

Non-Interactive Plaintext (In-)Equality Proofs and Group Signatures with Verifiable Controllable Linkability

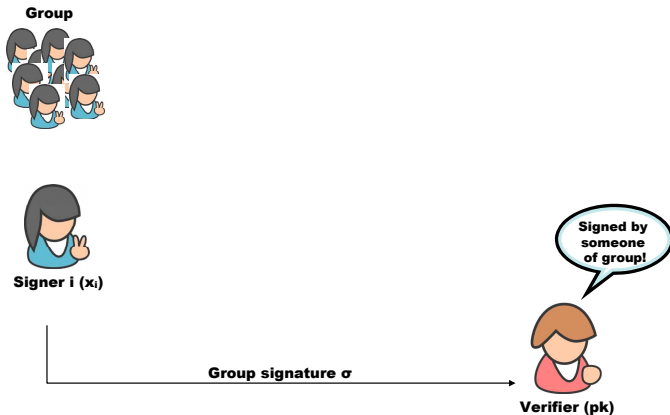
Olivier Blazy¹, David Derler², Daniel Slamanig², Raphael Spreitzer²

¹ **Université de Limoges, XLim, France**

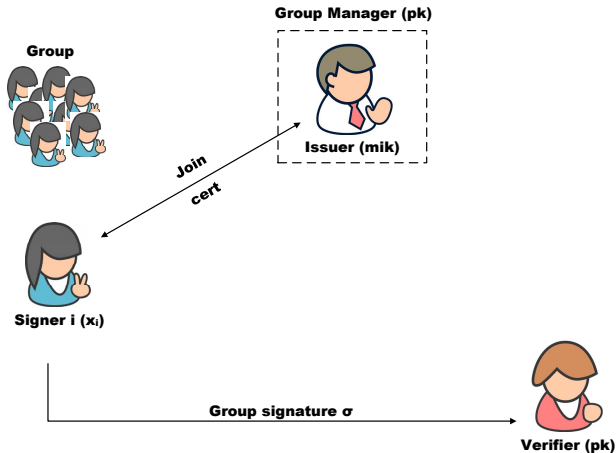
² **IAIK, Graz University of Technology, Austria**

CT-RSA 2016, San Francisco, 2nd March 2016

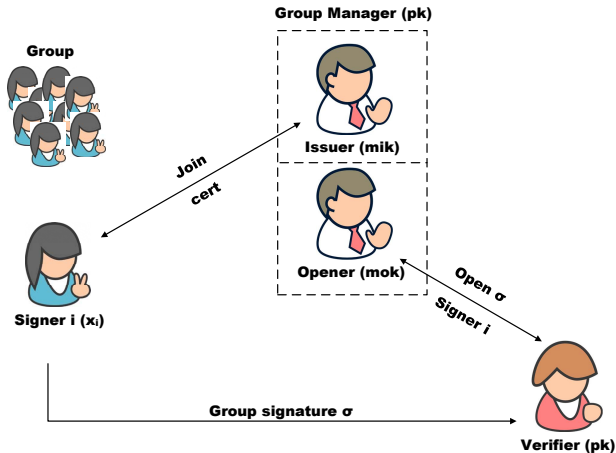
Group Signature Schemes [CvH91]



