Diffraction

Introduction, Theory

Ingmar
@ingmarr

Timo Volz
@magict1911

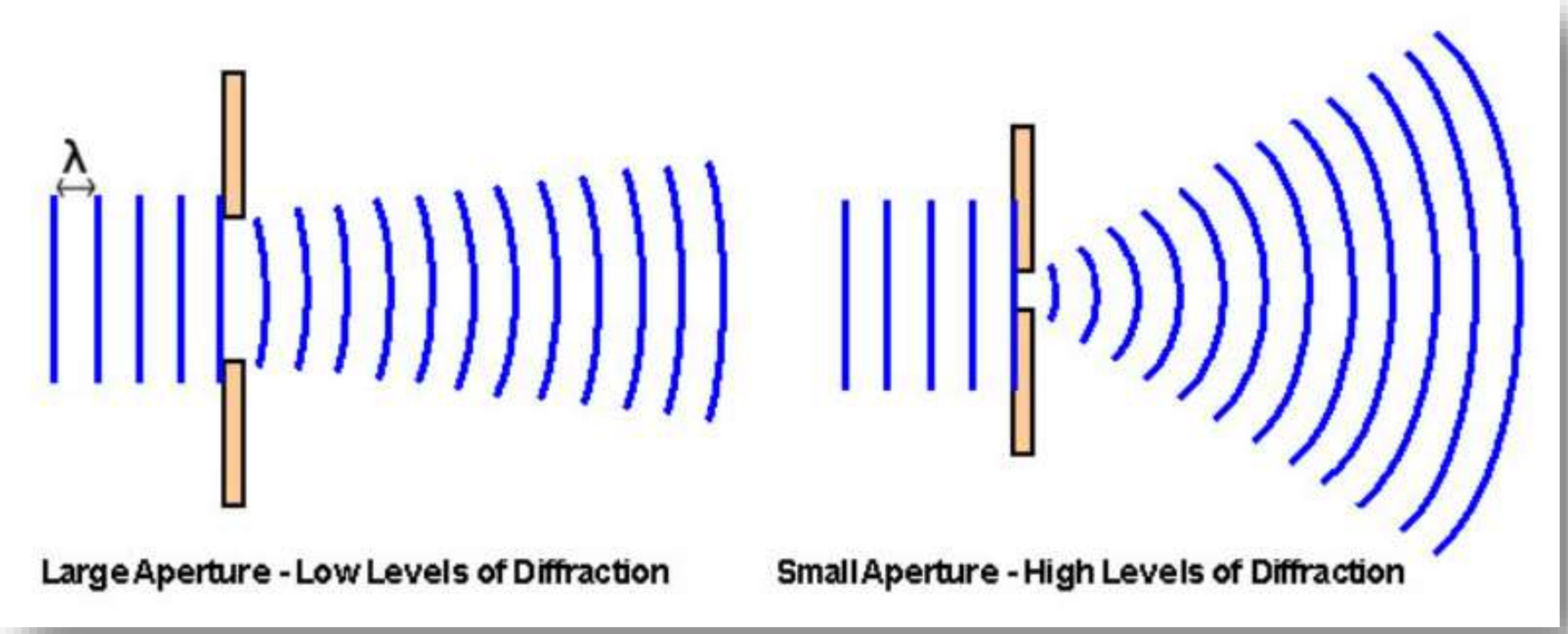
pixpoetry

@blackpoetry

Stephen Leonardi
@stephenleo1982

Greg Rosenke

@greg_rosenke



Source:

https://waves.neocities.org/diffractionpatterns.html

Visit the links below:

https://www.edumedia-sciences.com/en/media/160-diffraction

https://www.cyberphysics.co.uk/topics/light/A_level/diffraction_AS.htm

https://phet.colorado.edu/sims/html/wave-interference/latest/wave-interference_en.html https://www.youtube.com/watch?v=-mNQW5OShMA

https://www.schoolphysics.co.uk/age14-16/Wave%20properties/text/Diffraction_/index.html

https://i.pinimg.com/originals/1e/a6/c4/1ea6c43900ccb680f22cea079832e295.jpg

https://www.youtube.com/watch?v=lnZTS6yyxbA

https://www.youtube.com/watch?v=NazBRcMDOOo

Difference between interference and diffraction:

Interference	Diffraction
1. It is the result of interaction of light coming from two different wavefronts originating from two coherent sources.	1. It is the result of interaction of light coming from different parts of the same wavefront.
2. Interference fringes may or may not be of the same width.	2. Diffraction fringes are not of the same width.
3. Points of minimum intensity are perfectly dark.	3. Points of minimum intensity may not be perfectly dark.
4. All bright bands are of uniform intensity.	4. All bright bands are not of the same intensity.

Diffraction:

Definition:

The phenomenon of bending of light around the corners of an obstacle is called diffraction.

or

The encroachment of light in the geometrical shadow is called diffraction.

Condition for diffraction:

If the size or dimension of an obstacle is comparable to the size or dimension of the wave, diffraction is more pronounced.

