

by the formula  $\binom{n}{x} p^x (1-p)^{n-x}$ . For the problem at hand, it is given that each child is equally likely to be a boy or a girl, and so  $p = \frac{1}{2}$ . Thus, the probability of having exactly 2 girls born to a couple with 4 children is

$$\binom{4}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = \frac{4!}{2!2!} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 =$$

$$(6) \left(\frac{1}{4}\right) \left(\frac{1}{4}\right) = \frac{6}{16} = \frac{3}{8}.$$

**The correct answer is A.**

PS01564

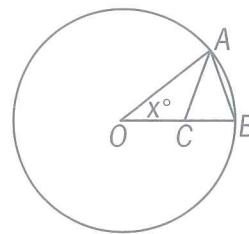
168. The closing price of Stock X changed on each trading day last month. The percent change in the closing price of Stock X from the first trading day last month to each of the other trading days last month was less than 50 percent. If the closing price on the second trading day last month was \$10.00, which of the following CANNOT be the closing price on the last trading day last month?
- (A) \$3.00  
 (B) \$9.00  
 (C) \$19.00  
 (D) \$24.00  
 (E) \$29.00

#### Arithmetic Applied problems; Percents

Let  $P$  be the first-day closing price, in dollars, of the stock. It is given that the second-day closing price was  $(1 + n\%)P = 10$ , so  $P = \frac{10}{1 + n\%}$ , for some value of  $n$  such that  $-50 < n < 50$ . Therefore,  $P$  is between  $\frac{10}{1 + 0.50} \approx 6.67$  and  $\frac{10}{1 - 0.50} = 20$ .

Hence, if  $Q$  is the closing price, in dollars, of the stock on the last day, then  $Q$  is between  $(0.50)(6.67) \approx 3.34$  (50% decrease from the lowest possible first-day closing price) and  $(1.50)(20) = 30$  (50% increase from the greatest possible first-day closing price). The only answer choice that gives a number of dollars not between 3.34 and 30 is the first answer choice.

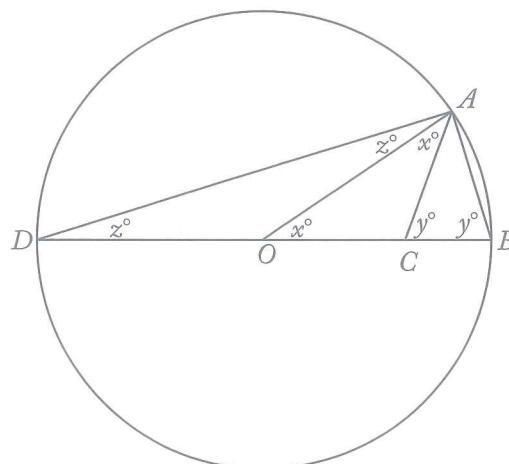
**The correct answer is A.**



PS02389

169. In the figure above, point  $O$  is the center of the circle and  $OC = AC = AB$ . What is the value of  $x$ ?
- (A) 40  
 (B) 36  
 (C) 34  
 (D) 32  
 (E) 30

#### Geometry Angles



Consider the figure above, where  $\overline{DB}$  is a diameter of the circle with center  $O$  and  $\overline{AD}$  is a chord. Since  $OC = AC$ ,  $\triangle OCA$  is isosceles and so the base angles,  $\angle AOC$  and  $\angle OAC$ , have the same degree measure. The measure of  $\angle AOC$  is given as  $x^\circ$ , so the measure of  $\angle OAC$  is  $x^\circ$ . Since  $AC = AB$ ,  $\triangle CAB$  is isosceles and so the base angles,  $\angle ACB$  and  $\angle ABC$ , have the same degree measure. The measure of each is marked as  $y^\circ$ . Likewise, since  $OD = OA$  and  $OA = OB$ ,  $\triangle DAO$  is isosceles with base angles,  $\angle DAO$  and  $\angle DAO$ , each measuring  $z^\circ$ . Each of the following statements is true:

- (i) The measure of  $\angle CAB$  is  $180 - 2y$  since the sum of the measures of the angles of  $\triangle CAB$  is 180.
- (ii)  $\angle DAB$  is a right angle (because  $\overline{DB}$  is a diameter of the circle) and so  $z + x + (180 - 2y) = 90$ , or, equivalently,  $2y - x - z = 90$ .
- (iii)  $z + 90 + y = 180$  since the sum of the measures of the angles of right triangle  $\triangle DAB$  is 180, or, equivalently,  $z = 90 - y$ .
- (iv)  $x = 2z$  because the measure of exterior angle  $\angle AOC$  to  $\triangle AOD$  is the sum of the measures of the two opposite interior angles,  $\angle ODA$  and  $\angle OAD$ .
- (v)  $y = 2x$  because the measure of exterior angle  $\angle ACB$  to  $\triangle OCA$  is the sum of the measures of the two opposite interior angles,  $\angle COA$  and  $\angle CAO$ .

Multiplying the final equation in (iii) by 2 gives  $2z = 180 - 2y$ . But,  $x = 2z$  in (iv), so  $x = 180 - 2y$ . Finally, the sum of the measures of the angles of  $\triangle CAB$  is 180 and so  $y + y + x = 180$ . Then from (v),  $2x + 2x + x = 180$ ,  $5x = 180$ , and  $x = 36$ .

**The correct answer is B.**

PS16967

170. An airline passenger is planning a trip that involves three connecting flights that leave from Airports A, B, and C, respectively. The first flight leaves Airport A every hour, beginning at 8:00 a.m., and arrives at Airport B  $2\frac{1}{2}$  hours later. The second flight leaves Airport B every 20 minutes, beginning at 8:00 a.m., and arrives at Airport C  $1\frac{1}{6}$  hours later. The third flight leaves Airport C every hour, beginning at 8:45 a.m. What is the least total amount of time the passenger must spend between flights if all flights keep to their schedules?

- (A) 25 min
- (B) 1 hr 5 min
- (C) 1 hr 15 min
- (D) 2 hr 20 min
- (E) 3 hr 40 min

### Arithmetic Operations on rational numbers

Since the flight schedules at each of Airports A, B, and C are the same hour after hour, assume that the passenger leaves Airport A at 8:00 and arrives at Airport B at 10:30. Since flights from Airport B leave at 20-minute intervals beginning on the hour, the passenger must wait 10 minutes at Airport B for the flight that leaves at 10:40 and arrives at Airport C  $1\frac{1}{6}$  hours or 1 hour  $\frac{6}{60}$  minutes later. Thus, the passenger arrives at Airport C at 11:50. Having arrived too late for the 11:45 flight from Airport C, the passenger must wait 55 minutes for the 12:45 flight. Thus, the least total amount of time the passenger must spend waiting between flights is  $10 + 55 = 65$  minutes, or 1 hour 5 minutes.

**The correct answer is B.**

PS07426

171. If  $n$  is a positive integer and  $n^2$  is divisible by 72, then the largest positive integer that must divide  $n$  is

- (A) 6
- (B) 12
- (C) 24
- (D) 36
- (E) 48

### Arithmetic Properties of numbers

Since  $n^2$  is divisible by 72,  $n^2 = 72k$  for some positive integer  $k$ . Since  $n^2 = 72k$ , then  $72k$  must be a perfect square. Since  $72k = (2^3)(3^2)k$ , then  $k = 2m^2$  for some positive integer  $m$  in order for  $72k$  to be a perfect square. Then,  $n^2 = 72k = (2^3)(3^2)(2m^2) = (2^4)(3^2)m^2 = [(2^2)(3)(m)]^2$ , and  $n = (2^2)(3)(m)$ . The positive integers that MUST divide  $n$  are 1, 2, 3, 4, 6, and 12. Therefore, the largest positive integer that must divide  $n$  is 12.

**The correct answer is B.**

PS16977

172. A certain grocery purchased  $x$  pounds of produce for  $p$  dollars per pound. If  $y$  pounds of the produce had to be discarded due to spoilage and the grocery sold the rest for  $s$  dollars per pound, which of the following represents the gross profit on the sale of the produce?

- (A)  $(x - y)s - xp$   
 (B)  $(x - y)p - ys$   
 (C)  $(s - p)y - xp$   
 (D)  $xp - ys$   
 (E)  $(x - y)(s - p)$

### Algebra Simplifying algebraic expressions; Applied problems

Since the grocery bought  $x$  pounds of produce for  $p$  dollars per pound, the total cost of the produce was  $xp$  dollars. Since  $y$  pounds of the produce was discarded, the grocery sold  $x - y$  pounds of produce at the price of  $s$  dollars per pound, yielding a total revenue of  $(x - y)s$  dollars. Then, the grocery's gross profit on the sale of the produce is its total revenue minus its total cost or  $(x - y)s - xp$  dollars.

The correct answer is A.

PS16990

173. If  $x$ ,  $y$ , and  $z$  are positive integers such that  $x$  is a factor of  $y$ , and  $x$  is a multiple of  $z$ , which of the following is NOT necessarily an integer?

- (A)  $\frac{x+z}{z}$   
 (B)  $\frac{y+z}{x}$   
 (C)  $\frac{x+y}{z}$   
 (D)  $\frac{xy}{z}$   
 (E)  $\frac{yz}{x}$

### Arithmetic Properties of numbers

Since the positive integer  $x$  is a factor of  $y$ , then  $y = kx$  for some positive integer  $k$ . Since  $x$  is a multiple of the positive integer  $z$ , then  $x = mz$  for some positive integer  $m$ .

Substitute these expressions for  $x$  and/or  $y$  into each answer choice to find the one expression that is NOT necessarily an integer.

A  $\frac{x+z}{z} = \frac{mz+z}{z} = \frac{(m+1)z}{z} = m+1$ , which

MUST be an integer

B  $\frac{y+z}{x} = \frac{y}{x} + \frac{z}{x} = \frac{kx}{x} + \frac{z}{mz} = k + \frac{1}{m}$ , which

NEED NOT be an integer

Because only one of the five expressions need not be an integer, the expressions given in C, D, and E need not be tested. However, for completeness,

C  $\frac{x+y}{z} = \frac{mz+kx}{z} = \frac{mz+k(mz)}{z} = \frac{mz(1+k)}{z} = m(1+k)$ , which MUST be an integer

D  $\frac{xy}{z} = \frac{(mz)y}{z} = my$ , which MUST be an integer

E  $\frac{yz}{x} = \frac{(kx)(z)}{x} = kz$ , which MUST be an integer

The correct answer is B.

PS08416

174. Running at their respective constant rates, Machine X takes 2 days longer to produce  $w$  widgets than Machine Y. At these rates, if the two machines together produce  $\frac{5}{4}w$  widgets in 3 days, how many days would it take Machine X alone to produce  $2w$  widgets?

- (A) 4  
 (B) 6  
 (C) 8  
 (D) 10  
 (E) 12

**Algebra Applied problems**

If  $x$ , where  $x > 2$ , represents the number of days Machine X takes to produce  $w$  widgets, then Machine Y takes  $x - 2$  days to produce  $w$  widgets. It follows that Machines X and Y can produce  $\frac{w}{x}$  and  $\frac{w}{x-2}$  widgets, respectively, in 1 day and together they can produce  $\frac{w}{x} + \frac{w}{x-2}$  widgets in 1 day. Since it is given that, together, they can produce  $\frac{5}{4}w$  widgets in 3 days, it follows that, together, they can produce  $\frac{1}{3}\left(\frac{5}{4}w\right) = \frac{5}{12}w$  widgets in 1 day. Thus,

$$\frac{w}{x} + \frac{w}{x-2} = \frac{5}{12}w$$

$$\left(\frac{1}{x} + \frac{1}{x-2}\right)w = \frac{5}{12}w$$

$$\left(\frac{1}{x} + \frac{1}{x-2}\right) = \frac{5}{12}$$

$$12x(x-2)\left(\frac{1}{x} + \frac{1}{x-2}\right) = 12x(x-2)\left(\frac{5}{12}\right)$$

$$12[(x-2) + x] = 5x(x-2)$$

$$12(2x-2) = 5x(x-2)$$

$$24x - 24 = 5x^2 - 10x$$

$$0 = 5x^2 - 34x + 24$$

$$0 = (5x-4)(x-6)$$

$$x = \frac{4}{5} \text{ or } 6$$

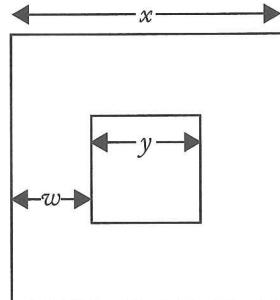
Therefore, since  $x > 2$ , it follows that  $x = 6$ . Machine X takes 6 days to produce  $w$  widgets and  $2(6) = 12$  days to produce  $2w$  widgets.

**The correct answer is E.**

PS07117

175. A square wooden plaque has a square brass inlay in the center, leaving a wooden strip of uniform width around the brass square. If the ratio of the brass area to the wooden area is 25 to 39, which of the following could be the width, in inches, of the wooden strip?

- I. 1
  - II. 3
  - III. 4
- (A) I only  
 (B) II only  
 (C) I and II only  
 (D) I and III only  
 (E) I, II, and III

**Geometry Area**

Note: Not drawn to scale.

Let  $x$  represent the side length of the entire plaque, let  $y$  represent the side length of the brass inlay, and  $w$  represent the uniform width of the wooden strip around the brass inlay, as shown in the figure above. Since the ratio of the area of the brass inlay to the area of the wooden strip is 25 to 39, the ratio of the area of the brass inlay to the area of the entire plaque is  $\frac{y^2}{x^2} = \frac{25}{25+39} = \frac{25}{64}$ .

Then,  $\frac{y}{x} = \sqrt{\frac{25}{64}} = \frac{5}{8}$  and  $y = \frac{5}{8}x$ . Also,  $x = y + 2w$  and  $w = \frac{x-y}{2}$ . Substituting  $\frac{5}{8}x$  for  $y$  into this

expression for  $w$  gives  $w = \frac{x - \frac{5}{8}x}{2} = \frac{\frac{3}{8}x}{2} = \frac{3}{16}x$ . Thus,

- I. If the plaque were  $\frac{16}{3}$  inches on a side, then the width of the wooden strip would be 1 inch, and so 1 inch is a possible width for the wooden strip.
- II. If the plaque were 16 inches on a side, then the width of the wooden strip would be 3 inches, and so 3 inches is a possible width for the wooden strip.
- III. If the plaque were  $\frac{64}{3}$  inches on a side, then the width of the wooden strip would be 4 inches, and so 4 inches is a possible width for the wooden strip.

The correct answer is E.

PS16963

$$176. \frac{2\frac{3}{5} - 1\frac{2}{3}}{\frac{2}{3} - \frac{3}{5}} =$$

- (A) 16  
 (B) 14  
 (C) 3  
 (D) 1  
 (E) -1

#### Arithmetic Operations on rational numbers

Work the problem:

$$\begin{aligned} & \frac{2\frac{3}{5} - 1\frac{2}{3}}{\frac{2}{3} - \frac{3}{5}} = \\ & \frac{\frac{13}{5} - \frac{5}{3}}{\frac{2}{3} - \frac{3}{5}} = \frac{\frac{39}{15} - \frac{25}{15}}{\frac{10}{15} - \frac{9}{15}} = \frac{\frac{14}{15}}{\frac{1}{15}} = \frac{14}{15} \times \frac{15}{1} = 14 \end{aligned}$$

The correct answer is B.

## **5.0** Data Sufficiency

## 5.0 Data Sufficiency

Data sufficiency questions appear in the Quantitative section of the GMAT® exam. Multiple-choice data sufficiency questions are intermingled with problem solving questions throughout the section. You will have 62 minutes to complete the Quantitative section of the GMAT exam, or about 2 minutes to answer each question. These questions require knowledge of the following topics:

- Arithmetic
- Elementary algebra
- Commonly known concepts of geometry

Data sufficiency questions are designed to measure your ability to analyze a quantitative problem, recognize which given information is relevant, and determine at what point there is sufficient information to solve a problem. In these questions, you are to classify each problem according to the five fixed answer choices, rather than find a solution to the problem.

Each data sufficiency question consists of a question, often accompanied by some initial information, and two statements, labeled (1) and (2), which contain additional information. You must decide whether the information in each statement is sufficient to answer the question or—if neither statement provides enough information—whether the information in the two statements together is sufficient. It is also possible that the statements, in combination do not give enough information to answer the question.

Begin by reading the initial information and the question carefully. Next, consider the first statement. Does the information provided by the first statement enable you to answer the question? Go on to the second statement. Try to ignore the information given in the first statement when you consider whether the second statement provides information that, by itself, allows you to answer the question. Now you should be able to say, for each statement, whether it is sufficient to determine the answer.

Next, consider the two statements in tandem. Do they, together, enable you to answer the question?

Look again at your answer choices. Select the one that most accurately reflects whether the statements provide the information required to answer the question.

## 5.1 Test-Taking Strategies

### 1. Do not waste valuable time solving a problem.

You only need to determine whether sufficient information is given to solve it.

### 2. Consider each statement separately.

First, decide whether each statement alone gives sufficient information to solve the problem. Be sure to disregard the information given in statement (1) when you evaluate the information given in statement (2). If either, or both, of the statements give(s) sufficient information to solve the problem, select the answer corresponding to the description of which statement(s) give(s) sufficient information to solve the problem.

### 3. Judge the statements in tandem if neither statement is sufficient by itself.

It is possible that the two statements together do not provide sufficient information. Once you decide, select the answer corresponding to the description of whether the statements together give sufficient information to solve the problem.

### 4. Answer the question asked.

For example, if the question asks, “What is the value of  $y$ ?” for an answer statement to be sufficient, you must be able to find one and only one value for  $y$ . Being able to determine minimum or maximum values for an answer (e.g.,  $y = x + 2$ ) is not sufficient, because such answers constitute a range of values rather than the specific value of  $y$ .

### 5. Be very careful not to make unwarranted assumptions based on the images represented.

Figures are not necessarily drawn to scale; they are generalized figures showing little more than intersecting line segments and the relationships of points, angles, and regions. For example, if a figure described as a rectangle looks like a square, do not conclude that it is actually a square just by looking at the figure.

If statement 1 is sufficient, then the answer must be **A or D**.

If statement 2 is not sufficient, then the answer must be **A**.

If statement 2 is sufficient, then the answer must be **D**.

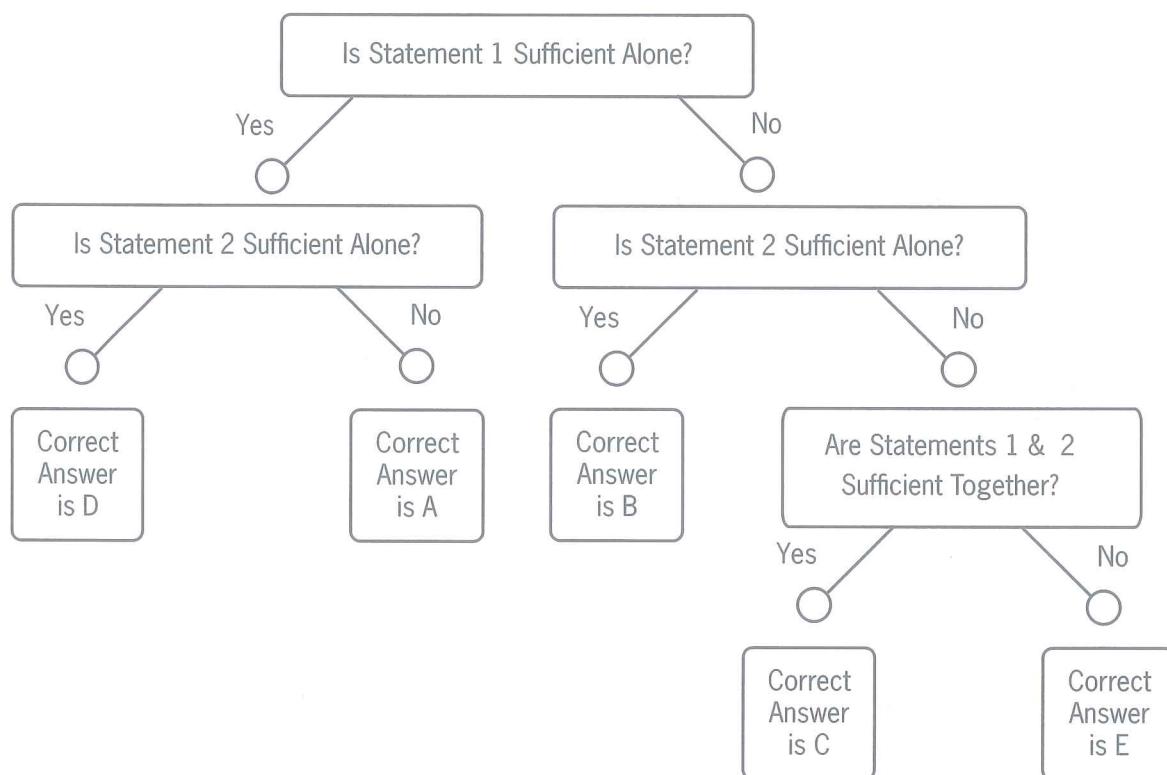
If statement 1 is not sufficient, then the answer must be **B, C, or E**.