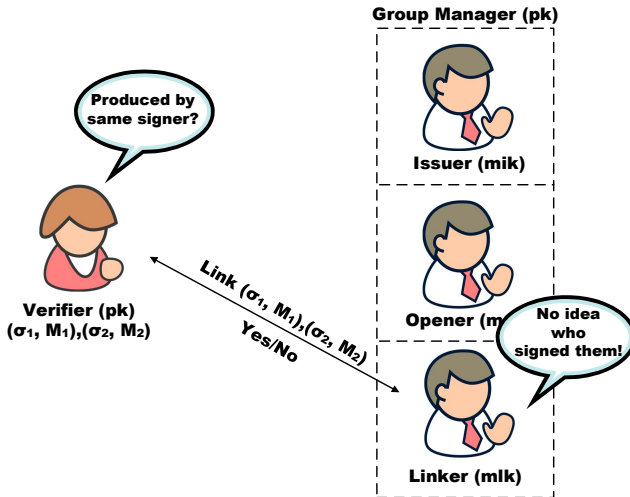
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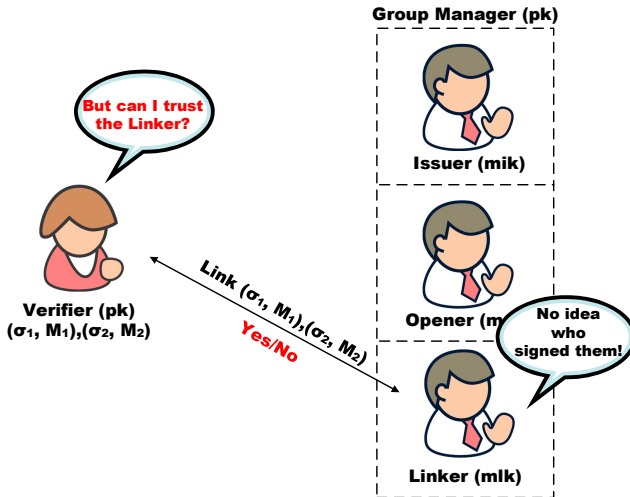
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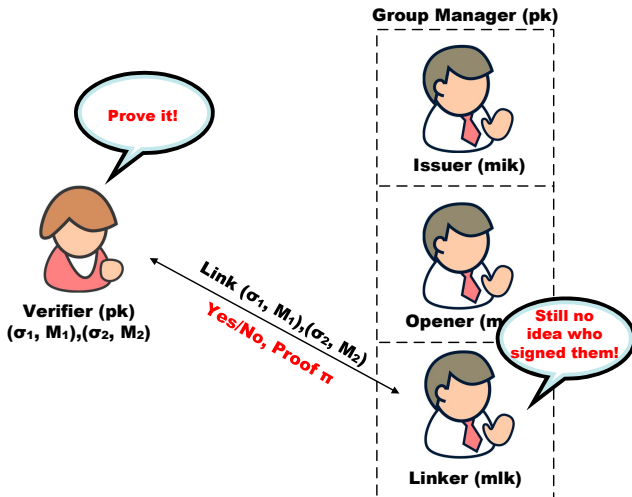
Controllable Linkability [HLhC⁺11, SSU14]



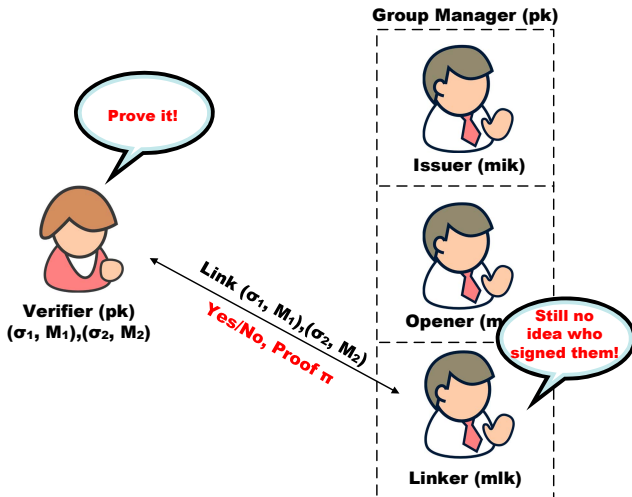
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Verifiable Controllable Linkability



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Primitive to prove plaintext (in-)equality

Contributions

1. Model generic **proof system** for plaintext (in-)equality

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2. **Efficient instantiation** of this proof system
3. Group signatures with **verifiable controllable linkability**
4. **Extend GSs** with verifiable controllable linkability (VCL)

Sign-Encrypt-Prove Paradigm

Basic building blocks

- $\mathcal{DS} = (\text{KG}_s, \text{Sign}, \text{Verify})$

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- Signature of Knowledge

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- $gpk \leftarrow (pk_e, pk_s),$

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- User's secret: x_i

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Keys

- $gpk \leftarrow (\textcolor{red}{pk}_e, \textcolor{blue}{pk}_s), gmsk \leftarrow \textcolor{red}{sk}_e, gmik \leftarrow \textcolor{blue}{sk}_s$

Join

- User's secret: x_i
- Issuer computes: $cert \leftarrow \text{Sign}(gmik, f(x_i))$

