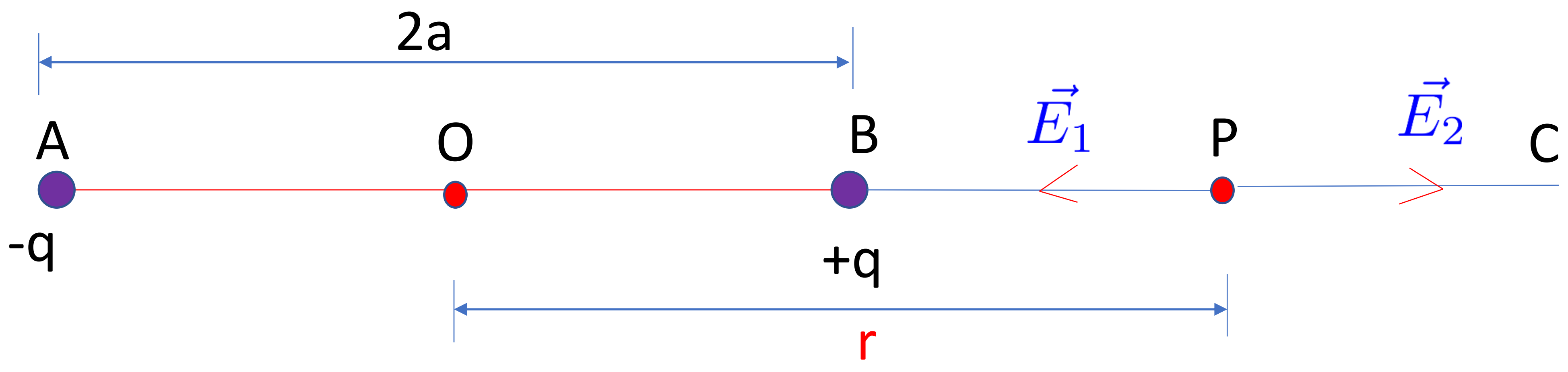
Electrostatics_4

Continued from electrostatics_3

Electric field due to a dipole on axial line:

Consider a dipole of length 2a having equal and opposite charges at the points A and B. We have to determine the electric field at point P which lies on the axis of the dipole.



Let O be the center of the dipole. Then OP = r, say.

Then the electric field at P due to –q charge is

$$E_1 = \frac{q}{4\pi\epsilon_0(r+a)^2} \quad (along\ \vec{PA})$$

Also, the electric field at P due to +q charge is given by

$$E_2 = \frac{q}{4\pi\epsilon_0 (r-a)^2} \quad (along\ \vec{PC})$$

Now, the resultant field at P is given by

$$E = E_2 - E_1 \ (along\ \vec{PC})$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{r^2 + 2ar + a^2 - r^2 + 2ar - a^2}{(r-a)^2(r+a)^2}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{4ar}{(r^2-a^2)^2}\right)$$

$$= \frac{(q.2a)2r}{4\pi\epsilon_0 (r^2-a^2)^2}$$

$$or,\ E = \frac{2pr}{4\pi\epsilon_0(r^2 - a^2)^2}\ \ (along\ \vec{PC})$$

where p =q.2a, is the dipole moment.

