Introduction to Frequent Itemset Mining

- *FIsM* $\rightarrow$ Frequent itemset mining
- *FIs* $\rightarrow$ Frequent itemset
- *DM* $\rightarrow$ Data mining
- *T* $\rightarrow$ Transaction
- *TID* $\rightarrow$ Transaction ID
- *Tids* $\rightarrow$ Transaction ID set / Tidset
- *I* $\rightarrow$ Item
- *Is* $\rightarrow$ Itemset
- *CIs* $\rightarrow$ Closed itemset
- *CFIs* $\rightarrow$ Closed frequent itemset
- *CIsM* $\rightarrow$ Closed itemset mining
- *CO* $\rightarrow$ Closure operator
- *MIs* $\rightarrow$ Maximal itemset
- *MFIs* $\rightarrow$ Maximal frequent itemset
- *Sp* $\rightarrow$ Support
- *db* $\rightarrow$ Database
- *A* $\rightarrow$ Apriori
- *AA* $\rightarrow$ Apriori algorithm
- *CA* $\rightarrow$ Charm algorithm
- # $\rightarrow$ number

FIsM is a cornerstone of *DM*, aimed at discovering sets of items that frequently co-occur in a transactional database. It is widely used in applications such as *market basket analysis* (e.g., identifying items frequently purchased together), *recommendation systems*, *bioinformatics* (e.g., finding co-occurring genes) and *web usage mining* (e.g., analyzing navigation patterns).

Basic Concepts

Fundamental definitions

Definition of items, itemsets, support and types of itemsets (frequent, closed, maximal)

Item $\rightarrow$ an *I* is a binary attribute in a *T* database, representing a single entity. For example, in a retail context, *I* could be products like *milk*, *bread* or *butter*.

The set of all items is denoted:

$$I = \{I_1, I_2, ..., I_m\}$$

where m is the number of distinct items

Itemset $\rightarrow$ an *Is* is a subset of items, $X \subset I$. For example, $X = \{\text{milk}, \text{bread}\}$ is an *Is*. The power set $P(I)$ contains all possible *Is* with $|P(I)| = 2^m$

Transactional database $\rightarrow$ a *T* database *D* is a collection of *T* where each *T* is an itemset associated with a unique transaction ID (*TID*)

Formally:

$$D = \{(t_1, X_1), (t_2, X_2), ..., (t_n, X_n)\}$$

where t_i is the *TID* and $X_i \subset I$ is the itemset

The example database

TID	Items
1	A, C, T, W
2	C, D, W

TID	Items
3	A, C, T, W
4	A, C, D, W
5	A, C, D, T, W
6	C, D, T

Here, $n = 6$ (# of transactions) and the *I* are $I = \{A, C, D, T, W\}$ so $m = 5$

Support $\rightarrow$ The *Sp* of an *Is* X, denoted $\sup(X)$, measures how frequently X appears in D. It is defined as

$$\sup(X) = \left(\frac{|\{t_i \in D | X \subset X_i\}|}{|D|}\right) \times 100$$

where

- $|\{t_i \in D | X \subset X_i\}| \rightarrow$ the # of *T* containing X
- $D \rightarrow$ the total # of *T*

The *Sp* count is

$$\sup\_\text{count}(X) = |\{t_i \in D | X \subset X_i\}|$$

For examples:

For $X = \{A\}$

- Appears in *TID*s 1, 3, 4, 5 $\rightarrow \sup\_\text{count} = 4$
- $\sup(A) = \frac{4}{6} \times 100 = 66.67\%$

For $X = \{A, C, D\}$

- Appears in *TID*s 4, 5 $\rightarrow \sup\_\text{count} = 2$
- $\sup(A, C, D) = \frac{2}{6} \times 100 = 33.33\%$

Frequent itemset $\rightarrow$ an *Is* X is frequent if its *Sp* meets or exceeds a user-defined minimum *Sp* threshold minsup

$$\sup(X) \geq \text{minsup}$$

or equivalently

$$\sup\_\text{count}(X) \geq \text{minsup} \times |D|$$

Example with $\text{minsup} = 70\%$

- $\sup(A) = 66.67\% < 70\% \rightarrow$ A is not frequent
- $\sup(C) = \frac{6}{6} \times 100 = 100\% \geq 70\% \rightarrow$ C is frequent

Closed Itemsets

Closed Itemsets $\rightarrow$ are a compact representation of *FIs*, capturing all *FIs* and their *Sp* without redundancy

Galois connection $\rightarrow$ a formal framework using a **Galois connection** to define *CIs*. Let:

- $I \rightarrow$ set of items
- $T \rightarrow$ set of transaction IDs
- $\phi \subset I \times T \rightarrow$ a binary relation where $(x, y) \in \phi$ means item x appears in transaction y.

