# Centroïd Center of Gravity Center of Mass Barycenter)

Hervé Abdi<sup>1</sup>

### 1 overview

The notion of centroïd generalizes the notion of a mean to multivariate analysis and multidimensional spaces. It applies to vectors instead of scalars, and it is computed by associating to each vector a mass which is a positive number taking values between 0 and 1 and such that the sum of all the masses is equal to 1. The centroïd of a set of vectors is also called the *center of gravity*, the *center of mass*, or the *barycenter* of this set.

## 2 Notations and definition

Let  $\mathcal{V}$  be a set of I vectors with each vector being composed of J elements

$$\mathcal{V} = \{\mathbf{v}_1, \dots, \mathbf{v}_i, \dots, \mathbf{v}_I\} \text{ with } \mathbf{v}_i = \left[v_{i,1}, \dots, v_{i,j}, \dots, v_{i,J}\right]^\mathsf{T}.$$
 (1)

<sup>1</sup>In: Neil Salkind (Ed.) (2007). Encyclopedia of Measurement and Statistics. Thousand Oaks (CA): Sage.

Address correspondence to: Hervé Abdi

Program in Cognition and Neurosciences, MS: Gr.4.1,

The University of Texas at Dallas,

Richardson, TX 75083-0688, USA

E-mail: herve@utdallas.edu http://www.utd.edu/~herve

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To each vector is associated a mass denoted  $m_i$  for vector i. These masses take values between 0 and 1 and the sum of these masses is equal to 1. The set of masses is a vector denoted  $\mathbf{m}$ . The centroïd of the set of vectors is denoted  $\mathbf{c}$ , it is defined as

$$\mathbf{c} = \sum_{i}^{I} m_{i} \mathbf{v}_{i} . \tag{2}$$

# 3 Examples

The mean of a set of numbers is the centroïd of this set of observations. Here, the mass of each number is equal to the inverse of the number of observations:  $m_i = \frac{1}{7}$ .

For multivariate data, the notion of centroïd generalizes the mean. For example, with the following three vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 10\\20 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 8\\24 \end{bmatrix}, \text{ and } \mathbf{v}_3 = \begin{bmatrix} 16\\32 \end{bmatrix},$$
 (3)

and the following set of masses:

$$m_1 = \frac{1}{2}$$
,  $m_2 = \frac{1}{8}$ , and  $m_3 = \frac{3}{8}$ , (4)

we obtain the following centroïd

$$\mathbf{c} = \sum_{i}^{I} m_{i} \mathbf{v}_{i} = \frac{1}{2} \begin{bmatrix} 10\\20 \end{bmatrix} + \frac{1}{8} \begin{bmatrix} 8\\24 \end{bmatrix} + \frac{3}{8} \begin{bmatrix} 16\\32 \end{bmatrix} = \begin{bmatrix} 12\\25 \end{bmatrix} . \tag{5}$$

In this example, if we plot the vectors in a two-dimensional space, the centroïd would be the center of gravity of the triangle made by these three vectors with masses assigned proportionally to their vector of mass. The notion of centroïd can be used with spaces of any dimensionality.

# 4 Properties of the centroïd

The properties of the centroïd of a set of vectors closely parallel the more familiar properties of the mean of a set of numbers. Recall

that a set of vectors defines a multidimensional space, and that to each multidimensional space is assigned a generalized Euclidean distance (see entry on distance). The core property of the centroïd is that: the centroïd of a set of vectors minimizes the weighted sum of the generalized squared Euclidean distances from the vectors to any point in the space. This quantity which generalizes the notion of variance, is called the *inertia* of the set of vectors relative to their centroïd.

Of additional interest for multivariate analysis, the *theorem of Huyghens* indicates that the weighted sum of the squared distances from a set of vectors to any vector in the space can be decomposed as a weighted sum of the squared distances from the vectors to their centroïd plus the (weighted) squared distance from the centroïd to this point. In term of inertia, Huyghens's theorem states that the inertia of a set of vectors to any point is equal to the inertia of the set of vectors to their centroïd plus the inertia of their centroïd to this point. As an obvious consequence of this theorem, the inertia of a set of vectors to their centroïd is minimal.