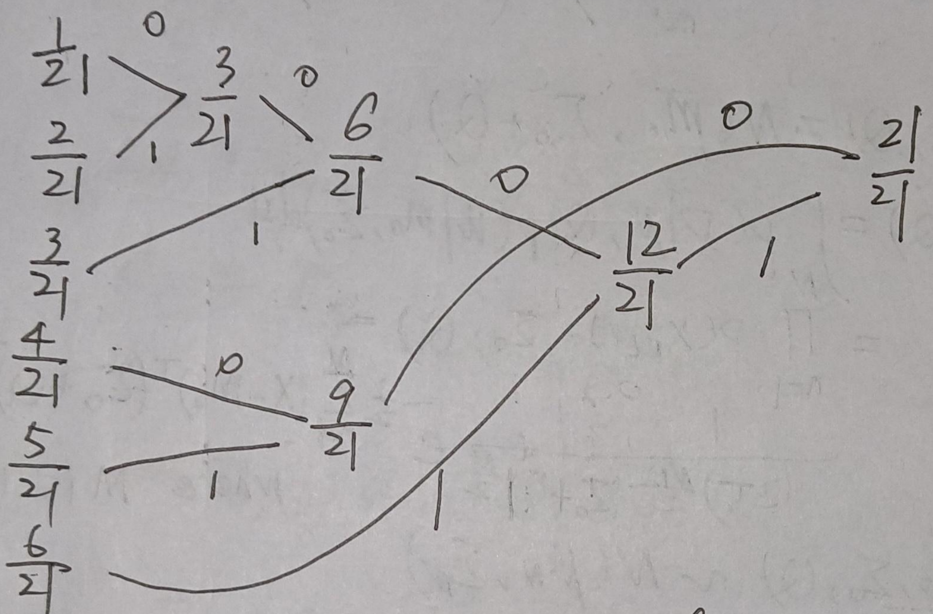
Homework 6 郑祖彬 12112328

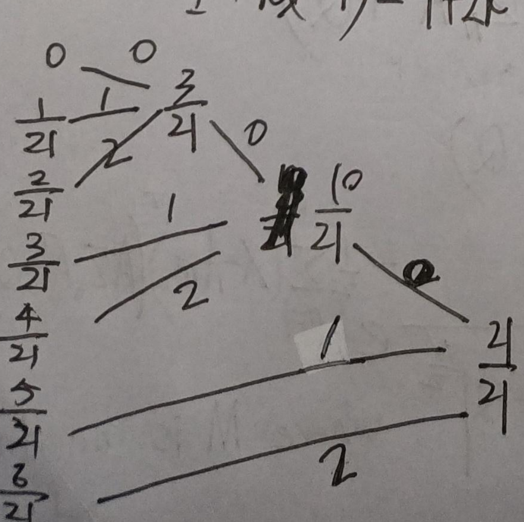
5.14.

(a)




hence for $X \sim p(\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21})$ the ~~bit~~ corresponding binary ~~code~~ Huffman code is 1000, 1001, 101, 00, 01, 11

(b) since $1+k(D-1) = 1+2k$ when $k=3$, $2k+1=7$, so




hence, $X \sim p(\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21})$ the corresponding ternary Huffman code is 001, 002, 01, 02, 1, 2

(c) in (a), $L_a=\sum p_i l_i = \frac{1}{21}\times4+\frac{2}{21}\times4+\frac{3}{21}\times3+\frac{4}{21}\times2+\frac{5}{21}\times2+\frac{6}{21}\times2$

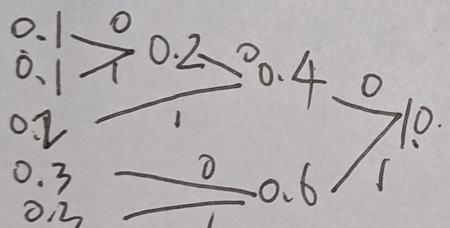
$= \frac{4+8+9+8+10+12}{21} = \frac{51}{21} = \frac{17}{7}$ bits

in (b) $L_b = \sum p_i l_i = \frac{1}{21}\times3+\frac{2}{21}\times3+\frac{3}{21}\times2+\frac{4}{21}\times2+\frac{5}{21}\times1+\frac{6}{21}\times1$

$= \frac{3+6+6+8+5+6}{21} = \frac{34}{21}$ ternary symbols

5.15.

