

# Market risk assessment: A multi-asset, agent-based approach applied to the 0VIX<sup>2</sup> lending protocol

Amit Chaudhary  
a.chaudhary.1@warwick.ac.uk

Daniele Pinna  
daniele@0vix.com

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## Abstract

We assess the market risk of the 0VIX lending protocol using a multi-asset agent-based model to simulate ensembles of users subject to price-driven liquidation risk. Our multi-asset methodology shows that the protocol's systemic risk is small under stress and that enough collateral is always present to underwrite active loans. Our simulations use a wide variety of historical data to model the market volatility and run the agent-based simulation to show that even if all the assets like ETH, BTC and MATIC increase their hourly volatility by more than 10x times, the protocol has less than 1% chance of default.

## 1 Introduction

DeFi lending protocols have seen a significant flow of capital. The lending system's stability will depend on the collateral value that the borrowers provide. At any point in time, the system must have adequate capital to become solvent. Recently <sup>1</sup>, research has attempted to estimate the financial risk on lending protocols associated with asset price fluctuations using Agent-based simulations. However, examples assume only two assets (one being the numeraire) being supplied and borrowed in the individual lending market. In reality, users can supply multi assets to the lending market and borrow multiple assets. Sometimes even the same asset is both borrowed and lent to capitalize on temporary incentive mechanisms aimed at attracting liquidity onto the lending market. This paper presents an enhanced multi-asset model where real-time liquidation calls are executed as a result of price turbulence and borrowers face periods where they need to raise cash to remain within the tolerance limit of the protocol parameters like collateral ratio. As a case study, we model these dynamics on the 0VIX<sup>2</sup> lending protocol.

We show how one can ensure the lending market's resilience to adverse shocks even when multiple assets become highly volatile simultaneously. This is done by exploring portions of the phase space of 0VIX's asset-specific parameters and optimizing them by requiring that over-collateralization is retained across a wide range of simulated price volatilities while minimizing the liquidation penalties to individual users. Analyses such as that presented here can be performed periodically on a running basis to offer individual users key insights into the risk of their portfolio positions, as well as propose recalibrations of protocol parameters for discerning governance participants. We believe

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<sup>1</sup>Kao, Chitra, Chiang, and Morrow (2020)

<sup>2</sup>0VIX is the decentralized, Polygon blockchain based open-source lending and borrowing protocol enhanced with veTokenomics, interest rate optimization curve beta ,DAO Treasury management.

this offers investors the confidence to participate with significant capital in OVIX lending pools.

Our model is motivated by the fact that a multi-asset portfolio understands the risks associated with lending protocols. The shock in the price of the single asset can amplify the risk in another pool. The trajectory taken by multi-asset liquidations under amplified market conditions can vary over time and depend on a number of factors. These must all be factored into the incentives which motivate independent and profit-driven liquidators.

We access the effect of the following factors 1) Available buy- and sell-side market liquidity across all asset pairs 2) Asset-specific liquidation incentives and maximal liquidation size offered to liquidators across all assets 3) Maximally allowed loan-to-value ratios for borrowing against specific assets deposited as collateral beyond which liquidations are permitted. These factors in our model contribute to the liquidation size, the collateral-loan assets chosen to liquidate, and whether a liquidation call will be made at all.

Our model considers agents (or users) with multi-asset portfolio allocations. The users are always constrained by the individual collateral ratio of the protocol defined Loan to value (LTV) ratios. The OVIX protocol stability is governed by the fact that all liabilities are redeemable. To maintain this balance, we model liquidation as a incentive mechanism where a liquidator is given a incentive to perform the foreclosure (Chatterjee & Eyigungor, 2015) like event. But whether it is profitable for the liquidator to perform the action is an critical feature deriving the stability of the system. The liquidators' profit is dependent on the traction cost of trading the bad debt in the market and the slippage cost associated with it. When the liquidator acts on the arbitrage opportunity in the bad state, the protocol benefits, decreasing the risk exposure. We stress the liquidation incentive to test the risk exposure of the protocol. The increase in liquidation incentive can reduce the systematic risk but also disincentivize the borrows as they see this as the potential penalty to their net borrowing cost. So protocols must optimally decide the incentive, keeping in mind the growth potential of the lending pools (Leshner & Hayes, 2019).

We assume that the strategic interaction between the liquidators are not present, and they are risk-averse (Schied & Schöneborn, 2009). This assumption represents the observed behaviour of liquidators where they immediately sell the debt in the market, assuming the risk of waiting is high enough to gain from any future price movements.

We model the slippage cost to provide real-life market conditions across crypto assets (Makarov & Schoar, 2020). There has been extensively studied in market impact models where liquidity and volatility are critical drivers of the execution cost (Tóth et al., 2011). Since we model the multi-asset model and include relatively lower liquidity assets like MATIC, the slippage costs becomes vital for the market participants. We assume the functional form of the slippage model and use it to calibrate our simulations. In the future, we aim to use the recent order book depth data across Centralized exchanges and liquidity across—decentralized exchanges (Lehar & Parlour, 2021) to model slippage more realistically.

