OR Final Project
Group G

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I Introduction : Background and Motivation

As urban areas continue to face challenges of parking scarcity, it is crucial to optimize the utilization of available parking spaces. In this proposal, we want to improve the space utilization of a parking lot and add flexibility to designation by allowing multiple vehicles parking. In each case, 30 ˚ , 45 ˚ , 60 ˚ ,75 ˚ and 90 ˚ parking angles were evaluated to determine the number of stalls for each parking angle.

We introduce a LP solution to optimize the parking grid allocation within a fixed parking space. Our objective function maximizes the number of parking grids in the parking lot and constraints are set according to the size of parking space and the parking grid shape of different vehicle types, which are cars, motorbikes and electric cars. The parking lot designer can choose the weights for each vehicle according to their needs. The proposed methodology can serve as a foundation of improving parking space management, leading to more efficient urban infrastructure and enhancing convenience for parking lot designers.

II Problem Description : The Parking Lot Problem

To solve our research problem, we develop a mathematical model with reference to the paper ”Parking Capacity Optimization Using Linear Programming” by Akmal S. Abdelfatah and MahmoudA. Taha. Our model is a Linear Program and the settings are as follow.

2-1 Parameters and variables

In this section, we will show the parameters and variables we defined in our model. For parameters setting, we are basically defining the shape of parking space and parking grid. Different shape of parking grid brings different Bay’s width and Bay’s depth. Our setting of car parking grid is a rectangle with width 2.5 meters and length 5 meters. For motorbikes, we assign a ratio of 0.4 to shrink the size of car parking grid, which sets are motorbikes parking size to 1 meter by 2 meters. For electric cars, we plus 1 to Bay’s depth for car to provide an additional one-meter space for electric car parking spots. The value and definition setting of C_1, C_3, A_2, D_1 for each angle are based on the paper ”Parking Capacity Optimization Using Linear Programming” by Akmal S. Abdelfatah and Mahmoud A. Taha.

Parameters

B : Width of parking lot.
L : Length of parking lot (Length of exterior rows).
w : Width of the road at top and bottom.
l : Length of interior rows. ($l = L - 2 \times w$)
num_{angles} : number of angles (set to 5)
I : Set of angles we considered (I ={90, 75, 60, 45, 30})
A_2 : Bay’s width for car, parallel to aisle.
A_3 : Bay’s width for motorbike, parallel to aisle. ($A_3 = A_2 \times 0.4$)

C_1 : Bay's depth to wall for car.
C_3 : Bay's depth to interlock for car.
C_4 : Bay's depth to wall for electric car. ($C_4 = C_1 + 1$)
C_5 : Bay's depth to interlock for electric car. ($C_5 = C_3 + 1$)
C_6 : Bay's depth to wall for motorbike. ($C_6 = C_1 \times 0.4$)
C_7 : Bay's depth to interlock for motorbike. ($C_7 = C_3 \times 0.4$)
D_1 : Aisle's width between bay lines for car.
D_2 : Aisle's width between bay lines for motorbike. ($D_2 = D_1 \times 0.4$)

Variables

$i \in I = \{90, 75, 60, 45, 30\}$.
X_i : number of full interior rows for car.
$X_{e,i}$: number of full exterior rows for car.
E_i : number of exterior rows for car.
$X_{Bev,i}$: number of full interior rows for electric car.
$X_{eBev,i}$: number of full exterior rows for electric car.
$E_{Bev,i}$: number of exterior rows for electric car.
$X_{Mot,i}$: number of full interior rows for motorbikes.
$X_{eMot,i}$: number of full exterior rows for motorbikes.
$E_{Mot,i}$: number of exterior rows for motorbikes.
n_i : number of full interior bays for car.
$n_{e,i}$: number of full exterior bays for car.
$n_{EE,i}$: number of exterior bays for car.
$n_{Bev,i}$: number of full interior bays for electric car.
$n_{eBev,i}$: number of full exterior bays for electric car.
$n_{EEBev,i}$: number of exterior bays for electric car.
$n_{Mot,i}$: number of full interior bays for motorbikes.
$n_{eMot,i}$: number of full exterior bays for motorbikes.
$n_{EEMot,i}$: number of exterior bays for motorbikes.

