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Highlights

- A regret-based robustness measure, that can be applied to all Pareto solutions.
- Extension of the minimax regret criterion, in multiobjective programming.
- Managerial usability for investment practitioners.
- Out-of-sample empirical testing with historical market data.

Robust multiobjective portfolio optimization: A minimax regret approach

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Abstract

An efficient frontier in the typical portfolio selection problem provides an illustrative way to express the tradeoffs between return and risk. Following the basic ideas of modern portfolio theory as introduced by Markowitz, security returns are usually extracted from past data. Our purpose in this paper is to incorporate future returns scenarios in the investment decision process. For representative points on the efficient frontier, the minimax regret portfolio is calculated, on the basis of the aforementioned scenarios. These points correspond to specific weight combinations. In this way, the areas of the efficient frontier that are more robust than others are identified. The underlying key-contribution is related to the extension of the conventional minimax regret criterion formulation, in multiobjective programming problems. The validity of the approach is verified through an illustrative empirical testing application on the Eurostoxx 50.

Keywords: Multiple objective programming; Portfolio optimization; Minimax regret; Robustness.

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1. Introduction & problem setting

The classic methodological framework proposed by Markowitz (1952, 1959) has been the primary influence for the majority of financial models designed to provide a solution to the portfolio selection problem. Exclusively based on the criteria of return and risk, investors seek to minimize the expected variance of their portfolio for certain levels of expected return. The crucial assumption for this classic bi-objective approach to work is the accuracy of the estimates of return and covariance matrices. Since the classic model is quite sensitive to its input parameters (Hodges, 1976), the existing noise in the available estimates of risk and return will result in erroneous portfolio selection output. In fact, little trust regarding accuracy may be given to the point estimates extracted by averaging past data and there is little guarantee that those estimates will match future values.

The need for more sophisticated financial tools has created space for exploring robust mathematical tools in order to protect a portfolio against input uncertainty. Robustness is a concept of crucial importance in financial decision making, especially when precarious instances are present. Thus, modeling processes for treating uncertainty are always necessary, when dealing with portfolio optimization problems. This feature typically leads to burdensome problem formulations, as robustness normally increases the number of constraints. It is, therefore, desired to maintain an acceptable balance between the robust modeling complexity and the overall efficiency of the results.

The conventional mean-variance formulation is a quadratic programming model and its solutions provides the efficient frontier or Pareto optimal set of portfolios. On this basis, we seek in this paper to build robust efficient frontiers, namely efficient portfolio sets that are close to optimal, under different scenarios. The field of multiobjective portfolio optimization offers very attractive linear programming modeling formulations that can be smoothly robustified (Mansini et al. 2014), while there is also the potential to include additional objectives and weight them accordingly.

More specifically, the main goal of this article is to develop a robust selection program that expands the concept of robust optimization, as it was proposed by Kouvelis and Yu (1997), to the multiobjective case. Kouvelis and Yu used the concept of “regret” to identify robust solutions in optimization problems. Regret is actually the deviation of an obtained solution from the optimum solution according to a specific scenario of parameters. In other words it can be defined as the difference between the obtained gain and the gain that we could get if we knew in advance which scenario will surely occur. Following Kouvelis and Yu ideas, we use the minimax regret criterion in order to identify robust areas in the Pareto front of multiobjective problems. We deal with input parameter uncertainty by considering time-varying alternatives for expressing a variety of market analysis horizons.

In this way we are in position to identify areas of the Pareto front that are more robust than other. The specific areas of the Pareto front are characterized by the weight combination used in the objective functions. Applying our model to data from the Eurostoxx 50, we gain evidence that certain objective areas (e.g. risk) display greater robustness than others (when robustness is measured in terms of the proposed version of regret). Moreover, the calculations of the minimax regret value inform us about the amount of benefit we trade for robustness, at each choice of weights. For explanatory purposes, informative graphs and tables throughout the paper summarize all of our empirical testing results.

The remainder of the paper is organized as follows: In Section 2, we review the history and applications of robust optimization models in finance. In particular, we classify and present key-research articles and theoretical notes on robust optimization, with focus on robust portfolio optimization. In Section 3, we present the underlying robust modeling framework and we extent the robust formulation to the multi-objective context. In Section 4, we test the proposed model with an illustrative application on the securities of the Eurostoxx 50. Finally, in Section 5, key findings and conclusions are given in an intuitive manner.

