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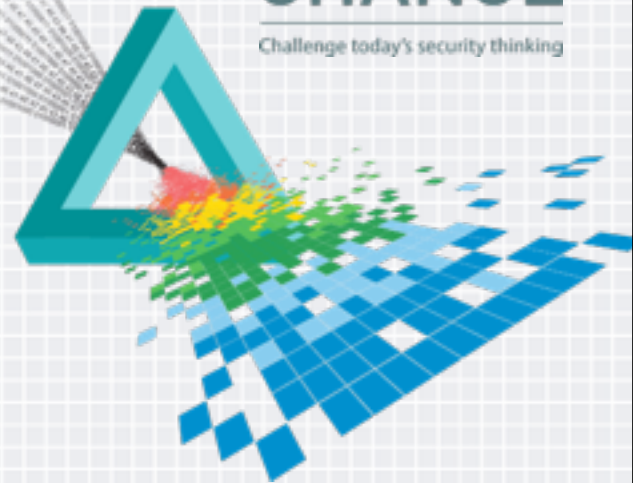
Communication Optimal Tardos-based Asymmetric Fingerprinting

Qiang Tang

University of Connecticut & University of Athens
joint work with Aggelos Kiayias, Nikos Leonardos, Helger Lipmaa, and Kateryna Pavlyk

CHANGE

Challenge today's security thinking



A Motivational Example

A Movie Producer



Cinema 1



Cinema 2

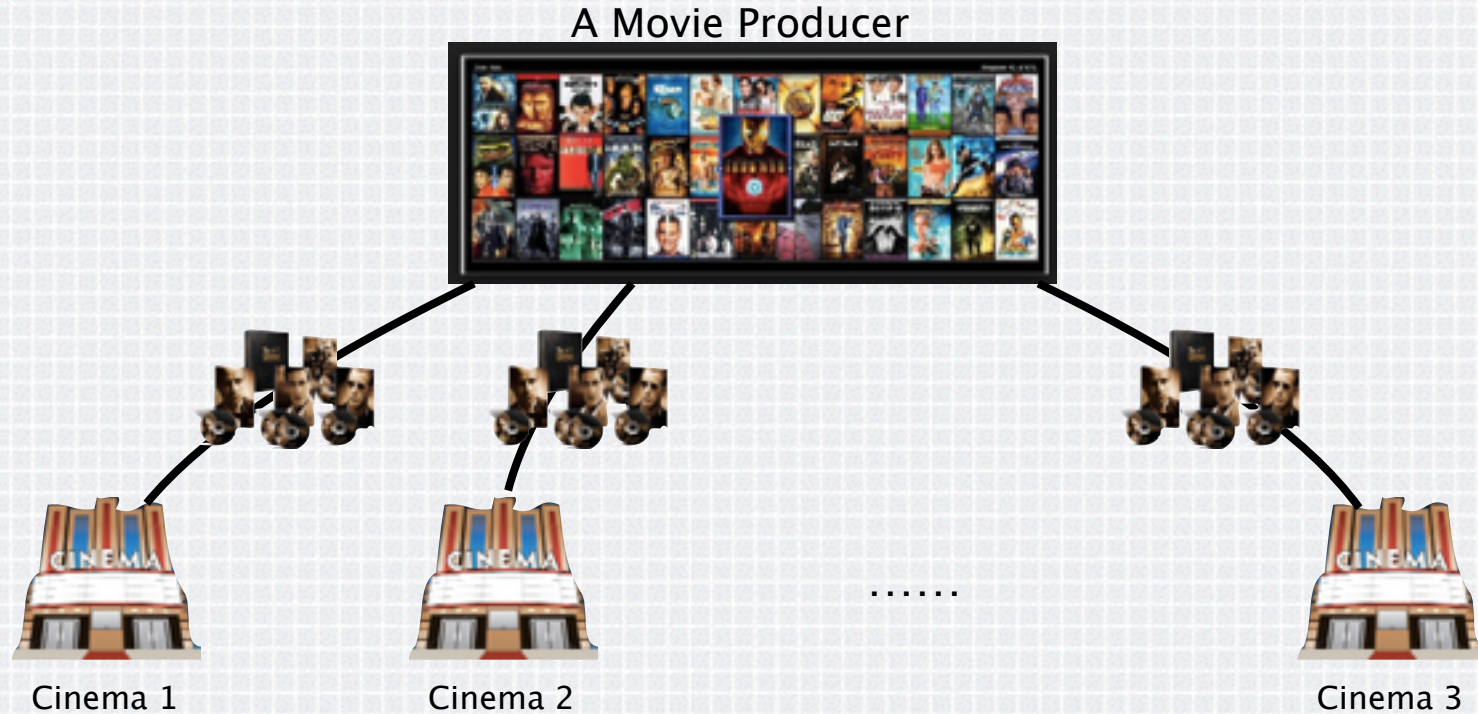
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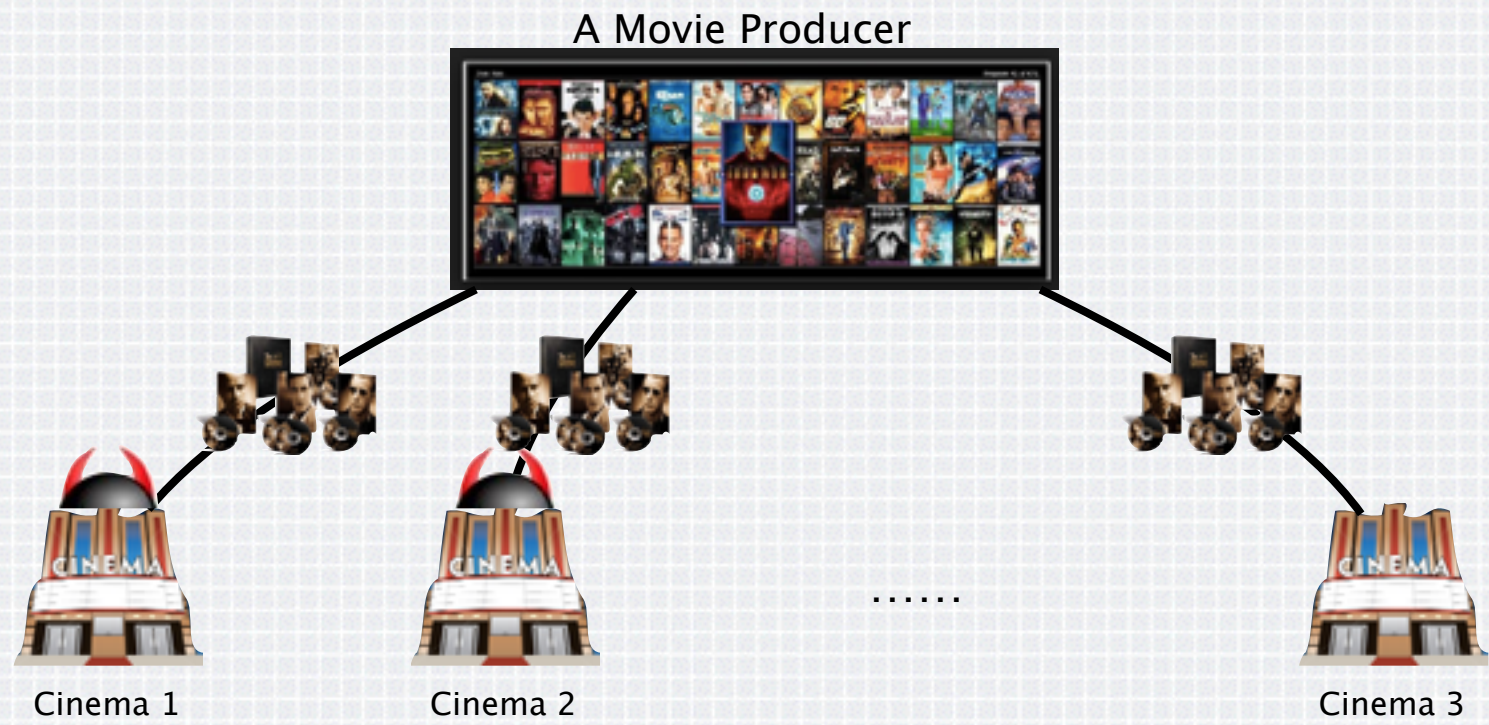
Cinema 3