Two mappings are defined

1 _ $t : P(I) \rightarrow P(T)$

$$t(X) = \{y \in T \mid \forall x \in X, (x, y) \in \phi\}$$

This returns the *TIDs* of transactions containing all items in X

2 _ $i: P(T) \to P(I)$

$$t(Y) = \{x \in I \mid \forall y \in Y, (x, y) \in \phi\}$$

This returns the items present in all transactions in Y

Closure Operator $\to$ The closure of an *Is* X is

$$c(X) = i(t(X))$$

An *Is* X is closed if

$$c(X) = X$$

This means X contains all items common to transactions that contain X

Closed Frequent Itemset $\to$ an *Is* is a *CFIs* if it is both closed ($c(X) = X$)
and frequent ($\sup(X) \geq \text{minsup}$)

For example with $\text{minsup} = 30\%$

For $X = \{A, W\}$

Proved it's closed

- $t(A, W) = \{1, 3, 4, 5\}$ (*TID*s where both A and W appear)
- $i(t(A, W)) = i(\{1, 3, 4, 5\}) = A, C, W$ (*Is* common to *TIDs* 1, 3, 4, 5)
- $c(A, W) = \{A, C, W\} \neq \{A, W\} \to (A, W)$ is not closed

For $X = \{C, D\}$

Proved it's closed

- $t(C, D) = \{2, 4, 5, 6\}$ (*TIDs* where both C and D appear)
- $i(t(C, D)) = i(\{2, 4, 5, 6\}) = \{C, D\}$ (*Is* common to *TIDs* 2, 4, 5, 6)
- $c(C, D) = \{C, D\} = \{C, D\} \to (C, D)$ is closed

Proved it's frequent

- $\sup(C, D) = \frac{4}{6} \times 100 = 66.67\% \geq 30\% \to (C, D)$ is a closed frequent items *CFIs*

Formal definition - A *CFIs* is a *FIs* X such that there exists no superset $Y \supset X$ with the same *Sp*

$$C = \{X | X \in F \text{ and } \nexists Y \supset X \text{ such that } \sup(X) = \sup(Y)\}$$

where $F = \{X \subset I | \sup(X) \geq \text{minsup}\} \to$ the set of all *FIs*

Maximal Itemsets

Maximal Itemsets $\to$ are the largest *FIs* providing the most compact representation

An *Is* X is a *MFIs* if it's frequent and has no frequent superset

$$M = \{X | X \in F \text{ and } \nexists Y \supset X \text{ such that } Y \in F\}$$

Given *FIs* $\{A, B\}, \{A, C\}, \{A, B, D\}$

- $\{A, C\}$ is maximal if no superset (e.g., $\{A, C, D\}$) is frequent
- $\{A, B, D\}$ is maximal if not superset (e.g., $\{A, B, D, E\}$) is frequent
- $\{A, B\}$ is not maximal because it is a subset of the frequent items $\{A, B, D\}$ (larger *FIs*)

Comparison of itemset types

The document compares three types of *Is*

- *FIs* **(F)** $\to$ all *Is* with $\sup(X) \geq \text{minsup}$
- *CFIs* **(C)** $\to$ *FIs* with no superset of equal ***Sp***

- *MFIs* **(M)** $\rightarrow$ *FIs* with no frequent superset

The relationship is

$$M \subset C \subset F$$

FIs $\rightarrow$ the largest set including all *Is* meeting or exceeding *Sp* threshold. For the large dataset, this set can be massive due to the combinatorial explosion of *Is* (2^m)

CIs $\rightarrow$ A subset of the *FIs*, reducing redundancy by excluding *Is* that have supersets with the same *Sp*. They retain all information about *FIs* and their *Sp*
MIs $\rightarrow$ The smallest set, containing only the largest *FIs*. They are subset of closed itemsets but may lose *Sp* information for subsets

Visual representation (described)

Imagine a lattice where each node is an *Is* and edges connect subsets to supersets

Bottom node $\emptyset$

Level 1 $\{A\}, \{C\}, \{D\}, \{T\}, \{W\}$

Level 2 $\{A, C\}, \{A, D\}, \{A, T\}, \{A, W\}, \{C, D\}, \ldots$

Level 3 $\{A, C, D\}, \{A, C, T\}, \{A, C, W\}, \ldots$

Top node $\{A, C, D, T, W\}$

F All nodes with $\text{support} \geq \text{minsup}$

C Nodes where no parent (superset) has the same *Sp*

M The highest nodes in the lattice that are frequent

Frequent itemset mining (FIsM)

Techniques and algorithms like Apriori

Problem definition

The goal is to find all *Is* in D with $\text{support} \geq \text{minsup}$

Input

- *T db* D
- Set of items I
- Minimum *Sp* threshold minsup (as a percentage or count)

Output

All *Is* $X \subset I$ such that $\sup(X) \geq \text{minsup}$

Parameters

$N = |D| \rightarrow$ # of *T*

$d = |I| \rightarrow$ # of distinct *I*

$w \rightarrow$ maximum # of I in a T

Challenges

Combinatorial explosion $\rightarrow$ the # of possible *Is* $2^{|i|} - 1$. For $|I| = 100$, this is $2^{100} - 1 \approx 10^{30}$

