PS01949

23. A company sells radios for \$15.00 each. It costs the company \$14.00 per radio to produce 1,000 radios and \$13.50 per radio to produce 2,000 radios. How much greater will the company's gross profit be from the production and sale of 2,000 radios than from the production and sale of 1,000 radios?

(A) \$500
(B) \$1,000
(C) \$1,500
(D) \$2,000
(E) \$2,500

Arithmetic Applied problems

If the company produces and sells 1,000 radios, its gross profit from the sale of these radios is equal to the total revenue from the sale of these radios minus the total cost. The total cost is equal to the number of radios produced multiplied by the production cost per radio: 1,000 × \$15.00. The total revenue is equal to the number of radios sold multiplied by the selling price: 1,000 × \$14.00. The gross profit in this case is therefore 1,000 × \$15.00 – 1,000 × \$14.00 = 1,000 × (\$15.00 – \$14.00) = 1,000 (\$1.00) = \$1,000. If 2,000 radios are produced and sold, the total cost is equal to 2,000 × \$13.50 and the total revenue is equal to 2,000 × \$15.00. The gross profit in this case is therefore 2,000 × \$15.00 – 2,000 × \$13.50 = 2,000 × (\$15.00 – \$13.50) = 2,000 × (\$1.50) = \$3,000. This profit of \$3,000 is \$2,000 greater than the gross profit of \$1,000 from producing and selling 1,000 radios.

The correct answer is D.

PS06555

24. Which of the following represent positive numbers?

I. $-3-(-5)$
II. $(-3)(-5)$
III. $-5-(-3)$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

Arithmetic Operations on integers

Find the value of each expression to determine if it is positive.

I. $-3-(-5)=-3+5=2$, which is positive.
II. $(-3)(-5)=15$, which is positive.
III. $-5-(-3)=-5+3=-2$, which is not positive.

The correct answer is D.

PS02948

25. If $\frac{x}{4}$ is 2 more than $\frac{x}{8}$, then $x=$

(A) 4
(B) 8
(C) 16
(D) 32
(E) 64

Algebra First-degree equations

Write an equation for the given information and solve for x.

$$\frac{x}{4}=2+\frac{x}{8}$$
$$(8)\left(\frac{x}{4}\right)=(8)\left(2+\frac{x}{8}\right)$$
$$2x=16+x$$
$$x=16$$

The correct answer is C.

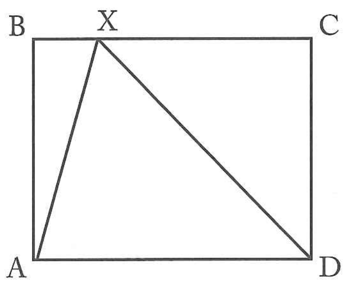
PS09983

26. Point X lies on side BC of rectangle $ABCD$, which has length 12 and width 8. What is the area of triangular region AXD?

(A) 96
(B) 48
(C) 32
(D) 24
(E) 20

Geometry Area

Note that, in rectangle $ABCD$, the sides BC and AD do not share an endpoint and must therefore be on opposite sides of the rectangle. We thus see that the point X, which is both on triangle AXD and on side BC of the rectangle, lies on the side of the rectangle that is opposite the side AD of the rectangle. AD is also a side of the triangle. So if the rectangle is drawn with AD horizontal and on the bottom (see the diagram, which is not drawn to scale), the vertical height of the triangle from the base AD is equal to the length of the sides on the rectangle that are adjacent to AD. Given the formula for the area of a triangle, $\frac{1}{2} \times \text{base} \times \text{height}$, the area of the triangle AXD is thus $\frac{1}{2} \times AD \times AB$ (or equivalently $\frac{1}{2} \times AD \times CD$).



Now, AD may be either a length or a width of the rectangle—equal to 12 or equal to 8. If AD is equal to 12, then AB is a width and is equal to 8; the area of the triangle is thus $\frac{1}{2} \times 12 \times 8$. If AD is instead equal to 8, then AB is equal to 12, and the formula for the area of the triangle results in the expression $\frac{1}{2} \times 8 \times 12$. In both cases, the area of the triangle is equal to 48.

The correct answer is B.

PS07659

27. A grocer has 400 pounds of coffee in stock, 20 percent of which is decaffeinated. If the grocer buys another 100 pounds of coffee of which 60 percent is decaffeinated, what percent, by weight, of the grocer's stock of coffee is decaffeinated?

(A) 28%
(B) 30%
(C) 32%
(D) 34%
(E) 40%

Arithmetic Percents

The grocer has 400 pounds of coffee in stock, of which $(400)(20\%) = 80$ pounds is decaffeinated coffee. Therefore, if the grocer buys 100 pounds of coffee, of which $(100)(60\%) = 60$ pounds is decaffeinated coffee, then the percent of the grocer's stock of coffee that is decaffeinated would be $\frac{80+60}{400+100} = \frac{140}{500} = \frac{28}{100} = 28\%$.

The correct answer is A.

PS05129

28. The toll T, in dollars, for a truck using a certain bridge is given by the formula $T = 1.50 + 0.50(x - 2)$, where x is the number of axles on the truck. What is the toll for an 18-wheel truck that has 2 wheels on its front axle and 4 wheels on each of its other axles?

(A) \$2.50
(B) \$3.00
(C) \$3.50
(D) \$4.00
(E) \$5.00

Algebra Operations on rational numbers

The 18-wheel truck has 2 wheels on its front axle and 4 wheels on each of its other axles, and so if A represents the number of axles on the truck in addition to the front axle, then $2 + 4A = 18$, from which it follows that $4A = 16$ and $A = 4$. Therefore, the total number of axles on the truck is $1 + A = 1 + 4 = 5$. Then, using $T = 1.50 + 0.50(x - 2)$, where x is the number of axles on the truck and $x = 5$, it follows that $T = 1.50 + 0.50(5 - 2) = 1.50 + 1.50 = 3.00$. Therefore, the toll for the truck is \$3.00.

The correct answer is B.

PS13917

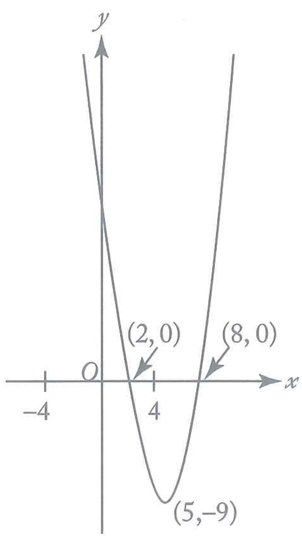
29. For what value of x between -4 and 4, inclusive, is the value of $x^2 - 10x + 16$ the greatest?

(A) −4
(B) −2
(C) 0
(D) 2
(E) 4

Algebra Second-degree equations

Given the expression $x^2 - 10x + 16$, a table of values can be created for the corresponding function $f(x) = x^2 - 10x + 16$ and the graph in the standard (x,y) coordinate plane can be sketched by plotting selected points:

x	$f(x)$
−4	72
−3	55
−2	40
−1	27
0	16
1	7
2	0
3	−5
4	−8
5	−9
6	−8
7	−5
8	0
9	7



It is clear from both the table of values and the sketch of the graph that as the value of x increases from −4 to 4, the values of $x^2 - 10x + 16$ decrease. Therefore, the value of $x^2 - 10x + 16$ is greatest when $x = -4$.

Alternatively, the given expression, $x^2 - 10x + 16$, has the form $ax^2 + bx + c$, where $a = 1$, $b = -10$, and $c = 16$. The graph in the standard (x,y) coordinate plane of the corresponding function $f(x) = ax^2 + bx + c$ is a parabola with vertex at $x = -\frac{b}{2a}$, and so the vertex of the graph of $f(x) = x^2 - 10x + 16$ is at

$$x = -\left(\frac{-10}{2(1)}\right) = 5.$$

Because $a = 1$ and 1 is positive, this parabola opens upward and values of $x^2 - 10x + 16$ decrease as x increases from −4 to 4. Therefore, the greatest value of $x^2 - 10x + 16$ for all values of x between −4 and 4, inclusive, is at $x = -4$.

The correct answer is A.

PS15994

30. If $x = -\frac{5}{8}$ and $y = -\frac{1}{2}$, what is the value of the expression $-2x - y^2$?

(A) $-\frac{3}{2}$

(B) -1

(C) 1

(D) $\frac{3}{2}$

(E) $\frac{7}{4}$

Algebra Fractions

If $x = -\frac{5}{8}$ and $y = -\frac{1}{2}$, then

$$-2x - y^2 = -2\left(-\frac{5}{8}\right) - \left(-\frac{1}{2}\right)^2 = \frac{5}{4} - \frac{1}{4} = \frac{4}{4} = 1.$$

The correct answer is C.

PS13686

31. If $x - y = R$ and $xy = S$, then $(x - 2)(y + 2) =$

(A) $R + S - 4$

(B) $R + 2S - 4$

(C) $2R - S - 4$

(D) $2R + S - 4$

(E) $2R + S$

Algebra Simplifying algebraic expressions; Substitution

$(x-2)(y+2)$	$= xy + 2x - 2y - 4$	multiply binomials
	$= xy + 2(x - y) - 4$	distributive principle
	$= S + 2R - 4$	substitution
	$= 2R + S - 4$	commutative principle

The correct answer is D.

PS01466

32. For positive integers a and b, the remainder when a is divided by b is equal to the remainder when b is divided by a. Which of the following could be a value of ab ?

I. 24
II. 30
III. 36

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Arithmetic Properties of integers

We are given that the remainder when a is divided by b is equal to the remainder when b is divided by a, and asked about possible values of ab. We thus need to find what our given condition implies about a and b.

We consider two cases: $a = b$ and $a \neq b$.

If $a = b$, then our given condition is trivially satisfied: the remainder when a is divided by a is equal to the remainder when a is divided by b. The condition thus allows that a be equal to b.

Now consider the case of $a \neq b$. Either a < b or $b < a$. Supposing that $a < b$, the remainder when a is divided by b is simply a. (For example, if 7 is divided by 10, then the remainder is 7.) However, according to our given condition, this remainder, a, is also the remainder when b is divided by a, which is impossible. If b is divided by a, then the remainder must be less than a. (For example, for any number that is divided by 10, the remainder cannot be 10 or greater.) Similar reasoning applies if we suppose that $b < a$. This is also impossible.

We thus see that a must be equal to b, and consider the statements I, II, and III.

I. Factored in terms of prime numbers, $24 = 3 \times 2 \times 2 \times 2$. Because "3" occurs only once in the factorization, we see that there is no integer a such that $a \times a = 24$. Based on the reasoning above, we see that 24 cannot be a value of ab.

II. Factored in terms of prime numbers, $30 = 5 \times 3 \times 2$. Because there is no integer a such that $a \times a = 30$, we see that 30 cannot be a value of ab.

III. Because $36 = 6 \times 6$, we see that 36 is a possible value of ab (with $a = b$).

The correct answer is B.

PS01867

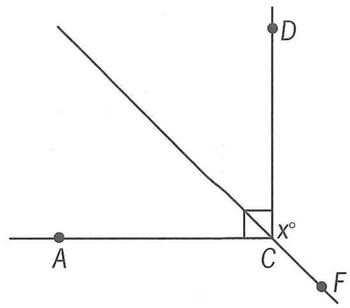
33. List S consists of the positive integers that are multiples of 9 and are less than 100. What is the median of the integers in S ?

(A) 36
(B) 45
(C) 49
(D) 54
(E) 63

Arithmetic Series and sequences

In the set of positive integers less than 100, the greatest multiple of 9 is 99 (9×11) and the least multiple of 9 is 9 (9×1). The sequence of positive multiples of 9 that are less than 100 is therefore the sequence of numbers $9 \times k$, where k ranges from 1 through 11. The median of the numbers k from 1 through 11 is 6. Therefore the median of the numbers $9 \times k$, where k ranges from 1 through 11, is $9 \times 6 = 54$.

The correct answer is D.

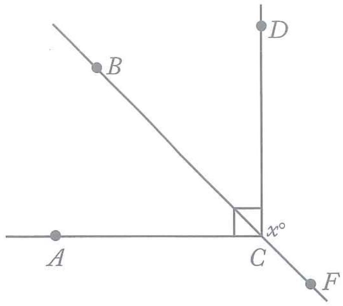


PS07397

34. In the figure above, if F is a point on the line that bisects angle ACD and the measure of angle DCF is $x°$, which of the following is true of x ?

(A) $90 \leq x < 100$
(B) $100 \leq x < 110$
(C) $110 \leq x < 120$
(D) $120 \leq x < 130$
(E) $130 \leq x < 140$

Geometry Angles



As shown in the figure above, if B is on the line that bisects $\angle ACD$ then the degree measure of $\angle DCB$ is $\frac{90}{2} = 45$. Then because B, C, and F are collinear, the sum of the degree measures of $\angle BCD$ and $\angle DCF$ is 180. Therefore, $x = 180 - 45 = 135$ and $130 \leq 135 < 140$.

The correct answer is E.

PS07380

35. A rope 20.6 meters long is cut into two pieces. If the length of one piece of rope is 2.8 meters shorter than the length of the other, what is the length, in meters, of the longer piece of rope?

(A) 7.5
(B) 8.9
(C) 9.9
(D) 10.3
(E) 11.7

Algebra First-degree equations

If x represents the length of the longer piece of rope, then $x - 2.8$ represents the length of the shorter piece, where both lengths are in meters. The total length of the two pieces of rope is 20.6 meters so,

$x + (x - 2.8) = 20.6$	given
$2x - 2.8 = 20.6$	add like terms
$2x = 23.4$	add 2.8 to both sides
$x = 11.7$	divide both sides by 2

Thus, the length of the longer piece of rope is 11.7 meters.

The correct answer is E.

PS01120

36. If x and y are integers and $x - y$ is odd, which of the following must be true?

I. xy is even.
II. $x^2 + y^2$ is odd.
III. $(x + y)^2$ is even.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

Arithmetic Properties of numbers

We are given that x and y are integers and that $x - y$ is odd, and then asked, for various operations on x and y, whether the results of the operations are odd or even. It is therefore useful to determine, given that $x - y$ is odd, whether x and y are odd or even. If both x and y are even—that is, divisible by 2—then $x - y = 2m - 2n = 2(m - n)$ for integers m and n. We thus see if both x and y are even then $x - y$ cannot be odd. And because $x - y$ *is* odd, we see that x and y cannot both be even. Similarly, if both x and y are odd, then, for integers j and k, $x = 2j + 1$ and $y = 2k + 1$. Therefore, $x - y = (2j + 1) - (2k + 1)$. The ones cancel, and we are left with $x - y = 2j - 2k = 2(j - k)$. Because $2(j - k)$ would be even, x and y cannot both be odd if $x - y$ is odd. It follows from all of this that one of x or y must be even and the other odd.

Now consider the statements I through III.

I. If one of x or y is even, then one of x or y is divisible by 2. It follows that xy is divisible by 2 and that xy is even.

II. Given that a number x or y is odd—not divisible by 2—we know that its product with itself is not divisible by 2 and is therefore odd. On the other hand, given that a number x or y is even, we know that its product with itself *is* divisible by 2 and is therefore even. The sum $x^2 + y^2$ is therefore the sum of an even number and an odd number. In such a case, the sum can be written as $(2m) + (2n + 1) = 2(m + n) + 1$, with m and n integers. It follows that $x^2 + y^2$ is not divisible by 2 and is therefore odd.

III. We know that one of x or y is even and the other is odd. We can therefore see from the discussion of statement II that $x + y$ is odd, and then also see, from the discussion of statement II, that the product of $x + y$ with itself, $(x + y)^2$, is odd.

The correct answer is D.

PS00335

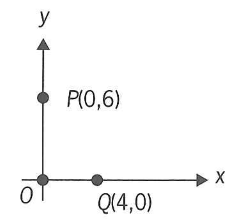
37. On Monday, the opening price of a certain stock was \$100 per share and its closing price was \$110 per share. On Tuesday the closing price of the stock was 10 percent less than its closing price on Monday, and on Wednesday the closing price of the stock was 4 percent greater than its closing price on Tuesday. What was the approximate percent change in the price of the stock from its opening price on Monday to its closing price on Wednesday?

(A) A decrease of 6%
(B) A decrease of 4%
(C) A decrease of 1%
(D) An increase of 3%
(E) An increase of 4%

Arithmetic Percents

The closing share price on Tuesday was 10% less than the closing price on Monday, \$110. 10% of \$110 is equal to $0.1 \times \$110 = \$11$, so the closing price on Tuesday was $\$110 - \$11 = \$99$. The closing price on Wednesday was 4% greater than this: $\$99 + (0.04 \times \$99) = \$99 + \$3.96 = \$102.96$. This value, \$102.96, is 2.96% greater than \$100, the opening price on Monday. The percentage change from the opening share price on Monday is therefore an increase of approximately 3%, which is the closest of the available answers to an increase of 2.96%.

The correct answer is D.



PS05109

38. In the rectangular coordinate system shown above, points O, P, and Q represent the sites of three proposed housing developments. If a fire station can be built at any point in the coordinate system, at which point would it be equidistant from all three developments?

(A) (3,1)
(B) (1,3)
(C) (3,2)
(D) (2,2)
(E) (2,3)

Geometry Coordinate geometry

Any point equidistant from the points (0,0) and (4,0) must lie on the perpendicular bisector of the segment with endpoints (0,0) and (4,0), which is the line with equation $x = 2$. Any point equidistant from the points (0,0) and (0,6) must lie on the perpendicular bisector of the segment with endpoints (0,0) and (0,6), which is the line with equation $y = 3$. Therefore, the point that is equidistant from (0,0), (4,0), and (0,6) must lie on both of the lines $x = 2$ and $y = 3$, which is the point (2,3).

Alternatively, let (x,y) be the point equidistant from (0,0), (4,0), and (0,6). Since the distance between (x,y) and (0,0) is equal to the distance between (x,y) and (4,0), it follows from the distance formula that $\sqrt{x^2 + y^2} = \sqrt{(x-4)^2 + y^2}$. Squaring both sides gives $x^2 + y^2 = (x - 4)^2 + y^2$. Subtracting y^2 from both sides of the last equation and then expanding the right side gives $x^2 = x^2 - 8x + 16$, or $0 = -8x + 16$, or $x = 2$. Also, since the distance between (x,y) and (0,0) is equal to the distance between (x,y) and (0,6), it follows from the distance formula that $\sqrt{x^2 + y^2} = \sqrt{x^2 + (y-6)^2}$. Squaring both sides of the last equation gives $x^2 + y^2 = x^2 + (y - 6)^2$. Subtracting x^2 from both sides and then expanding the right side gives $y^2 = y^2 - 12y + 36$, or $0 = -12y + 36$, or $y = 3$.

The correct answer is E.

PS05008

39. What is the perimeter, in meters, of a rectangular garden 6 meters wide that has the same area as a rectangular playground 16 meters long and 12 meters wide?

(A) 48
(B) 56
(C) 60
(D) 76
(E) 192

Geometry Perimeter and area

Let L represent the length, in meters, of the rectangular garden. It is given that the width of the garden is 6 meters and the area of the garden is the same as the area of a rectangular playground that is 16 meters long and 12 meters wide. It follows that $6L = (16)(12)$, and so $L = 32$. The perimeter of the garden is, then, $2(32 + 6) = 2(38) = 76$ meters.

The correct answer is D.

