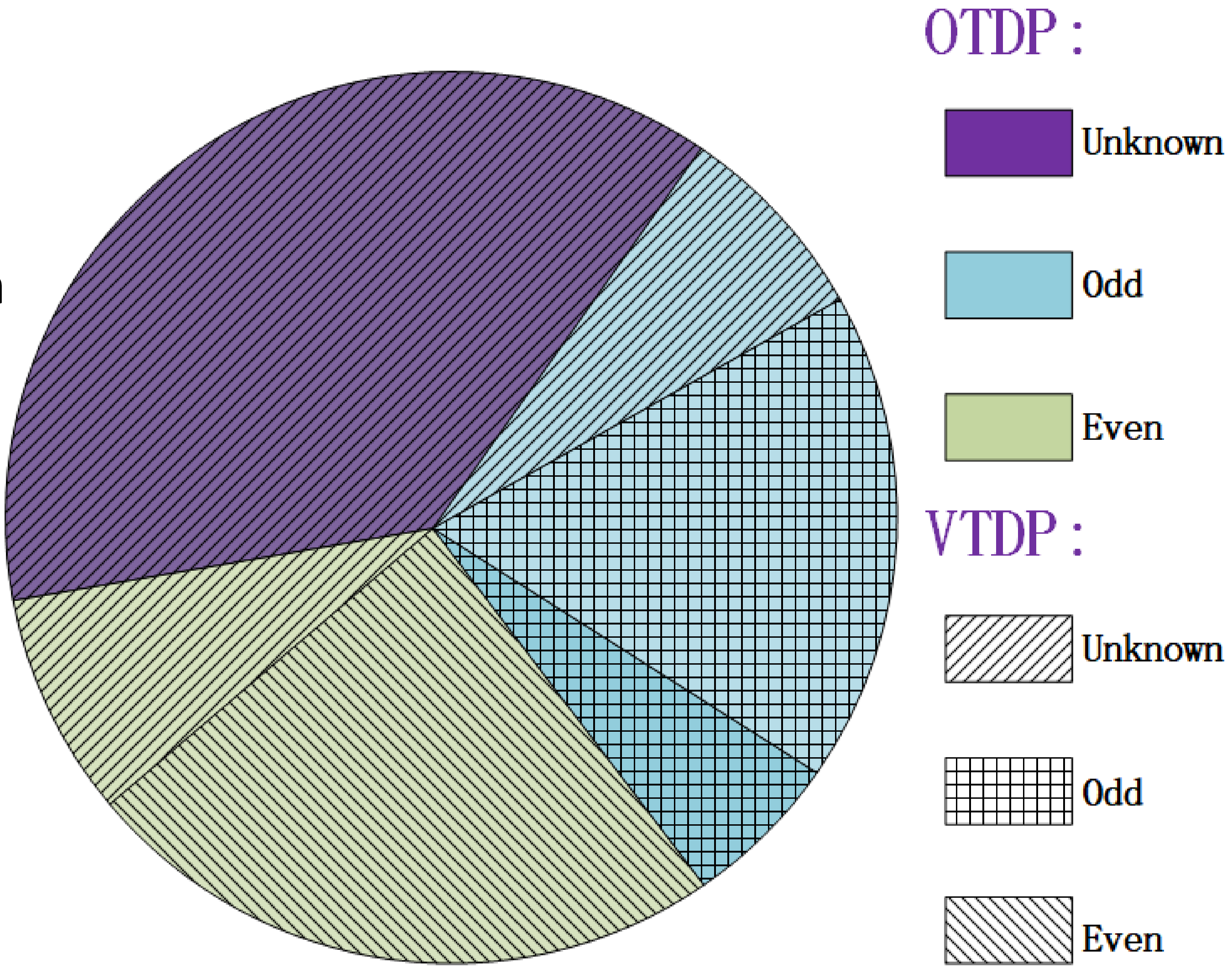


4 OTDP: even VTDP: odd



Relationship between VTDP and OTDP

- More bits are indicated unknown
- Some even-parity bits are indicated Odd-parity



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Propagation Rule of Key-XOR

- Some new vectors are **generated** from $\mathbb{L}$ and **appended** into $\mathbb{K}$
  - $l \in \mathbb{L}$, $l' = (l_0, l_1, \dots, \boldsymbol{l_i} \vee \mathbf{1}, \dots, l_{s-1})$ for $l_i = 0$

$$(\mathbf{0}, \mathbf{0}, \mathbf{1}, \mathbf{0}) \rightarrow \{(\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{0}), (\mathbf{0}, \mathbf{1}, \mathbf{1}, \mathbf{0}), (\mathbf{0}, \mathbf{0}, \mathbf{1}, \mathbf{1})\}$$

- Two problems in **automatic search model**?
  - **How to generate** the new vectors?
  - **How to insert them** into $\mathbb{K}$?

