2018 Tenth International Conference on Advanced Computational Intelligence (ICACI)
March 29–31, 2018, Xiamen, China

A new evolutionary algorithm based on MOEA/D for portfolio optimization

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***Abstract*—The portfolio optimization problem is a multi-objective problem which takes risk and return as optimization objectives. It is complicated in reality with many restrictions which results in an complex pareto front. MOEA/D is a popular multi-objective evolutionary algorithm framework with decomposition method, which has widely been used to solve multi-objective problems. In order to solve portfolio optimization problem with complex pareto front more effectively, we propose a new algorithm named MOEA/D-CP based on MOEA/D, which utilizes a new weight vector generation approach to generate a evenly distributed set of weight vectors. The experimental results show that the MOEA/D-CP performs much better than algorithm based on original MOEA/D.**

***Keywords*—portfolio optimization; weight vector generation; complex pareto front**

I. Introduction

The mean-variance model is a classical model in the modern portfolio theory It measures returns by expectation and measures investment risk by variance. The following models of portfolio optimization such as semi-variance model [1] [2] [3] and Mean-absolute deviation portfolio optimization model [4] are basically draws on their thinking.Over the years, scholars on the model have made the following improvements: In modern financial research, portfolio optimization problem is a well-known problem which has a great significance in reality. Portfolio optimization problem is a multi-objective optimization problem which requires the balance between maximum of returns and minimum of risk. In1952, through the quantitative study on the investment risk and profits, Markowitz established the mean-variance model and the best investment portfolio model which marked the birth of the modern portfolio theory [5] [6]. The theory explains that in the rational market, investors use the mean of return on assets to represent return on investment and the variance of return on assets to represent investment risk, and then seek Pareto optimal solution in many asset portfolios. All the possible optimal mean - variance combinations form a curve in the plane, and this curve is called efficient frontier or Pareto front.

With the development of the intelligent optimization algorithms, which have widely been used to solve all kinds of practical engineering problems including portfolio optimization. Nowadays, there are two main aspects to the research on this aspect: first, adding constraints to the model; secondly, turning the problem into a multi-objective problem for handling [7] [8]. Chen and Kou pointed out in this paper [9], There are about 400 titles [2] [3] cover the application of intelligent optimization algorithms in finance and economics, and most of them are about portfolio optimization.

However, as the number of restrictions in portfolio optimization problem increases a lot, the pareto front shape of this problem becomes more complex and the distribution of pareto optimal solutions is uneven. Therefore it may be difficult for current algorithms to handle. With the purpose to solve these portfolio optimization problems more efficiently , we propose an algorithm based on MOEA/D framework with a new weight vector generation approach.

The rest of the paper is organized as follows. In the next section, we present some basic knowledge of the theory and model of portfolio. The third section introduce the principle of multi-objective evolutionary algorithm with a proposed approach.Nextly,The fourth section elaborates a variety of experimental results of solving the portfolio optimization in

This work is supported by National Nature Science Foundation of China [Grant Nos. 61773296, 61573157]; Research Fund for Academic Team of Young Scholars at Wuhan University (Project No. Whu2016013).

reality. section V summarizes the paper and outlines some of the future work.

II. Multi-objective optimization mean variance model

A. Overview of the mean variance model

Over the years, scholars on the model have made the following improvements: In order to reduce the calculated amount of model parameters (covariance),Sharper gives a single factor model [10] for portfolio selection,Which use the single-factor income model to estimate the mean and covariance of risk assets, greatly reducing the number of parameter estimation and saving computing resources.The model is still a category of mean - variance analysis .In general, the excess return above the mean is actually what investors like, but it is treated as a risk in the mean variance model.A more precise risk characterization is the lower half variance, which is the expected value of the square of the negative deviation of the mean.Markowitz [11] discussed the mean one-half variance model. However, Konno and Yamazaki [12] depict the risks with the expected absolute deviation, and give a linear programming model of portfolio selection, which is often referred to as an absolute deviation model of mean value.

