2078/10/07

Date

Page

chapter-5 Transfer Function

Routh's – Hurwitz criterian ~~of~~ for stability checking.

system polynomial,

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n s^0$$

stable ??

Routh's Table:

s^n	a_0	a_2	a_4	a_6	$\cdots$
s^{n-1}	a_1	a_3	a_5	a_7	$\cdots$
s^{n-2}	b_1 (1st)	b_2	b_3		
s^{n-3}	c_1	c_2	c_3		
s^{n-4}	d_1	d_2	d_3		
$\vdots$	$\vdots$				
s^0	a_n				

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

If all elements of 1st column → +ve

The system is stable.

otherwise system is unstable.

Date ______
Page ______

Q. $p(s) = s^4 + 2s^3 + 6s^2 + 4s + 1 = 0$

Check the stability of the system described by the polynomial using Routh's H- criterion.

Sol. Routh's Table.

s^4 | 1 | 6 | 1
s^3 | 2 | 4 | 0
s^2 | $\frac{2*6 - 1*4}{2} = 4$ | $\frac{2*1 - 1*0}{2} = 1$ | 0
s^1 | $\frac{4*4 - 2*1}{4} = 14/4 = 7/2$ | 0
s^0 | $\frac{7/2*1 - 4*1\cdot 0}{7/2} = 1$

s^4	1	6	1
s^3	2	4	0
s^2	4	1	0
s^1	7/2	0	
s^0	1		

Here in 1st column all the elements are +ve so the system is stable.

Q. $H(s) = s^3 + s^2 + 2s + 24$

s^3	1	2	0
s^2	1	24	0
s	-22	0	
s^0	24		

$\frac{2\times1 - 1\times24}{1} = -22$

$\frac{-22\times24 - 1\times0}{-22}$

Here in first colum element are negetive so the system is unstable.

Date
Page

Q. Check for stability, Also find the no. of poles on the right half of s-plane. What is the no of poles in the given equation.

$s^5 + 6s^4 + 3s^3 + 2s^2 + s + 1$

Routh's Table.

s^5	1	3	1	0
s^4	6	2	1	0
s^3	$8/3$	$5/6$	0	
s^2	$1/18$	1	0	
s	-20.5	0		
s^0	1			

1 sign change

1 sign change.

$\frac{18-2}{6} \Rightarrow \frac{16}{6} = 8/3$

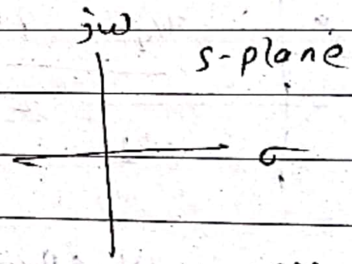
$\frac{6-1}{6}$

$\frac{8/3 \times 2 - 6 \times 5/6}{8/3}$

$\frac{16}{3} - 5$

$\frac{1/18 \times 5/6 - 8/3 \times 1}{1/18}$

Here the one element in first column is -ve so the system is unstable.



⇒ Number of sign changes = number of pole lie on right half.

Here total two sign changes so 2 poles lie on the right half of s-plane.

⇒ No. of poles = highest degree of eqn.

Here degree of eqn is 5 so no of poles is 5.

case - 1 when the 1st element of the row is zero.

eg. $P(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$

s^5	1	2	11	0
s^4	2	4	10	0
s^3	α	6	0	
s^2	$\frac{4\alpha - 12}{\alpha}$	10		
s^1	$(24\alpha - 72 - 10\alpha^2)/4\alpha - 12$			
s^0	10			

Assume α is a smallest +ve number

in case of 0 in 1st colum put $0 = \alpha$

The system is stable if $\frac{4\alpha - 12}{\alpha} > 0$

$4\alpha - 12 > 0$

$4\alpha > 12$

$\therefore \alpha > 3$

$q(s) = s^4 + s^3 + 2s^2 + 2s + 3$

Assume α be smallest +ve number.

s^4	1	2	3
s^3	1	2	0
s^2	α	3	0
s^1	$\frac{2\alpha - 3}{\alpha}$	0	
s^0	3		

The system is stable if

$\frac{2\alpha - 3}{\alpha} > 0$

$2\alpha - 3 > 0$

$2\alpha > 3$

$\Rightarrow \alpha > 3/2$

$$q(s) = s^4 + s^3 + s^2 + s + K$$

where, K is adjustable. Find the range of K so that the system is stable.

s^4	1	1	K
s^3	1	1	0
s^2	α	K	0
s^1	$\frac{\alpha - K}{\alpha}$	0	
s^0	K		

The system is stable only if $\frac{\alpha - K}{\alpha} > 0$

$\alpha - K > 0$

$\alpha > K \Rightarrow K < \alpha$

<u>case II</u>.

when all the elements of a row is zero.

e.g.

$$p(s) = s^5 + s^4 + 2s^3 + 2s^2 + s + 1$$

	s^5	1	2	1	0
eq^n	s^4	1	2	1	0
$\Rightarrow$	s^3	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	
	s^3	4	4	0	
eq^n	s^2	1	1	0	
$\Rightarrow$	s^1	$\boxed{0}$	$\boxed{0}$		
	s^1	2	0		
	s^0	1			

zero आएको row भन्दा माथि बाट एउटा equation बनाउने, त्यसको derivative गर्ने र coefficient लेख्ने।

$$q(s) = 1s^4 + 2s^2 + 1s^0$$

$$q'(s) = 4s^3 + 4s + 0$$

$$R(s) = 1s^2 + 1s^0$$

$$R'(s) = 2s + 0$$

All elements of 1st column are +ve.