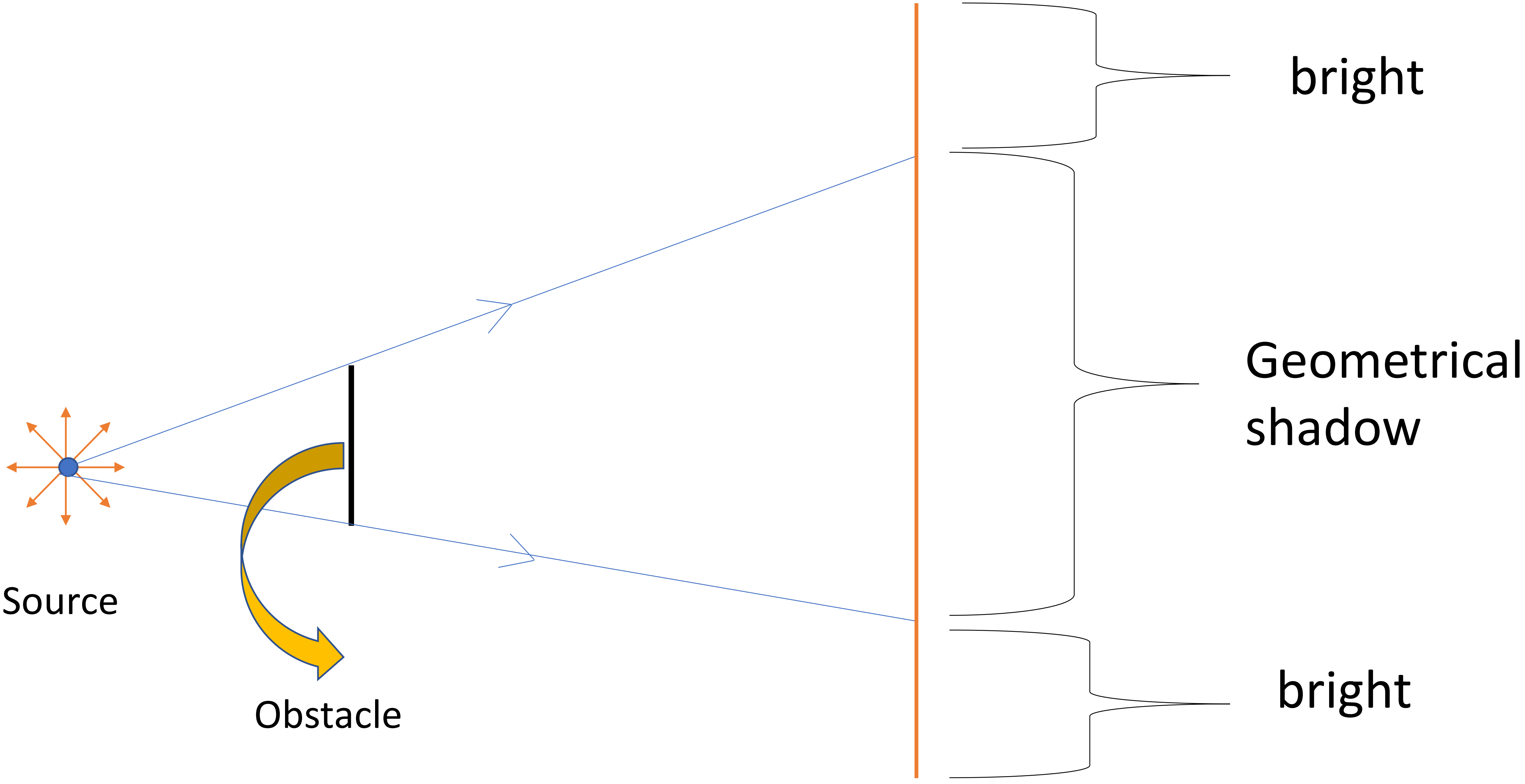


Fig (i): Diffraction of light

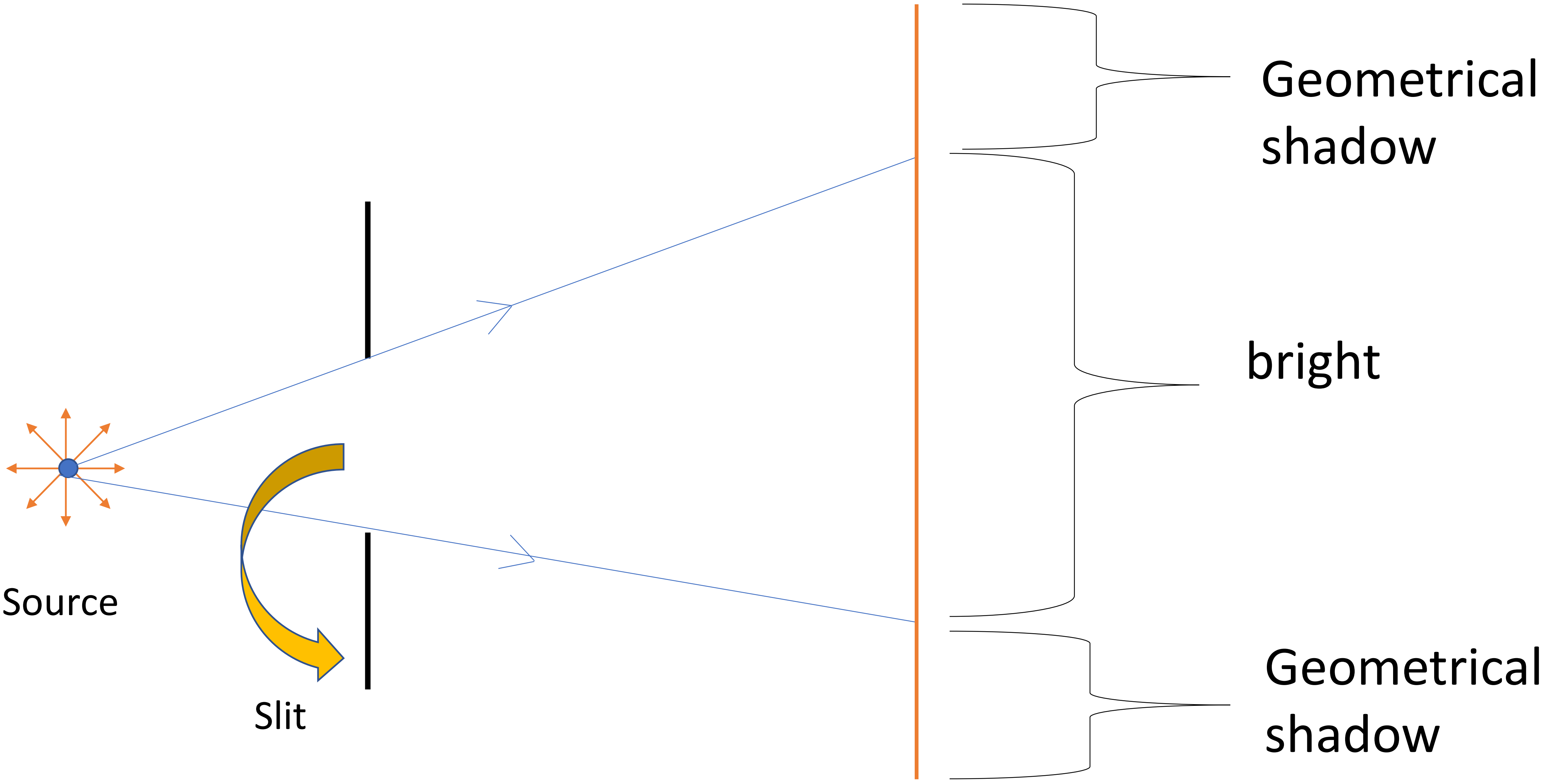
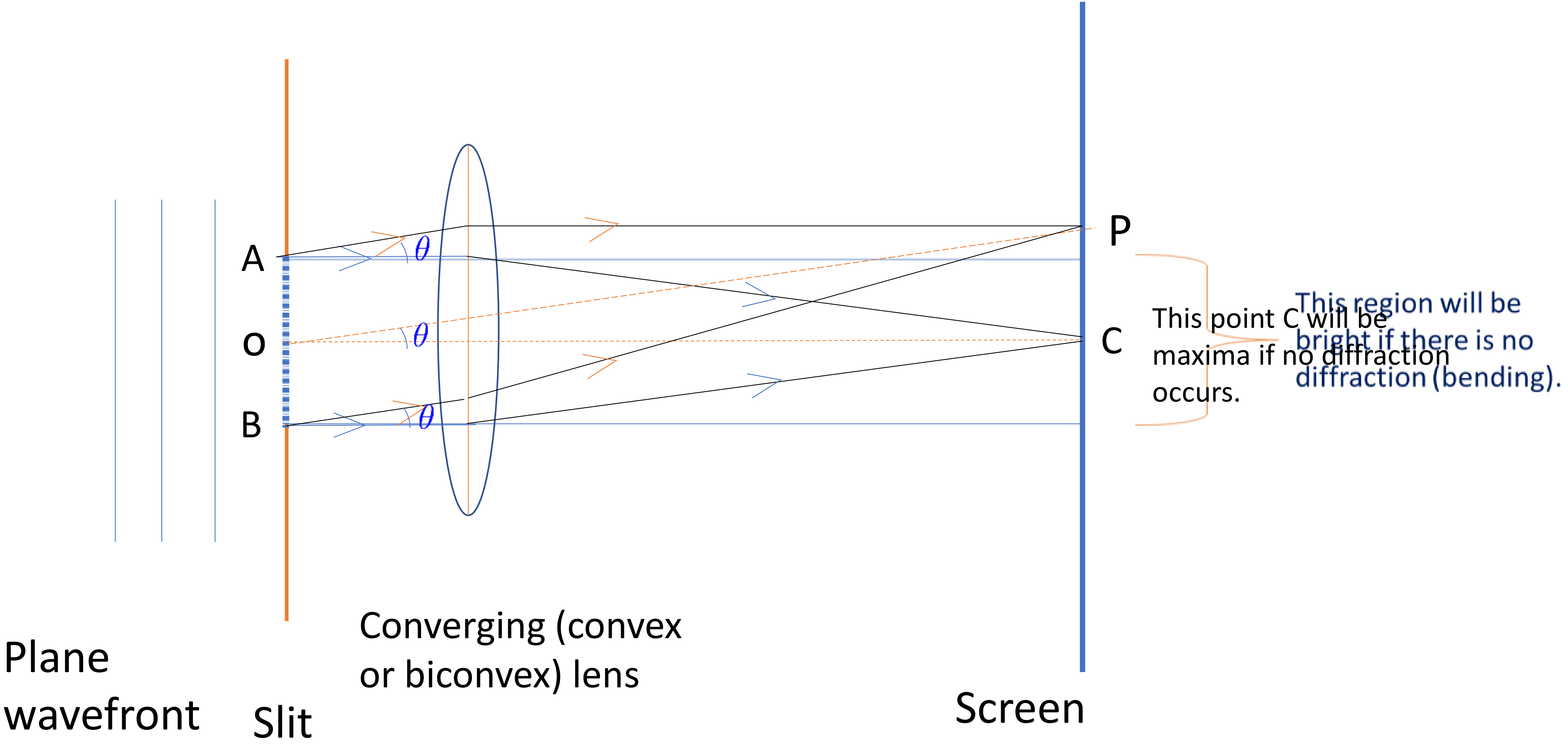