PS00918

40. $1 - 0.000001 =$

(A) (1.01)(0.99)
(B) (1.11)(0.99)
(C) (1.001)(0.999)
(D) (1.111)(0.999)
(E) (1.0101)(0.0909)

Arithmetic Place value

The task in this question is to find among the available answers the expression that is equal to $1 - 0.000001 = 0.999999$. In the case of option C, the first of the two factors, (1.001), is equal to $1 + 0.001$. One may therefore observe that $(1.001)(0.999) = (1 + 0.001)(0.999) = 0.999 + 0.000999 = 0.999999$. Option C is therefore a correct answer.

For option A, $(1.01)(0.99) = (1 + 0.01)(0.99) = 0.9999$. This option is therefore incorrect. For option B, $(1.11)(0.99) = (1 + 0.1 + 0.01)(0.99) = 0.99 + 0.099 + 0.0099 = 1.0989$. This option is therefore incorrect. For option D, $(1.111)(0.999) = 0.999 + 0.0999 + 0.00999 + 0.000999 = 1.109889$. This option is therefore incorrect. For option E, $(1.0101)(0.909) = 0.909 + 0.00909 + 0.0000909 = 0.9181809$. This option is therefore incorrect.

The correct answer is C.

PS04362

41. $|-4|(|-20|-|5|) =$

(A) −100
(B) −60
(C) 60
(D) 75
(E) 100

Arithmetic Absolute value

$|-4|(|-20|-|5|) = 4(20 - 5) = 4 \times 15 = 60$

The correct answer is C.

PS12934

42. Of the total amount that Jill spent on a shopping trip, excluding taxes, she spent 50 percent on clothing, 20 percent on food, and 30 percent on other items. If Jill paid a 4 percent tax on the clothing, no tax on the food, and an 8 percent tax on all other items, then the total tax that she paid was what percent of the total amount that she spent, excluding taxes?

(A) 2.8%
(B) 3.6%
(C) 4.4%
(D) 5.2%
(E) 6.0%

Arithmetic Applied problems

Let T represent the total amount Jill spent, excluding taxes. Jill paid a 4% tax on the clothing she bought, which accounted for 50% of the total amount she spent, and so the tax she paid on the clothing was $(0.04)(0.5T)$. Jill paid an 8% tax on the other items she bought, which accounted for 30% of the total amount she spent, and so the tax she paid on the other items was $(0.08)(0.3T)$. Therefore, the total amount of tax Jill paid was $(0.04)(0.5T) + (0.08)(0.3T) = 0.02T + 0.024T = 0.044T$. The tax as a percent of the total amount Jill spent, excluding taxes, was $\left(\frac{0.044T}{T} \times 100\right)\% = 4.4\%$.

The correct answer is C.

PS15469

43. How many integers x satisfy both $2 < x \leq 4$ and $0 \leq x \leq 3$?

(A) 5
(B) 4
(C) 3
(D) 2
(E) 1

Arithmetic Inequalities

The integers that satisfy $2 < x \leq 4$ are 3 and 4. The integers that satisfy $0 \leq x \leq 3$ are 0, 1, 2, and 3. The only integer that satisfies both $2 < x \leq 4$ and $0 \leq x \leq 3$ is 3, and so there is only one integer that satisfies both $2 < x \leq 4$ and $0 \leq x \leq 3$.

The correct answer is E.

PS09322

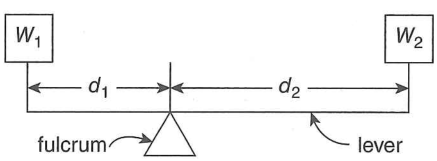
44. At the opening of a trading day at a certain stock exchange, the price per share of stock K was \$8. If the price per share of stock K was \$9 at the closing of the day, what was the percent increase in the price per share of stock K for that day?

(A) 1.4%
(B) 5.9%
(C) 11.1%
(D) 12.5%
(E) 23.6%

Arithmetic Percents

An increase from \$8 to \$9 represents an increase of $\left(\frac{9-8}{8} \times 100\right)\% = \frac{100}{8}\% = 12.5\%$.

The correct answer is D.



PS14237

45. As shown in the diagram above, a lever resting on a fulcrum has weights of w_1 pounds and w_2 pounds, located d_1 feet and d_2 feet from the fulcrum. The lever is balanced and $w_1d_1 = w_2d_2$. Suppose w_1 is 50 pounds and w_2 is 30 pounds. If d_1 is 4 feet less than d_2, what is d_2, in feet?

(A) 1.5
(B) 2.5
(C) 6
(D) 10
(E) 20

Algebra First-degree equations; Substitution

Given $w_1d_1 = w_2d_2$, $w_1 = 50$, $w_2 = 30$, and $d_1 = d_2 - 4$, it follows that $50(d_2 - 4) = 30d_2$, and so

$$\begin{aligned} 50(d_2-4) &= 30d_2 && \text{given} \\ 50d_2 - 200 &= 30d_2 && \text{distributive principle} \\ 20d_2 &= 200 && \text{add } 200 - 30d_2 \text{ to both sides} \\ d_2 &= 10 && \text{divide both sides by 20} \end{aligned}$$

The correct answer is D.

PS00037

46. The number of rooms at Hotel G is 10 less than twice the number of rooms at Hotel H. If the total number of rooms at Hotel G and Hotel H is 425, what is the number of rooms at Hotel G ?

(A) 140
(B) 180
(C) 200
(D) 240
(E) 280

Algebra Simultaneous equations

Let G be the number of rooms in Hotel G and let H be the number of rooms in Hotel H. Expressed in symbols, the given information is the following system of equations

$$\begin{cases} G = 2H - 10 \\ 425 = G + H \end{cases}$$

Solving the second equation for H gives $H = 425 - G$. Then, substituting $425 - G$ for H in the first equation gives

$$\begin{aligned} G &= 2(425 - G) - 10 \\ G &= 850 - 2G - 10 \\ G &= 840 - 2G \\ 3G &= 840 \\ G &= 280 \end{aligned}$$

The correct answer is E.

PS17036

47. $\frac{3}{100} + \frac{5}{1{,}000} + \frac{7}{100{,}000} =$

(A) 0.357
(B) 0.3507
(C) 0.35007
(D) 0.0357
(E) 0.03507

Arithmetic Operations on rational numbers

If each fraction is written in decimal form, the sum to be found is

$$\begin{array}{r} 0.03 \\ 0.005 \\ +0.00007 \\ \hline 0.03507 \end{array}$$

The correct answer is E.

PS01650

48. If r and s are positive integers such that $(2^r)(4^s) = 16$, then $2r + s =$

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

Algebra Exponents

Using the rules of exponents,

$(2^r)(4^s)$	$= 16$	given
$(2^r)(2^{2s})$	$= 2^4$	$4^s = (2^2)^s = 2^{2s}$, $16 = 2^4$
2^{r+2s}	$= 2^4$	addition property of exponents

Thus, $r + 2s = 4$. However, the problem asks for the value of $2r + s$. Since r and s are positive integers, $s < 2$; otherwise, r would not be positive. Therefore, $s = 1$, and it follows that $r + (2)(1) = 4$, or $r = 2$. The value of $2r + s$ is $(2)(2) + 1 = 5$.

Alternatively, since $(2^r)(4^s) = 16$ and both r and s are positive, it follows that $s < 2$; otherwise, $4^s \geq 16$ and r would not be positive. Therefore, $s = 1$ and $(2^r)(4) = 16$. It follows that $2^r = 4$ and $r = 2$. The value of $2r + s$ is $(2)(2) + 1 = 5$.

The correct answer is D.

PS06726

49. Three people each contributed x dollars toward the purchase of a car. They then bought the car for y dollars, an amount less than the total number of dollars contributed. If the excess amount is to be refunded to the three people in equal amounts, each person should receive a refund of how many dollars?

(A) $\frac{3x - y}{3}$
(B) $\frac{x - y}{3}$
(C) $\frac{x - 3y}{3}$
(D) $\frac{y - 3x}{3}$
(E) $3(x - y)$

Algebra Applied problems

The total to be refunded is equal to the total contributed minus the amount paid, or $3x - y$. If $3x - y$ is divided into three equal amounts, then each amount will be $\frac{3x - y}{3}$.

The correct answer is A.

$$\begin{aligned} 2x + 2y &= -4 \\ 4x + y &= 1 \end{aligned}$$

PS07331

50. In the system of equations above, what is the value of x ?

(A) -3
(B) -1
(C) $\frac{2}{5}$
(D) 1
(E) $1\frac{3}{4}$

Algebra Simultaneous equations

Solving the second equation for y gives $y = 1 - 4x$. Then, substituting $1 - 4x$ for y in the first equation gives

$$\begin{aligned} 2x + 2(1 - 4x) &= -4 \\ 2x + 2 - 8x &= -4 \\ -6x + 2 &= -4 \\ -6x &= -6 \\ x &= 1 \end{aligned}$$

The correct answer is D.

PS07080

51. Last week Jack worked 70 hours and earned \$1,260. If he earned his regular hourly wage for the first 40 hours worked, $1\frac{1}{2}$ times his regular hourly wage for the next 20 hours worked, and 2 times his regular hourly wage for the remaining 10 hours worked, what was his regular hourly wage?

(A) \$7.00
(B) \$14.00
(C) \$18.00
(D) \$22.00
(E) \$31.50

Algebra First-degree equations

If w represents Jack's regular hourly wage, then Jack's earnings for the week can be represented by the sum of the following amounts, in dollars: $40w$ (his earnings for the first 40 hours he worked), $(20)(1.5w)$ (his earnings for the next 20 hours he worked), and $(10)(2w)$ (his earnings for the last 10 hours he worked). Therefore,

$40w + (20)(1.5w) + (10)(2w) = 1{,}260$	given
$90w = 1{,}260$	add like terms
$w = 14$	divide both sides by 90

Jack's regular hourly wage was \$14.00.

The correct answer is B.

PS02402

52. If Mel saved more than \$10 by purchasing a sweater at a 15 percent discount, what is the smallest amount the original price of the sweater could be, to the nearest dollar?

(A) 45
(B) 67
(C) 75
(D) 83
(E) 150

Arithmetic; Algebra Percents; Inequalities; Applied problems

Letting P be the original price of the sweater in dollars, the given information can be expressed as $(0.15)P > 10$. Solving for P gives

$$\begin{aligned} (0.15)P &> 10 \\ P &> \frac{10}{0.15} = \frac{1{,}000}{15} = \frac{200}{3} \\ P &> 66\frac{2}{3} \end{aligned}$$

Thus, to the nearest dollar, the smallest amount P could have been is \$67.

The correct answer is B.

PS13426

53. If a and b are positive integers and $(2^a)^b = 2^3$, what is the value of $2^a 2^b$?

(A) 6
(B) 8
(C) 16
(D) 32
(E) 64

Algebra Exponents

It is given that $(2^a)^b = 2^3$, or $2^{ab} = 2^3$. Therefore, $ab = 3$. Since a and b are positive integers, it follows that either $a = 1$ and $b = 3$, or $a = 3$ and $b = 1$. In either case $a + b = 4$, and so $2^a 2^b = 2^{a+b} = 2^4 = 16$.

The correct answer is C.

PS03777

54. $\dfrac{1}{3-\dfrac{1}{3-\dfrac{1}{3-1}}}=$

(A) $\dfrac{7}{23}$

(B) $\dfrac{5}{13}$

(C) $\dfrac{2}{3}$

(D) $\dfrac{23}{7}$

(E) $\dfrac{13}{5}$

Arithmetic Operations with rational numbers

Perform each subtraction beginning at the lowest level in the fraction and proceeding upward.

$$\frac{1}{3-\dfrac{1}{3-\dfrac{1}{3-1}}}=\frac{1}{3-\dfrac{1}{3-\dfrac{1}{2}}}$$

$$=\frac{1}{3-\dfrac{1}{\dfrac{6}{2}-\dfrac{1}{2}}}$$

$$=\frac{1}{3-\dfrac{1}{\dfrac{5}{2}}}$$

$$=\frac{1}{3-\dfrac{2}{5}}$$

$$=\frac{1}{\dfrac{15}{5}-\dfrac{2}{5}}$$

$$=\frac{1}{\dfrac{13}{5}}$$

$$=\frac{5}{13}$$

The correct answer is B.

PS07386

55. After 4,000 gallons of water were added to a large water tank that was already filled to $\frac{3}{4}$ of its capacity, the tank was then at $\frac{4}{5}$ of its capacity. How many gallons of water does the tank hold when filled to capacity?

(A) 5,000
(B) 6,200
(C) 20,000
(D) 40,000
(E) 80,000

Algebra First-degree equations

Let C be the capacity of the tank. In symbols, the given information is $4,000+\frac{3}{4}C=\frac{4}{5}C$. Solve for C.

$$4,000+\frac{3}{4}C=\frac{4}{5}C$$

$$4,000=\left(\frac{4}{5}-\frac{3}{4}\right)C$$

$$4,000=\frac{16-15}{20}C$$

$$4,000=\frac{1}{20}C$$

$$20(4,000)=C$$

$$80,000=C$$

The correct answer is E.

PS01099

56. Five machines at a certain factory operate at the same constant rate. If four of these machines, operating simultaneously, take 30 hours to fill a certain production order, how many fewer hours does it take all five machines, operating simultaneously, to fill the same production order?

(A) 3
(B) 5
(C) 6
(D) 16
(E) 24

Arithmetic Applied problems

If 4 machines, working simultaneously, each work for 30 hours to fill a production order, it takes (4)(30) machine hours to fill the order. If 5 machines are working simultaneously, it will take $\frac{(4)(30)}{5} = 24$ hours. Thus, 5 machines working simultaneously will take $30 - 24 = 6$ fewer hours to fill the production order than 4 machines working simultaneously.

The correct answer is C.

PS01443

57. A certain toll station on a highway has 7 tollbooths, and each tollbooth collects \$0.75 from each vehicle that passes it. From 6 o'clock yesterday morning to 12 o'clock midnight, vehicles passed each of the tollbooths at the average rate of 4 vehicles per minute. Approximately how much money did the toll station collect during that time period?

(A) \$1,500
(B) \$3,000
(C) \$11,500
(D) \$23,000
(E) \$30,000

Arithmetic Rate problem

On average, 4 vehicles pass each tollbooth every minute. There are 7 tollbooths at the station, and each passing vehicle pays \$0.75. Therefore, the average rate, per minute, at which money is collected by the toll station is $\$\ (7 \times 4 \times 0.75) = \$\ (7 \times 4 \times \frac{3}{4}) = \$\ (7 \times 3) = \$21$. From 6 a.m. through midnight there are 18 hours. And because 18 hours is equal to 18×60 minutes, from 6 a.m. through midnight there are 1,080 minutes. The total amount of money collected by the toll station during this period is therefore $1{,}080 \times \$21 = \$22{,}680$, which is approximately \$23,000.

The correct answer is D.

PS13829

58. How many integers between 1 and 16, inclusive, have exactly 3 different positive integer factors? (Note: 6 is NOT such an integer because 6 has 4 different positive integer factors: 1, 2, 3, and 6.)

(A) 1
(B) 2
(C) 3
(D) 4
(E) 6

Arithmetic Properties of numbers

Using the process of elimination to eliminate integers that do NOT have exactly 3 different positive integer factors, the integer 1 can be eliminated since 1 has only 1 positive integer factor, namely 1 itself. Because each prime number has exactly 2 positive factors, each prime number between 1 and 16, inclusive, (namely, 2, 3, 5, 7, 11, and 13) can be eliminated. The integer 6 can also be eliminated since it was used as an example of an integer with exactly 4 positive integer factors. Check the positive integer factors of each of the remaining integers.

Integer	Positive integer factors	Number of factors
4	1, 2, 4	3
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
12	1, 2, 3, 4, 6, 12	6
14	1, 2, 7, 14	4
15	1, 3, 5, 15	4
16	1, 2, 4, 8, 16	5

Just the integers 4 and 9 have exactly 3 positive integer factors.

Alternatively, if the integer n, where $n > 1$, has exactly 3 positive integer factors, which include 1 and n, then n has exactly one other positive integer factor, say p. Since any factor of p would also be a factor of n, then p is prime, and so p is the only prime factor of n. It follows that $n = p^k$ for some integer $k > 1$. But if $k \geq 3$, then p^2 is a factor of n in addition to 1, p, and n, which contradicts the fact that n has exactly 3 positive integer factors. Therefore, $k = 2$ and $n = p^2$, which means that n is the square of a prime number. Of the integers between 1 and 16, inclusive, only 4 and 9 are the squares of prime numbers.

The correct answer is B.

PS06288

59. If $d = 2.0453$ and d^* is the decimal obtained by rounding d to the nearest hundredth, what is the value of $d^* - d$?

(A) –0.0053

(B) –0.0003

(C) 0.0007

(D) 0.0047

(E) 0.0153

Arithmetic Operations on rational numbers

Since $d = 2.0453$ rounded to the nearest hundredth is 2.05, $d^* = 2.05$; therefore, $d^* - d = 2.05 - 2.0453 = 0.0047$.

The correct answer is D.

PS14063

60. Stephanie has $2\frac{1}{4}$ cups of milk on hand and makes 2 batches of cookies, using $\frac{2}{3}$ cup of milk for each batch of cookies. Which of the following describes the amount of milk remaining after she makes the cookies?

(A) Less than $\frac{1}{2}$ cup

(B) Between $\frac{1}{2}$ cup and $\frac{3}{4}$ cup

(C) Between $\frac{3}{4}$ cup and 1 cup

(D) Between 1 cup and $1\frac{1}{2}$ cups

(E) More than $1\frac{1}{2}$ cups

Arithmetic Applied problems

In cups, the amount of milk remaining is

$2\frac{1}{4} - 2\left(\frac{2}{3}\right) = \frac{9}{4} - \frac{4}{3} = \frac{27-16}{12} = \frac{11}{12}$, which is greater than $\frac{3}{4} = \frac{9}{12}$ and less than 1.

The correct answer is C.

PS01656

61. The expression $n!$ is defined as the product of the integers from 1 through n. If p is the product of the integers from 100 through 299 and q is the product of the integers from 200 through 299, which of the following is equal to $\frac{p}{q}$?

(A) 99!

(B) 199!

(C) $\frac{199!}{99!}$

(D) $\frac{299!}{99!}$

(E) $\frac{299!}{199!}$

Arithmetic Series and sequences

The number p is equal to $100 \times 101 \times 102 \times \ldots \times 299$ and the number q is equal to $200 \times 201 \times 202 \times \ldots \times 299$. The number $\frac{p}{q}$ is thus equal to $\frac{100 \times 101 \times 102 \times \ldots \times 299}{200 \times 201 \times 202 \times \ldots \times 299} = \frac{100 \times 101 \times 102 \times \ldots \times 199 \times 200 \times 201 \times 202 \times \ldots \times 299}{200 \times 201 \times 202 \times \ldots \times 299}$.

Canceling $200 \times 201 \times 202 \times \ldots \times 299$ from the numerator and the denominator, we see that $\frac{p}{q} = 100 \times 101 \times 102 \times \ldots \times 199$. Note that the multiplication in this expression for $\frac{p}{q}$ begins with 100 (the smallest of the numbers being multiplied), whereas the multiplication in $n! = 1 \times 2 \times 3 \times \ldots \times n$ begins with 1. Starting with 199! as our numerator, we thus need to find a denominator that will cancel the undesired elements of the multiplication (in 199!). This number is $1 \times 2 \times 3 \times \ldots \times 99 = 99!$

That is, $\frac{p}{q} = 100 \times 101 \times 102 \times \ldots \times 199 = \frac{1 \times 2 \times 3 \times \ldots \times 99 \times 100 \times 101 \times 102 \times \ldots \times 199}{1 \times 2 \times 3 \times \ldots \times 99} = \frac{199!}{99!}$.

