Input or Arrival Pattern

→ The arrival Pattern describes the manner in Which the customers arrive and join the Queueing system. Since the customers arrive in a random order, the arrival pattern of Customers follows Poisson Process.

(i) Arrival Rate

→ Arrival Rate is The number of arrivals in one time period.

i.e Arrival Rate = λ customers per unit time.

(ii) Inter-Arrival time

→ The time between Consecutive arrivals is called inter-arrival time.

$$\text{Inter-Arrival time} = \frac{1}{\lambda}$$

Here, The number of customers may be from finite ~~and~~ or infinite Sources.

Also, the input Pattern should indicate the number of queues that are allowed to form, the maximum queue-length, the maximum number of customers requiring service,

(2) Service Mechanism

It is specified when it is known how many customers can be served at a given period of time.

(i) Service Rate.

The number of customers serviced in one time period.

i.e Service Rate = μ customers per unit time.

(ii) Inter service time

The time required to complete the service for a customer.

i.e Inter service time = $\frac{1}{\mu}$ time.

· The service pattern should also indicate the number of service channels (or servers) and may have one or many counters in series or in parallel.

No. of servers is denoted by S. (Sometimes c).

Queue Discipline

It is a manner (or rule) by which the customers are selected for service.

(i) FIFO or FCFS

↳ First In First out

This is the most commonly used procedure.

(ii) LIFO or LCFS.

↳ Last In First Out

This procedure is used in inventory systems.

(iii) SIRO → Selection in Random Order.

(iv) PIR → Service in Priority (or Priority in selection).

This procedure is used in manual transmission messaging systems.

System Capacity

The max^m number of Customers in the Queueing system can be either finite or infinite.

If any no. of Customers are allowed to join the Queue, then the system capacity is infinite. Sometimes ~~there is~~ only limited customers are allowed in the system, so that when the limiting value is reached, no further customers are allowed to enter the Queueing System.

Kendall's Notation of Queueing System

Kendall's notation provides a very convenient description of Queueing Systems.

This notation of a Queueing System has the form

$$(a/b/c):(d/e)$$

Where,

a = Inter-Arrival distribution.

b = Service time distribution.

c = Number of Channels or servers.

d = System capacity

e = Queue discipline.

In kendall's notation, a and b usually take one of the following symbols:

M = Markovian or Exponential distr

G = Arbitrary or General distribution

D = Fixed or Deterministic Distn.

The Four Important Queueing systems described by kendall's notation are as follows.

(i) $(M/M/1):(\infty/FIFO)$

(ii) $(M/M/s):(\infty/FIFO)$

(iii) $(M/M/1):(K/FIFO)$

(iv) $(M/M/s):(K/FIFO)$.

For a queue, the term line length and queue length are defined as:

Line length (or queue size) is the no. of customers in the queueing system.

Also, the queue length is defined as the no. of customers in the queue.

i.e Queue length = Line length − No. of customers being served.

Terminology

(i) n = Number of customers in the system (waiting and in service)

(ii) P_n = Probability of n customers are in the system.

(iii) λ = Mean arrival Rate in the system.

(iv) μ = Mean service rate.

(v) $\rho = \frac{\lambda}{\mu}$; traffic Intensity or server Utilization factor.

(vi) P_0 = Prob. of no customer in the system. (Prob. that server is ideal).

(vii) S = No. of service channels (or servers)

(viii) K or N = Max^m no. of customers allowed in the system.

(ix) L_s = Expected or Average number of customers in the system (waiting and in service).

x) L_q = Expected or Average no. of Customers in the queue (queue length)

(xi) W_s = Expected or Average waiting of a customer in the system. (waiting and in service).

(xii) W_q = Expected or Average waiting time of a customer in the queue.

(xiii) P_w = Prob. that an arriving Customers has to wait (system is busy).

To achieve steady-state Condition, it is necessary that $\lambda/\mu < 1$ {i.e arrival Rate is less than service rate}

Model II: $(M/M/s):(\infty/FIFO)$, multiple server with infinite capacity.

For this model, there are multiple servers s, working independently of each other.

If there are n customers in the system, then the following two cases may arise:

(i) If $n < s$, only n of the s servers will be busy and others will be idle.

Hence, mean service rate, $\mu_n = n\mu$.