Scalability $\rightarrow$ real-world datasets (e.g., Walmart with 100000 *I*, billions of T) require efficient algorithms

Memory abd I/O $\rightarrow$ storing and scanning large *db* is resource-intensive

The Apriori principle

The **Apriori principle**

The **Apriori principle** is a key optimization to reduce the search space:

- **Positive rule** - if an *Is* X is frequent, all its subsets are frequent
- **Negative rule** - if an *Is* X is not frequent, none of its supersets can be frequent

Mathematically,

$$\forall X, Y : (X \subset Y) \implies \sup(X) \geq \sup(Y)$$

This follows because adding items to an *Is* can only reduce the # of *T* containing it.

Implication

- When generating candidate k-itemsets (*parent*), only consider combinations of frequent $(k-1)$-itemsets (*child*)
- Prune candidates whose subsets are not frequent, avoiding unnecessary *sp* counting

Apriori algorithm

The *AA* is a level-wise (breadth-first) algorithm for mining *FIs*. It iteratively generates and tests candidate *Is*

Algorithm overview

1. **Find frequent 1-itemsets (L_1)** $\rightarrow$ scan D to count the sp of each item
2. **Iterate for** $k = 2, 3, \ldots$
    - Generate candidate k-itemsets (C_k) from frequent $(k-1)$-itemsets (L_{k-1})
    - Scan D to count the *sp* of each candidate
    - Keep candidates with $\text{support} \geq \text{minsup}$ as L_k
3. Stop when $L_k = \emptyset$
4. Output $L = \bigcup_k L_k$

Pseudocode

```
Input: D (transactional database), minsup (minimum support threshold)
Output: L (all frequent itemsets)

L_1 = {1-itemsets in D with support ≥ minsup}
for (k = 2; L_{k-1} ≠ ∅; k++) {
    C_k = apriori_gen(L_{k-1})  // Generate candidate k-itemsets
    for each transaction t in D {
        C_t = subset(C_k, t)  // Candidates in C_k contained in t
        for each c in C_t {
            c.count++  // Increment support count
        }
    }
    L_k = {c in C_k | c.count ≥ minsup}
}
return L = ∪_k L_k
```

`apriori_gen` function

The `apriori_gen` function generates C_k from L_{k-1}

1. **Join step** $\rightarrow$ combine pair of *Is* in L_{k-1} that share the **first** $k-2$ items. For example, $\{A, C\}$ and $\{A, D\}$ generate $\{A, C, D\}$
2. **Prune step** $\rightarrow$ remove candidates whose $(k-1)$-subsets are not in L_{k-1}. For $\{A, C, D\}$ check if $\{A, C\}$, $\{A, D\}$, $\{C, D\} \in L_{k-1}$

Detailed Example

Using the document's *db* with $\text{minsup} = 50\%$ (*sp* count ≥ 3)

TID	Items
1	A, C, T, W
2	C, D, W
3	A, C, T, W
4	A, C, D, W
5	A, C, D, T, W
6	C, D, T

Step 1 - Compute L_1 - Scan *D* to count each item's *sp*

Item	TIDs	Support Count	Support %
A	1, 3, 4, 5	4	66.67%
C	1, 2, 3, 4, 5, 6	6	100%
D	2, 4, 5, 6	4	66.67%
T	1, 3, 5, 6	4	66.67%
W	1, 2, 3, 4, 5	5	83.33%

All items have $\sup\_\text{count} \geq 3$, so:

$$L_1 = \{\{A\}, \{C\}, \{D\}, \{T\}, \{W\}\}$$

Step 2 - Compute C_2 and L_2

Join - Combine pair of L_1

$$C_2 = \{\{A, C\}, \{A, D\}, \{A, T\}, \{A, W\}, \{C, D\}, \{C, T\}, \{C, W\}, \{D, T\}, \{D, W\}, \{T, W\}\}$$

Count sp - Scan D

C_2	TIDs	Support Count	Support %
A, C	1, 3, 4, 5	4	66.67%
A, D	4, 5	2	33.33%
A, T	1, 3, 5	3	50%
A, W	1, 3, 4, 5	4	66.67%
C, D	2, 4, 5, 6	4	66.67%
C, T	1, 3, 5, 6	4	66.67%
C, W	1, 2, 3, 4, 5	5	83.33%
D, T	5, 6	2	33.33%
D, W	2, 4, 5	3	50%
T, W	1, 3, 5	3	50%