If statement 2 is sufficient, then the answer must be **B**.

If statement 2 is not sufficient, then the answer must be **C or E**.

If both statements together are sufficient, then the answer must be **C**.

If both statements together are still not sufficient, then the answer must be **E**.



## 5.2 The Directions

These directions are similar to those you will see for data sufficiency questions when you take the GMAT exam. If you read the directions carefully and understand them clearly before going to sit for the test, you will not need to spend much time reviewing them when you take the GMAT exam.

Each data sufficiency problem consists of a question and two statements, labeled (1) and (2), that give data. You have to decide whether the data given in the statements are *sufficient* for answering the question. Using the data given in the statements *plus* your knowledge of mathematics and everyday facts (such as the number of days in July or the meaning of *clockwise*), you must indicate whether the data given in the statements are sufficient for answering the questions and then indicate one of the following answer choices:

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked;
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked;
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient;
- (D) EACH statement ALONE is sufficient to answer the question asked;
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

**NOTE:** In data sufficiency problems that ask for the value of a quantity, the data given in the statements are sufficient only when it is possible to determine exactly one numerical value for the quantity.

**Numbers:** All numbers used are real numbers.

**Figures:** A figure accompanying a data sufficiency problem will conform to the information given in the question but will not necessarily conform to the additional information given in statements (1) and (2).

Lines shown as straight can be assumed to be straight and lines that appear jagged can also be assumed to be straight.

You may assume that the positions of points, angles, regions, and so forth exist in the order shown and that angle measures are greater than zero degrees.

All figures lie in a plane unless otherwise indicated.

To register for the GMAT exam go to [www.mba.com](http://www.mba.com)

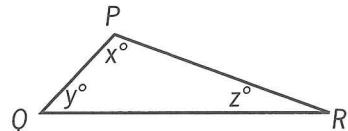
## 5.3 Sample Questions

Each **data sufficiency** problem consists of a question and two statements, labeled (1) and (2), which contain certain data. Using these data and your knowledge of mathematics and everyday facts (such as the number of days in July or the meaning of the word *counterclockwise*), decide whether the data given are sufficient for answering the question and then indicate one of the following answer choices:

- A** Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B** Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C** BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D** EACH statement ALONE is sufficient.
- E** Statements (1) and (2) TOGETHER are not sufficient.

**Note:** In data sufficiency problems that ask for the value of a quantity, the data given in the statements are sufficient only when it is possible to determine exactly one numerical value for the quantity.

**Example:**



In  $\triangle PQR$ , what is the value of  $x$ ?

- (1)  $PQ = PR$
- (2)  $y = 40$

**Explanation:** According to statement (1)  $PQ = PR$ ; therefore,  $\triangle PQR$  is isosceles and  $y = z$ . Since  $x + y + z = 180$ , it follows that  $x + 2y = 180$ . Since statement (1) does not give a value for  $y$ , you cannot answer the question using statement (1) alone. According to statement (2),  $y = 40$ ; therefore,  $x + z = 140$ . Since statement (2) does not give a value for  $z$ , you cannot answer the question using statement (2) alone. Using both statements together, since  $x + 2y = 180$  and the value of  $y$  is given, you can find the value of  $x$ . Therefore, BOTH statements (1) and (2) TOGETHER are sufficient to answer the questions, but NEITHER statement ALONE is sufficient.

**Numbers:** All numbers used are real numbers.

**Figures:**

- Figures conform to the information given in the question, but will not necessarily conform to the additional information given in statements (1) and (2).
- Lines shown as straight are straight, and lines that appear jagged are also straight.
- The positions of points, angles, regions, etc., exist in the order shown, and angle measures are greater than zero.
- All figures lie in a plane unless otherwise indicated.

\*DS05149

177. Does  $2x + 8 = 12$ ?

- (1)  $2x + 10 = 14$   
 (2)  $3x + 8 = 14$

DS01503

178. If  $M$  is a set of consecutive even integers, is 0 in set  $M$ ?

- (1) -6 is in set  $M$ .  
 (2) -2 is in set  $M$ .

DS15510

179. Rita's monthly salary is  $\frac{2}{3}$  Juanita's monthly salary.

What is their combined monthly salary?

- (1) Rita's monthly salary is \$4,000.  
 (2) Either Rita's monthly salary or Juanita's monthly salary is \$6,000.

DS13384

180. What is the value of the integer  $x$ ?

- (1)  $x$  rounded to the nearest hundred is 7,200.  
 (2) The hundreds digit of  $x$  is 2.

DS04644

181. Is  $2x > 2y$ ?

- (1)  $x > y$   
 (2)  $3x > 3y$

DS04636

182. If  $p$  and  $q$  are positive, is  $\frac{p}{q}$  less than 1?

- (1)  $p$  is less than 4.  
 (2)  $q$  is less than 4.

DS02779

183. In each quarter of 1998, Company M earned more money than in the previous quarter. What was the range of Company M's quarterly earnings in 1998?

- (1) In the 2nd and 3rd quarters of 1998, Company M earned \$4.0 million and \$4.6 million, respectively.  
 (2) In the 1st and 4th quarters of 1998, Company M earned \$3.8 million and \$4.9 million, respectively.

DS04510

184. In a certain factory, hours worked by each employee in excess of 40 hours per week are overtime hours and are paid for at  $1\frac{1}{2}$  times the employee's regular hourly pay rate. If an employee worked a total of 42 hours last week, how much was the employee's gross pay for the hours worked last week?

- (1) The employee's gross pay for overtime hours worked last week was \$30.  
 (2) The employee's gross pay for all hours worked last week was \$30 more than for the previous week.

DS01104

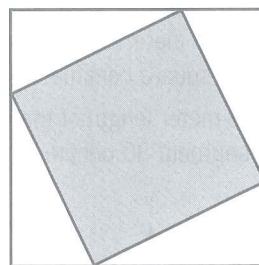
185. Is the integer  $p$  even?

- (1) The integer  $p^2 + 1$  is odd.  
 (2) The integer  $p + 2$  is even.

DS05172

186. If  $x > 0$ , what is the value of  $x^5$ ?

- (1)  $\sqrt{x} = 32$   
 (2)  $x^2 = 2^{20}$



DS17640

187. In the quilting pattern shown above, a small square has its vertices on the sides of a larger square. What is the side length, in centimeters, of the larger square?

- (1) The side length of the smaller square is 10 cm.  
 (2) Each vertex of the small square cuts 1 side of the larger square into 2 segments with lengths in the ratio of 1:2.

DS02589

188. Did Insurance Company K have more than \$300 million in total net profits last year?

- (1) Last year Company K paid out \$0.95 in claims for every dollar of premiums collected.  
 (2) Last year Company K earned a total of \$150 million in profits from the investment of accumulated surplus premiums from previous years.

\*These numbers correlate with the online test bank question number. See the GMAT Quantitative Review Online Index in the back of this book.

DS15349

189. How many hours would it take Pump A and Pump B working together, each at its own constant rate, to empty a tank that was initially full?
- Working alone at its constant rate, Pump A would empty the full tank in 4 hours 20 minutes.
  - Working alone, Pump B would empty the full tank at its constant rate of 72 liters per minute.

DS04573

190. What is the value of the integer  $N$ ?

- $101 < N < 103$
- $202 < 2N < 206$

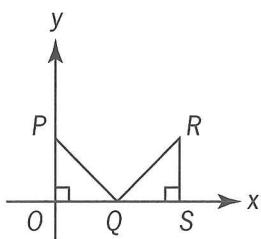
DS12033

191. Is  $zw$  positive?

- $z + w^3 = 20$
- $z$  is positive.

DS03006

192. On the scale drawing of a certain house plan, if 1 centimeter represents  $x$  meters, what is the value of  $x$ ?
- A rectangular room that has a floor area of 12 square meters is represented by a region of area 48 square centimeters.
  - The 15-meter length of the house is represented by a segment 30 centimeters long.



DS03939

193. In the rectangular coordinate system above, if  $\triangle OPQ$  and  $\triangle QRS$  have equal area, what are the coordinates of point  $R$ ?
- The coordinates of point  $P$  are  $(0,12)$ .
  - $OP = OQ$  and  $QS = RS$ .

DS07258

194. In a school that had a total of 600 students enrolled in the junior and senior classes, the students contributed to a certain fund. If all of the juniors but only half of the seniors contributed, was the total amount contributed more than \$740?
- Each junior contributed \$1 and each senior who contributed gave \$3.
  - There were more juniors than seniors enrolled in the school.

DS06650

195. How much did credit-card fraud cost United States banks in year X to the nearest \$10 million?

- In year X, counterfeit cards and telephone and mail-order fraud accounted for 39 percent of the total amount that card fraud cost the banks.
- In year X, stolen cards accounted for \$158.4 million, or 16 percent, of the total amount that credit-card fraud cost the banks.

DS17319

196. Is the positive integer  $n$  odd?

- $n^2 + (n+1)^2 + (n+2)^2$  is even.
- $n^2 - (n+1)^2 - (n+2)^2$  is even.

DS01130

197. In the  $xy$ -plane, circle  $C$  has center  $(1,0)$  and radius 2. If line  $k$  is parallel to the  $y$ -axis, is line  $k$  tangent to circle  $C$ ?

- Line  $k$  passes through the point  $(-1,0)$ .
- Line  $k$  passes through the point  $(-1,-1)$ .

DS14170

198. Company X's profits this year increased by 25% over last year's profits. Was the dollar amount of Company X's profits this year greater than the dollar amount of Company Y's?

- Last year, the ratio of Company Y's profits to Company X's profits was 5:2.
- Company Y experienced a 40% drop in profits from last year to this year.

DS09385

199. For all  $x$ , the expression  $x^*$  is defined to be  $ax + a$ , where  $a$  is a constant. What is the value of  $2^*$ ?

- $3^* = 2$
- $5^* = 3$

DS09260

200. Is  $k + m < 0$ ?

- $k < 0$
- $km > 0$

DS08352

201. The symbol  $\Delta$  represents which one of the following operations: addition, subtraction, or multiplication?

- $a \Delta (b \Delta c) \neq a \Delta (c \Delta b)$  for some numbers  $a$ ,  $b$ , and  $c$ .
- $a \Delta (b \Delta c) \neq (a \Delta b) \Delta c$  for some numbers  $a$ ,  $b$ , and  $c$ .

DS05989

202. What is the value of  $2^x + 2^{-x}$ ?

- (1)  $x > 0$   
 (2)  $4^x + 4^{-x} = 23$

DS13457

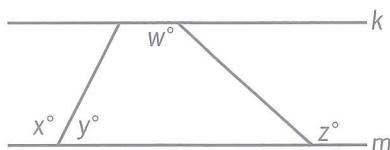
203. What is the ratio of  $c$  to  $d$ ?

- (1) The ratio of  $3c$  to  $3d$  is 3 to 4.  
 (2) The ratio of  $c + 3$  to  $d + 3$  is 4 to 5.

DS15099

204. A candle company determines that, for a certain specialty candle, the supply function is  $p = m_1x + b_1$  and the demand function is  $p = m_2x + b_2$ , where  $p$  is the price of each candle,  $x$  is the number of candles supplied or demanded, and  $m_1$ ,  $m_2$ ,  $b_1$ , and  $b_2$  are constants. At what value of  $x$  do the graphs of the supply function and demand function intersect?

- (1)  $m_1 = -m_2 = 0.005$   
 (2)  $b_2 - b_1 = 6$



DS12862

205. In the figure shown, lines  $k$  and  $m$  are parallel to each other. Is  $x = z$ ?

- (1)  $x = w$   
 (2)  $y = 180 - w$

DS13097

206. If  $k$  and  $\ell$  are lines in the  $xy$ -plane, is the slope of  $k$  less than the slope of  $\ell$ ?

- (1) The  $x$ -intercept of line  $k$  is positive, and the  $x$ -intercept of line  $\ell$  is negative.  
 (2) Lines  $k$  and  $\ell$  intersect on the positive  $y$ -axis.

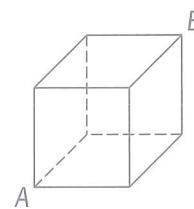
DS09642

207. When the wind speed is 9 miles per hour, the wind-chill factor  $w$  is given by

$$w = -17.366 + 1.19t,$$

where  $t$  is the temperature in degrees Fahrenheit. If at noon yesterday the wind speed was 9 miles per hour, was the wind-chill factor greater than 0?

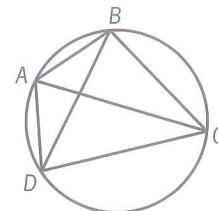
- (1) The temperature at noon yesterday was greater than 10 degrees Fahrenheit.  
 (2) The temperature at noon yesterday was less than 20 degrees Fahrenheit.



DS08852

208. What is the volume of the cube above?

- (1) The surface area of the cube is 600 square inches.  
 (2) The length of diagonal  $AB$  is  $10\sqrt{3}$  inches.



DS03989

209. In the figure shown, quadrilateral  $ABCD$  is inscribed in a circle of radius 5. What is the perimeter of quadrilateral  $ABCD$ ?

- (1) The length of  $AB$  is 6 and the length of  $CD$  is 8.  
 (2)  $AC$  is a diameter of the circle.

DS05766

210. How many members of a certain legislature voted against the measure to raise their salaries?

- (1)  $\frac{1}{4}$  of the members of the legislature did not vote on the measure.  
 (2) If 5 additional members of the legislature had voted against the measure, then the fraction of members of the legislature voting against the measure would have been  $\frac{1}{3}$ .

DS05986

211. If  $y \neq 0$ , is  $|x| = 1$ ?

- (1)  $x = \frac{y}{|y|}$   
 (2)  $|x| = -x$

DS08306

212. If  $x$  is a positive integer, what is the value of  $x$ ?

- (1)  $x^2 = \sqrt{x}$   
 (2)  $\frac{n}{x} = n$  and  $n \neq 0$ .

DS07568

213. Is the median of the five numbers  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  equal to  $d$ ?

- (1)  $a < c < e$   
 (2)  $b < d < c$

DS10383

214. During a certain bicycle ride, was Sherry's average speed faster than 24 kilometers per hour?  
(1 kilometer = 1,000 meters)

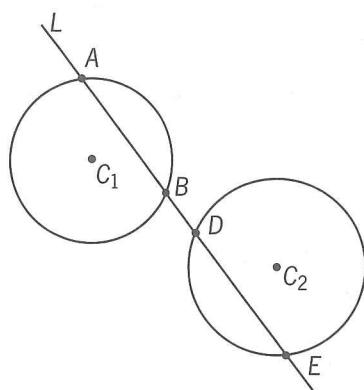
- Sherry's average speed during the bicycle ride was faster than 7 meters per second.
- Sherry's average speed during the bicycle ride was slower than 8 meters per second.

DS13907

215. Working together, Rafael and Salvador can tabulate a certain set of data in 2 hours. In how many hours can Rafael tabulate the data working alone?
- Working alone, Rafael can tabulate the data in 3 hours less time than Salvador, working alone, can tabulate the data.
  - Working alone, Rafael can tabulate the data in  $\frac{1}{2}$  the time that Salvador, working alone, can tabulate the data.

DS04039

216. If  $x$  and  $y$  are integers, what is the value of  $x$ ?
- $xy = 1$
  - $x \neq -1$



Note: Figure not drawn to scale.

DS18386

217. The figure above shows Line  $L$ , Circle 1 with center at  $C_1$ , and Circle 2 with center at  $C_2$ . Line  $L$  intersects Circle 1 at points  $A$  and  $B$ , Line  $L$  intersects Circle 2 at points  $D$  and  $E$ , and points  $C_1$  and  $C_2$  are equidistant from line  $L$ . Is the area of  $\triangle ABC_1$  less than the area of  $\triangle DEC_2$ ?
- The radius of Circle 1 is less than the radius of Circle 2.
  - The length of chord  $\overline{AB}$  is less than the length of chord  $\overline{DE}$ .

DS15938

218. Yesterday between 9:00 a.m. and 6:00 p.m. at Airport X, all flights to Atlanta departed at equally spaced times and all flights to New York City departed at equally spaced times. A flight to Atlanta and a flight to New York City both departed from Airport X at 1:00 p.m. yesterday. Between 1:00 p.m. and 3:00 p.m. yesterday, did another pair of flights to these 2 cities depart from Airport X at the same time?

- Yesterday at Airport X, a flight to Atlanta and a flight to New York City both departed at 10:00 a.m.
- Yesterday at Airport X, flights to New York City departed every 15 minutes between 9:00 a.m. and 6:00 p.m.

DS07206

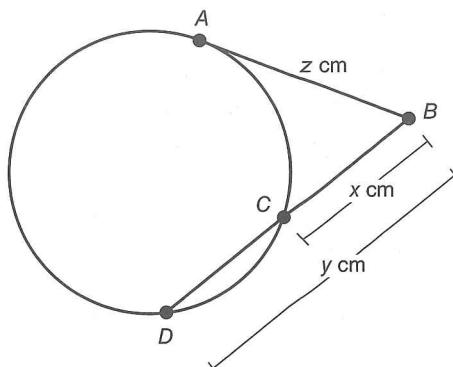
219. Of the total number of copies of Magazine X sold last week, 40 percent were sold at full price. What was the total number of copies of the magazine sold last week?
- Last week, full price for a copy of Magazine X was \$1.50 and the total revenue from full-price sales was \$112,500.
  - The total number of copies of Magazine X sold last week at full price was 75,000.

DS11614

220. If  $p$ ,  $s$ , and  $t$  are positive, is  $|ps - pt| > p(s - t)$ ?
- $p < s$
  - $s < t$

DS04468

221. Is  $x > y$ ?
- $x + y > x - y$
  - $3x > 2y$



DS17588

222. In the figure above,  $\overline{AB}$ , which has length  $z$  cm, is tangent to the circle at point  $A$ , and  $\overline{BD}$ , which has length  $y$  cm, intersects the circle at point  $C$ . If  $BC = x$  cm and  $z = \sqrt{xy}$ , what is the value of  $x$ ?
- $CD = x$  cm
  - $z = 5\sqrt{2}$

DS15863

223. Is the integer  $n$  a prime number?

- (1)  $24 \leq n \leq 28$   
 (2)  $n$  is not divisible by 2 or 3.

DS03615

224. What is the average (arithmetic mean) annual salary of the 6 employees of a toy company?

- (1) If the 6 annual salaries were ordered from least to greatest, each annual salary would be \$6,300 greater than the preceding annual salary.  
 (2) The range of the 6 annual salaries is \$31,500.

DS17503

225. In a certain order, the pretax price of each regular pencil was \$0.03, the pretax price of each deluxe pencil was \$0.05, and there were 50% more deluxe pencils than regular pencils. All taxes on the order are a fixed percent of the pretax prices. The sum of the total pretax price of the order and the tax on the order was \$44.10. What was the amount, in dollars, of the tax on the order?

- (1) The tax on the order was 5% of the total pretax price of the order.  
 (2) The order contained exactly 400 regular pencils.

DS06785

226. If  $m$  is an integer greater than 1, is  $m$  an even integer?

- (1) 32 is a factor of  $m$ .  
 (2)  $m$  is a factor of 32.

DS05657

227. If the set  $S$  consists of five consecutive positive integers, what is the sum of these five integers?

- (1) The integer 11 is in  $S$ , but 10 is not in  $S$ .  
 (2) The sum of the even integers in  $S$  is 26.

DS17543

228. If  $x > 0$ , what is the value of  $x$ ?

- (1)  $x^3 - x = 0$   
 (2)  $\sqrt[3]{x} - x = 0$

DS08307

229. A total of 20 amounts are entered on a spreadsheet that has 5 rows and 4 columns; each of the 20 positions in the spreadsheet contains one amount. The average (arithmetic mean) of the amounts in row  $i$  is  $R_i$  ( $1 \leq i \leq 5$ ). The average of the amounts in column  $j$  is  $C_j$  ( $1 \leq j \leq 4$ ). What is the average of all 20 amounts on the spreadsheet?

- (1)  $R_1 + R_2 + R_3 + R_4 + R_5 = 550$   
 (2)  $C_1 + C_2 + C_3 + C_4 = 440$

DS13132

230. Was the range of the amounts of money that Company Y budgeted for its projects last year equal to the range of the amounts of money that it budgeted for its projects this year?

- (1) Both last year and this year, Company Y budgeted money for 12 projects and the least amount of money that it budgeted for a project was \$400.  
 (2) Both last year and this year, the average (arithmetic mean) amount of money that Company Y budgeted per project was \$2,000.



DS01633

231. If  $a$ ,  $b$ ,  $c$ , and  $d$  are numbers on the number line shown and if the tick marks are equally spaced, what is the value of  $a + c$ ?