2. Literature review

Recent developments in the field of portfolio theory imply that the knowledge of future returns and variances, delivered by classic point-estimation techniques, cannot be thoroughly trusted. Since risk and return are characterized by randomness, one should keep in mind that problem data could be described by a set of scenarios. Mulvey et al. (1995) were the first to work on models of mathematical optimization where data values come in sets of scenarios, while explaining the concept of robust solutions and introducing the robust model formulation. In a more financially specialized setting, Vassiliadou-Zeniou and Zenios (1996) developed robust optimization tools for managing callable bond portfolios. Kouvelis and Yu (1997) published a book on robust discrete optimization. Their book addresses multiple aspects in the robust problem formulation process, such as uncertainty handling, computational complexity results and algorithmic developments. The scope of the book is much wider than financial or economic problems and extends to various robust discrete optimization cases, such as multi-period, multi-item and machine scheduling problems.

With regard to robust portfolio optimization, Tütüncü and Koenig (2004) described asset's risk and return using continuous uncertainty sets and developed a robust asset allocation program solved by a saddle-point algorithm. Also, Pinar and Tütüncü (2005) introduced the concept of robust profit opportunity in single-period and multi-period formulations. Likewise, multi-period portfolio optimization formulations with additional transactional constraints are found in Bertsimas and Pachamanova (2008).

While robust optimization is intended to protect the portfolio against uncertainty, Gregory et al. (2011) calculate that it comes with costs in terms of return. In terms of risk, Huo et al. (2012) propose robust covariance measures to be included in the portfolio optimization process, so as to generate covariance estimates stable and insensitive to outliers. In order to deal with output fluctuations and stress testing with respect to uncertainty in input data, a study of robustness of optimal portfolios under stochastic dominance constraints was conducted by Dupacova and Kopa (2014). Moreover, Maillet et al. (2015) perform a worst-case minimum variance optimization with respect to alternative covariance matrix estimators.

Kim et al. (2013a) explain what fundamental factors are in order to determine whether robust equity portfolios are more or less sensitive to factors than to individual assets' movements. Moving a step forward, Kim et al. (2014b) propose robust modeling that allows the control of the level of exposure portfolios have in a factor. Moreover, in a study of composition of robust equity portfolio Kim et al. (2013b) inspect the properties of the selected assets. Kim et al. (2014a) also survey developments of robust worst-case optimization, including robust counterparts for value-at-risk and conditional value-at-risk problems. Kim et al. (2015) discuss robust optimization performance with focus on worst market state returns. Another robust worst-case approach within the best value-at-risk Sharpe ratio context is found in Deng et al. (2013).

A holistic approach of the 60-year old history of the modern portfolio optimization is attempted in Kolm et al. (2014). The 20-year old history of robust portfolio optimization is included as well as new directions are discussed. Other research articles that summarize recent history and future trends of robust portfolio optimization are those of Fabozzi et al. (2007), Fabozzi et al. (2010), Mansini et al. (2014) and Scutella and Recchia (2013), where the relation between robustness and convex risk measures is also studied. A thorough inspection of both theoretical and practical research in robust optimization was made by Ben-Tal et al. (2009).

Cornuejols and Tütüncü (2006) wrote a book dedicated to optimization in finance. Within the book they go through topics of robust optimization in finance, analyzing the theory of robustness and taking a look at various types of uncertainty sets, different types of robustness (e.g. objective robustness, constraint robustness and relative robustness) and techniques such as sampling, conic optimization and saddle-point characterization. They formulate robust portfolio optimization problems in single-period, multi-period and relative portfolio selection contexts. The chapters are wrapped up with challenging exercises and extensions for the reader to solve. Besides, historical and theoretical reviews, useful guides for practitioners can be also found, such as Gorissen et al. (2015). In the robust multiobjective field, an effort to characterize the location of the robust Pareto frontier with respect to the corresponding original Pareto frontier using standard multiobjective optimization techniques was made by Fliege and Werner (2014).

As mentioned, the methodological contribution of the present work is that it expands the concept of the robust solution to the multiobjective case. We incorporate future scenarios for the return and risk, which are mainly based on the perspectives of the decision maker. It is an attempt to show how this information may be exploited in order to produce robust portfolios against a variety of future scenarios. The handling of future returns scenarios is made by using the concept of the minimax regret criterion.