Prune - Keep *Is* with $\sup\_\text{count} \geq 3$

$$L_2 = \{\{A, C\}, \{A, T\}, \{A, W\}, \{C, D\}, \{C, T\}, \{C, W\}, \{D, W\}, \{T, W\}\}$$

and 2 subsets are eliminated $\{A, D\}$ and $\{D, T\}$

Step 3 - Compute C_3 and L_3

Join - Combine pairs in L_2 *sharing* the **first item** ($k - 2 = 3 - 2 = 1$) (e.g., $\{A, C\}$ and $\{A, T\} \rightarrow \{A, C, T\}$)

$\Rightarrow C_3 = \{\{A, C, T\}, \{A, C, W\}, \{A, T, W\}, \{C, D, W\}, \{C, T, W\}, \{C, D, T\}\}$

Prune - Check if all ($k - 1$)-subsets are in L_2

- For $\{A, C, T\} \rightarrow$ subsets $\{A, C\}, \{A, T\}, \{C, T\} \in L_2$
- For $\{A, C, W\} \rightarrow$ subsets $\{A, C\}, \{A, W\}, \{C, W\} \in L_2$
- For $\{A, T, W\} \rightarrow$ subsets $\{A, T\}, \{A, W\}, \{T, W\} \in L_2$
- For $\{C, D, W\} \rightarrow$ subsets $\{C, D\}, \{C, W\}, \{D, W\} \in L_2$
- For $\{C, T, W\} \rightarrow$ subsets $\{C, T\}, \{C, W\}, \{T, W\} \in L_2$
- For $\{C, D, T\} \rightarrow$ subsets $\{C, D\}, \{C, T\} \in L_2$ but , $\{D, T\} \notin L_2$

Remove $\{C, D, T\}$ candidate and all remaining candidates are valid

Count sp - Scan *D*

C_3	TIDs	Support Count	Support %
A, C, T	1, 3, 5	3	50%
A, C, W	1, 3, 4, 5	4	66.67%
A, T, W	1, 3, 5	3	50%
C, D, W	2, 4, 5	3	50%
C, T, W	1, 3, 5	3	50%

Prune - Keep *Is* with $\sup\_\text{count} \geq 3$

$L_3 = \{\{A, C, T\}, \{A, C, W\}, \{A, T, W\}, \{C, D, W\}, \{C, T, W\}\}$

Step 4 - Compute C_4 and L_4

Join - Combine pairs in L_3 *sharing* the **two first items** ($k - 2 = 4 - 2 = 2$)

$\Rightarrow C_4 = \{A, C, T, W\}$ (combine $\{A, C, T\}$ and $\{A, C, W\}$)

Prune - Check if all ($k - 1$)-subsets are in L_3

- For $\{A, C, T, W\} \rightarrow$ subsets $\{A, C, T\}, \{A, C, W\}, \{A, T, W\}, \{C, T, W\} \in L_3$

Count sp - Scan *D*

C_4	TIDs	Support Count	Support %
A, C, T, W	1, 3, 5	3	50%

Prune - Keep *Is* with $\sup\_\text{count} \geq 3$

$L_4 = \{\{A, C, T, W\}\}$

Step 5 - Compute C_5 and L_5

No pair in L_4 share **three first items** ($k - 2 = 5 - 2 = 3$) so $C_5 = \emptyset$ and $L_5 = \emptyset$

Output

$L = L_1 \cup L_2 \cup L_3 \cup L_4 = \{\{A\}, \{C\}, \{D\}, \{T\}, \{W\}, \{A, C\}, \{A, T\}, \{A, W\}, \{C, D\}, \{C, T\}, \{C, W\}, \{D, W\}, \{T, W\}, \{A, C, T\}, \{$

$\Rightarrow$ **Total 19 FIs**

Visualization (Apriori)

TID	Items
1	A, C, T, W
2	C, D, W
3	A, C, T, W
4	A, C, D, W
5	A, C, D, T, W
6	C, D, T

C_1	Support
A	4
C	6
D	4
T	4
W	5

→

F_1	Support
A	4
C	6
D	4
T	4
W	5

C_2	Support
A,C	4
A,D	2
A,T	3
A,W	4
C,D	4
C,T	4
C,W	5
D,T	2
D,W	3
T,W	3

→

F_2	Support
A,C	4
A,T	3
A,W	4
C,D	4
C,T	4
C,W	5
D,W	3
T,W	3

→

C_3	Support
A,C,W	4
A,C,D	2
A,C,T	3
A,T,W	3
C,D,T	2
C,D,W	3
C,T,W	3

Tập con không phổ biến

F_3	Support
A,C,W	4
A,C,T	3
A,T,W	3
C,D,W	3
C,T,W	3

→

C_4	Support
A,C,T,W	3
C,D,T,W	1

→

F_4	Support
A,C,T,W	4

Lattice representation

The search space is represented as a **lattice**

- **Nodes** - all possible *Is* $(2^{|I|})$
- **Edges** - subset-superset relationships
- **Levels** - *Is* of size k (e.g., level 1: 1-itemsets, level 2: 2-itemsets)

```
          ∅
         /|\
       A  C  D  T  W
      /|\/|\ /|\ /|\
    AC (AD) AT AW CD CT CW (DT) DW TW
   /|\
  ACT ACW ATW CDW CTW
   |
 ACTW
```