The correct answer is C.

PS15753

62. A school club plans to package and sell dried fruit to raise money. The club purchased 12 containers of dried fruit, each containing $16\frac{3}{4}$ pounds. What is the maximum number of individual bags of dried fruit, each containing $\frac{1}{4}$ pounds, that can be sold from the dried fruit the club purchased?

(A) 50
(B) 64
(C) 67
(D) 768
(E) 804

Arithmetic Applied problems; Operations with fractions

The 12 containers, each containing $16\frac{3}{4}$ pounds of dried fruit, contain a total of $(12)\left(16\frac{3}{4}\right) = (12)\left(\frac{67}{4}\right) = (3)(67) = 201$ pounds of dried fruit, which will make $\frac{201}{\frac{1}{4}} = (201)(4) = 804$ individual bags that can be sold.

The correct answer is E.

Height	Price
Less than 5 ft	\$14.95
5 ft to 6 ft	\$17.95
Over 6 ft	\$21.95

PS02498

63. A nursery sells fruit trees priced as shown in the chart above. In its inventory 54 trees are less than 5 feet in height. If the expected revenue from the sale of its entire stock is estimated at \$2,450, approximately how much of this will come from the sale of trees that are at least 5 feet tall?

(A) \$1,730
(B) \$1,640
(C) \$1,410
(D) \$1,080
(E) \$810

Arithmetic Applied problems

If the nursery sells its entire stock of trees, it will sell the 54 trees that are less than 5 feet in height at the price per tree of \$14.95 shown in the chart. The expected revenue from the sale of the trees that are less than 5 feet tall is therefore $54 \times \$14.95 = \$807.30$. The revenue from the sale of the trees that are at least 5 feet tall is thus equal to the total revenue from the sale of the entire stock of trees minus \$807.30. The revenue from the sale of the entire stock of trees is estimated at \$2,450. Based on this estimate, the revenue from the sale of the trees that are at least 5 feet tall will be $\$2{,}450 - \$807.30 = \$1{,}642.70$, which is approximately \$1,640.

The correct answer is B.

PS10539

64. The sequence a_1, a_2, a_3, a_4, a_5 is such that $a_n = a_{n-1} + 5$ for $2 \le n \le 5$. If $a_5 = 31$, what is the value of a_1 ?

(A) 1
(B) 6
(C) 11
(D) 16
(E) 21

Algebra Sequences

Since $a_n = a_{n-1} + 5$, then $a_n - a_{n-1} = 5$. So,

$$a_5 - a_4 = 5$$
$$a_4 - a_3 = 5$$
$$a_3 - a_2 = 5$$
$$a_2 - a_1 = 5$$

Adding the equations gives

$$a_5 - a_4 + a_4 - a_3 + a_3 - a_2 + a_2 - a_1 = 5+5+5+5$$
$$a_5 - a_1 = 20$$

and substituting 31 for a_5 gives

$$31 - a_1 = 20$$
$$a_1 = 11.$$

The correct answer is C.

PS04971

65. A certain bridge is 4,024 feet long. Approximately how many minutes does it take to cross this bridge at a constant speed of 20 miles per hour? (1 mile = 5,280 feet)

(A) 1
(B) 2
(C) 4
(D) 6
(E) 7

Arithmetic Applied problems

First, convert 4,024 feet to miles since the speed is given in miles per hour:

$4{,}024\ \cancel{ft} \times \dfrac{1\text{ mi}}{5{,}280\ \cancel{ft}} = \dfrac{4{,}024}{5{,}280}$ mi.

Now, divide by 20 mph: $\dfrac{4{,}024}{5{,}280}$ mi $\div \dfrac{20\text{ mi}}{1\text{ hr}}$

$= \dfrac{4{,}024\ \cancel{mi}}{5{,}280} \times \dfrac{1\text{ hr}}{20\ \cancel{mi}} = \dfrac{4{,}024\text{ hr}}{(5{,}280)(20)}$.

Last, convert $\dfrac{4{,}024\text{ hr}}{(5{,}280)(20)}$ to minutes:

$\dfrac{4{,}024\ \cancel{hr}}{(5{,}280)(20)} \times \dfrac{60\ \text{min}}{1\ \cancel{hr}} = \dfrac{(4{,}024)(60)\ \text{min}}{(5{,}280)(20)} \approx$

$\dfrac{4{,}000}{5{,}000} \times \dfrac{60}{20}$ min. Then, $\dfrac{4{,}000}{5{,}000} \times \dfrac{60}{20}$ min $=$

$= 0.8 \times 3$ min ≈ 2 min. Thus, at a constant speed of 20 miles per hour, it takes approximately 2 minutes to cross the bridge.

The correct answer is B.

PS04009

66. If $S = \{0, 4, 5, 2, 11, 8\}$, how much greater than the median of the numbers in S is the mean of the numbers in S?

(A) 0.5
(B) 1.0
(C) 1.5
(D) 2.0
(E) 2.5

Arithmetic; Algebra Statistics; Concepts of sets

The median of S is found by ordering the values according to size (0, 2, 4, 5, 8, 11) and taking the average of the two middle numbers: $\dfrac{4+5}{2} = 4.5$.

The mean is $\dfrac{\text{sum of } n \text{ values}}{n} =$

$\dfrac{0+4+5+2+11+8}{6} = 5$.

The difference between the mean and the median is $5 - 4.5 = 0.5$.

The correct answer is A.

PS12657

67. The annual interest rate earned by an investment increased by 10 percent from last year to this year. If the annual interest rate earned by the investment this year was 11 percent, what was the annual interest rate last year?

(A) 1%
(B) 1.1%
(C) 9.1%
(D) 10%
(E) 10.8%

Arithmetic Percents

If L is the annual interest rate last year, then the annual interest rate this year is 10% greater than L, or $1.1L$. It is given that $1.1L = 11\%$. Therefore, $L = \dfrac{11\%}{1.1} = 10\%$. (Note that if the given information had been that the investment increased by *10 percentage points*, then the equation would have been $L + 10\% = 11\%$.)

The correct answer is D.

PS07394

68. A total of 5 liters of gasoline is to be poured into two empty containers with capacities of 2 liters and 6 liters, respectively, such that both containers will be filled to the same percent of their respective capacities. What amount of gasoline, in liters, must be poured into the 6-liter container?

(A) $4\frac{1}{2}$
(B) 4
(C) $3\frac{3}{4}$
(D) 3
(E) $1\frac{1}{4}$

Algebra Ratio and proportion

If x represents the amount, in liters, of gasoline poured into the 6-liter container, then $5 - x$ represents the amount, in liters, of gasoline poured into the 2-liter container. After the gasoline is poured into the containers, the 6-liter container will be filled to $\left(\frac{x}{6} \times 100\right)$% of its capacity and the 2-liter container will be filled to $\left(\frac{5-x}{2} \times 100\right)$% of its capacity. Because these two percents are equal,

$\frac{x}{6} = \frac{5-x}{2}$	given
$2x = 6(5 - x)$	multiply both sides by 12
$2x = 30 - 6x$	use distributive property
$8x = 30$	add $6x$ to both sides
$x = 3\frac{3}{4}$	divide both sides by 8

Therefore, $3\frac{3}{4}$ liters of gasoline must be poured into the 6-liter container.

The correct answer is C.

PS02775

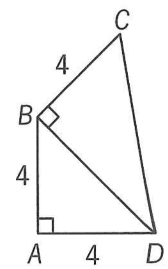
69. List S consists of 10 consecutive odd integers, and list T consists of 5 consecutive even integers. If the least integer in S is 7 more than the least integer in T, how much greater is the average (arithmetic mean) of the integers in S than the average of the integers in T?

(A) 2
(B) 7
(C) 8
(D) 12
(E) 22

Arithmetic Statistics

Let the integers in S be $s, s + 2, s + 4, \ldots, s + 18$, where s is odd. Let the integers in T be $t, t + 2, t + 4, t + 6, t + 8$, where t is even. Given that $s = t + 7$, it follows that $s - t = 7$. The average of the integers in S is $\frac{10s + 90}{10} = s + 9$, and, similarly, the average of the integers in T is $\frac{5t + 20}{5} = t + 4$. The difference in these averages is $(s + 9) - (t + 4) = (s - t) + (9 - 4) = 7 + 5 = 12$. Thus, the average of the integers in S is 12 greater than the average of the integers in T.

The correct answer is D.



PS05616

70. In the figure above, what is the area of triangular region BCD?

(A) $4\sqrt{2}$
(B) 8
(C) $8\sqrt{2}$
(D) 16
(E) $16\sqrt{2}$

Geometry Triangles; Area

By the Pythagorean theorem, $BD = \sqrt{4^2 + 4^2} = 4\sqrt{2}$. Then the area of ΔBCD is $\frac{1}{2}(4\sqrt{2})(4) = 8\sqrt{2}$.

The correct answer is C.

PS13882

71. What is the larger of the 2 solutions of the equation $x^2 - 4x = 96$?

(A) 8
(B) 12
(C) 16
(D) 32
(E) 100

Algebra Second-degree equations

It is given that $x^2 - 4x = 96$, or $x^2 - 4x - 96 = 0$, or $(x - 12)(x + 8) = 0$. Therefore, $x = 12$ or $x = -8$, and the larger of these two numbers is 12.

Alternatively, from $x^2 - 4x = 96$ it follows that $x(x - 4) = 96$. By inspection, the left side is either the product of 12 and 8, where the value of x is 12, or the product of -8 and -12, where the value of x is -8, and the larger of these two values of x is 12.

The correct answer is B.

PS10493

72. Of the goose eggs laid at a certain pond, $\frac{2}{3}$ hatched, and $\frac{3}{4}$ of the geese that hatched from those eggs survived the first month. Of the geese that survived the first month, $\frac{3}{5}$ did <u>not</u> survive the first year. If 120 geese survived the first year and if no more than one goose hatched from each egg, how many goose eggs were laid at the pond?

(A) 280
(B) 400
(C) 540
(D) 600
(E) 840

Arithmetic Operations with rational numbers

Let N represent the number of eggs laid at the pond. Then $\frac{2}{3}N$ eggs hatched and $\frac{3}{4}\left(\frac{2}{3}N\right)$ goslings (baby geese) survived the first month. Since $\frac{3}{5}$ of these goslings did not survive the first year, then $\frac{2}{5}$ did survive the first year. This means that $\frac{2}{5}\left(\frac{3}{4}\left(\frac{2}{3}N\right)\right)$ goslings survived the first year. But this number is 120 and so, $\frac{2}{5}\left(\frac{3}{4}\left(\frac{2}{3}N\right)\right)=120$, $\frac{1}{5}N=120$ and $N=5(120)=600$.

The correct answer is D.

PS09305

73. If $x^2-2x-15=0$ and $x>0$, which of the following must be equal to 0 ?

I. x^2-6x+9
II. $x^2-7x+10$
III. $x^2-10x+25$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

Algebra Second-degree equations

Since $x^2-2x-15=0$, then $(x-5)(x+3)=0$, so $x=5$ or $x=-3$. Since $x>0$, then $x=5$.

I. $5^2-6(5)+9=25-30+9=4\neq 0$
II. $5^2-7(5)+10=25-35+10=0$
III. $5^2-10(5)+25=25-50+25=0$

The correct answer is D.

PS10921

74. $\frac{(39{,}897)(0.0096)}{198.76}$ is approximately

(A) 0.02
(B) 0.2
(C) 2
(D) 20
(E) 200

Arithmetic Estimation

$$\frac{(39{,}897)(0.0096)}{198.76}\approx\frac{(40{,}000)(0.01)}{200}=(200)(0.01)=2$$

The correct answer is C.

PS13205

75. If a square region has area n, what is the length of the diagonal of the square in terms of n ?

(A) $\sqrt{2n}$
(B) $\sqrt{n}$
(C) $2\sqrt{n}$
(D) $2n$
(E) $2n^2$

Geometry Area; Pythagorean theorem

If s represents the side length of the square, then $n=s^2$. By the Pythagorean theorem, the length of the diagonal of the square is $\sqrt{s^2+s^2}=\sqrt{n+n}=\sqrt{2n}$.

The correct answer is A.

PS00817

76. The "prime sum" of an integer n greater than 1 is the sum of all the prime factors of n, including repetitions. For example, the prime sum of 12 is 7, since $12=2\times2\times3$ and $2+2+3=7$. For which of the following integers is the prime sum greater than 35 ?

(A) 440
(B) 512
(C) 620
(D) 700
(E) 750

Arithmetic Properties of numbers

A Since $440 = 2 \times 2 \times 2 \times 5 \times 11$, the prime sum of 440 is $2 + 2 + 2 + 5 + 11 = 22$, which is not greater than 35.

B Since $512 = 2^9$, the prime sum of 512 is $9(2) = 18$, which is not greater than 35.

C Since $620 = 2 \times 2 \times 5 \times 31$, the prime sum of 620 is $2 + 2 + 5 + 31 = 40$, which is greater than 35.

Because there can be only one correct answer, D and E need not be checked. However, for completeness,

D Since $700 = 2 \times 2 \times 5 \times 5 \times 7$, the prime sum of 700 is $2 + 2 + 5 + 5 + 7 = 21$, which is not greater than 35.

E Since $750 = 2 \times 3 \times 5 \times 5 \times 5$, the prime sum of 750 is $2 + 3 + 5 + 5 + 5 = 20$, which is not greater than 35.

The correct answer is C.

PS02256

77. Each machine at a toy factory assembles a certain kind of toy at a constant rate of one toy every 3 minutes. If 40 percent of the machines at the factory are to be replaced by new machines that assemble this kind of toy at a constant rate of one toy every 2 minutes, what will be the percent increase in the number of toys assembled in one hour by all the machines at the factory, working at their constant rates?

(A) 20%
(B) 25%
(C) 30%
(D) 40%
(E) 50%

Arithmetic Applied problems; Percents

Let n be the total number of machines working. Currently, it takes each machine 3 minutes to assemble 1 toy, so each machine assembles 20 toys in 1 hour and the total number of toys assembled in 1 hour by all the current machines is $20n$. It takes each new machine 2 minutes to assemble 1 toy, so each new machine assembles 30 toys in 1 hour. If 60% of the machines assemble 20 toys each hour and 40% assemble 30 toys each hour, then the total number of toys produced by the machines each hour is $(0.60n)(20) + (0.40n)(30) = 24n$.

The percent increase in hourly production is $\frac{24n - 20n}{20n} = \frac{1}{5}$ or 20%.

The correct answer is A.

PS10339

78. When a subscription to a new magazine was purchased for m months, the publisher offered a discount of 75 percent off the regular monthly price of the magazine. If the total value of the discount was equivalent to buying the magazine at its regular monthly price for 27 months, what was the value of m?

(A) 18
(B) 24
(C) 30
(D) 36
(E) 48

Algebra Percents

Let P represent the regular monthly price of the magazine. The discounted monthly price is then $0.75P$. Paying this price for m months is equivalent to paying the regular price for 27 months. Therefore, $0.75mP = 27P$, and so $0.75m = 27$. It follows that $m = \frac{27}{0.75} = 36$.

The correct answer is D.

PS10422

79. At a garage sale, all of the prices of the items sold were different. If the price of a radio sold at the garage sale was both the 15th highest price and the 20th lowest price among the prices of the items sold, how many items were sold at the garage sale?

(A) 33
(B) 34
(C) 35
(D) 36
(E) 37

Arithmetic Operations with integers

If the price of the radio was the 15th highest price, there were 14 items that sold for prices higher than the price of the radio. If the price of the radio was the 20th lowest price, there were 19 items that sold for prices lower than the price of the radio. Therefore, the total number of items sold is $14 + 1 + 19 = 34$.

The correct answer is B.

PS11738

80. Half of a large pizza is cut into 4 equal-sized pieces, and the other half is cut into 6 equal-sized pieces. If a person were to eat 1 of the larger pieces and 2 of the smaller pieces, what fraction of the pizza would remain uneaten?

(A) $\frac{5}{12}$

(B) $\frac{13}{24}$

(C) $\frac{7}{12}$

(D) $\frac{2}{3}$

(E) $\frac{17}{24}$

Arithmetic Operations with fractions

Each of the 4 equal-sized pieces represents $\frac{1}{8}$ of the whole pizza since each slice is $\frac{1}{4}$ of $\frac{1}{2}$ of the pizza. Each of the 6 equal-sized pieces represents $\frac{1}{12}$ of the whole pizza since each slice is $\frac{1}{6}$ of $\frac{1}{2}$ of the pizza. The fraction of the pizza remaining after a person eats one of the larger pieces and 2 of the smaller pieces is $1-\left[\frac{1}{8}+2\left(\frac{1}{12}\right)\right]=1-\left(\frac{1}{8}+\frac{1}{6}\right)=1-\frac{6+8}{48}=1-\frac{7}{24}=\frac{17}{24}$.

The correct answer is E.

PS14293

81. If $a=1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}$ and $b=1+\frac{1}{4}a$, then what is the value of $a-b$?

(A) $-\frac{85}{256}$

(B) $-\frac{1}{256}$

(C) $-\frac{1}{4}$

(D) $\frac{125}{256}$

(E) $\frac{169}{256}$

Arithmetic Operations with fractions

Given that $a=1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}$, it follows that $\frac{1}{4}a=\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}$ and so $b=1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}$. Then $a-b=\left(1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}\right)-\left(1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}\right)=-\frac{1}{256}$.

The correct answer is B.

PS10174

82. In a certain learning experiment, each participant had three trials and was assigned, for each trial, a score of either –2, –1, 0, 1, or 2. The participant's final score consisted of the sum of the first trial score, 2 times the second trial score, and 3 times the third trial score. If Anne received scores of 1 and –1 for her first two trials, not necessarily in that order, which of the following could NOT be her final score?

(A) –4

(B) –2

(C) 1

(D) 5

(E) 6

Arithmetic Applied problems

If x represents Anne's score on the third trial, then Anne's final score is either $1+2(-1)+3x=3x-1$ or $-1+2(1)+3x=3x+1$, where x can have the value –2, –1, 0, 1, or 2. The following table shows Anne's final score for each possible value of x.

x	$3x-1$	$3x+1$
–2	–7	–5
–1	–4	–2
0	–1	1
1	2	4
2	5	7

Among the answer choices, the only one not found in the table is 6.

The correct answer is E.

PS00111

83. For all positive integers m and v, the expression $m \ominus v$ represents the remainder when m is divided by v. What is the value of $((98 \ominus 33) \ominus 17) - (98 \ominus (33 \ominus 17))$?

(A) −10
(B) −2
(C) 8
(D) 13
(E) 17

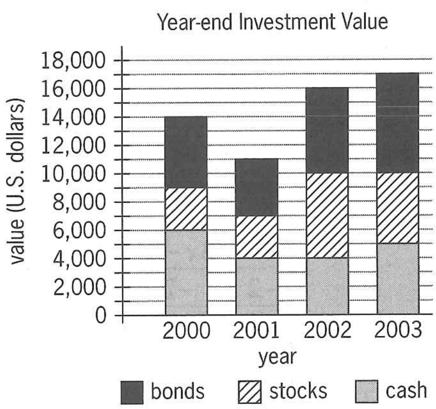
Arithmetic Operations with integers

First, for $((98 \ominus 33) \ominus 17)$, determine $98 \ominus 33$, which equals 32, since 32 is the remainder when 98 is divided by 33 $(98 = 2(33) + 32)$. Then, determine $32 \ominus 17$, which equals 15, since 15 is the remainder when 32 is divided by 17 $(32 = 1(17) + 15)$. Thus, $((98 \ominus 33) \ominus 17) = 15$.