A Motivational Example



A Motivational Example



A Motivational Example



The screenshot shows an eBay product listing for "Godfather-HD". The page layout includes the eBay logo, a user greeting "Hi, smeechiebutter! (Sign out)", and navigation tabs for "CATEGORIES", "ELECTRONICS", "FASHION", "MOTORS", "TICKETS", "DEALS", and "CLASSIFIEDS". Below the navigation, a breadcrumb trail reads: "Back to Daily Deals | Listed as Nexus 7 32GB, Wi-Fi, 7in - Black in category: Computers/Tablets & Networking > iPads, Tablets & ebook Readers".

The product title is "Godfather-HD", marked as a "FREE shipping" item. It has a "Like" button, a "Want" button (4), a "Own" button (2), a 5-star rating, and "14 product reviews". The item condition is "New". The quantity is set to 1, with a note "Limited quantity available / 1,228 sold". The price is \$5.99. There are two main buttons: "Buy It Now" and "Add to cart". A checkbox for "GeekSquad 2 yr warranty \$27.99" is shown, with a link to "See other plans from \$19.99". An "Add to Watch list" button is also present.

Below the product details, there is a "Bill Me Later" section with the text "Get 6 months to pay" and "Subject to credit approval. See terms". The shipping information states "Shipping: FREE Standard Shipping | See details", with the item location as "Elizabeth, New Jersey, United States" and ships to "United States See exclusions". The delivery estimate is "Estimated between Thu. Jan. 3 and Wed. Jan. 9", with a note to "Use One-day Shipping to get it by Jan. 3". The payment options listed are "PayPal, Bill Me Later | See details".

The product image shows a collection of DVD and Blu-ray cases and discs for the Godfather movies, featuring Al Pacino and Marlon Brando.

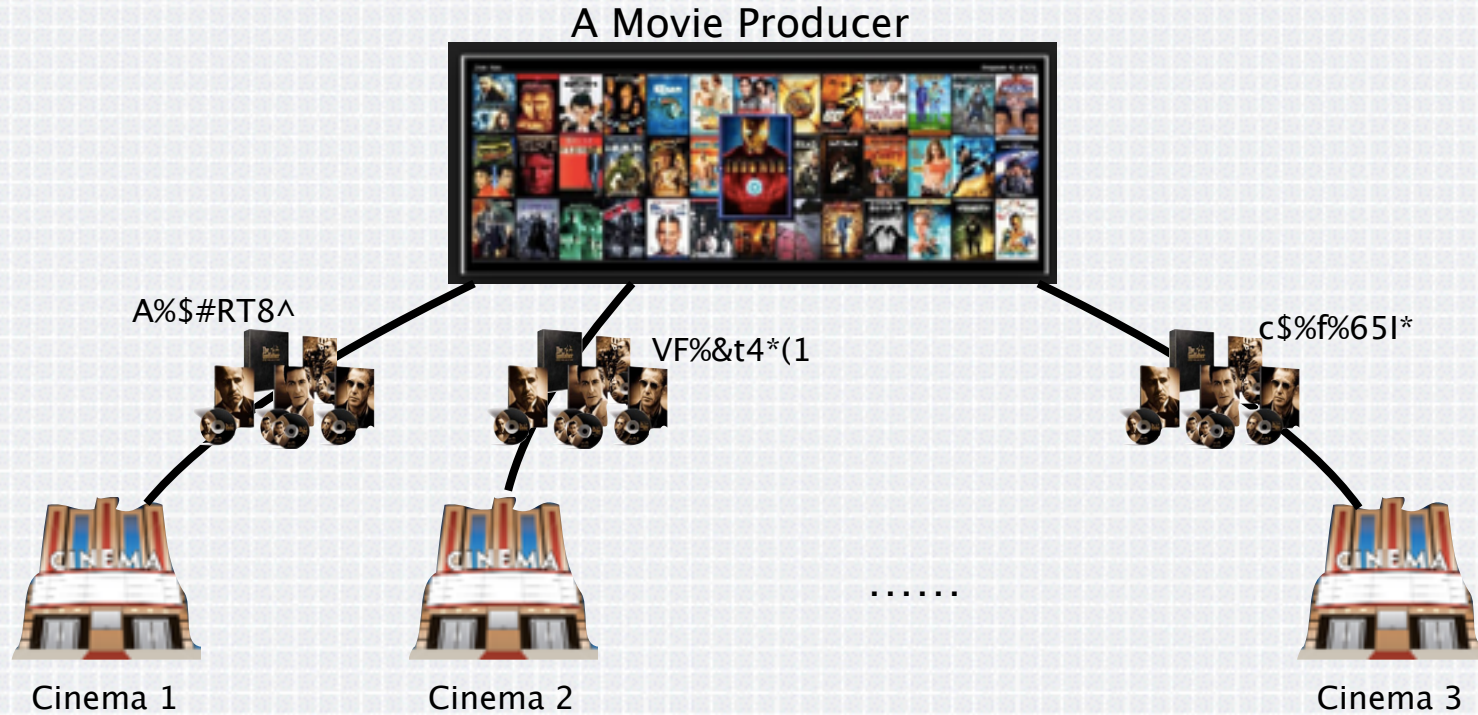


A Motivational Example

How to identify the source of the pirate?



Fingerprinting



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The product image shows the Blu-ray/DVD set for "The Godfather Part II", including the box set and several discs. Below the image, there is a "Bill Me Later" option: "Get 6 months to pay. Subject to credit approval. See terms".

The shipping information is: "Shipping: FREE Standard Shipping | See details". The item location is "Elizabeth, New Jersey, United States" and it ships to the "United States". The delivery estimate is "Estimated between Thu. Jan. 3 and Wed. Jan. 9". A note says "Use One-day Shipping to get it by Jan. 3". The payment options are "PayPal, Bill Me Later | See details".

