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1 (a) $h(X) = -\int_0^\infty \lambda e^{-\lambda x}\left[\ln\lambda + (-\lambda x)\right]dx$

$= -\int_0^\infty \lambda e^{-\lambda x}\ln\lambda \, dx + \int_0^\infty \lambda^2 e^{-\lambda x} x \, dx$

$= \ln\lambda \int_0^\infty e^{-\lambda x} d(-\lambda x) + \int_0^\infty e^{-\lambda x}(-\lambda x)\, d(-\lambda x)$.

$= \ln\lambda \int_{-\infty}^0 e^t dt + \int_{-\infty}^0 e^t t\, dt$ Let $g'(t) = e^t$ $f(t) = t$

$= \ln\lambda\left[e^0 - 0\right] + te^t\Big]_{-\infty}^0 - \int_{-\infty}^0 e^t dt$

$= \ln\lambda + 0 - [1-0] = \ln\lambda - 1$ nats

(b) $h(X) = -\int_{-\infty}^\infty \frac{1}{2}\lambda e^{-\lambda|x|}\left(\ln\frac{1}{2}\lambda + (-\lambda|x|)\right)dx$

$= -\int_{-\infty}^\infty \frac{1}{2}\lambda e^{-\lambda|x|}\ln\frac{1}{2}\lambda\, dx + \int_{-\infty}^\infty \frac{1}{2}\lambda^2 e^{-\lambda|x|}|x|\, dx$

$= -\int_0^\infty \lambda e^{-\lambda x}\ln\frac{1}{2}\lambda\, dx + \underbrace{\int_0^\infty \lambda^2 e^{-\lambda x} x\, dx}_{1}$

$= (\ln 2 - \ln\lambda)\underbrace{\int_0^\infty \lambda e^{-\lambda x}dx}_{1} + 1 = \ln 2 - \ln\lambda + 1$ nats

(c) $X = X_1 + X_2 \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$ Based on the collary from textbook

$h(X) = \frac{1}{2}\ln\left[2\pi e(\sigma_1^2+\sigma_2^2)\right]$ nats