3. Proposed model

It is well known that the minimax regret criterion is among the most popular criteria in decision sciences Savage (1954), along with the maximax, maximin, Hurwitz criterion etc., where different scenarios are present. It actually aims at selecting the solution or alternative which is under the worst case closer to its scenario optimum. The minimax regret criterion provides less conservative solutions than the "pessimistic" approach of the maximin criterion (also used to express "robustness"). The reason is that it takes into account the regret, i.e. the deviation of each solution from the best possible solution at each scenario. The regret is not an absolute measure of performance of the solutions -as it is the case in the maximin criterion- but it is relative to the best available performance for the specific scenario. That's why it is considered to provide less conservative solutions in the sense that they have not to be "safe" according to the worst realization of parameters but according to the relevant optimum of each scenario. We can find the maximum regret for each solution across the scenarios and then comparing these regrets we can find the solution with the minimum of these maximum regrets. This minimax regret solution is considered as the robust solution.

In order to explain the difference between maximin and minimax regret criterion consider the following example: Assume that we have 5 options that are evaluated in 3 scenarios: a pessimistic scenario, a most likely and an optimistic scenario. Imagine for example that we have 5 portfolios and the performances are the returns of each portfolio as shown in **Table 1**.

Table 1 Example of the maximin criterion with 5 options and 3 scenarios

	Pessimistic	Most likely	Optimistic	Min
portf1	3	10	17	3
portf2	5	9	15	5
portf3	5	6	12	5
portf4	**6**	**8**	**11**	**6**
portf5	4	7	16	4

According to the maximin criterion the selected portfolio should be portfolio 4 which has the best performance in the pessimistic scenario leaving the information from the other two scenarios actually unexploited. In the minimax regret approach, we first create the regret matrix as shown below by calculating the distance from the optimum for each one of the three scenarios (**Table 2**).

Table 2 The regret matrix and the minimax regret criterion

	Pessimistic	Most likely	Optimistic	Max
portf1	3	0	0	3
portf2	**1**	**1**	**2**	**2**
portf3	1	4	5	5
portf4	0	2	6	6
portf5	2	3	1	3

In this case the selected approach is the one with the minimum among the maximum regrets which is portfolio 2. With the minimax regret approach we exploit the information from all 3 scenarios and we obtain solutions that are more balanced. Compare for example portfolios 2 and 4: The only advantage of portfolio 4 is that it outperforms in the pessimistic scenario expressing a more conservative view.

The minimax regret criterion has been also introduced in mathematical programming formulations. Specific formulations have been developed in order to express this concept in problems where there are multiple scenarios for the model's parameters. In Hauser et al. (2013) a regret function is considered as the function that measures the difference between the performance of the solution with and without the benefit of hindsight. If we choose x as decision vector when s is the vector of realized parameter values (scenario), then the regret associated with having chosen x rather than the optimal solution associated with scenario s (i.e. $x^*(s)$) is defined as follows (assume maximization):

$$r(x, s) = f(x^*(s), s) - f(x, s)$$

The hindsight is considered as the prior knowledge of the parameter scenario that will occur. Therefore the optimal value with these parameters is the best outcome. Without this prior knowledge we can compute the minimax regret solution which is the one that has the minimum deviation from the optimal value under the worst case. Kouvelis and Yu (1997) accomplish this task for an infinite number of solutions according to the feasible region of the problem. Other attempts include the works of Loulou and Kanudia (1999), Mausser and Laguna (1999), Lobo (2000), Khodadadi et al. (2006) and Ehrgott et al. (2014).

Assume the following mathematical programming problem:

$$\begin{gathered} \max z = f(x) \\ \text{s.t.} \\ x \in F \end{gathered} \tag{1}$$

According to the above formulation, the objective function to be maximized is a combination of the decision variable *x*, where *x* belongs in set *F*. Assume now that we have a set *S* of scenarios for the objective function parameters (*S* contains a finite number of |*S*| scenarios), which means that the corresponding objective functions are denoted as $f^{s}(x)$. The minimax regret solution within the relative regret case is calculated from the following problem (see Kouvelis and Yu (1997), p. 29):

$$\begin{gathered} z_{\text{MMR}} = \min y \\ \text{s.t.} \\ f^{s}(x) \geq (1-y) z^{s} \quad s \in S \\ x \in F \end{gathered} \tag{2}$$

where z^s is the positive optimal value for the *s*-scenario and *y* is the variable that expresses the relative minimax regret.