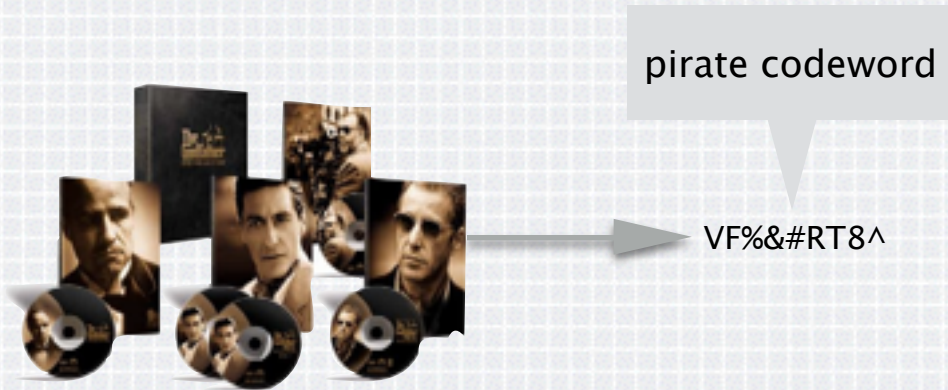
Fingerprinting



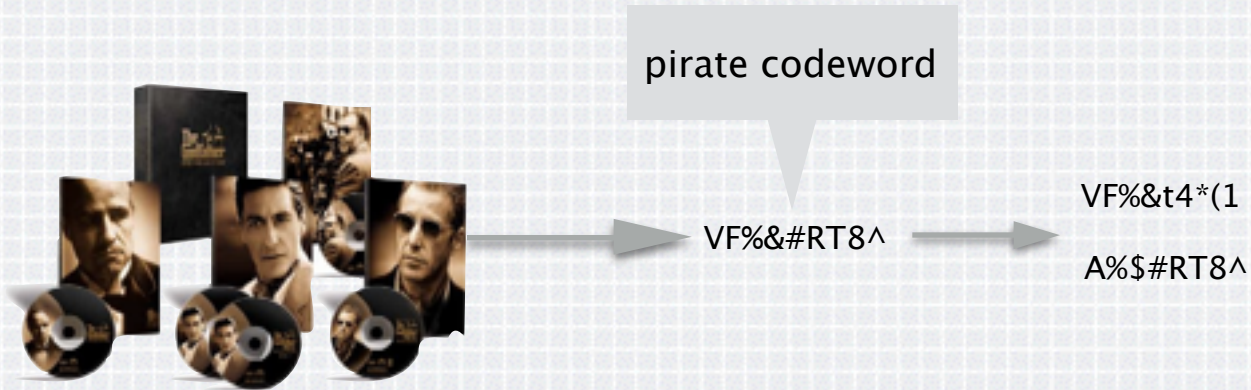
Fingerprinting



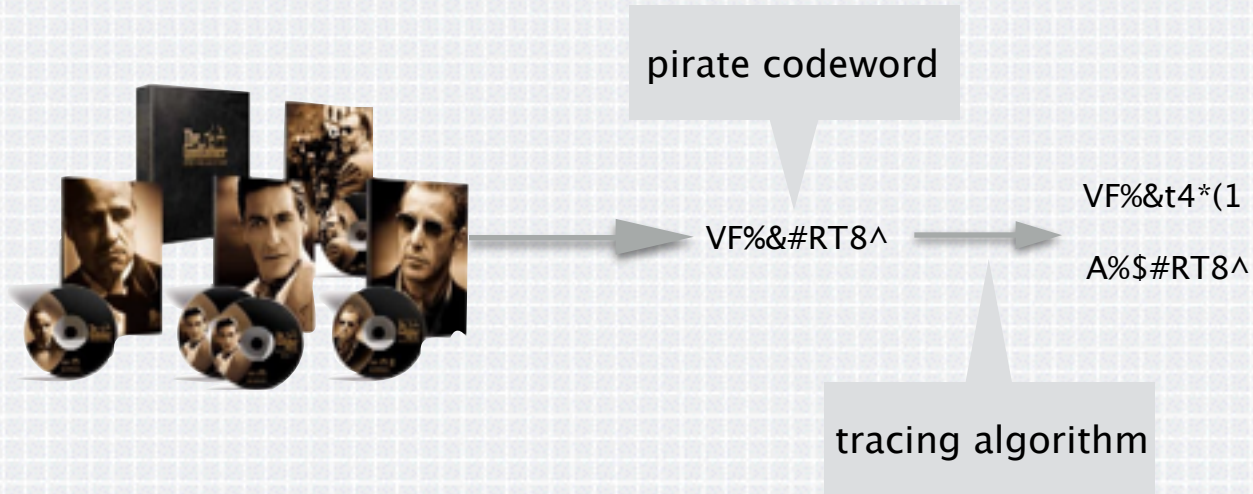
Fingerprinting



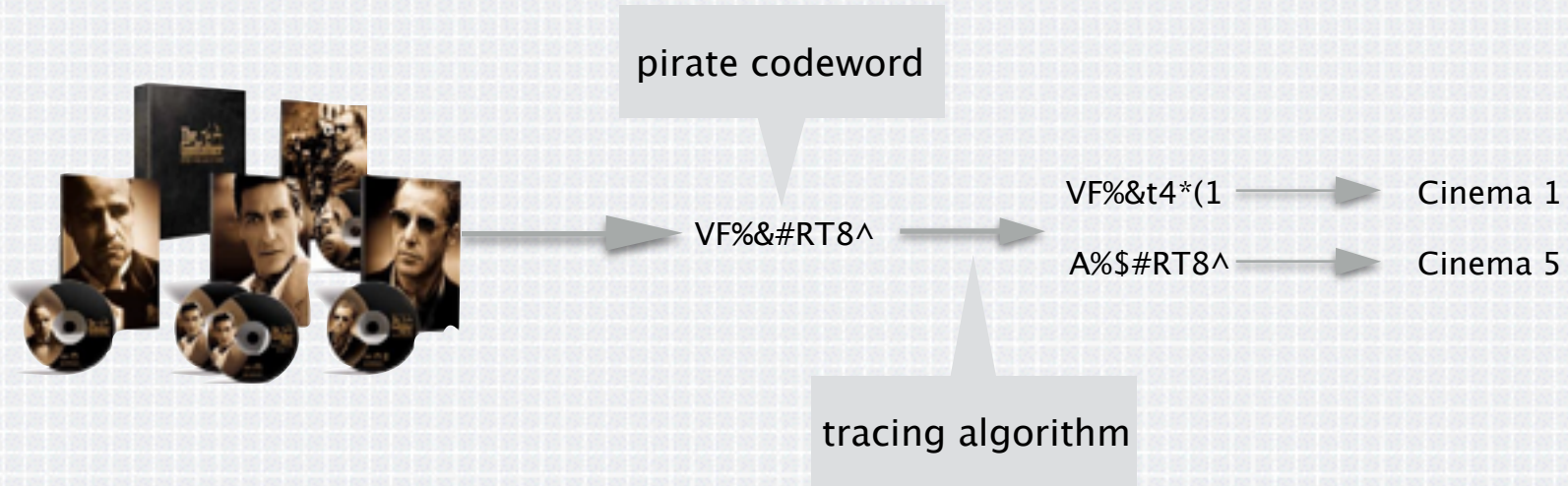
Fingerprinting



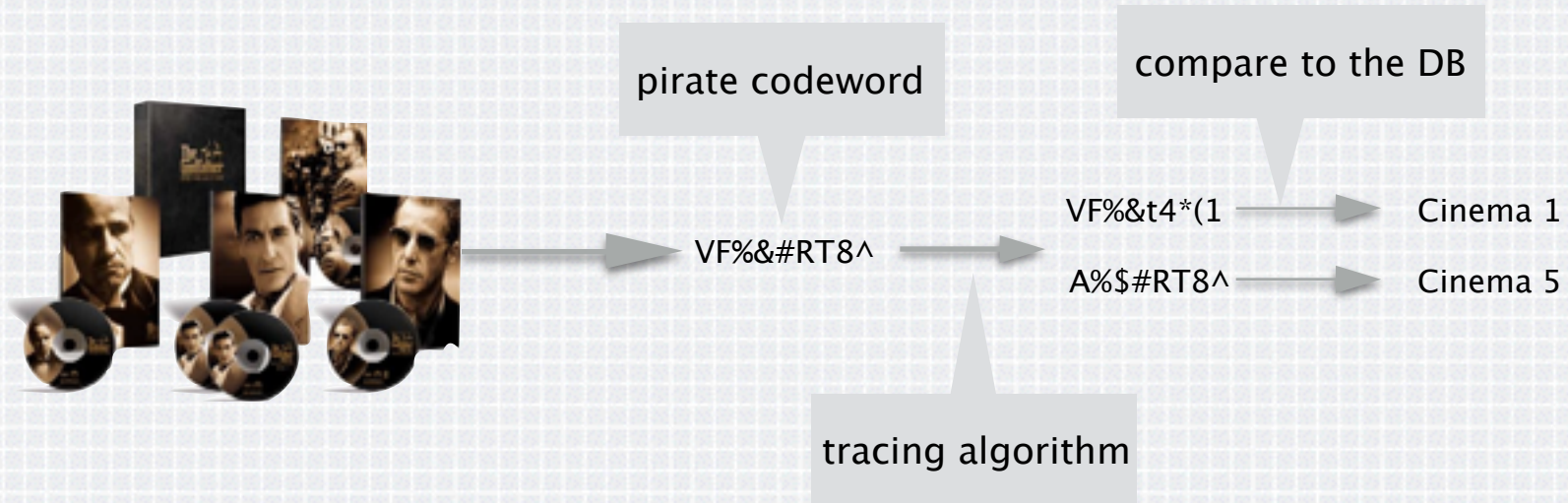
Fingerprinting



Fingerprinting



Fingerprinting



The Goals of Fingerprinting

- ◆ Individualize contents



The Goals of Fingerprinting

