

# Group Project

December 21, 2023

## Abstract

## 1 Introduction

## 2 Related work

## 3 Problem Formulation

Generally, considering a data stream  $\{X_t\}$ , where  $t \in \{1, 2, \dots\}$  is the test time step and  $X_t \in \mathbb{R}^d$  denotes the d-dimensional data feature arrived at time step  $t$ , the corresponding  $y_t$  to denote the true label of  $X_t$  and  $y_t \in \{1, \dots, c\}$  for  $c \geq 2$ , where  $c$  is the number of classes, however, the number of samples in each category in the data stream is uneven, i.e. data skewing occurs, online imbalance learning follows the conventional "test-then-train" online learning process: test sample arrives strictly one by one,  $X_t$  arrived at  $t$ , the aim of online imbalance learning is to predict its label with the latest model as

$$\hat{y}_t = \Theta_{t-1}(X_t),$$

where  $\Theta_{t-1}$  is the model trained after time  $t-1$ . Then, one can obtain the true label  $y_t$  before  $t+1$ , and the new training sample  $(X_t, y_t)$  is used to update model  $\Theta_{t-1}(\cdot)$  to  $\Theta_t(\cdot)$ .

## 4 Proposed algorithm: Instance Dependent Cost Online Classification

In this section, we design heuristics instance-dependent cost in online learning. To simplify cost matrix, we only consider the misclassification cost for each class (degree of freedom:  $k^2 - k$  down to  $k - 1$ ) and the misclassification cost for each instance (degree of freedom:  $n \times (k^2 - k)$  down to  $n \times (k - 1)$ ), where  $k$  is number of classes,  $n$  is number of instances.

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## 4.1 The original structure of the proposed algorithm

For each prediction, if our classifier predict correctly, we continue to train the classifier by fitting this instance by the class-dependent cost; Otherwise, i.e. the current classifier can not predict this instance correctly, the instance is difficult to the classifier that it can not handle, our current classifier should pay more attention to this instance. We continue to train the classifier by fitting this instance by the heuristics instance-dependent cost.

Note that in algorithm 1 we take the prediction error into consideration when calculating the heuristics instance-dependent cost. The prediction error term focus more on the instance that  $p$  nears to 0.5 but is misclassified as the opposite class. For such instances, our current classifier is almost able to predict correctly, so we give them a higher prediction error cost. However, for the instance that  $p$  nears to 0 or 1 but is misclassified as the opposite class, there are two possible reasons: 1) our current classifier is still weak 2) the instance is a noise. For such instance, we give them a lower prediction error cost to prevent overfitting or learning from noise. By receiving the feedback from prediction, the model is trained to strengthen the ability to correctly classify samples that are easy to classify incorrectly, thereby improving the overall performance of the classifier step by step.

**Input:** Input data, Input Labels

**Output:** Prediction

```

while Have more samples do
    datum, label <- next sample
    prediction label  $\hat{y}$  <- Predict by current classifier
    y <- the true label of this sample
    current ratio of Class y:  $CRC_y$  <-  $\frac{\# \text{ of current samples with label } y}{\# \text{ of all current samples}}$ 
    Class-dependent Cost:  $CDC_y$  <-  $\frac{1}{CRC_y}$ 
    if Prediction is correct then
        | train the classifier by fitting this instance with the
        |   class-dependent cost
    else
        | prediction error <-  $\alpha e^{\beta(1-y(1-p)-(1-y)p)}$ 
        | Instance-dependent Cost:  $IDC$  <-  $CDC_y$  + prediction error
        | train the classifier by fitting this instance with the heuristics
        |   instance-dependent cost
    end
end

```

**Algorithm 1:** Heuristics Instance-dependent Cost Online Classification 1

## 4.2 Change the structure of the proposed algorithm

We have modified the structure of the original algorithm in two aspects, one is the cost of using feedback based on instance prediction results each timestep,

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and the other one is the modification in the definition of prediction error term.

For the first aspect, we do not distinguish between correctly classified and incorrectly classified instance, using the same definition of instance-dependent cost as the cost for each instance. Since we found that, if the classifier correctly classifies each instance, it will degenerate into class-dependent cost classifier, which shows no obvious difference between class-dependent cost classifier and instance-dependent cost classifier.