Next, for $(98 \ominus (33 \ominus 17))$, determine $33 \ominus 17$, which equals 16, since 16 is the remainder when 33 is divided by 17 $(33 = 1(17) + 16)$. Then, determine $98 \ominus 16$, which equals 2, since 2 is the remainder when 98 is divided by 16 $(98 = 6(16) + 2)$. Thus, $(98 \ominus (33 \ominus 17)) = 2$.

Finally, $((98 \ominus 33) \ominus 17 - (98 \ominus (33 \ominus 17)) = 15 - 2 = 13$.

The correct answer is D.



PS13841

84. The chart above shows year-end values for Darnella's investments. For just the stocks, what was the increase in value from year-end 2000 to year-end 2003 ?

(A) \$1,000
(B) \$2,000
(C) \$3,000
(D) \$4,000
(E) \$5,000

Arithmetic Interpretation of graphs

From the graph, the year-end 2000 value for stocks is $9{,}000 - 6{,}000 = 3{,}000$ and the year-end 2003 value for stocks is $10{,}000 - 5{,}000 = 5{,}000$. Therefore, for just the stocks, the increase in value from year-end 2000 to year-end 2003 is $5{,}000 - 3{,}000 = 2{,}000$.

The correct answer is B.

PS05775

85. If the sum of the reciprocals of two consecutive odd integers is $\frac{12}{35}$, then the greater of the two integers is

(A) 3
(B) 5
(C) 7
(D) 9
(E) 11

Arithmetic Operations with fractions

The sum of the reciprocals of 2 integers, a and b, is $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$. Therefore, since $\frac{12}{35}$ is the sum of the reciprocals of 2 consecutive odd integers, the integers must be such that their sum is a multiple of 12 and their product is the same multiple of 35 so that the fraction reduces to $\frac{12}{35}$. Considering the simplest case where $a + b = 12$ and $ab = 35$, it is easy to see that the integers are 5 and 7 since 5 and 7 are the only factors of 35 that are consecutive odd integers. The larger of these is 7.

Algebraically, if a is the greater of the two integers, then $b = a - 2$ and

$$\frac{a+(a-2)}{a(a-2)} = \frac{12}{35}$$

$$\frac{2a-2}{a(a-2)} = \frac{12}{35}$$

$$\begin{aligned} 35(2a-2) &= 12a(a-2) \\ 70a - 70 &= 12a^2 - 24a \\ 0 &= 12a^2 - 94a + 70 \\ 0 &= 2(6a-5)(a-7) \end{aligned}$$

Thus, $6a - 5 = 0$, so $a = \frac{5}{6}$, or $a - 7 = 0$, so $a = 7$. Since a must be an integer, it follows that $a = 7$.

The correct answer is C.

PS05916

86. What is the sum of the odd integers from 35 to 85, inclusive?

(A) 1,560
(B) 1,500
(C) 1,240
(D) 1,120
(E) 1,100

Arithmetic Operations on integers

The odd integers from 35 through 85 form an arithmetic sequence with first term 35 and each subsequent term 2 more than the preceding term. Thus the sum $35 + 37 + 39 + \ldots + 85$ can be found as follows:

1st term	35	= 35		
2nd term	37	= 35	+	1(2)
3rd term	39	= 35	+	2(2)
4th term	41	= 35	+	3(2)
…	…	… …	…	…
26th term	85	= 35	+	25(2)
	Sum	= 35(26)	+	(1 + 2 + 3 + … + 25)(2)
		= 35(26)	+	$\frac{(25)(26)}{2}(2)$ See note below
		= 910	+	650
		= 1,560		

Note that if $s = 1 + 2 + 3 + \ldots + 25$, then $2s = (1 + 2 + 3 + \ldots + 25) + (25 + 24 + 23 + \ldots + 1)$, and so $2s = (1 + 25) + (2 + 24) + (3 + 23) + \ldots + (25 + 1) = (25)(26)$. Therefore, $s = \frac{(25)(26)}{2}$.

Alternatively, to determine the number of odd integers from 35 to 85, inclusive, consider that 3 of them (35, 37, and 39) have tens digit 3. Half of the integers with tens digit 4 are odd, so 5 of the odd integers between 35 and 85, inclusive, have tens digit 4. Similarly, 5 of the odd integers between 35 and 85, inclusive, have tens digit 5; 5 have tens digit 6; and 5 have tens digit 7. Finally, 3 have tens digit 8 (81, 83, and 85), and so the number of odd integers between 35 and 85, inclusive, is $3 + 5 + 5 + 5 + 5 + 3 = 26$. Now, let $S = 35 + 37 + 39 + \ldots + 85$. Then, $S = 85 + 83 + 81 + \ldots + 35$, and it follows that $2S = (35 + 85) + (37 + 83) + (39 + 81) + \ldots + (85 + 35) = (120)(26)$. Thus, $S = 35 + 37 + 39 + \ldots + 85 = \frac{(120)(26)}{2} = 1{,}560$.

The correct answer is A.

PS00777

87. In a certain sequence, each term after the first term is one-half the previous term. If the tenth term of the sequence is between 0.0001 and 0.001, then the twelfth term of the sequence is between

(A) 0.0025 and 0.025
(B) 0.00025 and 0.0025
(C) 0.000025 and 0.00025
(D) 0.0000025 and 0.000025
(E) 0.00000025 and 0.0000025

Arithmetic Sequences

Let a_n represent the nth term of the sequence. It is given that each term after the first term is $\frac{1}{2}$ the previous term and that $0.0001 < a_{10} < 0.001$.

Then for a_{11}, $\frac{0.0001}{2} < a_{11} < \frac{0.001}{2}$, or $0.00005 < a_{11} < 0.0005$. For a_{12}, $\frac{0.00005}{2} < a_{12} < \frac{0.0005}{2}$, or $0.000025 < a_{12} < 0.00025$. Thus, the twelfth term of the sequence is between 0.000025 and 0.00025.

The correct answer is C.

PS04765

88. A certain drive-in movie theater has a total of 17 rows of parking spaces. There are 20 parking spaces in the first row and 21 parking spaces in the second row. In each subsequent row there are 2 more parking spaces than in the previous row. What is the total number of parking spaces in the movie theater?

(A) 412
(B) 544
(C) 596
(D) 632
(E) 692

Arithmetic Operations on integers

Row	Number of parking spaces
1st row	20
2nd row	21
3rd row	21 + 1(2)
4th row	21 + 2(2)
...	... ...
17th row	21 + 15(2)

Then, letting S represent the total number of parking spaces in the theater,

$$\begin{aligned} S &= 20 + (16)(21) + (1 + 2 + 3 + \ldots + 15)(2) \\ &= 20 + 336 + \frac{(15)(16)}{2}(2) \quad \text{See note below} \\ &= 356 + 240 \\ &= 596 \end{aligned}$$

Note that if $s = 1 + 2 + 3 + \ldots + 15$, then $2s = (1 + 2 + 3 + \ldots + 15) + (15 + 14 + 13 + \ldots + 1)$, and so $2s = (1 + 15) + (2 + 14) + (3 + 13) + \ldots + (15 + 1) = (15)(16)$. Therefore, $s = \frac{(15)(16)}{2}$.

The correct answer is C.

PS10810

89. Ada and Paul received their scores on three tests. On the first test, Ada's score was 10 points higher than Paul's score. On the second test, Ada's score was 4 points higher than Paul's score. If Paul's average (arithmetic mean) score on the three tests was 3 points higher than Ada's average score on the three tests, then Paul's score on the third test was how many points higher than Ada's score?

(A) 9
(B) 14
(C) 17
(D) 23
(E) 25

Algebra Statistics

Let a_1, a_2, and a_3 be Ada's scores on the first, second, and third tests, respectively, and let p_1, p_2, and p_3 be Paul's scores on the first, second, and third tests, respectively. Then, Ada's average score is $\frac{a_1 + a_2 + a_3}{3}$ and Paul's average score is $\frac{p_1 + p_2 + p_3}{3}$. But, Paul's average score is 3 points higher than Ada's average score, so $\frac{p_1 + p_2 + p_3}{3} = \frac{a_1 + a_2 + a_3}{3} + 3$. Also, it is given that $a_1 = p_1 + 10$ and $a_2 = p_2 + 4$, so by substitution, $\frac{p_1 + p_2 + p_3}{3} = \frac{(p_1 + 10) + (p_2 + 4) + a_3}{3} + 3$. Then, $p_1 + p_2 + p_3 = (p_1 + 10) + (p_2 + 4) + a_3 + 9$ and so $p_3 = a_3 + 23$. On the third test, Paul's score was 23 points higher than Ada's score.

The correct answer is D.

PS06180

90. The price of a certain stock increased by 0.25 of 1 percent on a certain day. By what fraction did the price of the stock increase that day?

(A) $\frac{1}{2{,}500}$

(B) $\frac{1}{400}$

(C) $\frac{1}{40}$

(D) $\frac{1}{25}$

(E) $\frac{1}{4}$

Arithmetic Percents

It is given that the price of a certain stock increased by 0.25 of 1 percent on a certain day. This is equivalent to an increase of $\frac{1}{4}$ of $\frac{1}{100}$, which is $\left(\frac{1}{4}\right)\left(\frac{1}{100}\right)$, and $\left(\frac{1}{4}\right)\left(\frac{1}{100}\right) = \frac{1}{400}$.

The correct answer is B.

PS03831

91. For each trip, a taxicab company charges \$4.25 for the first mile and \$2.65 for each additional mile or fraction thereof. If the total charge for a certain trip was \$62.55, how many miles at most was the trip?

(A) 21
(B) 22
(C) 23
(D) 24
(E) 25

Arithmetic Applied problems

Subtracting the charge for the first mile leaves a charge of \$62.55 – \$4.25 = \$58.30 for the miles after the first mile. Divide this amount by \$2.65 to find the number of miles to which \$58.30 corresponds: $\frac{58.30}{2.65} = 22$ miles. Therefore, the total number of miles is at most 1 (the first mile) added to 22 (the number of miles after the first mile), which equals 23.

The correct answer is C.

PS12857

92. When 24 is divided by the positive integer n, the remainder is 4. Which of the following statements about n must be true?

I. n is even.
II. n is a multiple of 5.
III. n is a factor of 20.

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

Arithmetic Properties of numbers

Since the remainder is 4 when 24 is divided by the positive integer n and the remainder must be less than the divisor, it follows that $24 = qn + 4$ for some positive integer q and $4 < n$, or $qn = 20$ and $n > 4$. It follows that $n = 5$, or $n = 10$, or $n = 20$ since these are the only factors of 20 that exceed 4.

I. n is not necessarily even. For example, n could be 5.
II. n is necessarily a multiple of 5 since the value of n is either 5, 10, or 20.
III. n is a factor of 20 since $20 = qn$ for some positive integer q.

The correct answer is D.

PS12759

93. What is the thousandths digit in the decimal equivalent of $\frac{53}{5{,}000}$?

(A) 0
(B) 1
(C) 3
(D) 5
(E) 6

Arithmetic Place value

$\frac{53}{5{,}000} = \frac{106}{10{,}000} = 0.0106$ and the thousandths digit is 0.

The correct answer is A.

PS00986

94. The average (arithmetic mean) of the positive integers x, y, and z is 3. If $x < y < z$, what is the greatest possible value of z?

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

Algebra Inequalities

It is given that $\frac{x+y+z}{3} = 3$, or $x + y + z = 9$, or $z = 9 + (-x - y)$. It follows that the greatest possible value of z occurs when $-x - y = -(x + y)$ has the greatest possible value, which occurs when $x + y$ has the least possible value. Because x and y are different positive integers, the least possible value of $x + y$ occurs when $x = 1$ and $y = 2$. Therefore, the greatest possible value of z is $9 - 1 - 2 = 6$.

The correct answer is B.

PS14087

95. The product of 3,305 and the 1-digit integer x is a 5-digit integer. The units (ones) digit of the product is 5 and the hundreds digit is y. If A is the set of all possible values of x and B is the set of all possible values of y, then which of the following gives the members of A and B ?

	$\underline{A}$	$\underline{B}$
(A)	{1, 3, 5, 7, 9}	{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
(B)	{1, 3, 5, 7, 9}	{1, 3, 5, 7, 9}
(C)	{3, 5, 7, 9}	{1, 5, 7, 9}
(D)	{5, 7, 9}	{1, 5, 7}
(E)	{5, 7, 9}	{1, 5, 9}

Arithmetic Properties of numbers

Since the products of 3,305 and 1, 3,305 and 2, and 3,305 and 3 are the 4-digit integers 3,305, 6,610, and 9,915, respectively, it follows that x must be among the 1-digit integers 4, 5, 6, 7, 8, and 9. Also, since the units digit of the product of 3,305 and x is 5, it follows that x cannot be 4 (product has units digit 0), 6 (product has units digit 0), or 8 (product has units digit 0). Therefore, $A = \{5, 7, 9\}$. The possibilities for y will be the hundreds digits of the products (3,305)(5) = 16,525, (3,305)(7) = 23,135, and (3,305)(9) = 29,745. Thus, y can be 5, 1, or 7, and so $B = \{1, 5, 7\}$.

The correct answer is D.

PS05083

96. What is the largest integer n such that $\frac{1}{2^n} > 0.01$?

(A) 5
(B) 6
(C) 7
(D) 10
(E) 51

Arithmetic Exponents; Operations with rational numbers

Since $\frac{1}{2^n} > 0.01$ is equivalent to $2^n < 100$, find the largest integer n such that $2^n < 100$. Using trial and error, $2^6 = 64$ and $64 < 100$, but $2^7 = 128$ and $128 > 100$. Therefore, 6 is the largest integer such that $\frac{1}{2^n} > 0.01$.

The correct answer is B.

PS07001

97. If x and y are integers such that $2 < x \le 8$ and $2 < y \le 9$, what is the maximum value of $\frac{1}{x} - \frac{x}{y}$?

(A) $-3\frac{1}{8}$
(B) 0
(C) $\frac{1}{4}$
(D) $\frac{5}{18}$
(E) 2

Algebra Inequalities

Because x and y are both positive, the maximum value of $\frac{1}{x} - \frac{x}{y}$ will occur when the value of $\frac{1}{x}$ is maximum and the value of $\frac{x}{y}$ is minimum. The value of $\frac{1}{x}$ is maximum when the value of x is minimum or when $x = 3$. The value of $\frac{x}{y}$ is minimum when the value of x is minimum (or when $x = 3$) and the value of y is maximum (or when $y = 9$). Thus, the maximum value of $\frac{1}{x} - \frac{x}{y}$ is $\frac{1}{3} - \frac{3}{9} = 0$.

The correct answer is B.

PS01875

98. Items that are purchased together at a certain discount store are priced at \$3 for the first item purchased and \$1 for each additional item purchased. What is the maximum number of items that could be purchased together for a total price that is less than \$30 ?

(A) 25
(B) 26
(C) 27
(D) 28
(E) 29

Arithmetic Applied problems

After the first item is purchased, \$29.99 – \$3.00 = \$26.99 remains to purchase the additional items. Since the price for each of the additional items is \$1.00, a maximum of 26 additional items could be purchased. Therefore, a maximum of 1 + 26 = 27 items could be purchased for less than \$30.00.

The correct answer is C.

PS00774

99. What is the least integer z for which (0.000125)(0.0025)(0.00000125) $\times 10^z$ is an integer?

(A) 18
(B) 10
(C) 0
(D) −10
(E) −18

Arithmetic Decimals

Considering each of the three decimal numbers in parentheses separately, we know that 0.000125×10^6 is the integer 125, 0.0025×10^4 is the integer 25, and 0.00000125×10^8 is the integer 125. We thus know that $(0.000125) \times 10^6 \times (0.0025) \times 10^4 \times (0.00000125) \times 10^8 = (0.000125)(0.0025)(0.00000125) \times 10^6 \times 10^4 \times 10^8 = (0.000125)(0.0025)(0.00000125) \times 10^{6+4+8} = (0.000125)(0.0025)(0.00000125) \times 10^{18}$ is the integer $125 \times 25 \times 125$. We therefore know that if $z = 18$, then $(0.000125)(0.0025)(0.00000125) \times 10^z$ is an integer.

Now, if the product $125 \times 25 \times 125$ were divisible by 10, then for at least one integer z less than 18, $(0.000125)(0.0025)(0.00000125) \times 10^z$ would be an integer. However, each of the three numbers being multiplied in the product $125 \times 25 \times 125$ is odd (not divisible by 2). We thus know that $125 \times 25 \times 125$ is not divisible by 2 and is therefore odd. Because only even numbers are divisible by 10, we know that $125 \times 25 \times 125$ is not divisible by 10. We thus know that 18 is the *least* integer z such that $(0.000125)(0.0025)(0.00000125) \times 10^z$ is an integer.

Note that it is not necessary to perform the multiplication $125 \times 25 \times 125$.

The correct answer is A.

PS08407

100. The average (arithmetic mean) length per film for a group of 21 films is t minutes. If a film that runs for 66 minutes is removed from the group and replaced by one that runs for 52 minutes, what is the average length per film, in minutes, for the new group of films, in terms of t?

(A) $t+\frac{2}{3}$
(B) $t-\frac{2}{3}$
(C) $21t+14$
(D) $t+\frac{3}{2}$
(E) $t-\frac{3}{2}$

Arithmetic Statistics

Let S denote the sum of the lengths, in minutes, of the 21 films in the original group. Since the average length is t minutes, it follows that $\frac{S}{21} = t$.

If a 66-minute film is replaced by a 52-minute film, then the sum of the lengths of the 21 films in the resulting group is $S - 66 + 52 = S - 14$. Therefore, the average length of the resulting 21 films is $\frac{S-14}{21} = \frac{S}{21} - \frac{14}{21} = t - \frac{2}{3}$.

The correct answer is B.

PS08051

101. An open box in the shape of a cube measuring 50 centimeters on each side is constructed from plywood. If the plywood weighs 1.5 grams per square centimeter, which of the following is closest to the total weight, in kilograms, of the plywood used for the box? (1 kilogram = 1,000 grams)

(A) 2
(B) 4
(C) 8
(D) 13
(E) 19

Geometry Surface area

The total weight of the box is the sum of the weights of the 4 lateral sides of the box and the bottom of the box. Since the sides of the box all have the same area and the same density throughout, the total weight of the box is 5 times the weight of a side of the box. In grams, the weight of a side of the box is $(A\ \text{cm}^2)\left(1.5\ \dfrac{\text{g}}{\text{cm}^2}\right)$, where A is the area of a side of the box in square centimeters. Since $A = (50\text{ cm})(50\text{ cm}) = 2{,}500\text{ cm}^2$, the weight of a side of the box is $(2{,}500)(1.5) = 3{,}750$ grams $= 3.75$ kilograms. Therefore, the total weight of the box is $5(3.75) = 18.75$ kilograms.

The correct answer is E.

PS03614

102. A garden center sells a certain grass seed in 5-pound bags at \$13.85 per bag, 10-pound bags at \$20.43 per bag, and 25-pound bags at \$32.25 per bag. If a customer is to buy at least 65 pounds of the grass seed, but no more than 80 pounds, what is the least possible cost of the grass seed that the customer will buy?