Sign-Encrypt-Prove Paradigm I

Sign

- $T \leftarrow \text{Enc}(pk_e, cert)$

Sign-Encrypt-Prove Paradigm I

Sign

- $T \leftarrow \text{Enc}(\textcolor{red}{pk}_e, \text{cert})$
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Open

- $\text{cert} \leftarrow \text{Dec}(\textcolor{red}{sk}_e, T)$

Controllable Linkability

Public key encryption with **equality tests** [Tan12, SSU14]

- Conventional public key encryption scheme

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- One-way security for trapdoor holders

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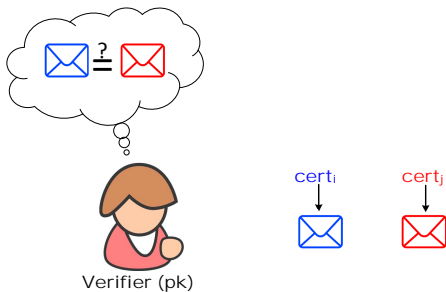
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\Rightarrow **ZK proof of knowledge of trapdoor for VCL**

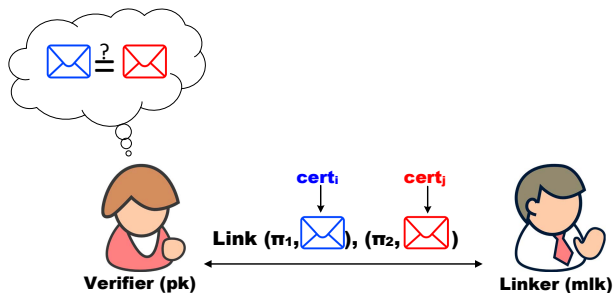
Setting



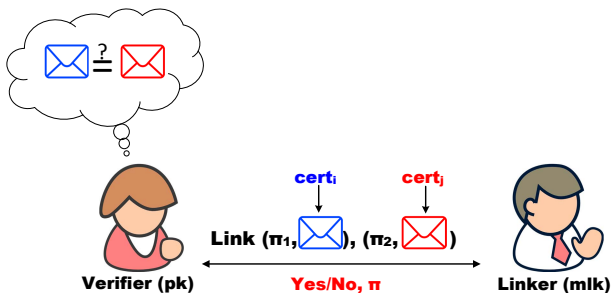
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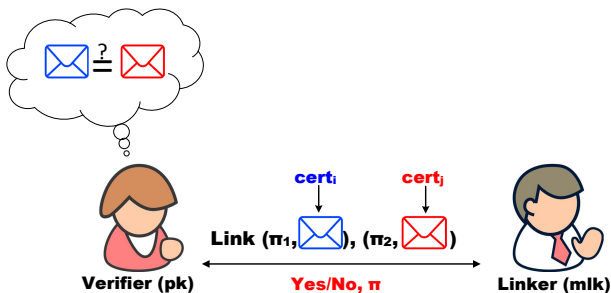
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Non-interactive plaintext (in-)equality proofs

Non-Interactive Plaintext (In-)Equality Proofs

Given any $\mathcal{PK}\mathcal{EQ}$ and ciphertexts T and T' under pk

Proof system Π

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Proof system Π

1. Prove knowledge of **trapdoor tk**
2. **Com** = 1 (membership) or **Com** = 0 (non-membership)
3. Without revealing trapdoor tk

(Non-)Membership Proofs

Com = 1 defines L for membership $((x, w) \in L_R)$

- Witnessed by trapdoor tk
- Standard techniques [GS08]

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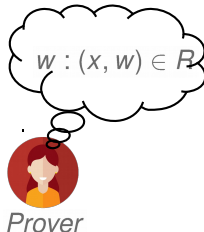
- Idea [BCV15]
 - Π_1 : Failing membership proof for L_R
 - Π_2 : Proof that Π_1 has been computed honestly
- Efficient instantiations (GS and SPHFs)
- Technicalities: m, r must be known [BCV15]

Smooth Projective Hash Functions (SPHF)

Hashing key: hk
 Projection key: hp



Statement: $x \in L_R$



hp

$$H \leftarrow \text{ProjHash}(hp, x, w)$$

H

$$H \stackrel{?}{=} \text{Hash}(hk, x)$$

If $x \in L_R : \text{Hash}(hk, x) \stackrel{!}{=} \text{ProjHash}(hp, x, w)$
 (Correctness)