Our market risk assessment of OVIX relies on an agent-based simulation that stresses

the lending protocol based on highly volatile price trajectories. Financial institutions, including banks and federal reserves, have been using such techniques to ascertain the economy's financial stability Ramadiah, Galbiati, and Soramäki (2021). The OVIX protocol will be on Polygon with significantly lower transaction costs, reducing the risk of liquidations and transactions failing due to costs. Past research has shown this bottleneck in the Ethereum Simulated EVM environment <sup>3</sup>

Our results show that the liquidation mechanism works, and the system remains stable even in the worst price history of MATIC, when it dropped 14% in a single day. Stressing the volatility of the assets has shown that the system remains within the safe LTV zone and can be scaled from simulated \$100 million to 10X without any significant rise in the solvency of OVIX. We also show that current protocols parameters are sufficient to face any unprecedented fall in asset prices. We test the OVIX stability across wide ranges of market volatility conditions on multiple collateral factors. We also test the robustness of liquidation incentives in the protocol and how the current incentives are sufficient to maintain the liquidity profiles in the pools where borrowers are optionally liquidated if they cross the protocol's loan to value thresholds. In particular, we verify a theoretical scaling between liquidation incentives and LTV ratios above which users become likely trapped into runaway LTV factors when subject to liquidations. This must be avoided at all costs as it may ultimately lead to undercollateralized protocols.

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<sup>3</sup>Polygon is the leading platform for Ethereum scaling and infrastructure development. Its growing suite of products offers developers easy access to all major scaling and infrastructure solutions: L2 solutions (ZK Rollups and Optimistic Rollups), sidechains, hybrid solutions, stand-alone and enterprise chains, data availability solutions, and more. Polygon's scaling solutions have seen widespread adoption with 3000+ applications hosted, 1B+ total transactions processed, 100M+ unique user addresses, and \$5B+ in assets secured.

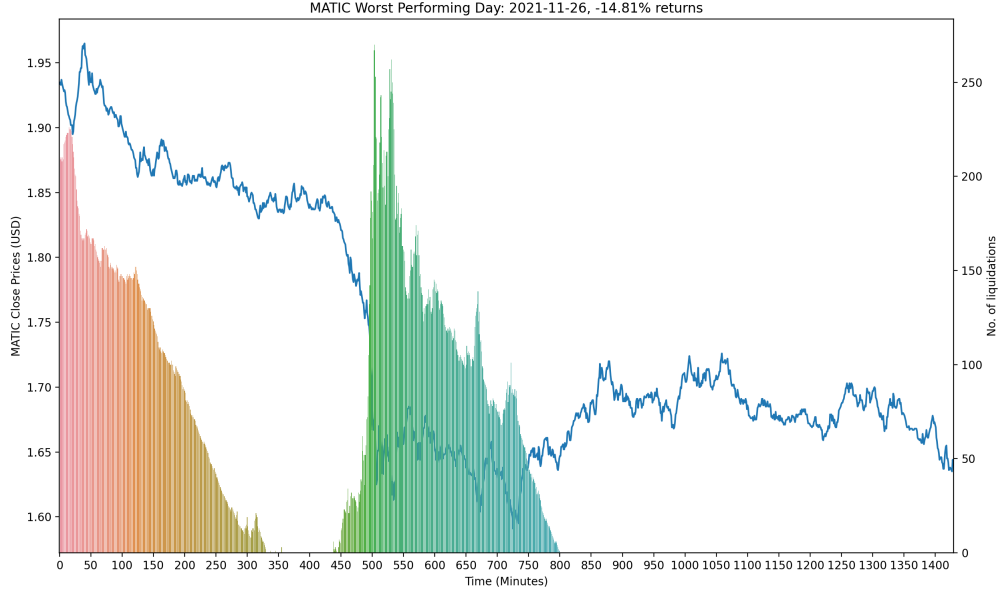


Figure 1: Simulated dual-asset MATICUSDC portfolio behavior across 1000 long-only protocol users during the worst price drawdown day of MATICUSD history. A bar plot of liquidation events overlays the price trajectory.

We also use Covid-pandemic time asset prices data to estimate and stress the OVIX protocol to find the likelihood of failure. The widespread economic damage caused by the COVID-19 pandemic provides a major test of the real recent stresses on crypto assets and the financial system. Our study uses the COVID data to run a market risk assessment on capital and liquidity requirements.

The remainder of this study is organized as follows. Section 2 elaborates the model of the market risk framework and protocol dynamics. Section 3 presents the data and agent-based simulation framework. Section 4 contains the results, and Section 5 concludes.

