## Lab02-Algorithm Analysis

Exercises for Algorithms by Nengjun Zhu, 2022-2023 Fall Semester.

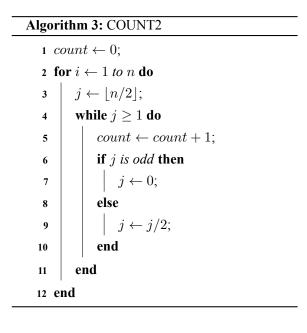
Name:	<b>Student ID:</b>	Email:

- 1. Consider the sorting algorithm shown in Alg.1, which is called BUBBLESORT.
  - (a) What is the minimum number of element comparisons? When is this minimum achieved?
  - (b) What is the maximum number of element comparisons? When is this maximum achieved?
  - (c) Express the running time of Alg.1 in terms of the O and  $\Omega$  notations.
  - (d) Can the running time of the algorithm be expressed in terms of the  $\Theta$  notation? Explain.

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Algorithm 1: BUBBLESORT
    input: An array A[1 \cdots n] of n elements.
    output: A[1 \cdots n] in nondecreasing order.
 i \leftarrow 1; sorted \leftarrow false;
 2 while i \leq n-1 and not sorted do
        sorted \leftarrow true;
 3
        for j \leftarrow n downto i + 1 do
            if A[j] < A[j-1] then
  5
                interchange A[j] and A[j-1];
                sorted \leftarrow false;
            end
        end
 9
        i \leftarrow i + 1
11 end
```

- 2. For Alg.2 and Alg.3 shown below, answer the following questions respectively.
  - (a) Give the maximum number of times Line 6 is executed in Alg.2 when n is a power of 3.
  - (b) Give the maximum number of times Line 5 is executed in Alg.3 when n is a power of 2.
  - (c) What is the time complexity of both algorithms expressed in the O and  $\Theta$  notations?

## Algorithm 2: COUNT1 1 $count \leftarrow 0$ ; $\mathbf{2} \ \ \mathbf{for} \ i \leftarrow 1 \ to \ n \ \mathbf{do}$ $j \leftarrow \lfloor n/3 \rfloor;$ while $j \geq 1$ do $\textbf{for } k \leftarrow 1 \textit{ to } i \textit{ do}$ $count \leftarrow count + 1;$ 6 if j is even then $j \leftarrow 0;$ 8 else $j \leftarrow \lfloor j/3 \rfloor;$ 10 end 11 end 12 end 13 14 end



3. Fill in the blanks with either true of false:

f(n)	g(n)	f = O(g)	$f = \Omega(g)$	$f = \Theta(g)$
$2n^3 + 3n$	$100n^2 + 2n + 100$			
$50n + \log n$	$10n + \log\log n$			
$50n\log n$	$10n \log \log n$			
$\log n$	$\log^2 n$			
n!	$5^n$			

4. Use the  $\prec$  relation to order the following functions by growth rate:

$$n^{1/100}, \sqrt{n}, \log n^{100}, n \log n, 5, \log \log n, \log^2 n, (\sqrt{n})^n, (1/2)^n, 2^{n^2}, n!$$