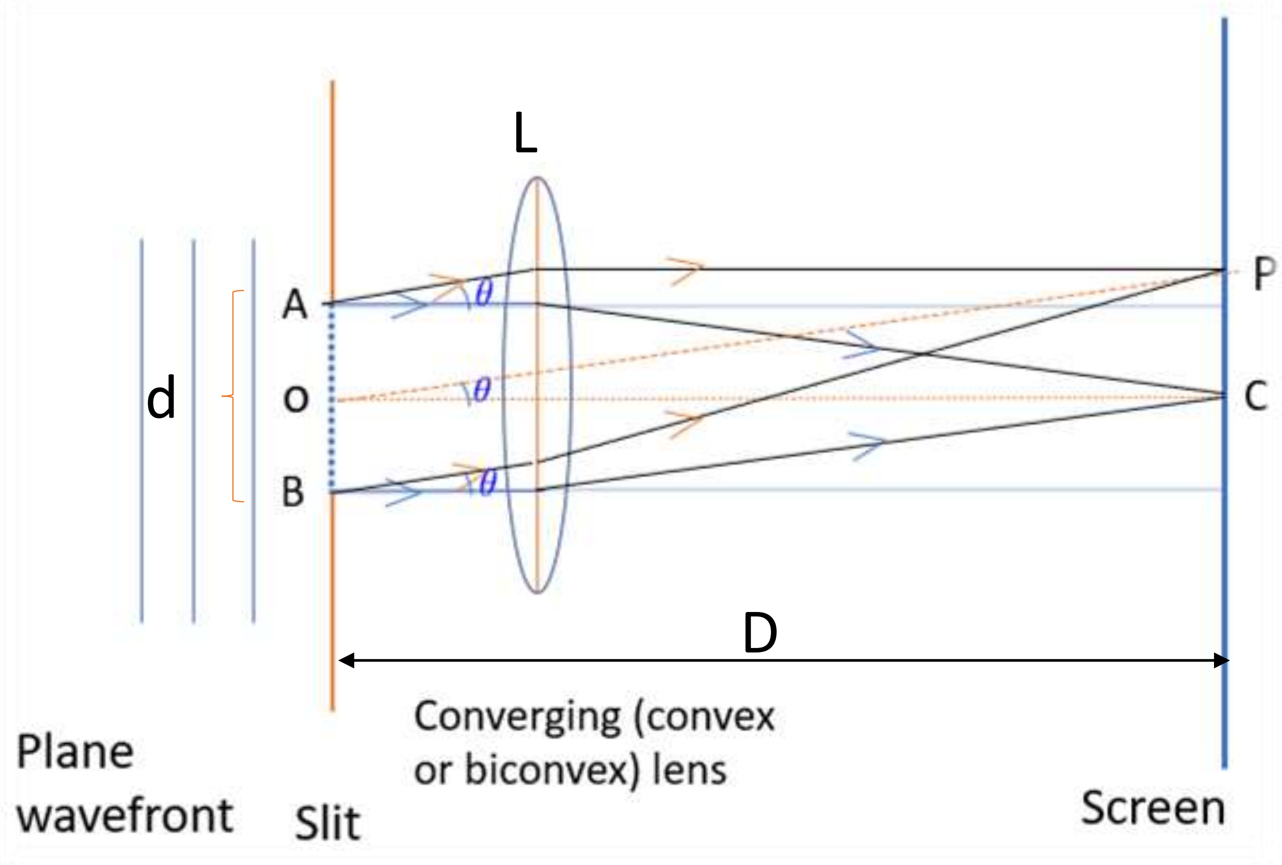
Fig (ii): Diffraction of light

Difference between Fresnel and Fraunhofer diffraction:

Fresnel diffraction	Fraunhofer diffraction
1. The light suffering diffraction is not modified by lenses or mirrors.	1. The light suffering diffraction is modified by lenses or mirrors.
2. In this case, either source or the screen or both are at finite distance from the obstacle or aperture.	2. In this case, either source or the screen or both are at infinite distance from the obstacle or aperture.
3. The incident wavefront is either spherical or cylindrical.	3. The incident and the diffracted wavefronts are plane.

Fraunhofer diffraction through a single slit:





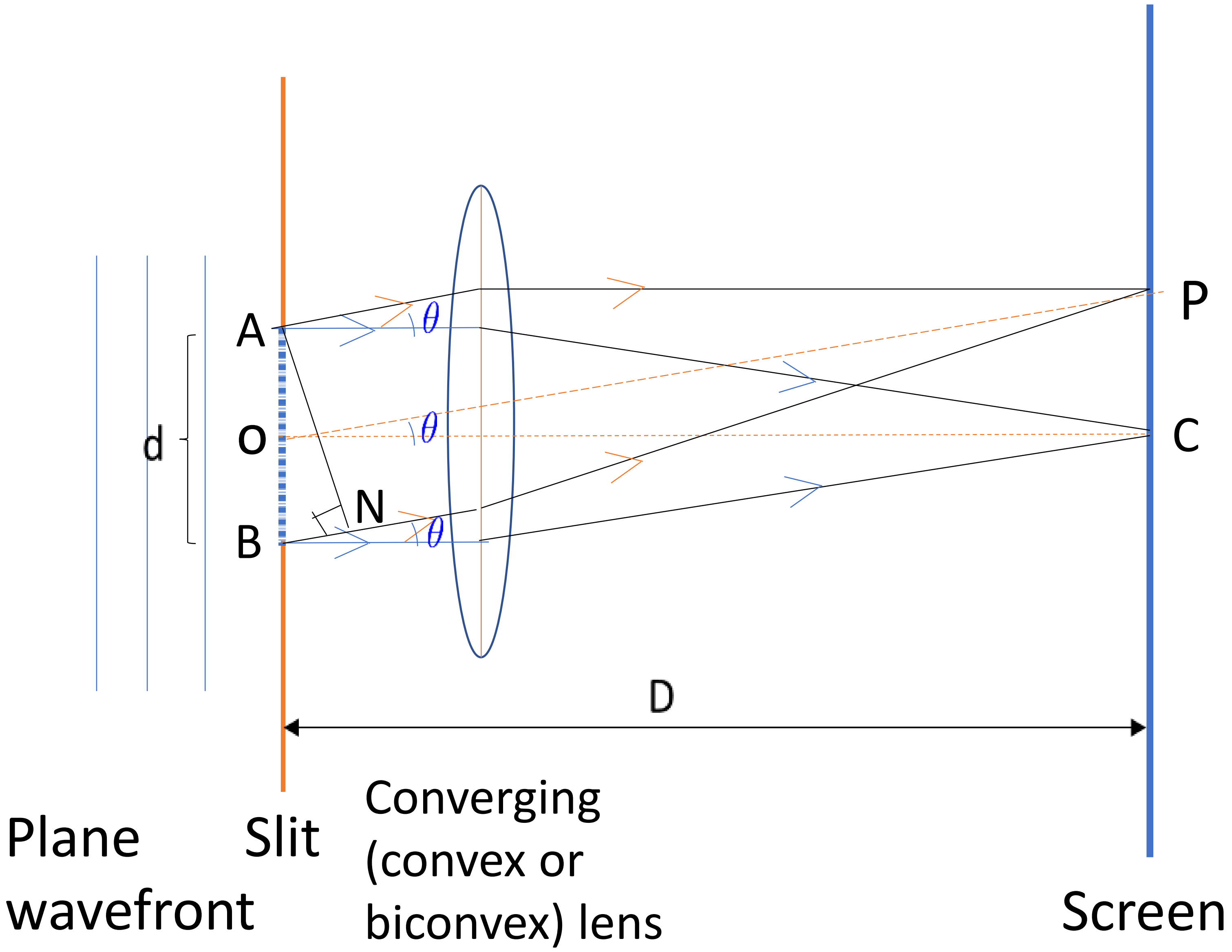
Let us consider a slit of width 'd'. Let a plane wavefront be incident on it.