## 2 Model

### 2.1 Assets and Users

We consider a set of users  $N_U$  participating on the platform, and  $N_A$  assets which users can deposit as collateral or borrow against other assets they have already deposited as collateral. At any given moment in time, each user  $k$  is fully determined by the collateral  $c_i^k$  and loan  $l_i^k$  amounts (in numeraire units) of each asset  $i$  they own. The total portfolio size and loan-to-value ratio (LTV) of their account can be derived as:

$$\begin{aligned}\text{portfolio}_k(t) &= \sum_{i=0}^{N_A} (c_i^k - l_i^k) \\ \text{LTV}_k(t) &= \frac{\sum_{i=0}^{N_A} l_i^k}{\sum_{i=0}^{N_A} c_i^k},\end{aligned}\tag{1}$$

where we omit the temporal dependence ( $t$ ) from the *rhs* for ease of legibility.

Each asset  $i$  on the protocol is defined by its maximal LTV  $\max_i^{\text{LTV}}$  beyond which the asset cannot collateralize anymore loans, a liquidation LTV  $\text{liq}_i^{\text{LTV}}$  beyond which any loans using the asset as collateral become liable for liquidation, and a closing factor  $\text{close}_i < 1$  denoting the maximal fraction of a loan portfolio consisting of that asset which can be liquidated. Given these protocol parameters  $\{\max_i^{\text{LTV}}, \text{liq}_i^{\text{LTV}}, \text{close}_i < 1\}$ , and a user's portfolio allocation  $\{c_i^k, l_i^k\}$ , one may derive each user's maximal and liquidation LTV:

$$\begin{aligned}\text{LTV}_k^{\max}(t) &= \frac{\sum_{i=0}^{N_A} c_i^k \max_i^{\text{LTV}}}{\text{portfolio}_k} \\ \text{LTV}_k^{\text{liq}}(t) &= \frac{\sum_{i=0}^{N_A} c_i^k \text{liq}_i^{\text{LTV}}}{\text{portfolio}_k}.\end{aligned}\tag{2}$$

Whenever a user's  $\text{LTV}_k > \text{LTV}_k^{\max}$  they may not borrow any more capital, and when  $\text{LTV}_k > \text{LTV}_k^{\text{liq}}$  a user becomes liquidatable.

Given a price trajectory for the model's assets, all the user portfolios and LTV values can be updated at each price tick by modifying the value of all the assets in their portfolio. This defines all further available actions on the protocol until the next price update.

## 2.2 Liquidators

Whenever a user's  $\text{LTV}_k > \text{LTV}_k^{\text{liq}}$ , a liquidator may attempt to liquidate an amount  $p_i < \text{close}_i \cdot l_i^k = l_i^k/2$  across one of any of the user's loan assets<sup>4</sup>. Execution is performed by utilizing any single collateral asset  $c_j^k$  to cover the amount<sup>5</sup>. To incentivize the active monitoring and liquidation calls performed by liquidators, the protocol assigns a percentage liquidation incentive  $\text{inc}_j$  to each asset  $j$  used as collateral to incentivize the liquidation of certain assets before others. Based on this, a liquidator will consider their potential gains minus any swap fees defined by the sum of three factors: transaction fees, trading fees, and slippage fees. Whereas the first two can in principle be absorbed inside the liquidation incentive factor by making it time-dependent  $\text{inc}_j \rightarrow \text{inc}_j(t)$ , we will drop them for notational simplicity by assuming them constant<sup>6</sup>.

<sup>4</sup>Where we set  $\text{close}_i = 1/2$  across all assets for simplicity

<sup>5</sup>In some cases, a user may have collateral deposited across a number of assets but loans concentrated in one asset only such that no single collateral asset can cover the maximal liquidatable amount  $l_i^k/2$

<sup>6</sup>Our multi-asset model simulator allows full time-dependent control of all such factors if desired.

### 2.2.1 Slippage Costs

The slippage fees are very relevant as they depend on the amount  $a_j^k$  of a user's collateral asset  $j$  being swapped to repay a liquidator's chosen  $l_i^k/2$  relative to the available sell-side liquidity  $V_{j \rightarrow i}$  in the  $(j, i)$  asset pair and the normalized distribution  $\rho_{j,i}(\epsilon)$  of such liquidity (as a function of deviation  $\epsilon$  from quoted market price) across all accessible markets. The percent slippage fee  $\sigma_{j,i}(a_j^k)$  is then defined by the following equations:

$$\frac{a_j^k}{V_{j \rightarrow i}} = \int_0^{\Delta P(a_j^k/V_{j \rightarrow i})} d\epsilon \rho_{j,i}(\epsilon) \quad (3)$$

$$\sigma_{j,i}(a_j^k) = \frac{V_{j \rightarrow i}}{a_j^k} \int_0^{\Delta P(a_j^k/V_{j \rightarrow i})} d\epsilon \epsilon \rho_{j,i}(\epsilon), \quad (4)$$

where the first equation self-consistently defines the total price slippage  $\Delta P(a_j^k/V_{j \rightarrow i})$  to swap  $a_j^k$ , and the second defines the percent lost due to slippage for this amount.