- ◆ Individualize contents
- ◆ Trace back to the sources



A Catch

Does fingerprinting really de-incentivize illegal content re-distribution?



A Catch

- ◆ **Both** the content provider and the content receiver can leak a copy



A Catch

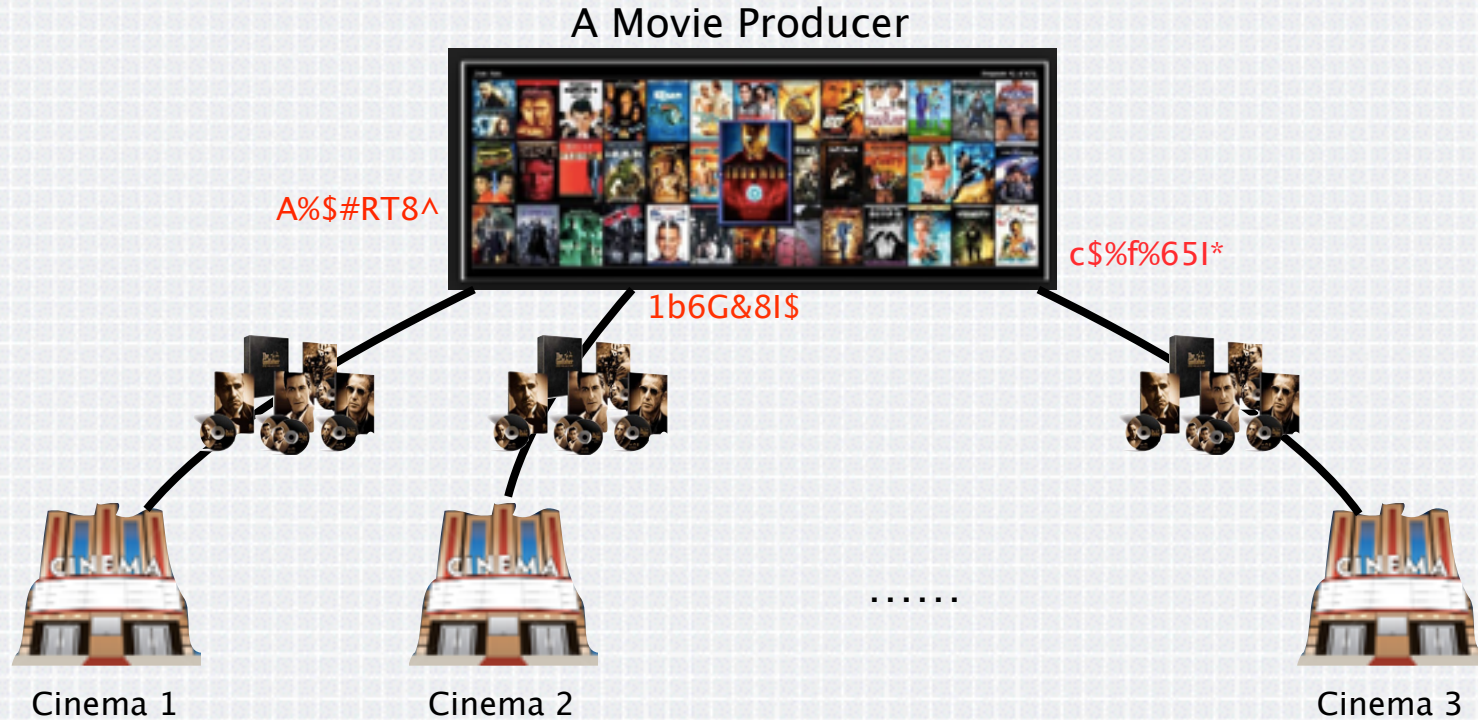
- ◆ **Both** the content provider and the content receiver can leak a copy
- ◆ The copy found in the public can not serve as a **undeniable** proof