(a)




| $p(x)$ | $C(x)$ |
|---|---|
| 0.3 | 10 |
| 0.3 | 11 |
| 0.2 | 01 |
| 0.1 | 000 |
| 0.1 | 001 |

$L_{avg} = 0.3\times2+0.3\times2+0.2\times2+0.1\times3\times2 = 2.2$ bits

(b) $L'_{avg} = 2p_1+2p_2+2p_3+3p_4+3p_5 = 2(p_1+p_2+p_3)+3(p_4+p_5)$

$H(p') = \sum_{i=1}^{5} p_i \log\frac{1}{p_i}$ subject to $\sum_{i=1}^{5} p_i = 1$, $L'_{avg} = H(p')$

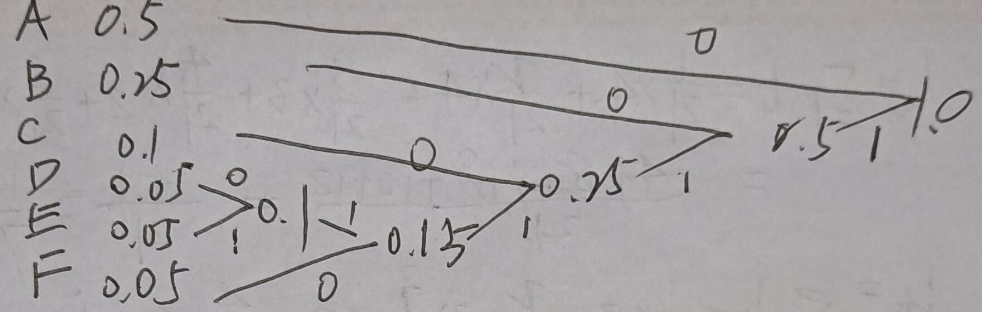
Since $H(p')_{max} = 2.32$ we Let $p_4=p_5=0.125$, $p_1=p_2=p_3=0.25$,

then $L'_{avg} = 2\times0.25\times3+3\times0.125 = 2.25$

$H(p') = 0.25\log_2 4\times3+0.125\log_2 8\times2 = 2.25$,

Hence, we have $L'_{avg} = H(p') = 2.25$, $p' = (0.25, 0.25, 0.25, 0.125, 0.125)$

5.16 (a)

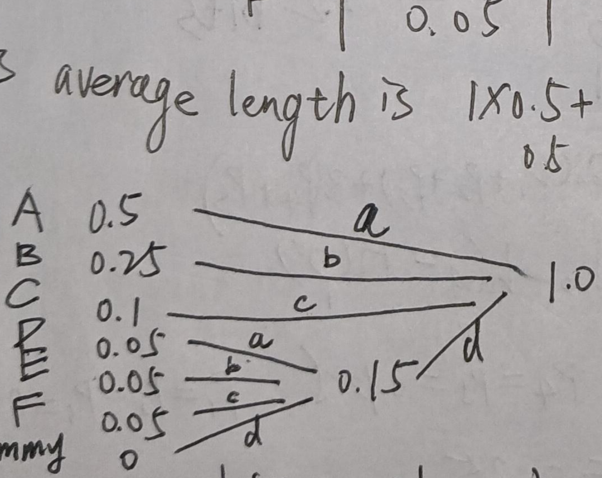



~~Hee~~. Hence, one of binary Huffman code is

| X | p(x) | C(x) |
|---|---|---|
| A | 0.5 | 0 |
| B | 0.25 | 10 |
| C | 0.1 | 110 |
| D | 0.05 | 11110 |
| E | 0.05 | 11111 |
| F | 0.05 | 1110 |

its average length is $\underset{0.5}{1\times0.5}+\underset{0.5}{2\times0.25}+\underset{0.3}{3\times0.1}+\underset{0.5}{5\times0.05\times2}+\underset{0.2}{4\times0.05}=2$ bits

