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Prove for $i \neq 1$, $\sum_{\forall C} Pr(C) Pr[e_i(C) | W=1] \leq 2^{-n[I(X;Y)-3\varepsilon]}$

Proof. $\sum_{\forall C} Pr(C) Pr[e_i(C) | W=1]$

$$= \sum_{\forall C} \left(\prod_{i=1}^{2^{nR}} Pr(x_C^n(i)) \right) Pr[e_i(C) | W=1]$$

$$= \sum_{X_i^n} \sum_{C: x_C^n(i) = x_i^n} \prod_{i=1}^{2^{nR}} Pr(x_C^n(i))\, Pr(X_i^n \text{ and } Y^n \text{ are joint typical} | W=1)$$

$$= \cancel{\sum_{X_i^n} Pr(x_C i} \sum_{X_i^n} Pr(X_i^n) Pr(X_i^n \text{ and } Y^n \text{ are joint typical} | W=1) \times \sum_{C: X_C^n(i) = X_i^n} \prod_{j \neq i}^{2^{nR}} Pr(X_C^n(j))$$

$$= \sum_{X_i^n} Pr(x_i^n) Pr(x_i^n \text{ and } Y^n \text{ are joint typical} | W=1)$$

$$= Pr(X_i^n \text{ and } Y^n \text{ are joint typical} | W=1) = Pr(E_i | W=1)$$

By the code generation process, $X^n(1)$ and $X^n(i)$ are independent for $i \neq 1$, so are Y^n and $X^n(i)$. Hence the probability that $X^n(i)$ and Y^n are jointly typical, i.e. $Pr(E_i | W=1) =$

$$Pr[(X^n(i), Y^n) \in A_\varepsilon^{(n)} | W=1] \leq 2^{-n[I(X;Y)-3\varepsilon]}$$