Whereas this quantity can be compiled from real-world data and used in our multi-asset model, it can be also effectively modelled for simulation purposes using  $\gamma$ -polynomial approximations depending on a slippage factor  $s_\gamma$  (to be fit alongside  $\gamma$ ) from historical data:

$$\sigma_{j,i}(a_j^k) \approx s_\gamma \cdot \left( \frac{a_j^k}{V_{j \rightarrow i}} \right)^\gamma \quad (5)$$

From this one can succinctly write the total profit a liquidator can make for swapping an amount  $a_j^k$  for a liquidated user's loan asset  $i$  as:

$$\text{liquidatorProfit}_{j,i}(a_j^k) = \left( inc_j - \sigma_{j,i}(a_j^k) \right) a_j^k, \quad (6)$$

and the total amount  $p_i$  of loan paid upon completing the swap:

$$p_i^k(a_j^k) = \left[ 1 - \left( inc_j + \sigma_{j,i}(a_j^k) \right) \right] a_j^k. \quad (7)$$

### 2.2.2 Liquidation Logic

Recently, it has been argued that real-world slippage behavior is suitably modelled in (5) by setting  $\gamma = 1$  (Kao et al., 2020). Under such an approximation,  $a_j^k$  can be explicitly solved as a function of  $p_i^k$  giving:

$$a_j^k(p_i^k) = \delta_{i,j} p_i^k + (1 - \delta_{i,j}) \frac{1 - inc_j}{2 \tilde{s}_{j,i}} \left[ 1 - \sqrt{1 - \frac{4 \tilde{s}_{j,i} p_i^k}{(1 - inc_j)^2}} \right] \quad (8)$$

where  $\delta_{i,j}$  is the Kronecker delta, and  $\tilde{s}_{j,i} \equiv s_0/V_{j \rightarrow i}$  represent an array of asset parameters<sup>7</sup> in the multi-asset model. When  $j \neq i$ , the swappable amount is well-defined as long as the

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<sup>7</sup>Potentially time-dependent also.

amount chosen to repay satisfies:

$$p_i^k < \frac{(1 - inc_j)^2}{4 \tilde{s}_{j,i}}. \quad (9)$$

In principle, if this inequality is violated, the total swap fees are effectively so large that no amount  $a_j^k$  can swap for  $p_i^k$ . In practice however, an optimal swap amount  $\bar{a}_j^k$  above which the liquidator profits decrease with increasing  $a_j^k$ . This sets a maximal swap/pay amount constraint from the liquidator's perspective which can be found by solving for the maximum of (6):

$$\begin{aligned} \bar{a}_j^k &= \frac{inc_j}{2 \tilde{s}_{j,i}} \\ \bar{p}_i^k &= p_i^k(\bar{a}_j^k). \end{aligned} \quad (10)$$

Combining (8), and (10), with the protocol imposed  $p_i^k < l_i^k/2$ , the liquidator's optimization strategy can be summarized as the following constrained minimization problem across all asset pairs  $(i, j)$  for every liquidatable user:

$$\begin{cases} \max_{i,j} & \text{liquidatorProfit}_{j,i}(a_j^k) \\ a_j^k & < \min [\bar{a}_j^k, c_j^k, a_j^k(l_i^k/2)] \end{cases} \quad (11)$$

### 2.3 Hard parametric constraints

The model just described presents large amounts of complexity resulting from the many free parameters which can be included. Their interactions and effects are generally non-trivial and only analyzable through extensive simulation efforts. However, some general hard constraints can be placed on two parameters in the system: the liquidation LTV  $LTV_k^{liq}$ , and the net fees  $inc_j + \sigma_{j,i}(a_j^k)$  paid.

As discussed, when a user's LTV crosses their liquidation threshold  $LTV_k > LTV_k^{liq}$ , liquidators become incentivized to liquidate a portion of their positions in an effort to sanitize their LTV health. This however requires that the LTV of the user will invariably decrease as a result of liquidations:

$$LTV_k(t+1) = \frac{\left(\sum_{i=0}^{N_A} l_i^k\right) - p_i^k(a_j^k)}{\left(\sum_{i=0}^{N_A} c_i^k\right) - a_k^j} < LTV_k(t) \quad (12)$$

More technically, given that this must be true for *any* amount  $a_k^j$  being liquidated, a healthy protocol must always demand that:

$$\partial_a LTV = \partial_a \frac{\left(\sum_{i=0}^{N_A} l_i^k\right) - \left[1 - (inc_j + \sigma_{j,i}(a_j^k))\right] a_j^k}{\left(\sum_{i=0}^{N_A} c_i^k\right) - a_k^j} < 0. \quad (13)$$