- (1)  $a + b = -8$   
 (2)  $a + d = 0$

DS06067

232. Is  $xm < ym$ ?

- (1)  $x > y$   
 (2)  $m < 0$

DS02899

233. If  $y = x^2 - 6x + 9$ , what is the value of  $x$ ?

- (1)  $y = 0$   
 (2)  $x + y = 3$

DS06810

234. What is the probability that Lee will make exactly 5 errors on a certain typing test?

- (1) The probability that Lee will make 5 or more errors on the test is 0.27.  
 (2) The probability that Lee will make 5 or fewer errors on the test is 0.85.

DS19208

235. If  $p$  is a positive integer, is  $2^p + 1$  a prime number?

- (1)  $p$  is a prime number.  
 (2)  $p$  is an even number.

DS02741

236. In the  $xy$ -plane, point  $(r,s)$  lies on a circle with center at the origin. What is the value of  $r^2 + s^2$ ?

- (1) The circle has radius 2.  
 (2) The point  $(\sqrt{2}, -\sqrt{2})$  lies on the circle.

DS06368

237. If  $r$ ,  $s$ , and  $t$  are nonzero integers, is  $r^5s^3t^4$  negative?
- $rt$  is negative.
  - $s$  is negative.

DS13706

238. Each Type A machine fills 400 cans per minute, each Type B machine fills 600 cans per minute, and each Type C machine installs 2,400 lids per minute. A lid is installed on each can that is filled and on no can that is not filled. For a particular minute, what is the total number of machines working?
- A total of 4,800 cans are filled that minute.
  - For that minute, there are 2 Type B machines working for every Type C machine working.

DS08660

239. If  $a$  and  $b$  are constants, what is the value of  $a$ ?
- $a < b$
  - $(t - a)(t - b) = t^2 + t - 12$ , for all values of  $t$ .

DS04474

240. If  $x$  is a positive integer, is  $\sqrt{x}$  an integer?
- $\sqrt{4x}$  is an integer.
  - $\sqrt{3x}$  is not an integer.

DS16456

241. If  $p$ ,  $q$ ,  $x$ ,  $y$ , and  $z$  are different positive integers, which of the five integers is the median?
- $p + x < q$
  - $y < z$

DS16277

242. If  $w + z = 28$ , what is the value of  $wz$ ?
- $w$  and  $z$  are positive integers.
  - $w$  and  $z$  are consecutive odd integers.

DS02474

243. If  $abc \neq 0$ , is  $\frac{a}{c} = \frac{a}{b}$ ?
- $a = 1$
  - $c = 1$

DS14471

244. The arithmetic mean of a collection of 5 positive integers, not necessarily distinct, is 9. One additional positive integer is included in the collection and the arithmetic mean of the 6 integers is computed. Is the arithmetic mean of the 6 integers at least 10?
- The additional integer is at least 14.
  - The additional integer is a multiple of 5.

DS11003

245. A certain list consists of 400 different numbers. Is the average (arithmetic mean) of the numbers in the list greater than the median of the numbers in the list?
- Of the numbers in the list, 280 are less than the average.
  - Of the numbers in the list, 30 percent are greater than or equal to the average.

DS03678

246. In a two-month survey of shoppers, each shopper bought one of two brands of detergent, X or Y, in the first month and again bought one of these brands in the second month. In the survey, 90 percent of the shoppers who bought Brand X in the first month bought Brand X again in the second month, while 60 percent of the shoppers who bought Brand Y in the first month bought Brand Y again in the second month. What percent of the shoppers bought Brand Y in the second month?
- In the first month, 50 percent of the shoppers bought Brand X.
  - The total number of shoppers surveyed was 5,000.

DS15902

247. If  $m$  and  $n$  are positive integers, is  $m + n$  divisible by 4?
- $m$  and  $n$  are each divisible by 2.
  - Neither  $m$  nor  $n$  is divisible by 4.

DS02940

248. What is the area of rectangular region  $R$ ?
- Each diagonal of  $R$  has length 5.
  - The perimeter of  $R$  is 14.

DS17137

249. How many integers  $n$  are there such that  $r < n < s$ ?
- $s - r = 5$
  - $r$  and  $s$  are not integers.

DS17147

250. If the total price of  $n$  equally priced shares of a certain stock was \$12,000, what was the price per share of the stock?
- If the price per share of the stock had been \$1 more, the total price of the  $n$  shares would have been \$300 more.
  - If the price per share of the stock had been \$2 less, the total price of the  $n$  shares would have been 5 percent less.

DS02865

251. If  $n$  is positive, is  $\sqrt{n} > 100$ ?

- (1)  $\sqrt{n-1} > 99$   
 (2)  $\sqrt{n+1} > 101$

DS17150

252. Is  $xy > 5$ ?

- (1)  $1 \leq x \leq 3$  and  $2 \leq y \leq 4$ .  
 (2)  $x + y = 5$

DS17151

253. In Year X, 8.7 percent of the men in the labor force were unemployed in June compared with 8.4 percent in May. If the number of men in the labor force was the same for both months, how many men were unemployed in June of that year?

- (1) In May of Year X, the number of unemployed men in the labor force was 3.36 million.  
 (2) In Year X, 120,000 more men in the labor force were unemployed in June than in May.

DS17112

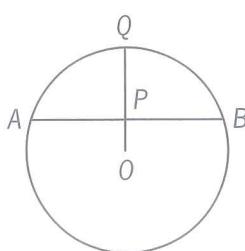
254. If  $x \neq 0$ , what is the value of  $\left(\frac{x^p}{x^q}\right)^4$ ?

- (1)  $p = q$   
 (2)  $x = 3$

DS17153

255. On Monday morning a certain machine ran continuously at a uniform rate to fill a production order. At what time did it completely fill the order that morning?

- (1) The machine began filling the order at 9:30 a.m.  
 (2) The machine had filled  $\frac{1}{2}$  of the order by 10:30 a.m. and  $\frac{5}{6}$  of the order by 11:10 a.m.



DS17107

256. What is the radius of the circle above with center O?

- (1) The ratio of  $OP$  to  $PQ$  is 1 to 2.  
 (2)  $P$  is the midpoint of chord  $AB$ .

DS15618

257. If  $a$  and  $b$  are positive integers, what is the value of the product  $ab$ ?

- (1) The least common multiple of  $a$  and  $b$  is 48.  
 (2) The greatest common factor of  $a$  and  $b$  is 4.

DS17095

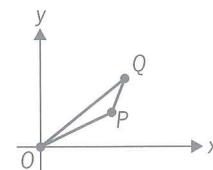
258. What is the number of 360-degree rotations that a bicycle wheel made while rolling 100 meters in a straight line without slipping?

- (1) The diameter of the bicycle wheel, including the tire, was 0.5 meter.  
 (2) The wheel made twenty 360-degree rotations per minute.

DS17168

259. In the equation  $x^2 + bx + 12 = 0$ ,  $x$  is a variable and  $b$  is a constant. What is the value of  $b$ ?

- (1)  $x - 3$  is a factor of  $x^2 + bx + 12$ .  
 (2) 4 is a root of the equation  $x^2 + bx + 12 = 0$ .



DS07715

260. In the figure above, line segment  $OP$  has slope  $\frac{1}{2}$  and line segment  $PQ$  has slope 2. What is the slope of line segment  $OQ$ ?

- (1) Line segment  $OP$  has length  $2\sqrt{5}$ .  
 (2) The coordinates of point  $Q$  are  $(5, 4)$ .

DS17164

261. In  $\triangle XYZ$ , what is the length of  $YZ$ ?

- (1) The length of  $XY$  is 3.  
 (2) The length of  $XZ$  is 5.

DS07217

262. If the average (arithmetic mean) of  $n$  consecutive odd integers is 10, what is the least of the integers?

- (1) The range of the  $n$  integers is 14.  
 (2) The greatest of the  $n$  integers is 17.

DS16044

263. If  $x$ ,  $y$ , and  $z$  are positive numbers, is  $x > y > z$ ?

- (1)  $xz > yz$   
 (2)  $yx > yz$

DS06644

264.  $K$  is a set of numbers such that

- (i) if  $x$  is in  $K$ , then  $-x$  is in  $K$ , and  
(ii) if each of  $x$  and  $y$  is in  $K$ , then  $xy$  is in  $K$ .

Is 12 in  $K$ ?

- (1) 2 is in  $K$ .  
(2) 3 is in  $K$ .

DS05637

265. If  $x^2 + y^2 = 29$ , what is the value of  $(x - y)^2$ ?

- (1)  $xy = 10$   
(2)  $x = 5$

DS16470

266. After winning 50 percent of the first 20 games it played, Team A won all of the remaining games it played. What was the total number of games that Team A won?

- (1) Team A played 25 games altogether.  
(2) Team A won 60 percent of all the games it played.

DS17181

267. Is  $x$  between 0 and 1?

- (1)  $x^2$  is less than  $x$ .  
(2)  $x^3$  is positive.

DS04083

268. If  $m$  and  $n$  are nonzero integers, is  $m^n$  an integer?

- (1)  $n^m$  is positive.  
(2)  $n^m$  is an integer.

DS16034

269. What is the value of  $xy$ ?

- (1)  $x + y = 10$   
(2)  $x - y = 6$

DS13189

270. If  $n$  is the least of three different integers greater than 1, what is the value of  $n$ ?

- (1) The product of the three integers is 90.  
(2) One of the integers is twice one of the other two integers.

DS16461

271. Is  $x^2$  greater than  $x$ ?

- (1)  $x^2$  is greater than 1.  
(2)  $x$  is greater than  $-1$ .

DS03503

272. Michael arranged all his books in a bookcase with 10 books on each shelf and no books left over. After Michael acquired 10 additional books, he arranged all his books in a new bookcase with 12 books on each shelf and no books left over. How many books did Michael have before he acquired the 10 additional books?

- (1) Before Michael acquired the 10 additional books, he had fewer than 96 books.  
(2) Before Michael acquired the 10 additional books, he had more than 24 books.

DS16469

273. If  $xy > 0$ , does  $(x - 1)(y - 1) = 1$ ?

- (1)  $x + y = xy$   
(2)  $x = y$

DS06842

274. Last year in a group of 30 businesses, 21 reported a net profit and 15 had investments in foreign markets. How many of the businesses did not report a net profit nor invest in foreign markets last year?

- (1) Last year 12 of the 30 businesses reported a net profit and had investments in foreign markets.  
(2) Last year 24 of the 30 businesses reported a net profit or invested in foreign markets, or both.

DS17110

275. Is the perimeter of square  $S$  greater than the perimeter of equilateral triangle  $T$ ?

- (1) The ratio of the length of a side of  $S$  to the length of a side of  $T$  is 4:5.  
(2) The sum of the lengths of a side of  $S$  and a side of  $T$  is 18.

DS17136

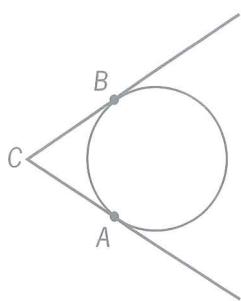
276. If  $x + y + z > 0$ , is  $z > 1$ ?

- (1)  $z > x + y + 1$   
(2)  $x + y + 1 < 0$

DS07832

277. For all  $z$ ,  $\lceil z \rceil$  denotes the least integer greater than or equal to  $z$ . Is  $\lceil x \rceil = 0$ ?

- (1)  $-1 < x < -0.1$   
(2)  $\lceil x + 0.5 \rceil = 1$



DS16464

278. The circular base of an above-ground swimming pool lies in a level yard and just touches two straight sides of a fence at points A and B, as shown in the figure above. Point C is on the ground where the two sides of the fence meet. How far from the center of the pool's base is point A?

- (1) The base has area 250 square feet.  
 (2) The center of the base is 20 feet from point C.

DS16050

279. If  $xy = -6$ , what is the value of  $xy(x + y)$ ?

- (1)  $x - y = 5$   
 (2)  $xy^2 = 18$

DS05519

280.  $[y]$  denotes the greatest integer less than or equal to  $y$ . Is  $d < 1$ ?

- (1)  $d = y - [y]$   
 (2)  $[d] = 0$

DS14052

281. If  $N$  is a positive odd integer, is  $N$  prime?

- (1)  $N = 2^k + 1$  for some positive integer  $k$ .  
 (2)  $N + 2$  and  $N + 4$  are both prime.

DS01140

282. If  $m$  is a positive integer, then  $m^3$  has how many digits?

- (1)  $m$  has 3 digits.  
 (2)  $m^2$  has 5 digits.

DS03308

283. What is the value of  $x^2 - y^2$ ?

- (1)  $(x - y)^2 = 9$   
 (2)  $x + y = 6$

DS1267

284. For each landscaping job that takes more than 4 hours, a certain contractor charges a total of  $r$  dollars for the first 4 hours plus  $0.2r$  dollars for each additional hour or fraction of an hour, where  $r > 100$ . Did a particular landscaping job take more than 10 hours?
- (1) The contractor charged a total of \$288 for the job.  
 (2) The contractor charged a total of  $2.4r$  dollars for the job.
- DS17600
285. If  $x^2 = 2^x$ , what is the value of  $x$ ?
- (1)  $2x = \left(\frac{x}{2}\right)^3$   
 (2)  $x = 2^{x-2}$
- DS01169
286. The sequence  $s_1, s_2, s_3, \dots, s_n, \dots$  is such that  $s_n = \frac{1}{n} - \frac{1}{n+1}$  for all integers  $n \geq 1$ . If  $k$  is a positive integer, is the sum of the first  $k$  terms of the sequence greater than  $\frac{9}{10}$ ?
- (1)  $k > 10$   
 (2)  $k < 19$
- DS05518
287. In the sequence  $S$  of numbers, each term after the first two terms is the sum of the two immediately preceding terms. What is the 5th term of  $S$ ?
- (1) The 6th term of  $S$  minus the 4th term equals 5.  
 (2) The 6th term of  $S$  plus the 7th term equals 21.
- DS01121
288. If 75 percent of the guests at a certain banquet ordered dessert, what percent of the guests ordered coffee?
- (1) 60 percent of the guests who ordered dessert also ordered coffee.  
 (2) 90 percent of the guests who ordered coffee also ordered dessert.
- DS05302
289. A tank containing water started to leak. Did the tank contain more than 30 gallons of water when it started to leak? (Note: 1 gallon = 128 ounces)
- (1) The water leaked from the tank at a constant rate of 6.4 ounces per minute.  
 (2) The tank became empty less than 12 hours after it started to leak.

DS12752

290. In the  $xy$ -plane, lines  $k$  and  $\ell$  intersect at the point  $(1,1)$ . Is the  $y$ -intercept of  $k$  greater than the  $y$ -intercept of  $\ell$ ?
- The slope of  $k$  is less than the slope of  $\ell$ .
  - The slope of  $\ell$  is positive.

DS14588

291. A triangle has side lengths of  $a$ ,  $b$ , and  $c$  centimeters. Does each angle in the triangle measure less than 90 degrees?
- The 3 semicircles whose diameters are the sides of the triangle have areas that are equal to  $3 \text{ cm}^2$ ,  $4 \text{ cm}^2$ , and  $6 \text{ cm}^2$ , respectively.
  - $c < a + b < c + 2$

DS00890

292. Each of the 45 books on a shelf is written either in English or in Spanish, and each of the books is either a hardcover book or a paperback. If a book is to be selected at random from the books on the shelf, is the probability less than  $\frac{1}{2}$  that the book selected will be a paperback written in Spanish?
- Of the books on the shelf, 30 are paperbacks.
  - Of the books on the shelf, 15 are written in Spanish.

DS06683

293. A small school has three foreign language classes, one in French, one in Spanish, and one in German. How many of the 34 students enrolled in the Spanish class are also enrolled in the French class?
- There are 27 students enrolled in the French class, and 49 students enrolled in either the French class, the Spanish class, or both of these classes.
  - One-half of the students enrolled in the Spanish class are enrolled in more than one foreign language class.

DS04910

294. If  $S$  is a set of four numbers  $w$ ,  $x$ ,  $y$ , and  $z$ , is the range of the numbers in  $S$  greater than 2?
- $w - z > 2$
  - $z$  is the least number in  $S$ .

DS12187

295. Last year  $\frac{3}{5}$  of the members of a certain club were males. This year the members of the club include all the members from last year plus some new members. Is the fraction of the members of the club who are males greater this year than last year?

- More than half of the new members are male.
- The number of members of the club this year is  $\frac{6}{5}$  the number of members last year.

DS13640

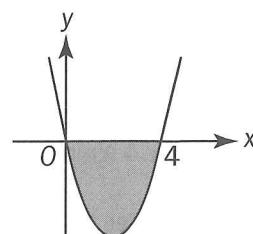
296. If  $a$ ,  $b$ , and  $c$  are consecutive integers and  $0 < a < b < c$ , is the product  $abc$  a multiple of 8?
- The product  $ac$  is even.
  - The product  $bc$  is a multiple of 4.

DS13837

297.  $M$  and  $N$  are integers such that  $6 < M < N$ . What is the value of  $N$ ?
- The greatest common divisor of  $M$  and  $N$  is 6.
  - The least common multiple of  $M$  and  $N$  is 36.

DS07575

298. Stations X and Y are connected by two separate, straight, parallel rail lines that are 250 miles long. Train P and train Q simultaneously left Station X and Station Y, respectively, and each train traveled to the other's point of departure. The two trains passed each other after traveling for 2 hours. When the two trains passed, which train was nearer to its destination?
- At the time when the two trains passed, train P had averaged a speed of 70 miles per hour.
  - Train Q averaged a speed of 55 miles per hour for the entire trip.



DS01613

299. In the  $xy$ -plane shown, the shaded region consists of all points that lie above the graph of  $y = x^2 - 4x$  and below the  $x$ -axis. Does the point  $(a,b)$  (not shown) lie in the shaded region if  $b < 0$ ?

- $0 < a < 4$
- $a^2 - 4a < b$

DS01685

300. If  $a$  and  $b$  are positive integers, is  $\sqrt[3]{ab}$  an integer?
- $\sqrt{a}$  is an integer.
  - $b = \sqrt{a}$

## 5.4 Answer Key

177.	D	208.	D	239.	C	270.	C
178.	E	209.	C	240.	A	271.	A
179.	A	210.	E	241.	E	272.	A
180.	E	211.	A	242.	B	273.	A
181.	D	212.	D	243.	B	274.	D
182.	E	213.	E	244.	C	275.	A
183.	B	214.	A	245.	D	276.	B
184.	A	215.	D	246.	A	277.	A
185.	D	216.	C	247.	C	278.	A
186.	D	217.	D	248.	C	279.	B
187.	C	218.	E	249.	C	280.	D
188.	E	219.	D	250.	D	281.	E
189.	E	220.	B	251.	B	282.	E
190.	D	221.	E	252.	E	283.	E
191.	E	222.	C	253.	D	284.	B
192.	D	223.	A	254.	A	285.	D
193.	C	224.	E	255.	B	286.	A
194.	E	225.	D	256.	E	287.	A
195.	B	226.	D	257.	C	288.	C
196.	D	227.	D	258.	A	289.	E
197.	D	228.	D	259.	D	290.	A
198.	C	229.	D	260.	B	291.	A
199.	D	230.	E	261.	E	292.	B
200.	C	231.	C	262.	D	293.	A
201.	D	232.	C	263.	E	294.	A
202.	B	233.	A	264.	C	295.	E
203.	A	234.	C	265.	A	296.	A
204.	C	235.	C	266.	D	297.	C
205.	D	236.	D	267.	A	298.	A
206.	C	237.	E	268.	E	299.	B
207.	E	238.	C	269.	C	300.	B

## 5.5 Answer Explanations

The following discussion of data sufficiency is intended to familiarize you with the most efficient and effective approaches to the kinds of problems common to data sufficiency. The particular questions in this chapter are generally representative of the kinds of data sufficiency questions you will encounter on the GMAT. Remember that it is the problem solving strategy that is important, not the specific details of a particular question.

\*DS05149

177. Does  $2x + 8 = 12$ ?

- (1)  $2x + 10 = 14$
- (2)  $3x + 8 = 14$

### Algebra First-degree equations

We need to determine, for each of statements 1 and 2, whether the statement is sufficient for determining whether  $2x + 8 = 12$ . Solving for  $x$ , we see that the equation  $2x + 8 = 12$  is equivalent to  $2x = 12 - 8 = 4$  and is thus equivalent to  $x = 2$ . We thus need to find whether the statements are sufficient for determining whether  $x = 2$ .

- (1) Given that  $2x + 10 = 14$ , it follows that  $2x = 14 - 10 = 4$ , and that  $x = 2$ ; SUFFICIENT.
- (2) Similarly, given that  $3x + 8 = 14$ , it follows that  $3x = 14 - 8 = 6$ , and that  $x = 2$ ; SUFFICIENT.