Then each point acts as the source of secondary disturbance.

L is the converging lens which converges the horizontal beam to the point C, which is the central maxima.

Let the angle of diffraction be θ .

The point P will be maxima or minima depending upon the path difference between the diffracted rays coming from the points A and B.



Now, draw AN normal to the diffracted ray coming from B.

Then the path difference between the diffracted rays coming from A and B is BN.

Now, in $\triangle$ ABN,

$$\sin\theta = \frac{BN}{AB}$$

So, $\boxed{BN = d\sin\theta}$

Now, divide the slit into two equal parts or zones, each of width d/2.

Then, the path difference between the light rays coming from the points A and O is

$$\frac{d}{2}\sin\theta$$

Then, by Fresnel's law (of half period zone),

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

$$\boxed{d\sin\theta = \lambda} \quad \text{(1}^{\text{st}}\text{ minima)}$$

Again, divide the slit into four equal parts or zones, each of width d/4.

Then, by Fresnel's law (of half period zone),

$$\frac{d}{4}\sin\theta = \frac{\lambda}{2}$$

$$\boxed{d \sin\theta = 2\lambda}$$ (2nd minima)

Similarly, if we divide the slit into 2n equal parts or zones, each of width d/2n,

$$\frac{d}{2n}\sin\theta = \frac{\lambda}{2}$$

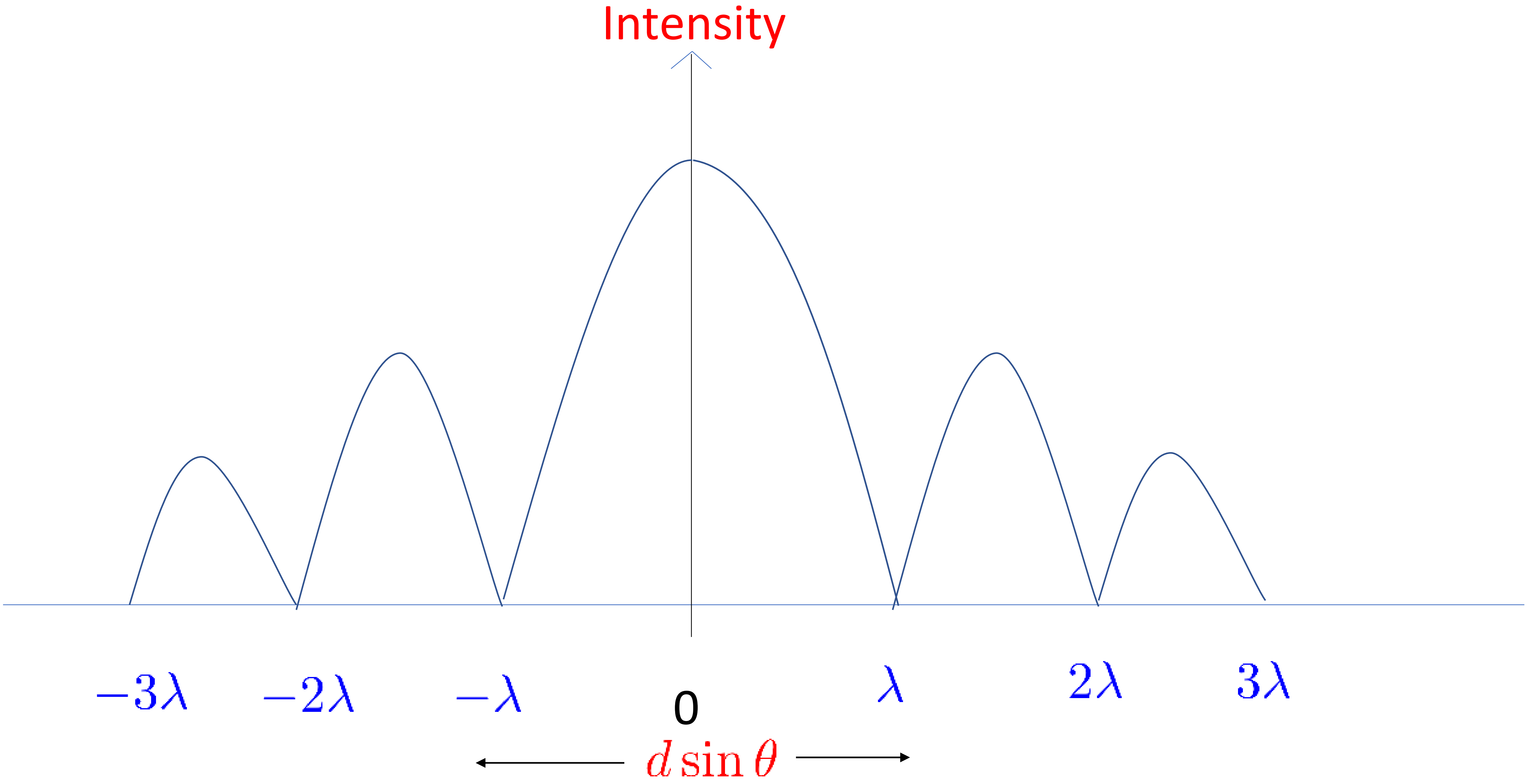
$$\boxed{d \sin\theta = n\lambda}$$ (n^{th} minima)

Hence in general, $\boxed{d \sin\theta = n\lambda}$, n = 1,2,3,……..