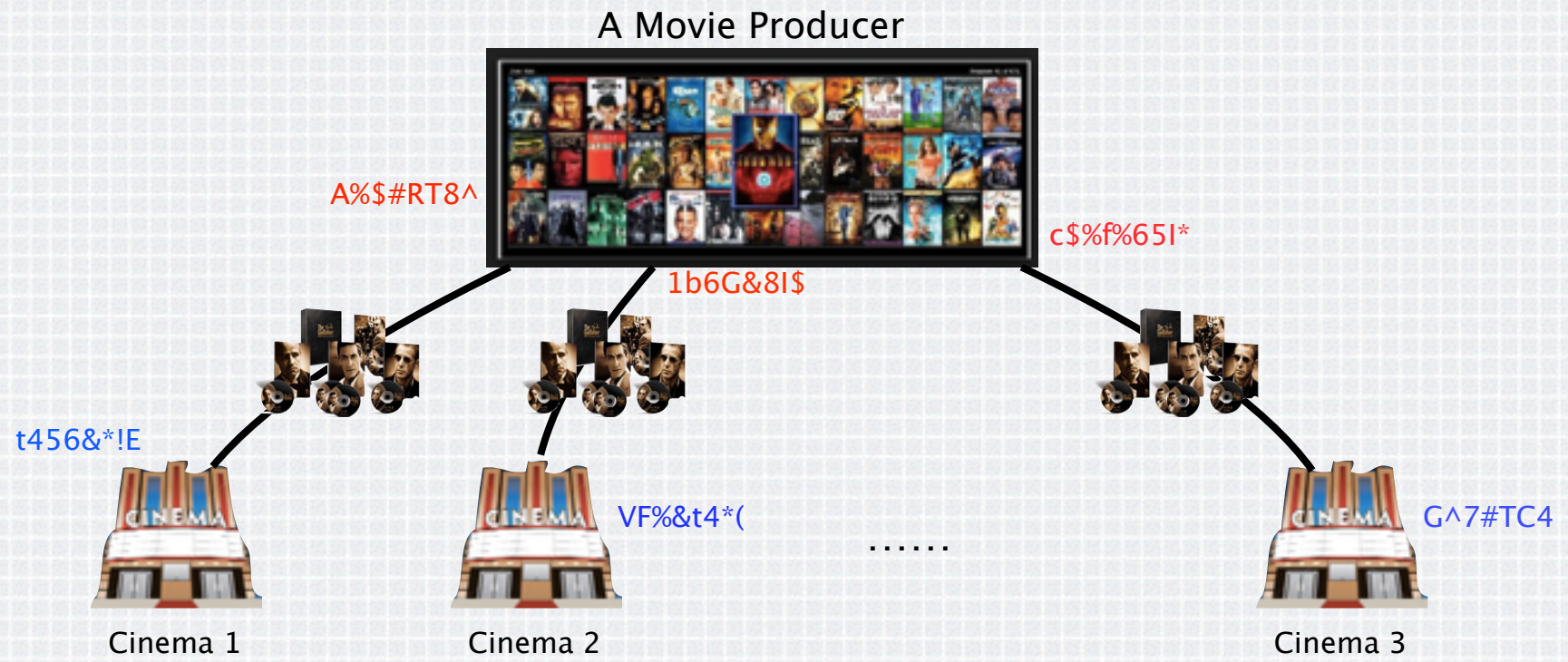
Asymmetric Fingerprinting



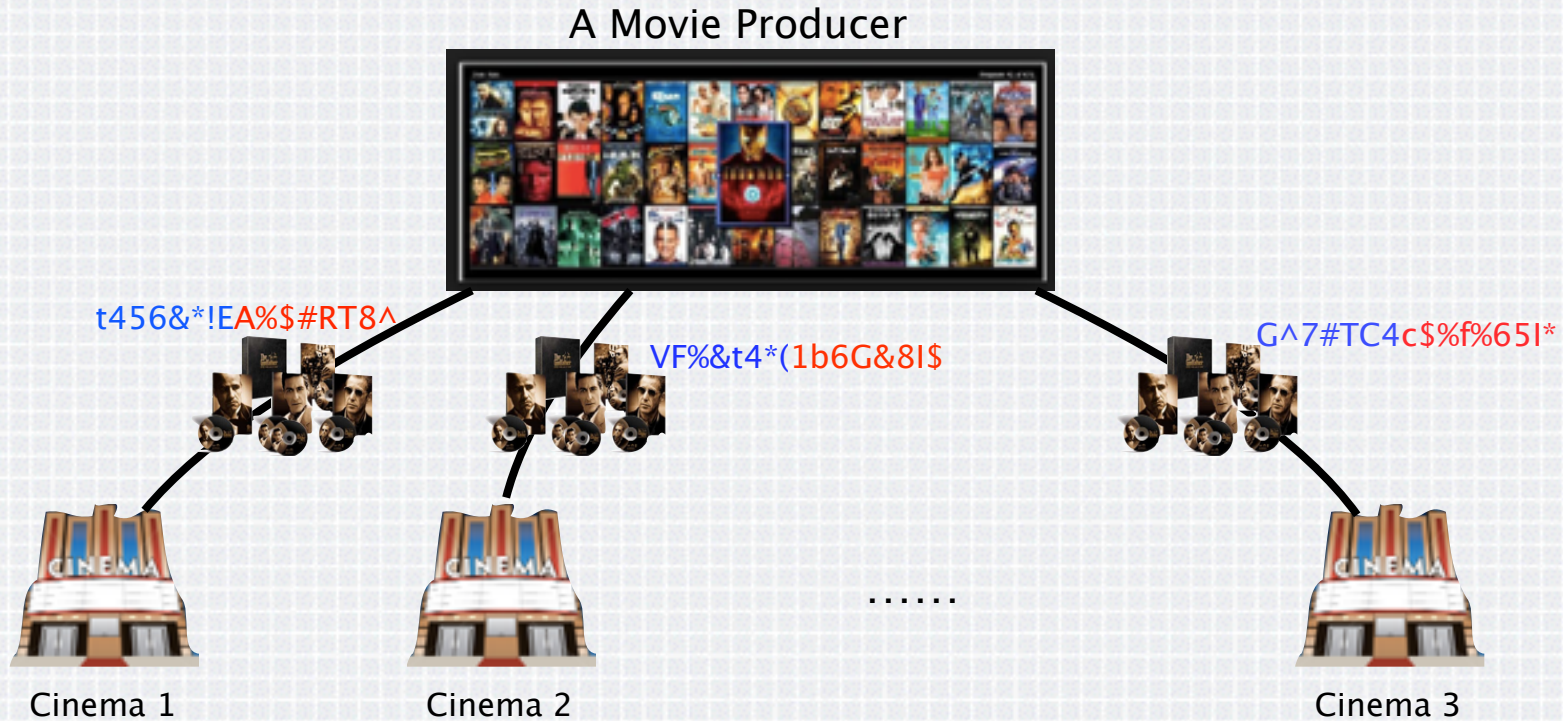
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ebay

Hi, smeechiebutler! (Sign out)

CATEGORIES ELECTRONICS FASHION MOTORS TICKETS DEALS CLASSIFIEDS

Back to Daily Deals | Listed as Nexus 7 32GB, Wi-Fi, 7in - Black in category: Computers/Tablets & Networking > iPads, Tablets & ebook Readers

Godfather-HD

FREE shipping

Item condition: **New**

Quantity: Limited quantity available / 1,228 sold

Price **\$5.99**

Buy It Now

Add to cart

☐ GeekSquad 2 yr warranty **\$27.99**
See other plans from **\$19.99**

Add to Watch list

Bill Me Later Get 6 months to pay
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Shipping: **FREE** Standard Shipping | See details
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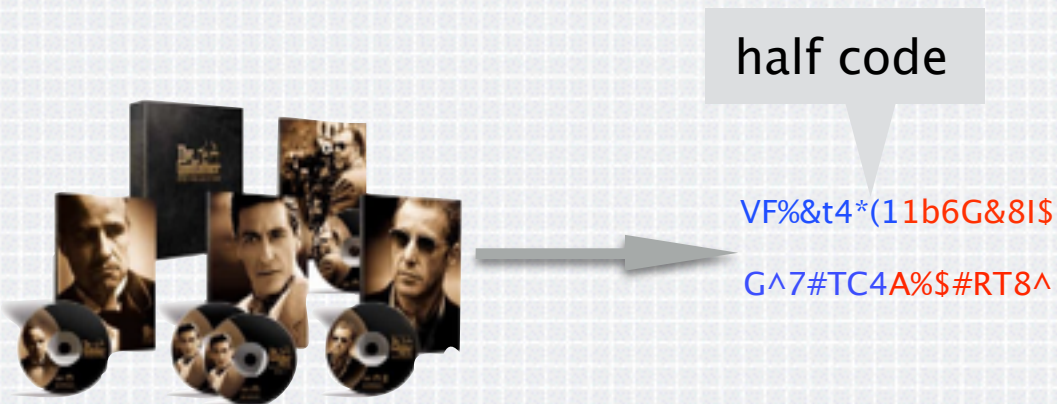
Delivery: Estimated between Thu. Jan. 3 and Wed. Jan. 9
Use One-day Shipping to get it by Jan. 3