(A) \$94.03
(B) \$96.75
(C) \$98.78
(D) \$102.07
(E) \$105.36

Arithmetic Applied problems

Let x represent the amount of grass seed, in pounds, the customer is to buy. It follows that $65 \le x \le 80$. Since the grass seed is available in only 5-pound, 10-pound, and 25-pound bags, then the customer must buy either 65, 70, 75, or 80 pounds of grass seed. Because the seed is more expensive per pound for smaller bags, the customer should minimize the number of the smaller bags and maximize the number of 25-pound bags to incur the least possible cost for the grass seed. The possible purchases are given in the table below.

x	Number of 25-pound bags	Number of 10-pound bags	Number of 5-pound bags	Total cost
65	2	1	1	\$98.78
70	2	2	0	\$105.36
75	3	0	0	\$96.75
80	3	0	1	\$110.60

The least possible cost is then 3(\$32.25) = \$96.75.

The correct answer is B.

PS12785

103. If $x = -|w|$, which of the following must be true?

(A) $x = -w$
(B) $x = w$
(C) $x^2 = w$
(D) $x^2 = w^2$
(E) $x^3 = w^3$

Algebra Absolute value

Squaring both sides of $x = -|w|$ gives $x^2 = (-|w|)^2$, or $x^2 = |w|^2 = w^2$.

Alternatively, if (x, w) is equal to either of the pairs $(-1,1)$ or $(-1,-1)$, then $x = -|w|$ is true. However, each of the answer choices except $x^2 = w^2$ is false for at least one of these two pairs.

The correct answer is D.

PS05965

104. Which of the following lines in the xy-plane does not contain any point with integers as both coordinates?

(A) $y = x$
(B) $y = x + \frac{1}{2}$
(C) $y = x + 5$
(D) $y = \frac{1}{2}x$
(E) $y = \frac{1}{2}x + 5$

Algebra; Arithmetic Substitution; Operations with rational numbers

A If x is an integer, y is an integer since $y = x$ Thus, the line given by $y = x$ contains points with integers as both coordinates.

B If x is an integer, then if y were an integer, then $y - x$ would be an integer. But, $y - x = \frac{1}{2}$ and $\frac{1}{2}$ is NOT an integer. Since assuming that y is an integer leads to a contradiction, then y cannot be an integer and the line given by $y = x + \frac{1}{2}$ does NOT contain any points with integers as both coordinates.

Since there can be only one correct answer, the lines in C, D, and E need not be checked, but for completeness,

C If x is an integer, $x + 5$ is an integer and so y is an integer since $y = x + 5$. Thus, the line given by $y = x + 5$ contains points with integers as both coordinates.

D If x is an even integer, $\frac{1}{2}x$ is an integer and so y is an integer since $y = \frac{1}{2}x$. Thus, the line given by $y = \frac{1}{2}x$ contains points with integers as both coordinates.

E If x is an even integer, $\frac{1}{2}x$ is an integer and $\frac{1}{2}x + 5$ is also an integer so y is an integer since $y = \frac{1}{2}x + 5$. Thus, the line given by $y = \frac{1}{2}x + 5$ contains points with integers as both coordinates.

The correct answer is B.

PS04160

105. A certain financial institution reported that its assets totaled \$2,377,366.30 on a certain day. Of this amount, \$31,724.54 was held in cash. Approximately what percent of the reported assets was held in cash on that day?

(A) 0.00013%
(B) 0.0013%
(C) 0.013%
(D) 0.13%
(E) 1.3%

Arithmetic Percents; Estimation

The requested percent can be estimated by converting the values into scientific notation.

$\frac{31{,}724.54}{2{,}377{,}366.30}$	value as fraction
$= \frac{3.172454 \times 10^4}{2.37736630 \times 10^6}$	convert to scientific notation
$= \frac{3.172454}{2.37736630} \times \frac{10^4}{10^6}$	arithmetic property of fractions
$= \frac{3.172454}{2.37736630} \times 10^{-2}$	subtract exponents
$\approx \frac{3}{2} \times 10^{-2}$	approximate
$= 1.5 \times 10^{-2}$	convert to decimal fraction
$= 0.015$	multiply
$= 1.5\%$	convert to percent

A more detailed computation would show that 1.3% is a better approximation. However, in order to select the best value from the values given as answer choices, the above computation is sufficient.

The correct answer is E.

$$\begin{array}{r} AB \\ +\ BA \\ \hline AAC \end{array}$$

PS09820

106. In the correctly worked addition problem shown, where the sum of the two-digit positive integers AB and BA is the three-digit integer AAC, and A, B, and C are different digits, what is the units digit of the integer AAC?

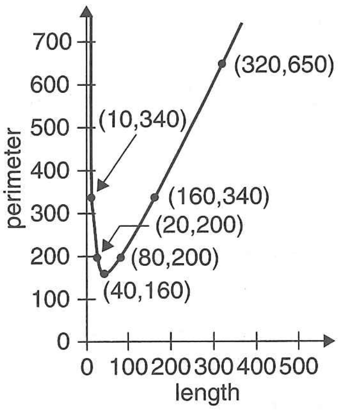
(A) 9
(B) 6
(C) 3
(D) 2
(E) 0

Arithmetic Place value

Determine the value of C.

It is given that $(10A + B) + (10B + A) = 100A + 10A + C$ or $11A + 11B = 110A + C$. Thus, $11B - 99A = C$, or $11(B - 9A) = C$. Therefore, C is divisible by 11, and 0 is the only digit that is divisible by 11.

The correct answer is E.



PS14060

107. Planning is in progress for a fenced, rectangular playground with an area of 1,600 square meters. The graph above shows the perimeter, in meters, as a function of the length of the playground. The length of the playground should be how many meters to minimize the perimeter and, therefore, the amount of fencing needed to enclose the playground?

(A) 10
(B) 40
(C) 60
(D) 160
(E) 340

Geometry Simple coordinate geometry

Since values of the perimeter are represented on the vertical axis, the point on the graph that corresponds to the minimum perimeter is the point on the graph that has the least y-coordinate, which is (40,160). This point corresponds to a length of 40 meters, which is what the question asked, and a perimeter of 160 meters.

The correct answer is B.

$$3r \le 4s + 5$$
$$|s| \le 5$$

PS06913

108. Given the inequalities above, which of the following CANNOT be the value of r?

(A) −20
(B) −5
(C) 0
(D) 5
(E) 20

Algebra Inequalities

Since $|s| \le 5$, it follows that $-5 \le s \le 5$. Therefore, $-20 \le 4s \le 20$, and hence $-15 \le 4s + 5 \le 25$. Since $3r \le 4s + 5$ (given) and $4s + 5 \le 25$ (end of previous sentence), it follows that $3r \le 25$. Among the answer choices, $3r \le 25$ is false only for $r = 20$.

The correct answer is E.

PS11647

109. If m is an even integer, v is an odd integer, and $m > v > 0$, which of the following represents the number of even integers less than m and greater than v?

(A) $\frac{m-v}{2} - 1$
(B) $\frac{m-v-1}{2}$
(C) $\frac{m-v}{2}$
(D) $m - v - 1$
(E) $m - v$

Arithmetic Properties of numbers

Since there is only one correct answer, one method of solving the problem is to choose values for m and v and determine which of the expressions gives the correct number for these values. For example, if $m = 6$ and $v = 1$, then there are 2 even integers less than 6 and greater than 1, namely the even integers 2 and 4. As the table below shows, $\frac{m-v-1}{2}$ is the only expression given that equals 2.

$$\frac{m-v}{2}-1=1.5$$

$$\frac{m-v-1}{2}=2$$

$$\frac{m-v}{2}=2.5$$

$$m-v-1=4$$

$$m-v=5$$

To solve this problem it is not necessary to show that $\frac{m-v-1}{2}$ always gives the correct number of even integers. However, one way this can be done is by the following method, first shown for a specific example and then shown in general. For the specific example, suppose $v = 15$ and $m = 144$. Then a list—call it the first list—of the even integers greater than v and less than m is 16, 18, 20, …, 140, 142. Now subtract 14 (chosen so that the second list will begin with 2) from each of the integers in the first list to form a second list, which has the same number of integers as the first list: 2, 4, 6, …, 128. Finally, divide each of the integers in the second list (all of which are even) by 2 to form a third list, which also has the same number of integers as the first list: 1, 2, 3, …, 64. Since the number of integers in the third list is 64, it follows that the number of integers in the first list is 64. For the general situation, the first list is the following list of even integers: $v + 1$, $v + 3$, $v + 5$, …, $m - 4$, $m - 2$. Now subtract the even integer $v - 1$ from (i.e., add $-v + 1$ to) each of the integers in the first list to obtain the second list: 2, 4, 6, …, $m - v - 3$, $m - v - 1$. (Note, for example, that $m - 4 - (v - 1) = m - v - 3$.) Finally, divide each of the integers (all of which are even) in the second list by 2 to obtain the third list: 1, 2, 3, …, $\frac{m-v-3}{2}$, $\frac{m-v-1}{2}$. Since the number of integers in the third list is $\frac{m-v-1}{2}$, it follows that the number of integers in the first list is $\frac{m-v-1}{2}$.

The correct answer is B.

PS02378

110. A positive integer is divisible by 9 if and only if the sum of its digits is divisible by 9. If n is a positive integer, for which of the following values of k is $25 \times 10^n + k \times 10^{2n}$ divisible by 9 ?

(A) 9
(B) 16
(C) 23
(D) 35
(E) 47

Arithmetic Properties of numbers

Since n can be any positive integer, let $n = 2$. Then $25 \times 10^n = 2{,}500$, so its digits consist of the digits 2 and 5 followed by two digits of 0. Also, $k \times 10^{2n} = k \times 10{,}000$, so its digits consist of the digits of k followed by four digits of 0. Therefore, the digits of $(25 \times 10^n) + (k \times 10^{2n})$ consist of the digits of k followed by the digits 2 and 5, followed by two digits of 0. The table below shows this for $n = 2$ and $k = 35$:

$$\begin{aligned} 25 \times 10^n &= \quad 2{,}500 \\ 35 \times 10^{2n} &= 350{,}000 \\ (25 \times 10^n) + (35 \times 10^{2n}) &= 352{,}500 \end{aligned}$$

Thus, when $n = 2$, the sum of the digits of $(25 \times 10^n) + (k \times 10^{2n})$ will be $2 + 5 = 7$ plus the sum of the digits of k. Of the answer choices, this sum of digits is divisible by 9 only for $k = 47$, which gives $2 + 5 + 4 + 7 = 18$. It can also be verified that, for each positive integer n, the only such answer choice is $k = 47$, although this additional verification is not necessary to obtain the correct answer.

The correct answer is E.

PS17806

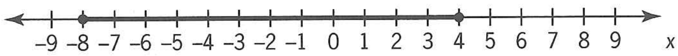
111. The perimeter of rectangle A is 200 meters. The length of rectangle B is 10 meters less than the length of rectangle A and the width of rectangle B is 10 meters more than the width of rectangle A. If rectangle B is a square, what is the width, in meters, of rectangle A ?

(A) 10
(B) 20
(C) 40
(D) 50
(E) 60

Geometry Rectangles; Perimeter

Let L meters and W meters be the length and width, respectively, of rectangle A. Then $(L - 10)$ meters and $(W + 10)$ meters are the length and width, respectively, of rectangle B. Since the perimeter of rectangle A is 200 meters, it follows that $2L + 2W = 200$, or $L + W = 100$. Since rectangle B is a square, it follows that $L - 10 = W + 10$, or $L - W = 20$. Adding the equations $L + W = 100$ and $L - W = 20$ gives $2L = 120$, or $L = 60$. From $L - W = 20$ and $L = 60$, it follows that $W = 40$, and so the width of rectangle A is 40 meters.

The correct answer is C.



PS08598

112. On the number line, the shaded interval is the graph of which of the following inequalities?

(A) $|x| \leq 4$

(B) $|x| \leq 8$

(C) $|x - 2| \leq 4$

(D) $|x - 2| \leq 6$

(E) $|x + 2| \leq 6$

Algebra Inequalities; Absolute value

The midpoint of the interval from -8 to 4, inclusive, is $\frac{-8+4}{2} = -2$ and the length of the interval from -8 to 4, inclusive, is $4 - (-8) = 12$, so the interval consists of all numbers within a distance of $\frac{12}{2} = 6$ from -2. Using an inequality involving absolute values, this can be described by $|x - (-2)| \leq 6$, or $|x + 2| \leq 6$.

Alternatively, the inequality $-8 \leq x \leq 4$ can be written as the conjunction $-8 \leq x$ and $x \leq 4$. Rewrite this conjunction so that the lower value, -8, and the upper value, 4, are shifted to values that have the same magnitude. This can be done by adding 2 to each side of each inequality, which gives $-6 \leq x + 2$ and $x + 2 \leq 6$. Thus, $x + 2$ lies between -6 and 6, inclusive, and it follows that $|x + 2| \leq 6$.

The correct answer is E.

PS12450

113. Last year members of a certain professional organization for teachers consisted of teachers from 49 different school districts, with an average (arithmetic mean) of 9.8 schools per district. Last year the average number of teachers at these schools who were members of the organization was 22. Which of the following is closest to the total number of members of the organization last year?

(A) 10^7

(B) 10^6

(C) 10^5

(D) 10^4

(E) 10^3

Arithmetic Statistics

There are 49 school districts and an average of 9.8 schools per district, so the number of schools is $(49)(9.8) \approx (50)(10) = 500$. There are approximately 500 schools and an average of 22 teachers at each school, so the number of teachers is approximately $(500)(22) \approx (500)(20) = 10{,}000 = 10^4$.

The correct answer is D.

PS09294

114. Of all the students in a certain dormitory, $\frac{1}{2}$ are first-year students and the rest are second-year students. If $\frac{4}{5}$ of the first-year students have not declared a major and if the fraction of second-year students who have declared a major is 3 times the fraction of first-year students who have declared a major, what fraction of all the students in the dormitory are second-year students who have not declared a major?

(A) $\frac{1}{15}$

(B) $\frac{1}{5}$

(C) $\frac{4}{15}$

(D) $\frac{1}{3}$

(E) $\frac{2}{5}$

Arithmetic Applied problems

Consider the table below in which T represents the total number of students in the dormitory. Since $\frac{1}{2}$ of the students are first-year students and the rest are second-year students, it follows that $\frac{1}{2}$ of the students are second-year students, and so the totals for the first-year and second-year columns are both $0.5T$. Since $\frac{4}{5}$ of the first-year students have not declared a major, it follows that the middle entry in the first-year column is $\frac{4}{5}(0.5T) = 0.4T$ and the first entry in the first-year column is $0.5T - 0.4T = 0.1T$. Since the fraction of second-year students who have declared a major is 3 times the fraction of first-year students who have declared a major, it follows that the first entry in the second-year column is $3(0.1T) = 0.3T$ and the second entry in the second-year column is $0.5T - 0.3T = 0.2T$. Thus, the fraction of students that are second-year students who have not declared a major is $\frac{0.2T}{T} = 0.2 = \frac{1}{5}$.

	First-year	Second-year	Total
Declared major	$0.1T$	$0.3T$	$0.4T$
Not declared major	$0.4T$	$0.2T$	$0.6T$
Total	$0.5T$	$0.5T$	T

The correct answer is B.

PS09050

115. If the average (arithmetic mean) of x, y, and z is $7x$ and $x \neq 0$, what is the ratio of x to the sum of y and z?

(A) 1:21
(B) 1:20
(C) 1:6
(D) 6:1
(E) 20:1

Algebra Ratio and proportion

Given that the average of x, y, and z is $7x$, it follows that $\frac{x+y+z}{3} = 7x$, or $x + y + z = 21x$, or $y + z = 20x$. Dividing both sides of the last equation by $20(y + z)$ gives $\frac{1}{20} = \frac{x}{y+z}$, so the ratio of x to the sum of y and z is 1:20.

The correct answer is B.

PS02352

116. In the coordinate plane, line k passes through the origin and has slope 2. If points $(3,y)$ and $(x,4)$ are on line k, then $x + y =$

(A) 3.5
(B) 7
(C) 8
(D) 10
(E) 14

Algebra Simple coordinate geometry

Since line k has slope 2 and passes through the origin, the equation of line k is $y = 2x$. If the point $(3,y)$ is on line k, then $y = 2(3) = 6$. If the point $(x,4)$ is on line k, then $4 = 2x$ and so $x = 2$. Therefore, $x + y = 6 + 2 = 8$.

The correct answer is C.

PS08661

117. If a, b, and c are constants, $a > b > c$, and $x^3 - x = (x - a)(x - b)(x - c)$ for all numbers x, what is the value of b?

(A) −3
(B) −1
(C) 0
(D) 1
(E) 3

Algebra Simplifying algebraic expressions

Since $(x - a)(x - b)(x - c) = x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1) = (x - 0)(x - 1)(x + 1)$ then a, b, and c are 0, 1, and −1 in some order. Since $a > b > c$, it follows that $a = 1$, $b = 0$, and $c = -1$.

The correct answer is C.

PS06273

118. $17^3 + 17^4 =$

(A) 17^7
(B) $17^3(18)$
(C) $17^6(18)$
(D) $2(17^3) + 17$
(E) $2(17^3) - 17$

Arithmetic Exponents

Since $17^3 = 17^3 \times 1$ and $17^4 = 17^3 \times 17$, then 17^3 may be factored out of each term. It follows that $17^3 + 17^4 = 17^3(1 + 17) = 17^3(18)$.

The correct answer is B.

PS02934

119. Company K's earnings were $12 million last year. If this year's earnings are projected to be 150 percent greater than last year's earnings, what are Company K's projected earnings this year?

(A) $13.5 million
(B) $15 million
(C) $18 million
(D) $27 million
(E) $30 million

Arithmetic Percents

If one quantity x is p percent greater than another quantity y, then $x = y + \left(\frac{p}{100}\right)y$. Let y represent last year's earnings and x represent this year's earnings, which are projected to be 150 percent greater than last year's earnings. Then, $x = y + \left(\frac{150}{100}\right)y = y + 1.5y = 2.5y$. Since last year's earnings were $12 million, this year's earnings are projected to be 2.5($12 million) = $30 million.

The correct answer is E.

PS05413

120. Jonah drove the first half of a 100-mile trip in x hours and the second half in y hours. Which of the following is equal to Jonah's average speed, in miles per hour, for the entire trip?

(A) $\frac{50}{x+y}$
(B) $\frac{100}{x+y}$
(C) $\frac{25}{x} + \frac{25}{y}$
(D) $\frac{50}{x} + \frac{50}{y}$
(E) $\frac{100}{x} + \frac{100}{y}$

Algebra Applied problems

Using average speed $= \frac{\text{total distance}}{\text{total time}}$, it follows that Jonah's average speed for his entire 100-mile trip is $\frac{100}{x + y}$.

The correct answer is B.

PS06135

121. What is the greatest number of identical bouquets that can be made out of 21 white and 91 red tulips if no flowers are to be left out? (Two bouquets are identical whenever the number of red tulips in the two bouquets is equal and the number of white tulips in the two bouquets is equal.)

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

Arithmetic Properties of numbers

Since the question asks for the greatest number of bouquets that can be made using all of the flowers, the number of bouquets will need to be the greatest common factor of 21 and 91. Since 21 = (3)(7) and 91 = (7)(13), the greatest common factor of 21 and 91 is 7. Therefore, 7 bouquets can be made, each with 3 white tulips and 13 red tulips.

The correct answer is E.