B. The basic theory of the mean variance model

In terms of model, this paper mainly refers to the mean variance model, combined with the actual problems of fund investment. The classical variance model is a single objective optimization problem that minimizes the risk in the case of fixed income or maximizes the return in the case of a fixed risk. Then the mean variance model of multi-objective optimization is established [13] [14]. Suppose there are N types of funds in the market and the data are processed to get the price data that measures the value of the fund:

$$F_i = (P_i^{t0}, P_i^{t1}, ..., P_i^{tm}), i = 1, 2, ..., N \tag{1}$$

where P_i^{tm} represents the price of the fund at time t^{m+1}, P_i^{t0} means that the price at the beginning of the fund is generally the earliest price in historical data as a reference price for the benefit. Therefore, the income data of fund F_i is expressed as:

$$R_i = (P_i^{t1} - P_i^{t0}, P_i^{t2} - P_i^{t1}, ..., P_i^{tm+1} - P_i^{tm}) i = 1, ..., N \tag{2}$$

The expected rate of return of fund F_i is:

$$\overline{R_i} = (\sum_{j=t_0}^{t_m} R_{i_j} / R_{i_{t_0}} / (t_m + 1) \tag{3}$$

The covariance of fund i and j is expressed as:

$$Cov(F_i, F_j) = E(R_i - \overline{R_i})(R_j - \overline{R_j}) \tag{4}$$

We can see that when i=j, the covariance is the variance of the fund F_i. Therefore, the mean variance model of fund portfolio can be fully expressed as:

$$max f_1(W) = W^T \overline{R} \tag{5}$$

$$min f_2(W) = W^T Cov(W) \tag{6}$$

$$s.t. \sum_{i=0}^{N} W_i = 1, i = 1, 2, ..., N \tag{7}$$

$$W_i \geqq 0 \tag{8}$$

among them, the vector W represents each fund investment weight vector, Cov represents the covariance matrix.

C. Mean variance model improvement

Moreover, the incomes of portfolio is also affected by transaction costs, in order to avoid high-frequency operation, under normal circumstances, a portfolio change need to deduct 1% fee. Some scholars have studied the problem of portfolio optimization considering transaction costs. Magill [3] studied the impact of transaction costs on capital market equilibrium. Norman [4] discusses the optimal consumer investment decision-making problem with transaction costs.Gennotte and Jung [15] have set up n kinds of venture capital asset market portfolio optimization problems.

In addition,Since the fund data information in the market can only reflect part of the real value and volatility information, the final Pareto solution set may be scattered or concentrated investment, which means that each fund will be a very small amount of investment or some small amount of funds take up a lot of investment.

The solution of these special cases does not meet the real investment needs. Generally, investors only invest a certain amount of funds. In other words, there are limits to the types of funds in the portfolio. In addition, there are upper and lower limits for each fund's share of investment.

At last, other investments need to consider the purchase of the same time, so the latest price data for investors to make investment decisions is very important, because the latest price data not only determines the amount of investment, but also the future price of the reference price calculation. So this paper applies the purchase price level $u_i, i = 1, 2, ..., N$which is used to measure the level of the current purchase price of the fund. The low value of the factor indicates that the lower the current fund price level, it may be an appropriate timing of the purchase. u can be measured by the rank of the current price in the historical price and obtained by the following equation:

$$u_i = \frac{p_i^{tm}}{(\sum F_i)/(t_{m+1})}, i = 1, 2, ..., N \tag{9}$$

In summary, the model is amended as follows:

$$max f_1(W) = W^T \overline{R}(1 - 1\%) \tag{10}$$

$$min f_2(W) = W^T Cov(W) \tag{11}$$

$$LL \leq W_i \leq UL \tag{12}$$

$$KL \leq \sum_{i=0}^{N} W_i \leq KU \tag{13}$$

Assuming a total investment of M yuan, N kinds of funds available for investment, The upper limit of each fund's investment quota is UL and the lower limit is LL. The maximum number of fund types in the portfolio is KU with a lower limit of KL. μ represents the purchase price level vector for N kinds of funds.

