$> t - k + 1$, then the length of the longest sequence of fault-free vertices is always greater than that of the longest sequence of faulty ones, and therefore the last vertex in the longest vertex chain corresponding to a subsyndrome of the form of $000 \cdots 01$ must be faulty according to the result restated above. Since the faulty vertex so identified is on the boundary of a faulty vertex sequence, the number of sequences of the remaining faulty vertices can not exceed k. In other words, we can identify at least one faulty vertex such that the number of sequences of the remaining vertices does not exceed k, as long as $\lceil (|V| - t)/k \rceil > t - k + 1$. Assume $\lceil (|V| - t)/k \rceil > t - k + 1$. Let $k_1 = (|V| - t - 1)/k$, we have $|V| - t = k_1 k + 1$. Therefore, $\lceil (|V| - t)/k \rceil = \lceil (k_1 k + 1)/k \rceil > t - k + 1$. By simple transformations we have the following inequalities one after another.

$$k_1 + 1 > t - k + 1$$

$$k_1 > t - k$$

$$(|V| - t - 1)/k > t - k$$

$$|V| > -k^2 + tk + t + 1$$

$$|V| \geq -k^2 + tk + t + 2.$$

Let $p = -k^2 + tk + t + 2$. It can be easily verified that p reaches its maximal value when $k = \lfloor t/2 \rfloor$ (considering t as constant) with the maximal value $p_{\max} = -\lfloor t/2 \rfloor^2 + t \lfloor t/2 \rfloor + t + 2$ which is also maximum. Notice that p increases monotonically as either of k and t increases ($k \in \{1, 2, \cdots, \lfloor t/2 \rfloor\}$). Therefore, if either of (1) and (2) is satisfied, then at least one faulty vertex can be identified such that the number of sequences of the remaining faulty vertices does not exceed k if a k'/t' fault set is present as long as $k' \leqslant k$ and $t' \leqslant t$. Hence, the theorem. Q.E.D.

We have not only given a sufficient condition for a single loop system to be sequentially k/t diagnosable but also pointed out an algorithm to diagnose such a system. Our result agrees with the previous one given in [2] if k is set equal to t.

VI. Conclusion

The k-component t fault diagnosis has been investigated in this correspondence. A system for k-component t fault diagnosability can have a simpler structure than that for general t fault diagnosability. To put the k/t diagnosis into practice the strategy of choosing k for a particular system should be studied in advance. We present an open problem searching for an optimal structure of k/t diagnosable systems for arbitrary value of k, which seems to be of interest.

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Linear Dependencies in Linear Feedback Shift Registers

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Abstract—**Linear feedback shift registers have been proposed to generate test patterns for the self-test of logic networks. The probability of linear dependence among k bit positions of a subset of k bits in a maximum length shift register sequence is an outstanding problem. In this correspondence, we derive a formula for the calculation of the probability.**

Index Terms—**Linear feedback shift registers, self-test, test pattern generation.**

Test Pattern Generation

Linear feedback shift registers (LFSR's) have been proposed to generate test patterns for the self-test of logic networks [1]–[3]. In this approach, the test patterns for a logic block with k inputs are obtained by gating a set of k bits, whose exact positions may be selected in random for VLSI circuits, from an LFSR. A single shift of the LFSR provides a test pattern to the logic block. Thus test patterns for all logic blocks can be efficiently generated by successively shifting the LFSR.

An m-stage LFSR is associated with a binary polynomial of degree m that specifies the taps for exclusive-or gates. Fig. 1 shows a four-stage LFSR associated with the polynomial $x^4 + x + 1$. If the LFSR is initially loaded with the pattern 1000 and is operated in an autonomous mode, then an infinite periodic sequence (see Fig. 2(a)) that repeats every 15 bits can be generated. The 15 successive subsequences of length 15 obtained from the infinite sequence (Fig. 2(b)) can be used as test patterns for logic self-test. In general, an LFSR associated with a primitive polynomial of degree m can be used to generate an infinite periodic sequence with a period of $n = 2^m - 1$ [4]–[6]. An n-bit subsequence of the infinite sequence is called a maximum-length sequence. A set of n distinct maximum-length sequences can be obtained by successively shifting a maximum-length sequence $n - 1$ times.

