Chapter 2

Wave Motion

Disturbance

The disturbance means change in pressure, density or displacement of the particles of the medium about their equilibrium position.

Wave Motion

Wave motion is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean positions. There is transfer of energy from one point to another through successive vibration of particles.

Characteristics of Wave Motion

1. The wave motion is the disturbance produced in the medium due to repeated periodic motion of the particles of the medium.
2. Wave travels in forward direction while the particles of the medium vibrate about their mean position.
3. In each vibration, a particle hands over some of its energy to the next particles which it receives from previous particle.
4. The velocity of wave (called wave velocity) is different from the velocity of particle (called particle velocity).

   wave velocity $v = f\lambda$

   particle velocity $u = \frac{dy}{dt} = \frac{d}{dt}(A\sin \omega t) = A\omega \cos \omega t$

6. Each particle in the medium starts vibrating a little later than the preceding one.
7. Wave velocity is uniform while particle velocity is different at different position. It is maximum at mean position and minimum at extreme position.

Types of Waves

a. **On the basis of modes of vibration of the particle of medium there are two types of waves.**

   1. **Transverse wave**

      When the particles of the medium vibrate about their mean position in a direction perpendicular to the direction of propagation of disturbance, the wave is called transverse wave. In a transverse wave motion, disturbance travels in the form of crest and trough.

Example: transverse wave produced in water when a stone is thrown in a pond, transverse wave produced in a stretched string when one end is fixed at wall and other end is moved up and down.

2. **Longitudinal wave**

If the particles of the medium propagating the wave motion vibrate in the direction of propagation, the wave is called longitudinal wave. In longitudinal wave motion, disturbance travels in the form of compression and rarefactions.

Example: Sound wave in air, longitudinal wave produced in spring when it is suddenly compressed and released.

b. **on the basis of necessity of medium for propagation there are two types of waves.**

1. **Mechanical wave:** The wave which require medium to propagate are called mechanical ware, Eg. Sound waves, water waves, seismic wave. It can be longitudinal or transverse in nature.

2. **Electromagnetic wave:** The wave which does not require medium to propagate are called non-mechanical or electro magnetic wave.

Eg: light waves, radio waves, x - ray, γ - ray etc.

It is transverse in nature.

c. **There are two types of wave on the basis of state of motion of particle of the medium.**

1. **Progressive wave:**

A wave that travels from one region to another region of medium through the successive vibration of the particle of the medium is called progressive wave.

In progressive wave motion there is transfer of energy from one point to another and no particle of the medium is permanently at rest. All those waves discussed above falls on this category.

2. **Stationary waves:**

When two progressive waves of the same amplitude and frequency travelling through a medium with the same speed but in opposite direction superimpose on each other, stationary or standing wave is formed. The wave is so called stationary because it does not travel in either direction and all the particles of the medium are permanently at rest. There is not flow of energy along the wave.

Equation of Plane Progressive Wave

Consider a wave travelling along +ve x-axis as shown in figure. Let a, ω and T be the amplitude, angular velocity and time period of the wave. This means all the particles vibrating in the medium have same amplitude a, same angular velocity ω and same time period T.

Let y be the displacement of the particle vibrating at 'O'. Then it's displacement at any instant 't' is given by.

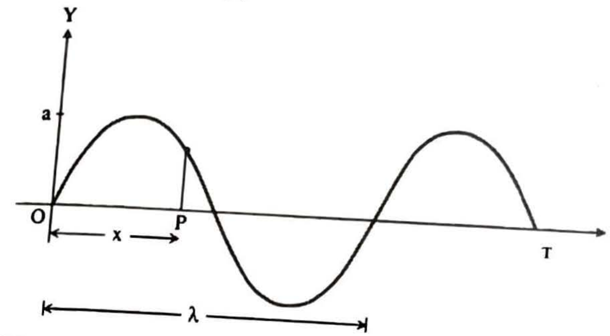
$$y = a \sin \omega t \quad ......(1)$$

Consider another particle at point 'P' at a distance x from 'O'.

This means path difference between the particle at 'P' and 'O' is x and let corresponding phase difference be ϕ.