Payments: **PayPal**, Bill Me Later | See details

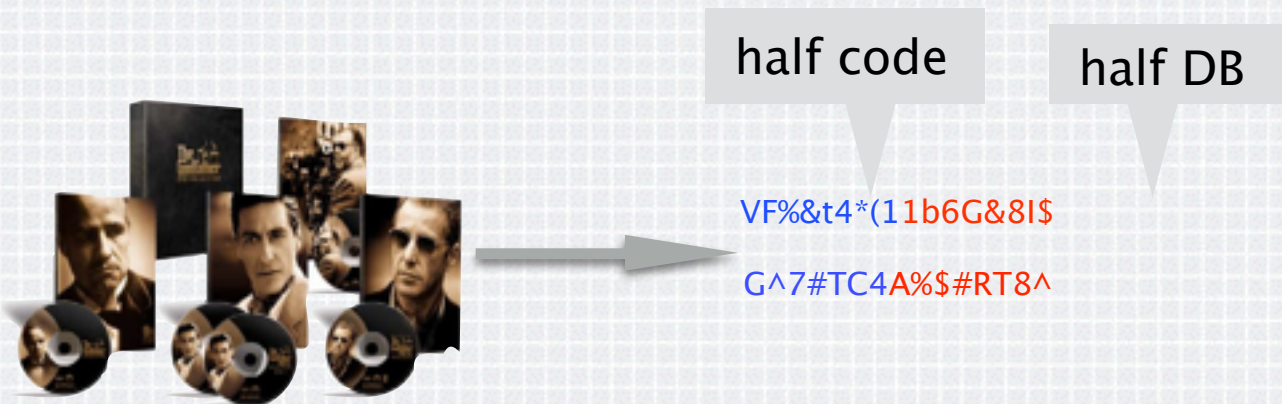
Asymmetric Fingerprinting



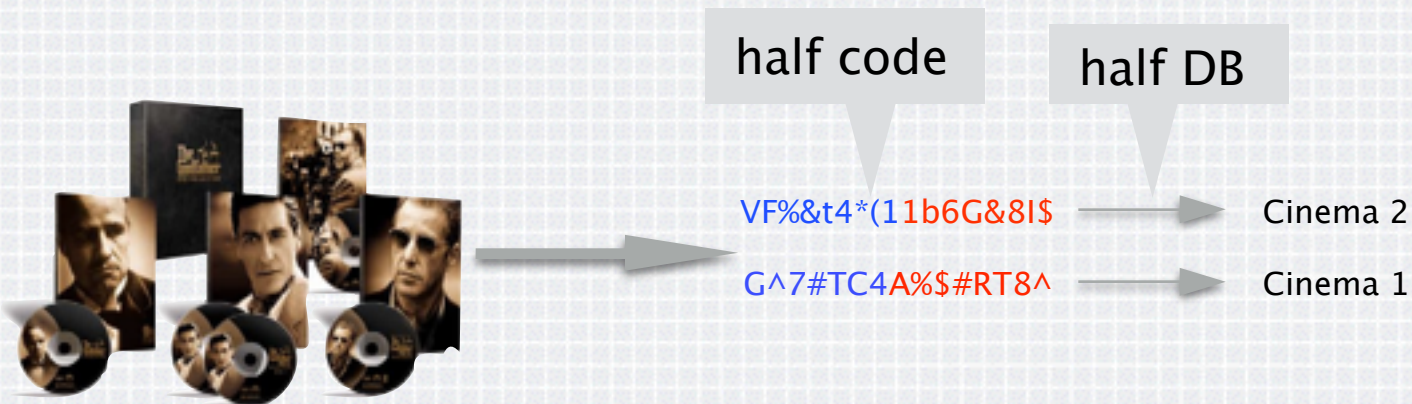
Asymmetric Fingerprinting



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Asymmetric Fingerprinting

VF%&t4*(11b6G&8I\$ → Cinema 2

G^7#TC4A%\$#RT8^ → Cinema 1



Asymmetric Fingerprinting

VF%&t4*(1b6G&8I\$ → Cinema 2

G^7#TC4A%\$#RT8^ → Cinema 1



Cinema 1

t456&*!EA%\$#RT8^



Cinema 2

VF%&t4*(1b6G&8I\$



Asymmetric Fingerprinting

VF%&t4*(11b6G&8I\$ → Cinema 2

G^7#TC4A%\$#RT8^ → Cinema 1



Cinema 1

t456&*!EA%\$#RT8^

t456&*!E

VF%&t4*(



Cinema 2

VF%&t4*(1b6G&8I\$



Asymmetric Fingerprinting

VF%&t4*(1b6G&8I\$ → Cinema 2

G^7#TC4A%\$#RT8^ → Cinema 1



Cinema 1

t456&*!EA%\$#RT8^

t456&*!E

VF%&t4*(



Cinema 2

VF%&t4*(1b6G&8I\$

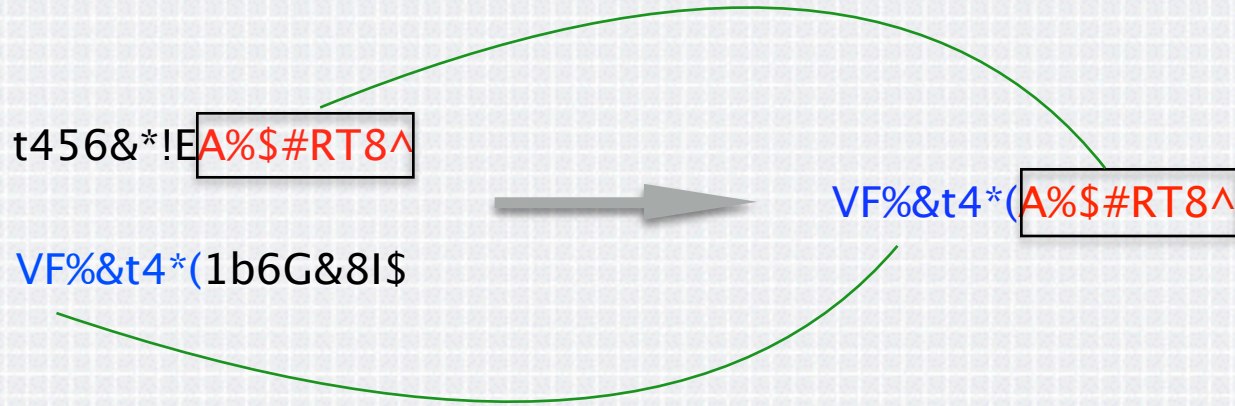


Cinema 1 ✗

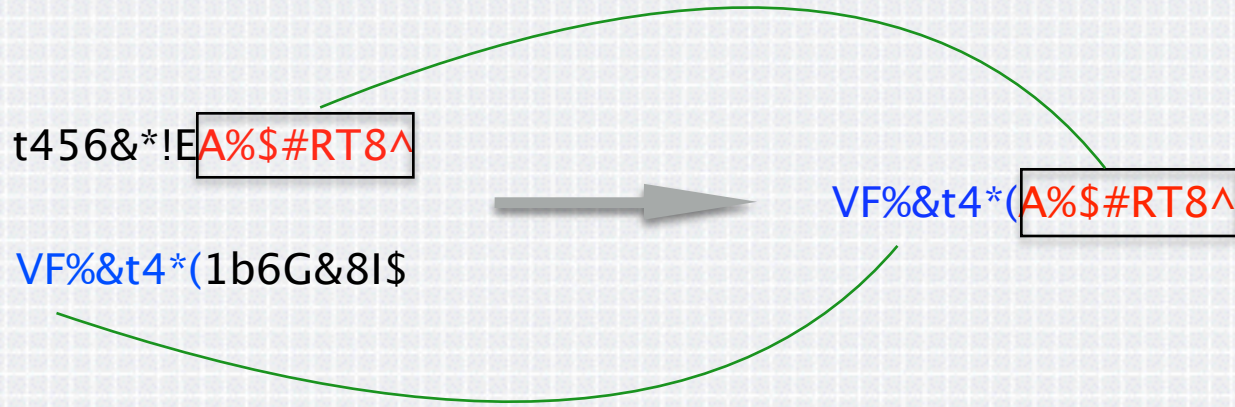
Cinema 2 ✓



A Subtle Security Consideration



A Subtle Security Consideration



Accusation Withdraw



A Subtle Security Consideration

1. User should not know how the two halves are mixed



A Subtle Security Consideration

1. User should not know how the two halves are mixed
2. Lower down the tracing parameter at the judge side



Important Efficiency Consideration

Even transmitting **2* movie size** kills the bandwidth



Important Efficiency Consideration

Even transmitting **2* movie size** kills the bandwidth

And will hinder the adoption of this technique



Important Efficiency Consideration

$$\text{Rate} = \frac{\text{Size of } \img alt="A stack of movie cases and discs, including Star Wars characters." data-bbox="488 372 582 482}}{\text{Size of actual transmission}}$$



Important Efficiency Consideration

$$\text{Rate} = \frac{\text{Size of } \begin{array}{c} \text{DVDs} \end{array}}{\text{Size of actual transmission}} \longrightarrow 1$$



Important Efficiency Consideration

Fight Piracy without Extra Bandwidth Cost !



High-level Construction Idea



Fingerprint Phase

- ◆ The content provider and a movie theater jointly **samples** a codeword (during the transmission of the movies), which is **oblivious** to the CP (using a conditional OT)



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- ◆ The CP and the movie theater runs OT protocols to see half of the codeword



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CP only knows half of the codeword



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CP only knows half of the codeword

Theaters don't know which part is known to the CP



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CP only knows half of the codeword

Theaters don't know which part is known to the CP

Rate optimal OT and COT are needed



Identify Phase

- ◆ Run the tracing algorithm of the underlying fingerprinting code on the half known to the CP



Dispute Phase

- ◆ The accused movie theaters submit the other halves of the codewords (with the proof)



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- ◆ The judge also runs the tracing algorithm with a **less** restrict parameter on these halves



Dispute Phase

- ◆ The accused movie theaters submit the other halves of the codewords (with the proof)
- ◆ The judge also runs the tracing algorithm with a **less** restrict parameter on these halves

Weaker judge side parameter is to avoid accusation withdraw



Communication Optimal Tardos-Based Asymmetric Fingerprinting



Linearly Homomorphic Encryption from DDH

Guilhem CASTAGNOS¹ Fabien LAGUILLAUMIE²

¹ Université de Bordeaux
INRIA Bordeaux - Sud-Ouest - LFANT
Institut de Mathématiques de Bordeaux UMR 5251,

² Université Claude Bernard Lyon 1
CNRS/ENSL/INRIA/UCBL LIP