The AA traverses this lattice level-wise, pruning non-frequent itemsets

Implementation considerations

Data Structures

- Use a hash table or trie to store candidate *Is* and their counts
- Represent *T* as bitsets for efficient subset checking

Optimizations

- **Transaction reduction** - remove *T* that can not contribute to new candidates
- **Prefix Trees** - store candidates in a prefix tree to reduce memory usage

Bottlenecks

- Multiple *db* scans (one per level)
- Generating and pruning candidates can be costly for large k

Closed itemset mining (CIsM)

Motivation

FIs can be numerous, esp in dense datasets. *CIs* reduce redundancy while preserving all information about *FIs* and their *sp*

Charm algorithm

The *CA* efficiently mines *CFIs* using a vertical data representation (tidsets) and tree-based search

Tidset (*Tids*) - for an *Is* X, $t(X)$ is the set of TIDs containing X. The *sp* is

$$\text{sup\_count}(X) = |t(X)|$$

For example, $t(\{A, C\}) = \{1, 3, 4, 5\} \rightarrow \text{sup\_count}(AC) = 4$

Charm process

1. **Initialize** - Start with a root node containing all 1-itemsets and their *Tids*
2. **Combine items** - for each *Is*, combine it with other *I* to form larger *Is*, computing *Tids* via intersection

$$t(X \cup Y) = t(X) \cap t(Y)$$

3. **Check closure** - for an *Is* X, compute $c(X) = i(t(X))$. If $c(X) = X$, X is closed
4. **Prune**

- If $t(X) = t(Y)$ for some Y, merge X into Y (since X is not closed)
- If $|t(X)| < \text{minsup}$, prune the branch

5. **Output** - collect all closed frequent itemsets *CFIs*

Example with $\text{minsup} = 50\% \rightarrow \text{support count} \geq 3$

Initial *Tids*

```
A: {1, 3, 4, 5}
C: {1, 2, 3, 4, 5, 6}
D: {2, 4, 5, 6}
T: {1, 3, 5, 6}
W: {1, 2, 3, 4, 5}
```

Combine, e.g., $t(A, C) = t(A) \cap t(C) = \{1, 3, 4, 5\}$

Check closure

For $AC \Rightarrow t(A, C) = \{1, 3, 4, 5\},\ i(\{1, 3, 4, 5\}) = \{A, C, W\}$

$\Rightarrow c(A, C) = ACW \neq AC \rightarrow$ **not closed**

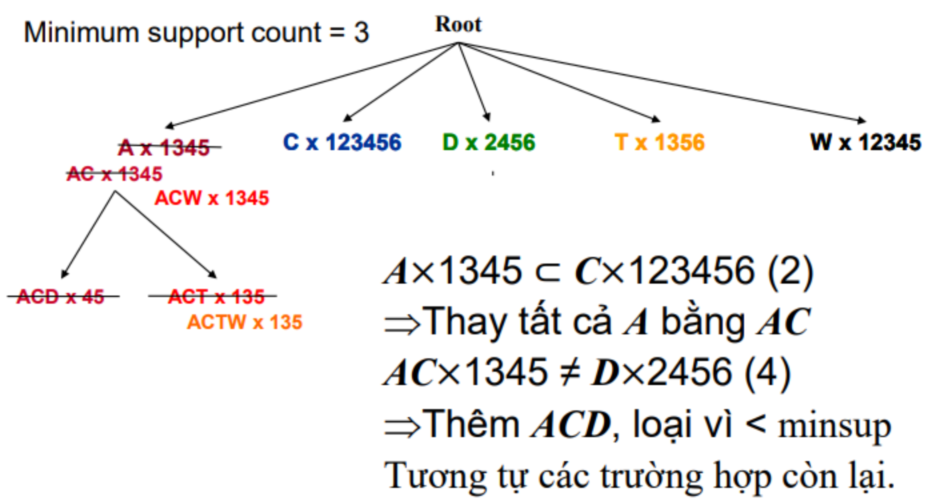
For $ACW \Rightarrow t(A, C, W) = \{1, 3, 4, 5\},\ i(\{1, 3, 4, 5\}) = \{A, C, W\}$