III. MOEA/D-CP

In this section, we make a discussion on the characteristic of weight vector generation approach in original MOEA/D when solving the multi-objective portfolio optimization problem which mentioned in section II. In order to solve the multi-objective portfolio optimization problem more effectively, we propose a new algorithm named MOEAD/CP with an approach of weight vector generation called circle partition(CP) approach. Then we present the detail of the new approach and the framework of the algorithm.

A. Original approach of generating weight vector

MOEA/D is a framework for solving multi-objective problem and it decomposes a multi-objective optimization problem into several subproblems by using a set of weight vectors [16]. A set of weight vector for *N* subproblems is denoted by $\lambda^1, ..., \lambda^N$. A subproblem which is produced by a weight vector λ^n corresponds to a pareto optimal solution. MOEA/D is a population-based framework, so it solves subproblems simultaneously. The aim of MOEA/D is to search a set of pareto optimal solutions which are evenly distributed on the pareto front. Because each subproblem and its corresponding Pareto optimal solution on the pareto front are determined by a precise weight vector, the approach of weight vector generation has a impact on the distribution of solutions.

It is well know that the approach of setting weight vector in original MOEA/D is determined by a parameter *H*, [17]. Each individual of a weight vector choose randomly from:

$$\{\frac{0}{H}, \frac{1}{H}, ..., \frac{H}{H}\} \tag{14}$$

For a multi-objective optimization problem with *m* single problems, relationship of the m individuals in a weight vector λ^n can be expressed as follows:

$$\sum_{i=1}^{m} \lambda_i^n = 1 \tag{15}$$

Thence, for a multi-objective optimization problem with m single problems, K weight vectors can be generated:

$$K = C_{H+m-1}^{m-1} \tag{16}$$

For the portfolio multi-objective optimization problem with 2 single problems mentioned above, H+1 different weight vectors can be generated. In order to generate a set of vectors which correspond to a set of pareto optimal solutions which are evenly distributed on the pareto front, we demand that H is large enough. However, it is hard to determine a suitable value for parameter *H*. In addition, the process of generating weight vector in the approach is stochastic, the performance of the algorithm can not be guaranteed when algorithm solves the problem with uneven pareto front.

B. Weight vector generation based on CP approach

To ensure that the solutions are distributed evenly on pareto front, we propose a new approach of generating weight vectors called circle partition(CP) approach. The CP can be regarded as a process that a number of weight vector parting a circle sequentially whose center is the reference point z^*, each weight vector corresponds to a sector with fixed angle. In this case, each weight vector passes through the same point z^* and shoots at different pareto optimal solution. The specific process can be described as follows. At the beginning of the process, $\frac{1}{2m}$ of the circle whose center is reference point is evenly divided into N-1 by N vector from reference point to pareto front, where m is the number of single objective problem. The angle between λ^n and λ^{n+1} is θ and θ is obtained by the equation:

$$\theta = (\frac{180}{mN})^o \tag{17}$$

where N is the number of vectors and the spread of all weight vectors is shown in the following picture:

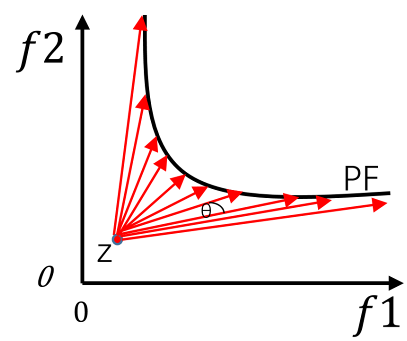

Fig. 1. Distribution of all weight vectors

It is obvious that the weight vector will evenly spread through the pareto front as long as the number of weight vector is large enough. It is promising for MOEA/D-CP to get a set of solutions more effectively than original MOEA/D. The detailed comparison on the portfolio MOP mentioned before will presented in next section.