<u>Case:</u> For a short dipole, $r \gg 2a,$

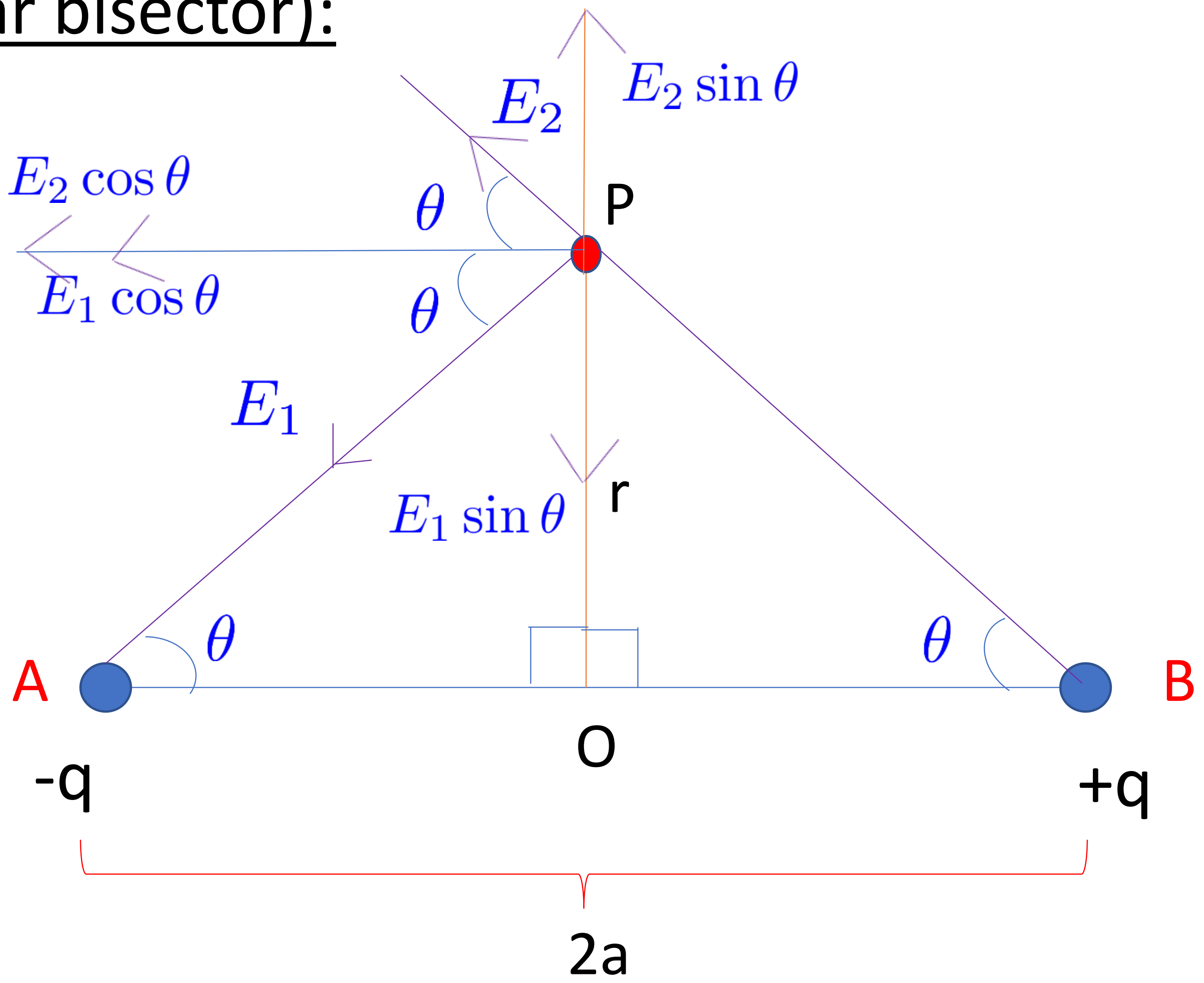
$$\rightarrow\ r \gg a$$

$$So,\ r^2 - a^2\ \approx\ r^2$$

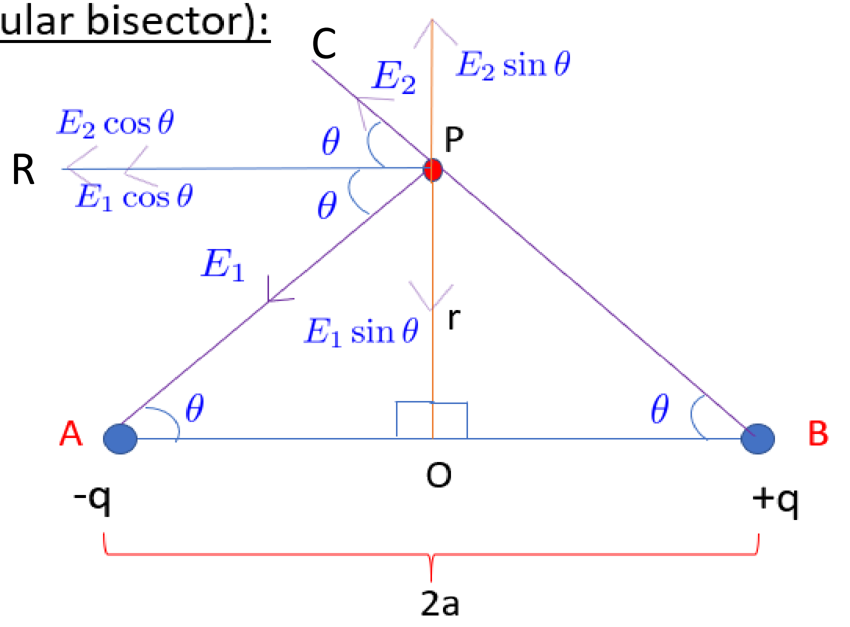
$$Then,\ E = \frac{2pr}{4\pi\epsilon_0 r^2} = \frac{2p}{4\pi\epsilon_0 r^3}$$

$$\rightarrow \boxed{E = \frac{2p}{4\pi\epsilon_0 r^3}}$$

Electric field due to a dipole on the equatorial line (perpendicular bisector):



Electric field due to a dipole on the equatorial line (perpendicular bisector):



Consider an electric dipole of length 2a. We have to find the electric field at P such that OP =r.

The electric field at P due to –q charge is

$$E_1 = \frac{q}{4\pi\epsilon_0 (AP)^2} \quad (along\ \vec{PA})$$

$$E_1 = \frac{q}{4\pi\epsilon_0 (r^2 + a^2)} \quad (along\ \vec{PA})$$

Also, The electric field at P due to +q charge is

$$E_2 = \frac{q}{4\pi\epsilon_0 (r^2 + a^2)} \quad (along\ \vec{PC})$$

Here, the vertical components of E_1 and E_2 cancel out each other, while the horizontal components add up to provide the resultant electric field at P.

$$Hence,\ E = E_1 \cos\theta + E_2 \cos\theta$$
$$= 2E_1 \cos\theta$$

