Lab02-Algorithm Analysis

Exercises for Algorithms by Nengjun Zhu, 2022-2023 Fall Semester.

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1. Consider the sorting algorithm shown in Alg.1, which is called BUBBLESORT.

   (a) What is the minimum number of element comparisons? When is this minimum achieved?

   (b) What is the maximum number of element comparisons? When is this maximum achieved?

   (c) Express the running time of Alg.1 in terms of the O and Ω notations.

   (d) Can the running time of the algorithm be expressed in terms of the Θ notation? Explain.

Algorithm 1: BUBBLESORT

input : An array $A[1 \cdots n]$ of n elements.
output: $A[1 \cdots n]$ in nondecreasing order.

```
1  i ← 1; sorted ← false;
2  while i ≤ n − 1 and not sorted do
3  |  sorted ← true;
4  |  for j ← n downto i + 1 do
5  |  |  if A[j] < A[j − 1] then
6  |  |  |  interchange A[j] and A[j − 1];
7  |  |  |  sorted ← false;
8  |  |  end
9  |  end
10 |  i ← i + 1
11 end
```

2. For Alg.2 and Alg.3 shown below, answer the following questions respectively.

   (a) Give the maximum number of times Line 6 is executed in Alg.2 when n is a power of 3.

   (b) Give the maximum number of times Line 5 is executed in Alg.3 when n is a power of 2.

   (c) What is the time complexity of both algorithms expressed in the O and Θ notations?

Algorithm 2: COUNT1

```
1  count ← 0;
2  for i ← 1 to n do
3  |  j ← ⌊n/3⌋;
4  |  while j ≥ 1 do
5  |  |  for k ← 1 to i do
6  |  |  |  count ← count + 1;
7  |  |  |  if j is even then
8  |  |  |  |  j ← 0;
9  |  |  |  else
10 |  |  |  |  j ← ⌊j/3⌋;
11 |  |  |  end
12 |  |  end
13 |  end
14 end
```

Algorithm 3: COUNT2

```
1  count ← 0;
2  for i ← 1 to n do
3  |  j ← ⌊n/2⌋;
4  |  while j ≥ 1 do
5  |  |  count ← count + 1;
6  |  |  if j is odd then
7  |  |  |  j ← 0;
8  |  |  else
9  |  |  |  j ← j/2;
10 |  |  end
11 |  end
12 end
```

3. Fill in the blanks with either true of false:

$f(n)$	$g(n)$	$f = O(g)$	$f = \Omega(g)$	$f = \Theta(g)$
$2n^3 + 3n$	$100n^2 + 2n + 100$			
$50n + \log n$	$10n + \log\log n$			
$50n\log n$	$10n\log\log n$			
$\log n$	$\log^2 n$			
$n!$	5^n			

4. Use the $\prec$ relation to order the following functions by growth rate:

$$n^{1/100}, \sqrt{n}, \log n^{100}, n\log n, 5, \log\log n, \log^2 n, (\sqrt{n})^n, (1/2)^n, 2^{n^2}, n!$$