Let T be a matrix that contains as rows all n distinct maximum-length sequences obtained by shifting a maximum-length sequence, and let $T(k)$ be a matrix formed by a set of k distinct columns of T. One approach in self-testing VLSI circuits is to use the n k-bit row vectors of $T(k)$ as a test set for a logic block with k inputs. By tapping

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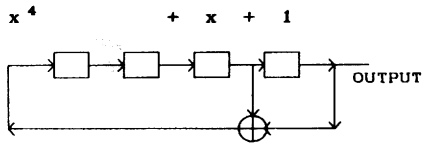

Fig. 1. An LFSR associated with $x^4 + x + 1$.

⟵ period ⟶

.....11001000111101011001000111101011001000

(a)

$$T = \left|\begin{array}{c}
111101011001000\\
011110101100100\\
001111010110010\\
000111101011001\\
100011110101100\\
010001111010110\\
001000111101011\\
100100011110101\\
110010001111010\\
011001000111101\\
101100100011110\\
010110010001111\\
101011001000111\\
110101100100011\\
111010110010001
\end{array}\right|$$

(b)

Fig. 2. (a) An output sequence. (b) A test set of 15 patterns.

different sets of bit positions the same matrix T would provide test sets for many logic blocks. The positions tapped for a set of k inputs to a logic block are in general not predetermined and thus can be considered as random.

The row vectors of $T(k)$ may or may not contain all 2^k binary k-tuples depending on whether the columns of $T(k)$ are linearly independent. It can be shown [4]–[6] that the row vectors of T plus the all zeros vector form an m-dimensional vector subspace of the n-dimensional binary vector space. Thus, if m is greater than k and $T(k)$ contains k linearly independent columns, then $T(k)$ contains all possible binary k-tuples as rows. If the columns of $T(k)$ are not linearly independent, some binary k-tuples do not appear as row vectors of $T(k)$. For example, let $T(4)$ be the submatrix obtained from columns 1, 3, 4, and 12 of the matrix T of Fig. 2(b). As can be seen from Fig. 3, the (mod 2) sum of the four columns of $T(4)$ is the all zeros vector. That is, the four columns are linearly dependent. Clearly all odd-weight 4-tuples are precluded in the test set $T(4)$.

It can be shown that any 2 columns of T are linearly independent [4]. For $k \geq 3$, the columns of $T(k)$ are not necessarily linearly independent. In this correspondence, we derive an analytic expression for the probability of linear dependence of the columns of $T(k)$ for a given k. The result can be used to judge the quality of maximum-length sequences used as test stimuli in a self-test design.

The problem of fault coverage in testing logic networks with random input patterns has been investigated [7]–[9]. In general, the fault detection probability increases as the number of distinct test patterns increases. The calculation of the detection probability is made under the assumption that test patterns are random, at least in the sense that the test patterns are distinct. This assumption is also used to compute the confidence level of detecting faults for a fixed number of test patterns. If a set of maximum-length sequences is used as a test set and the columns of $T(k)$ are not linearly independent, then the test patterns are far from random. The test patterns for a particular logic block may consist of only a small number of distinct patterns and certain test patterns are precluded in the test set. As a result, the techniques in analyzing fault coverage may not be applicable. On the other hand, if the probability of linear dependence among k columns of $T(k)$ is small, the results of analytic calculation based on random test pattern assumption may still be valid. Thus, the probability of linear dependence of the columns of $T(k)$ can be used to qualify the validity of the calculation of fault coverage based on random pattern test assumption.