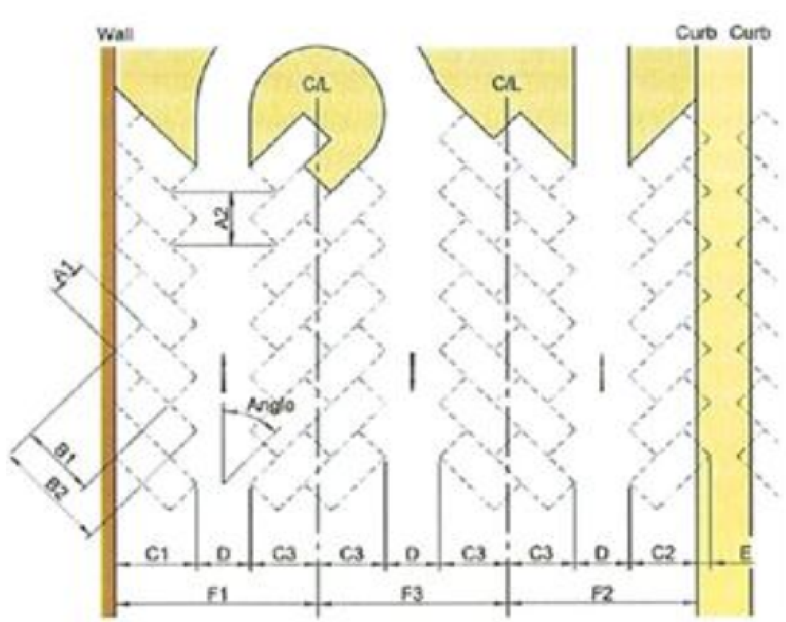

Figure 1: Definition of Parameters shown in Graph

Parameters / Angle	90°	75°	60°	45°	30°
A_2	2.5	2.7	2.8	3.5	5.6
A_3	1.0	1.08	1.12	1.4	2.24
C_1	5.0	5.5	5.6	5.3	4.5
C_3	5.0	5.15	5.05	4.4	3.4
C_4	6.0	6.5	6.6	6.3	5.5
C_5	6.0	6.15	6.05	5.4	4.4
C_6	2.0	2.2	2.24	2.12	1.8
C_7	2.0	2.06	2.02	1.76	1.36
D_1	7.0	6.0	4.5	3.75	3.5
D_2	2.8	2.4	1.8	1.5	1.4

Table 1: Value Setting of Parameters

2-2 Objective function and constraints

In this section we show the formulation of two problem settings. In the first setting, we only have one type of parking grid; while in the second setting we have multiple types of parking grid (car, electric car, motorbike).

Setting 1

$$
\begin{aligned}
\max \quad & \sum_{i \in I} n_i + n_{e,i} + n_{EE,i} \\
\text{s.t.} \quad & \sum_{i \in I} (2 \cdot C_{3,i} + D_{1,i}) \cdot X_i + (C_{1,i} + C_{3,i} + D_{1,i}) \cdot X_{e,i} + (C_{1,i} + D_{1,i}) \cdot E_i \ \leq \ B \ - (1) \\
& \frac{2l}{A_{2,i}} \cdot X_i \ \geq \ n_i \quad \forall\, i \in I \ - (2) \\
& \frac{l}{A_{2,i}} \cdot X_{e,i} + \frac{L}{A_{2,i}} \cdot X_{e,i} \ \geq \ n_{e,i} \quad \forall\, i \in I \ - (3) \\
& \frac{L}{A_{2,i}} \cdot E_i \ \geq \ n_{EE,i} \quad \forall\, i \in I \ - (4) \\
& \sum_{i \in I} X_{e,i} + E_i \ = \ 2 \ - (5)
\end{aligned}
$$

Our objective function tries to maximize the number of parking grids within the given parking space. Constraint 1 means that total width of full interior rows, full exterior rows, and exterior rows should be less than the width of the parking lot. Constraint 2,3,4 means the number of spots in each rows should be less than its upper bound, which is the maximum number of capacity. The upper bound of number of spots in each rows can be calculated by (max length of row / width of a spot) times the number of rows. Constraint 5 shows that the sum of numbers of full exterior and exterior rows will be 2 because there only exist two width edges in parking lot.