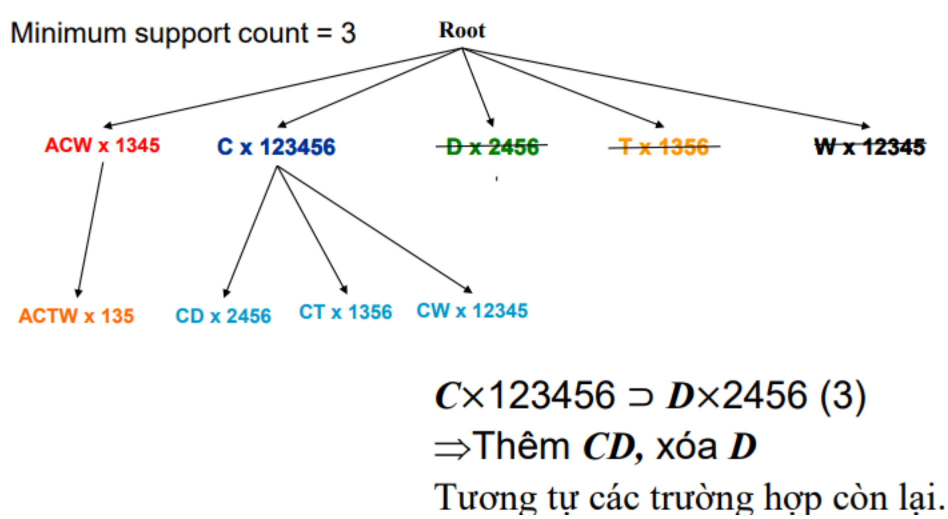
$\Rightarrow c(A, C, W) = ACW \rightarrow$ **closed**

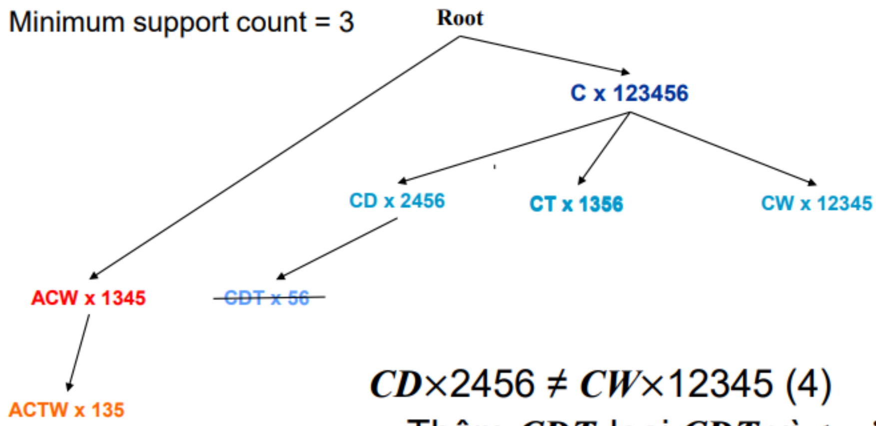
Result 7 *CFIs*

```
C × {1, 2, 3, 4, 5, 6}
ACT × {1, 3, 5}
ACW × {1, 3, 4, 5}
ATW × {1, 3, 5}
CDW × {2, 4, 5}
CTW × {1, 3, 5}
ACTW × {1, 3, 5}
```

Visualization (Charm)

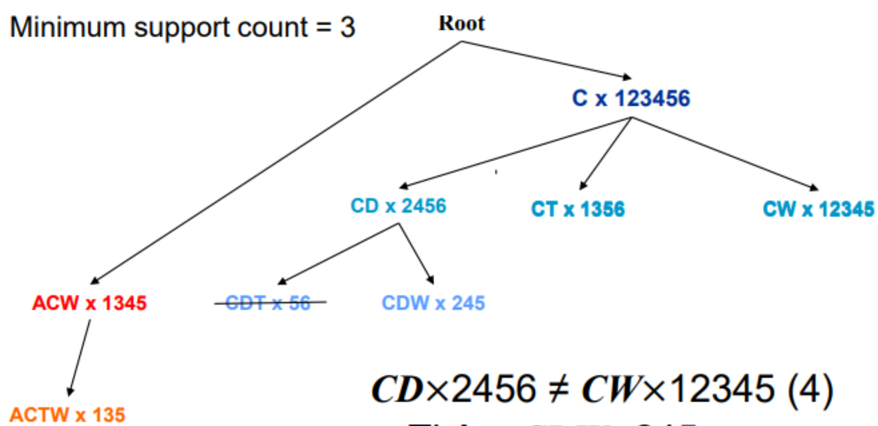






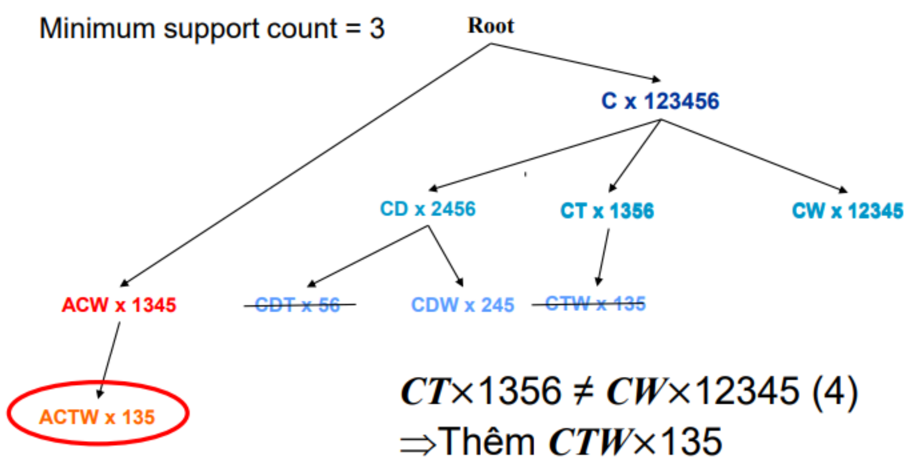
$CD \times 2456 \neq CW \times 12345$ (4)