Hence system is stable.

Date
Page

$q(s) = s^5 + s^4 + 4s^3 + 24s^2 + 3s + 63$

s^5	1	4	3
s^4	1	24	63
s^3	-20	-60	0
s^2	21	63	0
s^1	0	0	$\leftarrow$ all zero case. so $R(s) = 21s^2 + 63$ $= 42$
s^1	42	0	
s^0	63		

Here all the elements in 1st column are not +ve so the system is not stable.

$$\text{If} = \frac{s^2+3s+2}{s^3+3s^2+3s+A}$$

only consider $s^3 + 3s^2 + 3s + A$ and solve for it.

Denomenator नीचे लिखें।

2078/10/09

Date
Page

Pole-zero diagram and their plot.

$$H(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\ldots(s-z_n)}{(s-p_1)(s-p_2)(s-p_3)---(s-p_m)} \quad \begin{matrix}\text{(Numerator)}\\ \text{(denominator)}\end{matrix}$$

$s-z_1=0$ root लाई zeros भनेर भनिन्छ ।

$H(s)=0$

$s-z_2=0$ $\quad$ $s-z_3=0$

$s=z_2$ $\quad$ $s=z_3$

(zeros).

Denominator,

$s-p_1=0$

$$H(s) = \frac{---}{0} = \infty \quad \text{(pole)}$$

$s-p_2=0$ $\quad$ $s-p_3=0$

$s=p_2$ $\quad$ $s=p_3$

poles

$$H(s) = \frac{10(s+1)(s+9)}{(s+3)(s+7)}$$

plot the pole-zero diagram and also find the time domain response from the plot zero diagram.

Sol^n scaling factor = 10

zeros

$s+1=0$

$s=-1$

$s+4=0$

$s=-4$

poles

$s+3=0$

$s=-3$

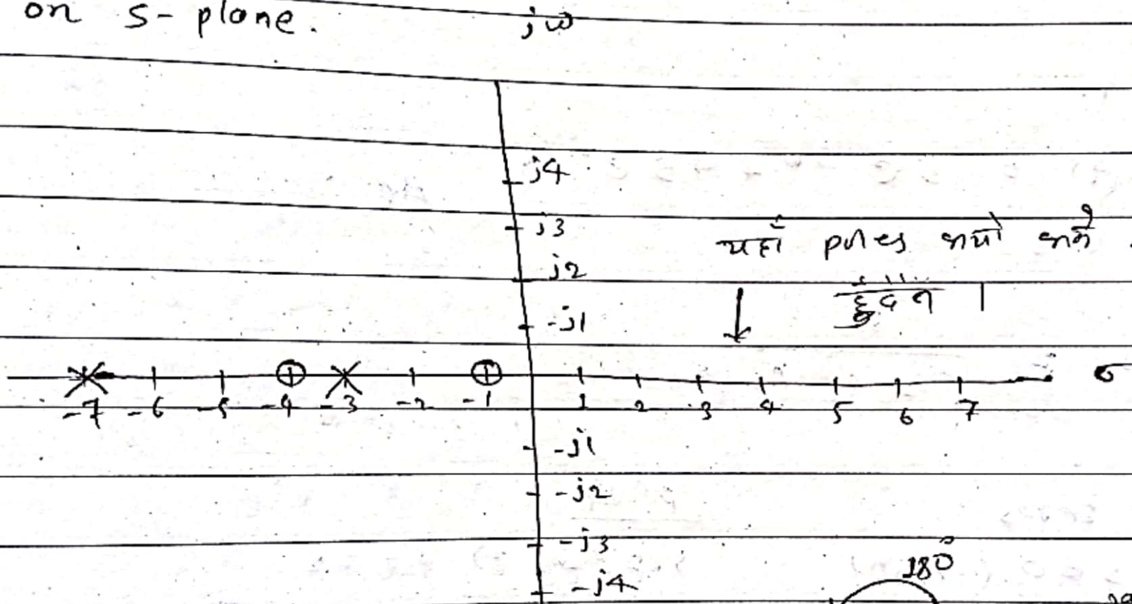
$s+7=0$

$s=-7$

zeros → O

poles → × or *

plot on s-plane.



180°

Always right to left.