C. Framework of MOEA/D-CP

MOEA/D is a framework with the method of decomposition. In this section we append CP approach to MOEA/D. The algorithm can be described as follows.

where $F^n(x)$ is the subproblem function defined by the weight vector λ^n, FV^n is the value of $F^n(x^n)$.

As we can see, the ***CP*** approach is executed before algorithm's iteration, so the extra cost of algorithm produced by the approach is very small comparing with the all algorithm.

Algorithm 1 Procedure of MOEA/D-CP

Initialize:
for each $n \in [1, N]$ **do**
  for each $m \in [1, M]$ **do**
    $\frac{\lambda_m^n}{\lambda_{m-1}^n} = \tan(n \cdot \theta)$;
  end for
end for
Randomly generate a population $x^1, ...x^N$ in the decision space;
Calculate the value FV^n of subproblem corresponding to x^n;
Update:
while Stop condition is not satisfied **do**
  for each $n \in [1, N]$ **do**
    Randomly select two solution to generate the x^n's offspring using DE operators y^n;
    Repair the solution y^n using a problem-specific method;
    If the subproblems value of $F_n(y^n) < F_n(x^n)$, replace x^n with y^n ;
  end for
end while

Moreover, the algorithm's complexity is determined by the number of population N,and it's algorithm complexity is $O(N)$.

IV. Experiment studies

In this section, we test MOEA/D-CP and original MOEA/D algorithm on the portfolio optimization described in section II. We compare the experimental results obtained by two algorithms and then make a conclusion on the features of MOEA/D-CP.

A. Data sets

To verify the precise performance of algorithms when solving the portfolio optimization problem,we choose four problems from the publicly available OR-Library (http://people.brunel.ac.uk/ mastjjb/jeb/orlib/portinfo.html). These data sets includes data from four major stock market around the world from the March,1992 to September,1997.They are Hang Seng (Hong Kong), DAX 100 (Germany), FTSE 100 (UK) and S&P 100 (USA). We had 291 values for each stock from which to calculate (weekly) returns and covariances and the size of our four test problems ranged from N=31 (Hang Seng) to N=98 (S&P 100). These four stock markets' name and number of stock is respectively:

- Hang Seng, 31
- DAX 100, 85
- FTSE 100, 89.
- S&P 100, 89

the original date in these data sets is comprised with the mean and standard deviation of the stock and the covariance between two stocks.

B. Parameter setting

In terms of the parameter of MOEA/D and MOEA/D-CP, the population size is set 100, the number of neighbor of each solution is 5, the stopping criterion is 1000 iterations. The Tchebycheff approach is used to decompose the MOP into a number of subproblems. The differential evolution algorithm (DE) [18] is applied as a method of crossover and mutation, the crossover rate is set to 0.5, the step size is set as 0.5, the mutation rate is $\frac{1}{N}$ where N is the number of population. To make it fair for two algorithms, all results are obtained after 20 independent runs.

C. Results Comparison

In this section, the results of four experiments are shown in Figure 2,3,4,5.

The black line in picture stands for the pareto front of the problem. Need to emphasize that pareto front in each of the four portfolio optimization problems is not evenly, the risk will increase slowly with the increase of return at the beginning and then increase quickly. Therefore, these four MOPs may be difficult for original MOEA/D to solve.