Models of Key-XOR for Three-subset Division Property

Model the VTDP for Key-XOR

1. Allocate n-bit variables $\mathcal{V}_j$ $(j \in \{0, 1, 2, \ldots, s-1\})$. Check each bit of $\mathcal{L}$, i.e., $\mathcal{L}[0], \mathcal{L}[1], \ldots, \mathcal{L}[s-1]$, and assign $\mathcal{V}_j$ as follows,

$$\mathcal{V}_j = \begin{cases} \mathcal{L} \vee \vec{e}_j, & \text{if } \mathcal{L}[j] = 0, \\ \vec{1}, & otherwise, \end{cases}$$

STP ASSERT $\mathcal{L}^j =$ IF $\mathcal{L}[j] = 0$ THEN $\mathcal{L} \vee \vec{e}_j$ ELSE $\vec{1}$ ENDIF;

2. Let $\{\mathcal{K}'\} = \{\mathcal{K}\} \cup \{\mathcal{V}_0\} \cup \{\mathcal{V}_1\} \cup \cdots \cup \{\mathcal{V}_{s-1}\}$.

STP ASSERT $\mathcal{K}' = \mathcal{K}$ OR $\mathcal{K}' = \mathcal{V}_0$ OR $\mathcal{K}' = \mathcal{V}_1$ OR $\ldots$ OR $\mathcal{K}' = \mathcal{V}_{s-1}$;

Initial Rules for Three-subset Division Property

Initial Rules

Let $((\mathcal{K}_0^0, \mathcal{K}_1^0, \ldots, \mathcal{K}_{n-1}^0), (\mathcal{L}_0^0, \mathcal{L}_1^0, \ldots, \mathcal{L}_{n-1}^0))$ denote the initial division property, where n is the block size. The constraints on $\mathcal{K}_i^0$ and $\mathcal{L}_i^0$ are

$$\mathcal{K}_i^0 = 1, \text{for } i = 0, 1, 2, \ldots, n-1.$$

$$\mathcal{L}_i^0 = \begin{cases} 1, & \text{if the } i\text{-th bit is active,} \\ 0, & \text{otherwise.} \end{cases}$$

Stopping Rules for Three-subset Division Property

Stopping Rules

1 examine whether there is a unit vector $\vec{e}_{i_0} \in \mathbb{K}$:

$$\mathcal{K}_i^r = \begin{cases} 1, & \text{if } i = i_0, \\ 0, & \text{otherwise.} \end{cases}$$

2 If not stopped. Check whether there is a unit vector $\vec{e}_{i_0} \in \mathbb{L}$:

$$\mathcal{L}_i^r = \begin{cases} 1, & \text{if } i = i_0, \\ 0, & \text{otherwise.} \end{cases}$$

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Applications on Some Ciphers

Cipher	Data	Round	bits	Time	Reference
SIMON32	2^{31}	14	32		TM16@FSE'16, XZBL@Asiacrpt'16
		15	3	27s	TM16@FSE'16, Ours
SIMON32(102)	2^{31}	20	1		XZBL@Asiacrpt'16
		20	3	25s	Ours
SIMON48(102)	2^{47}	28	1		XZBL@Asiacrpt'16
		28	3	9.3s	Ours
SIMON64(102)	2^{63}	36	1		XZBL@Asiacrpt'16
		36	3	1.1h	Ours
KATAN/KTANTAN32	2^{31}	99	1		SWLW@eprint
		101	1	5.6h	Ours
KATAN/KTANTAN48	2^{47}	63.5	1		SWLW@eprint
		64	1	16h	Ours
KATAN/KTANTAN64	2^{63}	72.3	1		SWLW@eprint
		72.3	2	18h	Ours
SPECK32	2^{31}	6	1		SWW@eprint
		6	2	3.5m	Ours

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Summary

- **A new division property** that can find more distinguishers
- **Automatic search model** of the variant division property
- It may bring **some new insights** into bit-based division property

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Thanks for Your Attention!