PS11454

122. In the xy-plane, the points (c,d), $(c,-d)$, and $(-c,-d)$ are three vertices of a certain square. If $c < 0$ and $d > 0$, which of the following points is in the same quadrant as the fourth vertex of the square?

(A) (–5,–3)
(B) (–5,3)
(C) (5,–3)
(D) (3,–5)
(E) (3,5)

Geometry Coordinate geometry

Because the points (c,d) and $(c,-d)$ lie on the same vertical line (the line with equation $x = c$), one side of the square has length $2d$ and is vertical. Therefore, the side of the square opposite this side has length $2d$, is vertical, and contains the vertex $(-c,-d)$. From this it follows that the remaining vertex is $(-c,d)$, because $(-c,d)$ lies on the same vertical line as $(-c,-d)$ (the line with equation $x = -c$) and these two vertices are a distance $2d$ apart. Because $c < 0$ and $d > 0$, the point $(-c,d)$ has positive x-coordinate and positive y-coordinate. Thus, the point $(-c,d)$ is in Quadrant I. Of the answer choices, only (3,5) is in Quadrant I.

The correct answer is E.

PS05470

123. If the amount of federal estate tax due on an estate valued at \$1.35 million is \$437,000 plus 43 percent of the value of the estate in excess of \$1.25 million, then the federal tax due is approximately what percent of the value of the estate?

A. 30%
B. 35%
C. 40%
D. 45%
E. 50%

Arithmetic Percents; Estimation

The amount of tax divided by the value of the estate is

$$\frac{[0.437 + (0.43)(1.35 - 1.25)]\ \cancel{\text{million}}}{1.35\ \cancel{\text{million}}} \quad \text{value as fraction}$$

$$= \frac{0.437 + (0.43)(0.1)}{1.35} \quad \text{arithmetic}$$

$$= \frac{0.48}{1.35} = \frac{48}{135} \quad \text{arithmetic}$$

By long division, $\frac{48}{135}$ is approximately 35.6, so the closest answer choice is 35%.

Alternatively, $\frac{48}{135}$ can be estimated by $\frac{48}{136} = \frac{6}{17} \approx \frac{6}{18} = \frac{1}{3} \approx 33\%$, so the closest answer choice is 35%. Note that $\frac{48}{135}$ is greater than $\frac{48}{136}$, and $\frac{6}{17}$ is greater than $\frac{6}{18}$, so the correct value is greater than 33%, which rules out 30% being the closest.

The correct answer is B.

PS05924

124. If $\frac{3}{10^4} = x\%$, then $x =$

(A) 0.3
(B) 0.03
(C) 0.003
(D) 0.0003
(E) 0.00003

Arithmetic Percents

Given that $\frac{3}{10^4} = x\%$, and writing $x\%$ as $\frac{x}{100}$, it follows that $\frac{3}{10^4} = \frac{x}{100}$. Multiplying both sides by 100 gives $x = \frac{300}{10^4} = \frac{300}{10{,}000} = \frac{3}{100} = 0.03$.

The correct answer is B.

PS01285

125. What is the remainder when 3^{24} is divided by 5 ?

(A) 0

(B) 1

(C) 2

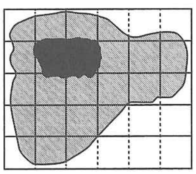
(D) 3

(E) 4

Arithmetic Properties of numbers

A pattern in the units digits of the numbers $3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243$, etc., can be found by observing that the units digit of a product of two integers is the same as the units digit of the product of the units digit of the two integers. For example, the units digit of $3^5 = 3 \times 3^4 = 3 \times 81$ is 3 since the units digit of 3×1 is 3, and the units digit of $3^6 = 3 \times 3^5 = 3 \times 243$ is 9 since the units digit of 3×3 is 9. From this it follows that the units digit of the powers of 3 follow the pattern 3, 9, 7, 1, 3, 9, 7, 1, etc., with a units digit of 1 for $3^4, 3^8, 3^{12}, \ldots, 3^{24}, \ldots$. Therefore, the units digit of 3^{24} is 1. Thus, 3^{24} is 1 more than a multiple of 10, and hence 3^{24} is 1 more than a multiple of 5, and so the remainder when 3^{24} is divided by 5 is 1.

The correct answer is B.



PS11692

126. In the figure shown, a square grid is superimposed on the map of a park, represented by the shaded region, in the middle of which is a pond, represented by the black region. If the area of the pond is 5,000 square yards, which of the following is closest to the area of the park, in square yards, including the area of the pond?

(A) 30,000

(B) 45,000

(C) 60,000

(D) 75,000

(E) 90,000

Geometry Estimation; Area

Let s be the side length, in yards, represented by each of the squares that form the square grid. By inspection, the map of the pond fills approximately 2 squares, so the area of the pond is approximately $2s^2$ yd^2. Since it is given that the area of the pond is 5,000 yd^2, it follows that $2s^2 = 5{,}000$, or $s^2 = 2{,}500$, or $s = 50$. The entire rectangular figure has a horizontal length of 6 squares, which represents 6(50 yd) = 300 yd, and a vertical length of 5 squares, which represents 5(50 yd) = 250 yd, so the area of the entire rectangular figure represents (300 yd)(250 yd) = 75,000 yd^2.

In the rectangular figure, less area is not shaded than is shaded, so to estimate the area represented by the shaded portion it will be easier to estimate the area represented by the portion that is not shaded and subtract this estimate from 75,000 yd^2, the area represented by the entire rectangular figure. By inspection, the area not shaded in the upper right corner represents approximately 2 squares, the area not shaded in the lower right corner represents approximately 6 squares, and the area not shaded on the left side represents approximately 2 squares. Thus, the area not shaded represents approximately $(2 + 6 + 2)$ squares, or approximately 10 squares, or approximately 10(2,500 yd^2) = 25,000 yd^2. Therefore, the area of the park is approximately 75,000 yd^2 – 25,000 yd^2 = 50,000 yd^2, and of the values available, 45,000 is the closest.

The correct answer is B.

PS03623

127. If the volume of a ball is 32,490 cubic millimeters, what is the volume of the ball in cubic centimeters? (1 millimeter = 0.1 centimeter)

(A) 0.3249

(B) 3.249

(C) 32.49

(D) 324.9

(E) 3,249

Arithmetic Measurement conversion

Since 1 mm = 0.1 cm, it follows that 1 mm^3 = $(0.1)^3$ cm^3 = 0.001 cm^3. Therefore, 32,490 mm^3 = (32,490)(0.001) cm^3 = 32.49 cm^3.

The correct answer is C.

PS07058

128. David used part of \$100,000 to purchase a house. Of the remaining portion, he invested $\frac{1}{3}$ of it at 4 percent simple annual interest and $\frac{2}{3}$ of it at 6 percent simple annual interest. If after a year the income from the two investments totaled \$320, what was the purchase price of the house?

(A) \$96,000
(B) \$94,000
(C) \$88,000
(D) \$75,000
(E) \$40,000

Algebra Applied problems; Percents

Let x be the amount, in dollars, that David used to purchase the house. Then David invested $(100{,}000 - x)$ dollars, $\frac{1}{3}$ at 4% simple annual interest and $\frac{2}{3}$ at 6% simple annual interest. After one year the total interest, in dollars, on this investment was $\frac{1}{3}(100{,}000 - x)(0.04) + \frac{2}{3}(100{,}000 - x)(0.06) = 320$. Solve this equation to find the value of x.

$\frac{1}{3}(100{,}000 - x)(0.04) + \frac{2}{3}(100{,}000 - x)(0.06) = 320$	given
$(100{,}000 - x)(0.04) + 2(100{,}000 - x)(0.06) = 960$	multiply both sides by 3
$4{,}000 - 0.04x + 12{,}000 - 0.12x = 960$	distributive property
$16{,}000 - 0.16x = 960$	combine like terms
$16{,}000 - 960 = 0.16x$	add $0.16x - 960$ to both sides
$100{,}000 - 6{,}000 = x$	divide both sides by 0.16
$94{,}000 = x$	

Therefore, the purchase price of the house was \$94,000.

The correct answer is B.

PS11537

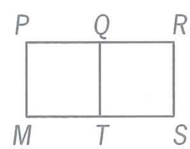
129. In the sequence $x_0, x_1, x_2, \ldots, x_n$, each term from x_1 to x_k is 3 greater than the previous term, and each term from x_{k+1} to x_n is 3 less than the previous term, where n and k are positive integers and $k < n$. If $x_0 = x_n = 0$ and if $x_k = 15$, what is the value of n ?

(A) 5
(B) 6
(C) 9
(D) 10
(E) 15

Algebra Sequences

Since $x_0 = 0$ and each term from x_1 to x_k is 3 greater than the previous term, then $x_k = 0 + (k)(3)$. Since $x_k = 15$, then $15 = 3k$ and $k = 5$. Since each term from x_{k+1} to x_n is 3 less than the previous term, then $x_n = x_k - (n - k)(3)$. Substituting the known values for x_k, x_n, and k gives $0 = 15 - (n - 5)(3)$, from which it follows that $3n = 30$ and $n = 10$.

The correct answer is D.



Note: Not drawn to scale.

PS11145

130. In the figure shown above, line segment QR has length 12, and rectangle $MPQT$ is a square. If the area of rectangular region $MPRS$ is 540, what is the area of rectangular region $TQRS$?

(A) 144
(B) 216
(C) 324
(D) 360
(E) 396

Geometry; Algebra Area; Second-degree equations

Since $MPQT$ is a square, let $MP = PQ = x$. Then $PR = PQ + QR = x + 12$. The area of $MPRS$ can be expressed as $x(x + 12)$. Since the area of $MPRS$ is given to be 540,

$$\begin{aligned} x(x+12) &= 540 \\ x^2 + 12x &= 540 \\ x^2 + 12x - 540 &= 0 \\ (x-18)(x+30) &= 0 \\ x &= 18 \text{ or } x = -30 \end{aligned}$$

Since x represents a length and must be positive, $x = 18$. The area of $TQRS$ is then $(12)(18) = 216$.

As an alternative to solving the quadratic equation, look for a pair of positive numbers such that their product is 540 and one is 12 greater than the other. The pair is 18 and 30, so $x = 18$ and the area of $TQRS$ is then $(12)(18) = 216$.

The correct answer is B.

PS09439

131. A certain manufacturer sells its product to stores in 113 different regions worldwide, with an average (arithmetic mean) of 181 stores per region. If last year these stores sold an average of 51,752 units of the manufacturer's product per store, which of the following is closest to the total number of units of the manufacturer's product sold worldwide last year?

(A) 10^6
(B) 10^7
(C) 10^8
(D) 10^9
(E) 10^{10}

Arithmetic Estimation

$$\begin{aligned} (113)(181)(51{,}752) &\approx (100)(200)(50{,}000) \\ &= 10^2 \times (2\times 10^2) \times (5 \times 10^4) \\ &= (2\times 5)\times 10^{2+2+4} \\ &= 10^1 \times 10^8 = 10^9 \end{aligned}$$

The correct answer is D.

PS17708

132. Andrew started saving at the beginning of the year and had saved \$240 by the end of the year. He continued to save and by the end of 2 years had saved a total of \$540. Which of the following is closest to the percent increase in the amount Andrew saved during the second year compared to the amount he saved during the first year?

(A) 11%
(B) 25%
(C) 44%
(D) 56%
(E) 125%

Arithmetic Percents

Andrew saved \$240 in the first year and \$540 – \$240 = \$300 in the second year. The percent increase in the amount Andrew saved in the second year compared to the amount he saved in the first year is $\left(\dfrac{300-240}{240}\times 100\right)\% = \left(\dfrac{60}{240}\times 100\right)\% = \left(\dfrac{1}{4}\times 100\right)\% = 25\%$.

The correct answer is B.

PS19062

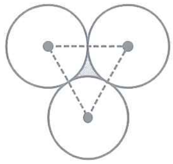
133. Two numbers differ by 2 and sum to S. Which of the following is the greater of the numbers in terms of S?

(A) $\dfrac{S}{2} - 1$
(B) $\dfrac{S}{2}$
(C) $\dfrac{S}{2} + \dfrac{1}{2}$
(D) $\dfrac{S}{2} + 1$
(E) $\dfrac{S}{2} + 2$

Algebra First-degree equations

Let x represent the greater of the two numbers that differ by 2. Then, $x - 2$ represents the lesser of the two numbers. The two numbers sum to S, so $x + (x - 2) = S$. It follows that $2x - 2 = S$, or $2x = S + 2$, or $x = \dfrac{S}{2} + 1$.

The correct answer is D.



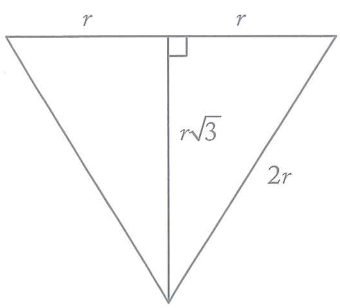
PS00904

134. The figure shown above consists of three identical circles that are tangent to each other. If the area of the shaded region is $64\sqrt{3} - 32\pi$, what is the radius of each circle?

(A) 4
(B) 8
(C) 16
(D) 24
(E) 32

Geometry Circles; Triangles; Area

Let r represent the radius of each circle. Then the triangle shown dashed in the figure is equilateral with sides $2r$ units long. The interior of the triangle is comprised of the shaded region and three circular sectors. The area of the shaded region can be found as the area of the triangle minus the sum of the areas of the three sectors. Since the triangle is equilateral, its side lengths are in the proportions as shown in the diagram below. The area of the interior of the triangle is $\frac{1}{2}(2r)\left(r\sqrt{3}\right) = r^2\sqrt{3}$.



Each of the three sectors has a central angle of 60° because the central angle is an angle of the equilateral triangle. Therefore, the area of each sector is $\frac{60}{360} = \frac{1}{6}$ of the area of the circle. The sum of the areas of the three sectors is then $3\left(\frac{1}{6}\pi r^2\right) = \frac{1}{2}\pi r^2$. Thus, the area of the shaded region is $r^2\sqrt{3} - \frac{1}{2}\pi r^2 = r^2\left(\sqrt{3} - \frac{1}{2}\pi\right)$. But, this area is given as $64\sqrt{3} - 32\pi = 64\left(\sqrt{3} - \frac{1}{2}\pi\right)$.

Thus $r^2 = 64$, and $r = 8$.

The correct answer is B.

PS02053

135. In a numerical table with 10 rows and 10 columns, each entry is either a 9 or a 10. If the number of 9s in the nth row is $n - 1$ for each n from 1 to 10, what is the average (arithmetic mean) of all the numbers in the table?

(A) 9.45
(B) 9.50
(C) 9.55
(D) 9.65
(E) 9.70

Arithmetic Operations with integers

There are $(10)(10) = 100$ entries in the table. In rows $1, 2, 3, \ldots, 10$, the number of 9s is $0, 1, 2, \ldots, 9$, respectively, giving a total of $0 + 1 + 2 + \ldots + 9 = 45$ entries with a 9. This leaves a total of $100 - 45 = 55$ entries with a 10. Therefore, the sum of the 100 entries is $45(9) + 55(10) = 405 + 550 = 955$, and the average of the 100 entries is $\frac{955}{100} = 9.55$

The correct answer is C.

PS08485

136. A positive integer n is a perfect number provided that the sum of all the positive factors of n, including 1 and n, is equal to $2n$. What is the sum of the reciprocals of all the positive factors of the perfect number 28 ?

(A) $\frac{1}{4}$
(B) $\frac{56}{27}$
(C) 2
(D) 3
(E) 4

Arithmetic Properties of numbers

The factors of 28 are 1, 2, 4, 7, 14, and 28. Therefore, the sum of the reciprocals of the factors of 28 is $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}=$

$$\frac{28}{28}+\frac{14}{28}+\frac{7}{28}+\frac{4}{28}+\frac{2}{28}+\frac{1}{28}=$$

$$\frac{28+14+7+4+2+1}{28}=\frac{56}{28}=2.$$

The correct answer is C.

PS11430

137. The infinite sequence $a_1, a_2, \ldots, a_n, \ldots$ is such that $a_1 = 2$, $a_2 = -3$, $a_3 = 5$, $a_4 = -1$, and $a_n = a_{n-4}$ for $n > 4$. What is the sum of the first 97 terms of the sequence?

(A) 72
(B) 74
(C) 75
(D) 78
(E) 80

Arithmetic Sequences and series

Because $a_n = a_{n-4}$ for $n > 4$, it follows that the terms of the sequence repeat in groups of 4 terms:

Values for n	Values for a_n
1, 2, 3, 4	2, −3, 5, −1
5, 6, 7, 8	2, −3, 5, −1
9, 10, 11, 12	2, −3, 5, −1
13, 14, 15, 16	2, −3, 5, −1

Thus, since $97 = 24(4) + 1$, the sum of the first 97 terms can be grouped into 24 groups of 4 terms each, with one remaining term, which allows the sum to be easily found:

$$(a_1+a_2+a_3+a_4)+(a_5+a_6+a_7+a_8)+\ldots+(a_{93}+a_{94}+a_{95}+a_{96})+a_{97}$$

$$=(2-3+5-1)+(2-3+5-1)+\ldots+(2-3+5-1)+2$$

$$=24(2-3+5-1)+2=24(3)+2=74$$

The correct answer is B.

PS09901

138. The sequence $a_1, a_2, \ldots, a_n, \ldots$ is such that $a_n = 2a_{n-1} - x$ for all positive integers $n \geq 2$ and for a certain number x. If $a_5 = 99$ and $a_3 = 27$, what is the value of x?

(A) 3
(B) 9
(C) 18
(D) 36
(E) 45

Algebra Sequences and series

An expression for a_5 that involves x can be obtained using $a_3 = 27$ and applying the equation $a_n = 2a_{n-1} - x$ twice, once for $n = 4$ and once for $n = 5$.

$a_4 = 2a_3 - x$	using $a_n = 2a_{n-1} - x$ for $n = 4$
$= 2(27) - x$	using $a_3 = 27$
$a_5 = 2a_4 - x$	using $a_n = 2a_{n-1} - x$ for $n = 5$
$= 2[2(27) - x] - x$	using $a_4 = 2(27) - x$
$= 4(27) - 3x$	combine like terms

Therefore, using $a_5 = 99$, we have

$99 = 4(27) - 3x$	given
$3x = 4(27) - 99$	adding $(3x - 99)$ to both sides
$x = 4(9) - 33$	dividing both sides by 3
$x = 3$	arithmetic

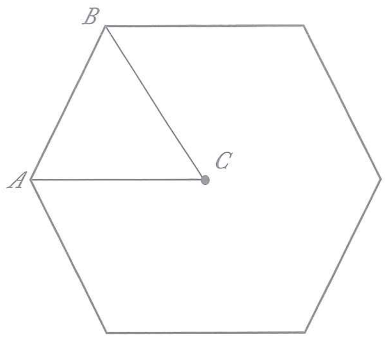
The correct answer is A.

PS03779

139. A window is in the shape of a regular hexagon with each side of length 80 centimeters. If a diagonal through the center of the hexagon is w centimeters long, then $w =$

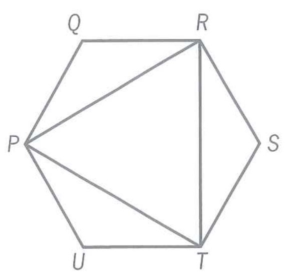
(A) 80
(B) 120
(C) 150
(D) 160
(E) 240

Geometry Polygons



Let A and B be the endpoints of one of the sides of the hexagon and let C be the center of the hexagon. Then the degree measure of $\angle ACB$ is $\frac{360}{6} = 60$ and the sum of the degree measures of $\angle ABC$ and $\angle BAC$ is $180 - 60 = 120$. Also, since $AC = BC$, the degree measures of $\angle ABC$ and $\angle BAC$ are equal. Therefore, the degree measure of each of $\angle ABC$ and $\angle BAC$ is 60. Thus, ΔABC is an equilateral triangle with side length $AB = 80$. It follows that the length of a diagonal through the center of the hexagon is $2(AC) = 2(80) = 160$.