Similarly, secondary maxima are given by the equation

$$\boxed{d \sin\theta = (2n+1)\frac{\lambda}{2}}$$

Intensity pattern in Fraunhofer diffraction due to a single slit:



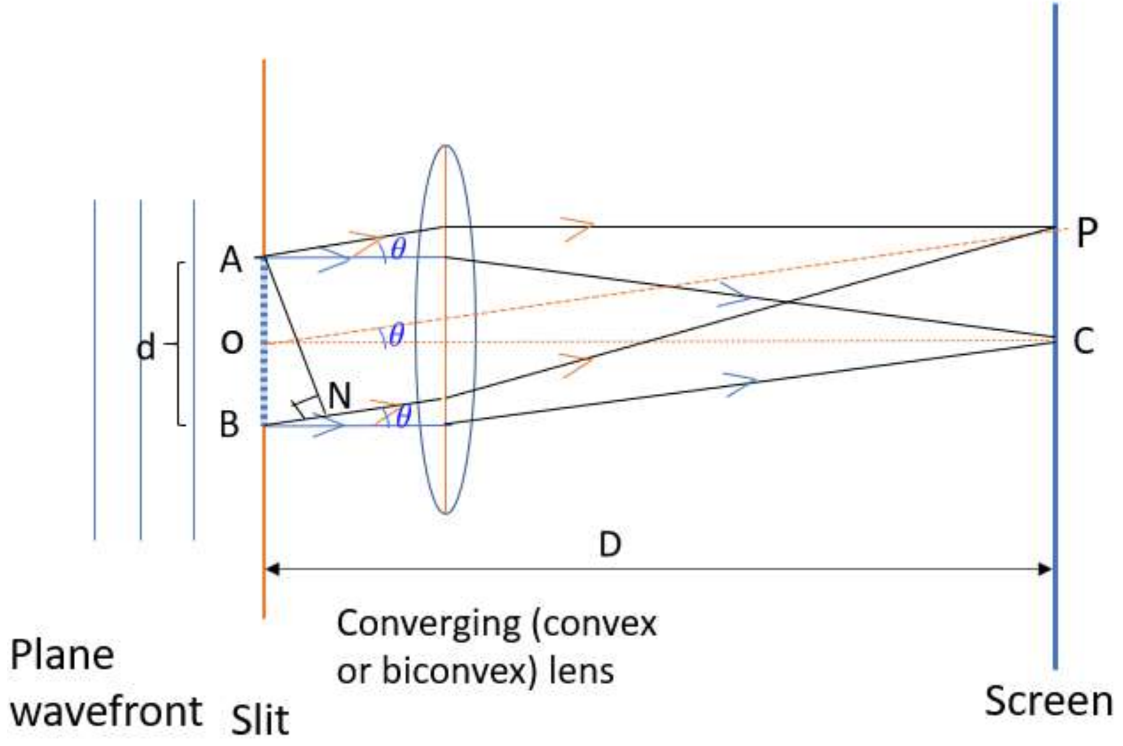
To find the distance of the nth minima from the central maxima:

For the nth minima, $\boxed{d \sin\theta = n\lambda}$

$$\sin\theta = \frac{n\lambda}{d}$$

From $\triangle$ PCO, $\tan\theta = \dfrac{x}{D}$

where CP = x



For small angle θ $\quad \sin\theta \approx \theta \approx \tan\theta$

,

So, $\dfrac{x}{D} = \dfrac{n\lambda}{d}$

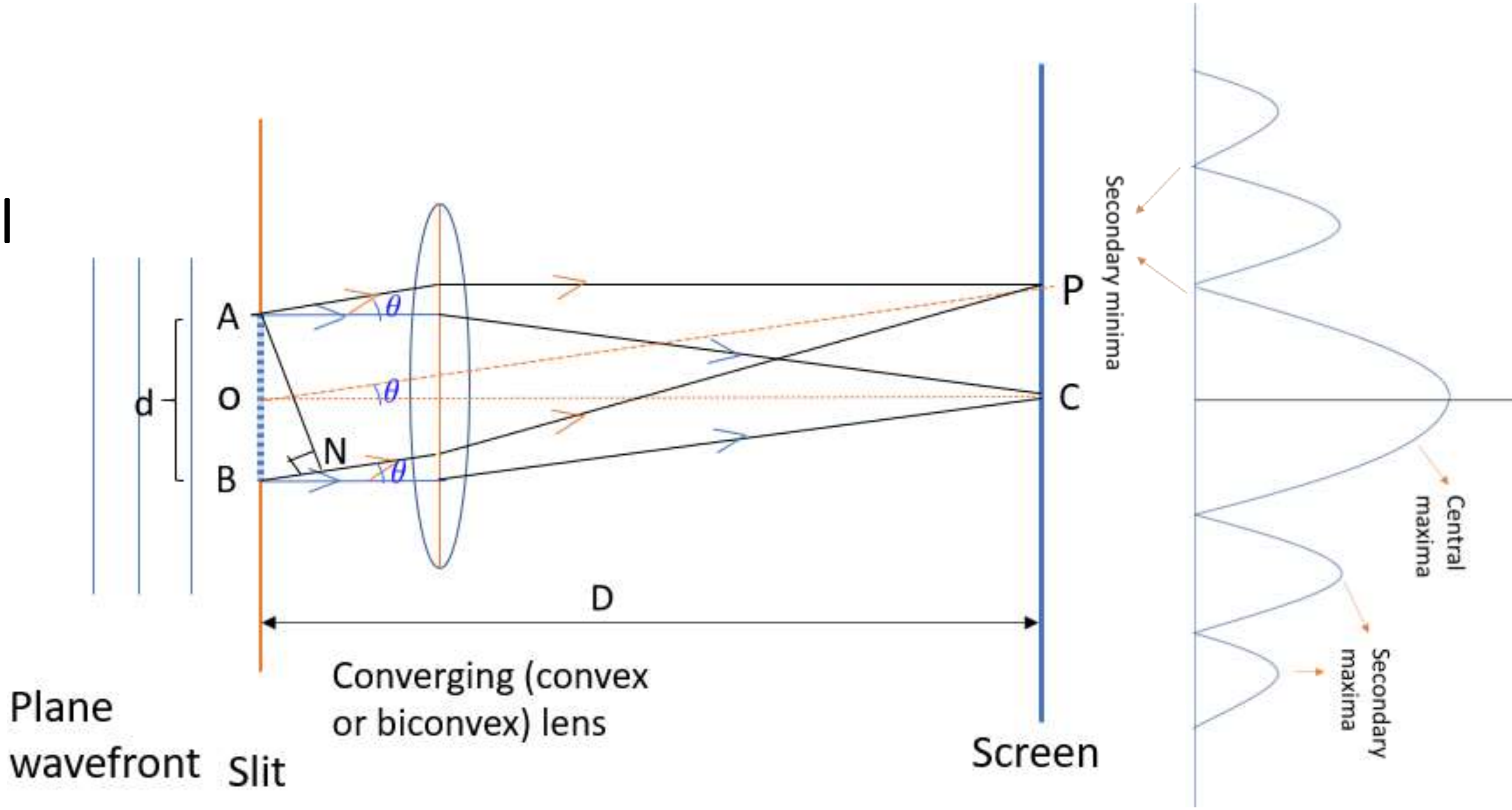
$$x = \frac{n\lambda D}{d}$$

$$\boxed{x_n = \frac{n\lambda D}{d}}$$

This gives the distance of the nth minima from the central maxima.