Alternatively, for both statements 1 and 2, it is only necessary to determine that it is possible to solve each of 1 and 2 to produce a unique value for  $x$ . Care must be taken with such an approach (there are cases such as, for example,  $3y = 5 + 3y$ , that cannot be solved for a unique value of the variable). However, the approach can save time.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS01503

178. If  $M$  is a set of consecutive even integers, is 0 in set  $M$ ?

- (1)  $-6$  is in set  $M$ .
- (2)  $-2$  is in set  $M$ .

### Arithmetic Series and sequences

For a set  $M$  of consecutive even integers, can we determine whether  $M$  contains the number 0?

- (1) Given that  $-6$  is in the set  $M$ ,  $M$  could be a set of strictly negative integers or a set that contains both negative integers and zero (and perhaps positive integers). For example, it could be the set  $\{-8, -6, -4\}$  or the set  $\{-8, -6, -4, -2, 0, 2\}$ ; NOT sufficient.
- (2) Similarly, given that  $-2$  is in  $M$ ,  $M$  could be a set of strictly negative integers or a set that contains both negative integers and zero (and perhaps positive integers). For example, it could be the set  $\{-4, -2\}$  or the set  $\{-8, -6, -4, -2, 0, 2\}$ ; NOT sufficient.

Furthermore we can see that statements 1 and 2 together are not sufficient. For example,  $\{-8, -6, -4, -2, 0, 2\}$  contains  $-6$ ,  $-2$ , and 0, while  $\{-8, -6, -4, -2\}$  contains  $-6$  and  $-2$  but not 0.

**The correct answer is E;**  
**both statements together are not sufficient.**

DS15510

179. Rita's monthly salary is  $\frac{2}{3}$  Juanita's monthly salary.  
What is their combined monthly salary?

- (1) Rita's monthly salary is \$4,000.
- (2) Either Rita's monthly salary or Juanita's monthly salary is \$6,000.

### Arithmetic Applied problems

Let  $R$  and  $J$  be Rita's and Juanita's monthly salaries, respectively, in dollars. It is given that  $R = \frac{2}{3}J$ . Determine the value of their combined salary, which can be expressed as  $R + J = \frac{2}{3}J + J = \frac{5}{3}J$ .

\*These numbers correlate with the online test bank question number. See the GMAT Quantitative Review Online Index in the back of this book.

- (1) Given that  $R = 4,000$ , it follows that  $4,000 = \frac{2}{3}J$ , or  $J = \frac{3}{2}(4,000) = 6,000$ . Therefore,  $\frac{5}{3}J = \frac{5}{3}(6,000) = 10,000$ ; SUFFICIENT.

- (2) Given that  $R = 6,000$  or  $J = 6,000$ , then  $J = \frac{3}{2}(6,000) = 9,000$  or  $J = 6,000$ . Thus,  $\frac{5}{3}J = \frac{5}{3}(9,000) = 15,000$  or  $\frac{5}{3}J = \frac{5}{3}(6,000) = 10,000$ , and so it is not possible to determine the value of  $\frac{5}{3}J$ ; NOT sufficient.

**The correct answer is A;**  
**statement 1 alone is sufficient.**

DS13384

180. What is the value of the integer  $x$ ?

- (1)  $x$  rounded to the nearest hundred is 7,200.  
(2) The hundreds digit of  $x$  is 2.

### Arithmetic Rounding

- (1) Given that  $x$  rounded to the nearest hundred is 7,200, the value of  $x$  cannot be determined. For example,  $x$  could be 7,200 or  $x$  could be 7,201; NOT sufficient.  
(2) Given that the hundreds digit of  $x$  is 2, the value of  $x$  cannot be determined. For example,  $x$  could be 7,200 or  $x$  could be 7,201; NOT sufficient.

Taking (1) and (2) together is of no more help than either (1) or (2) taken separately because the same examples were used in both (1) and (2).

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS04644

181. Is  $2x > 2y$ ?

- (1)  $x > y$   
(2)  $3x > 3y$

### Algebra Inequalities

- (1) It is given that  $x > y$ . Thus, multiplying both sides by the positive number 2, it follows that  $2x > 2y$ ; SUFFICIENT.

- (2) It is given that  $3x > 3y$ . Thus, multiplying both sides by the positive number  $\frac{2}{3}$ , it follows that  $2x > 2y$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS04636

182. If  $p$  and  $q$  are positive, is  $\frac{p}{q}$  less than 1?

- (1)  $p$  is less than 4.  
(2)  $q$  is less than 4.

### Arithmetic Properties of numbers

- (1) Given that  $p$  is less than 4, then it is not possible to determine whether  $\frac{p}{q}$  is less than 1. For example, if  $p = 1$  and  $q = 2$ , then  $\frac{p}{q} = \frac{1}{2}$  and  $\frac{1}{2}$  is less than 1. However, if  $p = 2$  and  $q = 1$ , then  $\frac{p}{q} = 2$  and 2 is not less than 1; NOT sufficient.  
(2) Given that  $q$  is less than 4, then it is not possible to determine whether  $\frac{p}{q}$  is less than 1. For example, if  $p = 1$  and  $q = 2$ , then  $\frac{p}{q} = \frac{1}{2}$  and  $\frac{1}{2}$  is less than 1. However, if  $p = 2$  and  $q = 1$ , then  $\frac{p}{q} = 2$  and 2 is not less than 1; NOT sufficient.

Taking (1) and (2) together is of no more help than either (1) or (2) taken separately because the same examples were used in both (1) and (2).

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS02779

183. In each quarter of 1998, Company M earned more money than in the previous quarter. What was the range of Company M's quarterly earnings in 1998?

- (1) In the 2nd and 3rd quarters of 1998, Company M earned \$4.0 million and \$4.6 million, respectively.  
(2) In the 1st and 4th quarters of 1998, Company M earned \$3.8 million and \$4.9 million, respectively.

**Arithmetic Statistics**

We know that for each of the quarters in 1998, Company M earned more money than in the previous quarter. Is it possible to determine the range of the company's quarterly earnings in 1998?

- (1) Although we are told the value of the earnings for the 2nd and 3rd quarters, Company M's 4th quarter earnings could, consistent with statement 1, be any amount that is greater than the 3rd quarter earnings. Likewise, the company's 1st quarter earnings could be any positive amount that is less than the company's 2nd quarter earnings. The difference between these two values would be the range, and we see that it cannot be determined; NOT sufficient.
- (2) We are given the earnings for the 1st and 4th quarters, and we already know that, from quarter to quarter, the earnings in 1998 have always increased. We can thus infer that Company M's earnings for the 2nd and 3rd quarters are less than the 4th quarter earnings but greater than the 1st quarter earnings. The difference between the greatest quarterly earnings and the least quarterly earnings for 1998 is thus the difference between the 4th quarter earnings and the 1st quarter earnings—the values \$4.9 million and \$3.8 million, respectively, that we have been given; SUFFICIENT.

**The correct answer is B;**  
**statement 2 alone is sufficient.**

DS04510

184. In a certain factory, hours worked by each employee in excess of 40 hours per week are overtime hours and are paid for at  $1\frac{1}{2}$  times the employee's regular hourly pay rate. If an employee worked a total of 42 hours last week, how much was the employee's gross pay for the hours worked last week?
- (1) The employee's gross pay for overtime hours worked last week was \$30.
  - (2) The employee's gross pay for all hours worked last week was \$30 more than for the previous week.

**Arithmetic Applied problems**

If an employee's regular hourly rate was  $\$R$  and the employee worked 42 hours last week, then the employee's gross pay for hours worked last week was  $40R + 2(1.5R)$ . Determine the value of  $40R + 2(1.5R) = 43R$ , or equivalently, the value of  $R$ .

- (1) Given that the employee's gross pay for overtime hours worked last week was \$30, it follows that  $2(1.5R) = 30$  and  $R = 10$ ; SUFFICIENT.
- (2) Given that the employee's gross pay for all hours worked last week was \$30 more than for the previous week, the value of  $R$  cannot be determined because nothing specific is known about the value of the employee's pay for all hours worked the previous week; NOT sufficient.

**The correct answer is A;**  
**statement (1) alone is sufficient.**

DS01104

185. Is the integer  $p$  even?

- (1) The integer  $p^2 + 1$  is odd.
- (2) The integer  $p + 2$  is even.

**Arithmetic Properties of integers**

- (1) For any odd number  $m$ ,  $m - 1$  must be even. Therefore, given that  $p^2 + 1$  is odd,  $p^2$  must be even. Now, if in this case  $p$  were odd,  $p$  would not be divisible by 2 and so would not have 2 as one of its prime factors.  $p^2$  would also not have 2 as one of its prime factors, and so, if  $p$  were odd,  $p^2$  would be odd. Therefore, given that  $p^2 + 1$  is odd (and  $p^2$  is even),  $p$  must not be odd. That is,  $p$  must be even; SUFFICIENT.
- (2) Given that  $p + 2$  is even, it follows that  $p + 2$  is divisible by 2. That is,  $p + 2 = 2k$ , where  $k$  is an integer. Thus,  $p = 2k - 2 = 2(k - 1)$ , where  $k - 1$  is an integer. The integer  $p$  is thus divisible by 2 and is therefore even; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS05172

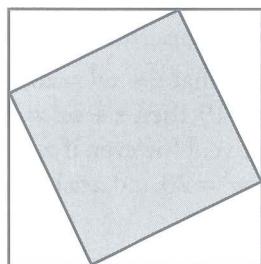
186. If  $x > 0$ , what is the value of  $x^5$ ?

- (1)  $\sqrt{x} = 32$
- (2)  $x^2 = 2^{20}$

### Algebra Exponents

- (1) Given that  $\sqrt{x} = 32$ , it follows that  $x = 32^2$  and  $x^5 = (32^2)^5$ ; SUFFICIENT.
- (2) Given that  $x^2 = 2^{20}$ , since  $x$  is positive, it follows that  $x = \sqrt{2^{20}} = 2^{10}$  and  $x^5 = (2^{10})^5$ ; SUFFICIENT.

The correct answer is D;  
each statement alone is sufficient.

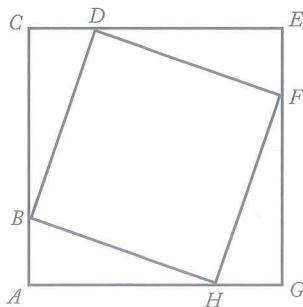


DS17640

187. In the quilting pattern shown above, a small square has its vertices on the sides of a larger square. What is the side length, in centimeters, of the larger square?

- (1) The side length of the smaller square is 10 cm.
- (2) Each vertex of the small square cuts 1 side of the larger square into 2 segments with lengths in the ratio of 1:2.

### Geometry Triangles; Pythagorean theorem



Determine the side length of the larger square or, in the figure above, determine  $AG = AH + HG$ . Note that  $\triangle BAH$ ,  $\triangle DCB$ ,  $\triangle FED$ , and  $\triangle HGF$  are the same size and shape and that  $AB = CD = EF = GH$  and  $BC = DE = FG = HA$ .

- (1) This indicates that  $HF = 10$ , but it is possible that  $HG = 6$  and  $GF = 8$  ( $\sqrt{6^2 + 8^2} = 10$ ), from which it follows that the side length of the larger square is  $6 + 8 = 14$ , and it is possible that  $HG = 1$  and  $GF = \sqrt{99}$  ( $\sqrt{1^2 + (\sqrt{99})^2} = 10$ ), from which it follows that the side length of the larger square is  $1 + \sqrt{99}$ ; NOT sufficient.
- (2) This indicates that if  $HG = x$ , then  $AH = 2x$ . If  $x = 2$ , then the side length of the larger square is  $2 + 2(2) = 6$ , but if  $x = 5$ , then the side length of the larger square is  $5 + 2(5) = 15$ ; NOT sufficient.

Taking (1) and (2) together,  $10 = \sqrt{x^2 + (2x)^2}$ , which can be solved for  $x$ . Then taking 3 times the value of  $x$  gives the side length of the larger square.

The correct answer is C;  
both statements together are sufficient.

DS02589

188. Did Insurance Company K have more than \$300 million in total net profits last year?

- (1) Last year Company K paid out \$0.95 in claims for every dollar of premiums collected.
- (2) Last year Company K earned a total of \$150 million in profits from the investment of accumulated surplus premiums from previous years.

### Arithmetic Applied problems

Letting  $R$  and  $E$ , respectively, represent the company's total revenue and total expenses last year, determine if  $R - E > \$300$  million.

- (1) This indicates that, for  $\$x$  in premiums collected, the company paid  $\$0.95x$  in claims, but gives no information about other sources of revenue or other types of expenses; NOT sufficient.
- (2) This indicates that the company's profits from the investment of accumulated surplus premiums was \$150 million last year, but gives no information about other sources of revenue or other types of expenses; NOT sufficient.

Taking (1) and (2) together gives information on profit resulting from collecting premiums and paying claims as well as profit resulting from investments from accumulated surplus premiums, but gives no indication whether there were other sources of revenue or other types of expenses.

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS15349

189. How many hours would it take Pump A and Pump B working together, each at its own constant rate, to empty a tank that was initially full?

- (1) Working alone at its constant rate, Pump A would empty the full tank in 4 hours 20 minutes.
- (2) Working alone, Pump B would empty the full tank at its constant rate of 72 liters per minute.

#### Arithmetic Applied problems

Determine how long it would take Pumps A and B working together, each at its own constant rate, to empty a full tank.

- (1) This indicates how long it would take Pump A to empty the tank, but gives no information about Pump B's constant rate; NOT sufficient.
- (2) This indicates the rate at which Pump B can empty the tank, but without information about the capacity of the tank or Pump A's rate, it is not possible to determine how long both pumps working together would take to empty the tank; NOT sufficient.

Taking (1) and (2) together gives the amount of time it would take Pump A to empty the tank and the rate at which Pump B can empty the tank, but without knowing the capacity of the tank, it is not possible to determine how long the pumps working together would take to empty the tank.

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS04573

190. What is the value of the integer  $N$ ?

- (1)  $101 < N < 103$
- (2)  $202 < 2N < 206$

#### Arithmetic Inequalities

- (1) Given that  $N$  is an integer and  $101 < N < 103$ , it follows that  $N = 102$ ; SUFFICIENT.
- (2) Given that  $N$  is an integer and  $202 < 2N < 206$ , it follows that  $101 < N < 103$  and  $N = 102$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS12033

191. Is  $zw$  positive?

- (1)  $z + w^3 = 20$
- (2)  $z$  is positive.

#### Arithmetic Properties of numbers

- (1) Given that  $z + w^3 = 20$ , if  $z = 1$  and  $w = \sqrt[3]{19}$  then  $z + w^3 = 20$  and  $zw$  is positive. However, if  $z = 20$  and  $w = 0$ , then  $z + w^3 = 20$  and  $zw$  is not positive; NOT sufficient.
- (2) Given that  $z$  is positive, if  $z = 1$  and  $w = \sqrt[3]{19}$ , then  $zw$  is positive. However, if  $z = 20$  and  $w = 0$ , then  $zw$  is not positive; NOT sufficient.

Taking (1) and (2) together is of no more help than either (1) or (2) taken separately because the same examples were used in both (1) and (2).

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS03006

192. On the scale drawing of a certain house plan, if 1 centimeter represents  $x$  meters, what is the value of  $x$ ?

- (1) A rectangular room that has a floor area of 12 square meters is represented by a region of area 48 square centimeters.
- (2) The 15-meter length of the house is represented by a segment 30 centimeters long.

#### Arithmetic Ratio and proportion

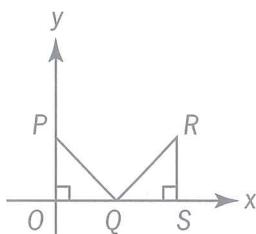
It is given that on the scale drawing, 1 centimeter represents  $x$  meters. Determine the value of  $x$ . Note that 1 cm<sup>2</sup> represents  $x^2$  m<sup>2</sup>.

- (1) This indicates that an area of 12 m<sup>2</sup> is represented by an area of 48 cm<sup>2</sup>. Then,

dividing both 12 and 48 by 48, it follows that an area of  $\frac{12}{48} = \frac{1}{4}$  m<sup>2</sup> is represented by an area of  $\frac{48}{48} = 1$  cm<sup>2</sup> and so  $x^2 = \frac{1}{4}$  or  $x = \frac{1}{2}$ ; SUFFICIENT.

- (2) This indicates that a length of 15 m is represented by a length of 30 cm. Then, dividing both 15 and 30 by 30, it follows that a length of  $\frac{15}{30} = \frac{1}{2}$  m is represented by a length of  $\frac{30}{30} = 1$  cm and so  $x = \frac{1}{2}$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**



DS03939

193. In the rectangular coordinate system above, if  $\Delta OPQ$  and  $\Delta QRS$  have equal area, what are the coordinates of point R?

- (1) The coordinates of point P are (0,12).  
(2)  $OP = OQ$  and  $QS = RS$ .

#### Geometry Coordinate geometry; Triangles

Since the area of  $\Delta OPQ$  is equal to the area of  $\Delta QRS$ , it follows that  $\frac{1}{2}(OQ)(OP) = \frac{1}{2}(QS)(SR)$ , or  $(OQ)(OP) = (QS)(SR)$ . Also, if both  $OQ$  and  $SR$  are known, then the coordinates of point R will be known.

- (1) Given that the y-coordinate of P is 12, it is not possible to determine the coordinates of point R. For example, if  $OQ = QS = SR = 12$ , then the equation  $(OQ)(OP) = (QS)(SR)$  becomes  $(12)(12) = (12)(12)$ , which is true, and the x-coordinate of R is  $OQ + QS = 24$  and the y-coordinate of R is  $SR = 12$ . However, if

$OQ = 12$ ,  $QS = 24$ , and  $SR = 6$ , then the equation  $(OQ)(OP) = (QS)(SR)$  becomes  $(12)(12) = (24)(6)$ , which is true, and the x-coordinate of R is  $OQ + QS = 36$  and the y-coordinate of R is  $SR = 6$ ; NOT sufficient.

- (2) Given that  $OP = OQ$  and  $QS = RS$ , it is not possible to determine the coordinates of point R, since everything given would still be true if all the lengths were doubled, but doing this would change the coordinates of point R; NOT sufficient.

Taking (1) and (2) together, it follows that  $OP = OQ = 12$ . Therefore,  $(OQ)(OP) = (QS)(SR)$  becomes  $(12)(12) = (QS)(SR)$ , or  $144 = (QS)(SR)$ . Using  $QS = RS$  in the last equation gives  $144 = (QS)^2$ , or  $12 = QS$ . Thus,  $OQ = QS = SR = 12$  and point R has coordinates (24,12).

**The correct answer is C;**  
**both statements together are sufficient.**

DS07258

194. In a school that had a total of 600 students enrolled in the junior and senior classes, the students contributed to a certain fund. If all of the juniors but only half of the seniors contributed, was the total amount contributed more than \$740?

- (1) Each junior contributed \$1 and each senior who contributed gave \$3.  
(2) There were more juniors than seniors enrolled in the school.

#### Arithmetic Applied problems

The task in this question is to determine whether the respective statements are sufficient for answering the question of whether the total amount contributed was more than \$740. In making this determination, it is important to remember that we are to use only the information that has been given. For example, it may seem plausible to assume that the number of seniors at the school is roughly equal to the number of juniors. However, because no such information has been provided, we cannot assume that this assumption holds. With this in mind, consider statements 1 and 2.

- (1) If it were the case that half of the 600 students were seniors, then, given that half of the 300 seniors would have contributed

\$3, there would have been  $150 \times \$3 = \$450$  in contributions from the seniors and  $300 \times \$1 = \$300$  in contributions from the juniors, for a total of \$750—more than the figure of \$740 with which the question is concerned. However, as noted, we cannot make such an assumption. To test the conditions that we have actually been given, we can consider extreme cases, which are often relatively simple. For example, given the information provided, it is possible that only two of the students are seniors and the other 598 students are juniors. If this were the case, then the contributions from the juniors would be \$598 (\$1 per student) and the contributions from the seniors would be \$3 (\$3 for the one senior who contributes, given that only half of the 2 seniors contribute). The total contributions would then be  $\$598 + \$3 = \$601$ ; NOT sufficient.

- (2) Merely with this statement—and not statement 1—we have no information as to how much the students contributed. We therefore cannot determine the total amount contributed; NOT sufficient.

We still need to consider whether statements 1 and 2 are sufficient *together* for determining whether a minimum of \$740 has been contributed. However, note that the reasoning in connection with statement 1 applies here as well. We considered there the possibility that the 600 students included only two seniors, with the other 598 students being juniors. Because this scenario also satisfies statement 2, we see that statements 1 and 2 taken together are not sufficient.

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS06650

195. How much did credit-card fraud cost United States banks in year X to the nearest \$10 million?

- (1) In year X, counterfeit cards and telephone and mail-order fraud accounted for 39 percent of the total amount that card fraud cost the banks.
- (2) In year X, stolen cards accounted for \$158.4 million, or 16 percent, of the total amount that credit-card fraud cost the banks.