$\Rightarrow$Thêm CDT, loại CDT vì < $minsup$



$CD \times 2456 \neq CW \times 12345$ (4)

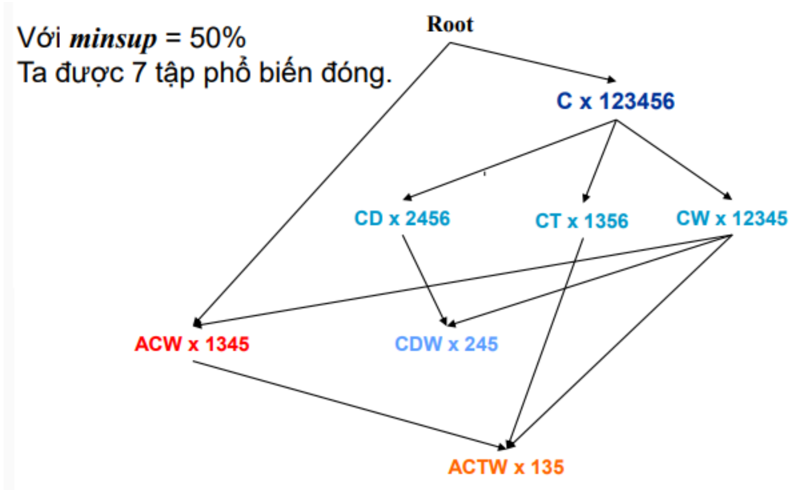
$\Rightarrow$Thêm $CDW \times 245$



$CT \times 1356 \neq CW \times 12345$ (4)

$\Rightarrow$Thêm $CTW \times 135$

Loai vì bi bao bởi $ACTW \times 135$



Định lý 1: Đặt $X_i \times t(X_i)$ và $X_j \times t(X_j)$ là hai thành viên bất kỳ của một lớp [P]. Bốn thuộc tính sau là:

1. Nếu t(Xi) = t(Xj), thì c(Xi)= c(Xj) = c(Xi $\cup$ Xj).
2. Nếu t(Xi) $\subset$ t(Xj), thì c(Xi)$\neq$ c(Xj), nhưng c(Xi)= c(Xi $\cup$ Xj)
3. Nếu t(Xi) $\supset$ t(Xj), thì c(Xi)$\neq$ c(Xj), nhưng c(Xj)= c(Xi $\cup$ Xj)
4. Nếu t(Xi) $\not\subset$ t(Xj)và t(Xj) $\not\subset$ t(Xi), thì c(Xi)$\neq$c(Xj)$\neq$ c(Xi $\cup$ Xj

CHARM-PROPERTY $(X \times Y, X_i, X_j, P_i, [P_i], [P])$
if $(\sigma(X) \geq minsup)$ **then**
  if $t(X_i) = t(X_j)$ **then (1)**
    remove X_j from $[P]$
    $P_i = P_i \cup X_j$
  else if $t(X_i) \subset t(X_j)$ **then (2)**
    $P_i = P_i \cup X_j$
  else if $t(X_i) \supset t(X_j)$ **then (3)**
    remove X_j from $[P]$
    Add $X \times Y$ to $[P_i]$
  else if $t(X_i) \neq t(X_j)$ **then (4)**
    Add $X \times Y$ to $[P_i]$

Advantages

Compactness - fewer *Is* than *FIs*

Lossless - retains all *sp* information

Efficient rules - association rules from *CFIs* are non-redundant

Implementation notes

Vertical format - store *Tids* as bitsets or lists for fast intersection

Diffsets - optimize by storing differences between *Tids* to reduce memory

Challenges - dense datasets may produce many *CFIs*, requiring pruning strategies

Maximal itemset mining

Motivation

Maximal itemsets are the most compact representation, ideal for dense datasets where even closed itemsets are numerous

GenMax algorithm

The **GenMax algorithm** mines *MFIs* using backtracking and *Tids*

Algorithm overview

- **Approach** $\longrightarrow$ depth-first search with backtracking
- **Data structure** $\longrightarrow$ *Tids* for *sp* counting
- **Pruning** $\longrightarrow$ eliminates branches leading to non-*MIs*