Then, the distance of the 1st minima from the central maxima is give by

$$x_1 = \frac{\lambda D}{d}$$



Also, half width of the central maxima

$$= x_1 = \frac{\lambda D}{d}$$

And width of the central maxima $= 2x_1 = \dfrac{2\lambda D}{d}$

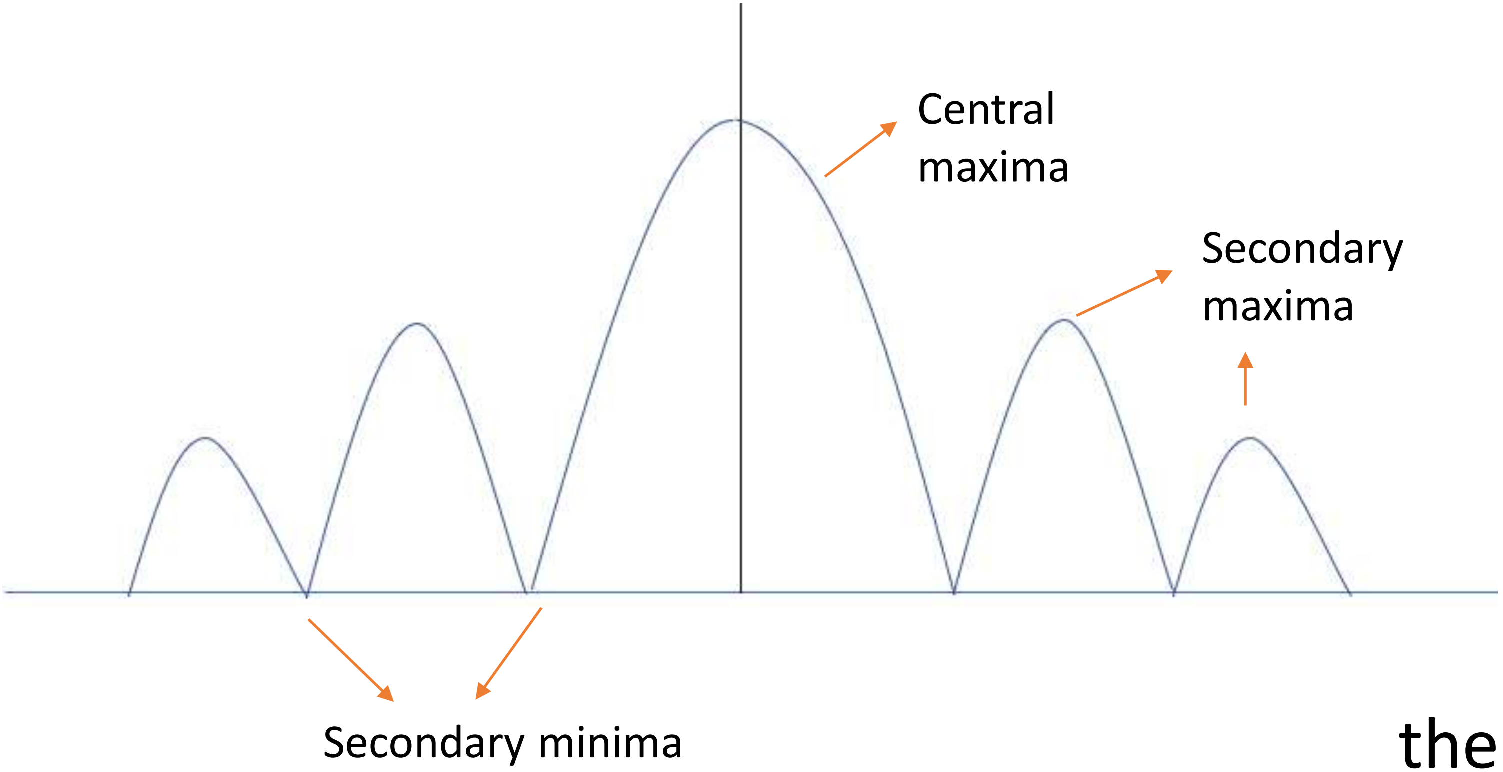
Half angular width of the central maxima = θ

Angular width of the central maxima = 2θ

Now, the distance between the consecutive minima = $x_{n+1} - x_n$

$$= \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$= \frac{\lambda D}{d}$$



which is same as the half width of the central maxima

or

the width of the secondary maxima.

Numericals:

1. In Fraunhofer's diffraction due to a narrow slit, a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of the central maxima, find the wavelength of light.

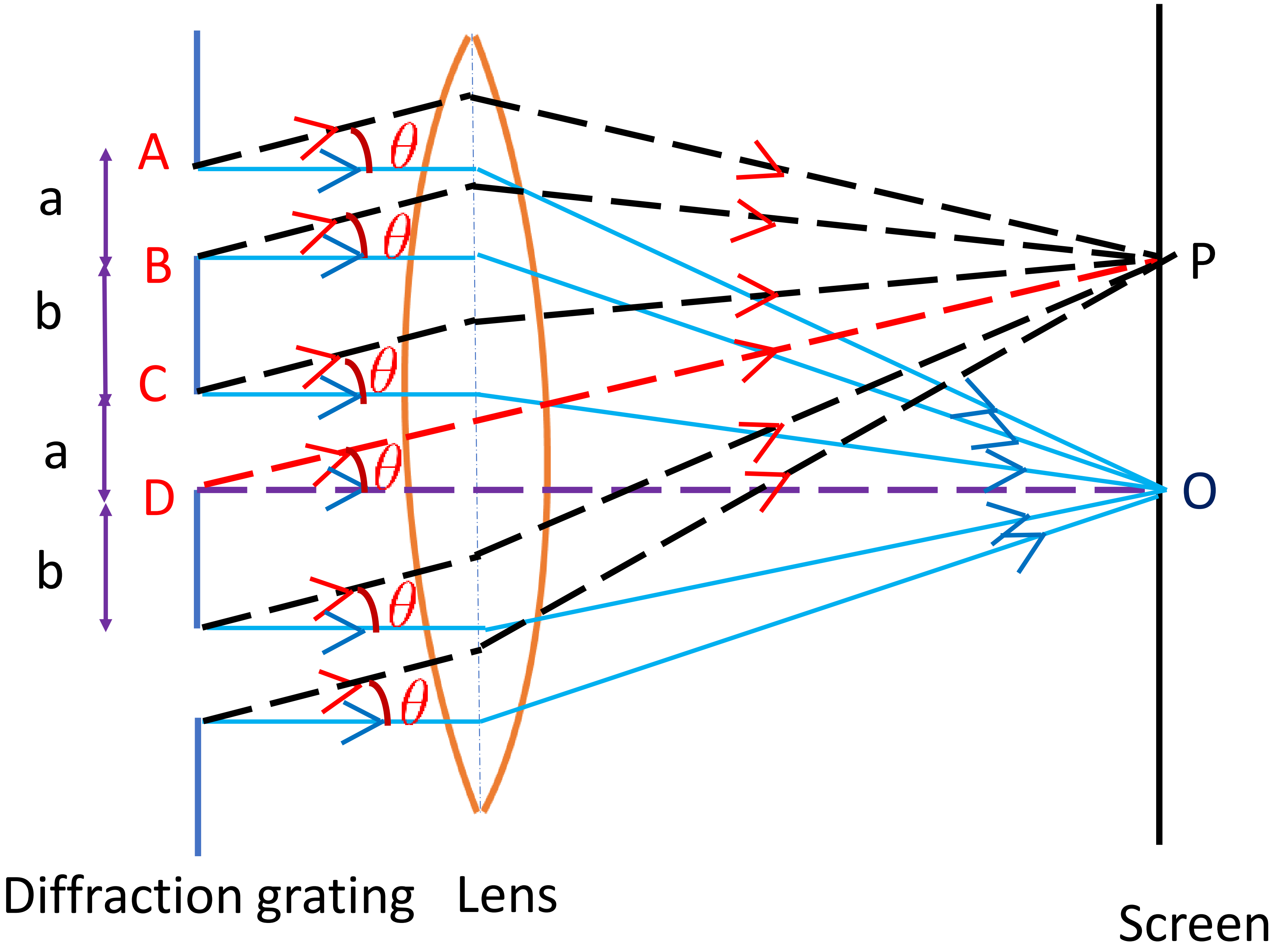
Ans: 5000 A^0

2. Light of wavelength 5000 A^0 is incident on a slit of width 0.30 mm. The screen is placed 2m away from the slit. Find the position of the first dark fringe and width of the central bright fringe. (Ans: 4x10^-3 m, 8 mm)