The current research on the design of test sets for exhaustive coverage of inputs has not provided a practical solution for a moderate large values of n and k [10]–[12]. An alternative is to use test patterns that can provide an "almost" exhaustive coverage. Maximum-length sequences are natural candidates for this approach because of their simplicity in implementation. The question is how close is a test set constructed from maximum-length sequences to an exhaustive coverage. The investigation of linear dependence of $T(k)$ would provide an answer for a given n and k. If the probability of linear dependence of $T(k)$ is small, the test set of $T(k)$ may be considered as an exhaustive test set for practical purposes. This is another reason to investigate the probability of linear dependence for columns of $T(k)$.

$$T(4) = \left|\begin{array}{cccc}
1 & 1 & 1 & 1\\
0 & 1 & 1 & 0\\
0 & 1 & 1 & 0\\
0 & 0 & 1 & 1\\
1 & 0 & 0 & 1\\
0 & 0 & 0 & 0\\
0 & 1 & 0 & 1\\
1 & 0 & 1 & 0\\
1 & 0 & 0 & 1\\
0 & 1 & 0 & 1\\
1 & 1 & 1 & 1\\
0 & 0 & 1 & 1\\
1 & 1 & 0 & 0\\
1 & 0 & 1 & 0\\
1 & 1 & 0 & 0
\end{array}\right|$$

Fig. 3. A submatrix from columns 1, 3, 4, and 12 of T.

Probability of Linear Dependence

The row vectors of T are not linearly independent. In fact, there are only m linearly independent rows [4]–[6]. Let G be the matrix formed by a set of m linearly independent rows of T. Then the columns of G are all distinct and form the set of all possible nonzero m-tuples. The columns of G and the all-zeros m-tuple form an m-dimensional vector space. In addition, all possible nonzero linear combinations of the row vectors of G are row vectors of T. For example, let G be the matrix of the first four rows of T in Fig. 2(b). Then the four rows of G are linearly independent, and every row of T in Fig. 2(b) can be expressed as a linear combination of the rows of G.

Recall that $T(k)$ is a matrix formed by a set of k columns of T. Let $G(k)$ be the matrix formed by the corresponding k columns of G. Then the columns of $T(k)$ are linearly independent if and only if $G(k)$ are linearly independent. Thus, the problem of determining the linear dependence of $T(k)$ is the same as the problem of determining the linear dependence of $G(k)$.

Since mxtrix G has m linearly independent rows, the column rank of G is m. Thus, any set of $(m + 1)$ or more columns of G are linearly dependent. That is, the probability of linear dependence of the columns of $G(k)$ is 1 for $k > m$. In the following discussion we will assume that $k \leq m$.

For a given k, we can enumerate all possible sets of k linearly independent columns in G. Since there are n columns in G, there are n ways of selecting the first of a set of k columns, and there are $(n - 1)$ ways of selecting the second column. Let $c(1)$ and $c(2)$ be the first two columns selected. Then $c(1) + c(2)$ is dependent of $c(1)$ and $c(2)$ and thus cannot be selected as the third column. Third column $c(3)$ can be selected in $(n - 3)$ ways. Now, $c(1)$, $c(2)$, and $c(3)$ form a basis of a 3-dimensional subspace. Any column in G excluding the columns that are elements of the 3-dimensional subspace is linearly independent of $c(1)$, $c(2)$, and $c(3)$. Since there are 7 nonzero elements in the 3-dimensional subspace, the fourth column can be selected in $(n - 7)$ ways. In general, the ith column can be selected in $(n + 1 - 2^{i-1})$ ways. Therefore, the number of possible sets of k linearly independent columns in G is

$$N(k) = n(n-1)(n-3)(n-7) \cdots (n+1-2^{k-1})/k!.$$

The probability that a set of k columns of G are linearly independent is equal to $N(k)/\binom{n}{k}$ where $\binom{n}{k}$ is the number of combinations of k out of n objects. Therefore, the probability that a set of k columns of G are linearly dependent is equal to $1 - N(k)/\binom{n}{k}$.