Then the displacement of the particle at 'P' at same time 't' is,

$$y = a \sin (\omega t - \phi) \quad .....(2)$$



We know, path difference, λ = phase difference, 2π

path difference, 1 = phase difference, $2\pi/\lambda$

path difference, x = phase difference, $\frac{2\pi}{\lambda}.x$

Therefore, $\phi = \frac{2\pi}{\lambda}.x \quad ...(3)$

Substituting ϕ for equation (2) $y = a \sin (\omega t - \frac{2\pi}{\lambda}.x)$

$\Rightarrow \quad y = a\sin (\omega t - kx) \quad ......(4)$

where $k = \frac{2\pi}{\lambda}$ is called the wave number or propagation constant.

or, $\quad y = a \sin (2\pi ft - \frac{2\pi}{\lambda}.x)$

$= a \sin \frac{2\pi}{\lambda} (f\lambda.t - x)$

$= a \sin \frac{2\pi}{\lambda} (vt - x) = a \sin k (vt - x) ....(5)$

Equation (4) or (5) represents plane progressive wave. If the wave travels along -ve x direction or from right to left, it will arrive at 'P' before 'O'. So the vibration at P will lead 'O' and wave equation becomes.

$$y = a \sin (\omega t + kx) = a \sin \frac{2\pi}{\lambda} (vt + x)$$

Relation between Wave Velocity and Particle Velocity

The equation of plane progressive wave is

$$y = a\sin(\omega t - kx) \quad ...(1)$$

Differentiating this equation w.r.t 't' and x.

$$\frac{dy}{dt} = a\omega\cos(\omega t - kx) \;.....(2) \text{ and } \frac{dy}{dx} = a(-k)\,.\cos(\omega t - kx) \;..(3)$$

Dividing equation (2) by equation (3)

$$\frac{\frac{dy}{dt}}{\frac{dy}{dx}} = -\frac{\omega}{k} \Rightarrow \frac{dy}{dt} = -\frac{\omega}{k}\frac{dy}{dx}$$

or,
$$\frac{dy}{dt} = -\frac{2\pi f}{2\pi/\lambda}\frac{dy}{dx} = -f\lambda\frac{dy}{dx} = -v\frac{dy}{dx}$$

$$\frac{dy}{dt} = -v\frac{dy}{dx}$$

Therefore, particle velocity = - (wave velocity) × slope of displacement curve.

Particle acceleration

The equation of plane progressive wave is, $y = a\sin(\omega t - kx)$

Particle velocity, $v_p = \frac{dy}{dt} = a\omega\cos(\omega t - kx)$

Particle acceleration, $a_p = \frac{d^2y}{dt^2} = -a\omega^2\sin(\omega t - kx)$

$a_p = -\omega^2 y.$

Therefore, maximum velocity, $(v_p)_{max} = a\omega$

and maximum acceleration, $(a_p)_{max} = -\omega^2 a$

Differential equation of wave motion

The equation of plane progressive wave is,

$$y = a\sin(\omega t - kx) \;......\;(1)$$

Differentiating w.r.t. time 't'

$$\frac{dy}{dt} = a\omega\cos(\omega t - kx)$$

Again differentiating w.r.t. to time 't'

$$\frac{d^2y}{dt^2} = -a\omega^2\sin(\omega t - kx) = -\omega^2 y \;.......\;(2)$$

Differentiating equation (1) w.r.t. x.

$$\frac{dy}{dx} = a(-k)\cos(\omega t - kx)$$

Again differentiating with 'x'

$$\frac{d^2y}{dx^2} = -a(-k)^2\sin(\omega t - kx) = -k^2 y \;.....\;(3)$$

Dividing equation (2) by (3)

$$\frac{d^2y/dt^2}{d^2y/dx^2} = \frac{\omega^2}{k^2} = \left(\frac{2\pi f}{2\pi/\lambda}\right)^2 = (f\lambda)^2 = v^2$$

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2} \quad ......\;(4)$$

This is the general (differential) equation of wave motion. It's solution is, $y = a\sin(\omega t - kx)$.

Energy, Power, Intensity of plane progressive wave

In progressive wave there is no transfer of the medium but there is always transfer of energy in the direction of propagation of the wave. Energy transfer in wave is done by vibrating particle of the medium.

The potential energy is given by

$$\text{P.E.} = -\int_0^y F.dy = -\int_0^y -kydy = \frac{ky^2}{2} = \frac{1}{2}ky^2$$

Here k is force constant and given by $k = m\omega^2$

Therefore, $\text{P.E} = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 a^2\sin^2(\omega t - kx) \quad ....(1)$

Potential energy per unit volume $= \frac{1}{2}\rho\omega^2 a^2\sin^2(\omega t - kx)$

The kinetic energy is given by

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\left\{\frac{d}{dt}[a\sin(\omega t - kx)]\right\}^2$$

$$\text{K.E.} = \frac{1}{2}m a^2\omega^2\cos^2(\omega t - kx)$$

$$\text{K.E. per unit volume} = \frac{1}{2}\rho a^2\omega^2\cos^2(\omega t - kx)$$