3. Diffraction pattern of a single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wavelength = 4890 A^0

(Ans : 1.596x10^-2mm)

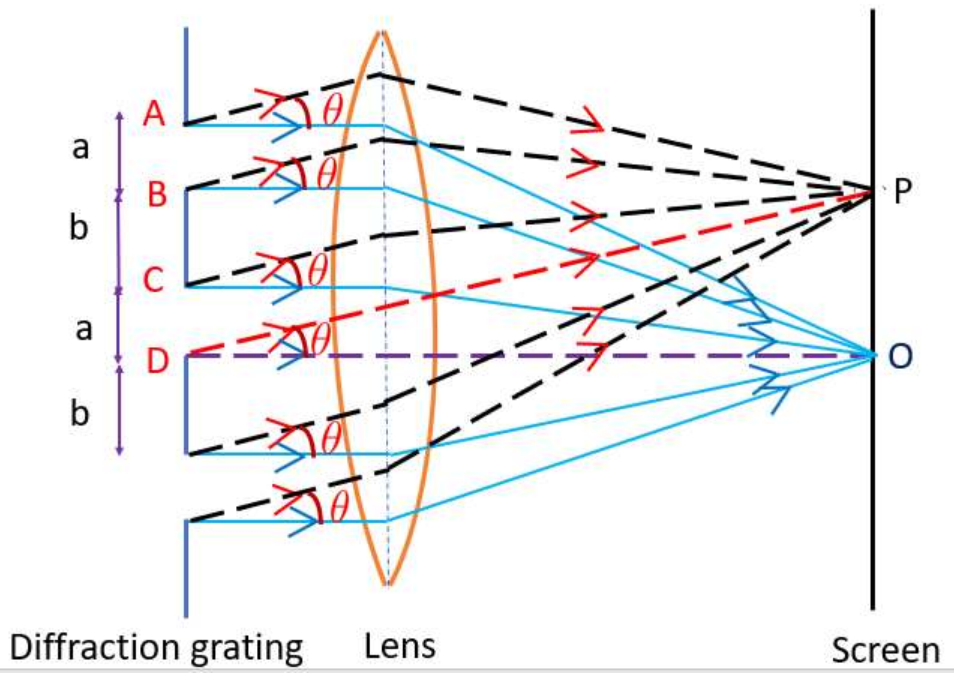
Plane transmission grating (Diffraction grating):



Definition:

It is a glass plate having a large number of lines such that each line blocks light and the spacing between the lines transmits light.

Plane transmission grating (Diffraction grating):



Consider a diffraction grating having N number of lines per inch.

Then the grating element is given by

$$a + b = \frac{1}{N}\ inch$$

$$a + b = \frac{2.54\ cm}{N}$$

However, if N is the number of lines per cm or per mm or per m,

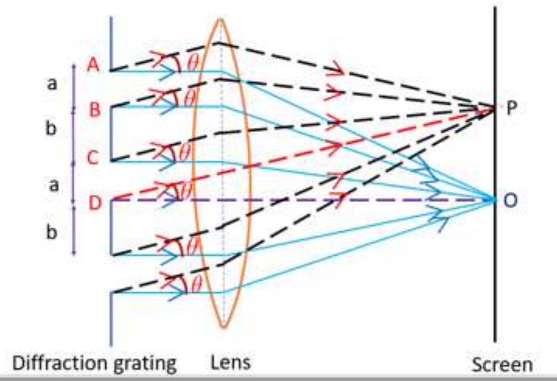
$$a + b = \frac{1}{N}\ cm\ or\ mm\ or\ m$$

Here,

$$a = \ slit\ width\ or\ transparency$$

$$b = \ line\ width\ or\ opacity$$

Plane transmission grating (Diffraction grating):



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$$a+b=\frac{1}{N}\ cm\ or\ mm\ or\ m$$

Here,

$a=$ slit width or transparency

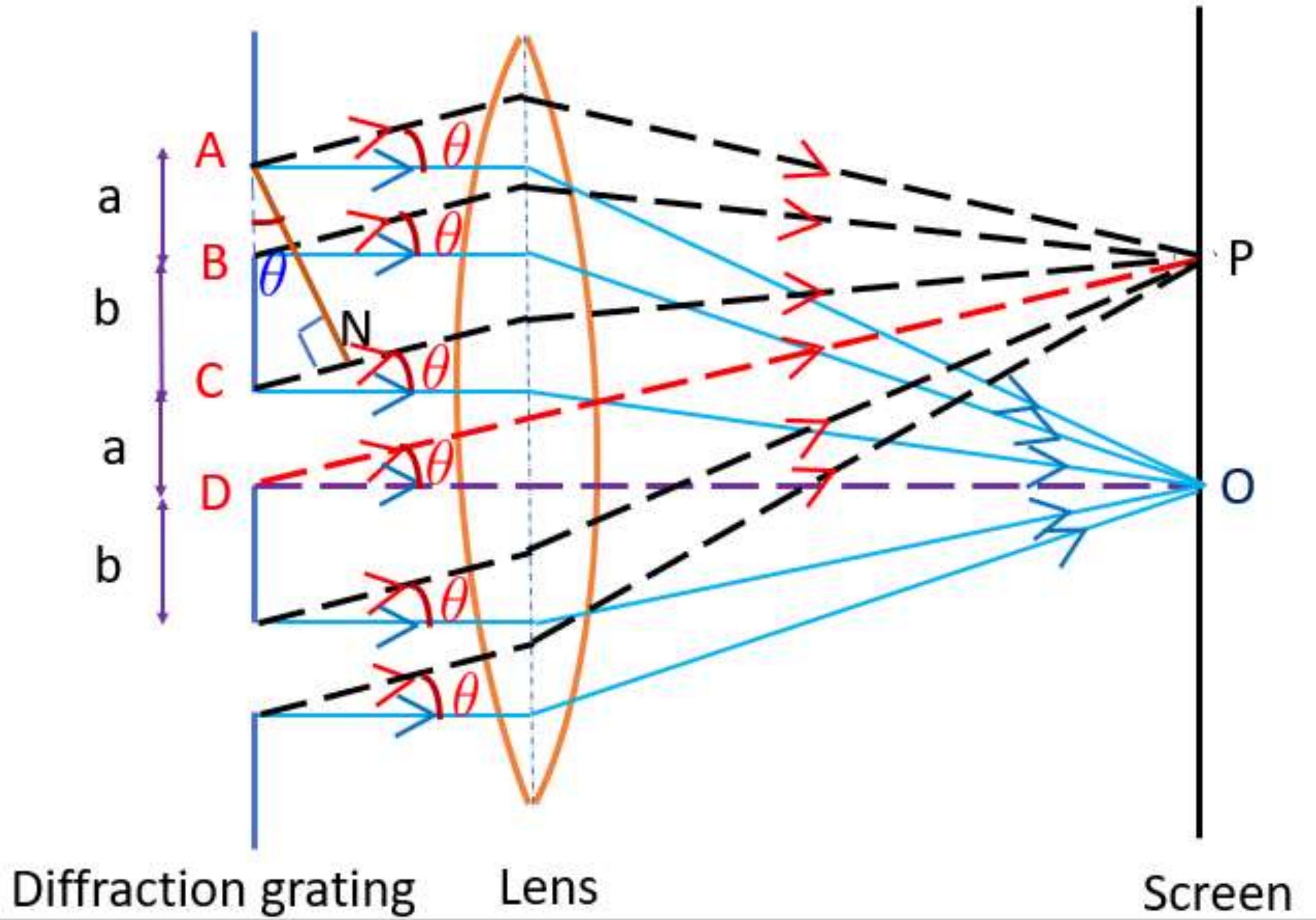
$b=$ line width or opacity

The lens converges the horizontal rays to the point O.