To qualitatively characterize the importance of this constraint, let us perform the derivative in (13) under the assumption that all liquidation fees are constant and independent of  $a_j^k$ , one has:

$$\begin{aligned}
\partial_a \text{LTV} &= \partial_a \frac{L - (1-x)a}{C - a} \\
&= \frac{-(1-x)(C - a) + L - (1-x)a}{(C - a)^2} \\
&= \frac{L - (1-x)C}{(C - a)^2} < 0 \\
\implies \frac{L}{C} = \text{LTV} &< 1 - x,
\end{aligned} \tag{14}$$

where all indices have been dropped for ease of legibility  $a_k^j \rightarrow a$ , the sum of collateral/loan assets have been compressed to  $C/L$  respectively, and all liquidation fees have been grouped into the term  $x$ .

We find that a universal relationship exists between the maximal LTV reached on the protocol and the average liquidation cost which, if violated leads to the systemic creation of undercollateralized users. Equation (14) is a direct prediction of our model whose verification can be seen in Fig. 4.

### 3 Data and Simulations

#### 3.1 User Initialization

Agents in our model are considered passive. Their portfolio is randomly allocated at the beginning of the simulation and assumed to not be adjusted throughout the course of the simulation. To begin the initialization process, collateral and loan asset values  $\{c_i^k, l_i^k\}_{i=0}^{N_A}$  are randomly assigned. For the results presented in this paper, we choose to neglect portfolios where users borrow and collateralize identical assets. To impose this, we unwind portfolio allocations after having generated them.

To begin a simulation, each user is assigned a portfolio size  $\text{portfolio}_k(0)$  and LTV value  $\text{LTV}_k(0)$ . These are drawn randomly from lognormal distributions targeting mean portfolio values of \$5000 and mean LTV values of 0.6.

The initial portfolio asset values can then be individually rescaled  $\{c_i^k, l_i^k\}_{i=0}^{N_A} \rightarrow \{r_C^k \cdot c_i^k, r_L^k \cdot l_i^k\}_{i=0}^{N_A}$  such that they respect  $(\text{portfolio}_k(0), \text{LTV}_k(0))$  assigned. The rescaling factors are given by:

$$r_C^k = \frac{\text{portfolio}_k(0)}{(1 - \text{LTV}_k(0)) \sum_{i=0}^{N_A} c_i^k} \tag{15}$$

$$r_L^k = r_C^k \cdot \text{LTV}_k(0) \cdot \frac{\sum_{i=0}^{N_A} c_i^k}{\sum_{i=0}^{N_A} l_i^k}. \tag{16}$$



A typical initial portfolio ensemble is shown in the left figure of Fig.6

### 3.2 Price Trajectories

To run our model, we have collected the past 3 years of price data across Bitcoin, Ethereum, Matic, and USDC with minute tick level resolution. This allows us to both simulate our model under actual past price dynamics, as well as generate new cross-asset price dynamics by randomly selecting portions of the historical data. Since our agents aren't allowed to modify their portfolio allocations, simulations are run with only one day's worth of price data. Simulating beyond this timescale is obviously possible but likely unrealistic. To simulate specific price volatility, each randomly filtered price data sequence is individually rescaled to match the desired *hourly* target volatility prior to commencing the simulation run.