The correct answer is D.



PS03695

140. In the figure shown, *PQRSTU* is a regular polygon with sides of length x. What is the perimeter of triangle *PRT* in terms of x ?

(A) $\frac{x\sqrt{3}}{2}$

(B) $x\sqrt{3}$

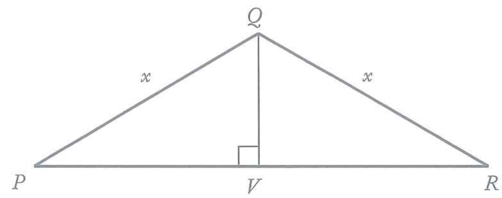
(C) $\frac{3x\sqrt{3}}{2}$

(D) $3x\sqrt{3}$

(E) $4x\sqrt{3}$

Geometry Polygons

Since $PQRSTU$ is a regular hexagon, ΔPQR, ΔRST, and ΔTUP are the same size and shape, so $PR = RT = TP$ and the perimeter of ΔPRT is $3(PR)$. Note that in the figure above, $PQRSTU$ is partitioned into four triangles. The sum of the degree measures of the interior angles of each triangle is 180°. The total of the degree measures of the interior angles of these four triangles is equal to the sum of the degree measures of the six interior angles of $PQRSTU$. Since $PQRSTU$ is a regular hexagon, each of $\angle UPQ$, $\angle PQR$, $\angle QRS$, $\angle RST$, $\angle STU$, and $\angle TUP$ has the same measure, which is $\frac{(4)(180°)}{6} = 120°$.



In the figure above, ΔPQR is isosceles with $PQ = QR = x$. The measure of $\angle PQR$ is 120°, and the measure of $\angle P$ = the measure of $\angle R = \frac{180° - 120°}{2} = 30°$. $\overline{QV}$ is perpendicular to $\overline{PR}$ and $PV = VR$. Since ΔPVQ is a 30°–60°–90° triangle, its side lengths are in the ratio $1{:}\sqrt{3}{:}2$, and so $PV = \frac{x\sqrt{3}}{2}$ and $PR = x\sqrt{3}$. Therefore, the perimeter of ΔPRT is $3(x\sqrt{3}) = 3x\sqrt{3}$.

The correct answer is D.

PS11755

141. In a certain medical survey, 45 percent of the people surveyed had the type A antigen in their blood and 3 percent had both the type A antigen and the type B antigen. Which of the following is closest to the percent of those with the type A antigen who also had the type B antigen?

(A) 1.35%

(B) 6.67%

(C) 13.50%

(D) 15.00%

(E) 42.00%

Arithmetic Applied problems; Percents

Let n be the total number of people surveyed. Then, the proportion of the people who had type A who also had type B is $\frac{(3\%)\cancel{n}}{(45\%)\cancel{n}} = \frac{3}{45} = \frac{1}{15}$, which as a percent is approximately 6.67%. Note that by using $\frac{1}{15} = \frac{1}{3} \times \frac{1}{5}$, which equals $\frac{1}{3}$ of 20%, we can avoid dividing by a 2-digit integer.

The correct answer is B.

PS05146

142. On a certain transatlantic crossing, 20 percent of a ship's passengers held round-trip tickets and also took their cars aboard the ship. If 60 percent of the passengers with round-trip tickets did not take their cars aboard the ship, what percent of the ship's passengers held round-trip tickets?

(A) $33\frac{1}{3}\%$

(B) 40%

(C) 50%

(D) 60%

(E) $66\frac{2}{3}\%$

Arithmetic Percents

Since the number of passengers on the ship is immaterial, let the number of passengers on the ship be 100 for convenience. Let x be the number of passengers that held round-trip tickets. Then, since 20 percent of the passengers held a round-trip ticket and took their cars aboard the ship, $0.20(100) = 20$ passengers held round-trip tickets and took their cars aboard the ship. The remaining passengers with round-trip tickets did not take their cars aboard, and they represent $0.6x$ (that is, 60 percent of the passengers with round-trip tickets). Thus $0.6x + 20 = x$, from which it follows that $20 = 0.4x$, and so $x = 50$. The percent of passengers with round-trip tickets is, then, $\frac{50}{100} = 50\%$.

The correct answer is C.

PS03696

143. If x and k are integers and $(12^x)(4^{2x+1}) = (2^k)(3^2)$, what is the value of k?

(A) 5

(B) 7

(C) 10

(D) 12

(E) 14

Arithmetic Exponents

Rewrite the expression on the left so that it is a product of powers of 2 and 3.

$$\begin{aligned}(12^x)(4^{2x+1}) &= [(3\cdot 2^2)^x][(2^2)^{2x+1}]\\ &= (3^x)[(2^2)^x][2^{2(2x+1)}]\\ &= (3^x)(2^{2x})(2^{4x+2})\\ &= (3^x)(2^{6x+2})\end{aligned}$$

Then, since $(12^x)(4^{2x+1}) = (2^k)(3^2)$, it follows that $(3^x)(2^{6x+2}) = (2^k)(3^2) = (3^2)(2^k)$, so $x = 2$ and $k = 6x + 2$. Substituting 2 for x gives $k = 6(2) + 2 = 14$.

The correct answer is E.

PS11024

144. If S is the sum of the reciprocals of the 10 consecutive integers from 21 to 30, then S is between which of the following two fractions?

(A) $\frac{1}{3}$ and $\frac{1}{2}$

(B) $\frac{1}{4}$ and $\frac{1}{3}$

(C) $\frac{1}{5}$ and $\frac{1}{4}$

(D) $\frac{1}{6}$ and $\frac{1}{5}$

(E) $\frac{1}{7}$ and $\frac{1}{6}$

Arithmetic Estimation

The value of $\frac{1}{21} + \frac{1}{22} + \frac{1}{23} + \ldots + \frac{1}{30}$ is LESS than $\frac{1}{20} + \frac{1}{20} + \frac{1}{20} + \ldots + \frac{1}{20}$ (10 numbers added), which equals $10\left(\frac{1}{20}\right) = \frac{1}{2}$, and GREATER than $\frac{1}{30} + \frac{1}{30} + \frac{1}{30} + \ldots + \frac{1}{30}$

(10 numbers added), which equals $10\left(\frac{1}{30}\right)=\frac{1}{3}$.
Therefore, the value of $\frac{1}{21}+\frac{1}{22}+\frac{1}{23}+\ldots+\frac{1}{30}$ is between $\frac{1}{3}$ and $\frac{1}{2}$.

The correct answer is A.

PS08729

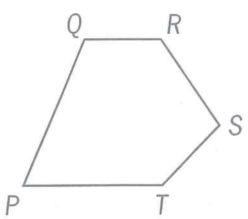
145. For every even positive integer m, $f(m)$ represents the product of all even integers from 2 to m, inclusive. For example, $f(12)=2\times4\times6\times8\times10\times12$. What is the greatest prime factor of $f(24)$?

(A) 23
(B) 19
(C) 17
(D) 13
(E) 11

Arithmetic Properties of numbers

Rewriting $f(24)=2\times4\times6\times8\times10\times12\times14\times\ldots\times20\times22\times24$ as $2\times4\times2(3)\times8\times2(5)\times12\times2(7)\times\ldots\times20\times2(11)\times24$ shows that all of the prime numbers from 2 through 11 are factors of $f(24)$. The next prime number is 13, but 13 is not a factor of $f(24)$ because none of the even integers from 2 through 24 has 13 as a factor. Therefore, the largest prime factor of $f(24)$ is 11.

The correct answer is E.



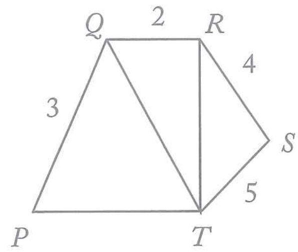
Note: Not drawn to scale.

PS08572

146. In pentagon $PQRST$, $PQ=3$, $QR=2$, $RS=4$, and $ST=5$. Which of the lengths 5, 10, and 15 could be the value of PT ?

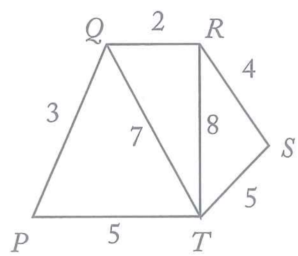
(A) 5 only
(B) 15 only
(C) 5 and 10 only
(D) 10 and 15 only
(E) 5, 10, and 15

Geometry Polygons; Triangles



Note: Not drawn to scale.

In the figure above, diagonals $\overline{TQ}$ and $\overline{TR}$ have been drawn in to show ΔTRS and ΔTRQ. Because the length of any side of a triangle must be less than the sum of the lengths of the other two sides, $RT<5+4=9$ in ΔTRS, and $QT<RT+2$ in ΔTRQ. Since $RT<9$, then $RT+2<9+2=11$, which then implies $QT<11$. Now, $PT<QT+3$ in ΔTQP, and since $QT<11$, $QT+3<11+3=14$. It follows that $PT<14$. Therefore, 15 cannot be the length of $\overline{PT}$ since $15 \not< 14$.



Note: Not drawn to scale.

To show that 5 can be the length of $\overline{PT}$, consider the figure above. For ΔTQP, the length of any side is less than the sum of the lengths of the other two sides as shown below.

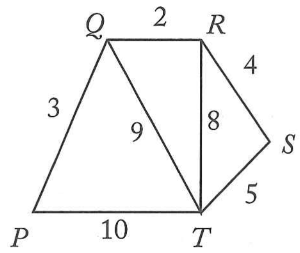
$$QT=7<8=5+3=PT+PQ$$
$$PQ=3<12=5+7=PT+TQ$$
$$PT=5<10=3+7=PQ+TQ$$

For ΔRQT, the length of any side is less than the sum of the lengths of the other two sides as shown below.

$$RT=8<9=7+2=QT+QR$$
$$RQ=2<15=7+8=QT+RT$$
$$QT=7<10=2+8=QR+RT$$

For ΔRST, the length of any side is less than the sum of the lengths of the other two sides as shown below.

$$RS = 4 < 13 = 8 + 5 = TR + TS$$
$$RT = 8 < 9 = 5 + 4 = ST + SR$$
$$ST = 5 < 12 = 8 + 4 = TR + RS$$



Note: Not drawn to scale.

To show that 10 can be the length of $\overline{PT}$, consider the figure above. For ΔTQP, the length of any side is less than the sum of the lengths of the other two sides as shown below.

$$QT = 9 < 13 = 10 + 3 = PT + PQ$$
$$PQ = 3 < 19 = 10 + 9 = PT + TQ$$
$$PT = 10 < 12 = 3 + 9 = PQ + TQ$$

For ΔRQT, the length of any side is less than the sum of the lengths of the other two sides as shown below.

$$RT = 8 < 11 = 9 + 2 = QT + QR$$
$$RQ = 2 < 17 = 9 + 8 = QT + RT$$
$$QT = 9 < 10 = 2 + 8 = QT + RT$$

For ΔRST, the length of any side is less than the sum of the lengths of the other two sides as shown below.

$$RS = 4 < 13 = 8 + 5 = TR + TS$$
$$RT = 8 < 9 = 5 + 4 = ST + SR$$
$$ST = 5 < 12 = 8 + 4 = TR + RS$$

Therefore, 5 and 10 can be the length of $\overline{PT}$, and 15 cannot be the length of $\overline{PT}$.

The correct answer is C.

$$3, k, 2, 8, m, 3$$

PS07771

147. The arithmetic mean of the list of numbers above is 4. If k and m are integers and $k \neq m$ what is the median of the list?

(A) 2
(B) 2.5
(C) 3
(D) 3.5
(E) 4

Arithmetic Statistics

Since the arithmetic mean $= \dfrac{\text{sum of values}}{\text{number of values}}$, then $\dfrac{3+k+2+8+m+3}{6} = 4$, and so $\dfrac{16+k+m}{6} = 4$, $16 + k + m = 24$, $k + m = 8$. Since $k \neq m$, then either $k < 4$ and $m > 4$ or $k > 4$ and $m < 4$. Because k and m are integers, either $k \leq 3$ and $m \geq 5$ or $k \geq 5$ and $m \leq 3$.

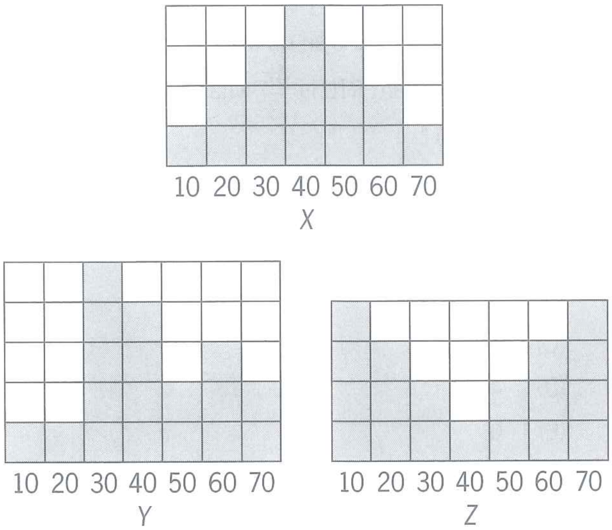
Case (i): If $k \leq 2$, then $m \geq 6$ and the six integers in ascending order are k, 2, 3, 3, m, 8 or k, 2, 3, 3, 8, m. The two middle integers are both 3 so the median is $\dfrac{3+3}{2} = 3$.

Case (ii): If $k = 3$, then $m = 5$ and the six integers in ascending order are 2, k, 3, 3, m, 8. The two middle integers are both 3 so the median is $\dfrac{3+3}{2} = 3$.

Case (iii): If $k = 5$, then $m = 3$ and the six integers in ascending order are 2, m, 3, 3, k, 8. The two middle integers are both 3 so the median is $\dfrac{3+3}{2} = 3$.

Case (iv): If $k \geq 6$, then $m \leq 2$ and the six integers in ascending order are m, 2, 3, 3, k, 8 or m, 2, 3, 3, 8, k. The two middle integers are both 3 so the median is $\dfrac{3+3}{2} = 3$.

The correct answer is C.



PS04987

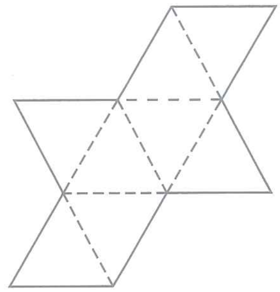
148. If the variables X, Y, and Z take on only the values 10, 20, 30, 40, 50, 60, or 70 with frequencies indicated by the shaded regions above, for which of the frequency distributions is the mean equal to the median?

(A) X only
(B) Y only
(C) Z only
(D) X and Y
(E) X and Z

Arithmetic Statistics

The frequency distributions for both X and Z are symmetric about 40, and thus both X and Z have mean = median = 40. Therefore, any answer choice that does not include both X and Z can be eliminated. This leaves only answer choice E.

The correct answer is E.

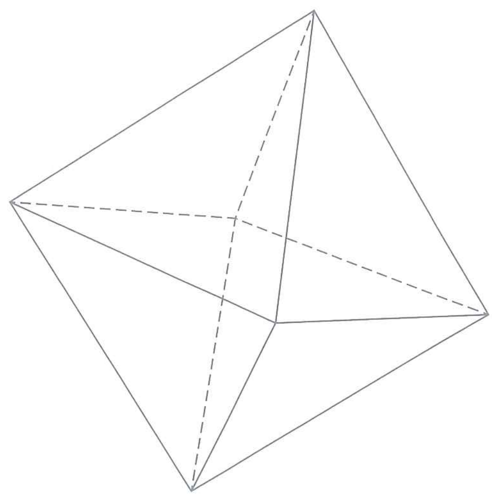


PS15538

149. When the figure above is cut along the solid lines, folded along the dashed lines, and taped along the solid lines, the result is a model of a geometric solid. This geometric solid consists of 2 pyramids, each with a square base that they share. What is the sum of the number of edges and the number of faces of this geometric solid?

(A) 10
(B) 18
(C) 20
(D) 24
(E) 25

Geometry Solids



A geometric solid consisting of 2 pyramids, each with a square base that they share, is shown in the figure above. From the figure it can be seen that the solid has 12 edges and 8 faces. Therefore, the sum of the number of edges and the number of faces of the solid is $12 + 8 = 20$.

Alternatively, the solid has $7 + 5 = 12$ edges because each edge in the solid is generated from either a dashed segment (there are 7 dashed segments) or from a pair of solid segments taped together (there are $\frac{10}{2} = 5$ such pairs of solid segments), and the solid has 8 faces because there are 8 small triangles in the given figure. Therefore, the sum of the number of edges and the number of faces of the solid is $12 + 8 = 20$.

The correct answer is C.

$$2x + y = 12$$
$$|y| \le 12$$

PS03356

150. For how many ordered pairs (x,y) that are solutions of the system above are x and y both integers?

(A) 7
(B) 10
(C) 12
(D) 13
(E) 14

Algebra Absolute value

From $|\,y\,| \le 12$, if y must be an integer, then y must be in the set $S = \{\pm 12, \pm 11, \pm 10, \ldots, \pm 3, \pm 2, \pm 1, 0\}$.

Since $2x + y = 12$, then $x = \frac{12 - y}{2}$. If x must be an integer, then $12 - y$ must be divisible by 2; that is, $12 - y$ must be even. Since 12 is even, $12 - y$ is even if and only if y is even. This eliminates all odd integers from S, leaving only the even integers $\pm 12, \pm 10, \pm 8, \pm 6, \pm 4, \pm 2$, and 0. Thus, there are 13 possible integer y-values, each with a corresponding integer x-value and, therefore, there are 13 ordered pairs (x,y), where x and y are both integers, that solve the system.

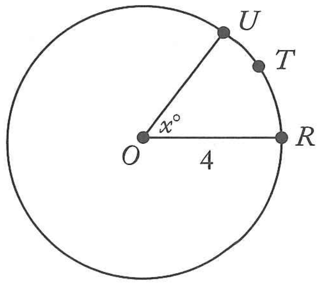
The correct answer is D.

PS08859

151. The points R, T, and U lie on a circle that has radius 4. If the length of arc RTU is $\frac{4\pi}{3}$ what is the length of line segment RU?

(A) $\frac{4}{3}$
(B) $\frac{8}{3}$
(C) 3
(D) 4
(E) 6

Geometry Circles; Triangles; Circumference



In the figure above, O is the center of the circle that contains R, T, and U and x is the degree measure of $\angle ROU$. Since the circumference of the circle is $2\pi(4) = 8\pi$ and there are 360° in the circle, the ratio of the length of arc RTU to the circumference of the circle is the same as the ratio of x to 360. Therefore, $\frac{\frac{4\pi}{3}}{8\pi} = \frac{x}{360}$. Then $x = \frac{\frac{4\pi}{3}(360)}{8\pi} = \frac{480\pi}{8\pi} = 60$. This means that ΔROU is an isosceles triangle with side lengths $OR = OU = 4$ and vertex angle measuring 60°. The base angles of ΔROU must have equal measures and the sum of their measures must be $180° - 60° = 120°$. Therefore, each base angle measures 60°, ΔROU is equilateral, and $RU = 4$.