k	m=10	m=20	m=30
1	.000000000	.000000000	.000000000
2	.000000000	.000000000	.000000000
3	.000979432	.000000954	.000000001
4	.004897160	.000004768	.000000005
5	.015639192	.000015259	.000000015
6	.040780038	.000040054	.000000039
7	.094541629	.000094412	.000000092
8	.201485531	.000208843	.000000204
9	.395803830	.000444353	.000000434
10	.694922644	.000922889	.000000902
11		.001888080	.000001845
12		.003826117	.000003741
13		.007705119	.000007544
14		.015444278	.000015160
15		.030814096	.000030405
16		.061086829	.000060906
17		.119754631	.000121922
18		.229772081	.000243961
19		.422318593	.000488024
20		.711153787	.000976048
21			.001951638
22			.003900931
23			.007791922
24			.015543525
25			.030925635
26			.061209186
27			.119883590
28			.229898121
29			.422423575
30			.711211780

Fig. 4. Probability of k-bit linear dependence using an m-bit LFSR.

REMARKS

Fig. 4 shows the probability of k-bit linear dependence of m-bit maximum-length sequences for $m = 10$, 20, and 30. In the case of $m = 30$, the probability of 20-bit linear dependence is less than 0.00098. If the number of inputs to each logic block is no more than 20, then the probability that an input pattern is missing from the test set is insignificant. The set of $2^{30} - 1$ patterns may be considered as an exhaustive test set for $k \leq 20$. A subset of these patterns may be used as a random test set, whose fault coverage may be calculated as in [7]–[9].

LFSR's can be designed to generate test sets that exhaust all possible k-tuples in any k out of n bit positions [11], [12]. In general, the polynomials associated with the LFSR's are not primitive and the size of a test set may be substantially greater than n.

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Spectral Signature Testing of Multiple Stuck-at Faults in Irredundant Combinational Networks

P. K. LUI AND J. C. MUZIO

Abstract—**Earlier spectral signature testing methods are extended to the multiple stuck-at fault model. The testability condition for multiple-input faults is established and a minimal spanning signature (MSS) is defined to cover all these faults. It is then shown that an MSS, which in most cases contains a single spectral coefficient, will detect over 99 percent of all input and internal multiple faults. An approach is described to obtain a complete signature for all multiple faults in any irredundant combinational network with comparatively small numbers of fan-outs. Tree networks that include XOR/XNOR gates are shown to be easily tested. Internally fan-out-free and general irredundant networks are also considered. A design approach is proposed to enable a network to be tested for all single and most multiple faults using a single coefficient, with the possible overhead being an extra control input.**

Index Terms—**Combinational network, fault detection, multiple stuck-at faults, spectral signature testing, syndrome testing.**

I. INTRODUCTION

The traditional test vector approach to fault detection in a combinational network has in recent years given way to exhaustive testing techniques, where all input combinations are applied to the network under test (NUT) and the output response vector is compressed. This new approach drastically reduces the data volume to be handled at test time and is practical for networks of up to 20 inputs. The avoidance of test sets makes the method feasible for built-in test.

Under one such technique called spectral signature testing [8], [10], [11], [14], the test consists of verifying one or more spectral coefficients of the output response vector. This set of test coefficients is termed a spectral signature of the NUT. The actual test verification is straightforward with very little extra hardware required (see [8] or [11] for details).

This correspondence describes the derivation of a complete spectral signature to detect all multiple (stuck-at) faults in the NUT. Although the present discussion is limited to single output irredundant combinational networks, results are easily extended to multiple-output networks and to certain sequential networks which behave combinationally during testing (e.g., level-sensitive scan design LSSD [5]).

The use of spectral coefficients for fault detection has been considered by a number of authors, primarily for single stuck-at faults. Tzidon *et al.* [16] introduce the weight as the one's count of the output response vector. The weight is in fact the simplest of all possible coefficients. Savir [12] employs the syndrome, which is equivalent to the weight. Susskind [14] adds a second coefficient called C_{all}. Carter [3] investigates parity testing when the weight is odd. Miller and Muzio [10], [11] give a more general treatment by considering all possible spectral coefficients. We extend the analysis to cover multiple stuck-at faults for two important reasons. In practice, with the increasing density of VLSI chips, the common assumption that only single faults can occur is no longer valid. Indeed, a number of physical defects on the chip manifest themselves as multiple faults. Theoretically speaking, a single fault is a special case of a multiple fault.

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