# INT-RUP Analysis of Block-cipher Based Authenticated Encryption Schemes

Avik Chakraborti, Nilanjan Datta and Mridul Nandi

Indian Statistical Institute, Kolkata

CTRSA-2016, San Francisco, USA



# Outline of the talk

- Introduction.
- Our Contribution.
- Conclusion

- Introduction
- Our Contribution
- 3 Conclusions

# Authenticated Encryption (AE)

# Why AE?

- Privacy of Plaintext.
- Authenticity of the plaintext/ ciphertext and associated data.

## More Formally....

- Tagged-encryption : AE.enc :  $\mathcal{M} \times \mathcal{D} \times \mathcal{N} \times \mathcal{K} \rightarrow (\mathcal{C} \times \mathcal{T})$
- $\bullet \ \, \textbf{Verified} \text{-decryption} : \ \, \textbf{AE.dec} : (\mathcal{C} \times \mathcal{T}) \times \mathcal{D} \times \mathcal{N} \times \mathcal{K} \rightarrow \mathcal{M} \cup \bot$

# Intigrity Security AE

# Intigrity Security of AE

- Integrity Security of AE when adversary is given Encryption and Verification oracle.
- Encryption Query:  $(N_i, D_i, M_i) \rightarrow (C_i, T_i)$ Verificationtion Query:  $(N_i, D_i, (C_i, T_i)) \rightarrow M_i / \bot$
- $Adv_{\pi}^{int-ctxt}(A) = |Pr[K \in_{R} \mathcal{K} : A^{\mathcal{E}_{K},\mathcal{V}_{K}} \neq \bot]|$

# INT-RUP Security and rate of Block Cipher based AE

# INT-RUP Security of AE Construction (Andreeva et.al.)

- Adversary is given both Encryption, Decryption and Verification oracle.
- Decryption Query:  $(N_i, D_i, C_i) \rightarrow M_i$  (no  $T_i$  in the query)
- $\mathsf{Adv}^{int-rup}_{\pi}(A) = |Pr[K \in_R \mathcal{K} : A^{\mathcal{E}_K,\mathcal{D}_K,\mathcal{V}_K} \neq \bot]|$
- Used for low-end devices with limited buffer.

#### Rate of a Construction

- Messages blocks processed per block-cipher call.
- Rate-1 means efficient construction.



# Affine Mode AE

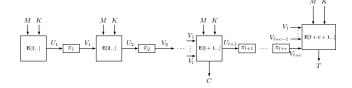


Figure: Structure of Affine Mode AE Schemes

# Affine Mode AE - Encryption

# Matrix Representation

$$E. \left( \begin{array}{c} L \\ M \\ Y^* = \begin{pmatrix} Y \\ Y_{tag} \end{pmatrix} \right) = \left( \begin{array}{c} X^* = \begin{pmatrix} X \\ X_{tag} \end{pmatrix} \\ Z = \begin{pmatrix} C \\ T \end{pmatrix} \end{array} \right)$$

# Encryption Matrix Representation

- E: encryption matrix, L: key vector, M is message vector
- Y: Intermediate output from  $\pi$  during M Processing
- $Y_{tag}$ : Intermediate output from  $\pi$  during tag Processing
- X : Intermediate input to  $\pi$  during M Processing
- $X_{tag}$  Intermediate input to  $\pi$  during tag Processing
- C : ciphertext vector, T : tag vector

- Introduction
- Our Contribution
  - INT-RUP Insecurity of Affine mode AE
  - INT-RUP Insecurity of CPFB
  - mCPFB: A rate  $\frac{3}{4}$  INT-RUP secure construction
- Conclusions

# Our Contribution

#### Result 1.

rate-1 Affine mode Authenticated Encryption mode is INT-RUP insecure.

# Significance of the Result

Guideline: To achieve INT-RUP security, one has to compromise efficiency.

# Our Contribution

#### Result 2.

CPFB (rate  $\frac{3}{4}$ ) is INT-RUP insecure.

#### Questions

- How much efficiency we have to loose to get INT-RUP security?
- Can we have an INT-RUP secure scheme with rate  $\frac{3}{4}$ ?

# Our Contribution

#### Result 3.

m-CPFB (rate  $\frac{3}{4}$ ) is INT-RUP insecure.

# Significance

- INT-RUP comes with small degrade in efficiency.
- "rate-1" a borderline criteria for INT-RUP security.