$$[since\ |E_1| = |E_2|, in\ magnitude]$$

$$E = 2\frac{q}{4\pi\epsilon_0(r^2 + a^2)} \cdot \frac{a}{\sqrt{(r^2 + a^2)}}$$

$$E = \frac{p}{4\pi\epsilon_0(r^2 + a^2)^{\frac{3}{2}}}$$

Case: For a short dipole, $r \gg 2a,$

$r \gg a,$

$$So, \boxed{E = \frac{p}{4\pi\epsilon_0 r^3}}$$

$$So, \frac{E_{axial}}{E_{equatorial}} = 2 : 1$$

Summary on electric field and potential due to monopole, dipole and quadrupole:

Due to →	monopole	dipole	quadrupole
(i) Potential (V)	$V = \frac{q}{4\pi\epsilon_0 r}$ $V \propto \frac{1}{r}$	$V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$ (at any point) $V \propto \frac{1}{r^2}$	$V = \frac{qa^2}{4\pi\epsilon_0 r^3}(3\cos^2\theta - 1)$ $V_{axial} = \frac{Q}{4\pi\epsilon_0 r^3}$ $V \propto \frac{1}{r^3}$
(ii) Electric field (E)	$E = \frac{q}{4\pi\epsilon_0 r^2}$ $E \propto \frac{1}{r^2}$	$E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$ (at any point) $E \propto \frac{1}{r^3}$	$E_{axial} = \frac{3Q}{4\pi\epsilon_0 r^4}$ $E \propto \frac{1}{r^4}$

The above values of the potential and electric field show that they decrease rapidly for dipole and quadrupole in comparison with those values for monopole.