(ii) If $n \geq s$; all the servers will be busy.

Hence, the mean service rate, $\mu_n = s\mu$.

To derive the results for this model, we have

$$\lambda_n = \lambda \,, \; \forall n \geq 0.$$

and

$$\mu_n = \begin{cases} n\mu & \text{if } n < s \\ s\mu & \text{if } n \geq s \end{cases}$$

Also, the mean arrival rate λ is less than $s\mu$. i.e $\lambda < s\mu$.

Now, Traffic Intensity,

$$\rho = \frac{\lambda}{s\mu}.$$

(i) P_0 = The Prob. that the system is idle.

i.e $$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \cdot \frac{s\mu}{s\mu-\lambda}\right]^{-1}$$

or.

$$P_0 = \left[1 + \frac{s\rho}{1!} + \frac{(s\rho)^2}{2!} + \cdots + \frac{(s\rho)^s}{s!(1-\rho)}\right]^{-1}$$

(ii) P_n = The Prob. of n customers in the System.

$$P_n = \begin{cases} \frac{s^n \cdot \rho^n}{n!} P_0 ; & \text{if } n \leq s \\ \frac{s^s \cdot \rho^n}{s!} P_0 ; & \text{if } n > s \end{cases}$$

Where $\rho = \lambda/s\mu$.

(iii) $P(N \geq s)$ = Prob. that All Servers are busy. (utilization factor)

$$= \frac{(s\rho)^s}{s!(1-\rho)} \cdot P_0$$

Example: A supermarket has a single cashier. During peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be processed by the cashier is 24 per hour.

Calculate:

(i) The probability that the cashier is idle.

(ii) Average number of customers in the system.

(iii) Average number of customers in the queue.

(iv) Average time a customer spends in the system.

(v) Average time a customer spends in the queue for service.

(vi) Expected waiting time for a Customer in the System

$$W_s = \frac{1}{\mu - \lambda} \left[\text{or } \frac{L_s}{\lambda}\right] \left[\text{or } W_q + \frac{1}{\mu}\right]$$

(vii) Expected waiting time for a Customer in the queue.

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} \left[\text{or } \frac{L_q}{\lambda}\right]$$

(viii) Prob. that the queue is non-empty.

$$P(n>1) = 1 - P_0 - P_1$$

$$= \left(\frac{\lambda}{\mu}\right)^2$$

(ix) Prob. that the number of Customers, n in the system exceeds a given number k

i.e $P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^k$

and $P(n>k) = \left(\frac{\lambda}{\mu}\right)^{k+1}$.

(x) $P(W_s > t) = e^{(\lambda-\mu)t}$

(xi) $P(W_q > t) = \rho . e^{(\lambda-\mu)t}$

Model I

$(M/M/1):(\infty/FIFO)$; Single Server with Infinite Capacity.

This model is based on certain assumptions.

(i) Arrivals are described by poisson distn and come from infinite ~~source~~ input source.

(ii) Single waiting line and each arrival waits to be served.

(iii) Queue discipline — FIFO or FCFS.

(iv) Single server and service time follows exponential distn.

(v) Customers arrival is independent but the arrival rate does not change over time.

(vi) The average service rate is more than

Performance Measures or Characteristics (Formula)

Here, λ = Mean arrival Rate.

$1/\lambda$ = Inter-Arrival time.

and μ = Mean servical rate

$1/\mu$ = Service time.

Then (i) Traffic Intensity,

$$\rho = \frac{\lambda}{\mu}$$

(ii) P_0 = Prob. of no customer in the system (Prob. that server is ideal)

i.e $P_0 = 1 - \rho \cdot \left(1 - \frac{\lambda}{\mu}\right)$

(iii) P_n = Prob. of n Customers in the System

i.e $P_n = \rho^n (1-\rho)$ $\left[\text{or } \left(\frac{\lambda}{\mu}\right)^n P_0\right]$

(iv) L_s = Average No. of Customers in the System

i.e $L_s = \frac{\rho}{1-\rho}$ $\left[\text{or } \frac{\lambda}{\mu - \lambda}\right]$

(v) L_q = Average no. of customers in the queue.

i.e $L_q = \frac{\rho^2}{1-\rho}$ $\left[\text{or } \frac{\lambda^2}{\mu(\mu-\lambda)}\right]$