For the second aspect, the prediction error is redefined as:

$$\text{prediction error} = (1 - p_t)^\alpha, \quad (1)$$

where  $p_t$  is the classification probability as the true label, defined as:

$$p_t = \begin{cases} p, & \text{if } y=1 \\ 1-p, & \text{if } y=0 \end{cases} \quad (2)$$

In the above  $y \in \{0, 1\}$  specifies the ground-truth class and  $p \in [0, 1]$  is the model's estimated probability for the class with label  $y = 1$ .

Then the instance-dependent cost is redefined as

$$\text{instance-dependent cost} = (1 + \text{prediction error}) \times \text{class-dependent cost} \quad (3)$$

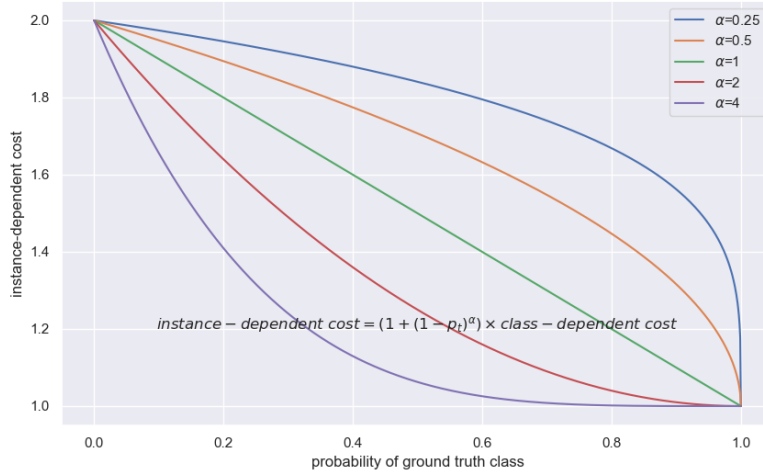


Figure 1: instance-dependent cost with different value of  $\alpha$

The instance-dependent cost is visualized for several values of  $\alpha \in [0.25, 4]$  in 1. We note two properties of the instance-dependent cost. (1) When an instance is misclassified and  $p_t$  is small, the prediction error term nears to 1, which contributes a large instance-dependent cost. As  $p_t$  nears to 1, the

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prediction error goes to 0 and the instance-dependent cost for well-classified instance is down-weighted to class-dependent cost. (2) The hypothesis parameter  $\alpha$  smoothly adjusts the rate at which hard instances are focused on more. Hence, compare with the former definition of prediction error focusing the instances with  $p_t$  nears to 0.5, the new definition of prediction error focuses on the instances with  $p_t$  nears to 0, which are considered as hard instances, and slightly adjusted by the hypothesis parameter  $\alpha$ .

The whole process with new structure is summarized in algorithm 2.

**Input:** Input data, Input Labels

**Output:** Prediction

```

while Have more samples do
    datum, label <- next sample
    prediction label  $\hat{y}$  <- Predict by current classifier
    y <- the true label of this sample
    current ratio of Class y:  $CRC_y <- \frac{\# \text{ of current samples with label } y}{\# \text{ of all current samples}}$ 
    Class-dependent Cost:  $CDC_y <- \frac{1}{CRC_y}$ 
    if y is 1 then
        |  $p_t <- p$ 
    else
        |  $p_t <- 1 - p$ 
    end
    prediction error <-  $(1 - p_t)^\alpha$ 
    Instance-dependent Cost:  $IDC <- (1 + \text{prediction error}) \times CDC_y$ 
    train the classifier by fitting this instance with the heuristics
    instance-dependent cost
end

```

**Algorithm 2:** Heuristics Instance-dependent Cost Online Classification 2

## 5 Experiments

### 5.0.1 Experimental Setup

In this subsection, we illustrate our experimental dataset setting, parameter setting, and comparison methods.

For comparison method, we design a naive version of our method, which does not further fine-tuning on the human designed cost matrix. The human designed matrix is totally determined by tracked imbalance ratio .

Following existing works of online class imbalance learning, we select Hoeffding Tree as the base classifier and the number of base learners is set to 10. We set the interval between each evolution 100 data samples with a maximum generation of 5 in each evolution process. The number of individuals are set to 20 for all evolution processes. The metric we used for fitness evaluation is the difference between the maximum recall and the minimum recall.