# INT-RUP Attack

# Queries of INT-RUP Adversary

- Encryption Query:  $(N, AD, M^0 = (M_1^0, M_2^0, ..., M_I^0))$ . Let,  $C^0 = (C_1^0, C_2^0, ..., C_I^0, T^0)$  be the tagged ciphertext.
- Unverified Plaintext Query:  $(N, AD, C^1 = (C_1^1, C_2^1, \dots, C_l^1))$ . Let  $M^1 = (M_1^1, M_2^1, \dots, M_l^1)$  be the corresponding plaintext.
- Forged Query:  $(N, AD, C^f = (C_1^f, C_2^f, \dots, C_l^f), T^f)$ , which realizes a  $\delta = (\delta_1, \dots, \delta_l)$  sequence.

# $C^f$ realizes a $\delta = (\delta_1, \dots, \delta_l)$ -sequence

$$\forall i \leq I, \ U_i^f = U_i^{\delta_i} \ \text{and} \ \forall i > I, \ U_i^f = U_i^0.$$

# Structure of Decryption Matrix

# **During Message Process**

- Observed by Enc and Dec queries
- $\Delta C^{ij}$ ,  $\Delta V^{ij} \Delta U^{ij}$  and  $\Delta M^{ij}$  are observed differences.

$$\left(\begin{array}{cc} D_{12} & D_{13} \\ D_{32} & D_{33} \end{array}\right) \cdot \left(\begin{array}{c} \Delta C^{ij} \\ \Delta V^{ij} \end{array}\right) = \left(\begin{array}{c} \Delta U^{ij} \\ \Delta M^{ij} \end{array}\right), \quad i = 0, j \in \{1, f\}$$

# **During Tag Process**

•  $\Delta C^{0f}, \Delta V^{0f}, \Delta V_{tag}^{0f} U_{tag}^{0f}$ , and  $\Delta T^{0f}$  are observed differences.

$$\begin{pmatrix} D_{22} & D_{23} & D_{24} \\ D_{42} & D_{43} & D_{44} \end{pmatrix} \cdot \begin{pmatrix} \Delta C^{0f} \\ \Delta V^{0f} \\ \Delta V^{0f}_{tag} \end{pmatrix} = \begin{pmatrix} \Delta U^{0f}_{tag} \\ \Delta T^{0f} \end{pmatrix}$$

# INT-RUP Attack (Construction of Forged Query)

# Step I: Find $\Delta V^{01}$

$$\Delta V^{01} = D_{33}^{-1} (\Delta M^{01} + D_{32} \Delta C^{01})$$

# Step II: Find $\Delta C^{0f}$ in terms of $\delta$

$$\Delta C^{0f} = D_{12}^{-1} \cdot (\Delta U^{0f} + D_{32} \Delta V^{0f})$$
  
=  $D_{12}^{-1} (\delta \cdot U^{01} + D_{32} \delta \cdot V^{01})$   
=  $D^* \cdot \delta$ 

# INT-RUP Attack (Construction of Forged Query)

# Step III: Find $\delta$ that makes $\Delta U_{tag}^{0f}=0$

Solve the following set of equations to find a  $\delta$ :

$$D_{22}\Delta C^{0f} + D_{23}\Delta V^{0f} = 0$$

This equation has at least one solution as long as l > (c-1).n

# Step IV: Find $\Delta C^{0f}$ and $\Delta T^{0f}$

Put  $\delta = \delta^*$  in the following equations:

$$\begin{array}{lcl} \Delta C^{0f} & = & D_{12}^{-1}.D^*.\delta \\ \Delta T^{0f} & = & D_{42}\Delta C_{0f} + D_{43}\Delta V_{0f} \end{array}$$

# Revisit CPFB

# Encryption and Tag Genration of CPFB

# INT-RUP Attack on CPFB

#### INT-RUP Attack on CPFB

- **1** Encryption query:  $(N, A, M^0)$ ,  $|M^0| = I = 129$ . Let  $C^0$  be the ciphertext
- **2** Unverified Plaintext decryption query:  $(N, A, C^1)$  of length I. Let,  $M^1$  be the corresponding plaintext.
- **3** Compute Y values:  $Y_1^0, \dots, Y_l^0$  and  $Y_1^1, \dots, Y_l^1$  from the two queries (by  $M^0 + C^0$  and  $M^1 + C^1$ ).
- Find the  $\delta$ -sequence:  $\delta = (\delta_1, \dots, \delta_l)$ , with  $\delta_1 = 0$  such that,  $\sum_{i=2}^{l} (Y_i^{\delta_i} || Z_i^{\delta_i}) = \sum_{i=2}^{l} (Y_i^0 || Z_i^0).$

Expect  $2^{32}$ -many such  $\delta$ -sequences.