(angle)

$$H(s) = \frac{A}{s+3} + \frac{B}{s+7}$$

Residue, Residue

$$A = \text{scaling factor} \times \frac{\text{multiplication of pole to zeros distance}}{\text{multiplication of pole to pole distance}}$$

(angle)

~~$10 \times \frac{2\angle 0^\circ \times 1\angle 180^\circ}{4}$~~

180° ←• R→L

•→ 0° L→R

$$= 10 \times \frac{2\angle 0^\circ \times 1\angle 180^\circ}{4\angle 180^\circ}$$

$$= 5\angle 0^\circ$$

$$B = 10 \times \frac{6\angle 0^\circ \times 3\angle 0^\circ}{4\angle 0^\circ}$$

$$= 45$$

Now,

$$H(s) = \frac{5}{s+3} + \frac{45}{s+7}$$

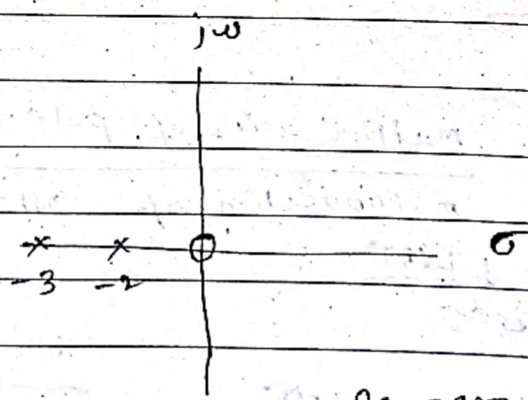
ILT.

$$h(t) = 5e^{-3t} + 45e^{-7t}$$ Ans.

Q. $$H(s) = \frac{3s}{(s+2)(s+3)}$$

Sol^n

zeros	poles
$s = 0$ (origin)	$s+2=0 \Rightarrow s=-2$
	$s+3=0 \Rightarrow s=-3$



pole zero diagram.

$$I(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = 3 \times \frac{2\angle 0^\circ}{1\angle 180^\circ} \Rightarrow -6$$

Date
Page

$$B = 3 \times \frac{3 \angle 0^\circ}{1 \angle 0^\circ}$$

$$= 9$$

$$I(s) = \frac{-6}{s+2} + \frac{9}{s+3}$$

ILT.

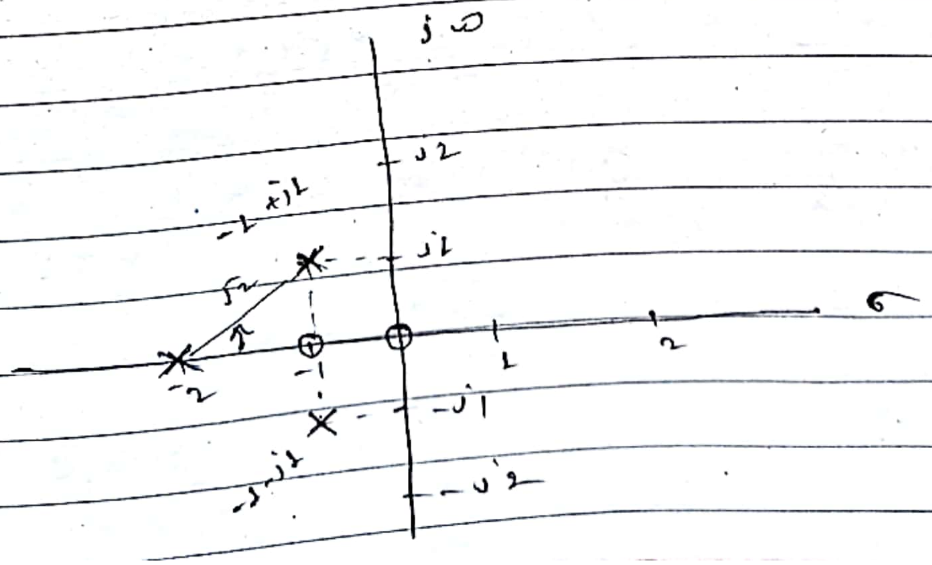
$$i(t) = -6e^{-2t} + 9e^{-3t}$$

Q. $$V(s) = \frac{10(s+1)s\cancel{(s+2)}}{(s+2)(s^2+2s+2)}$$

plot the pole-zero diagram on s-plane and also find the time domain response from the pole-zero diagram.

Sol^n Scaling factor = 10

zeros	poles
$s+1=0$ &	$s+2=0 \Rightarrow s=-2$
$s=-1$	$s^2+2s+2=0$
$s=0$	$s=-1+j1$
	$s=-1-j1$



$$V(s) = \frac{10\, s(s+1)}{(s+2)(s^2+2s+2)}$$

शुन्य point बाट distance नाप्ने हो त्यहाँको angle लिने । (Anticlockwise)

$$V(s) = \frac{10\, s\,(s+1)}{(s+2)\,(s+1+j1)\,(s+1-j1)}$$

$$V(s) = \frac{A}{s+2} + \frac{B}{s+1-j1} + \frac{C}{s+1+j1}$$

$$A = 10 * \frac{1\angle 0^\circ \times 2\angle 0^\circ}{\sqrt{2}\angle 45^\circ \times \sqrt{2}\angle 315^\circ}$$

$$= 10$$

$$B = 10 * \frac{1\angle 270^\circ \times \sqrt{2}\angle 315^\circ}{\sqrt{2}\angle 225^\circ \times 2\angle 270^\circ}$$