Therefore, Total energy per unit volume $= \frac{1}{2}\rho a^2\omega^2$

Average K.E. over a complete wave length

$$= \frac{1}{\lambda}\int_0^\lambda \frac{1}{2}\rho a^2\omega^2\cos^2(\omega t - kx)\,dx$$

$$= \frac{1}{\lambda}\left(\frac{1}{2}\rho a^2\omega^2\right)\int_0^\lambda \left[\frac{1}{2}(1 + \cos 2(\omega t - kx))\right]dx$$

$$= \frac{1}{4}\frac{\rho a^2\omega^2}{\lambda}\left[\int_0^\lambda dx + \int_0^\lambda \cos\{2(\omega t - kx)\}\,dx\right]$$

$$= \frac{1}{4}\frac{\rho a^2\omega^2}{\lambda} \times \lambda = \frac{1}{4}\rho a^2\omega^2 = \frac{1}{2} \text{ total energy,}$$

= constant

Average P.E. over a complete wave length.

$$= \frac{1}{\lambda}\int_0^\lambda \frac{1}{2}\rho a^2\omega^2 \sin^2(\omega t - kx)\,dx$$

$$= \frac{1}{2}\frac{\rho a^2\omega^2}{\lambda}\int_0^\lambda \left[\frac{1}{2}(1 - \cos 2(\omega t - kx))\right]dx$$

$$= \frac{1}{4}\frac{\rho a^2\omega^2}{\lambda}.\lambda = \frac{1}{4}\rho a^2\omega^2 = \frac{1}{2} \text{ Total energy}$$

= constant.

This shows that the average K.E per unit volume and P.E per unit volume are equal and eq equal to half the total energy per unit volume.

Here.It is seen that both K.E and P.E depend upon the values of x and t, but K.E per unit volum P.E. per unit volume and Total energy density are independent of either i.e. constants.

Power of progressive wave: The total energy transported by wave per unit time is called power

Here, total energy per unit volume.

$$E/V = \frac{1}{2}\rho a^2\omega^2$$

$$\Rightarrow E = \frac{1}{2}V\rho a^2\omega^2,$$

where V is the volume of medium it is given by, $V = A \times l$

A - cross sectional area through which wave travels.

l - is the distance travelled by wave.

Also, $l = vt$, v is the speed of wave and t is the time

Therefore, $V = A \times vt$

Substituting V for E,

$$E = \frac{1}{2}A\,v\,t\,\rho\,a^2\omega^2$$

Rate of energy transfer or power $P = E/t = \frac{1}{2}A\,v\,\rho\,a^2\omega^2$

Intensity of progressive ware:

The rate of flow of energy per unit area of crosssection through which wave travels is cal energy current or energy flux or intensity of progressive wave.

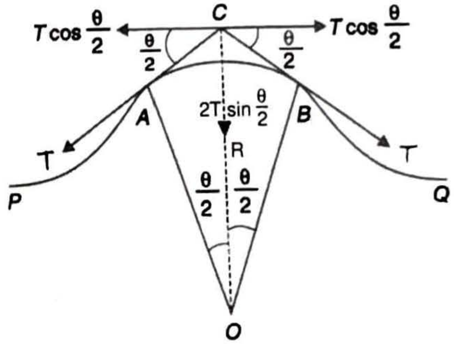
$$I = \frac{P}{A} = \frac{1}{A}\left(\frac{1}{2}A\,v\,\rho\,a^2\omega^2\right)$$

$$I = \frac{1}{2}v\rho a^2\omega^2$$

Hence, both power and intensity of travelling wave is directly proportional to square of amplitude.

Velocity of Transverse Wave along a Stretched String

Consider a portion PABQ of a stretched string in which a transverse wave is travelling from left to right with velocity v. consider a small string element AB of length ΔL forming on arc of a circle of radius 'R' and subtending an angle AOB = θ at the centre of that circle.



Let T be the tension at A and B. The directions of these tensions are tangential to the element at A and B.

Resolving the tension at A into two rectangular components T sin $\theta/2$ as vertical and T cos $\theta/2$ as horizontal component.

Similarly the tension at B can be resolved into two components T sin $\theta/2$ as vertical and T cos $\theta/2$ as horizontal component.