# INT-RUP Attack on CPFB

#### INT-RUP Attack on CPFB

Perform the following for all such  $\delta$ -sequence:

- Set  $C_1^f = C_1^0$ . For all 1 < i < I, set  $C_i^f = C_i^{\delta_i}$  if  $\delta_{i-1} = \delta_i$  and  $C_i^{\delta_i} + Y_i^0 + Y_i^1$ , otherwise.
- ② Set  $C_I^f = C_I^0$  if  $\delta_I = 0$ . Else, set  $C_I^f = C_I^0 + Y_I^0 + Y_I^1$ .
- **3** Return  $(C_1^f, C_2^f, \dots, C_I^f, T^0)$  as forged Ciphertext.

# Building an INT-RUP Secure rate- $\frac{3}{4}$ Construction

#### Potential Weakness of CPFB

- $Y_i$  values can be observed. Only  $Z_i$ -values are unknown.
- $\bigcirc$   $Z_i$  has only 32-bit entropy on the Tag.

## Requirement of the New Construction

- Ensure 128-bit entropy of Z-values on the tag.
- Ensure at-least 4 different Z-values for 2 messages of same length.

# mCPFB: modified CPFB

#### Introduce ECC Code

Expand  $M = (M_1, ..., M_l)$  by a Distance 4 Error Correcting Code ECCode:

ECCode(
$$M$$
) = ( $M_1, ..., M_l, M_{l+1}, M_{l+2}, M_{l+3}$ )  
( $M_{l+1}, M_{l+2}, M_{l+3}$ ) =  $V_{\beta}^{(3,l)} \cdot M$ 

# Produce 128-bit entropy of *Z*-values during Tag Generation:

Update  $Z^M$  as follows:

$$Z_M = V_{\alpha}^{(4,l+3)} \cdot (Z_2, Z_3, \cdots, Z_{l+3}, Z_{l+4}) \oplus (0^{32} || (Y_2 \oplus \cdots \oplus Y_{l+3}))$$

# mCPFB: modified CPFB

# Changes in the keys

- $\kappa_0$  is used as the masking key only.
- $\kappa_1$  is used as block-cipher key for AD processing.
- $\kappa_1, \ldots, \kappa_{-2}$  is used as block-cipher keys for message processing.
- $\kappa_{-1}$  is used as block-cipher key for tag and producing L-values.

# INT-RUP Security of mCPFB

#### Claim 1

Consider the function f that takes N, I and i as input and outputs O such that  $O = E_{\kappa[i]}(I||(i \mod 2^{32}) + \kappa_0)$  where  $\kappa[i] = E_K(N||j||I)$ ,  $j = \lceil \frac{i}{2^{32}} \rceil$ . f is assumed to have  $(q,\epsilon)$ -PRF security where  $\epsilon$  is believed to achieve beyond birthday security.

# INT-RUP advantage

 $f: (q_e + q_r, \epsilon)$ -PRF. Any adversary  $\mathcal{A}$  with  $q_e$  many encryption query and  $q_r$  many unverified plaintext queries, one forgery attempts, has the advantage:

$$Adv_{mCPFB}^{int\_rup}(\mathcal{A}) \leq rac{5}{2^{128}} + \epsilon$$

# **Proof Sketch**

# Argument for Different Cases

- (Case A)  $\forall i, N^* \neq N_i$ : Through randomness of  $\kappa_{-1}$ .
- (Case B)  $\exists$  unique  $i \ni N^* = N_i, T^* \neq T_i$ : Through randomness of  $\kappa_{-1}$ .
- (Case C)  $\exists$  unique  $i \ni N^* = N_i$ ,  $T^* = T_i$ ,  $|C_i| = |C^*|$ : Through randomness of  $Z_i$ 's.
- (Case D)  $\exists$  unique  $i \ni N^* = N_i$ ,  $T^* = T_i$ ,  $|C_i| \neq |C^*|$ : Through randomness of  $\kappa_{-1}$ .