Pseudocode (MFI-backtrack)

```
Input: I_l (current itemset), C_l (combinable items), l (length)
Output: MFI (maximal frequent itemsets)

MFI-backtrack(I_l, C_l, l):
for each x in C_l:
    I_{l+1} = I_l ∪ {x}
    P_{l+1} = {y in C_l | y > x}  // Items after x (lexicographic order)
    if I_{l+1} ∪ P_{l+1} has a superset in MFI:
        return  // Prune branch
    C_{l+1} = FI-combine(I_{l+1}, P_{l+1})
    if C_{l+1} is empty:
        if I_{l+1} has no superset in MFI:
            MFI = MFI ∪ I_{l+1}
    else:
        MFI-backtrack(I_{l+1}, C_{l+1}, l+1)
```

FI-combine function

```
FI-combine(I_{l+1}, P_{l+1}):
C = ∅
for each y in P_{l+1}:
    if sup(I_{l+1} ∪ {y}) ≥ minsup:
        C = C ∪ {y}
return C
```

Detailed example (with $\text{minsup} = 50\%$)

Initial Tidsets

```
A: {1, 3, 4, 5}
C: {1, 2, 3, 4, 5, 6}
D: {2, 4, 5, 6}
T: {1, 3, 5, 6}
W: {1, 2, 3, 4, 5}
```

Start with A

- Combinable items $C_1 = \{C, T, W\}$
- Generate $AC \rightarrow t(AC) = \{1, 3, 4, 5\}, \text{sup\_count} = 4 \geq 3$
- Generate $AT \rightarrow t(AT) = \{1, 3, 5\}, \text{sup\_count} = 3 \geq 3$
- Generate $AW \rightarrow t(AW) = \{1, 3, 4, 5\}, \text{sup\_count} = 4 \geq 3$

$\Rightarrow C_2 = \{AC, AT, AW\}$

For AC

- Combinable items $P_2 = \{T, W\}$
- Generate $ACT \rightarrow t(ACT) = \{1, 3, 5\}, \sup\_\text{count} = 3 \geq 3$
- Generate $ACW \rightarrow t(ACW) = \{1, 3, 4, 5\}, \sup\_\text{count} = 4 \geq 3$

$\Rightarrow C_3 = \{ACT, ACW\}$

For ACT

- Combinable items $P_3 = \{W\}$
- Generate $ACTW \rightarrow t(ACTW) = \{1, 3, 5\}, \sup\_\text{count} = 3 \geq 3$

$\Rightarrow C_4 = \{ACTW\}$

For $ACTW$

- No combinable items $C_5 = \emptyset$
- Check maximality $\rightarrow$ no superset in *MFIs* yet so add $ACTW$

Backtract to ACW

- Superset $ACTW$ exists so prune ACW

Result 2 *MFIs*

```
ACTW × {1, 3, 5}
CDW × {2, 4, 5}
```

Advantages

- **Minimal output** $\rightarrow$ Only the largest frequent itemsets
- **Efficent for dense data** $\rightarrow$ Reduces output size significantly

Implementation notes

- **Tidset Intersection** $\rightarrow$ use bitsets for fast computation.
- **Pruning** $\rightarrow$ maintain a set of maximal itemsets to check for supersets.
- **Challenges** $\rightarrow$ ensuring no maximal itemset is missed requires careful pruning.

Observations and comparisons

Hierarchy $M \subset C \subset F$

Closed itemsets

- Lossless representation of all frequent itemsets
- Ideal for generating concise association rules

Maximal itemsets

- Most compact but loss *sp* information for subsets
- Best for dense datasets with many *CIs*

Trade-offs

- **F** $\rightarrow$ Comprehensive but redundant and large.
- **C** $\rightarrow$ Balances compactness and information.
- **M** $\rightarrow$ Minimal but may require reconstruction of subsets.

Practical applications

- **Market Basket Analysis** $\rightarrow$ place frequently co-purchased items together or offer bundles.
- **Recommendation Systems** $\rightarrow$ suggest items based on frequent patterns.

- **Bioinformatics** $\longrightarrow$ identify co-occurring genes or proteins.
  Web Mining: Analyze common navigation paths.

Advanced Topics and Extensions

Other algorithms

FP-Growth

- Uses a Frequent Pattern tree (FP-tree) to avoid candidate generation
- More efficient than Apriori for large, sparse datasets

Eclat

- Vertical data format with *Tids*
- Depth-first search, similar to **Charm** and **GenMax**

LCM (Linear time Closed itemset Miner)

- Optimized for closed itemset mining, faster than **Charm** in some cases

Dense vs Sparse datasets

- **Sparse datasets** $\longrightarrow$ few items per transactions (e.g., retail). Many *FIs* but **Apriori** or **FP-Growth** works well
- **Dense datasets** $\longrightarrow$ many items per transaction (e.g., bioinformatics). Fewer but larger itemsets, **Charm** or **GenMax** is preferred