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Intro to Machine Learning	September 22

Predicting Customer Purchase using logistic regression

Customers	time on site	Pages Viewed	purchase	Predicted 0.168	Loss 0.184
A	1	4	0		
B	2	3	0	0.231	0.253
C	3	7	1	0.769	0.263
d	5	2	1	0.69	0.371
e	6	6	1	0.961	0.04

$m_1 = 0.8$

$m_2 = 0.4$

$b = -21$

$z = m_1(x_1) + m_2(x_2) + b$

$y = \frac{1}{1 + e^{-(z)}}$

$\text{loss}_i = (y_i \times \ln(\hat{y}_i) + (1 - y_i) \times \ln(1 - \hat{y}_i))$

Computation of probability per customer

Customer A

$$z = (0.8(1)) + (0.4(4)) - 4$$
$$= -1.6$$

$$y = \frac{1}{1 + e^{-(-1.6)}}$$

$$= 0.168$$

Customer b

$$z = (0.8(2)) + (0.4(3)) - 4$$
$$= -1.2$$

$$y = \frac{1}{1 + e^{-(-1.2)}}$$

$$= 0.231$$

Customer C

$$z = (0.8(3)) + (0.4(7)) - 4$$
$$= 1.2$$

$$y = \frac{1}{1 + e^{-(1.2)}}$$
$$= 0.769$$

Customer d

$$z = (0.8(5)) + (0.4(2)) - 4$$
$$= 0.8$$

$$y = \frac{1}{1 + e^{-(0.8)}}$$
$$= 0.69$$

Customer E

$$z = (0.8(6)) + (0.4(6)) - 4$$
$$= 3.2$$

$$y = \frac{1}{1 + e^{-(3.2)}}$$
$$= 0.961$$

Computation of average loss per customer

Customer A

$$\text{Loss}_i = (0(\ln(0.168)) + (1-0)(\ln(1-0.168)))$$
$$= (0 + 1(\ln(1-0.168)))$$
$$= 0.184$$

Customer b

$$\text{Loss}_i = (0(\ln(0.231)) + (1-0)(\ln(1-0.231)))$$
$$= (0 + 1(\ln(1-0.231)))$$
$$= 0.268$$

Customer C

$$\text{Loss}_i = (1(\ln(0.769)) + (1-1)(\ln(1-0.769)))$$
$$= (\ln(0.769) + (\ln(1-0.769)))$$
$$= 0.263$$

Customer D

$$\text{Loss}_i = (1(\ln(0.69)) + (1-1)(\ln(1-0.69)))$$
$$= (\ln(0.69) + (\ln(1-0.69)))$$
$$= 0.371$$

Customer E

$$\text{Loss}_i = (1(\ln(0.961)) + (1-1)(\ln(1-0.961)))$$
$$= (\ln(0.961) + (\ln(1-0.961)))$$
$$= 0.04$$

Average loss $= \frac{1}{2} \sum \text{loss}_i$

$$= \frac{0.184 + 0.263 + 0.263 + 0.321 + 0.041}{5}$$

$$= 0.2242$$

$$\frac{dL}{dm_1} = \frac{1}{5}\left(0.168(1) + 0.231(2) + (-0.231(3)) + (-0.31(5)) + (-0.039(6))\right)$$

$$= \frac{1}{5}(-1.897)$$

$$= -0.3794$$

$$\frac{dL}{dm_2} = \frac{1}{5}\left((0.168(4)) + (0.231(2)) + (-0.231(7)) + (-0.31(2)) + (-0.039(6))\right)$$

$$= \frac{1}{5}(-1.106)$$

$$= -0.2212$$

$$\frac{dL}{db} = \frac{1}{5}\left((0.168) + (0.231) + (-0.231) + (-0.31) + (-0.039)\right)$$

$$= \frac{1}{5}(-0.181)$$

$$= 0.0362$$

$new_{m_1} = m_1 - \eta \frac{d_1}{dm_1}$ learning rate ($\eta = 0.1$)

$= 0.8 + 0.036941$

$= 0.836941$

$new_{m_2} = 0.4 - (0.1)(-0.2212)$

$= 0.4 + 0.02212$

$= 0.42212$

$newb = -4 - (0.1)(-0.0362)$

$= -4 + 0.00362$

$= -3.996638$

Compute new probabilities for each customer

Customer A

$z = ((0.83694(1)) + (0.42212(4))) - 3.9964$

$= -1.4791$

$y = \frac{1}{1+e^{-(-1.4791)}}$

$= 0.187$

Customer B

$z = ((0.83694(2)) + (0.42212(3))) - 3.9964$

$= -1.056$

$y = \frac{1}{1+e^{-(-1.056)}}$

$= 0.258$

Customer C

$$z = ((0.83694(3)) + (0.42212(7))) - 3.9964$$
$$= 1.469$$

$$y = \frac{1}{1+e^{-(1.469)}}$$

$$= 0.813$$

Customer d

$$z = ((0.83694(5)) + (0.42212(2))) - 3.9964$$
$$= 1.033$$

$$y = \frac{1}{1+e^{-(1.033)}}$$

$$= 0.737$$

Customer e

$$z = ((0.83697(6)) + (0.42212(6))) - 3.9964$$
$$= 3.558$$

$$y = \frac{1}{1+e^{-(3.558)}}$$

$$= 0.972$$

Compute new loss for each customer

$$\text{loss}_i = -(0(\ln(0.187)) + (1-0)(\ln(1-0.187)))$$
$$= 0.207$$

Customer b

$$\text{Loss}_i = (0(\ln(0.258)) + (1-0)(\ln(1-0.258)))$$
$$= 0.298$$

Customer c

$$\text{loss}_i = (1(\ln(0.813)) + (1-1)(\ln(1-0.813)))$$
$$= 0.207$$

Customer d

$$\text{Loss}_i = (1(\ln(0.737)) + (1-1)(\ln(1-0.737)))$$
$$= 0.305$$

Customer e

$$\text{loss}_i = (1(\ln(0.972)) + (1-1)(\ln(1-0.972)))$$
$$= 0.028$$

$$\text{Average} = \frac{0.207 + 0.298 + 0.207 + 0.305 + 0.028}{5}$$

$$= 0.209$$

Old average loss = 0.2242
New average loss = 0.209