Laboratoire de l'Informatique du Parallélisme

CT-RSA 2015



Outline

Linearly Homomorphic Encryption

Class Groups of Imaginary Quadratic Fields

New proposal

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Linearly Homomorphic Encryption

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New proposal

Linearly Homomorphic Encryption ?

- Public key encryption scheme with the following properties:
- Suppose that the set of plaintexts \mathcal{M} is a ring
- $c \leftarrow \text{Encrypt}(pk, m), c' \leftarrow \text{Encrypt}(pk, m')$
- $c_1 \leftarrow \text{EvalSum}(pk, c, c')$ s.t.

$$\text{Decrypt}(sk, c_1) = m + m'$$

- For $\alpha \in \mathcal{M}$, $c_2 \leftarrow \text{EvalScal}(pk, c, \alpha)$ s.t.

$$\text{Decrypt}(sk, c_2) = \alpha m$$

- Applications: Electronic Voting, Private Information Retrieval, Mix-Net, Oblivious Transfer, Fingerprinting...

Examples from Factoring

- Goldwasser Micali (84)
 - Plaintext space $\mathcal{M} = \mathbb{Z}/2\mathbb{Z}$
 - Ciphertext space : $\mathbb{Z}/N\mathbb{Z}$ where $N = pq$ is an RSA integer
- Paillier (99)
 - Plaintext space $\mathcal{M} = \mathbb{Z}/N\mathbb{Z}$
 - Ciphertext space : $\mathbb{Z}/N^2\mathbb{Z}$ where $N = pq$ is an RSA integer
 - Plaintext encoding :

$$m \in \mathbb{Z}/N\mathbb{Z} \mapsto (1 + N)^m \equiv 1 + mN \pmod{N^2}$$

From DDH: ElGamal “in the exponent”

- Folklore message encoding: $m \in \mathbf{N} \mapsto g^m$
- $(c_1, c_2) = (g^r, h^r g^m) \leftarrow \text{Encrypt}(pk, m)$
- $\text{Decrypt}(pk, c) : c_2/c_1^x = g^m \rightsquigarrow m$
- m must be small. Can only do a bounded number of homomorphic operations:
 - $(c_1, c_2) = (g^r, h^r g^m) \leftarrow \text{Encrypt}(pk, m),$
 - $(c'_1, c'_2) = (g^{r'}, h^{r'} g^{m'}) \leftarrow \text{Encrypt}(pk, m'),$

$$(c_1 c'_1, c_2 c'_2) = (g^{r+r'}, h^{r+r'} g^{m+m'})$$

$$(c_1^\alpha, c_2^\alpha) = (g^{r\alpha}, h^{r\alpha} g^{m\alpha})$$

DDH group with an easy DL subgroup

- $(G, \times) = \langle g \rangle$ a cyclic group of order n
- $n = ps$, $\gcd(p, s) = 1$
- $\langle f \rangle = F \subset G$ subgroup of G of order p
- The DL problem is easy in F : There exists, Solve, a deterministic polynomial time algorithm s.t.

$$\text{Solve}(p, f, f^x) \rightsquigarrow x$$

- The DDH problem is hard in G even with access to the Solve algorithm

A Generic Linearly Homomorphic Encryption Scheme

- $\mathcal{M} = \mathbb{Z}/p\mathbb{Z}$
- $pk : h = g^x, sk : x$, where g has order $n = ps$ for an unknown s
- Encrypt : $c = (c_1, c_2) = (g^r, f^m h^r)$, where $f \in \langle g \rangle$ has order p
- Decrypt : $A \leftarrow c_2/c_1^x$, $\text{Solve}(p, f, A) \rightsquigarrow m$
- EvalSum :

$$(c_1 c'_1, c_2 c'_2) = (g^{r+r'}, h^{r+r'} f^{m+m'})$$

- EvalScal :

$$(c_1^\alpha, c_2^\alpha) = (g^{r\alpha}, h^{r\alpha} f^{m\alpha})$$

An Unsecure Instantiation

- p a prime and $G = \langle g \rangle = (\mathbb{Z}/p^2\mathbb{Z})^\times$ of order $n = p(p-1)$
- $f = 1 + p \in G$, $F = \langle f \rangle = \{1 + kp, k \in \mathbb{Z}/p\mathbb{Z}\}$
- $f^m = 1 + mp$.
- There exist a unique $(\alpha, r) \in (\mathbb{Z}/p\mathbb{Z}, (\mathbb{Z}/p\mathbb{Z})^\times)$ such that $g = f^\alpha r^p$

$$g^{p-1} = f^{\alpha(p-1)} = f^{-\alpha}$$

- Public key : $h = g^x$,

$$h^{p-1} = f^{-\alpha x} \rightsquigarrow x \pmod{p}$$

- $(c_1, c_2) = (g^r, h^r f^m)$

$$c_1^{p-1} = f^{-\alpha r} \rightsquigarrow r \pmod{p}$$

$$c_2^{p-1} = f^{-\alpha x r - m} \rightsquigarrow m \pmod{p}$$

Partial Discrete Logarithm Problem

- $(G, \times) = \langle g \rangle$ a cyclic group of order n
- $n = ps$, $\gcd(p, s) = 1$
- $\langle f \rangle = F \subset G$ subgroup of G of order p
- Partial Discrete Logarithm (PDL) Problem:

Given $X = g^x$ compute $x \bmod p$.