In the present work we extend the conventional formulation to the multiobjective case. Specifically, we use the weighting method in order to calculate the Pareto optimal solutions of the Pareto front. Assume that we have a problem with *P* objective functions:

$$\max(f_1(x), f_2(x), \ldots, f_P(x)) \ \text{ s.t. } \ x \in F \tag{3}$$

Using the weighting method (see e.g. Steuer, 1989) we can calculate efficient (non-dominated) points solving the following single objective problem that has as objective function the weighting sum of the objective functions at hand (assume all objectives are for maximization):

$$\max z = \sum_{p=1}^{P} w_p \times f_p(x) \ \text{ s.t. } \ x \in F \tag{4}$$

In order to be meaningful the weights and independent of the scale of the objective functions, it is better to use the normalization formulas for the objective functions as follows:

$$\max z = \sum_{p=1}^{P} w_p \times \frac{f_p(x) - f_{p,\min}}{f_{p,\max} - f_{p,\min}} \ \text{ s.t. } \ x \in F \tag{5}$$

where $f_{p,min}$ and $f_{p,max}$ are the minimum and the maximum values of the objective functions as obtained from the payoff table (the payoff table is a *p* × *p* table that includes the individual optimization values of the objective functions). The solution of this problem corresponds to a Pareto optimal solution of the multi-objective problem. Varying the weights we obtain a representative set of the Pareto optimal solutions of the multi-objective problem. It must be noted that with the weighting method the Pareto set is approximated (not all the Pareto optimal solutions are found, which is the “exact solution”). Usually, the more the weight combinations that are used, the better is the approximation.

The concept of our method is to apply the Kouvelis and Yu (1997) formulation to each combination of the weights. In this way, we obtain the minimax regret solutions that correspond to different areas of the Pareto front. Assuming that we have $|S|$ scenarios for the objective function coefficients, we discretize the weight space to g weight combinations (vectors) and we solve the following problem:

$$\begin{gathered} \mathrm{MMR}(g) = \min y_g \\ \text{s.t.} \\ \sum_{p=1}^{P} w_p^g \times \frac{f_p^s(x) - f_{p,\min}^s}{f_{p,\max}^s - f_{p,\min}^s} \geq (1 - y_g) z_g^s \\ s \in S \\ x \in F \end{gathered} \tag{6}$$

and we solve the above problem for every g obtaining the minimax regret solution at representative points of the Pareto front. According to the value of the minimax regret solution (y_g) we can draw conclusions about the areas of higher or lower robustness of the Pareto front.

Applying the above method to the portfolio optimization problem we use two objectives: the Mean Absolute Deviation (MAD), as a risk measure to be minimized and the expected portfolio return, as an objective to be maximized. For a universe of N assets and T historical periods (past horizon time- length), the objective functions are given in the formulas below:

$$\min \mathrm{z}_1 = \frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{N} x_i \left(R_{it} - \bar{R}_i \right) \right| \tag{7a}$$

$$\max \mathrm{z}_2 = \sum_{i=1}^{N} x_i \bar{R}_i \tag{7b}$$

where $\bar{R}_i = \frac{1}{T} \sum_{t=1}^{T} R_{it}$ and R_{it} is the return of the i-th asset during the t-th historical period.

The linearity of the model regarding the first objective function is persevered through the Konno and Yamazaki (1991) transformation. On this basis, T additional positive continuous variables y_t are used for the representation of each period's absolute deviation from the mean, resulting in $2 \times T$ constraints:

$$\begin{aligned} &\sum_{i=1}^{N} x_i \times \left(R_{it} - \bar{R}_i\right) + y_t \geq 0, \quad t = 1, 2, \ldots, T \\ &\sum_{i=1}^{N} x_i \times \left(R_{it} - \bar{R}_i\right) - y_t \leq 0, \quad t = 1, 2, \ldots, T \end{aligned} \tag{8}$$

Then, the objective function is transformed to:

$$\min \mathrm{z}_1 = \frac{1}{T} \sum_{t=1}^{T} y_t \tag{9}$$

Therefore, for each weight combination g, we solve the $|S|$ problems declared in equation (10) to identify the optimum value of the weighted sum for every scenario s.

(Model 1)

$$\begin{aligned} &\forall s \in S: \\ &z_g^s = \max\left(w_1^g \frac{f_{1,\max}^s - f_1^s(x)}{f_{1,\max}^s - f_{1,\min}^s} + w_2^g \frac{f_2^s(x) - f_{2,\min}^s}{f_{2,\max}^s - f_{2,\min}^s}\right) \\ &st \quad x \in F \end{aligned} \tag{10}$$

Observe in equation (10) that the first term corresponds to the normalization of an objection function to be minimized (MAD in our case). After the calculation of the optimal values z_g^s for the weight combination g, we put them as parameters in the model of equation (6) in order to solve the problem that calculates the minimax regret for the specific weight combination using equation (11).