The horizontal components are equal and opposite so they cancelled each other and the vertical component gets added since they are along same direction along CO to give resultant tension,

$$T\sin\theta/2 + T\sin\theta/2 = 2T\sin\theta/2$$

As θ is small, $\sin\theta/2 \approx \theta/2$

Therefore, resultant tension $= 2\,T.\,\theta/2 = T\theta$ ....(1)

Since the string segment AB is moving in an arc of circle. This resultant tension is balanced by the centripetal force towards the centre of the circle.

i.e. $T\,\theta$ = centripetal force $= \frac{mv^2}{R}$ ...(2)

Where m = mass of string element, If μ is mass per unit length then $m = \mu\,\Delta L$

Also from figure $\Delta L = R\theta \Rightarrow \theta = \frac{\Delta L}{R}$

Substituting m and ΔL for equation (2),

$$T.\frac{\Delta l}{R}=\frac{\mu.\Delta l}{R}v^2$$

$$\Rightarrow T=\mu v^2$$

Therefore, $v=\sqrt{\frac{T}{\mu}}$ ....(3)

Which is the expression for velocity of transverse wave along stretched string. This shows velocity of wave depends upon the tension and linear density of the string, not on the freque of the wave.

Frequency of wave is given by, $f=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$.

Standing Wave and Resonance

When a string stretched between two clamps is made to oscillate at certain frequency interference produces standing wave pattern with nodes and large antinodes. This condition called resonance and the string is said to resonate at these certain frequencies called reso frequencies.

Let a string be stretched between two clamps by a fixed distance 'l'. A node must exist at each its ends because each end is fixed and cannot oscillate. A simplest pattern that meets requirement is shown in fig (1).

For this pattern, $\lambda/2=l$

$\lambda=2l$

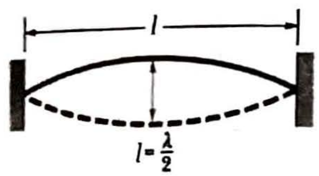


Figure (1)

A second simple pattern meeting the requirement of nodes at fixed string is shown in figur For this pattern.

$\lambda=l=2\frac{l}{2}$

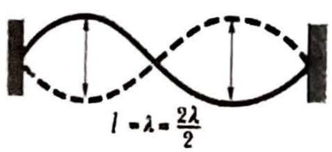


Figure (2)

A third pattern is as shown in figure (3)

Here $l=3\lambda/2$

$\lambda=2\frac{l}{3}$

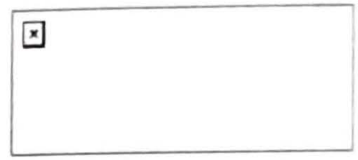


Figure (3)

Continuing this process we observe that if the left going and right going traveling waves are to set up resonance by their interference, they must have the wave length

$$\lambda=\frac{2l}{n},\ n=1,2,3\ .....$$

The resonant frequencies are given by the relation,

$$f=\frac{v}{\lambda}=n.\frac{v}{2l}\ \text{for}\ n=1,2,3,\ .....(1)$$

The oscillation made with lowest resonant frequency $f_1=\frac{v}{2l}$ for n = 1 is called fundamental mode or first harmonic. The second harmonic is the oscillation mode with n = 2 and so on.

Solved Examples

1. **A stretched string has linear density 525 gm/m and is under tension of 45N. A sinusoidal wave with frequency 120 Hz and amplitude 8.5 mm is sent along the string from one end. At what average rate does the wave transport energy.**

Solution:

Here, μ = 525 gm/m = 0.525 kg/m, T = 45 N

Velocity of wave $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{45}{0.525}}=9.26$ m/sec

f = 120 Hz, amplitude, a = 8.5 mm = 8.5×10^{-3}m

The average rate at which wave transport energy i.e. intensity wave is

$$I=\frac{1}{2}v\rho\omega^2a^2=\frac{1}{2}v\mu\omega^2a^2$$

$$=\frac{1}{2}\times9.26\times0.525\times(2\pi\times120)^2\times(8.5\times10^{-3})^2$$

= 99.74 J/sec = 99.74 Watt.

2. **A source of sound has a frequency of 512 Hz and amplitude of 0.25 cm. What is the flow of energy across a square cm per second. If the velocity of sound in air is 340m/sec and density of air is 0.00129 gm/cm³.**

Solution:

Here frequency, f = 512 Hz,

Amplitude, a = 0.25cm = 0.25×10^{-2}m, velocity of sound in air, v = 340m/sec, density of air, ρ = 0.00129 gm/cm^3

$$= \frac{0.00129\times10^{-3}}{10^{-6}} \text{ kg/m}^3 = 1.29 \text{ kg/m}^3$$

The flow of energy per unit area per unit time, i.e. intensity of wave, $I = \frac{1}{2}v\rho\omega^2a^2 = \frac{1}{2}\times$ $340\times1.29\times(2\pi\times512)^2\times(0.25\times10^{-2})^2$