- The knowledge of s makes the PDL problem easy.

s must be hidden or unknown !

A Secure Instantiation

- Bresson, Catalano, Pointcheval (03)
- Let N be an RSA integer, $G = \langle g \rangle \subset (\mathbb{Z}/N^2\mathbb{Z})^\times$
- $n = \text{Card}(G) = Ns$ with $s \mid \varphi(N)$,
- $f = 1 + N \in G$, $F = \langle f \rangle = \{1 + kN, k \in \mathbb{Z}/N\mathbb{Z}\}$, of order N
- Public key : $h = g^x$, x secret key
- $(c_1, c_2) = (g^r, h^r f^m)$
- Based on DDH in $(\mathbb{Z}/N^2\mathbb{Z})^\times$ and the Factorisation problem.
- The factorisation of N acts as a second trapdoor.

Outline

Linearly Homomorphic Encryption

Class Groups of Imaginary Quadratic Fields

New proposal

Definitions

Imaginary Quadratic Fields

- $K = \mathbf{Q}(\sqrt{\Delta_K}), \Delta_K < 0$
- Fundamental Discriminant:
 - $\Delta_K \equiv 1 \pmod{4}$ square-free
 - $\Delta_K \equiv 0 \pmod{4}$ and $\Delta_K/4 \equiv 2, 3 \pmod{4}$ square-free
- Non Fundamental Discriminant:
 - $\Delta_\ell = \ell^2 \Delta_K$
 - ℓ is the conductor

Class Group of Discriminant Δ

- Finite Group denoted $C(\Delta)$
- Elements: Equivalence classes of Ideals
- Class Number: $h(\Delta) \approx \sqrt{|\Delta|}$

ElGamal in Class Group

- Buchmann and Williams (88): Diffie-Hellman key exchange and ElGamal
- Düllmann, Hamdy, Möller, Pohst, Schielzeth, Vollmer (90-07): Implementation
- Size of Δ_K ? Index calculus algorithm to compute $h(\Delta_K)$ and Discrete Logarithm in $C(\Delta_K)$
- Security Estimates from Biasse, Jacobson and Silvester (10):
 - Complexity conjectured $L_{|\Delta_K|}(1/2, o(1))$
 - Δ_k : 1348 bits as hard as factoring a 2048 bits RSA integer
 - Δ_k : 1828 bits as hard as factoring a 3072 bits RSA integer

Map between two Class Groups

- Let Δ_K be a fundamental negative discriminant, $\Delta_K \neq -3, -4$, ℓ a conductor, and $\Delta_\ell = \ell^2 \Delta_K$
- There exists a surjective morphism, denoted $\bar{\varphi}_\ell$, between $C(\Delta_\ell)$ and $C(\Delta_K)$
- $\bar{\varphi}_\ell$ is effective, can be computed if ℓ is known
- Used by the NICE cryptosystem by Paulus and Takagi (00), $\Delta_K = -q$, $\Delta_p = -qp^2$, p, q primes, p is the trapdoor
- C., Laguillaumie (09) :

In each non trivial class of $\ker \bar{\varphi}_p$, there exists an ideal of the

$$\text{form } \left[p^2 \mathbf{Z} + \frac{bp + \sqrt{\Delta_p}}{2} \mathbf{Z} \right]$$

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A Subgroup with an Easy DL Problem

- $\Delta_K = -pq$, $\Delta_p = -qp^3$, p, q primes and $pq \equiv 3 \pmod{4}$

$$h(\Delta_p) = p \times h(\Delta_K)$$

- Let $f = \left[p^2 \mathbf{Z} + \frac{p + \sqrt{\Delta_p}}{2} \mathbf{Z} \right] \in \mathcal{C}(\Delta_p)$

- $F = \ker \bar{\varphi}_p = \langle f \rangle$ is of order p , and

$$f^m = \left[p^2 \mathbf{Z} + \frac{[m^{-1} \bmod p]p + \sqrt{\Delta_p}}{2} \mathbf{Z} \right]$$

A New Linearly Homomorphic Encryption Scheme

- $\Delta_K = -pq$, $\Delta_p = -qp^3$, p, q primes and $pq \equiv 3 \pmod{4}$ and $(p/q) = -1$, $q > 4p$
- Let g be an element of $C(\Delta_p)$, $h = g^x$ where x secret key
- g has order ps for an unknown $s|h(\Delta_K)$
- $(c_1, c_2) = (g^r, h^r f^m)$ where f has order p
- Based on DDH in $C(\Delta_p)$ (and the Class number problem).
- Linearly homomorphic over $\mathbf{Z}/p\mathbf{Z}$ where p can be chosen (almost) independently from the security parameter

Some Variants

- **Faster Variant:** most of the work in $C(\Delta_K)$ (based on a non standard problem)
- **More general message spaces:**
 - $\mathbf{Z}/N\mathbf{Z}$ with $N = \prod_{i=1}^n p_i$, with a discriminant of the form $\Delta_K = -Nq$
 - $\mathbf{Z}/p^t\mathbf{Z}$ for $t > 1$, with discriminants of the form $\Delta_{p^t} = p^{2t} \Delta_K$, and $\Delta_K = -pq$

Performance comparison

Cryptosystem	Parameter	Message Space	Encryption (ms)	Decryption (ms)
Paillier	2048 bits modulus	2048 bits	28	28
BCPo3	2048 bits modulus	2048 bits	107	54
New Proposal	1348 bits Δ_K	80 bits	93	49
Fast Variant	1348 bits Δ_K	80 bits	82	45
Fast Variant	1348 bits Δ_K	256 bits	105	68
Paillier	3072 bits modulus	3072 bits	109	109
BCPo3	3072 bits modulus	3072 bits	427	214
New Proposal	1828 bits Δ_K	80 bits	179	91
Fast Variant	1828 bits Δ_K	80 bits	145	78
Fast Variant	1828 bits Δ_K	512 bits	226	159
Fast Variant	1828 bits Δ_K	912 bits	340	271

Timings performed with Sage and PARI/GP.

Linearly Homomorphic Encryption from DDH

Guilhem CASTAGNOS¹ Fabien LAGUILLAUMIE²

¹ Université de Bordeaux
INRIA Bordeaux - Sud-Ouest - LFANT
Institut de Mathématiques de Bordeaux UMR 5251,

² Université Claude Bernard Lyon 1
CNRS/ENSL/INRIA/UCBL LIP
Laboratoire de l'Informatique du Parallélisme

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