### Arithmetic Percents

- (1) It is given that certain parts of the total fraud cost have a total that is 39% of the total fraud cost, but since no actual dollar amounts are specified, it is not possible to estimate the total fraud cost to the nearest \$10 million; NOT sufficient.
- (2) Given that \$158.4 million represents 16% of the total fraud cost, it follows that the total fraud cost equals \$158.4 million divided by 0.16; SUFFICIENT.

**The correct answer is B;**  
**statement 2 alone is sufficient.**

DS17319

196. Is the positive integer  $n$  odd?

- (1)  $n^2 + (n + 1)^2 + (n + 2)^2$  is even.
- (2)  $n^2 - (n + 1)^2 - (n + 2)^2$  is even.

### Arithmetic Properties of numbers

The positive integer  $n$  is either odd or even. Determine if it is odd.

- (1) This indicates that the sum of the squares of three consecutive integers,  $n^2$ ,  $(n + 1)^2$ , and  $(n + 2)^2$ , is even. If  $n$  is even, then  $n + 1$  is odd and  $n + 2$  is even. It follows that  $n^2$  is even,  $(n + 1)^2$  is odd, and  $(n + 2)^2$  is even and, therefore, that  $n^2 + (n + 1)^2 + (n + 2)^2$  is odd. But, this contradicts the given information, and so,  $n$  must be odd; SUFFICIENT.
- (2) This indicates that  $n^2 - (n + 1)^2 - (n + 2)^2$  is even. Adding the even number represented by  $2(n + 1)^2 + 2(n + 2)^2$  to the even number represented by  $n^2 - (n + 1)^2 - (n + 2)^2$  gives the even number represented by  $n^2 + (n + 1)^2 + (n + 2)^2$ . This is Statement (1); SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS01130

197. In the  $xy$ -plane, circle C has center  $(1, 0)$  and radius 2. If line k is parallel to the  $y$ -axis, is line k tangent to circle C?

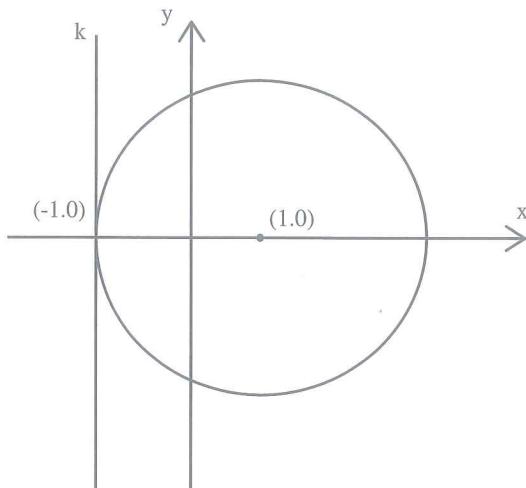
- (1) Line k passes through the point  $(-1, 0)$ .
- (2) Line k passes through the point  $(-1, -1)$ .

DS14170

**Geometry Coordinate geometry**

Can we determine whether line  $k$ , which is parallel to the  $y$ -axis, is tangent to the circle  $C$ ?

- (1) Given that line  $k$  passes through the point  $(-1,0)$ , we can represent the scenario in the following diagram, which is not drawn to scale.



The points  $(1,0)$  and  $(-1,0)$  are two units apart on the  $x$  axis. We therefore know that circle  $C$ , with center  $(1,0)$  and radius 2, passes through the point  $(-1,0)$  and that, given that  $k$  passes through the point  $(-1,0)$ , the circle intersects line  $k$  at this point. Furthermore, the radial line from the center  $(1,0)$  of the circle to point  $(-1,0)$  on the circle rests on the  $x$ -axis and is therefore perpendicular to the  $y$ -axis. Line  $k$ , being parallel to the  $y$ -axis, must also be perpendicular to this radial line. Therefore, because line  $k$  both intersects the circle at a point at which a radial line intersects the circle and is perpendicular to this radial line, we see that that line  $k$  must be tangent to circle  $C$ ; SUFFICIENT.

- (2) The sufficiency of statement 2 follows from the sufficiency of statement 1. For, if  $k$  is perpendicular to the  $y$ -axis and passes through point  $(-1,-1)$ , then  $k$  must also pass through point  $(-1,0)$ ;  $k$  is simply the line  $x = -1$ . The reasoning for statement 1 now applies; SUFFICIENT.

**The correct answer is D;**  
**Each statement alone is sufficient.**

198. Company X's profits this year increased by 25% over last year's profits. Was the dollar amount of Company X's profits this year greater than the dollar amount of Company Y's?

- (1) Last year, the ratio of Company Y's profits to Company X's profits was  $5:2$ .
- (2) Company Y experienced a 40% drop in profits from last year to this year.

**Algebra Applied problems**

Let  $P_X$  and  $P'_X$ , respectively, be the profits of Company X last year and this year, and let  $P_Y$  and  $P'_Y$ , respectively, be the profits of Company Y last year and this year. Then  $P'_X = 1.25P_X$ . Is  $P'_X > P'_Y$ ?

- (1) Given that  $\frac{P_Y}{P_X} = \frac{5}{2}$ , it is not possible to determine whether  $P'_X > P'_Y$  because nothing is known about the value of  $P'_Y$  other than  $P'_Y$  is positive; NOT sufficient.
- (2) Given that  $P'_Y = 0.6P_Y$ , it is not possible to determine whether  $P'_X > P'_Y$  because nothing is known that relates the profits of Company X for either year to the profits of Company Y for either year; NOT sufficient.

Taking (1) and (2) together, it is given that

$$P'_X = 1.25P_X \text{ and from (1) it follows that } \frac{P_Y}{P_X} = \frac{5}{2},$$

or  $P_X = \frac{2}{5}P_Y$ , and thus  $P'_X = (1.25)\left(\frac{2}{5}P_Y\right)$ . From

(2) it follows that  $P'_Y = 0.6P_Y$ , or  $P_Y = \frac{1}{0.6}P'_Y$ ,

and thus  $P'_X = (1.25)\left(\frac{2}{5}\right)\left(\frac{1}{0.6}P'_Y\right)$ . Since

the last equation expresses  $P'_X$  as a specific number times  $P'_Y$ , it follows that it can be determined whether or not  $P'_X > P'_Y$ . Note that

$$(1.25)\left(\frac{2}{5}\right)\left(\frac{1}{0.6}\right) = \left(\frac{5}{4}\right)\left(\frac{2}{5}\right)\left(\frac{5}{3}\right) = \frac{5}{6}, \text{ and so the}$$

answer to the question "Is  $P'_X > P'_Y$ " is no.

**The correct answer is C;**  
**both statements together are sufficient.**

DS09385

199. For all  $x$ , the expression  $x^*$  is defined to be  $ax + a$ , where  $a$  is a constant. What is the value of  $2^*$ ?

- (1)  $3^* = 2$   
 (2)  $5^* = 3$

### Algebra Linear equations

Determine the value of  $2^* = (a)(2) + a = 3a$ , or equivalently, determine the value of  $a$ .

- (1) Given that  $3^* = 2$ , it follows that  $(a)(3) + a = 2$ , or  $4a = 2$ , or  $a = \frac{1}{2}$ ; SUFFICIENT.  
 (2) Given that  $5^* = 3$ , it follows that  $(a)(5) + a = 3$ , or  $6a = 3$ , or  $a = \frac{1}{2}$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS09260

200. Is  $k + m < 0$ ?

- (1)  $k < 0$   
 (2)  $km > 0$

### Arithmetic Properties of numbers

- (1) Given that  $k$  is negative, it is not possible to determine whether  $k + m$  is negative. For example, if  $k = -2$  and  $m = 1$ , then  $k + m$  is negative. However, if  $k = -2$  and  $m = 3$ , then  $k + m$  is not negative; NOT sufficient.  
 (2) Given that  $km$  is positive, it is not possible to determine whether  $k + m$  is negative. For example, if  $k = -2$  and  $m = -1$ , then  $km$  is positive and  $k + m$  is negative. However, if  $k = 2$  and  $m = 1$ , then  $km$  is positive and  $k + m$  is not negative; NOT sufficient.

Taking (1) and (2) together,  $k$  is negative and  $km$  is positive, it follows that  $m$  is negative. Therefore, both  $k$  and  $m$  are negative, and hence  $k + m$  is negative.

**The correct answer is C;**  
**both statements together are sufficient.**

DS08352

201. The symbol  $\Delta$  represents which one of the following operations: addition, subtraction, or multiplication?

- (1)  $a \Delta (b \Delta c) \neq a \Delta (c \Delta b)$  for some numbers  $a$ ,  $b$ , and  $c$ .  
 (2)  $a \Delta (b \Delta c) \neq (a \Delta b) \Delta c$  for some numbers  $a$ ,  $b$ , and  $c$ .

### Arithmetic Arithmetic operations

Can we determine which of the operations—addition, subtraction, or multiplication—is the operation  $\Delta$ ?

- (1) Given the condition that  $\Delta$  has the property that, for some numbers  $a$ ,  $b$ , and  $c$ ,  $a \Delta (b \Delta c) \neq a \Delta (c \Delta b)$ , we can infer that, for some numbers  $b$  and  $c$ ,  $b \Delta c \neq c \Delta b$ . Both addition and multiplication have the commutative property, whereby, for any numbers  $x$  and  $y$ ,  $x + y = y + x$  and  $x \times y = y \times x$ . For example,  $7 + 2 = 9 = 2 + 7$ , and  $7 \times 2 = 14 = 2 \times 7$ . We thus see that, for all numbers  $x$ ,  $y$ , and  $z$ , both of the statements  $x + (y + z) = x + (z + y)$  and  $x \times (y \times z) = x \times (z \times y)$  are true. The operation  $\Delta$  therefore cannot be addition or multiplication.

Subtraction, on the other hand, lacks the commutative property; for example,  $7 - 2 = 5$  and  $2 - 7 = -5$ . The operation  $\Delta$  could therefore be subtraction. Subtraction is therefore the one operation among addition, subtraction, and multiplication that satisfies statement 1; SUFFICIENT.

- (2) The reasoning in this case is similar to the reasoning for statement 1, but concerning the associative property rather than the commutative property. Both addition and multiplication have this property. For any numbers  $x$ ,  $y$ , and  $z$ , the statements  $x + (y + z) = (x + y) + z$  and  $x \times (y \times z) = (x \times y) \times z$  are always true. For example, in the case of multiplication,  $2 \times (3 \times 5) = 2 \times 15 = 30 = 6 \times 5 = (2 \times 3) \times 5$ . However, in contrast to addition and multiplication, the operation of subtraction does not have the associative property. For example, for the numbers 2, 3, and 5,  $2 - (3 - 5) = 2 - (-2) = 4$ , whereas  $(2 - 3) - 4 = -1 - 4 = -5$ . Subtraction is therefore the one operation among addition, subtraction, and multiplication that satisfies statement 2; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS05989

202. What is the value of  $2^x + 2^{-x}$ ?

- (1)  $x > 0$
- (2)  $4^x + 4^{-x} = 23$

### Algebra Equations

Can we determine the value of  $2^x + 2^{-x}$ ?

- (1) The condition  $x > 0$  by itself is not sufficient for determining the value of  $2^x + 2^{-x}$ . For example, if  $x = 1$ , then  $2^x + 2^{-x} = 2^1 + 2^{-1} = 2\frac{1}{2}$ . And if  $x = 2$ , then  $2^x + 2^{-x} = 2^2 + 2^{-2} = 4\frac{1}{4}$ ; NOT sufficient.
- (2) Given  $4^x + 4^{-x} = 23$ , it may be tempting to reason that this is an equation with only one unknown, and that it is therefore possible to determine the value of  $x$  and then the value of  $2^x + 2^{-x}$ . However, this reasoning can often produce erroneous results. For example the equation  $(y - 1)(y - 3) = 0$  has only one unknown but is consistent with two values for  $y$  (1 and 3). To be sure that the statement  $4^x + 4^{-x} = 23$  is sufficient for determining the value of  $2^x + 2^{-x}$ , consider first the square of  $2^x + 2^{-x}$ .

$$\begin{aligned}(2^x + 2^{-x})^2 &= (2^x)^2 + (2^{-x})^2 + 2(2^x)(2^{-x}) \\&= 2^{2x} + 2^{-2x} + 2(2^{x-x}) \\&= (2^2)^x + (2^{-2})^x + 2 \\&= 4^x + 4^{-x} + 2.\end{aligned}$$

So  $4^x + 4^{-x} = (2^x + 2^{-x})^2 - 2$ . The condition that  $4^x + 4^{-x} = 23$  thus becomes

$(2^x + 2^{-x})^2 - 2 = 23$ , or  $(2^x + 2^{-x})^2 = 25$ . And because we know that  $2^x + 2^{-x} > 0$  (because both  $2^x > 0$  and  $2^{-x} > 0$ ), we see that statement 1 implies  $(2^x + 2^{-x}) = \sqrt{25} = 5$ ; SUFFICIENT.

The correct answer is B;  
statement 2 alone is sufficient.

DS13457

203. What is the ratio of  $c$  to  $d$ ?

- (1) The ratio of  $3c$  to  $3d$  is 3 to 4.
- (2) The ratio of  $c + 3$  to  $d + 3$  is 4 to 5.

### Arithmetic Ratio and proportion

Determine the value of  $\frac{c}{d}$ .

- (1) Given that  $\frac{3c}{3d} = \frac{3}{4}$ , it follows that  $\frac{3c}{3d} = \frac{c}{d} = \frac{3}{4}$ ; SUFFICIENT.
- (2) Given that  $\frac{c+3}{d+3} = \frac{4}{5}$ , then it is not possible to determine the value of  $\frac{c}{d}$ . For example, if  $c = 1$  and  $d = 2$ , then  $\frac{c+3}{d+3} = \frac{4}{5}$  and  $\frac{c}{d} = \frac{1}{2}$ . However, if  $c = 5$  and  $d = 7$ , then  $\frac{c+3}{d+3} = \frac{8}{10} = \frac{4}{5}$  and  $\frac{c}{d} = \frac{5}{7}$ ; NOT sufficient.

The correct answer is A;  
statement (1) alone is sufficient.

DS15099

204. A candle company determines that, for a certain specialty candle, the supply function is  $p = m_1x + b_1$  and the demand function is  $p = m_2x + b_2$ , where  $p$  is the price of each candle,  $x$  is the number of candles supplied or demanded, and  $m_1$ ,  $m_2$ ,  $b_1$ , and  $b_2$  are constants. At what value of  $x$  do the graphs of the supply function and demand function intersect?

- (1)  $m_1 = -m_2 = 0.005$
- (2)  $b_2 - b_1 = 6$

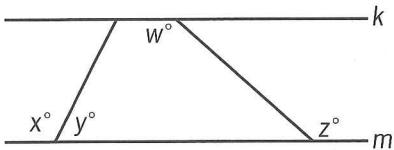
### Algebra First-degree equations

The graphs will intersect at the value of  $x$  such that  $m_1x + b_1 = m_2x + b_2$  or  $(m_1 - m_2)x = b_2 - b_1$ .

- (1) This indicates that  $m_1 = -m_2 = 0.005$ . It follows that  $m_1 - m_2 = 0.01$ , and so  $0.01x = b_2 - b_1$  or  $x = 100(b_2 - b_1)$ , which can vary as the values of  $b_2$  and  $b_1$  vary; NOT sufficient.
- (2) This indicates that  $b_2 - b_1 = 6$ . It follows that  $(m_1 - m_2)x = 6$ . This implies that  $m_1 \neq m_2$ , and so  $x = \frac{b_2 - b_1}{m_1 - m_2} = \frac{6}{m_1 - m_2}$ , which can vary as the values of  $m_1$  and  $m_2$  vary; NOT sufficient.

Taking (1) and (2) together,  $m_1 - m_2 = 0.01$  and  $b_2 - b_1 = 6$  and so the value of  $x$  is  $\frac{6}{0.01} = 600$ .

The correct answer is C;  
both statements together are sufficient.

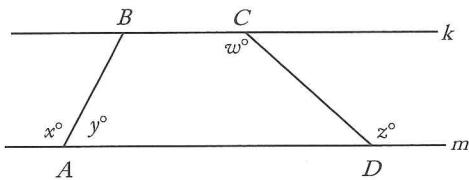


DS12862

205. In the figure shown, lines  $k$  and  $m$  are parallel to each other. Is  $x = z$ ?

- (1)  $x = w$
- (2)  $y = 180 - w$

### Geometry Angles



Since lines  $k$  and  $m$  are parallel, it follows from properties of parallel lines that in the diagram above  $x$  is the degree measure of  $\angle ABC$  in quadrilateral  $ABCD$ . Therefore, because  $y = 180 - x$ , the four interior angles of quadrilateral  $ABCD$  have degree measures  $(180 - x)$ ,  $x$ ,  $w$ , and  $(180 - z)$ .

- (1) Given that  $x = w$ , then because the sum of the degree measures of the angles of the quadrilateral  $ABCD$  is 360, it follows that  $(180 - x) + x + x + (180 - z) = 360$ , or  $x - z = 0$ , or  $x = z$ ; SUFFICIENT.
- (2) Given that  $y = 180 - w$ , then because  $y = 180 - x$ , it follows that  $180 - w = 180 - x$ , or  $x = w$ . However, it is shown in (1) that  $x = w$  is sufficient; SUFFICIENT.

**The correct answer is D;**  
each statement alone is sufficient.

DS13097

206. If  $k$  and  $\ell$  are lines in the  $xy$ -plane, is the slope of  $k$  less than the slope of  $\ell$ ?

- (1) The  $x$ -intercept of line  $k$  is positive, and the  $x$ -intercept of line  $\ell$  is negative.
- (2) Lines  $k$  and  $\ell$  intersect on the positive  $y$ -axis.

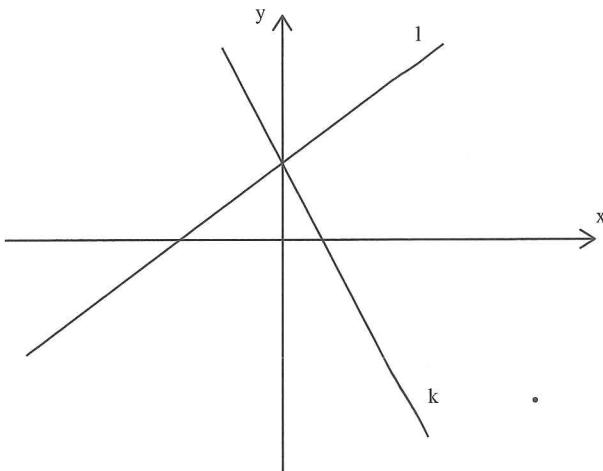
### Geometry Simple coordinate geometry

Can we determine, for lines  $k$  and  $\ell$  in the  $xy$ -plane, whether the slope of  $k$  is less than the slope of  $\ell$ ?

- (1) Given that the  $x$ -intercept of  $k$  is positive and the  $x$ -intercept of line  $\ell$  is negative, we cannot determine whether the slope of  $k$  is less than the slope of  $\ell$ . For example, in the case of line  $k$ , we have only been told where (within a certain range) line  $k$  intersects another line (the  $x$ -axis). Although a line with only a single  $x$ -intercept would not be horizontal, the line  $k$  could have any non-horizontal slope. Likewise in the case of line  $\ell$ . For example, the slope of  $k$  could be positive and the slope of  $\ell$  negative, or vice versa; NOT sufficient.

- (2) Given that  $k$  and  $\ell$  intersect on the positive  $y$ -axis, we cannot determine whether the slope of  $k$  is less than the slope of  $\ell$ . The point here is the same as the point with statement 1. With statement 2, we have only been given, for each of lines  $k$  and  $\ell$ , a condition on where the two lines intersect. And, given only that a line passes through a particular point (and regardless of whether another line happens to pass through this point), the line could have any slope. For example, again, the slope of  $k$  could be positive and the slope of  $\ell$  negative, or vice versa; NOT sufficient.

Considering statements 1 and 2 together, we have, for each of lines  $k$  and  $\ell$ , a condition on two of the points that the line passes through. As illustrated in the diagram, the two conditions together are sufficient for determining the relationship in question.



Because  $k$  intersects the positive  $y$ -axis and the positive  $x$ -axis, its slope must be downward (negative). And because  $\ell$  intersects the negative  $x$ -axis and the positive  $y$ -axis, its slope must be

upward (positive). The slope of  $k$  is therefore less than the slope of  $l$ .

**The correct answer is C;**  
**both statements together are sufficient.**

DS09642

207. When the wind speed is 9 miles per hour, the wind-chill factor  $w$  is given by

$$w = -17.366 + 1.19t,$$

where  $t$  is the temperature in degrees Fahrenheit. If at noon yesterday the wind speed was 9 miles per hour, was the wind-chill factor greater than 0?

- (1) The temperature at noon yesterday was greater than 10 degrees Fahrenheit.
- (2) The temperature at noon yesterday was less than 20 degrees Fahrenheit.