(Model 2)

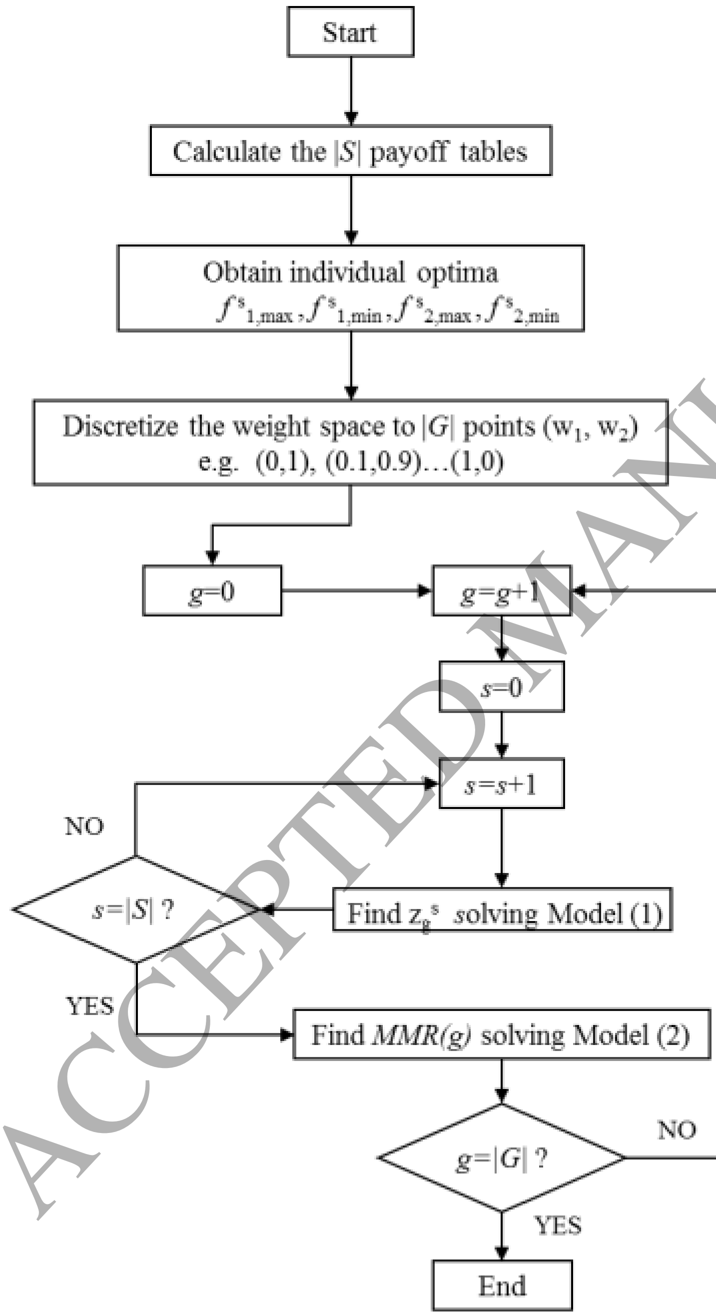
$$\begin{aligned} &MMR(g) = \min y_g \\ &st \\ &w_1^g \frac{f_{1,\max}^s - f_1^s(x)}{f_{1,\max}^s - f_{1,\min}^s} + w_2^g \frac{f_2^s(x) - f_{2,\min}^s}{f_{2,\max}^s - f_{2,\min}^s} \geq (1 - y_g) z_g^s \qquad s \in S \\ &x \in F \end{aligned} \tag{11}$$

Subsequently, we move forward to the next weight combination and we repeat the process described with Model 1 and Model 2. In this way we scan all the weight combinations calculating the minimax regret solution for each one of them. In total

(|S|+1)×|G| problems are solved. The smaller the minimax regret, the more robust is the corresponding efficient solution.

The flowchart of the proposed algorithm for the specific case study of portfolio optimization using risk and return as objective functions is depicted in **Figure 1**.

Figure 1 The flowchart of the method for the minimax regret criterion



The method is not restricted to two objective problems. However, when more than two objective functions are considered the computation burden will severely increase. This has to do mainly with the increased number of weight combinations needed to adequately represent the multi-dimension Pareto front. However the number of weight combinations per objective function (referred also as *grid points*) can be adjusted in accordance with the number of objective function in order to keep the computational time in tractable limits. The discretization of the weight space for more than two objective functions is straightforward and it can be efficiently performed with a loop process across the criteria keeping the rule that the sum of the weights should be always equal to 1. For example, assume a three objective problem with 11 grid points per objective function. In this case we need 66 grid points in total and this is multiplied by (|S|+1) runs of an LP or MILP model.