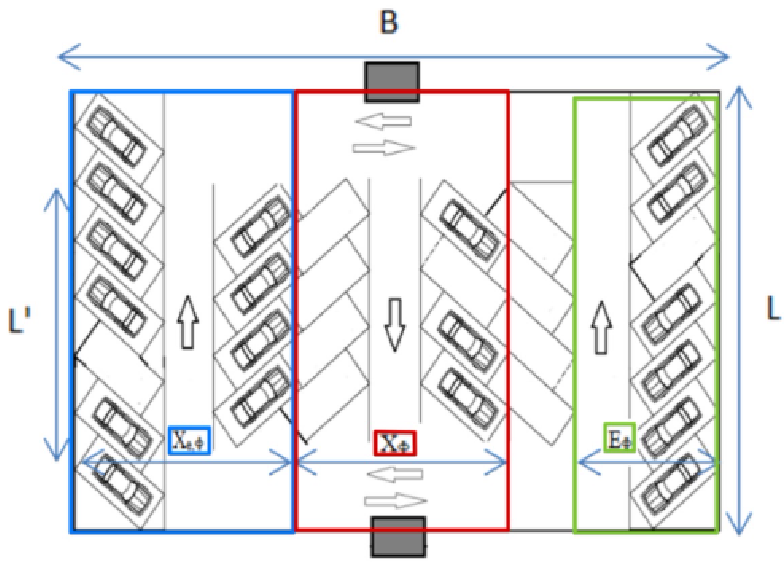

Figure 2: Parking Layout in our formulation

Setting 2

$$
\begin{aligned}
\max \quad & \sum_{i \in I} (n_i) + (n_{Mot,i} + n_{eMot,i} + n_{EEMot,i}) + (n_{Bev,i} + n_{eBev,i} + n_{EEBev,i}) \\
\text{s.t.} \quad & \sum_{i \in I} (2 \cdot C_{3,i} + D_{1,i}) \cdot X_i + (2 \cdot C_{7,i} + D_{2,i}) \cdot X_{Mot,i} + (2 \cdot C_{5,i} + D_{1,i}) \cdot X_{Bev,i} \\
& \quad + (C_{6,i} + C_{7,i} + D_{2,i}) \cdot X_{eMot,i} + (C_{4,i} + C_{5,i} + D_{1,i}) \cdot X_{eBev,i} \\
& \quad + (C_{6,i} + D_{1,i}) \cdot E_{Mot,i} + (C_{4,i} + D_{1,i}) \cdot E_{Bev,i} \ \leq \ B \\
& \frac{2l}{A_{2,i}} \cdot X_i \ \geq \ n_i \quad \forall\, i \in I \\
& \frac{2l}{A_{2,i}} \cdot X_{Bev,i} \ \geq \ n_{Bev,i} \quad \forall\, i \in I \\
& \frac{l}{A_{2,i}} \cdot X_{eBev,i} + \frac{L}{A_{2,i}} \cdot X_{eBev,i} \ \geq \ n_{eBev,i} \quad \forall\, i \in I \\
& \frac{L}{A_{2,i}} \cdot E_{Bev,i} \ \geq \ n_{EEBev,i} \quad \forall\, i \in I \\
& \frac{2l}{A_{3,i}} \cdot X_{Mot,i} \ \geq \ n_{Mot,i} \quad \forall\, i \in I \\
& \frac{l}{A_{3,i}} \cdot X_{eMot,i} + \frac{L}{A_{2,i}} \cdot X_{eMot,i} \ \geq \ n_{eMot,i} \quad \forall\, i \in I \\
& \frac{L}{A_{3,i}} \cdot E_{Mot,i} \ \geq \ n_{EEMot,i} \quad \forall\, i \in I \\
& \sum_{i \in I} X_{eMot,i} + E_{Mot,i} \ = \ 1 \\
& \sum_{i \in I} X_{eBev,i} + E_{Bev,i} \ = \ 1
\end{aligned}
$$

Our new formulation is similar to the first setting. Our objective function still maximize the number of parking grids within the given parking space and the meaning of constraints are the same as setting 1. The only difference is we add more types of parking grid, which provides more flexibility in real word problem but our framework and concept remains the same.