- Introduction
- 2 Our Contribution
- 3 Conclusions

# Conclusions

- INT-RUP attack on any "Rate-1" affine AE mode
- INT-RUP attack on a "Rate- $\frac{3}{4}$ " AE scheme CPFB
- Proposal of mCPFB: an INT-RUP secure scheme

# Thank you

# From Stateless to Stateful: Generic Authentication and Authenticated Encryption Constructions with Application to TLS

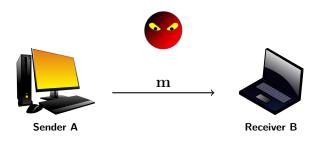
Colin Boyd<sup>1</sup> Britta Hale<sup>1</sup> Stig Frode Mjølsnes<sup>1</sup> Douglas Stebila<sup>2</sup>

 $^{1}$ Norwegian University of Science and Technology  $^{2}$ Queensland University of Technology

1 March 2016

#### AUTHENTICATION PROTOCOLS

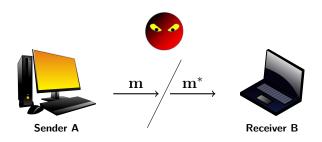
#### What is data authentication?



Is m from A?

#### AUTHENTICATION PROTOCOLS

#### What is data authentication?



Is m from A?

Has m been modified?

#### ACHIEVING AUTHENTICATION

- Message Authentication Code (MAC)
  - HMAC, etc...

message MAC tag

- Signatures
  - DSA, Elliptic Curve DSA, etc...

message <mark>signature</mark>

- Authenticated Encryption with Associated Data (AEAD)
  - Galois Counter Mode (GCM), etc...

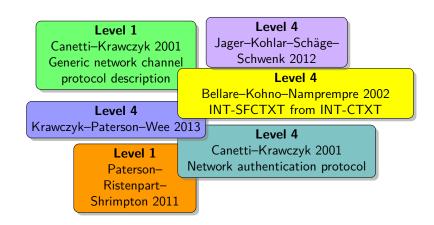
ciphertext



#### AUTHENTICATION HIERARCHY

Example		Sender	Receiver
			Auth., No Replays, Strictly Incr., No Drops
TLS	Level 4	$m_0, m_1, m_2, m_3, m_4, m_5$	$\longrightarrow m_0, m_1, m_2, m_3, m_4, m_5$
			Auth., No Replays, Strictly Incr.
802.11	Level 3	$m_0, m_1, m_2, m_3, m_4, m_5$	$m_0, m_2, m_3, m_5$
			Auth., No Replays
DTLS*	Level 2	$m_0, m_1, m_2, m_3, m_4, m_5$	$m_3, m_0, m_5, m_2$
			Authentication only
DTLS	Level 1	$m_0, m_1, m_2, m_3, m_4, m_5$	$m_3, m_5, m_3, m_2$

#### SECURE CHANNEL VARIATIONS



#### HIERARCHY OF AUTHENTICATION

# $\operatorname{Exp}^{\operatorname{auth}_i}_{\Pi.\mathcal{A}}()$ :

#### Oracle Send(m):

- 1: k <sup>\$</sup> Kgn()
- 2:  $st_{\rm E} \leftarrow \bot, st_{\rm D} \leftarrow \bot$ 3:  $u \leftarrow 0$ ,  $v \leftarrow 0$
- 4: r ← 0 5: A<sup>Send(·),Recv(·)</sup>()
- 6: return r

- 1:  $u \leftarrow u + 1$ 

  - 3: return  $sent_n$  to A
- 2:  $(sent_u, st_E) \leftarrow Snd(k, m, st_E)$

# Oracle Recv(c):

- 1: v ← v + 1
- 2:  $rcvd_v \leftarrow c$ 3:  $(m, \alpha, st_D) \leftarrow \text{Rcv}(k, c, st_D)$
- 4: if  $(\alpha = 1) \land (cond_i)$  then
- 5: r ← 1
  - return r to A
- 7: end if
- 8: return

- Basic authentication:  $cond_1 = (\nexists w : c = sent_w)$
- Basic authentication, no replays:  $cond_2 = (\nexists w : c = sent_w) \lor (\exists w < v : c = rcvd_w)$
- Basic authentication, no replays, strictly increasing:  $\operatorname{cond}_3 = (\nexists w : c = \operatorname{sent}_w) \vee (\exists w, x, y : (w < v) \wedge (\operatorname{sent}_x = \operatorname{rcvd}_w) \wedge (\operatorname{sent}_u = \operatorname{rcvd}_w)) \wedge (\operatorname{sent}_u = \operatorname{rcvd}_w)$  $rcvd_v) \wedge (x \geq y)$
- Basic authentication, no replays, strictly increasing, no drops:  $cond_A = (u < v) \lor (c \neq sent_v)$