One part of the datasets "contraceptive", "segment0", "yeast-0-2-5-6 vs 3-7-8-9" and "yeast1" come from Keel repository . Other datasets comes from synthesizing. All the dataset information are summarized in Table 1. Here IR means the imbalance ratio (ratio between instance number of maximum class and instance number of minimum class).

The metric we used is online GMean following , which give instantaneous GMean for each time step. The overall GMean is calculated by simply averaging the GMean on every time steps.

datasets	IR	#ins.	#fea.	datasets	IR	#ins.	#fea.	datasets	IR	#ins.	#fea.
synthesize1	5.00	1200	10	segment0	6.02	2308	19	yeast1	2.46	1484	8
synthesize2	5.00	1200	5	segment0-5-1tra	6.02	1846	19	yeast1-5-1tra	2.46	1187	8
synthesize3	5.00	1200	10	segment0-5-2tra	6.02	1846	19	yeast1-5-2tra	2.46	1187	8
synthesize4	10.00	1100	12	segment0-5-3tra	6.02	1846	19	yeast1-5-3tra	2.46	1187	8
synthesize5	5.00	3000	25	segment0-5-4tra	6.00	1847	19	yeast1-5-4tra	2.46	1187	8
synthesize6	5.00	3000	50	segment0-5-5tra	6.02	1847	19	yeast1-5-5tra	2.45	1188	8
synthesize7	5.00	4800	50	segment0-5-1tst	6.00	462	19	yeast1-5-1tst	2.45	297	8
synthesize8	5.00	4800	30	segment0-5-2tst	6.00	462	19	yeast1-5-2tst	2.45	297	8
				segment0-5-3tst	6.00	462	19	yeast1-5-3tst	2.45	297	8
				segment0-5-4tst	6.09	461	19	yeast1-5-4tst	2.45	297	8
				segment0-5-5tst	5.98	461	19	yeast1-5-5tst	2.48	296	8

Table 1: Overview of used datasets

### 5.0.2 Overall Performance Comparison

datasets	class-dependent cost	instance-dependent cost				
		$\alpha=0.25$	$\alpha=0.5$	$\alpha=1$	$\alpha=2$	$\alpha=4$
synthesize1	0.897	0.899	<b>0.901</b>	0.899	0.899	0.897
synthesize2	0.741	0.730	0.656	<b>0.751</b>	<b>0.751</b>	0.745
synthesize3	0.831	<b>0.841</b>	0.818	0.809	0.827	0.833
synthesize4	0.757	0.751	0.745	<b>0.771</b>	0.713	0.706
synthesize5	0.651	0.629	0.683	0.571	0.684	<b>0.691</b>
synthesize6	0.701	0.715	0.659	<b>0.767</b>	0.743	0.754
synthesize7	0.730	0.721	<b>0.774</b>	0.751	0.768	0.741
synthesize8	0.644	0.686	0.704	0.699	<b>0.764</b>	0.688

Table 2: Comparison with class-dependent cost in terms of overall GMean, the better algorithm is denoted as bold, synthesize benchmark

We conduct the experiment on aforementioned datasets, the performances of our method and comparison method are summarized in Table 2.

For the second aspect, the prediction error is redefined as:

$$\text{prediction error} = (1 - p_t)^\alpha, \quad (4)$$

where  $p_t$  is the classification probability as the true label, defined as:

$$p_t = \begin{cases} p, & \text{if } y=1 \\ 1 - p, & \text{if } y=0 \end{cases} \quad (5)$$