4. Empirical testing

We use the 50 stocks of the Eurostoxx 50 the Europe's leading blue-chip index for the Eurozone, provides a high capitalization representation of supersector leaders in the Eurozone. The index covers 50 stocks from 12 Eurozone countries. The model is presented in Xidonas and Mavrotas (2014). We use five scenarios of return and risk evolution, all of which conceived in close cooperation with a team of portfolio managers. In the absence of actual decision makers we create 5 scenarios for the return and the risk as follows: We used historical data of 80, 60, 40, 20, and 10 weeks, extracting the average return and MAD from the corresponding data. Therefore, Scenario 1 that corresponds to 80 weeks past horizon denotes a more long-term point of view than Scenarios 2, 3, 4 and 5 that denote a short-term behavior. The five efficient frontiers are depicted in **Figure 2** and the five payoff tables are shown in **Table 3**.

Table 3 The payoff table in the 5 scenarios

	scen #1		scen#2		scen#3		scen#4		scen#5	
	MAD	Return	MAD	Return	MAD	Return	MAD	Return	MAD	Return
min MAD	1.45	12.40	1.38	9.96	1.39	8.36	0.83	20.11	0.62	10.86
max Return	2.49	58.73	2.26	44.63	2.27	47.82	1.98	90.46	2.25	94.11

The obtained results using the minimax regret model are shown in **Table 4**. We used 11 weight combinations, namely (0, 1), (0.1, 0.9), (0.2, 0.8),…, (0.9, 0.1), (1, 0). The optimum of each scenario for the weight combinations (0, 1), (0.1, 0.9) and (1, 0), along with the minimax regret solution are shown in **Table 4**, where the first objective function is the minimization of risk and the second one is the maximization of return. Moreover, in **Table 4**, four out of the eleven weight combinations are presented. Namely, the combinations (0, 1), (0.1, 0.9), (0.9, 0.1) and (1, 0). For each one of them we see the outputs of the 5 scenarios in terms of Wsum (= the weighted sum of objective functions according to equation 5), Return, MAD, the number of stocks in the portfolio and the weights of each one of the 50 securities. The minimax regret solution is presented in the 7-th line of each scenario (with bold fonts). It has to be mentioned that the minimax regret figure expresses how far we are from the individual optima of each scenario in the worst case and it is expressed as fraction from 0 to 1. The smaller the minimax regret, the more robust is the solution. Robust solutions are attractive because no matter which scenario will eventually occur, we will be close to the optimum of the occurred scenario.

Regarding **Figure 2**, the 5 Pareto fronts correspond to each one of the considered scenarios. They are dissimilar because they correspond to different scenarios for the returns. For example scenarios 4 and 5 correspond to higher returns than scenarios 1, 2 and 3 as it can be seen for the maximum return regions (upper right of chart). The reviewer's insight can be interpreted as follows: In the upper right part of the chart where the driver is the maximization of return the dispersion is greater than the lower left part where the minimization of risk is the driver. Maybe this is an indication why the more robust areas are in the latter region (lower left).

In **Table 4** we can see that modifying the weights is crucial to the composition of the portfolios. As we increase the weight of "risk" we see that more securities enter the portfolios. There are 19 stocks that are never present in any portfolio at any case (either scenarios or weight combination). If we quantify the steadiness or robustness of the portfolios by the magnitude of the minimax regret figure we can identify regions of the Pareto set that are more robust (lower minimax regret values). It is noteworthy that in most cases the minimax regret portfolio includes more stocks than the optimal portfolios of the individual scenarios.

When the weights of the objective functions are moving from max return to min risk (i.e. from $w_1 = 0$ to $w_1 = 1$), the stocks that have the highest return are losing weight in the optimal portfolios and they are mostly replaced by stocks that are less profitable but they are also less correlated with each other contributing to lowering the risk (expressed by MAD). That's why in **Table 4** we can see stocks that are in their upper bound (10% of the portfolio) to gradually reduce their share and other stocks to gradually increase their weight.

Moreover, we are in position to observe that the minimax regret solution in all the weight combinations contains more stocks in the final portfolio, than the individual scenarios optima. This means that we have a more dispersed allocation of the total investment universe. The minimax regret solution across the Pareto front is obtained from the minimax regret values for the specific weight combinations. Hence, we are able to detect areas of the Pareto front that present relatively increased robustness in relation to other areas. Finally, we calculate the minimax regret solutions for the 11 weight coefficients of the relative minimax regret criterion. The results are shown in **Figure 3**.