Construction - Non-Membership Proof

Hashing key: hk

Projection key: hp



Verifier

Statement: $x \in L_R$



Prover

hp

$H \leftarrow \text{ProjHash}(hp, x, w)$

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$? \\ H \neq \text{Hash}(hk, x)$

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H $\pi_1 = Proof((hp, x, H), w)$

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\xrightarrow{hp}

$H \leftarrow \text{ProjHash}(hp, x, w)$

$\xleftarrow{H, \pi_1}$ $\pi_1 = \text{Proof}((hp, x, H), w)$

$$H \stackrel{?}{\neq} \text{Hash}(hk, x) \wedge \text{Verify}((hp, x, H), \pi_1) \stackrel{?}{=} 1$$

Construction - Non-Membership Proof

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Prover

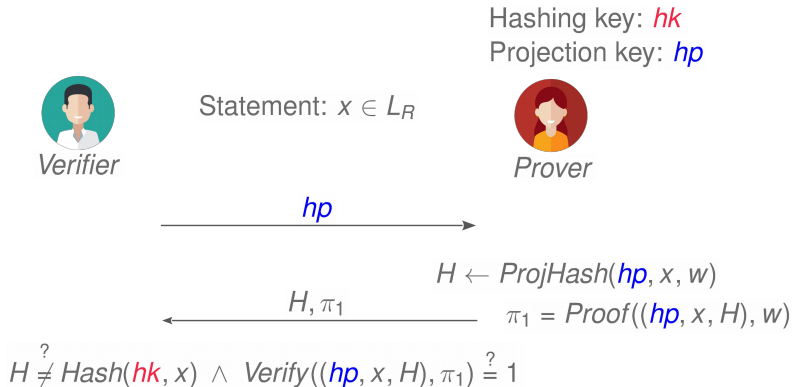
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Construction - Non-Membership Proof



Construction - Non-Membership Proof



Verifier

Statement: $x \in L_R$

Hashing key: *hk*
Projection key: *hp*



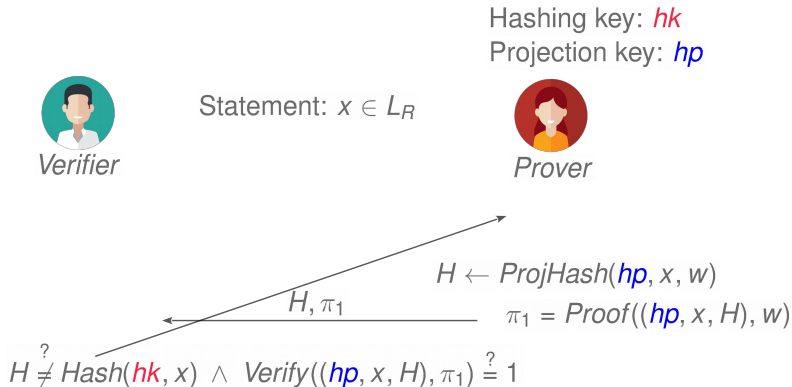
Prover

$H \leftarrow \text{ProjHash}(\textcolor{blue}{hp}, x, w)$

$\xleftarrow{H, \pi_1} \quad \pi_1 = \text{Proof}((\textcolor{blue}{hp}, x, H), w)$

$H \stackrel{?}{\neq} \text{Hash}(\textcolor{red}{hk}, x) \wedge \text{Verify}((\textcolor{blue}{hp}, x, H), \pi_1) \stackrel{?}{=} 1$

Construction - Non-Membership Proof



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← H, H', hp, π_1

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Example of Efficient Instantiation

ElGamal with equality tests (as in [SSU14])

- Keypair: $(\textcolor{red}{sk}, pk) \leftarrow (\textcolor{red}{x}, g^{\textcolor{red}{x}}) \in \mathbb{Z}_p \times \mathbb{G}_1$
- Trapdoor: $(\hat{r}, \hat{r}^{\textcolor{red}{x}}) \in \mathbb{G}_2 \times \mathbb{G}_2$
- Encryption of m : $(g^{\textcolor{blue}{r}}, m \cdot g^{\textcolor{red}{x} \cdot \textcolor{blue}{r}}) \in \mathbb{G}_1 \times \mathbb{G}_1$

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- Encryption of m : $(g^r, m \cdot g^{x \cdot r}) \in \mathbb{G}_1 \times \mathbb{G}_1$