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datasets	class-dependent cost	instance-dependent cost				
		$\alpha=0.25$	$\alpha=0.5$	$\alpha=1$	$\alpha=2$	$\alpha=4$
segment0	0.957	0.966	0.962	0.962	0.972	<b>0.973</b>
segment0-5-1tra	0.931	0.948	<b>0.975</b>	0.973	<b>0.975</b>	<b>0.975</b>
segment0-5-2tra	0.931	0.963	0.960	0.949	<b>0.968</b>	0.914
segment0-5-3tra	0.956	0.967	0.946	0.970	0.972	<b>0.973</b>
segment0-5-4tra	0.948	0.929	0.946	0.946	<b>0.952</b>	0.950
segment0-5-5tra	0.975	<b>0.976</b>	0.975	0.974	0.974	0.974
segment0-5-1tst	0.902	0.870	0.887	<b>0.907</b>	0.901	0.892
segment0-5-2tst	0.864	<b>0.869</b>	<b>0.869</b>	<b>0.869</b>	<b>0.869</b>	<b>0.869</b>
segment0-5-3tst	0.821	<b>0.886</b>	0.883	0.848	0.848	0.843
segment0-5-4tst	0.874	<b>0.922</b>	0.908	0.913	0.903	0.911
segment0-5-5tst	0.861	<b>0.866</b>	<b>0.866</b>	0.865	0.863	0.863

Table 3: Comparison with class-dependent cost in terms of overall GMean, the better algorithm is denoted as bold, segment benchmark

datasets	class-dependent cost	instance-dependent cost				
		$\alpha=0.25$	$\alpha=0.5$	$\alpha=1$	$\alpha=2$	$\alpha=4$
yeast1	0.598	0.661	0.655	0.636	0.643	<b>0.671</b>
yeast1-5-1tra	0.553	<b>0.570</b>	0.537	0.537	0.529	0.536
yeast1-5-2tra	0.617	<b>0.623</b>	0.601	0.610	0.619	0.598
yeast1-5-3tra	<b>0.623</b>	0.587	0.584	0.604	0.599	0.585
yeast1-5-4tra	0.566	0.570	<b>0.676</b>	0.653	0.650	0.651
yeast1-5-5tra	<b>0.683</b>	0.647	0.598	0.620	0.572	0.573
yeast1-5-1tst	0.602	0.608	0.612	0.625	0.625	<b>0.637</b>
yeast1-5-2tst	<b>0.632</b>	0.619	0.591	0.589	0.629	0.594
yeast1-5-3tst	0.455	<b>0.471</b>	0.433	0.447	0.449	0.431
yeast1-5-4tst	<b>0.554</b>	0.502	0.515	0.531	0.509	0.480
yeast1-5-5tst	0.492	<b>0.625</b>	0.616	0.591	0.581	0.578

Table 4: Comparison with class-dependent cost in terms of overall GMean, the better algorithm is denoted as bold, yeast benchmark

In the above  $y \in \{0, 1\}$  specifies the ground-truth class and  $p \in [0, 1]$  is the model's estimated probability for the class with label  $y = 1$ .

Then the instance-dependent cost is redefined as

$$\text{instance-dependent cost} = (\text{threshold} + \text{prediction error}) \times \text{class-dependent cost} \quad (6)$$

$$\hat{y}_t = \arg \max \Theta_{t-1}(X_t) = \arg \max \sum_{i=1}^{42} \text{weight}_{i,t} \times \theta_{i,t-1}(X_t)$$

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where  $\text{weight}_{i,t} = \frac{\text{gmean}_{i,t-1}}{\sum_{j=1}^{42} \text{gmean}_{j,t-1}}$

datasets	class-dependent cost	ensemble	instance-dependent cost				
			$\alpha=0.25$	$\alpha=0.5$	$\alpha=1$	$\alpha=2$	$\alpha=4$
synthesize1	0.897	<b>0.902</b>	0.899	0.901	0.899	0.899	0.897
synthesize2	0.741	0.744	0.730	0.656	<b>0.751</b>	<b>0.751</b>	0.745
synthesize3	0.831	<b>0.855</b>	0.841	0.818	0.809	0.827	0.833
synthesize4	0.757	<b>0.791</b>	0.751	0.745	0.771	0.713	0.706
synthesize5	0.651	<b>0.712</b>	0.629	0.683	0.571	0.684	0.691
synthesize6	0.701	<b>0.771</b>	0.715	0.659	0.767	0.743	0.754
synthesize7	0.730	<b>0.806</b>	0.721	0.774	0.751	0.768	0.741
synthesize8	0.644	<b>0.769</b>	0.686	0.704	0.699	0.764	0.688

Table 5: Comparison with class-dependent cost in terms of overall GMean, the better algorithm is denoted as bold, synthesize benchmark

