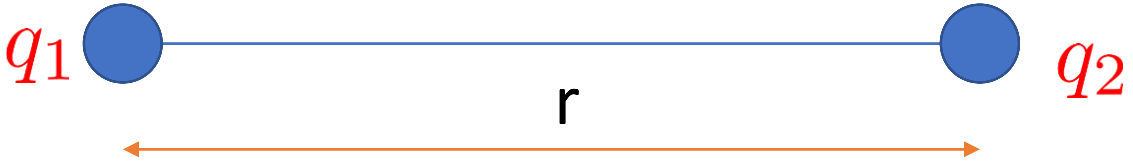
Electrostatics

[$\underbrace{\text{1 long question}}_{\text{9 marks}} + \underbrace{\text{1 numerical}}_{\text{6 marks}}$ = 15 marks]

$\boxed{Coulomb's\ law}$:



The force acting between the two charges is given by

$$F \propto q_1 . q_2$$

$$and\ F \propto \frac{1}{r^2}$$

Combinining we get,

$$F \propto \frac{q_1 . q_2}{r^2}$$

In SI System, $$\boxed{F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}}$$

$$where\ \epsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$$
$$= permittivity\ of\ free\ space\ or\ vacuum\ or\ air$$
$$and\ \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$$

$If\ the\ charges\ are\ placed\ in\ a\ medium,\ force\ is\ given\ by$

$$F_m = \frac{1}{4\pi\epsilon_m}\frac{q_1q_2}{r^2}$$
$$where\ \epsilon_m = permitivity\ of\ medium$$
$$= \epsilon_0.\epsilon_r[\epsilon_r\ being\ the\ relative\ permittivity]$$

$Now\ dividing\ we\ get,$

$$\frac{F}{F_m} = \epsilon_r$$
$$i.e. \boxed{\frac{F_{air}}{F_{med}} = \epsilon_r = \frac{\epsilon_m}{\epsilon_0}}$$

$$Also, \quad \boxed{\epsilon_r = k = \frac{F_{air}}{F_{med}} = \frac{\epsilon_m}{\epsilon_0}}$$

$$where\ k = \underbrace{dielectric}_{insulator}\ constant\ of\ the\ medium$$

Electric Field Intensity or Electric Field Strength :

A charge (a point charge) creates a field of influence around it. The force exerted on a unit positive test charge due to a given charge is called the the electric field intensity/strength or simply the electric field. It is denoted by E and is given by

$$\boxed{E = \frac{F}{q_0}} \quad where\ q_0 = test\ charge$$

Note :

(i) Electric force is a vector.

(ii) Electric field is a vector.

(iii) Electric charge is a scalar.

(iv) Electric potential is a scalar.

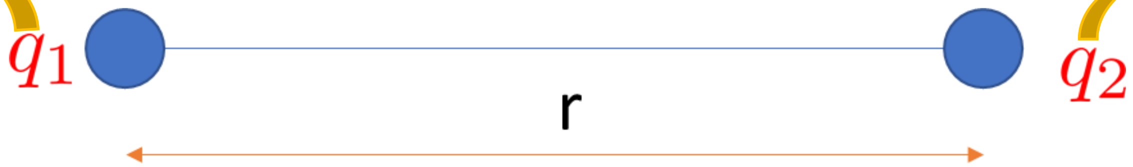
Definition of E :

$$\boxed{E = \frac{F}{q_0}}$$

Use this formula to write the unit of E

Unit : N/C

Expression (Formula) for E :



$$\boxed{q_1 = q}$$

given charge

$$\boxed{q_2 = q_0 = +1\ C}$$

unit positive test charge

$$Then, \quad F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \quad becomes$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

$\Downarrow$

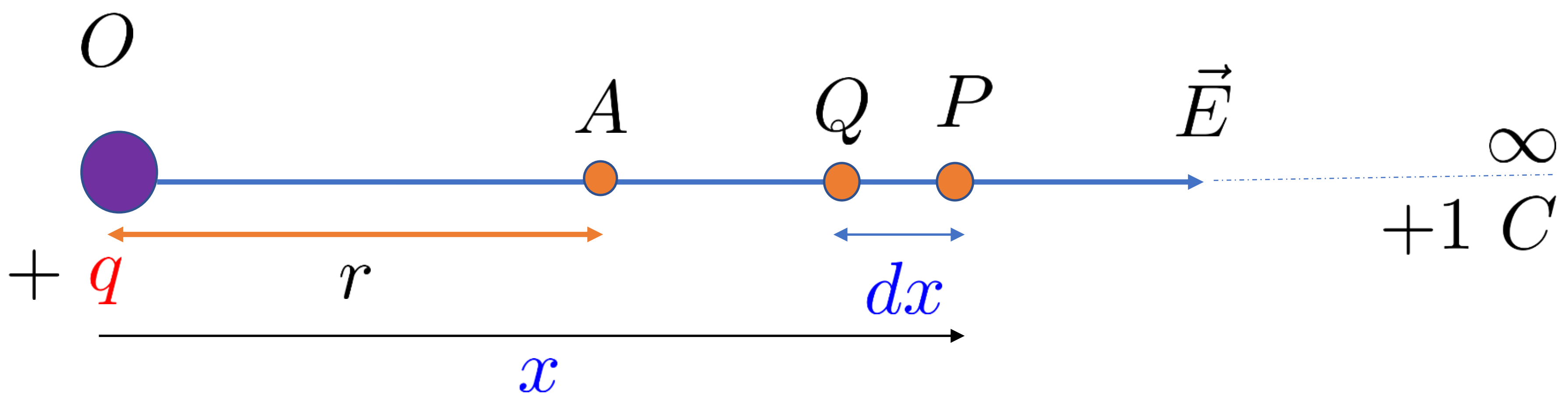
[We use this formula for theory derivation and numerical calculation]

Electric Potential (V):

It is defined as the amount of work done in bringing a unit positive test charge from infinity to a given point within the electric field region of the given charge.

It is denoted by V and is a scalar.

$\underline{Expression\ (Derivation)\ for\ V}$:



Consider a point charge q *at the point* O*. This charge is considered to be positive. Then the direction of the electric field is away from it, as shown in the figure. Then* $+1\ C$ *is brought from infinity to a point* A *such that* $OA = r$.

Consider an intermediate point P *such that* $OP = x$. *Let another point* Q *be such that* $PQ = dx$.

By the definition of potential, the potential at A is given by,

$$V = V_A = \text{Work done in bringing } +1C \text{ from infinity to } A$$

$$= \int_{\infty}^{r} dW = \int_{\infty}^{r} \vec{E}.\vec{dx} = -\int_{\infty}^{r} E\, dx$$

where E is the electric field at the pont P, $E = \dfrac{q}{4\pi\epsilon_0 x^2}$

and the negative sign is due to the opposite direction of $\vec{E}$ *and* $\vec{dx}$

$$\text{Then } V = -\int_{\infty}^{r} \frac{q}{4\pi\epsilon_0 x^2} dx$$

$$= -\frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} x^{-2} dx$$

$$or,\ V = -\frac{q}{4\pi\epsilon_0}\left|(-1)x^{-1}\right|_{\infty}^{r}$$

$$or,\ V = \frac{q}{4\pi\epsilon_0}\left|x^{-1}\right|_{\infty}^{r}$$

$$or,\ V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$So,\ \boxed{V = \frac{q}{4\pi\epsilon_0 r}}$$

$\boxed{Note}$: *Potential(V) is a scalar and unit is Volt(V).*