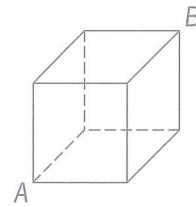
### Algebra Applied problems

Determine whether  $-17.366 + 1.19t$  is greater than 0.

- (1) Given that  $t > 10$ , it follows that  $-17.366 + 1.19t > -17.366 + 1.19(10)$ , or  $-17.366 + 1.19t > -5.466$ . However, it is not possible to determine whether  $-17.366 + 1.19t$  is greater than 0. For example, if  $t = 19$ , then  $-17.366 + 1.19t = 5.244$  is greater than 0. However, if  $t = 11$ , then  $-17.366 + 1.19t = -4.276$ , which is not greater than 0; NOT sufficient.
- (2) Given that  $t < 20$ , the same examples used in (1) show that it is not possible to determine whether  $-17.366 + 1.19t$  is greater than 0; NOT sufficient.

Taking (1) and (2) together is of no more help than either (1) or (2) taken separately because the same examples were used in both (1) and (2).

**The correct answer is E;**  
**both statements together are still not sufficient.**



DS08852

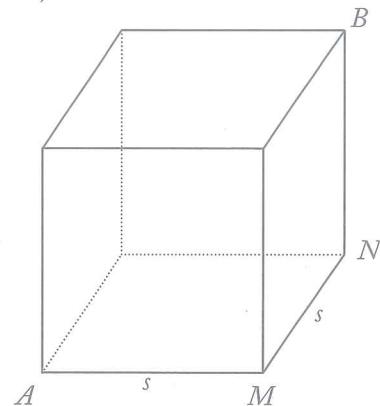
208. What is the volume of the cube above?

- (1) The surface area of the cube is 600 square inches.
- (2) The length of diagonal  $AB$  is  $10\sqrt{3}$  inches.

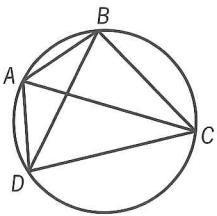
### Geometry Volume

This problem can be solved by determining the side length,  $s$ , of the cube.

- (1) This indicates that  $6s^2 = 600$ , from which it follows that  $s^2 = 100$  and  $s = 10$ ; SUFFICIENT.
- (2) To determine diagonal  $AB$ , first determine diagonal  $AN$  by applying the Pythagorean theorem to  $\triangle AMN$ :  $AN = \sqrt{s^2 + s^2} = \sqrt{2s^2}$ . Now determine  $AB$  by applying the Pythagorean theorem to  $\triangle ANB$ :  $AB = \sqrt{(AN)^2 + (NB)^2} = \sqrt{2s^2 + s^2} = \sqrt{3s^2} = s\sqrt{3}$ . It is given that  $AB = 10\sqrt{3}$ , and so  $s = 10$ ; SUFFICIENT.



**The correct answer is D;**  
**each statement alone is sufficient.**



DS03989

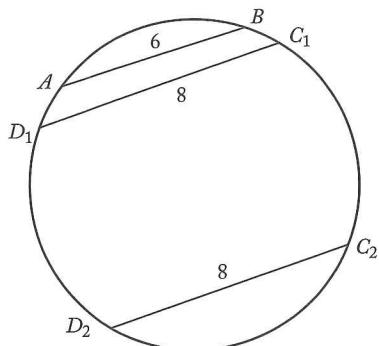
209. In the figure shown, quadrilateral  $ABCD$  is inscribed in a circle of radius 5. What is the perimeter of quadrilateral  $ABCD$ ?

- (1) The length of  $AB$  is 6 and the length of  $CD$  is 8.
- (2)  $AC$  is a diameter of the circle.

**Geometry Quadrilaterals; Perimeter; Pythagorean theorem**

Determine the perimeter of quadrilateral  $ABCD$ , which is given by  $AB + BC + CD + DA$ .

- (1) This indicates that  $AB = 6$  and  $CD = 8$ , but gives no information about  $BC$  or  $DA$ .



For example, the perimeter of  $ABC_1D_1$  is clearly different than the perimeter of  $ABC_2D_2$  and  $\overline{CD}$  could be positioned where  $\overline{C_1D_1}$  is on the diagram or it could be positioned where  $\overline{C_2D_2}$  is on the diagram; NOT sufficient.

- (2) This indicates that  $AC = 2(5) = 10$  since  $AC$  is a diameter of the circle and the radius of the circle is 5. It also indicates that  $\angle ABC$  and  $\angle ADC$  are right angles since each is inscribed in a semicircle. However, there is no information about  $AB$ ,  $BC$ ,  $CD$ , or  $DA$ . For example, if  $AB = CD = 6$ , then  $BC = DA = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$  and the perimeter of  $ABCD$  is  $2(6 + 8) = 28$ . However, if  $AB = DA = 2$ , then  $BC = CD = \sqrt{10^2 - 2^2} = \sqrt{96}$  and the perimeter of  $ABCD = 2(2 + \sqrt{96})$ ; NOT sufficient.

Taking (1) and (2) together,  $\triangle ABC$  is a right triangle with  $AC = 10$  and  $AB = 6$ . It follows from the Pythagorean theorem that  $BC = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$ . Likewise,  $\triangle ADC$  is a right triangle with  $AC = 10$  and  $CD = 8$ . It follows from the Pythagorean theorem that  $DA = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$ . Thus, the perimeter of quadrilateral  $ABCD$  can be determined.

**The correct answer is C;  
both statements together are sufficient.**

DS05766

210. How many members of a certain legislature voted against the measure to raise their salaries?

- (1)  $\frac{1}{4}$  of the members of the legislature did not vote on the measure.
- (2) If 5 additional members of the legislature had voted against the measure, then the fraction of members of the legislature voting against the measure would have been  $\frac{1}{3}$ .

**Arithmetic Ratio and proportion**

The task in this question is to determine whether, on the basis of statements 1 and 2, it is possible to calculate the number of members of the legislature who voted against a certain measure.

- (1) This statement, that  $\frac{1}{4}$  of the members of the legislature did not vote on the measure, is compatible with any number of members of the legislature voting against the measure. After all, any number among the  $\frac{3}{4}$  of the remaining members could have voted against the measure. Furthermore, based on statement 1, we do not know the number of members of the legislature (although we do know, based on this statement, that the number of members of the legislature is divisible by 4); NOT sufficient.
- (2) This statement describes a scenario, of 5 additional members of the legislature voting against the measure, and stipulates that  $\frac{1}{3}$  of the members of the legislature would have voted against the measure in the scenario. Given this condition, we know that the number of members of the legislature was divisible by 3, and that the

legislature had at least 15 members (to allow for the “5 additional members of the legislature” that could have voted against the measure, for a total of  $\frac{1}{3}$  of the members voting against it). However, beyond this we know essentially nothing from statement 2. In particular, depending on the number of members of the legislature (which we have not been given), any number of members could have voted against the measure. For example, exactly one member could have voted against the measure, in which case the legislature would have had  $(1 + 5) \times 3 = 18$  members. Exactly two members could have voted against the measure, in which case the legislature would have had  $(2 + 5) \times 3 = 21$  members, and so on for 3 members voting against, 4 members voting against, etc.; NOT sufficient.

Considering the statements 1 and 2 together, the reasoning is similar to the reasoning for statement 2, but with the further condition that the total number of members of the legislature is divisible by 12 (so as to allow that both exactly  $\frac{1}{4}$  of the members did not vote on the measure while exactly  $\frac{1}{3}$  could have voted against the measure). For example, it could have been the case that the legislature had 24 members. In this case,  $\frac{1}{3}$  of the members would have been 8 members, and, consistent with statements 1 and 2, 3 of the members (8 – 5) could have voted against the measure. Or the legislature could have had 36 members, in which case, consistent with statements 1 and 2,  $\frac{1}{3}(36) - 5 = 12 - 5 = 7$  members could have voted against the measure.

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS05986

211. If  $y \neq 0$ , is  $|x| = 1$ ?

(1)  $x = \frac{y}{|y|}$

(2)  $|x| = -x$

**Algebra Absolute value**Can we determine whether  $|x| = 1$ ?

- (1) Given that  $x = \frac{y}{|y|}$ , we consider two cases:  
 $y > 0$  and  $y < 0$ . If  $y > 0$ , then  $|y| = y$  and  $\frac{y}{|y|} = \frac{y}{y} = 1$ . So if  $y > 0$ , then  $x = 1$  and, of course,  $|x| = 1$ . If  $y < 0$ , then  $|y| = (-1)y$  and  $x = \frac{y}{|y|} = \frac{y}{(-1)y} = (-1)\frac{y}{y} = (-1)(1) = -1$ . So  $|x| = 1$ . If  $y < 0$ , then  $|x| = 1$ . In both of the two cases,  $|x| = 1$ ; SUFFICIENT.
- (2) Given that  $|x| = -x$ , all we know is that  $x$  is not positive. For example, both  $-4$  and  $-5$  satisfy this condition on  $x$ :  $|-4| = 4 = -(-4)$  and  $|-5| = 5 = -(-5)$ ; NOT sufficient.

**The correct answer is A;**  
**statement 1 alone is sufficient.**

DS08306

212. If  $x$  is a positive integer, what is the value of  $x$ ?

(1)  $x^2 = \sqrt{x}$

(2)  $\frac{n}{x} = n$  and  $n \neq 0$ .

**Algebra Operations with radicals**

- (1) It is given that  $x$  is a positive integer. Then,

$$\begin{aligned} x^2 &= \sqrt{x} && \text{given} \\ x^4 &= x && \text{square both sides} \\ x^4 - x &= 0 && \text{subtract } x \text{ from both sides} \\ x(x-1)(x^2+x+1) &= 0 && \text{factor left side} \end{aligned}$$

Thus, the positive integer value of  $x$  being sought will be a solution of this equation. One solution of this equation is  $x = 0$ , which is not a positive integer. Another solution is  $x = 1$ , which is a positive integer. Also,  $x^2 + x + 1$  is a positive integer for all positive integer values of  $x$ , and so  $x^2 + x + 1 = 0$  has no positive integer solutions. Thus, the only possible positive integer value of  $x$  is 1; SUFFICIENT.

- (2) It is given that  $n \neq 0$ . Then,

$$\frac{n}{x} = n \quad \text{given}$$

$$n = nx \quad \text{multiply both sides by } x$$

$$1 = x \quad \text{divide both sides by } n, \text{ where } n \neq 0$$

Thus,  $x = 1$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS07568

213. Is the median of the five numbers  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  equal to  $d$ ?

- (1)  $a < c < e$
- (2)  $b < d < c$

### Arithmetic Statistics

Determine if the median of the five numbers,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , is equal to  $d$ .

- (1) This indicates that  $a < c < e$ , but does not indicate a relationship of  $b$  and  $d$  with  $a$ ,  $c$ , and  $e$ . For example, if  $a = 5$ ,  $b = 1$ ,  $c = 10$ ,  $d = 7$ , and  $e = 15$ , then  $a < c < e$ , and  $d$  is the median. However, if  $a = 5$ ,  $b = 1$ ,  $c = 10$ ,  $d = 2$ , and  $e = 15$ , then  $a < c < e$ , and  $a$ , not  $d$ , is the median; NOT sufficient.
- (2) This indicates that  $b < d < c$ , but does not indicate a relationship of  $a$  and  $e$  with  $b$ ,  $d$ , and  $c$ . For example, if  $a = 5$ ,  $b = 1$ ,  $c = 10$ ,  $d = 7$ , and  $e = 15$ , then  $b < d < c$ , and  $d$  is the median. However, if  $a = 5$ ,  $b = 1$ ,  $c = 10$ ,  $d = 2$ , and  $e = 15$ , then  $b < d < c$ , and  $a$ , not  $d$ , is the median; NOT sufficient.

Taking (1) and (2) together is of no more help than either (1) or (2) taken separately since the same examples used to show that (1) is not sufficient also show that (2) is not sufficient.

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS10383

214. During a certain bicycle ride, was Sherry's average speed faster than 24 kilometers per hour? (1 kilometer = 1,000 meters)

- (1) Sherry's average speed during the bicycle ride was faster than 7 meters per second.
- (2) Sherry's average speed during the bicycle ride was slower than 8 meters per second.

### Arithmetic Applied problems

This problem can be solved by converting 24 kilometers per hour into meters per second. First, 24 kilometers is equivalent to 24,000 meters

and 1 hour is equivalent to 3,600 seconds. Then, traveling 24 kilometers in 1 hour is equivalent to traveling 24,000 meters in 3,600 seconds, or  $\frac{24,000}{3,600} = 6\frac{2}{3}$  meters per second.

- (1) This indicates that Sherry's average speed was faster than 7 meters per second, which is faster than  $6\frac{2}{3}$  meters per second and, therefore, faster than 24 kilometers per hour; SUFFICIENT.
- (2) This indicates that Sherry's average speed was slower than 8 meters per second. Her average speed could have been 7 meters per second (since  $7 < 8$ ), in which case her average speed was faster than  $6\frac{2}{3}$  meters per second and, therefore, faster than 24 kilometers per hour. Or her average speed could have been 5 meters per second (since  $5 < 8$ ), in which case her average speed was not faster than  $6\frac{2}{3}$  meters per second and, therefore, not faster than 24 kilometers per hour; NOT sufficient.

**The correct answer is A;**  
**statement 1 alone is sufficient.**

DS13907

215. Working together, Rafael and Salvador can tabulate a certain set of data in 2 hours. In how many hours can Rafael tabulate the data working alone?

- (1) Working alone, Rafael can tabulate the data in 3 hours less time than Salvador, working alone, can tabulate the data.
- (2) Working alone, Rafael can tabulate the data in  $\frac{1}{2}$  the time that Salvador, working alone, can tabulate the data.

### Algebra Simultaneous equations

We are given that Rafael and Salvador, working together, can tabulate the set of data in two hours. That is, if Rafael tabulates data at the rate of  $R$  units of data per hour and Salvador tabulates the data at the rate of  $S$  units per hour, then, if the set of data is made up of  $D$  units, then  $2R + 2S = D$ . Can we determine how much time, in hours, it takes Rafael to tabulate the data if working alone?

- (1) First of all, note that the choice of units used to measure the amounts of data doesn't matter. In particular, we can define one unit of data to be  $D$ . Thus,  $2R + 2S = 1$ . With this in mind, consider the condition that Rafael, when working alone, can tabulate the data in 3 hours less time than Salvador can when working alone. Given that Rafael tabulates  $R$  units of data per unit time, he takes  $\frac{1}{R}$  units of time to tabulate one unit of data. Similarly, Salvador takes  $\frac{1}{S}$  units of time to tabulate one unit of data. This unit, as defined, is simply the entire set of data. Our given condition thus becomes  $\frac{1}{R} = \frac{1}{S} - 3$ , and we have the set of simultaneous equations made up of this equation and the equation  $2R + 2S = 1$ .

One way to determine the number of hours it would take Rafael to tabulate the data is to solve one of these equations for  $S$  and then substitute this solution into the other equation. Considering the first of these equations, we multiply both sides by  $RS$  and then manipulate the result as follows.

$$\begin{aligned} S &= R - 3RS \\ S + 3RS &= R \\ S(1 + 3R) &= R \\ S &= \frac{R}{1 + 3R} \end{aligned}$$

Substituting into the equation  $2R + 2S = 1$ ,

$$2R + \frac{2R}{1 + 3R} = 1$$

Multiplying both sides by  $1 + 3R$  to eliminate the fraction,

$$\begin{aligned} 2R(1 + 3R) + 2R &= 1 + 3R \\ 2R + 6R^2 &= 1 + R \\ 6R^2 + R - 1 &= 0 \\ (3R - 1)(2R + 1) &= 0 \end{aligned}$$

This equation has two solutions,  $-\frac{1}{2}$  and  $\frac{1}{3}$ . However, because the rate  $R$  cannot be negative, we find that Rafael tabulates  $\frac{1}{3}$  of a unit of data every hour. Since one unit is the entire set, it takes Rafael 3 hours to tabulate the entire set; SUFFICIENT.

- (2) We are given that Rafael, working alone, can tabulate the data in  $\frac{1}{2}$  the amount of time it takes Salvador, working alone, to tabulate the data. As in the discussion of statement 1, we have that Rafael tabulates  $R$  units of data every hour, and takes  $\frac{1}{R}$  hours to tabulate one unit of data. Similarly, it takes Salvador  $\frac{1}{S}$  hours to tabulate one unit of data. One unit of data has been defined to be the size of the entire set to be tabulated, so statement 2 becomes the expression

$$\frac{1}{R} = \frac{1}{2} \times \frac{1}{S} = \frac{1}{2S}$$

We thus have  $2S = R$ . Substituting this value for  $2S$  in the equation  $2R + 2S = 1$ , we have  $R + 2R = 1$ , and  $3R = 1$ . Solving for  $R$  we get  $\frac{1}{3}$ ; SUFFICIENT.

Note that, for both statements 1 and 2, it would have been possible to stop calculating once we had determined whether it was possible to find a unique value for  $R$ . The ability to make such judgments accurately is part of what the test has been designed to measure.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS04039

216. If  $x$  and  $y$  are integers, what is the value of  $x$ ?

- (1)  $xy = 1$   
(2)  $x \neq -1$

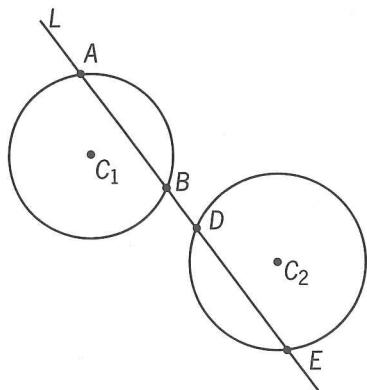
### Arithmetic Properties of integers

Given that  $x$  and  $y$  are integers, determine the value of  $x$ .

- (1) If  $x = y = -1$ , then  $xy = 1$ , and if  $x = y = 1$ , then  $xy = 1$ ; NOT sufficient.  
(2) Given that  $x \neq -1$ , the value of  $x$  could be any other integer; NOT sufficient.

Taking (1) and (2) together, since the two possibilities for the value of  $x$  are  $x = -1$  or  $x = 1$  by (1), and  $x \neq -1$  by (2), then  $x = 1$ .

**The correct answer is C;**  
**both statements together are sufficient.**



Note: Figure not drawn to scale.

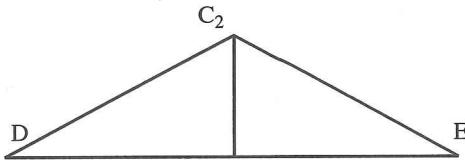
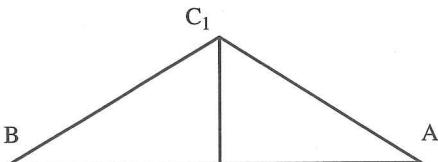
DS18386

217. The figure above shows Line  $L$ , Circle 1 with center at  $C_1$ , and Circle 2 with center at  $C_2$ . Line  $L$  intersects Circle 1 at points  $A$  and  $B$ , Line  $L$  intersects Circle 2 at points  $D$  and  $E$ , and points  $C_1$  and  $C_2$  are equidistant from line  $L$ . Is the area of  $\triangle ABC_1$  less than the area of  $\triangle DEC_2$ ?
- The radius of Circle 1 is less than the radius of Circle 2.
  - The length of chord  $\overline{AB}$  is less than the length of chord  $\overline{DE}$ .

### Geometry Triangles

We are given various elements of information that apply regardless of whether we assume that statements 1, 2, or both are true, and asked whether it is possible, when considering one or both of these statements, to determine if the area of triangle  $\triangle ABC_1$  is less than the area of  $\triangle DEC_2$ .

- Given the condition that the radius of Circle 1 is less than the radius of Circle 2, it may be useful to consider the following diagrams of the triangles, in which they have been rotated so as to have the sides  $AB$  and  $DE$  represented as horizontal and on the bottom. The diagrams are not drawn to scale.



Note that the radius of Circle 1 is equal to (the length)  $C_1B (= C_1A)$  and that the radius of Circle 2 is equal to  $C_2D (= C_2E)$ . Furthermore, because line  $L$  is equidistant from points  $C_1$  and  $C_2$ , we know that the respective heights of the triangles (distances from  $C_1$  and  $C_2$  to the respective bases  $BA$  and  $DE$ ) are the same. However, because (with statement 1) the radius of Circle 1 is less than the radius of Circle 2, we know that  $C_1B$  and  $C_1A$  are less than  $C_2D$  and  $C_2E$ . Because the triangles have the same height, triangle  $\triangle ABC_1$  must be less "wide" than  $\triangle DEC_2$  and must thus have a lesser base (length). And because the area of a triangle is always  $\frac{1}{2} \times \text{base} \times \text{height}$ , we can infer that the area of  $\triangle ABC_1$  is less than the area of  $\triangle DEC_2$ ; SUFFICIENT.