(b)




$1+k(D-1)=1+k(4-1)=3k+1 \quad k=2$

its average length is $0.5+0.25+0.1$
$+0.05\times2\times3$
$=$ ~~2.7~~ 1.15 quaternary symbols

one of quaternary Huffman code:

| X | p(x) | C(x) |
|---|---|---|
| A | 0.5 | a |
| B | 0.25 | b |
| C | 0.1 | c |
| D | 0.05 | da |
| E | 0.05 | db |
| F | 0.05 | dc |

(c) average length: $(0.5+0.25+0.1)\times 2+(0.05\times 2\times 3)\times 2$
$= 2.3$ bits

(d) proof. Let $L_Q$ be the ~~a first~~ average ~~len~~ length code constructed by building a quaternary ~~Huffman code~~ Huffman codes

we have $L_{QB} = 2L_Q$

since Huffman code is optimal, i.e, the best prefix code, and the binary code constructed from the quaternary code is also prefix ~~do~~ code, we have the lower bound:

$$L_H \le L_{QB}.$$

Consider the upper bound: $H_4(X) \le L_Q < H_4(X)+1$.

where $H_4(X)$ denotes that $H_4(X) = \sum_{i=1}^{n} p_i \log_4 \frac{1}{p_i}$

$H_2(X) \le L_H < H_2(X)+1$, where ~~$H_4($~~ $H_2(X) = \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i}$

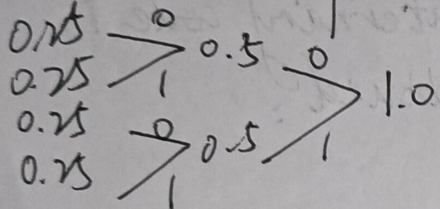
since $H_2(X) = 2H_4(X)$ we have

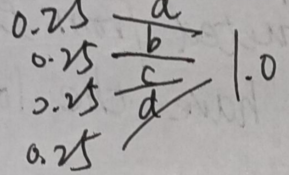
$$2H_4(X) \le 2L_Q < 2H_4(X)+2$$
$$H_2(X) \le L_{QB} < H_2(X)+2$$

Hence, $L_H \le L_{QB} < L_H + 2$

(e). example:

| X | p(X) | binary Huffman code | quaternary Huffman code → QB. |
|---|---|---|---|
| A | 0.25 | 00 | a → 00 |
| B | 0.25 | 01 | b → 01 |
| C | 0.25 | 10 | c → 10 |
| D | 0.25 | 11 | d → 11 |




Hence $L_H = 0.25\times2\times4$
$= 2$

$L_{QB} = 2\cdot(0.25\times1\times4)$
$= 2$

$1+k(4-1)=3k+1$ | $k=1$

in this example, $L_H = L_{QB}$

(f). proof. ~~in (e), we have~~

~~$H_2(X) \le L_{QB} < H_2(X)+2$~~

Suppose ~~all of~~ the binary Huffman code of X, some of its codewords have ~~odd~~ the code word length which is odd, then inverse the mapping: B → Q

where n is the number of codewords whose length is odd.

we have $L_{BQ} = \frac{1}{2}\left(L_H + \sum_{i=1}^{n} p_i\right) \le \frac{1}{2}(L_H+1)$ but not ~~ins~~ sufficiently prefix code.

since $L_Q$ is prefix code, so we have $L_{BQ} \ge L_Q$

so $L_{QB} = 2L_Q \le 2L_{BQ} \le L_H+1$. Hence, $L_{QB} \le L_H+1$

example.

| X | p(X) | binary Huffman code | quaternary Huffman code → QB |
|---|---|---|---|
| A | 0.5 | 0 | a → 00 |
| B | 0.5 | 1 | b → 01 |

Hence, $L_H = 0.5\times1\times2 = 1$

$L_{QB} = 0.5\times2\times2 = 2$

$L_{QB} = L_H + 1$