Then O is the central maxima.

Let θ be the angle of diffraction such that the diffracted rays are converged to the point P.

Then P may be the point of maxima or minima depending upon the path difference between the diffracted rays.



Now, draw AN perpendicular to the diffracted ray coming from the point C.

Then, CN gives the path difference between the diffracted rays coming from the points A and C.

$$In \triangle ACN,$$

$$\sin\theta = \frac{CN}{AC}$$

$$\rightarrow \quad CN = AC\ sin\theta$$

$$\rightarrow \quad CN = (a+b)\ sin\theta$$

For P to be maxima,

Path difference (CN) = $n\lambda$

Hence the diffraction grating equation is

$$\boxed{(a+b)\sin\theta = n\lambda}, n = 1, 2, 3, ...$$

This equation gives the point of maxima.

Dispersive power:

It is denoted by $\dfrac{d\theta}{d\lambda}$ and is defined as the ratio of the difference in angles of diffraction to the difference in wavelengths of the neighboring spectral lines.

Expression: Since the diffraction grating equation is

$$\boxed{(a+b)\sin\theta = n\lambda}, n = 1, 2, 3, ...$$

Differentiating w.r.t. 'λ', we get

$$\frac{d\sin\theta}{d\lambda} = \frac{n}{a+b}\frac{d\lambda}{d\lambda}$$

Or, $$cos\theta\,\frac{d\theta}{d\lambda} = \frac{n}{a+b}$$

Hence, $$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$

Resolving power: It is denoted by $\frac{\lambda}{d\lambda}$ and is defined as the ratio of the wavelength of a spectral line to the difference in wavelengths of neighboring spectral lines.

It is given by

$$\boxed{\frac{\lambda}{d\lambda} = nN'}$$

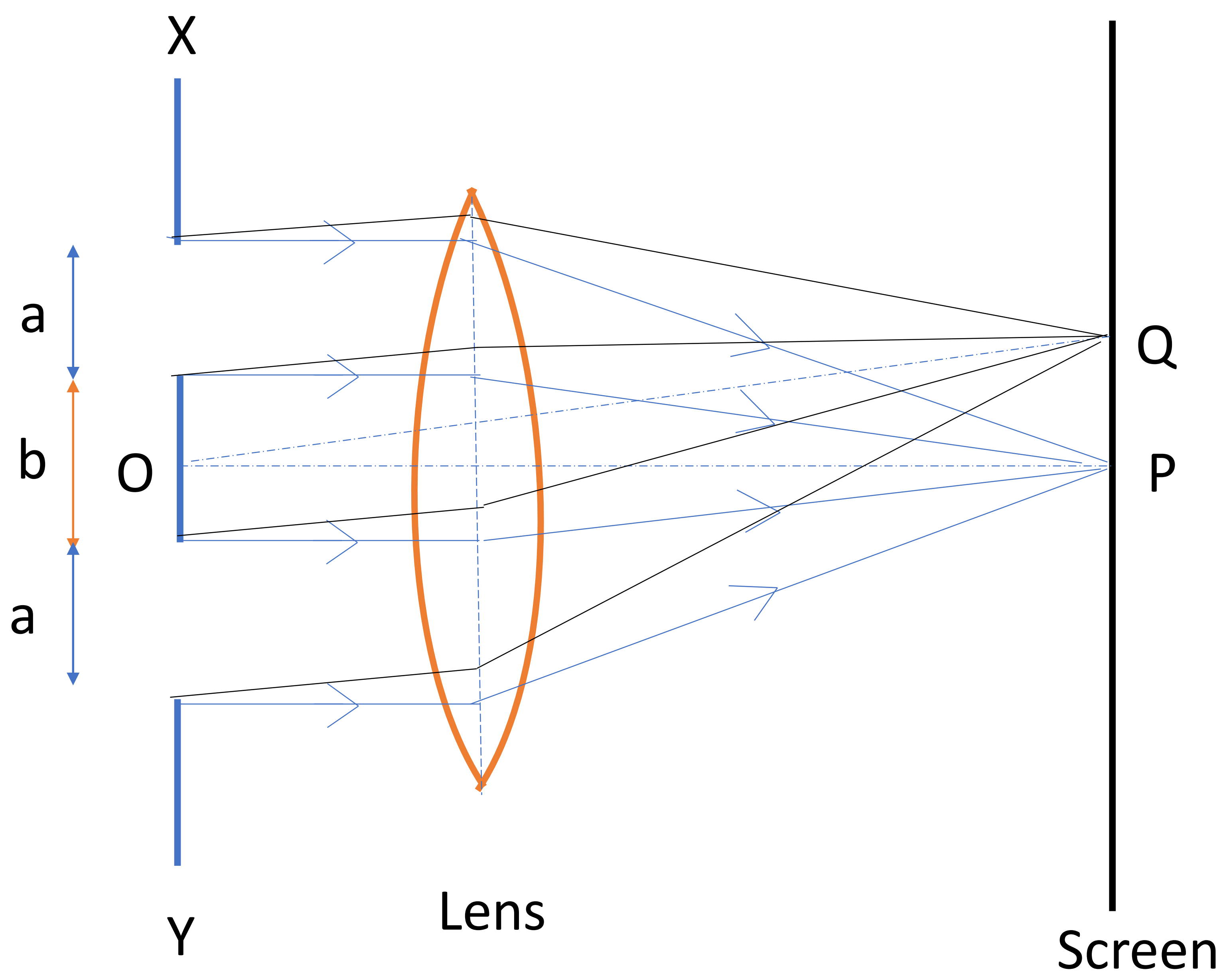
where $\lambda = \textit{smaller wavelength}$

$d\lambda = \textit{difference in wavelengths of the neighboring spectral lines}$

$n = \textit{order of the maxima}$

$N' = \textit{minimum no. of lines required for resolution}$

Fraunhofer diffraction through double slits:



In this case, both interference and diffraction occur.

(i) <u>For interference:</u>

Let α be the angle of inclination for maxima, then, as in diffraction grating, the path difference is given by

$$(a+b)\sin\alpha = n\lambda,\ \ n = 1, 2, 3, \ldots.$$

$$\rightarrow (a+b)\sin\alpha_n = n\lambda,\ \ n = 1, 2, 3, \ldots.$$

Let β be the angle of inclination for minima, then the path difference is given by

$$(a+b)\sin\beta = (2n+1)\frac{\lambda}{2},\ \ n = 1, 2, 3, \ldots.$$

$$\rightarrow (a+b)\sin\beta_n = (2n+1)\frac{\lambda}{2},\ \ n = 1, 2, 3, \ldots.$$

The angular separation between any two successive maxima or minima is given by

$$\sin\alpha_2 - \sin\alpha_1 = \sin\beta_2 - \sin\beta_1 = \frac{\lambda}{a+b}$$

(ii) For diffraction:

As in Fraunhofer's diffraction through single slit,

The diffraction minima is given by

$$\boxed{a\sin\theta = n\lambda}, \quad n = 1, 2, 3, \ldots$$

And the diffraction maxima is given by

$$\boxed{a\sin\theta = (2n+1)\frac{\lambda}{2}}$$

And:

Don’t forget to watch the lecture by Walter Lewin:

https://www.youtube.com/watch?v=sKO8n_-xtDc

Exam

Q. If a and b be slit width and the line width respectively in Fraunhofer's diffraction through double slits, determine the missing orders.