#### HIERARCHY OF AEAD

$$\begin{array}{l} \operatorname{Exp}_{\Pi,A}^{-1} & (): \\ \hline 1: \ k \xleftarrow{} \operatorname{Kgn}() \\ 2: \ st_{\operatorname{E}} \leftarrow \bot, \ st_{\operatorname{D}} \leftarrow \bot \\ 3: \ u \leftarrow 0, \ v \leftarrow 0 \\ 4: \ \operatorname{phase} \leftarrow 0 \\ 5: \ b' \xleftarrow{} \xleftarrow{} A^{\operatorname{Encrypt}(\cdot)}, \operatorname{Decrypt}(\cdot)() \\ 6: \ \operatorname{return} \ b' \end{array}$$

```
Oracle Encrypt(I, ad, m_0, m_1):
```

- 6:  $(ad, m, \alpha, st_D)$  $\leftarrow D(k, ad, c, st_D)$ 7: if  $(\alpha = 1) \wedge \text{cond}_i$  then
- $phase \leftarrow 1$ 9: end if

Oracle Decrypt(ad, c):

return  $\perp$ 

1: **if** b = 0 **then** 

4:  $v \leftarrow v + 1$ 

5:  $rcvd.c... \leftarrow c$ 

3: end if

- = 10: if phase = 1 then
  - return m
  - 12: end if 13: return ⊥

- Basic authenticated encryption:  $cond_1 = (\nexists w : (c = sent.c_w) \land (ad = sent.ad_w))$
- Basic authenticated encryption, no replays:  $\operatorname{cond}_2 = (\nexists w : (c = sent.c_w) \land (\operatorname{ad} = sent.ad_w)) \lor (\exists w < v : c = rcvd.c_w)$

8: return sent.c.

Basic authenticated encryption, no replays, strictly increasing:  $\mathsf{cond}_3 = (\nexists w : (c = sent.c_w) \land (\mathsf{ad} = sent.ad_w)) \lor (\exists w, x, y : (w < v) \land (sent.c_x = sent.ad_w)) \lor (\exists w, x, y : (w < v) \land (sent.c_x = sent.ad_w))$  $rcvd.c_w) \land (sent.c_y = rcvd.c_v) \land (x \ge y))$ 

7:  $(sent.ad_u, sent.c_u, st_E)$ 

 $(ad, sent.c^{(b)}, st_n^{(b)})$ 

Basic authenticated encryption, no replays, strictly increasing, no drops:  $cond_A = (u < v) \lor (c \neq sent.c_v) \lor (ad \neq sent.ad_v)$ 

6: end if



#### SECURE CHANNELS WITH TLS

Paterson-Ristenpart-Shrimpton 2011

MEE-TLS encoding – CBC (message len.) + (tag len.) > (block len.) 
$$-8$$
 TLS satisfies Level 1 AEAD

#### SECURE CHANNELS WITH TLS

Authenticated and Confidential Channel Establishment (ACCE)

Jager–Kohlar–Schäge–Schwenk 2012

Stateful length-hiding AEAD at Level 4 ACCE security for TLS (Suites: TLS-DHE)

Krawczyk-Paterson-Wee 2013

Stateful length-hiding AEAD at Level 4
Constrained chosen ciphertext security

ACCE security for TLS
Suites: TLS-RSA,
TLS-CCA, TLS-DH, TLS-DHE

#### IMPLICATIONS BETWEEN AUTHENTICATION LEVELS

$$st_{
m E}'$$
 and  $st_{
m D}'$ :