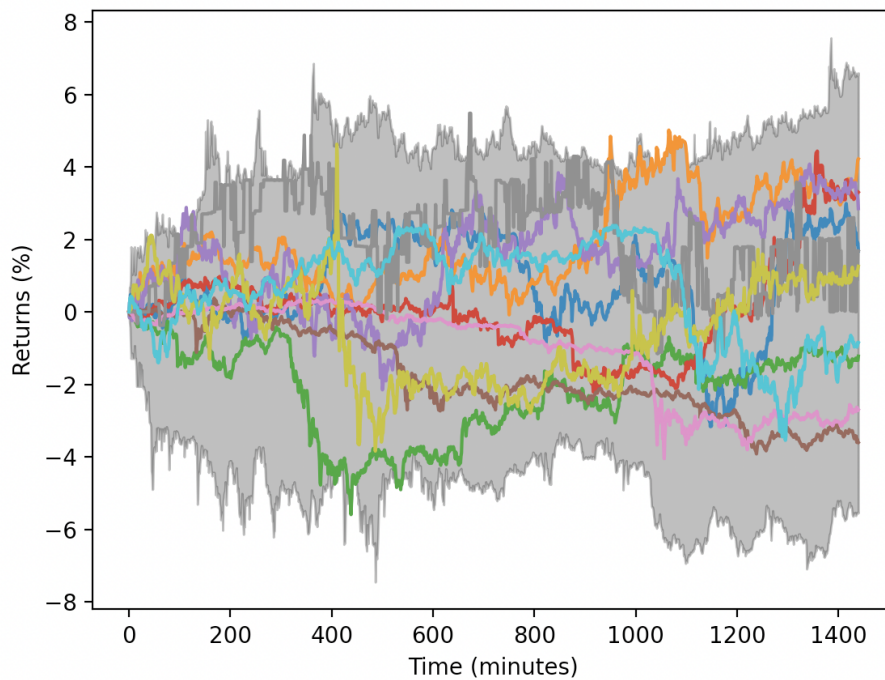


Figure 2: Example ensemble of generated price trajectories for the ETH asset. Shaded areas represent bounds of 100 generated trajectories of which only 10 are plotted for visual convenience.

### 3.3 Data

To collect statistically significant data, 1000 simulation runs are performed for each set of desired protocol parameters. Each simulation run tracks the evolution of 1000 user portfolios across Bitcoin, Ethereum, Matic, and USDC regenerating their individual allocation at each run such that results aren't biased to specific portfolio ensembles. Throughout the simulations we assume that the total available sell-side market volume for swaps is \$100 M for all X-MATIC pairs (where 'X' stands for BTC, ETH, and USDC), and \$1000 M for the rest.

## 4 Results

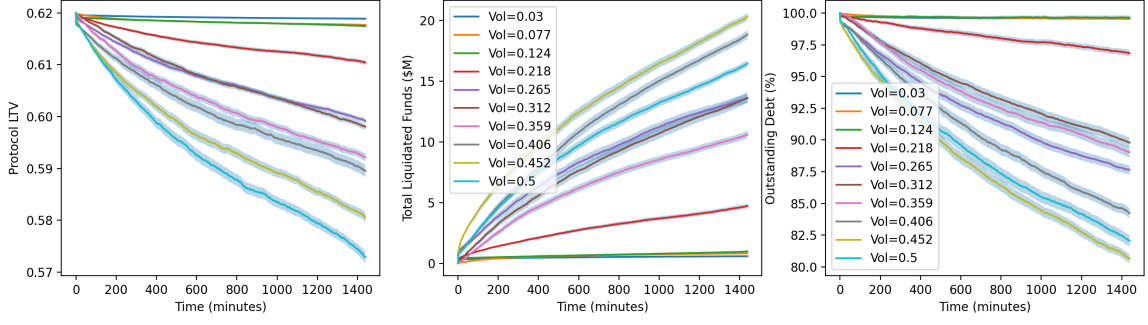


Figure 3: Simulated MATICUSDC dual-asset portfolio behavior across 1000 protocol users subject to 1000 randomly generated daily price trajectories with varying degree of hourly volatility (see Section 3.2 for details). Shaded regions correspond to 95% confidence bands. *Left figure:* Users' collective portfolio loan-to-value ratios over time. *Middle figure:* Total liquidated funds over time. *Right figure:* Users' collective outstanding debt as a percentage of their initial allocation.