The last two constraints are set because we want at least one row of motorbike and one row of electric cars in the parking lot. Also, when arranging the spaces we want the parking area of the motorbike and electric cars can be arranged to two sides, which is more likely to be the arrangement in real world case.

III Method and Analysis

We solve our LP using "Gurobi" optimizer and test our model on different type of instances. In this section we will analyze the changes of parking layout as the length and width of parking space varies and perform a weight setting discussion to observe the variation of the parking grid number for general vehicles, electric vehicles, and motorcycles under different weight scenarios.

Parking Space Discussion

In the following discussion, we modify the width and length of the parking space, which are B and L in our LP formulation. We divide our instances into 3 groups. Group1 has size of 90*160; Group2 has size of 120*120; Group3 has size of 160*90.

*Discussion 1 : Road Width and Spot Size

We analyze the changes of parking grid number of each group as road width varies and the changes of of parking grid number of each road width as group varies. From figure 3 and 4, we know that our LP formulation gives different solution when area of parking lot is the same. Group1 outperforms Group3 in every road width. This provides insight to our model user to assign the length of longer edge to value B so that they can acquire a better result of space allocation. More space can be utilize when they set small road width.

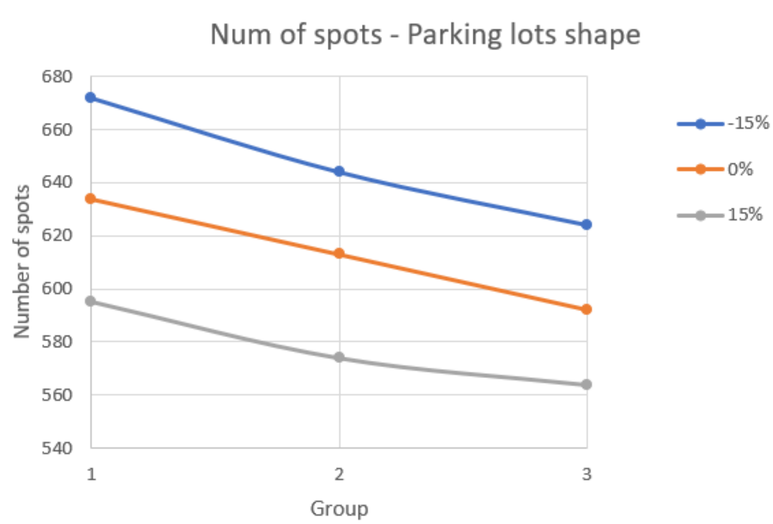

Figure 3: Different Groups

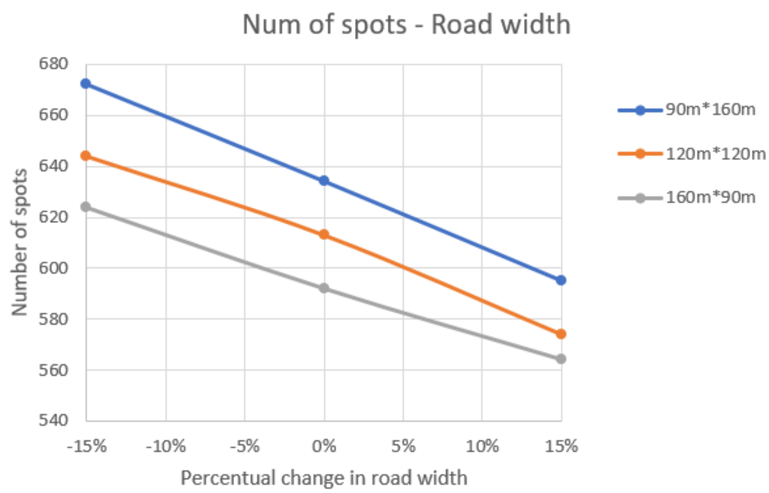

Figure 4: Different Road Width

***Discussion 2 : Angle Selection**

We analyze the parking angle of parking spot in different space settings. From table 2, our model select parking rows of 90° and 60° the most in different case of instances. This is because the larger the degree the more spots we can assign to a single row. Most of the stalls are with 60-degree parking angle since it has a reasonable traffic lane width.