$= 14170.26$ Watt/m^2 = 1.417 Watt/cm^2

3. **A stretched string has a linear mass density of 5.0 gm/cm and a tension of 10 N. A wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is travelling in –ve x direction. Write the wave equation with appropriate units.**

Solution:

Here, $\mu = 5\text{gm/cm} = \frac{5\times10^{-3}}{10^{-2}} = 0.5$ kg/m

Tension, T = 10 N

Velocity, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{0.5}} = 4.47$ m/sec

Here, amplitude, a = 0.12 mm

Frequency, f = 100 Hz

Since, $v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{4.47}{100} = 0.0447$m

Therefore, $k = \frac{2\pi}{\lambda} = 141 \text{ m}^{-1}$

The equation of wave travelling in –ve x direction is

$y = a \sin(\omega t + kx)$

$= (0.12\text{mm}) \sin[(2\pi f)\, t + (141\text{m}^{-1})\, x]$

$= 0.12 \text{ mm} \sin[(628\text{s}^{-1})\, t + (141 \text{ m}^{-1})\, x]$

4. **If the intensity of wave is 1.0×10^6 W/m^2 at 50km from a source, what is the intensity at 10 km from the source.**

Solution:

Since Intensity, $I \propto \frac{1}{(\text{distance})^2}$ i.e. $I \propto \frac{1}{r^2}$

Therefore, $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \Rightarrow \frac{1\times10^6}{I_2} = \frac{(10\times10^3)^2}{(50\times10^3)^2} = \frac{1}{25}$

$I_2 = 25\times10^6$ Watt/m^2 = 2.5×10^7 Watt/m^2

5. ***The equation of transverse wave on a string is y = (2.0mm) sin [(20m^{-1}) x – (600 S^{-1}) t]. The tension on the string is 15 N. i) What is the wave speed ii) Find the linear density of the string in grams per meter.***

Solution:

The given equation is,

$y = (2.0 \text{ mm}) \sin[(20 \text{ m}^{-1})\, x - (600 \text{ S}^{-1})\, t]$

Comparing this equation with general displacement equation

$y = A \sin(kx - \omega t)$

$k = 20 \text{ m}^{-1}$ and $\omega = 600 \text{ S}^{-1}$

Therefore, velocity, $v = \frac{\omega}{k} = \frac{600}{20} = 30$ m/S

Here, tension on the string, T = 15 N

Since, $v = \sqrt{\frac{T}{\mu}} \Rightarrow v^2 = \frac{T}{\mu} \Rightarrow \mu = \frac{T}{v^2}$

$\mu = \frac{15}{30^2} = \frac{1}{60}$ kg/m $= \frac{1000}{60}$ gm/m = 16.67 gm/m

6. **Calculate the ratio of intensity of following two waves y_1 = 6 sin (0.4t - 25 x) cm and y_2 = 2.5 sin (3.2 t - 200 x) cm.**

Solution:

For first wave, $y_1 = 6 \sin(0.4\, t - 25\, x)$

$a = 6\text{cm}, \omega = 0.4, k = 25 \Rightarrow v = \frac{\omega}{k} = \frac{0.4}{25} = 0.016$ m/s

For second wave, $y_2 = 2.5 \sin(3.2\, t - 200\, x)$

$a = 2.5 \text{ cm}, \omega = 3.2, k = 200 \Rightarrow v = \frac{3.2}{200} = 0.016$ m/s

$\therefore \frac{I_1}{I_2} = \frac{1/2\, v_1\rho\omega_1^2a_1^2}{1/2\, v_2\rho\omega_2^2a_2^2} = \frac{0.4^2\times6^2}{2.5^2\times3.2^2} = 0.09$

7. **Here are the equation of three waves**
   1) y (x, t) = 2 sin (4x – 2t) 2) y (x,t) = 2 sin (3x – 4t) and 3) y (x, t) = 2 sin (3x - 3t). Write down the equation in accordance with (1) wave speed (2) maximum particle velocity (maximum transverse speed) in descending order.

Solution:

For first wave, y (x, t) = 2 sin (4x - 2t)

Comparing this equation with $y = A \sin(kx - \omega t)$

A = 2, ω = 2, k = 4

Wave speed $= \frac{\omega}{k} = \frac{2}{4} = 0.5$

Maximum particle velocity = $A\omega = 2\times2 = 4$

For second wave,

A = 1, k = 3, ω = 4

Wave speed = $\omega/k = 4/3 = 1.33$

Maximum particle velocity = $A\omega = 2\times4 = 8$

For third wave.