- $lackbox{ } st'_{
  m E}:st'_{
  m E}.$ substate  $:=st_{
  m E}$ , where  $st_{
  m E}$  is the state in  $\Pi,\,st'_{
  m E}.$ counter
- $lacksymbol{s} t_{
  m D}': st_{
  m D}'$  .substate  $:= st_{
  m D}$  , where  $st_{
  m D}$  is the state in  $\Pi$  ,  $st_{
  m D}'$  .status,  $st_{
  m D}'$  .sqnlist

```
Rcv'(k, c, st'_D):
Kgn'():
 1: return Π.Kgn()
                                                                                        1: if st'_D.status = failed then
                                                                                                return (\bot, 0, st_D)
\operatorname{Snd}'(k, m, st'_{\operatorname{E}}):
                                                                                        3. end if
 1: (c, st'_{r}.substate)
                                                                                        4: (m_{\Pi}, \alpha, st'_{D}.substate)
      \leftarrow \Pi.\mathrm{Snd}(k, \mathsf{Ecd}(st'_{\mathsf{E}}.\mathsf{counter}, m), st'_{\mathsf{E}}.\mathsf{substate})
                                                                                             \leftarrow \Pi.\text{Rcv}(k, c, st'_{D}.\text{substate})
                                                                                        5: if \alpha = 1 then
 2: st'_{\mathbf{E}}.counter \leftarrow st'_{\mathbf{E}}.counter +1
                                                                                                   (\operatorname{sqn}, m, \alpha) \leftarrow \operatorname{Dcd}(st'_{\Pi}, \operatorname{sqnlist}, m_{\Pi})
 3: return (c, st'_{E})
                                                                                        7 end if
                                                                                        8: if (\alpha = 0) \vee \text{TEST}i then
                                                                                                  st'_{D} .status = failed
                                                                                                   return (\bot, 0, st'_{D})
                                                                                       10:
                                                                                       11: end if
                                                                                       12: st'_{D}.sqnlist = st'_{D}.sqnlist||sqn
                                                                                       13: return (m, \alpha, st'_D)
```

- Basic authentication, no replays:  $TEST2 = (\exists j : sqn = st'_{D}.sqnlist_{j})$
- Basic authentication, no replays, strictly increasing: TEST3 =  $(\exists j : \operatorname{sqn} \not > st'_{\mathcal{D}}.\operatorname{sqnlist}_i)$
- Basic authentication, no replays, strictly increasing, no drops:  $\texttt{TEST4} = (\exists j : \mathtt{sqn} \not > st'_D.\mathtt{sqnlist}_i) \lor (\mathtt{sqn} \not = \mathtt{max}\{st'_D.\mathtt{sqnlist}_i\} + 1)$



#### ANALYSIS METHOD

#### **Computational Analysis:**

Complexity-theoretic reduction proofs

- Protocol specification
- Adversary capabilities
- Adversary winning conditions

Security is reducible to that of an underlying *hard* problem



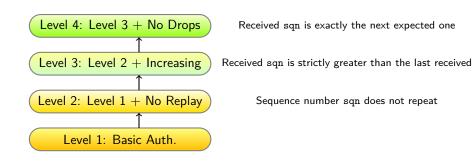
#### IMPLICATIONS BETWEEN AUTHENTICATION LEVELS

#### THEOREM

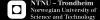
Let  $\Pi$  be a secure level-1 authentication scheme and Coding be an authentication encoding scheme with collision-resistant encoding. Let  $i \in \{2,3,4\}$ . Then  $\Pi'_i = P(\Pi, \operatorname{Coding}, \operatorname{TEST}i)$  is a secure level-i authentication scheme. Specifically, let  $\mathcal A$  be an adversary algorithm that runs in time t and asks  $q_s$  Send queries and  $q_r$  Recv queries, and let  $q = q_s + q_r$ . Then there exists an adversary  $\mathcal B$  that runs in time  $t_{\mathcal B} \approx t$  and asks no more than  $q_{\mathcal B} = \frac12 q_s(q_s - 1)$  queries, and an adversary  $\mathcal F$  that runs in time  $t_{\mathcal F} \approx t$  and asks  $q_{\mathcal F} = q$  queries, such that

$$\mathbf{Adv}^{\mathsf{auth}_i}_{P(\Pi,\mathtt{Coding},\mathtt{TEST}i)}(\mathcal{A}) \leq \mathbf{Adv}^{\mathsf{auth}_1}_\Pi(\mathcal{F}) + \mathbf{Adv}^{\mathsf{collision}}_{\mathtt{Ecd}}(\mathcal{B}) \;.$$

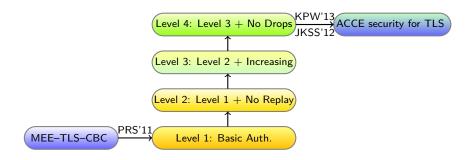
#### IMPLICATIONS BETWEEN AUTHENTICATION LEVELS



Sequence number can be included implicitly or explicitly



#### AUTHENTICATION LEVELS APPLIED – TLS



#### AUTHENTICATION LEVELS APPLIED

- Protocol Analysis
  - Selection of appropriate authentication experiment

- Building Authentication Protocols
  - Encoding and checking sequence numbers to achieve desired level

# Questions