Figure 2 Representation of the 5 efficient frontiers

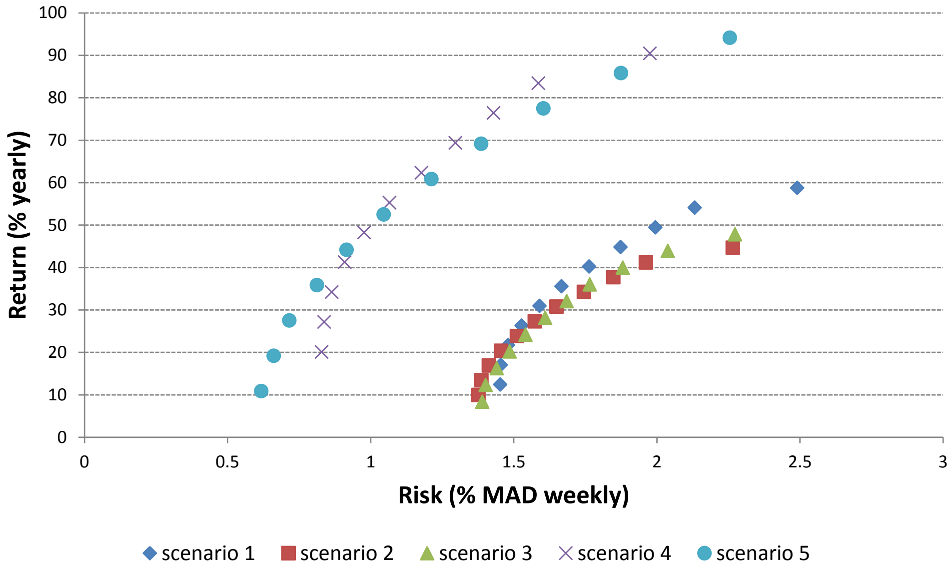
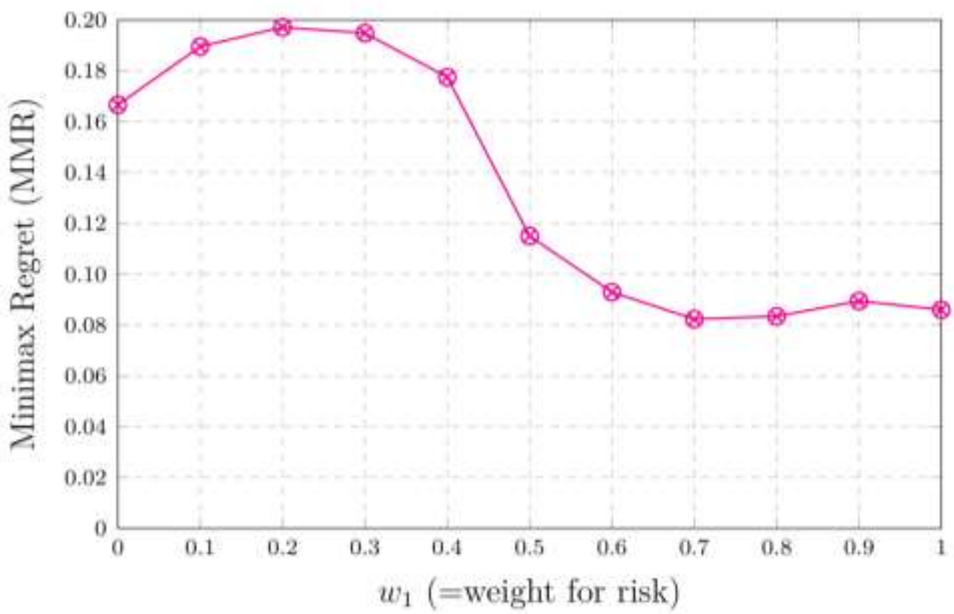