$A = 2, K = 3, \omega = 3$

Wave speed $= \omega/k = 3/3 = 1$

Maximum particle velocity $= A\omega = 2 \times 3 = 6$

The descending order is, 2nd, 3rd and 1st wave.

8. **A wave of frequency 500 cycles /sec has a phase velocity of 350 ms^{-1} (i) How far apart are two points 60° out phase (ii) what is the phase difference between two displacements at certain points at times 10^{-3} sec apart.**

Solution:

Here, $f = 500$ cycles/sec, $v = 350\ ms^{-1}$

Since, $v = f\lambda \Rightarrow \lambda = \frac{v}{f}$

$$\lambda = \frac{350}{500} = 0.7\ m$$

i) We have, phase difference, 2π = path difference, λ

Phase difference, 1 = path difference, $\frac{\lambda}{2\pi}$

Phase difference, 60° = path difference, $\frac{\lambda}{2\pi} \times 60$

Therefore, path difference $= \frac{0.7}{2 \times 180} \times 60 = 0.116\ m$

ii) Here the time period, $T = \frac{1}{f} = \frac{1}{500} = 2 \times 10^{-3}$ sec

For time period 'T' seconds the phase difference $= 2\pi$

For time period 1 sec the phase difference $= \frac{2\pi}{T}$

For time 10^{-3} sec the phase difference $= \frac{2\pi}{T} \times 10^{-3}$

$$= \frac{2\pi}{2 \times 10^{-3}} \times 10^{-3} = \pi \text{ rad}$$

9. ***One end of 14 m long rubber tube with total mass 0.8 kg is fastened to a fixed support. A cord Attached to the other end passes over a pulley and supports an object with a mass of 7.5 kg. The tube struck a transverse blow at one end. Find the time required for the pulse to reach the other end.***

Solution:

Mass of rubber tube, m = 0.8 kg, length l = 14 m

Mass per unit length, $\mu = \frac{m}{l} = 0.057$ kg/m

Mass of object, M = 7.5 kg

Tension on rubber tube, $T = Mg = 7.5 \times 9.8 = 73.5$ N

Therefore, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{73.5}{0.057}} = 35.9$ m/sec

Thus, time required for the pulse to reach the other end,

$$T = \frac{l}{v} = \frac{14}{35.9} = 0.39 \text{ sec}$$

10. **Show that for a wave on string, the kinetic energy per unit length of string is K.E. = $\frac{T}{2}$, where T is the tension on the string.**

Solution:

We know, $K.E = \frac{1}{2} m v^2$

K.E. per unit length $= \frac{1}{2}\frac{m}{l}v^2 = \frac{1}{2}\mu v^2$

$$= \frac{1}{2}\mu . \frac{T}{\mu} \qquad \left[\text{Since, } v = \sqrt{\frac{T}{\mu}}\right]$$

$$= \frac{T}{2}$$

11. **Two strings have been tied together with a knot and then stretched between two rigid supports. The string have linear density $\mu_1 = 1.4 \times 10^{-4}$ kg/m and $\mu_2 = 2.8 \times 10^{-4}$ kg/m. Their lengths are $L_1 = 3.0$ m and $L_2 = 2.0$ m, and first string is under a tension of 400 N. Simultaneously, on each string a pulse is sent from the rigid support end towards the knot, which pulse reaches the knot first?**

Solution:

For first string, $\mu_1 = 1.4 \times 10^{-4}$ kg/m, length $L_1 = 3$m, $T = 400$ N

Therefore, $v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{400}{1.4 \times 10^{-4}}} = 1690.3$ m/sec

The time required to pulse to reach the knot, $t_1 = \frac{L_1}{v_1} = \frac{3}{1690.3}$

$\Rightarrow \quad t_1 = 1.77 \times 10^{-3}$ sec

For second string, $\mu_2 = 2.8 \times 10^{-4}$ kg/m, $L_2 = 2$m, $T = 400$ N

Therefore, $v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{400}{2.8 \times 10^{-4}}} = 1195.228$ m/.sec

The time required to pulse on 2nd string to reach the knot,

$$t_2 = \frac{L_2}{v_2} = \frac{2}{1195.228} = 1.673 \times 10^{-3} \text{ sec}$$

Since $t_2 < t_1$ thus the pulse on second string reaches the knot first.

12. **Calculate the wave length, frequency, speed of the wave and the maximum particle velocity in the wave represented by, y = 10 sin (8πt – 0.08 πx). The value of x and y are in CGS system.**

Solution.