In case of having different angles (i.e. 60 and 90 degrees) within the same parking facility, engineering judgment and experience will be used to arrange the rows. It's recommended to arrange the rows with the same parking angle next to each other to avoid confusion for parking users.

Settings / Angle	**90°**	**75°**	**60°**	**45°**	**30°**
G_1/-15%	3	0	3	0	0
G_1/0%	0	0	6	0	0
G_1/15%	1	0	5	0	0
G_2/-15%	4	0	4	0	0
G_2/0%	1	0	7	0	0
G_2/15%	4	1	2	0	0
G_3/-15%	3	0	8	0	0
G_3/0%	0	0	10	1	0
G_3/15%	2	0	8	0	0

Table 2: Value Setting of Parameters

Weight Discussion

In the following discussion, we modify the objective function to obtain variations in the number of parking spaces for different vehicles under different weight scenarios. Constraints are same as the Setting 2 in the previous section. We defined three new variables, which are w_c, w_{Mot}, w_{BEV} denoting the weight of cars, motorcycles and electric vehicles and slightly change our objective function as follows.

$$\max \quad \sum_{i \in I} w_c(n_i) + w_{Mot}(n_{Mot,i} + n_{eMot,i} + n_{EEMot,i}) + w_{BEV}(n_{Bev,i} + n_{eBev,i} + n_{EEBev,i})$$

***Discussion 1 : Motorbikes'weight**

We set w_c and w_{BEV} to 1 and search w_{Mot} in range from $[0, 1]$ with step of 0.04. From Figure 5, the proportion of motorbikes exhibits a sharp increase when the motorbikes'weight is set to 0.16, which aligns with our expectations. This is because the ratio of motorbike area to cars area is exactly 1 : 0.16. Battery electric vehicles consistently show lower quantities, as battery electric vehicles occupy more depth than cars by one meter.

***Discussion 2 : Electric Vehicles'weight**

We set w_c to 1 and w_{Mot} to 0.16 and search w_{BEV} in range from $[1, 1.5]$ with step of 0.025. From Figure 6, the proportion of electric vehicles exhibits a significant jump at 1.175 and 1.375. When the weight for electric vehicles is set to 1, both cars and battery electric vehicles exhibit a very low quantity. However, the number of battery electric vehicles is not zero because we guarantee at least one row of parking spaces for battery electric vehicles and motorbikes. Cars consistently show a low quantity throughout.

***Discussion 3 : Cars'weight**

We set w_{BEV} to 1.2 and w_{Mot} to 0.16 and search w_c in range from $[1, 1.25]$ with step of 0.025. From Figure 7, the proportion of cars exhibits a significant jump when the weight for cars is set to 1.05. It can also be observed that when the weight for cars is set to 1.05, battery electric vehicles to 1.2, and motorbikes to 0.16, all three categories can achieve a relatively balanced number of parking spaces compared to the initial setup.

***Discussion Result**

Under the assumption of parking lot shape is a square of length equal to 120, width of the top and bottom road equals to 7. We find the optimal weight $(w_c, w_{Mot}, w_{BEV}) = (1.05, 0.16, 1.2)$, under this weight the number of three types of parking spaces are closest to uniform distribution. Therefore, parking lot owners can make adjustments around these weights based on local preferences for different vehicle types and government policies, aiming to achieve the most efficient and convenient allocation of parking spaces. Moreover, our discussion can support our future works in analyzing the optimal ratio of different vehicles based on local preferences and government policies, which enhance the generality of our model.