Table 4 Details of the obtained solutions for 4 weight coefficients

$w_1 = 0$	Scen#	Wsum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	50
	1	0,999	2.4902	58.731	10	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	2	0,999	2.265	44.632	10	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	3	0,999	2.2734	47.821	10	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	4	0,999	1.9764	90.459	10	0	0.1	0	0	0	0	0	0	0	0	0.1	…	0
	5	0,999	2.2553	94.112	10	0	0.1	0	0	0	0.1	0	0	0	0	0.1	…	0
	MMR =	**0.1666**			**11**	**0**	**0.1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.1**	**…**	**0**
$w_1 = 0.1$	Scen#	Wsum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	50
	1	0,9061	2.2974	58.089	10	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	2	0,9073	2.0772	44.1	10	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	3	0,9	2.2734	47.821	10	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	4	0,9097	1.8486	90.348	10	0	0.1	0	0	0	0	0	0	0	0	0.1	…	0
	5	0,9	2.2553	94.112	10	0	0.1	0	0	0	0.1	0	0	0	0	0.1	…	0
	MMR =	**0.1895**			**12**	**0**	**0.1**	**0**	**0**	**0**	**0.06**	**0**	**0.088**	**0**	**0**	**0.1**	**…**	**0**
…	…		…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
$w_1 = 0.9$	Scen#	Wsum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	50
	1	0,9078	1,4552	16,9617	14	0	0	0,052	0	0,015	0,1	0	0,082	0	0	0	…	0
	2	0,9014	1,381	11,9187	12	0	0	0	0	0	0,072	0	0,1	0	0	0	…	0
	3	0,903	1,392	10,1132	13	0	0	0,089	0	0	0,057	0	0,1	0	0	0	…	0.018
	4	0,9033	0,8331	24,6355	14	0	0,064	0	0	0,01	0	0	0	0	0	0	…	0
	5	0,9003	0,6187	11,3294	11	0	0,1	0	0	0	0	0	0,096	0	0	0	…	0
	MMR =	**0.089**			**13**	**0**	**0,037**	**0**	**0**	**0**	**0,07**	**0**	**0,1**	**0**	**0**	**0**	**…**	**0**
$w_1 = 1$	Scen#	Wsum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	50
	1	0,999	1.4528	12.402	12	0	0	0.1	0	0	0.1	0	0.042	0	0	0	…	0
	2	0,999	1.3768	9.962	12	0	0	0.047	0	0	0.099	0	0.1	0	0	0	…	0
	3	0,999	1.3905	8.3581	13	0	0	0.06	0	0	0.07	0	0.1	0	0	0	…	0.067
	4	0,999	0.8291	20.106	13	0	0	0	0	0.01	0.057	0	0	0	0	0	…	0
	5	0,999	0.6182	10.865	11	0	0.1	0	0	0	0	0	0.1	0	0	0	…	0
	MMR =	**0.086**			**13**	**0**	**0.02**	**0**	**0**	**0**	**0.068**	**0**	**0.1**	**0**	**0**	**0**	**…**	**0**

Figure 3 The MMR values across the Pareto front (relative MMR)



In **Figure 3**, the lower the relative minimax regret the more robust is the specific area of the Pareto front. Thus, it is obvious that there are areas in the Pareto front with higher robustness based on the 5 scenarios. For example the Pareto optimal solutions that correspond to weights varying from 0.1 to 0.4 are less robust than the Pareto optimal solutions that correspond to weights varying from 0.6 to 1 (robust area of the Pareto front). It is clear that when minimizing risk is weighted more (i.e. $w_1 \geq 0.5$), the minimax regret value drops from a level of 18% to a level of 9%. Hence, the area of the Pareto front that corresponds to minimizing risk against maximizing return, provide more robust solutions in terms of the minimax regret criterion.

5. Concluding remarks and discussion

Advances in portfolio management research highlight the growing momentum of robust portfolio optimization. In order to explore the portfolio stability and protect it against input uncertainty, new and effective robust tools need to be deployed and tested. Robust tools may not only be useful in theoretical research, but they also should come in hand for practical investors, as they will allow them to define uncertainty in input portfolio parameters, as they perceive it. In this work we equip the multiobjective portfolio analysis tool with robust techniques. In particular, we extend the conventional formulation for the minimax regret criterion in multiobjective programming problems.

More specifically, we apply the proposed model in real-world data from Eurostoxx 50 with 5 scenarios for the corresponding returns. The efficient frontier is approximated with 11 points each one of them corresponding to a specific weight coefficient for return and risk, our two objective functions (risk is quantified with the MAD). The obtained results are meaningful since they suggest the areas of the Pareto front that are more robust. The smaller the minimax regret for each weight combination the more robust is the specific Pareto optimal solution. In the specific empirical testing case that examines, it was found that the robust areas of the Pareto front are those where the weight of risk minimization is increased.

Therefore, by using the weighting method for generating the Pareto optimal solutions, we can detect the robust asset selection results and the robust areas of the efficient frontier, which is valuable information for the decision maker in the presence of uncertainty in terms of scenarios. The idea of detecting the robust Pareto optimal solutions in the presence of multiple objective functions and multiple scenarios is a new approach.

Future research might be driven towards examining the effectiveness of the method in portfolio optimization for more objective functions and also in other multiobjective problems. In addition, other robustness models in the same context of the minimax regret criterion may be developed in combination with other multiobjective techniques appropriate for generating representations of the Pareto front (e.g. the constraint method).

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