Here, $y = 10 \sin(8\pi t - 0.08\ \pi x)$ .... (1)

Comparing this equation with

$y = A \sin(\omega t - kx)$ ....(2)

We get,

$A = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}, \omega = 8\pi, k = 0.08\pi$

(i) Frequency $(f) = \frac{\omega}{2\pi} = \frac{8\pi}{2\pi} = 4 \text{ Hz}$

(ii) $k = \frac{2\pi}{\lambda}$

or, Wave length $(\lambda) = \frac{2\pi}{k} = \frac{2\pi}{0.08\pi} = 25 \text{ cm} = 25 \times 10^{-2} \text{ m}$

(iii) Speed of the wave $(v) = f\lambda = 4 \times 25 \times 10^{-2} = 1 \text{ m/sec}$

(iv) Maximum particle velocity $(v_{max}) = A\omega = 10 \times 10^{-2} \times 8\pi = 2.512 \text{ m/sec}$

13. A transverse sinusoidal wave is generated at one end of a long horizontal string by a bar which moves up and down through a distance of 0.5 m. The motion is continuous and repeated regularly twice each second. If the string has linear mass density of 0.005 kg/m and is kept under a tension of 2 N. Find the speed, amplitude, time period and wave length of the wave motion.

Solution.

$f = 2 \text{ Hz}, T = 2N, x = 0.5 \text{ m}, \mu = 0.005 \text{ kg/m}$

We have,

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{2 \text{ N}}{0.005 \text{ kg/m}}}$$

Speed $(v) = 20 \text{ m/s}$

Also, $v = \omega\sqrt{A^2 - x^2}$

Or, $20 = 2\pi f\sqrt{A^2 - (0.5)^2}$

Or, $400 = 4\pi^2 f^2 [A^2 - (0.5)^2]$

Or, $\frac{400}{4\pi^2 (2^2)} = A^2 - 0.25 \Rightarrow A^2 = 2.785$

$\Rightarrow$ Amplitude $(A) = 1.67 \text{ m}$

Time period $(T) = \frac{1}{f} = \frac{1}{2} = 0.5 \text{ sec}$

Wave length $(\lambda) = \frac{v}{f} = \frac{20}{2} = 10 \text{ m}$

14. Calculate the wavelength, frequency, speed of the wave and maximum particle velocity in the wave represented by y = 20 sinπ (2t − 0.05x), the values of x and y are in centimeters.

Solution.

We have,

$y = 20 \sin\pi (2t - 0.05x)$ .... (1)

Comparing equation (1) with the equation of progressive wave $y = A \sin(\omega t - kx)$

$A = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$

$\omega = 2\pi \Rightarrow 2\pi f = 2\pi \Rightarrow f = 1 \text{ Hz}$

$k = \pi\, 0.05 \,(\text{cm}^{-1}) = \pi\, 0.05 \times 100 \text{ m}^{-1} = 5\pi \text{ m}^{-1}$

i. Wave length, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{5\pi} = 0.4 \text{ m}$

ii. Speed of wave, $v = f\lambda = 1 \times 0.4 = 0.4 \text{ m/sec}$

iii. Maximum particle velocity $(V_{max}) = A\omega = 20 \times 10^{-2} \times 2\pi = 1.25 \text{ m/sec}$

15. Calculate frequency of vibration of air particles in plane progressive wave of amplitude 2.18×10^{-10} m and intensity 10^{-10} W/m², the velocity of sound in air is 340 m/S and density of air is 0.00129 gm/cc.

Solution.

$A = 2.18 \times 10^{-10} \text{ m}, I = 10^{-10} \text{ W/m}^2, v = 340 \text{ m/sec},$

$\rho = 0.00129 \text{ gm/cc} = \frac{0.00129 \times 10^{-3}}{10^{-6}} = 1.29 \text{ kg/m}^3$

We have,

$$I = \frac{1}{2} v\rho\, \omega^2 a^2$$

$$\omega^2 = \frac{2I}{v\rho a^2}$$

$$\omega = \sqrt{\frac{2I}{v\rho a^2}}$$

$$2\pi f = \sqrt{\frac{2I}{v\rho a^2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2I}{v\rho a^2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2 \times 10^{-10}}{340 \times 1.29 \times (2.18 \times 10^{-10})^2}}$$

$f = 493 \text{ Hz}.$

16. A progressive and stationary, simple harmonic wave having frequency 250 Hz and each having same velocity 30 m/S

(i) Determine the phase difference between two vibrating point in a progressive wave at a distance of 10 cm.

(ii) Wave equation of progressive wave if amplitude is 0.03 m.