## 6 Conclusion

Then the instance-dependent cost is redefined as

$$\text{instance-dependent cost} = (\text{threshold} + \text{prediction error}) \times \text{class-dependent cost} \quad (7)$$

$$\hat{y}_t = \arg \max_{\text{Label}} \Theta_{t-1}(\mathbf{X}_t) = \arg \max_{\text{Label}} \sum_{i=1}^{42} w_{i,t} \Theta_{i,t-1}(\mathbf{X}_t)$$

where  $w_{i,t} = \frac{\text{gmean}_{i,t-1}}{\sum_{j=1}^{42} \text{gmean}_{j,t-1}}$ ,  $\Theta_{i,t-1}$  is the  $i^{th}$  base classifier obtained at timestep  $t-1$ ,  $\Theta_{i,t-1}(\mathbf{X}_t)$  outputs a vector containing the probability distribution of each predicted label.

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**Input:** Input data, Input Labels

**Output:** Prediction

Initialization:  $\gamma \leftarrow$  decay rate,  $N \leftarrow$  num of baseclassifier

**while** *Have more samples* **do**

$t \leftarrow$  timestep

    datum, label  $\leftarrow$  next sample

$\rho \leftarrow$  a random number  $\in [0, 1]$

**if**  $\rho < e^{-\gamma t}$  **then**

$\mathbb{S} \leftarrow \{1, 2, \dots, N\}$  //mass strategy

**else**

$\mathbb{S} \leftarrow \arg \max_{\text{base indices}, j=1,2,\dots,N} \text{gmean}_{j,t-1}$  //elite strategy

**end**

$w_{i,t} \leftarrow \frac{\text{gmean}_{i,t-1}}{\sum_{j \in \mathbb{S}} \text{gmean}_{j,t-1}}, i \in \mathbb{S}$

$\hat{y}_t \leftarrow \arg \max_{\text{Label}} \Theta_{t-1}(\mathbf{X}_t) = \arg \max_{\text{Label}_{i \in \mathbb{S}}} \sum w_{i,t} \Theta_{i,t-1}(\mathbf{X}_t)$

$y_t \leftarrow$  the true label of this sample

    update current ratio of class  $y_t$ :  $R_{y_t} \leftarrow \frac{\# \text{ of current samples with label } y_t}{\# \text{ of all current samples}}$

    class-dependent cost  $\leftarrow \frac{1}{R_{y_t}}$

$p_{t,i} \leftarrow [\Theta_{i,t-1}(\mathbf{X}_t)]_{y_t}, i \in \{1, 2, \dots, N\}$

    prediction error  $_i \leftarrow (1 - p_{t,i})^{\alpha_i}, i \in \{1, 2, \dots, N\}$

    instance-dependent cost  $_i \leftarrow$

        (threshold  $_i$  + prediction error  $_i$ ) class-dependent cost,  $i \in \{1, 2, \dots, N\}$

$\Theta_{i,t} \leftarrow \text{train}(\Theta_{i,t-1}, \text{instance-dependent cost}_i), i \in \{1, 2, \dots, N\}$

**end**

Note that  $\Theta_{i,t-1}$  is the  $i^{th}$  base classifier obtained at timestep  $t - 1$ ,

$\Theta_{i,t-1}(\mathbf{X}_t)$  outputs a vector containing the probability distribution of each predicted label

**Algorithm 3:** Mass and Elite Strategy Based Instance-dependent Cost Online Classification(MAESIC)



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datasets	class-dependent cost	instance-dependent cost	
		Mass Strategy	Mass and Elite Strategy
synthesize1	0.897	<b>0.907</b>	<b>0.907</b>
synthesize2	0.741	0.799	<b>0.820</b>
synthesize3	0.831	<b>0.851</b>	0.848
synthesize4	0.757	<b>0.773</b>	0.758
synthesize5	0.651	0.734	<b>0.744</b>
synthesize6	0.701	<b>0.793</b>	0.780
synthesize7	0.730	<b>0.825</b>	0.820
synthesize8	0.644	0.778	<b>0.796</b>

Table 6: Comparison with class-dependent cost in terms of overall GMean, the better algorithm is denoted as bold, synthesize benchmark