- Given that  $AB$  is less than  $DE$ , we can infer that the area of  $\triangle ABC_1$  is less than the area of  $\triangle DEC_2$ . After all, we know that the heights of the two triangles are the same (because, as discussed in connection with statement 1, line  $L$  is equidistant from  $C_1$  and  $C_2$ ). The formula for the area of a triangle,  $\frac{1}{2} \times \text{base} \times \text{height}$ , thus allows us to make our inference; SUFFICIENT.

**The correct answer is D;  
each statement alone is sufficient.**

DS15938

218. Yesterday between 9:00 a.m. and 6:00 p.m. at Airport X, all flights to Atlanta departed at equally spaced times and all flights to New York City departed at equally spaced times. A flight to Atlanta and a flight to New York City both departed from Airport X at 1:00 p.m. yesterday. Between 1:00 p.m. and 3:00 p.m. yesterday, did another pair of flights to these 2 cities depart from Airport X at the same time?
- Yesterday at Airport X, a flight to Atlanta and a flight to New York City both departed at 10:00 a.m.
  - Yesterday at Airport X, flights to New York City departed every 15 minutes between 9:00 a.m. and 6:00 p.m.

**Arithmetic Applied problems**

It is useful to note that although the departures discussed all lie between 9:00 a.m. and 6:00 p.m., there is no information concerning when the first departures took place during this time other than what is necessary for the information to be consistent. For example, since departures to both Atlanta and New York City took place at 1:00 p.m., the first departure to either of these cities could not have occurred after 1:00 p.m.

- (1) Given that departures to both Atlanta and New York City took place at 10:00 a.m., it is not possible to determine whether simultaneous departures to these cities occurred between 1:00 p.m. and 3:00 p.m. For example, it is possible that departures to both Atlanta and New York City took place every 15 minutes beginning at 9:15 a.m., and thus it is possible that simultaneous departures to both these cities occurred between 1:00 p.m. and 3:00 p.m. However, it is also possible that departures to Atlanta took place every 3 hours beginning at 10:00 a.m. and departures to New York City took place every 15 minutes beginning at 9:15 a.m., and thus it is possible that no simultaneous departures to these cities occurred between 1:00 p.m. and 3:00 p.m.; NOT sufficient.
- (2) Given that departures to New York City took place every 15 minutes, the same examples used in (1) can be used to show that it is not possible to determine whether simultaneous departures to these cities occurred between 1:00 p.m. and 3:00 p.m.; NOT sufficient.

Taking (1) and (2) together, it is still not possible to determine whether simultaneous departures to these cities occurred between 1:00 p.m. and 3:00 p.m. because both (1) and (2) are true for the examples above.

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS07206

219. Of the total number of copies of Magazine X sold last week, 40 percent were sold at full price. What was the total number of copies of the magazine sold last week?

- (1) Last week, full price for a copy of Magazine X was \$1.50 and the total revenue from full-price sales was \$112,500.
- (2) The total number of copies of Magazine X sold last week at full price was 75,000.

**Algebra Applied problems**

For the copies of Magazine X sold last week, let  $n$  be the total number of copies sold and let  $\$p$  be the full price of each copy. Then for Magazine X last week, a total of  $0.4n$  copies were each sold at price  $\$p$ . What is the value of  $n$ ?

- (1) Given that  $\$p = 1.50$  and  $(0.4n)(\$p) = \$112,500$ , it follows that  $(0.4n)(1.5) = 112,500$ , or  $0.6n = 112,500$ , or  $n = \frac{112,500}{0.6}$ ; SUFFICIENT.
- (2) Given that  $0.4n = 75,000$ , it follows that  $n = \frac{75,000}{0.4}$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS11614

220. If  $p$ ,  $s$ , and  $t$  are positive, is  $|ps - pt| > p(s - t)$ ?

- (1)  $p < s$
- (2)  $s < t$

**Algebra Absolute value**

Since  $p$  is positive, it follows that  $|p(s - t)| = |p||s - t| = p|s - t|$ . Therefore, the task is to determine if  $|s - t| > s - t$ . Since  $|s - t| = s - t$  if and only if  $s - t \geq 0$ , it follows that  $|s - t| > s - t$  if and only if  $s - t < 0$ .

- (1) This indicates that  $p < s$  but does not provide information about the relationship between  $s$  and  $t$ . For example, if  $p = 5$ ,  $s = 10$ , and  $t = 15$ , then  $p < s$  and  $s < t$ , but if  $p = 5$ ,  $s = 10$ , and  $t = 3$ , then  $p < s$  and  $s > t$ ; NOT sufficient.
- (2) This indicates that  $s < t$ , or equivalently,  $s - t < 0$ ; SUFFICIENT.

**The correct answer is B;**  
**statement 2 alone is sufficient.**

DS04468

221. Is  $x > y$ ?

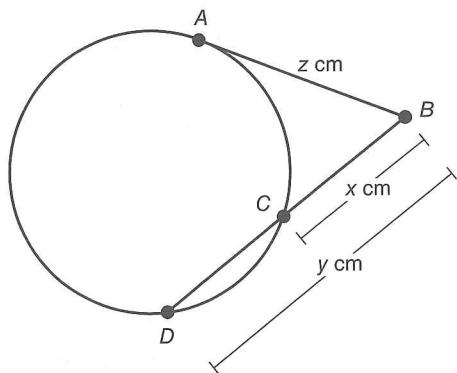
- (1)  $x + y > x - y$   
 (2)  $3x > 2y$

**Algebra Inequalities**

- (1) Given that  $x + y > x - y$ , it follows that  $y > -y$ , or  $2y > 0$ , or  $y > 0$ . However, nothing is known about the value of  $x$ . If  $x = 2$  and  $y = 1$ , then  $x + y > x - y$  and the answer to the question is yes. However, if  $x = 1$  and  $y = 1$ , then  $x + y > x - y$  and the answer to the question is no; NOT sufficient.
- (2) Given that  $3x > 2y$ , then  $x = 2$  and  $y = 1$  is possible and the answer to the question is yes. However, if  $3x > 2y$ , then  $x = 1$  and  $y = 1$  is also possible and the answer to the question is no; NOT sufficient.

Taking (1) and (2) together is of no more help than either (1) or (2) taken separately because the same examples used to show that (1) is not sufficient also show that (2) is not sufficient.

**The correct answer is E;**  
**both statements together are still not sufficient.**



DS17588

222. In the figure above,  $\overline{AB}$ , which has length  $z$  cm, is tangent to the circle at point  $A$ , and  $\overline{BD}$ , which has length  $y$  cm, intersects the circle at point  $C$ . If  $BC = x$  cm and  $z = \sqrt{xy}$ , what is the value of  $x$ ?

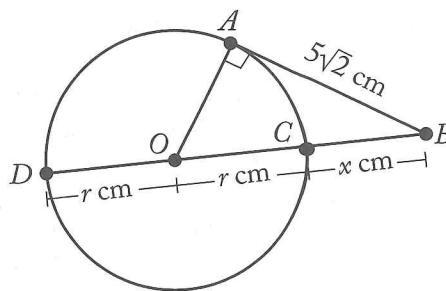
- (1)  $CD = x$  cm  
 (2)  $z = 5\sqrt{2}$

**Geometry Circles**

- (1) Given that  $CD = x$  cm, it is not possible to determine the value of  $x$  because all the given information continues to hold when

all the parts of the figure increase in length by any given nonzero factor; NOT sufficient.

- (2) Given that  $z = 5\sqrt{2}$ , the value of  $x$  will vary when the radius of the circle varies and  $\overline{CD}$  is a diameter and thus passes through the center of the circle. To see this, let  $r$  be the radius, in centimeters, of the circle and let  $O$  be the center of the circle, as shown in the figure below. Then, because  $\overline{CD}$  is a diameter, it follows that  $CD = 2r$  and  $y = x + CD = x + 2r$ . Also,  $\Delta OAB$  is a right triangle and the Pythagorean theorem gives  $(OA)^2 + (AB)^2 = (OB)^2$ , or  $r^2 + (5\sqrt{2})^2 = (x + r)^2$ , or  $r^2 + 50 = x^2 + 2xr + r^2$ , or  $x(x + 2r) = 50$ , which implies that  $xy = z^2$  and  $z = \sqrt{xy}$ , since  $y = x + 2r$  and  $z = 5\sqrt{2}$ . Therefore, if  $z = 5\sqrt{2}$  and  $\overline{CD}$  is a diameter, then  $z = \sqrt{xy}$  holds, and the value of  $x$  can vary.



This can be seen by considering the equation  $x(x + 2r) = 50$ , or  $x = \frac{50}{x + 2r}$ . If the value of  $r$  changes slightly to a new value  $R$ , then the value of  $x$  must also change. Otherwise, there would be two different numbers, namely  $\frac{50}{x + 2r}$  and  $\frac{50}{x + 2R}$ , equal to each other, which is a contradiction; NOT sufficient.

Taking (1) and (2) together,  $y = x + CD = x + x = 2x$  and  $z = 5\sqrt{2}$ , so  $z = \sqrt{xy}$  becomes  $5\sqrt{2} = \sqrt{x(2x)}$ , or  $(5\sqrt{2})^2 = (\sqrt{x(2x)})^2$ , or  $50 = x(2x)$ , or  $x^2 = 25$ , or  $x = 5$ .

**The correct answer is C;**  
**both statements together are sufficient.**

DS15863

223. Is the integer  $n$  a prime number?

- (1)  $24 \leq n \leq 28$
- (2)  $n$  is not divisible by 2 or 3.

### Arithmetic Properties of numbers

Determine if the integer  $n$  is a prime number.

- (1) This indicates that  $n$  is between 24 and 28, inclusive. It follows that the value of  $n$  can be 24, 25, 26, 27, or 28. Each of these is NOT a prime number. Thus, it can be determined that  $n$  is NOT a prime number; SUFFICIENT.
- (2) This indicates that  $n$  is not divisible by 2 or 3. If  $n = 7$ , then  $n$  is not divisible by 2 or 3 and is a prime number. However, if  $n = 25$ , then  $n$  is not divisible by 2 or 3 and is a not prime number since 25 has a factor, namely 5, other than 1 and itself; NOT sufficient.

**The correct answer is A;**  
**statement 1 alone is sufficient.**

DS03615

224. What is the average (arithmetic mean) annual salary of the 6 employees of a toy company?

- (1) If the 6 annual salaries were ordered from least to greatest, each annual salary would be \$6,300 greater than the preceding annual salary.
- (2) The range of the 6 annual salaries is \$31,500.

### Arithmetic Statistics

Can we determine the arithmetic mean of the annual salaries of the 6 employees?

- (1) Given only that the 6 annual salaries can be put into a sequence from least to greatest, with a difference of \$6,300 between adjacent members of the sequence, we can infer certain things about the mean of the salaries. For example, because none of the salaries would be negative, we know from statement 1 that the mean of the salaries is greater than or equal to

$$0 + \$6,300 + \$12,600 + \$18,900 + \$25,200 + \$31,500 .$$

6

(It is not necessary to perform this calculation.) However, depending on what the least of the salaries is—that is, the value at which the sequence of salaries begins—

the average of the salaries could, consistent with condition 1, take on any value greater than this quotient; NOT sufficient.

- (2) Given the statement that the range of the salaries is \$31,500, reasoning similar to the reasoning for statement 1 applies. A difference between least salary and greatest salary of \$31,500 is consistent with any value for the least salary, so long as the greatest salary is \$31,500 greater than the least salary. Furthermore, even if we knew what the least and the greatest salaries are, it would be impossible to determine the mean merely from the range; NOT sufficient.

As reflected in the numerator of the quotient in the discussion of statement 1, we can see that statement 1 implies statement 2. In the sequence of 6 salaries with a difference of \$6,300 between adjacent members of the sequence, the difference between the least salary and the greatest salary is  $5 \times \$6,300 = \$31,500$ . Therefore, because statement 1 is insufficient for determining the mean of the salaries, the combination of statement 1 and statement 2 is also insufficient for determining the mean of the salaries.

**The correct answer is E;**  
**both statements together are not sufficient.**

DS17503

225. In a certain order, the pretax price of each regular pencil was \$0.03, the pretax price of each deluxe pencil was \$0.05, and there were 50% more deluxe pencils than regular pencils. All taxes on the order are a fixed percent of the pretax prices. The sum of the total pretax price of the order and the tax on the order was \$44.10. What was the amount, in dollars, of the tax on the order?

- (1) The tax on the order was 5% of the total pretax price of the order.
- (2) The order contained exactly 400 regular pencils.

### Arithmetic Percents

Let  $n$  be the number of regular pencils in the order and let  $r\%$  be the tax rate on the order as a percent of the pretax price. Then the order contains  $1.5n$  deluxe pencils, the total pretax price of the order is  $(\$0.03)n + (\$0.05)(1.5n) = \$0.105n$ , and the sum of the total pretax price of the order and the tax

on the order is  $\left(1 + \frac{r}{100}\right) (\$0.105n)$ . Given that  $\left(1 + \frac{r}{100}\right) (\$0.105n) = \$44.10$ , what is the value of  $\left(\frac{r}{100}\right) (\$0.105n)$ ?

- (1) Given that  $r = 5$ , then  $\left(1 + \frac{r}{100}\right) (\$0.105n) = \$44.10$  becomes  $(1.05)(0.105n) = 44.10$ , which is a first-degree equation that can be solved for  $n$ . Since the value of  $r$  is known and the value of  $n$  can be determined, it follows that the value of  $\left(\frac{r}{100}\right) (\$0.105n)$  can be determined; SUFFICIENT.

- (2) Given that  $n = 400$ , then

$\left(1 + \frac{r}{100}\right) (\$0.105n) = \$44.10$  becomes  $\left(1 + \frac{r}{100}\right) (0.105)(400) = 44.10$ , which is a first-degree equation that can be solved for  $r$ . Since the value of  $r$  can be determined and the value of  $n$  is known, it follows that the value of  $\left(\frac{r}{100}\right) (\$0.105n)$  can be determined; SUFFICIENT.

The correct answer is D;  
each statement alone is sufficient.

DS06785

226. If  $m$  is an integer greater than 1, is  $m$  an even integer?

- (1) 32 is a factor of  $m$ .  
(2)  $m$  is a factor of 32.

#### Arithmetic Properties of numbers

- (1) Given that 32 is a factor of  $m$ , then each of the factors of 32, including 2, is a factor of  $m$ . Since 2 is a factor of  $m$ , it follows that  $m$  is an even integer; SUFFICIENT.
- (2) Given that  $m$  is a factor of 32 and  $m$  is greater than 1, it follows that  $m = 2, 4, 8, 16$ , or 32. Since each of these is an even integer,  $m$  must be an even integer; SUFFICIENT.

The correct answer is D;  
each statement alone is sufficient.

DS05657

227. If the set  $S$  consists of five consecutive positive integers, what is the sum of these five integers?

- (1) The integer 11 is in  $S$ , but 10 is not in  $S$ .  
(2) The sum of the even integers in  $S$  is 26.

#### Arithmetic Sequences

- (1) This indicates that the least integer in  $S$  is 11 since  $S$  consists of consecutive integers and 11 is in  $S$ , but 10 is not in  $S$ . Thus, the integers in  $S$  are 11, 12, 13, 14, and 15, and their sum can be determined; SUFFICIENT.
- (2) This indicates that the sum of the even integers in  $S$  is 26. In a set of 5 consecutive integers, either two of the integers or three of the integers are even. If there are three even integers, then the first integer in  $S$  must be even. Also, since  $\frac{26}{3} = 8\frac{2}{3}$ , the three even integers must be around 8. The three even integers could be 6, 8, and 10, but are not because their sum is less than 26; or they could be 8, 10, and 12, but are not because their sum is greater than 26. Therefore,  $S$  cannot contain three even integers and must contain only two even integers. Those integers must be 12 and 14 since  $12 + 14 = 26$ . It follows that the integers in  $S$  are 11, 12, 13, 14, and 15, and their sum can be determined; SUFFICIENT.

Alternately, if  $n, n + 1, n + 2, n + 3$ , and  $n + 4$  represent the five consecutive integers and three of them are even, then  $n + (n + 2) + (n + 4) = 26$ , or  $3n = 20$ , or  $n = \frac{20}{3}$ , which is not an integer.

On the other hand, if two of the integers are even, then  $(n + 1) + (n + 3) = 26$ , or  $2n = 22$ , or  $n = 11$ . It follows that the integers are 11, 12, 13, 14, and 15, and their sum can be determined; SUFFICIENT.

The correct answer is D;  
each statement alone is sufficient.

DS17543

228. If  $x > 0$ , what is the value of  $x$ ?

- (1)  $x^3 - x = 0$   
(2)  $\sqrt[3]{x} - x = 0$

### Algebra Factoring; Operations with radical expressions

- (1) Given that  $x^3 - x = 0$ , factoring gives  $x(x^2 - 1) = x(x - 1)(x + 1) = 0$ . Hence,  $x = 0$ ,  $x = 1$ , or  $x = -1$ . Since  $x > 0$ , the value of  $x$  cannot be 0 or  $-1$ , and so  $x = 1$ ; SUFFICIENT.
- (2) Given that  $\sqrt[3]{x} - x = 0$ , it follows that  $\sqrt[3]{x} = x$ , or  $(\sqrt[3]{x})^3 = x^3$ , or  $x = x^3$ . Therefore,  $x^3 - x = 0$  and the discussion in (1) shows that the only positive value of  $x$  is  $x = 1$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS08307

229. A total of 20 amounts are entered on a spreadsheet that has 5 rows and 4 columns; each of the 20 positions in the spreadsheet contains one amount. The average (arithmetic mean) of the amounts in row  $i$  is  $R_i$  ( $1 \leq i \leq 5$ ). The average of the amounts in column  $j$  is  $C_j$  ( $1 \leq j \leq 4$ ). What is the average of all 20 amounts on the spreadsheet?

- (1)  $R_1 + R_2 + R_3 + R_4 + R_5 = 550$   
 (2)  $C_1 + C_2 + C_3 + C_4 = 440$

### Arithmetic Statistics

It is given that  $R_i$  represents the average of the amounts in row  $i$ . Since there are four amounts in each row,  $4R_i$  represents the total of the amounts in row  $i$ . Likewise, it is given that  $C_j$  represents the average of the amounts in column  $j$ . Since there are five amounts in each column,  $5C_j$  represents the total of the amounts in column  $j$ .

- (1) It is given that  $R_1 + R_2 + R_3 + R_4 + R_5 = 550$ , and so  $4(R_1 + R_2 + R_3 + R_4 + R_5) = 4R_1 + 4R_2 + 4R_3 + 4R_4 + 4R_5 = 4(550) = 2,200$ . Therefore, 2,200 is the sum of all 20 amounts (4 amounts in each of 5 rows), and the average of all 20 amounts is  $\frac{2,200}{20} = 110$ ; SUFFICIENT.

- (2) It is given that  $C_1 + C_2 + C_3 + C_4 = 440$ , and so  $5(C_1 + C_2 + C_3 + C_4) = 5C_1 + 5C_2 + 5C_3 + 5C_4 = 5(440) = 2,200$ . Therefore, 2,200 is the sum of all 20 amounts (5 amounts in each of

4 columns), and the average of all 20 amounts is  $\frac{2,200}{20} = 110$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS13132

230. Was the range of the amounts of money that Company Y budgeted for its projects last year equal to the range of the amounts of money that it budgeted for its projects this year?

- (1) Both last year and this year, Company Y budgeted money for 12 projects and the least amount of money that it budgeted for a project was \$400.  
 (2) Both last year and this year, the average (arithmetic mean) amount of money that Company Y budgeted per project was \$2,000.

### Arithmetic Statistics

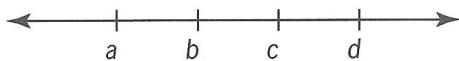
Let  $G_1$  and  $L_1$  represent the greatest and least amounts, respectively, of money that Company Y budgeted for its projects last year, and let  $G_2$  and  $L_2$  represent the greatest and least amounts, respectively, of money that Company Y budgeted for its projects this year. Determine if the range of the amounts of money Company Y budgeted for its projects last year is equal to the range of amounts budgeted for its projects this year; that is, determine if  $G_1 - L_1 = G_2 - L_2$ .

- (1) This indicates that  $L_1 = L_2 = \$400$ , but does not give any information about  $G_1$  or  $G_2$ ; NOT sufficient.  
 (2) This indicates that the average amount Company Y budgeted for its projects both last year and this year was \$2,000 per project, but does not give any information about the least and greatest amounts that it budgeted for its projects either year; NOT sufficient.

Taking (1) and (2) together, it is known that  $L_1 = L_2 = \$400$  and that the average amount Company Y budgeted for its projects both last year and this year was \$2,000 per project, but there is no information about  $G_1$  or  $G_2$ . For example, if, for each year, Company Y budgeted \$400 for each of 2 projects and \$2,320 for each of the 10 others, then (1) and (2) are true and the

range for each year was  $\$2,320 - \$400 = \$1,920$ . However, if, last year, Company Y budgeted \$400 for each of 2 projects and \$2,320 for each of the 10 others, and, this year, budgeted \$400 for each of 11 projects and \$19,600 for 1 project, then (1) and (2) are true, but the range for last year was \$1,920 and the range for this year was  $\$19,600 - \$400 = \$19,200$ .