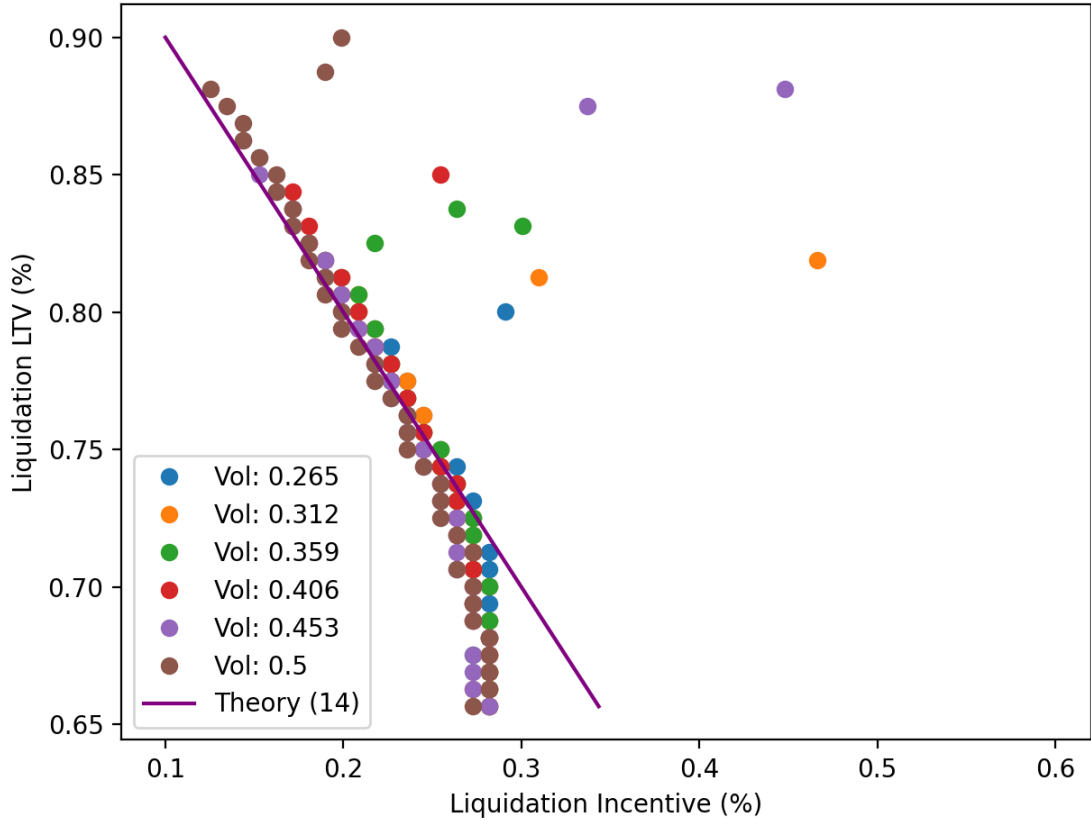


Figure 4: Undercollateralization frontier: Scatter plot of pair values (liquidation LTV, liquidation incentive) at which the number of undercollateralized users on the protocol exceeds  $> 1\%$ . For each such pair, 1000 MATICUSD user portfolios were simulated 100 times each to collect sufficient statistics. Solid purple line represents the theoretical prediction based on our model discussed in the lead-up to equation (14)