Pairing based equality test:

- Ciphertexts: $(g^r, m \cdot g^{x \cdot r}), (g^{r'}, m' \cdot g^{x \cdot r'})$

$$m = m' \iff \frac{e(m \cdot g^{x \cdot r}, \hat{r})}{e(g^r, \hat{r}^x)} = \frac{e(m' \cdot g^{x \cdot r'}, \hat{r})}{e(g^{r'}, \hat{r}^x)}$$

Instantiation of Π_{ϵ}

Com = 1: plaintext **equality proof**

$$((g^r, m \cdot g^{x \cdot r}), (g^{r'}, m' \cdot g^{x \cdot r'}), g^x) \in L_{\epsilon} \iff$$

$$\frac{e(m \cdot g^{x \cdot r}, \hat{r})}{e(g^r, \hat{r}^x)} = \frac{e(m' \cdot g^{x \cdot r'}, \hat{r})}{e(g^{r'}, \hat{r}^x)} \wedge$$

$$e(g, \hat{r}^x) = e(g^x, \hat{r})$$

Instantiation of Π_{\in}

Com = 1: plaintext **equality proof**

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$$e(g, \hat{r}^x) = e(g^x, \hat{r})$$

$$\prod_{i=1}^2 e(A_i, \underline{\hat{Y}_i}) = \frac{e(m \cdot g^{x \cdot r} \cdot (m' \cdot g^{x \cdot r'})^{-1}, \hat{r})}{e(g^r \cdot g^{-r'}, \hat{r}^x)} = 1_{\mathbb{G}_T}$$

Instantiation of Π_{\neq}

Com = 0: plaintext **inequality proof**

$$((g^r, m \cdot g^{x \cdot r}), (g^{r'}, m' \cdot g^{x \cdot r'}), g^x) \in L_{\neq} \iff$$

$$\frac{e(m \cdot g^{x \cdot r}, \hat{r})}{e(g^r, \hat{r}^x)} \neq \frac{e(m' \cdot g^{x \cdot r'}, \hat{r})}{e(g^{r'}, \hat{r}^x)} \wedge$$

$$e(g, \hat{r}^x) = e(g^x, \hat{r})$$

Instantiation of Π_{\notin}

Com = 0: plaintext **inequality proof**

$$((g^r, m \cdot g^{x \cdot r}), (g^{r'}, m' \cdot g^{x \cdot r'}), g^x) \in L_{\notin} \iff$$

$$\frac{e(m \cdot g^{x \cdot r}, \hat{r})}{e(g^r, \hat{r}^x)} \neq \frac{e(m' \cdot g^{x \cdot r'}, \hat{r})}{e(g^{r'}, \hat{r}^x)} \wedge$$

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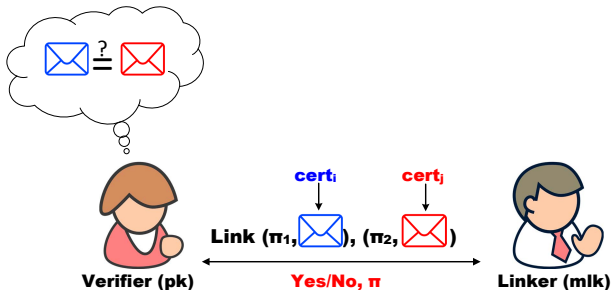
\Rightarrow Our construction for non-membership proofs

NIPEI Proof System

Proof system $\Pi = (\Pi_{\in}, \Pi_{\notin})$

NIPEI Proof System

Proof system $\Pi = (\Pi_{\in}, \Pi_{\notin})$



GSSs with Verifiable Controllable Linkability

Extended security model for VCL-GS

- Algorithms: Link and $\text{Link}_{\text{Judge}}$
- Property: **linking soundness**

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Instantiation based on **NIPEI**

- Link : **$\Pi.\text{Proof}$**
- $\text{Link}_{\text{Judge}}$: **$\Pi.\text{Verify}$**

Take-Home Message

- Proposed generic approach for (in-)equality proof

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- Also works for DLIN and CCA-secure ElGamal variants
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- Novel application
 - GSSs with verifiable controllable linkability

Non-Interactive Plaintext (In-)Equality Proofs and Group Signatures with Verifiable Controllable Linkability

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