The correct answer is D.

PS02955

152. A certain university will select 1 of 7 candidates eligible to fill a position in the mathematics department and 2 of 10 candidates eligible to fill 2 identical positions in the computer science department. If none of the candidates is eligible for a position in both departments, how many different sets of 3 candidates are there to fill the 3 positions?

(A) 42
(B) 70
(C) 140
(D) 165
(E) 315

Arithmetic Elementary combinatorics

To fill the position in the math department, 1 candidate will be selected from a group of 7 eligible candidates, and so there are 7 sets of 1 candidate each to fill the position in the math department. To fill the positions in the computer science department, any one of the 10 eligible candidates can be chosen for the first position and any of the remaining 9 eligible candidates can be chosen for the second position, making a total of $10 \times 9 = 90$ sets of 2 candidates to fill the computer science positions. But, this number includes the set in which Candidate A was chosen to fill the first position and Candidate B was chosen to fill the second position as well as the set in which Candidate B was chosen for the first position and Candidate A was chosen for the second position. These sets are not different essentially since the positions are identical and in both sets Candidates A and B are chosen to fill the 2 positions. Therefore, there are $\frac{90}{2} = 45$ sets of 2 candidates to fill the computer science positions. Then, using the multiplication principle, there are $7 \times 45 = 315$ different sets of 3 candidates to fill the 3 positions.

The correct answer is E.

PS06189

153. A survey of employers found that during 1993 employment costs rose 3.5 percent, where employment costs consist of salary costs and fringe-benefit costs. If salary costs rose 3 percent and fringe-benefit costs rose 5.5 percent during 1993, then fringe-benefit costs represented what percent of employment costs at the beginning of 1993 ?

(A) 16.5%
(B) 20%
(C) 35%
(D) 55%
(E) 65%

Algebra; Arithmetic First-degree equations; Percents

Let E represent employment costs, S represent salary costs, and F represent fringe-benefit costs. Then $E = S + F$. An increase of 3 percent in salary costs and a 5.5 percent increase in fringe-benefit costs resulted in a 3.5 percent increase in employment costs. Therefore $1.03S + 1.055F = 1.035E$. But, $E = S + F$, so $1.03S + 1.055F = 1.035(S + F) = 1.035S + 1.035F$.

Combining like terms gives $(1.055 - 1.035)F = (1.035 - 1.03)S$ or $0.02F = 0.005S$. Then, $S = \frac{0.02}{0.005}F = 4F$. Thus, since $E = S + F$, it follows that $E = 4F + F = 5F$. Then, F as a percent of E is $\frac{F}{E} = \frac{F}{5F} = \frac{1}{5} = 20\%$.

The correct answer is B.

PS02528

154. The subsets of the set $\{w, x, y\}$ are $\{w\}$, $\{x\}$, $\{y\}$, $\{w, x\}$, $\{w, y\}$, $\{x, y\}$, $\{w, x, y\}$, and $\{\ \}$ (the empty subset). How many subsets of the set $\{w, x, y, z\}$ contain w ?

(A) Four
(B) Five
(C) Seven
(D) Eight
(E) Sixteen

Arithmetic Sets

As shown in the table, the subsets of $\{w, x, y, z\}$ can be organized into two columns, those subsets of $\{w, x, y, z\}$ that do not contain w (left column) and the corresponding subsets of $\{w, x, y, z\}$ that contain w (right column), and each of these collections has the same number of sets. Therefore, there are 8 subsets of $\{w, x, y, z\}$ that contain w.

subsets not containing w	subsets containing w
$\{\ \}$	$\{w\}$
$\{x\}$	$\{w, x\}$
$\{y\}$	$\{w, y\}$
$\{z\}$	$\{w, z\}$
$\{x, y\}$	$\{w, x, y\}$
$\{x, z\}$	$\{w, x, z\}$
$\{y, z\}$	$\{w, y, z\}$
$\{x, y, z\}$	$\{w, x, y, z\}$

The correct answer is D.

PS10309

155. There are 5 cars to be displayed in 5 parking spaces, with all the cars facing the same direction. Of the 5 cars, 3 are red, 1 is blue, and 1 is yellow. If the cars are identical except for color, how many different display arrangements of the 5 cars are possible?

(A) 20
(B) 25
(C) 40
(D) 60
(E) 125

Arithmetic Elementary combinatorics

There are 5 parking spaces from which 3 must be chosen to display the 3 identical red cars. Thus, there are $\binom{5}{3} = \frac{5!}{3!2!} = 10$ different arrangements of the 3 identical red cars in the parking spaces. There are 2 spaces remaining for displaying the single blue car and 1 space left for displaying the single yellow car. Therefore, there are $(10)(2)(1) = 20$ arrangements possible for displaying the 5 cars in the 5 parking spaces.

The correct answer is A.

PS17461

156. The number $\sqrt{63 - 36\sqrt{3}}$ can be expressed as $x + y\sqrt{3}$ for some integers x and y. What is the value of xy?

(A) −18
(B) −6
(C) 6
(D) 18
(E) 27

Algebra Operations on radical expressions

Squaring both sides of $\sqrt{63 - 36\sqrt{3}} = x + y\sqrt{3}$ gives $63 - 36\sqrt{3} = x^2 + 2xy\sqrt{3} + 3y^2 = (x^2 + 3y^2) + (2xy)\sqrt{3}$, which implies that $-36 = 2xy$, or $xy = -18$. Indeed, if $-36 \neq 2xy$, or equivalently, if $36 + 2xy \neq 0$, then we could write $\sqrt{3}$ as a quotient of the two integers $63 - x^2 - 3y^2$ and $36 + 2xy$, which is not possible because $\sqrt{3}$ is an irrational number. To be more explicit, $63 - 36\sqrt{3} = x^2 + 2xy\sqrt{3} + 3y^2$ implies $63 - x^2 - 3y^2 = (36 + 2xy)\sqrt{3}$, and if $36 + 2xy \neq 0$, then we could divide both sides of the equation $63 - x^2 - 3y^2 = (36 + 2xy)\sqrt{3}$ by $36 + 2xy$ to get $\frac{63 - x^2 - 3y^2}{36 + 2xy} = \sqrt{3}$.

The correct answer is A.

PS01334

157. There are 10 books on a shelf, of which 4 are paperbacks and 6 are hardbacks. How many possible selections of 5 books from the shelf contain at least one paperback and at least one hardback?

(A) 75
(B) 120
(C) 210
(D) 246
(E) 252

Arithmetic Elementary combinatorics

The number of selections of 5 books containing at least one paperback and at least one hardback is equal to $T - N$, where T is the total number of selections of 5 books and N is the number of selections that do not contain both a paperback and a hardback. The value of T is

$$\binom{10}{5} = \frac{10!}{5!(10-5)!} = \frac{(6)(7)(8)(9)(10)}{(1)(2)(3)(4)(5)}$$
$$= (7)(2)(9)(2) = 252.$$

To find the value of N, first note that no selection of 5 books can contain all paperbacks, since there are only 4 paperback books. Thus, the value of N is equal to the number of selections of 5 books that contain all hardbacks, which is equal to 6 since there are 6 ways that a single hardback can be left out when choosing the 5 hardback books. It follows that the number of selections of 5 books containing at least one paperback and at least one hardback is $T - N = 252 - 6 = 246$.

The correct answer is D.

PS03774

158. If x is to be chosen at random from the set {1, 2, 3, 4} and y is to be chosen at random from the set {5, 6, 7}, what is the probability that xy will be even?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) $\frac{5}{6}$

Arithmetic; Algebra Probability; Concepts of sets

By the principle of multiplication, since there are 4 elements in the first set and 3 elements in the second set, there are $(4)(3) = 12$ possible products of xy, where x is chosen from the first set and y is chosen from the second set. These products will be even EXCEPT when both x and y are odd. Since there are 2 odd numbers in the first set and 2 odd numbers in the second set, there are $(2)(2) = 4$ products of x and y that are odd. This means that the remaining $12 - 4 = 8$ products are even. Thus, the probability that xy is even is $\frac{8}{12} = \frac{2}{3}$.

The correct answer is D.

PS04254

159. The function f is defined for each positive three-digit integer n by $f(n) = 2^x 3^y 5^z$, where x, y, and z are the hundreds, tens, and units digits of n, respectively. If m and v are three-digit positive integers such that $f(m) = 9f(v)$, then $m - v =$

(A) 8

(B) 9

(C) 18

(D) 20

(E) 80

Algebra Place value

Let the hundreds, tens, and units digits of m be A, B, and C, respectively; and let the hundreds, tens, and units digits of v be a, b, and c, respectively. From $f(m) = 9f(v)$ it follows that $2^A 3^B 5^C = 9(2^a 3^b 5^c) = 3^2(2^a 3^b 5^c) = 2^a 3^{b+2} 5^c$. Therefore, $A = a$, $B = b + 2$, and $C = c$. Now calculate $m - v$.

$m - v =$	$(100A + 10B + C) - (100a + 10b + c)$	place value property
$=$	$(100a + 10(b + 2) + c) - (100a + 10b + c)$	obtained above
$=$	$10(b + 2) - 10b$	combine like terms
$=$	$10b + 20 - 10b$	distributive property
$=$	20	combine like terms

The correct answer is D.

PS06312

160. If $10^{50} - 74$ is written as an integer in base 10 notation, what is the sum of the digits in that integer?

(A) 424

(B) 433

(C) 440

(D) 449

(E) 467

Arithmetic Properties of numbers

$10^2 - 74$	=	$100 - 74$	=	26
$10^3 - 74$	=	$1{,}000 - 74$	=	926
$10^4 - 74$	=	$10{,}000 - 74$	=	9,926
$10^5 - 74$	=	$100{,}000 - 74$	=	99,926
$10^6 - 74$	=	$1{,}000{,}000 - 74$	=	999,926

From the table above it is clear that $10^{50} - 74$ in base 10 notation will be 48 digits of 9 followed by the digits 2 and 6. Therefore, the sum of the digits of $10^{50} - 74$ is equal to $48(9) + 2 + 6 = 440$.

The correct answer is C.

PS09056

161. A certain company that sells only cars and trucks reported that revenues from car sales in 1997 were down 11 percent from 1996 and revenues from truck sales in 1997 were up 7 percent from 1996. If total revenues from car sales and truck sales in 1997 were up 1 percent from 1996, what is the ratio of revenue from car sales in 1996 to revenue from truck sales in 1996 ?

(A) 1:2
(B) 4:5
(C) 1:1
(D) 3:2
(E) 5:3

Algebra; Arithmetic First-degree equations; Percents

Let C_{96} and C_{97} represent revenues from car sales in 1996 and 1997, respectively, and let T_{96} and T_{97} represent revenues from truck sales in 1996 and 1997, respectively. A decrease of 11 percent in revenue from car sales from 1996 to 1997 can be represented as $(1 - 0.11)C_{96} = C_{97}$, and a 7 percent increase in revenue from truck sales from 1996 to 1997 can be represented as $(1 + 0.07)T_{96} = T_{97}$. An overall increase of 1 percent in revenue from car and truck sales from 1996 to 1997 can be represented as $C_{97} + T_{97} = (1 + 0.01)(C_{96} + T_{96})$. Then, by substitution of expressions for C_{97} and T_{97} that were derived above, $(1 - 0.11)C_{96} + (1 + 0.07)T_{96} = (1 + 0.01)(C_{96} + T_{96})$ and so $0.89C_{96} + 1.07T_{96} = 1.01(C_{96} + T_{96})$ or $0.89C_{96} + 1.07T_{96} = 1.01C_{96} + 1.01T_{96}$. Then, combining like terms gives $(1.07 - 1.01)T_{96} = (1.01 - 0.89)C_{96}$ or $0.06T_{96} = 0.12C_{96}$. Thus $\frac{C_{96}}{T_{96}} = \frac{0.06}{0.12} = \frac{1}{2}$. The ratio of revenue from car sales in 1996 to revenue from truck sales in 1996 is 1:2.

The correct answer is A.

PS14267

162. Becky rented a power tool from a rental shop. The rent for the tool was \$12 for the first hour and \$3 for each additional hour. If Becky paid a total of \$27, excluding sales tax, to rent the tool, for how many hours did she rent it?

(A) 5
(B) 6
(C) 9
(D) 10
(E) 12

Arithmetic Applied problems

Becky paid a total of \$27 to rent the power tool. She paid \$12 to rent the tool for the first hour and \$27 – \$12 = \$15 to rent the tool for the additional hours at the rate of \$3 per additional hour. It follows that she rented the tool for $\frac{15}{3} = 5$ additional hours and a total of $1 + 5 = 6$ hours.

The correct answer is B.

PS06959

163. If $4 < \frac{7 - x}{3}$, which of the following must be true?

I. $5 < x$
II. $|x + 3| > 2$
III. $-(x + 5)$ is positive.

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Algebra Inequalities

Given that $4 < \frac{7-x}{3}$, it follows that $12 < 7 - x$. Then, $5 < -x$ or, equivalently, $x < -5$.

I. If $4 < \frac{7-x}{3}$, then $x < -5$. If $5 < x$ were true then, by combining $5 < x$ and $x < -5$, it would follow that $5 < -5$, which cannot be true. Therefore, it is not the case that, if $4 < \frac{7-x}{3}$, then Statement I must be true. In fact, Statement I is never true.

II. If $4 < \frac{7-x}{3}$, then $x < -5$, and it follows that $x + 3 < -2$. Since $-2 < 0$, then $x + 3 < 0$ and $|x+3| = -(x+3)$. If $x+3 < -2$, then $-(x+3) > 2$ and by substitution, $|x + 3| > 2$. Therefore, Statement II must be true for every value of x such that $x < -5$. Therefore, Statement II must be true if $4 < \frac{7-x}{3}$.

III. If $4 < \frac{7-x}{3}$, then $x < -5$ and $x + 5 < 0$. But if $x + 5 < 0$, then it follows that $-(x + 5) > 0$ and so $-(x + 5)$ is positive. Therefore Statement III must be true if $4 < \frac{7-x}{3}$.

The correct answer is D.

PS08654

164. A certain right triangle has sides of length x, y, and z, where $x < y < z$. If the area of this triangular region is 1, which of the following indicates all of the possible values of y ?

(A) $y > \sqrt{2}$

(B) $\frac{\sqrt{3}}{2} < y < \sqrt{2}$

(C) $\frac{\sqrt{2}}{3} < y < \frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{3}}{4} < y < \frac{\sqrt{2}}{3}$

(E) $y < \frac{\sqrt{3}}{4}$

Geometry; Algebra Triangles; Area; Inequalities

Since x, y, and z are the side lengths of a right triangle and $x < y < z$, it follows that x and y are the lengths of the legs of the triangle and so the area of the triangle is $\frac{1}{2}xy$. But, it is given that the area is 1 and so $\frac{1}{2}xy = 1$. Then, $xy = 2$ and $y = \frac{2}{x}$. Under the assumption that x, y, and z are all positive since they are the side lengths of a triangle, $x < y$ implies $\frac{1}{x} > \frac{1}{y}$ and then $\frac{2}{x} > \frac{2}{y}$. But, $y = \frac{2}{x}$, so by substitution, $y > \frac{2}{y}$, which implies that $y^2 > 2$ since y is positive. Thus, $y > \sqrt{2}$.

Alternatively, if $x < \sqrt{2}$ and $y < \sqrt{2}$ then $xy < 2$. If $x > \sqrt{2}$ and $y > \sqrt{2}$, then $xy > 2$. But, $xy = 2$ so one of x or y must be less than $\sqrt{2}$ and the other must be greater than $\sqrt{2}$. Since $x < y$, it follows that $x < \sqrt{2} < y$ and $y > \sqrt{2}$.

The correct answer is A.

PS14397

165. On a certain day, a bakery produced a batch of rolls at a total production cost of \$300. On that day, $\frac{4}{5}$ of the rolls in the batch were sold, each at a price that was 50 percent greater than the average (arithmetic mean) production cost per roll. The remaining rolls in the batch were sold the next day, each at a price that was 20 percent less than the price of the day before. What was the bakery's profit on this batch of rolls?

(A) \$150

(B) \$144

(C) \$132

(D) \$108

(E) \$90

Arithmetic Applied problems

Let n be the number of rolls in the batch and p be the average production price, in dollars, per roll. Then the total cost of the batch is $np = 300$ dollars, and the total revenue from selling the rolls in the batch is $\left(\frac{4}{5}n\right)(1.5p) + \left(\frac{1}{5}n\right)(0.8)(1.5p) = \left(\frac{4}{5}n\right)\left(\frac{3}{2}p\right) + \left(\frac{1}{5}n\right)\left(\frac{4}{5}\right)\left(\frac{3}{2}p\right) = \left(\frac{6}{5} + \frac{6}{25}\right)np = \left(\frac{36}{25}\right)np$. Therefore, the profit from selling the rolls in the batch is $\left(\frac{36}{25}\right)np - np = \left(\frac{11}{25}\right)np = \left(\frac{11}{25}\right)(300)$ dollars $= 132$ dollars.

The correct answer is C.

PS05972

166. A set of numbers has the property that for any number t in the set, $t + 2$ is in the set. If -1 is in the set, which of the following must also be in the set?

I. -3
II. 1
III. 5

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

Arithmetic Properties of numbers

It is given that -1 is in the set and, if t is in the set, then $t + 2$ is in the set.

I. Since $\{-1, 1, 3, 5, 7, 9, 11, \ldots\}$ contains -1 and satisfies the property that if t is in the set, then $t + 2$ is in the set, it is not true that -3 must be in the set.

II. Since -1 is in the set, $-1 + 2 = 1$ is in the set. Therefore, it must be true that 1 is in the set.

III. Since -1 is in the set, $-1 + 2 = 1$ is in the set. Since 1 is in the set, $1 + 2 = 3$ is in the set. Since 3 is in the set, $3 + 2 = 5$ is in the set. Therefore, it must be true that 5 is in the set.

The correct answer is D.

PS04780

167. A couple decides to have 4 children. If they succeed in having 4 children and each child is equally likely to be a boy or a girl, what is the probability that they will have exactly 2 girls and 2 boys?

(A) $\frac{3}{8}$
(B) $\frac{1}{4}$
(C) $\frac{3}{16}$
(D) $\frac{1}{8}$
(E) $\frac{1}{16}$

Arithmetic Probability

Representing the birth order of the 4 children as a sequence of 4 letters, each of which is B for boy and G for girl, there are 2 possibilities (B or G) for the first letter, 2 for the second letter, 2 for the third letter, and 2 for the fourth letter, making a total of $2^4 = 16$ sequences. The table below categorizes some of these 16 sequences.

# of boys	# of girls	Sequences	# of sequences
0	4	GGGG	1
1	3	BGGG, GBGG, GGBG, GGGB	4
3	1	GBBB, BGBB, BBGB, BBBG	4
4	0	BBBB	1

The table accounts for $1 + 4 + 4 + 1 =$ 10 sequences. The other 6 sequences will have 2Bs and 2Gs. Therefore the probability that the couple will have exactly 2 boys and 2 girls is $\frac{6}{16} = \frac{3}{8}$.

For the mathematically inclined, if it is assumed that a couple has a fixed number of children, that the probability of having a girl each time is p, and that the sex of each child is independent of the sex of the other children, then the number of girls, x, born to a couple with n children is a random variable having the binomial probability distribution. The probability of having exactly x girls born to a couple with n children is given