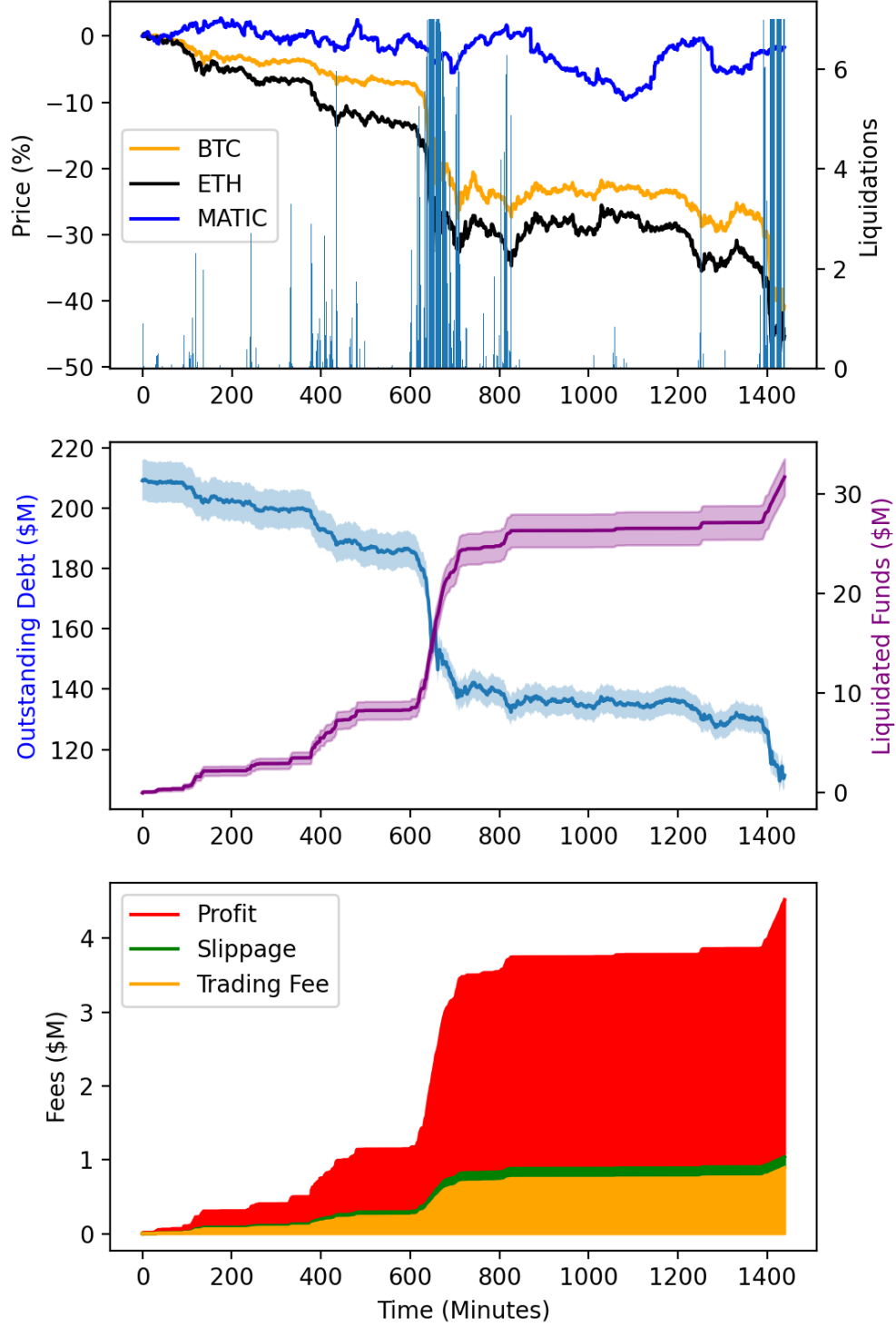


Figure 5: Simulated portfolio behavior across 1000 protocol users throughout the ‘COVID Crash’ day (20 Feb. 2020). User portfolios are initialized by randomly assigning collateral/loan position values (ETH, BTC, MATIC, and USDC assets were considered) consistent with a target loan-to-value ratio and portfolio size drawn from log-normal distributions for each of them (see Section 3.1). 100 such simulations are then repeated (with a different initialization each time) to collect average statistics. All figures are plotted versus time in minutes. *Top figure:* Liquidation events vs price drawdowns. *Middle figure:* Dual axis plot of users’ collective outstanding debt and total liquidated funds (shaded areas represent 95% confidence intervals). *Bottom figure:* Total liquidator profits, trading, and slippage fees.

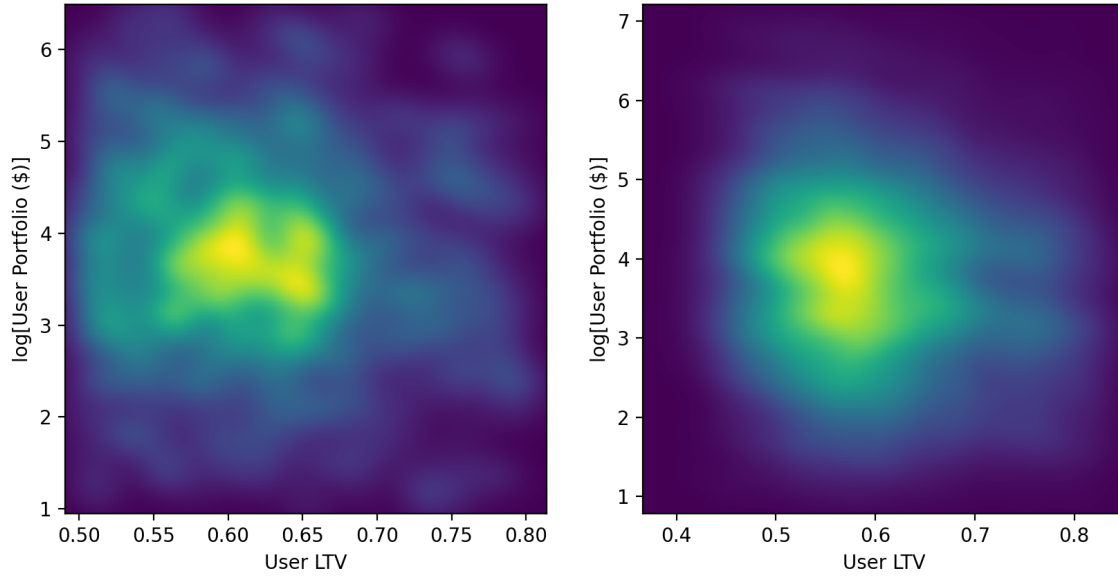


Figure 6: Initial (*left figure*) and final (*right figure*) user portfolio distributions corresponding to a single MATICUSD volatility simulation (40% volatility results are shown). Users are collectively characterized according to their loan-to-value ratios and portfolio size.

## 5 Discussion and Conclusions

We have built an agent-based, multi-asset simulator for lending markets which allows for extensive testing and aggregation of portfolio statistics across varying stressors and protocol parameters. We have demonstrated its principal characteristics by simulating portfolio behaviors subject to volatile real-world data such as the COVID market crash of 20 Feb 2020, as well as artificially generated high-volatility price scenarios. In principle, such a framework allows for a detailed protocol parameter exploration under various circumstances, allowing for their optimization. What our model does not include is dynamical re-allocation of user portfolios during market unwinding events. We have argued for their neglect by remarking that our simulations are valid only in a limit of passive user behavior. As such, we have refrained from running our models for longer than a daily (1440 minute) timescale. In the future we plan to add intra-day user interactions characterized by an intrinsic user-specific timescale describing the frequency with which they may re-balance their portfolios. Similarly to the loan-to-value ratio and portfolio sizes, this too can be drawn from a suitable distribution. Overall, we believe our framework is ideally suited for decentralized lending markets such as the soon-to-be-launched 0VIX, due to the potential for generating data-driven protocol upgrade proposals which governance token holders can evaluate and vote on.

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