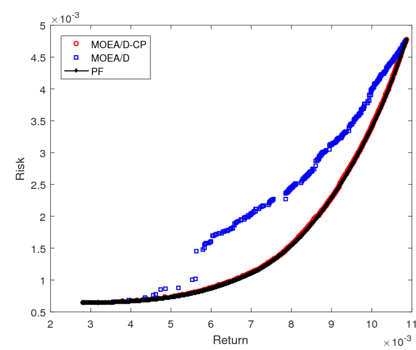

Fig. 2. Hang Seng

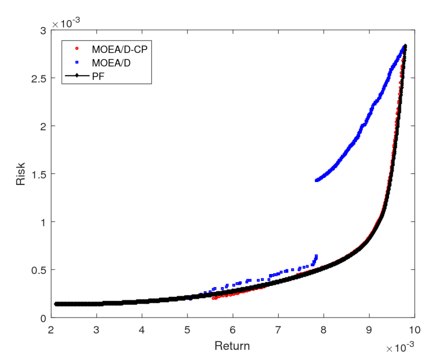

Fig. 3. DAX 100

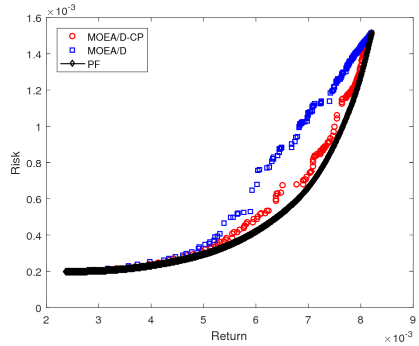

Fig. 4. FTSE 100

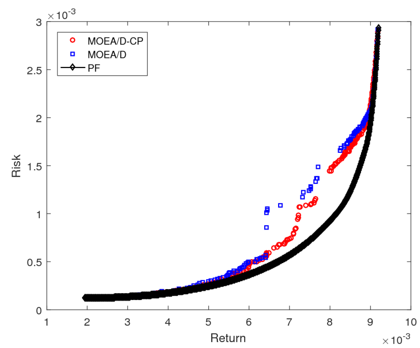

Fig. 5. S&P 100

In order to compare the convergence and diversity of these algorithms, we use the hypervolume [19] as a performance metric to evaluate solutions got by these algorithms. The hypervolume values are the volume of shape which are formed by solutions and reference point in the objective space, a larger hypervolume can reflect that solutions are more approximate to the true pareto front with a ideal diversity. The hypervolume values got by these algorithms on four portfolio problems are shown in below Table I.

TABLE I. Hypervolume Values

	MOEA/D	MOEA/D-CP
HangSeng	3.13E-04	5.76E-04
DAX 100	4.47E-04	6.78E-04
FTSE 100	3.36E-04	5.96E-04
S&P 100	2.57E-05	8.67E-05

It is obvious that the hypervolume values of solutions got by MOEA/D-CP are more larger on four portfolio problems which suggests that MOEA/D-CP can get a set of solutions with better diversity than original MOEA/D. In other words, the solutions generated by MOEA/D-CP are more evenly distributed than the solutions got by original MOEA/D. Moreover, in the first and second experiments, the MOEA/D-CP gets a better result whose solutions is more approximate to pareto front. Benefitting from the CP approach, the multi-objective optimization problem is divided into several subproblems whose corresponding pareto optimal solutions are evenly distributed on pareto front. So all subproblems' solutions in each iteration can search the solution space more uniformly, in which case the algorithm can get more information about solution space and accelerate the efficiency of solving subproblem. In the third and fourth experiments, the MOEA/D-CP performs better, the distribution of solutions in original MOEA/D are sparse around the knee point, in contrast the solutions generated by MOEA/D-CP are uniformly distributed especially around the knee point of pareto front as a comparison.

As a results, we can say that the MOEA/D-CP performs much better than original MOEA/D when solving these portfolio multi-objective optimization problems.

V. Conclusion and Future work

In this paper, we summarized the importance of weight vector in MOEA/D to algorithm's performance. The pareto front of MOP in reality is complexity and uneven, so we propose CP approach to generate a set of weight vectors with the purpose of enhancing the performance of MOEA/D when dealing with MOP with uneven pareto front. It has been proved that MOEA/D-CP can get more excellent solutions on four portfolio optimization problems. This work also shows the impact of weight vector on algorithm with method of decomposition.

In the future, we will unceasingly modify the approach to improve algorithm's capability of solving more complexity MOP.

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