(iii) Distance between nodes in stationary wave.

Solution.

Here, $f = 250 \text{ Hz}, v = 30 \text{ m/S}$

We have, $v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{30}{250}$

$\lambda = 0.12 \text{ m}$

i. Since, path difference, λ= Phase difference, 2π

Path difference, 1 = Phase difference, $2\pi/\lambda$

Path difference 10 cm (= 0.1m) = Phase difference, $\frac{2\pi}{\lambda} \times 0.1$

Therefore, phase difference $= \frac{2\pi}{\lambda} \times 0.1$

$$= \frac{2\pi}{0.12} \times 0.1 = 5.24°$$

ii. Here, amplitude, A = 0.03 m

The wave equation is given by,

$y = A \sin(\omega t - kx)$

$y = 0.03 \sin\left(2\pi ft - \frac{2\pi}{\lambda} x\right)$

$y = 0.03 \sin\left(2\pi \times 250\, t - \frac{2\pi}{0.12} x\right)$

$y = 0.03 \sin(500\, \pi t - 52.36x)$

$y = 0.03 \sin(1570.8\, t - 52.36\, x)$

iii. The distance between two adjacent nodes in a stationary wave is $\frac{\lambda}{2} = \frac{0.12}{2} = 0.06$ m.

Exercise

1. How standing waves are produced on a string? Write down the difference between travelling wave and standing wave.
2. Show that for a plane progressive wave, on the average, half the energy is kinetic and half-potential.
3. What is wave motion? Write the equation of sine wave travelling on the string; apply the theory of supper position to find the maxima and minima of interference pattern on the string.
4. Calculate amount of energy transmitted along a stretched string when a wave passes through it.
5. Differentiate between plane progressive wave and stationary wave. Show that intensity of a particular wave for a medium remains constant.
6. Deduce an expression for velocity of transverse waves in a stretched string.
7. What is super position of wave? Describe interference of wave.
8. In the progressive wave, show that the potential energy and kinetic energy of every particle will change with time but the average K.E per unit volume and P.E. per unit volume remains constant.

9. Distinguish between wave velocity and particle velocity. Obtain the relation for acceleration of the particle in wave motion.
10. The six strings of a guitar are of the same length and are under same tension, but have different thickness. On which string the wave travels the fastest? Answer with mathematical expression.
11. Define wave velocity and particle velocity, obtain the relation between them.
12. Derive a relation for speed of transverse wave in a stretched string and show that the average rate of energy transfer is $\frac{1}{2}\mu v\omega^2 A^2$, where symbol carry their usual meaning.
13. Derive the differential equation of transverse wave of a stretched string with applied tension T and mass per unit length μ. Also find the velocity of the wave propagating through the string.
14. Starting from the progressive wave equation $y = a \sin \frac{2\pi}{\lambda}(vt - x)$. Prove that the differential equation.

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

15. What is energy density of wave? Derive its expression.
16. What is meant by wave motion? What are its characteristics? Explain how it transports energy and derive a relation for the average rate at which the wave transports the energy along the string.
17. Differentiate between travelling wave and mechanical wave. Find an expression for the energy and power of a traveling wave.
18. What power is transmitted by a transverse wave of amplitude 1 cm and frequency 2 Hz propagating through a string of linear mass density 1 gm/m and under tension 20 N.
19. A string 2.72m long has a mass of 263 gm. The tension in the string is 36.1 N. What must be the frequency of traveling waves of amplitude 2.5 mm in order that average transmitted power is 85.5 W?
20. Calculate the minimum intensity of audibility in watts per square cm from a note of 1000 Hz if the amplitude of vibration is 10^{-9}cm. Given density of air is 0.0013 gm/cc and velocity of sound in air is 340 m/s.
21. A piano wire with mass 3gm and length 80 cm is stretched with tension of 25 N. A wave with frequency 120 Hz and amplitude 1.6 mm travels along the wire a) calculate the average power carried by the wave b) what happens to the power if the wave amplitude is halved.
22. The speed of a transverse wave on a string is 170 m/s, when the string tension is 120 N. To what value must the tension be changed to raise the wave speed to 180 m/sec?

23. A rod vibrating at 12 Hz generates harmonic waves with amplitude of 1.5 mm in a string of linear mass density 2 gm/m. If the tension in the string is 15 N, what is the average power supplied by the source.

24. A sound wave of frequency 300 Hz has an intensity of 1 μ W/m^2. If density of air is 1.29 kg/m^3 and speed of sound in air is 330 m/sec. What is the amplitude of air oscillation caused by this wave?

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