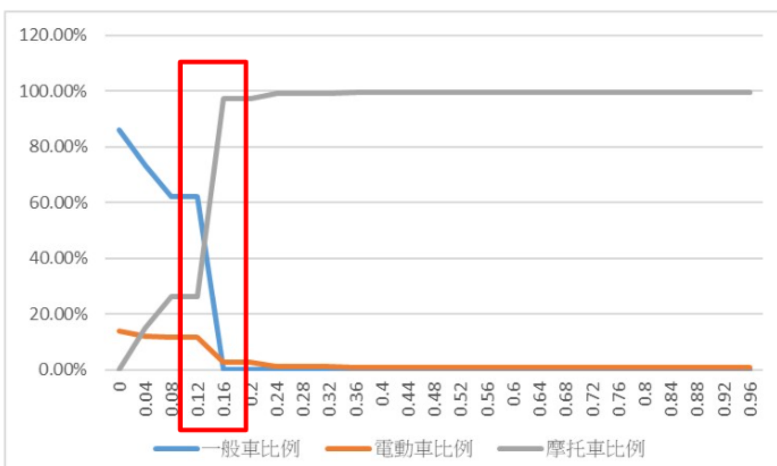

Figure 5: Discussion 1 : Vehicles weight

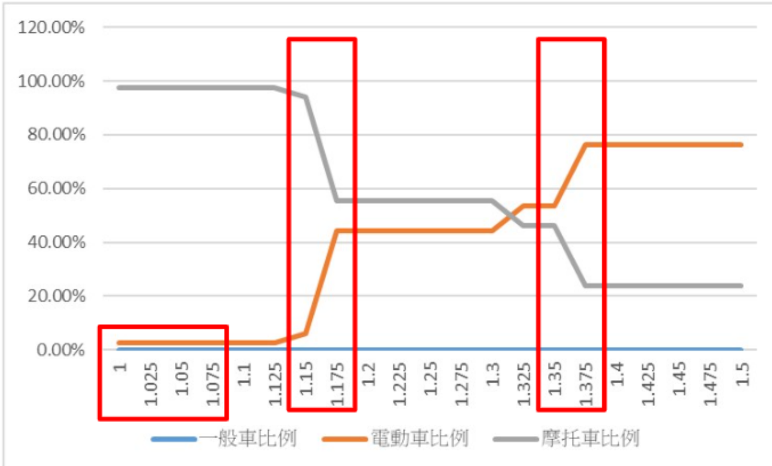

Figure 6: Discussion 2 : Vehicles weight

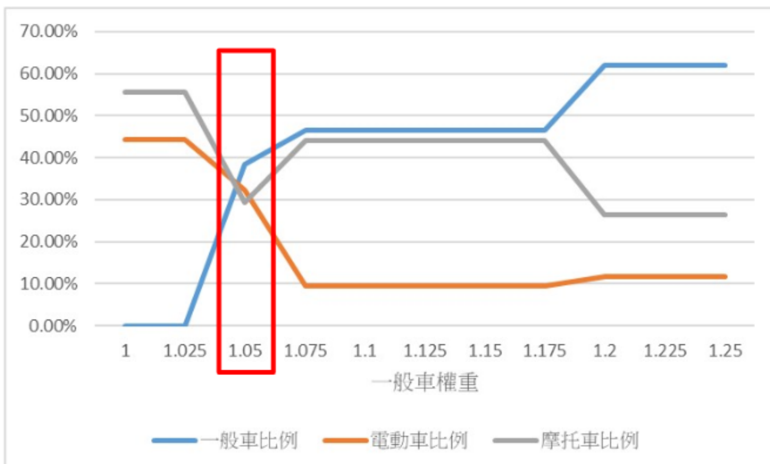

Figure 7: Discussion 3 : Vehicles weight

IV Conclusion and Future Works

In this proposal, Integer Linear Programming has been used to find the optimal parking layout for a parking lot with a given dimensions. We introduce different types of parking grids in our formulation, so that real world user can easily design their parking space according to their needs. Moreover, we discuss the changes of parking layout under different parking space size and weights of vehicles.
Future works are as follows :

- Route Planning in Parking Lot: Incorporate road's width into the optimization process.
- Survey the proportions of electric vehicles, gasoline vehicles, and motorcycles in the vicinity of the parking lot and arrange the parking lot in the most efficient manner based on the optimal ratio.
- The formulation can be modified to suit irregular shapes, which may require a non-linear formulation to be considered for such cases.

V References

1. "Parking Capacity Optimization Using Linear Programming" by Akmal S. Abdelfatah and MahmoudA. Taha.

2. "Optimisation of Parking Layout-A Mixed Integer Linear Programming Formulation for Maximum Number of Parking Spots, Applicable for Evaluation of Autonomous Parking Benefits" by MALIN KARLSSON and RICHARD PETERSSON