**The correct answer is E;**  
**both statements together are still not sufficient.**



DS01633

231. If  $a$ ,  $b$ ,  $c$ , and  $d$  are numbers on the number line shown and if the tick marks are equally spaced, what is the value of  $a + c$ ?

- (1)  $a + b = -8$   
(2)  $a + d = 0$

### Algebra Sequences

It is given that the distance between  $a$  and  $b$  is the same as the distance between  $b$  and  $c$ , which is the same as the distance between  $c$  and  $d$ . Letting  $q$  represent this distance, then  $b = a + q$ ,  $c = a + 2q$ , and  $d = a + 3q$ . The value of  $a + c$  can be determined if the value of  $a + (a + 2q) = 2a + 2q$  can be determined.

- (1) It is given that  $a + b = -8$ . Then,  $a + (a + q) = 2a + q = -8$ . From this, the value of  $2a + 2q$  cannot be determined. For example, the values of  $a$  and  $q$  could be  $-5$  and  $2$ , respectively, or they could be  $-6$  and  $4$ , respectively; NOT sufficient.  
(2) It is given that  $a + d = 0$ . Then,  $a + (a + 3q) = 2a + 3q = 0$ . From this, the value of  $2a + 2q$  cannot be determined. For example, the values of  $a$  and  $q$  could be  $-3$  and  $2$ , respectively, or they could be  $-6$  and  $4$ , respectively; NOT sufficient.

Taking (1) and (2) together, adding the equations,  $2a + q = -8$  and  $2a + 3q = 0$  gives  $4a + 4q = -8$  and so  $2a + 2q = \frac{-8}{2} = -4$ .

**The correct answer is C;**  
**both statements together are sufficient.**

DS06067

232. Is  $xm < ym$ ?

- (1)  $x > y$   
(2)  $m < 0$

### Algebra Inequalities

- (1) Given that  $x > y$ , the inequality  $xm < ym$  can be true (for example, if  $m = -1$ , then  $xm < ym$  becomes  $-x < -y$ , or  $x > y$ , which is true by assumption) and it is possible that the inequality  $xm < ym$  can be false (for example, if  $m = 0$ , then  $xm < ym$  becomes  $0 < 0$ , which is false); NOT sufficient.  
(2) Given that  $m < 0$ , the inequality  $xm < ym$  can be true (for example, if  $m = -1$ ,  $x = 2$ , and  $y = 1$ , then  $xm < ym$  becomes  $-2 < -1$ , which is true) and it is possible that the inequality  $xm < ym$  can be false (for example, if  $m = -1$ ,  $x = 1$ , and  $y = 2$ , then  $xm < ym$  becomes  $-1 < -2$ , which is false); NOT sufficient.

Taking (1) and (2) together, multiplying both sides of the inequality  $x > y$  by  $m$  reverses the inequality sign (since  $m < 0$ ), which gives  $xm < ym$ .

**The correct answer is C;**  
**both statements together are sufficient.**

DS02899

233. If  $y = x^2 - 6x + 9$ , what is the value of  $x$ ?

- (1)  $y = 0$   
(2)  $x + y = 3$

### Algebra Second-degree equations

Given that  $y = x^2 - 6x + 9 = (x - 3)^2$ , what is the value of  $x$ ?

- (1) Given that  $y = 0$ , it follows that  $(x - 3)^2 = 0$ , or  $x = 3$ ; SUFFICIENT.  
(2) Given that  $x + y = 3$ , or  $y = 3 - x$ , then  $x = 3$  and  $y = 0$  are possible, since  $y = (x - 3)^2$  becomes  $0 = (3 - 3)^2$ , which is true, and  $y = 3 - x$  becomes  $0 = 3 - 3$ , which is true. However,  $x = 2$  and  $y = 1$  are also possible, since  $y = (x - 3)^2$  becomes  $1 = (2 - 3)^2$ , which is true, and  $y = 3 - x$  becomes  $1 = 3 - 2$ , which is true; NOT sufficient.

Note: The values for  $x$  and  $y$  used in (2) above can be found by solving  $(x - 3)^2 = 3 - x$ , which can be rewritten as  $x^2 - 6x + 9 = 3 - x$ , or  $x^2 - 5x + 6 = 0$ , or  $(x - 3)(x - 2) = 0$ .

**The correct answer is A;**  
**statement 1 alone is sufficient.**

DS06810

234. What is the probability that Lee will make exactly 5 errors on a certain typing test?

- (1) The probability that Lee will make 5 or more errors on the test is 0.27.
- (2) The probability that Lee will make 5 or fewer errors on the test is 0.85.

### Arithmetic Probability

- (1) Given that 0.27 is the probability that Lee will make 5 or more errors on the test, it is clearly not possible to determine the probability that Lee will make exactly 5 errors on the test; NOT sufficient.
- (2) Given that 0.85 is the probability that Lee will make 5 or fewer errors on the test, it is clearly not possible to determine the probability that Lee will make exactly 5 errors on the test; NOT sufficient.

Taking (1) and (2) together, let  $E$  be the event that Lee will make 5 or more errors on the test and let  $F$  be the event that Lee will make 5 or fewer errors on the test. Then  $P(E \text{ or } F) = 1$ , since it will always be the case that, when taking the test, Lee will make at least 5 errors or at most 5 errors. Also, (1) and (2) can be expressed as  $P(E) = 0.27$  and  $P(F) = 0.85$ , and the question asks for the value of  $P(E \text{ and } F)$ . Using the identity  $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$ , it follows that  $1 = 0.27 + 0.85 - P(E \text{ and } F)$ , or  $P(E \text{ and } F) = 0.27 + 0.85 - 1 = 0.12$ . Therefore, the probability that Lee will make exactly 5 errors on the test is 0.12.

**The correct answer is C;**  
**both statements together are sufficient.**

DS19208

235. If  $p$  is a positive integer, is  $2^p + 1$  a prime number?

- (1)  $p$  is a prime number.
- (2)  $p$  is an even number.

### Arithmetic Properties of integers

Given that  $p$  is a positive integer, can we determine whether  $2^p + 1$  is a prime number?

- (1) Given that  $p$  is a prime number, we don't have enough information to determine whether  $2^p + 1$  is a prime number. To see this, it best to consider some cases. If  $p = 2$ , then  $2^p + 1 = 2^2 + 1 = 2 \times 2 + 1 = 4 + 1 = 5$ , which is prime. And if  $p = 3$ , then  $2^p + 1 = 2^3 + 1 = 2 \times 2 \times 2 + 1 = 8 + 1 = 9$ , which is not prime (it is equal to  $3 \times 3$ ); NOT sufficient.
- (2) Given that  $p$  is an even number, we can again consider cases and see that it is impossible to determine whether  $2^p + 1$  is a prime number. If  $p = 2$  then  $2^p + 1 = 2^2 + 1 = 4 + 1 = 5$ , which, again, is prime. And if  $p = 6$ , then  $2^p + 1 = 2^6 + 1 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 + 1 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) + 1 = 8 \times 8 + 1 = 64 + 1 = 65$ , which is not prime (it is equal to  $13 \times 5$ ); NOT sufficient.

Considering the two statements together, we have that  $p$  is both prime and even. The only even number that is not divisible by some other positive integer besides 1 is 2. That is, the only prime even integer is 2.  $2^p + 1$  is therefore equal to  $2^2 + 1 = 5$ , which is prime.

**The correct answer is C;**  
**both statements together are sufficient.**

DS02741

236. In the  $xy$ -plane, point  $(r,s)$  lies on a circle with center at the origin. What is the value of  $r^2 + s^2$ ?

- (1) The circle has radius 2.
- (2) The point  $(\sqrt{2}, -\sqrt{2})$  lies on the circle.

### Geometry Simple coordinate geometry

Let  $R$  be the radius of the circle. A right triangle with legs of lengths  $|r|$  and  $|s|$  can be formed so that the line segment with endpoints  $(r,s)$  and  $(0,0)$  is the hypotenuse. Since the length of the hypotenuse is  $R$ , the Pythagorean theorem for this right triangle gives  $R^2 = r^2 + s^2$ . Therefore, to determine the value of  $r^2 + s^2$ , it is sufficient to determine the value of  $R$ .

- (1) It is given that  $R = 2$ ; SUFFICIENT.

- (2) It is given that  $(\sqrt{2}, -\sqrt{2})$  lies on the circle. A right triangle with legs each of length  $\sqrt{2}$  can be formed so that the line segment with endpoints  $(\sqrt{2}, -\sqrt{2})$  and  $(0,0)$  is the hypotenuse. Since the length of the hypotenuse is the radius of the circle, which is  $R$ , where  $R^2 = r^2 + s^2$ , the Pythagorean theorem for this right triangle gives  $R^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2 + 2 = 4$ . Therefore,  $r^2 + s^2 = 4$ ; SUFFICIENT.

**The correct answer is D;**  
**each statement alone is sufficient.**

DS06368

237. If  $r$ ,  $s$ , and  $t$  are nonzero integers, is  $r^5s^3t^4$  negative?

- (1)  $rt$  is negative.  
 (2)  $s$  is negative.

#### Arithmetic Properties of numbers

Since  $r^5s^3t^4 = (rt)^4rs^3$  and  $(rt)^4$  is positive,  $r^5s^3t^4$  will be negative if and only if  $rs^3$  is negative, or if and only if  $r$  and  $s$  have opposite signs.

- (1) It is given that  $rt$  is negative, but nothing can be determined about the sign of  $s$ . If the sign of  $s$  is the opposite of the sign of  $r$ , then  $r^5s^3t^4 = (rt)^4rs^3$  will be negative. However, if the sign of  $s$  is the same as the sign of  $r$ , then  $r^5s^3t^4 = (rt)^4rs^3$  will be positive; NOT sufficient.
- (2) It is given that  $s$  is negative, but nothing can be determined about the sign of  $r$ . If  $r$  is positive, then  $r^5s^3t^4 = (rt)^4rs^3$  will be negative. However, if  $r$  is negative, then  $r^5s^3t^4 = (rt)^4rs^3$  will be positive; NOT sufficient.

Given (1) and (2), it is still not possible to determine whether  $r$  and  $s$  have opposite signs. For example, (1) and (2) hold if  $r$  is positive,  $s$  is negative, and  $t$  is negative, and in this case  $r$  and  $s$  have opposite signs. However, (1) and (2) hold if  $r$  is negative,  $s$  is negative, and  $t$  is positive, and in this case  $r$  and  $s$  have the same sign.

**The correct answer is E;**  
**both statements together are still not sufficient.**

DS13706

238. Each Type A machine fills 400 cans per minute, each Type B machine fills 600 cans per minute, and each Type C machine installs 2,400 lids per minute. A lid is installed on each can that is filled and on no can that is not filled. For a particular minute, what is the total number of machines working?
- (1) A total of 4,800 cans are filled that minute.  
 (2) For that minute, there are 2 Type B machines working for every Type C machine working.

#### Algebra Simultaneous equations

- (1) Given that 4,800 cans were filled that minute, it is possible that 12 Type A machines, no Type B machines, and 2 Type C machines were working, for a total of 14 machines, since  $(12)(400) + (0)(600) = 4,800$  and  $(2)(2,400) = 4,800$ . However, it is also possible that no Type A machines, 8 Type B machines, and 2 Type C machines were working, for a total of 10 machines, since  $(0)(400) + (8)(600) = 4,800$  and  $(2)(2,400) = 4,800$ ; NOT sufficient.
- (2) Given that there are 2 Type B machines working for every Type C machine working, it is possible that there are 6 machines working—3 Type A machines, 2 Type B machines, and 1 Type C machine. This gives  $3(400) + 2(600) = 2,400$  cans and  $1(2,400) = 2,400$  lids. It is also possible that there are 12 machines working—6 Type A machines, 4 Type B machines, and 2 Type C machines. This gives  $6(400) + 4(600) = 4,800$  cans and  $2(2,400) = 4,800$  lids; NOT sufficient.

Taking (1) and (2) together, since there were 4,800 cans filled that minute, there were 4,800 lids installed that minute. It follows that 2 Type C machines were working that minute, since  $(2)(2,400) = 4,800$ . Since there were twice this number of Type B machines working that minute, it follows that 4 Type B machines were working that minute. These 4 Type B machines filled  $(4)(600) = 2,400$  cans that minute, leaving  $4,800 - 2,400 = 2,400$  cans to be filled by Type A machines. Therefore, the number of Type A machines working that minute was  $\frac{2,400}{400} = 6$ ,

and it follows that the total number of machines working that minute was  $2 + 4 + 6 = 12$ .

**The correct answer is C;  
both statements together are sufficient.**

DS08660

239. If  $a$  and  $b$  are constants, what is the value of  $a$ ?

- (1)  $a < b$
- (2)  $(t - a)(t - b) = t^2 + t - 12$ , for all values of  $t$ .

### Algebra Second-degree equations

- (1) Given that  $a < b$ , it is not possible to determine the value of  $a$ . For example,  $a < b$  is true when  $a = 1$  and  $b = 2$ , and  $a < b$  is true when  $a = 2$  and  $b = 3$ ; NOT sufficient.
- (2) By factoring, what is given can be expressed as  $(t - a)(t - b) = (t + 4)(t - 3)$ , so either  $a = -4$  and  $b = 3$ , or  $a = 3$  and  $b = -4$ ; NOT sufficient.

Taking (1) and (2) together, the relation  $a < b$  is satisfied by only one of the two possibilities given in the discussion of (2) above, namely  $a = -4$  and  $b = 3$ . Therefore, the value of  $a$  is  $-4$ .

**The correct answer is C;  
both statements together are sufficient.**

DS04474

240. If  $x$  is a positive integer, is  $\sqrt{x}$  an integer?

- (1)  $\sqrt{4x}$  is an integer.
- (2)  $\sqrt{3x}$  is not an integer.

### Algebra Radicals

- (1) It is given that  $\sqrt{4x} = n$ , or  $4x = n^2$ , for some positive integer  $n$ . Since  $4x$  is the square of an integer, it follows that in the prime factorization of  $4x$ , each distinct prime factor is repeated an even number of times. Therefore, the same must be true for the prime factorization of  $x$ , since the prime factorization of  $x$  only differs from the prime factorization of  $4x$  by two factors of 2, and hence by an even number of factors of 2; SUFFICIENT.
- (2) Given that  $\sqrt{3x}$  is not an integer, it is possible for  $\sqrt{x}$  to be an integer (for example,  $x = 1$ ) and it is possible for  $\sqrt{x}$  to not be an integer (for example,  $x = 2$ ); NOT sufficient.

**The correct answer is A;  
statement 1 alone is sufficient.**

DS16456

241. If  $p$ ,  $q$ ,  $x$ ,  $y$ , and  $z$  are different positive integers, which of the five integers is the median?

- (1)  $p + x < q$
- (2)  $y < z$

### Arithmetic Statistics

Since there are five different integers, there are two integers greater and two integers less than the median, which is the middle number.

- (1) No information is given about the order of  $y$  and  $z$  with respect to the other three numbers; NOT sufficient.
- (2) This statement does not relate  $y$  and  $z$  to the other three integers; NOT sufficient.

Because (1) and (2) taken together do not relate  $p$ ,  $x$ , and  $q$  to  $y$  and  $z$ , it is impossible to tell which is the median. For example, if  $p = 3$ ,  $x = 4$ ,  $q = 8$ ,  $y = 9$ , and  $z = 10$ , then the median is 8, but if  $p = 3$ ,  $x = 4$ ,  $q = 8$ ,  $y = 1$ , and  $z = 2$ , then the median is 3.

**The correct answer is E;  
both statements together are still not sufficient.**

DS16277

242. If  $w + z = 28$ , what is the value of  $wz$ ?

- (1)  $w$  and  $z$  are positive integers.
- (2)  $w$  and  $z$  are consecutive odd integers.

### Arithmetic Arithmetic operations

- (1) The fact that  $w$  and  $z$  are both positive integers does not allow the values of  $w$  and  $z$  to be determined because, for example, if  $w = 20$  and  $z = 8$ , then  $wz = 160$ , and if  $w = 10$  and  $z = 18$ , then  $wz = 180$ ; NOT sufficient.
- (2) Since  $w$  and  $z$  are consecutive odd integers whose sum is 28, it is reasonable to consider the possibilities for the sum of consecutive odd integers: ...,  $(-5) + (-3) = -8$ ,  $(-3) + (-1) = -4$ ,  $(-1) + 1 = 0$ ,  $1 + 3 = 4$ , ...,  $9 + 11 = 20$ ,  $11 + 13 = 24$ ,  $13 + 15 = 28$ ,  $15 + 17 = 32$ , .... From this list it follows that only one pair of consecutive odd integers has 28 for its sum, and hence there is exactly one possible value for  $wz$ .

This problem can also be solved algebraically by letting the consecutive odd integers  $w$  and

$z$  be represented by  $2n + 1$  and  $2n + 3$ , where  $n$  can be any integer. Since  $28 = w + z$ , it follows that

$$28 = (2n+1) + (2n+3)$$

$$28 = 4n + 4 \quad \text{simplify}$$

$$24 = 4n \quad \text{subtract 4 from both sides}$$

$$6 = n \quad \text{divide both sides by 4}$$

Thus,  $w = 2(6) + 1 = 13$ ,  $z = 2(6) + 3 = 15$ , and hence exactly one value can be determined for  $wz$ ; SUFFICIENT.

The correct answer is B;  
statement 2 alone is sufficient.

- DS02474
243. If  $abc \neq 0$ , is  $\frac{\frac{a}{b}}{c} = \frac{a}{\frac{b}{c}}$ ?
- (1)  $a = 1$
- (2)  $c = 1$

### Algebra Fractions

Since  $\frac{\frac{a}{b}}{c} = \frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$  and

$\frac{a}{c} = a \div c = a \times \frac{c}{b} = \frac{ac}{b}$ , it is to be

determined whether  $\frac{a}{bc} = \frac{ac}{b}$ .

- (1) Given that  $a = 1$ , the equation to be investigated,  $\frac{a}{bc} = \frac{ac}{b}$ , is  $\frac{1}{bc} = \frac{c}{b}$ . This equation can be true for some nonzero values of  $b$  and  $c$  (for example,  $b = c = 1$ ) and false for other nonzero values of  $b$  and  $c$  (for example,  $b = 1$  and  $c = 2$ ); NOT sufficient.
- (2) Given that  $c = 1$ , the equation to be investigated,  $\frac{a}{bc} = \frac{ac}{b}$ , is  $\frac{a}{b} = \frac{a}{b}$ . This equation is true for all nonzero values of  $a$  and  $b$ ; SUFFICIENT.

The correct answer is B;  
statement 2 alone is sufficient.

DS14471

244. The arithmetic mean of a collection of 5 positive integers, not necessarily distinct, is 9. One additional positive integer is included in the collection and the arithmetic mean of the 6 integers is computed. Is the arithmetic mean of the 6 integers at least 10?

- (1) The additional integer is at least 14.
- (2) The additional integer is a multiple of 5.

### Arithmetic Statistics

Since the arithmetic mean of the 5 integers is 9, the sum of the 5 integers divided by 5 is equal to 9, and hence the sum of the 5 integers is equal to  $(5)(9) = 45$ . Let  $x$  be the additional positive integer. Then the sum of the 6 integers is  $45 + x$ , and the arithmetic mean of the 6 integers is  $\frac{45+x}{6}$ . Determine whether  $\frac{45+x}{6} \geq 10$ , or equivalently, whether  $45 + x \geq 60$ , or equivalently, whether  $x \geq 15$ .

- (1) Given that  $x \geq 14$ , then  $x$  could equal 14 and  $x \geq 15$  is not true, or  $x$  could equal 15 and  $x \geq 15$  is true; NOT sufficient.
- (2) Given that  $x$  is a multiple of 5, then  $x$  could equal 10 and  $x \geq 15$  is not true, or  $x$  could equal 15 and  $x \geq 15$  is true; NOT sufficient.

Taking (1) and (2) together, then  $x$  is a multiple of 5 that is greater than or equal to 14, and so  $x$  could equal one of the numbers 15, 20, 25, 30, . . . . Each of these numbers is greater than or equal to 15.

The correct answer is C;  
both statements together are sufficient.

DS11003

245. A certain list consists of 400 different numbers. Is the average (arithmetic mean) of the numbers in the list greater than the median of the numbers in the list?

- (1) Of the numbers in the list, 280 are less than the average.
- (2) Of the numbers in the list, 30 percent are greater than or equal to the average.

### Arithmetic Statistics

In a list of 400 numbers, the median will be halfway between the 200th and the 201st numbers in the list when the numbers are ordered from least to greatest.